Psychol. Statistics using R

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# Hypothesis Testing

# Comparing Two Means


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# Statistics

1. **Concepts of Hypothesis Testing**
   - Hypothesis/effect size/decision
   - Five steps of testing
2. **Comparing two samples**
   - Pooled sample variance (equal / unequal)
3. **[R] Independent & paired t-test**
   - Examples for practices

Theories

Practice

Assignment

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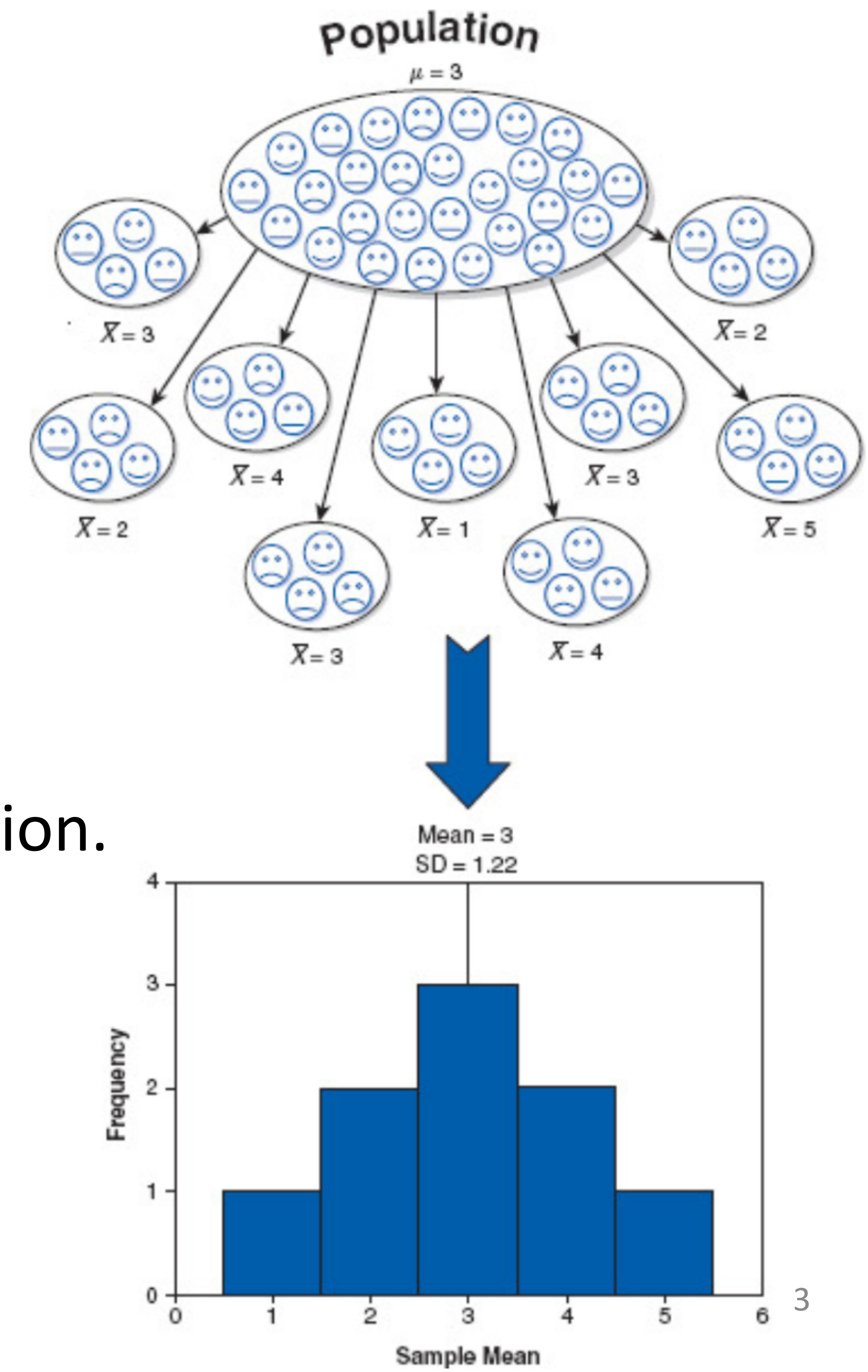
# Standard Error of the Mean (SEM)

- How to measure the **Unreliability** from random samples?
- Unreliability depends on:
  - ***<u>variance</u>*** ($s^2$)
  - ***<u>sample size</u>*** (n)
- Standard error (SE) indicates:

➡ The variability *across* sample means from multi-data of the same population.

➡ How large the error is for the conclusion?

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# KEY CONCEPTS

$$\text{test statistic} = \frac{\text{variance explained by the model}}{\text{variance not explained by the model}} = \frac{\text{effect}}{\text{error}}$$

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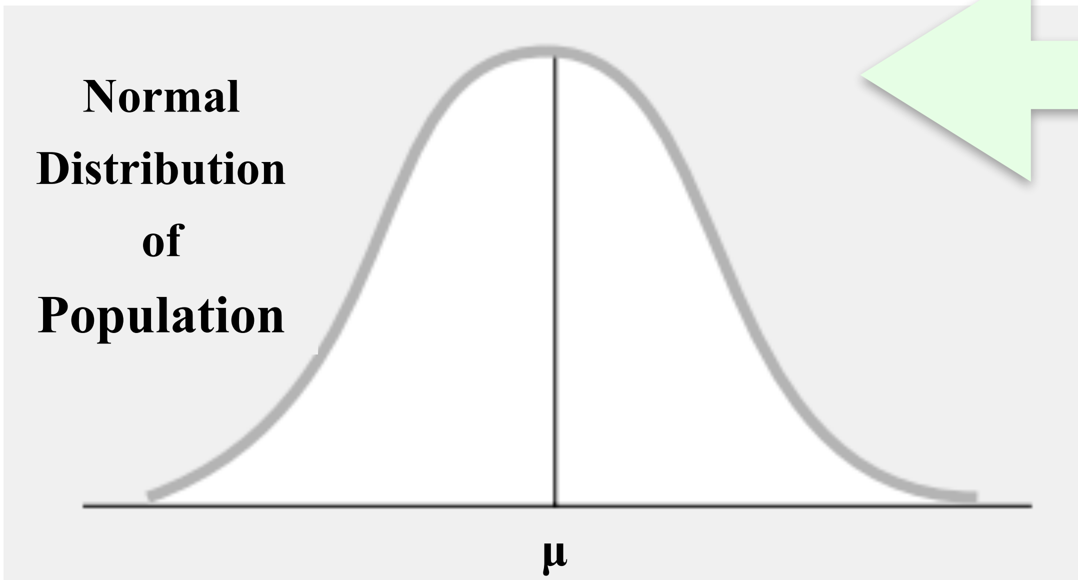
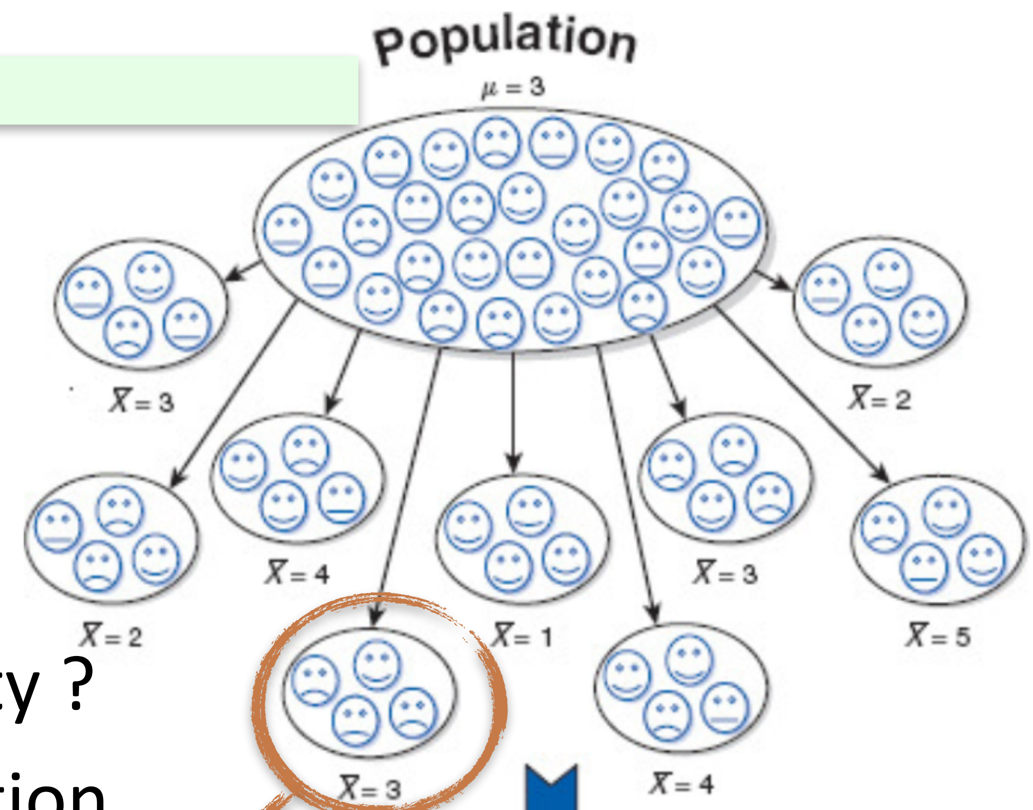
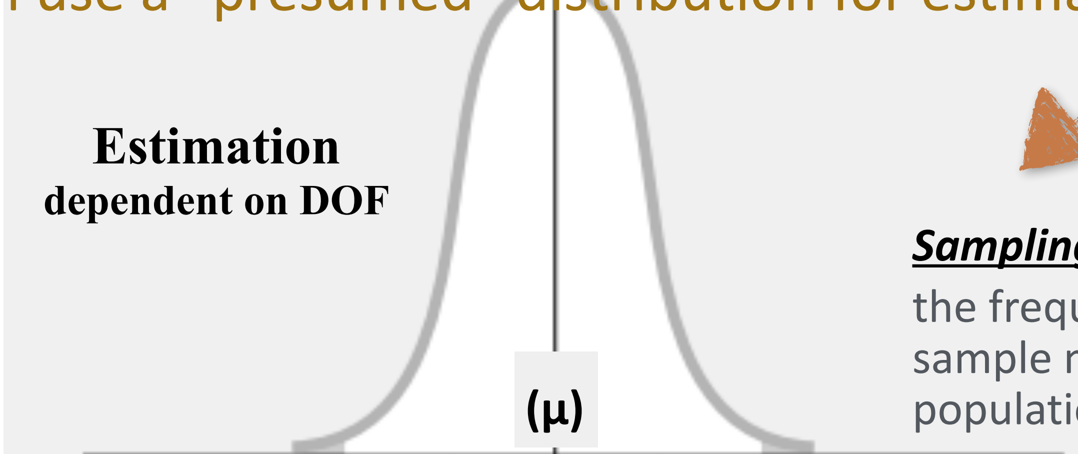
# Parametric Methods



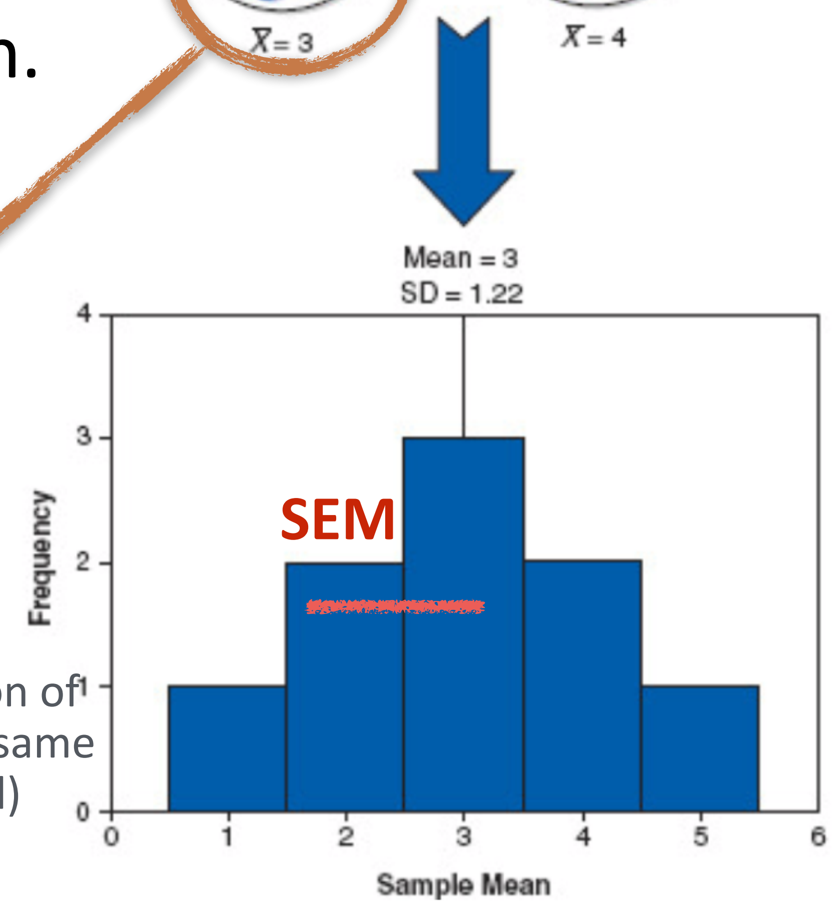




- W/o multi-data, how to estimate (un)reliability ?
  ➡ Presume a quasi-normal sampling distribution.
  ➡ Set the $H_0$ from the target μ. $H_0\!: \mu = 0$

In reality, we can only access one data
→ then use a" presumed" distribution for estimation




***Sampling distribution***:
the frequency distribution of sample means from the same population. (symmetrical)

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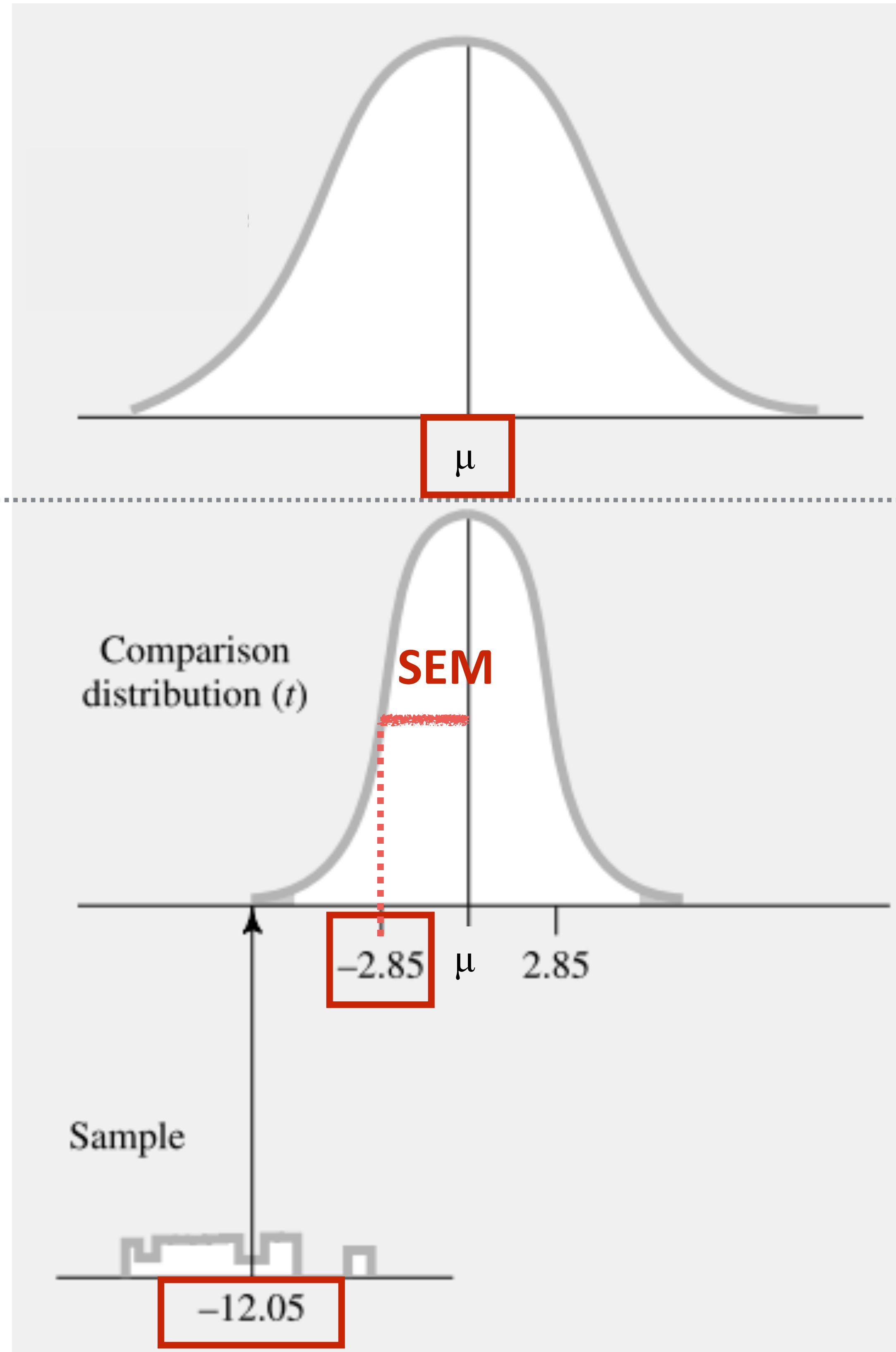
# Inferential Statistics (→ Population)

➡ Compare the sample mean to the estimated mean **($H_0$ distribution)**

$H_0\colon\ \mu = 0$




**Population**
***null hypothesis ($H_0$)***

*from real data for inference*

**Estimation**
***unreliability for estimation***
*from the presumed sampling distribution (e.g., t-distribution)*

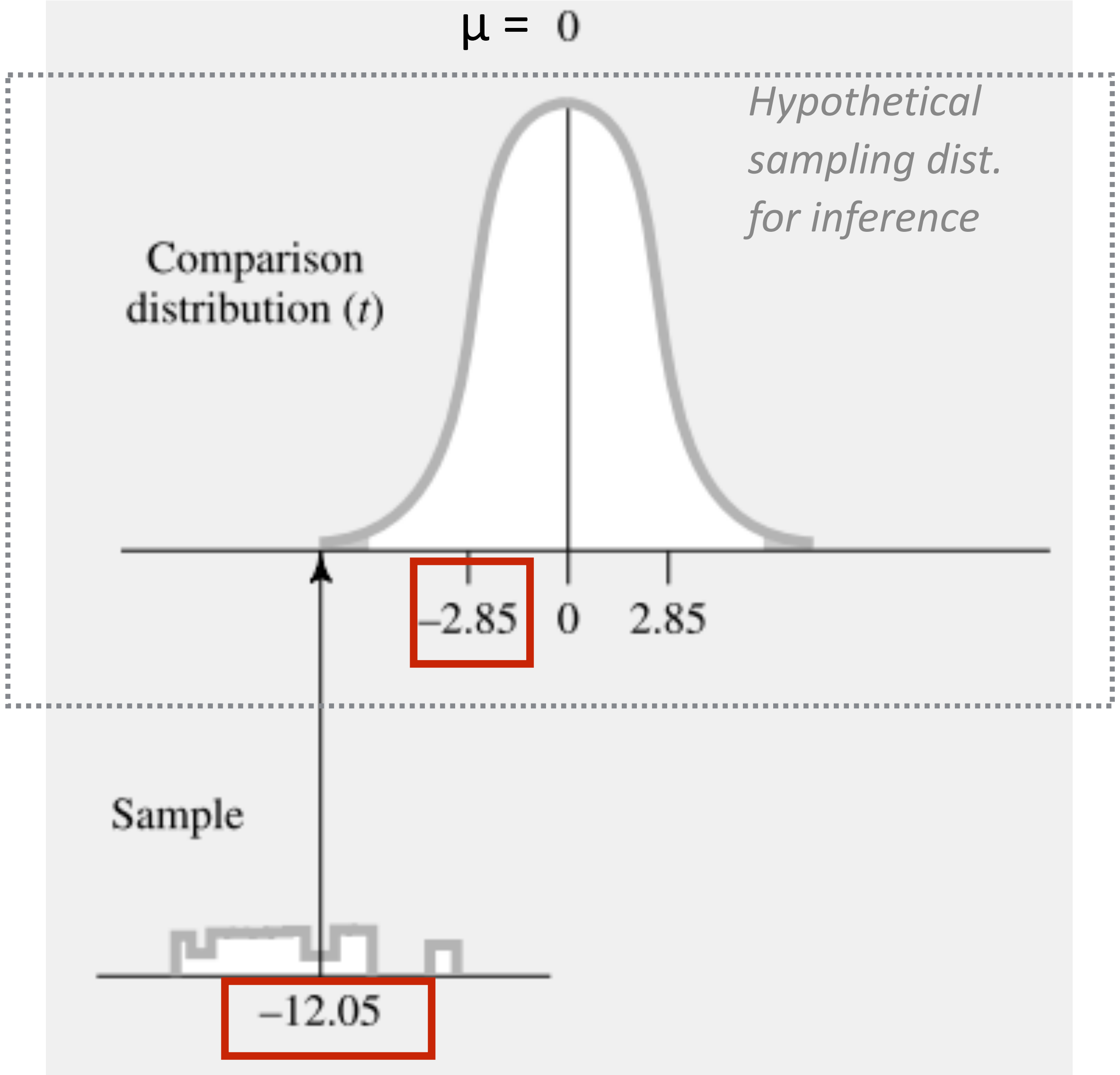
**Sample**
***descriptive stat.***

- **Compare the effect with the SEM**

$$\frac{-12.05}{2.85} = -4.2$$

# One-sample t Test

$$t = \frac{\overline{X} - \mu}{\sqrt{\frac{s^2}{n}}} = \frac{\textbf{Effect}}{\textbf{Error}}$$




**Question**: Is the sample different from μ=0 ?

$H_a$ ➡ The sample mean ≠ 0

➡ compare the **effect** (difference of means) to the **unreliability** in the estimated t-distribution

$\text{CDF}(-4.2) < 0.05$

*assume n=11*

The sample mean falls within the CDF less than **5% of the chance** that sample mean is same as μ.

$H_0$ (An unwanted opposite scenario) Assume the sample mean = 0

# Statistical Hypothesis

$$\begin{array}{|ll|} H_0\text{: } \mu = c & E(t) = 0 \\ H_a\text{: } \mu \neq c & E(t) \neq 0 \end{array}$$

## $H_0$

*Null hypothesis*

*No effect*. Samples may come from the same population but sample means fluctuate a lot.

**is the unwanted situation**

## $H_a$

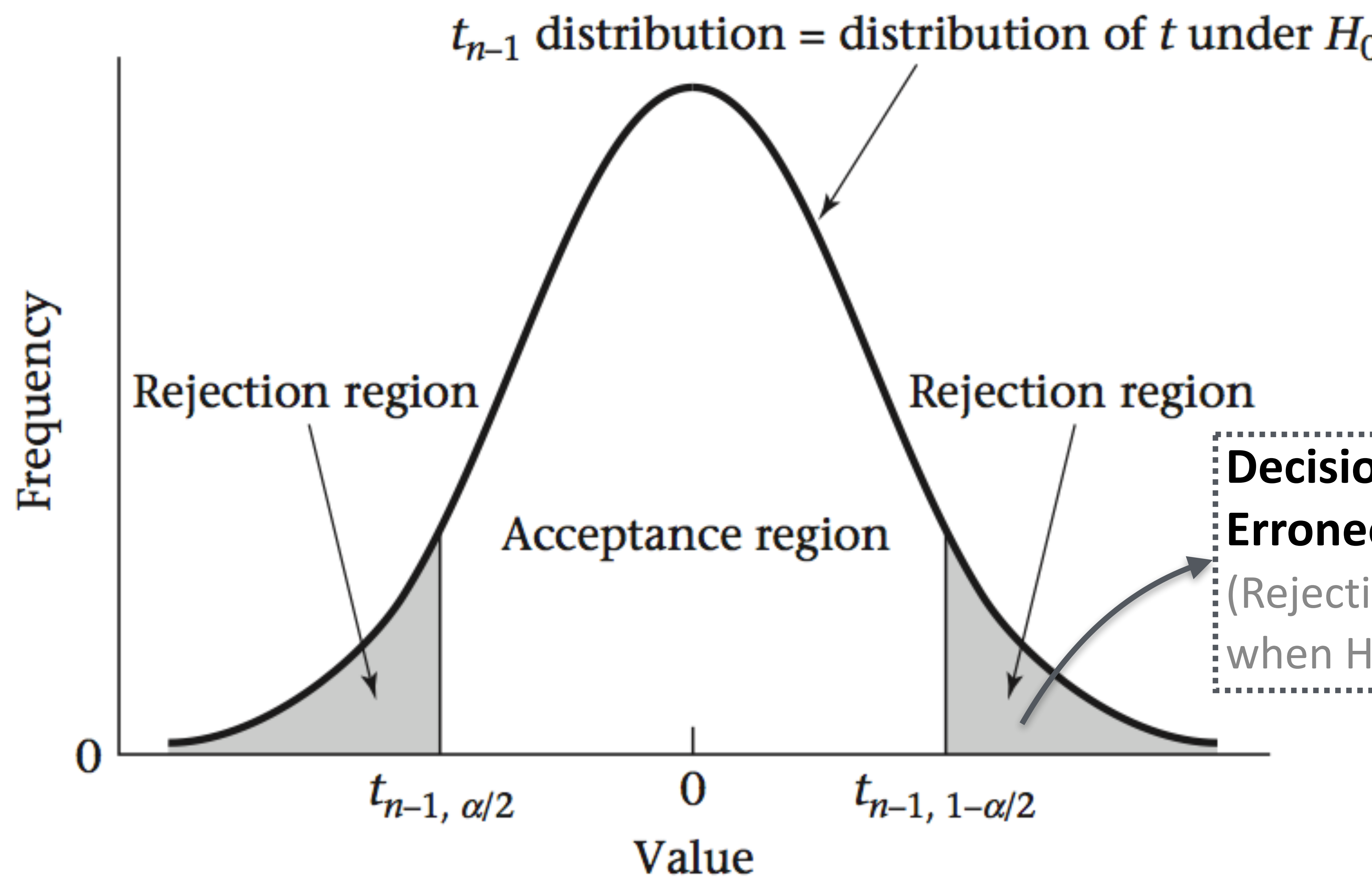
*Alternative hypothesis*

*Certain effect*. Samples may come from different population from the expectation.

**is the expected situation**

# Estimated p-value: *5% CDF*

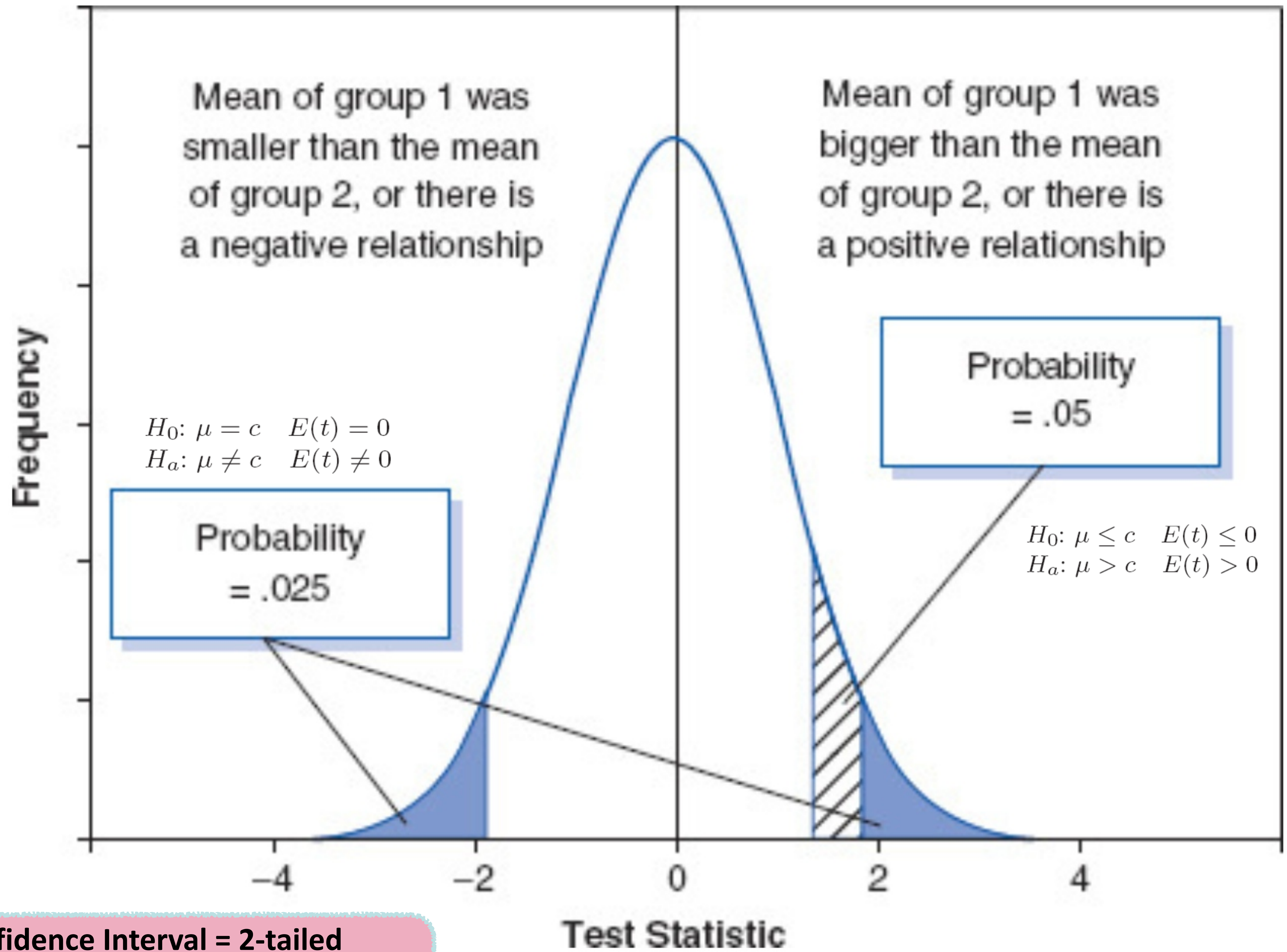
$$\boxed{\begin{array}{ll} H_0\colon \mu = c & E(t) = 0 \\ H_a\colon \mu \neq c & E(t) \neq 0 \end{array}}$$



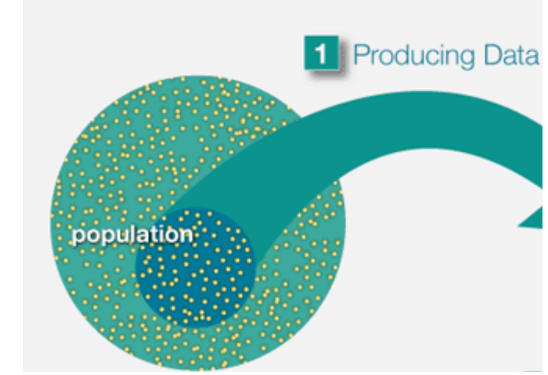

**Decision may be Erroneous**
(Rejecting $H_0$ when $H_0$ is true)

# p-value: 1-tail / 2-tail

➡ Which one is easier to pass $p<0.05$? One-tail or Two-tail?




Confidence Interval = 2-tailed




# Types of Variables

- **Numeric variables**
  - <u>Equal-interval variables</u> (interval scales)
    - **Continuous variable**: e.g., GPA; height
    - Discrete counts: e.g., the number of time visiting dentist
    - Proportions: e.g., percentage, rate
  - <u>Rank-order variables</u> (ordinal/discrete scales)
    - e.g., order of finishing a race; birth order of children
    - Physical activity level (low, moderate and high)
- **Nominal variables**
  - Gender (male, female)
  - Ethnicity (Caucasian, African American, Asian and Hispanic)
  - Profession (surgeon, doctor, nurse, dentist)

Demo

# Growth of Stinging Trees

**EXAMPLE 6.1.** A forest ecologist, studying regeneration of rainforest communities in gaps caused by large trees falling during storms, read that stinging tree, *Dendrocnide excelsa*, seedlings will grow 1.5 m/yr in direct sunlight in such gaps. In the gaps in her study plot she identified 9 specimens of this species and measured them in 2005 and again 1 year later. Listed below are the changes in height for the 9 specimens. Do her data support the published contention that seedlings of this species will average 1.5 m of growth per year in direct sunlight?

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| 1.9 | 2.5 | 1.6 | 2.0 | 1.5 | 2.7 | 1.9 | 1.0 | 2.0 |

① Hypothesis

$H_0$: $\mu_d = 1.5$ m/year

$H_a$: $\mu_d \neq 1.5$ m/year

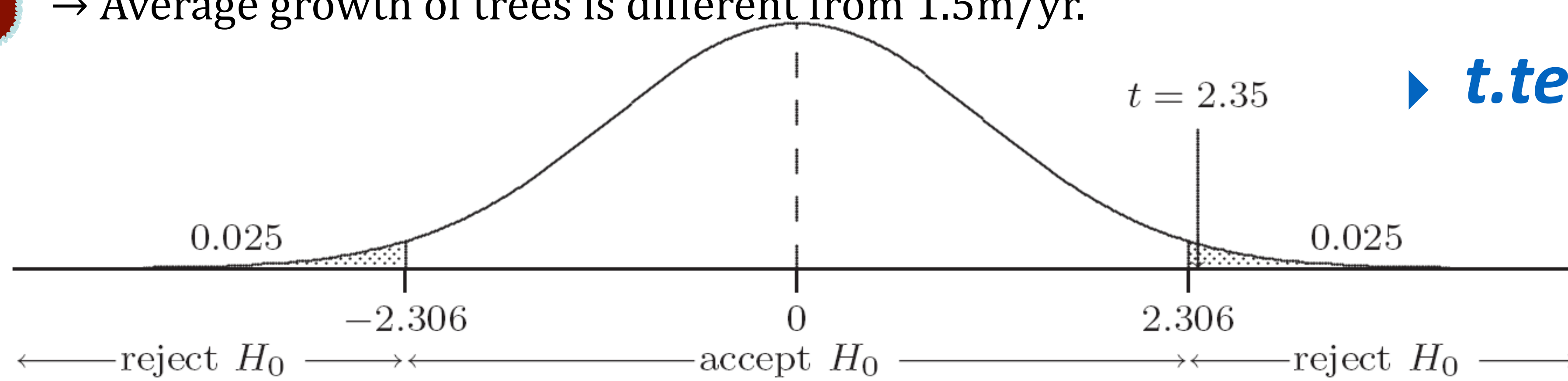
③ Testing

$$t = \frac{\overline{X} - \mu}{\frac{s}{\sqrt{n}}} = \frac{1.90 - 1.50}{\frac{0.51}{\sqrt{9}}} = \frac{0.40}{\frac{0.51}{3}} = 2.35.$$

⑤ Decision

Reject H0 based on the comparison.

→ Average growth of trees is different from 1.5m/yr.




➡ **Try to use R for this testing**

- ***t.test(data, mu=1.5)***



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# Five Steps of Hypothesis Testing

| | | |
|---|---|---|
| | a) $H_0$: $\mu = c$ $E(t) = 0$; $H_a$: $\mu \neq c$ $E(t) \neq 0$    b) $H_0$: $\mu \geq c$ $E(t) \geq 0$; $H_a$: $\mu < c$ $E(t) < 0$    c) $H_0$: $\mu \leq c$ $E(t) \leq 0$; $H_a$: $\mu > c$ $E(t) > 0$, | |
| ***Step 1*** | • Define a research hypothesis ($H_a$) and set a null hypothesis ($H_0$) according to the research question | ① Hypothesis |
| ***Step 2*** | • Determine the characteristic of the estimated distribution and check model assumption | ② Assumption |
| ***Step 3*** | • Compare the sample position and the critical value (threshold) on the estimated distribution | ③ Testing |
| ***Step 4*** | • Report the effect size of/between the samples | ④ Effect Size |
| ***Step 5*** | • Reject $H_0$ or Reject $H_a$<br>• Make the final decision | ⑤ Decision |

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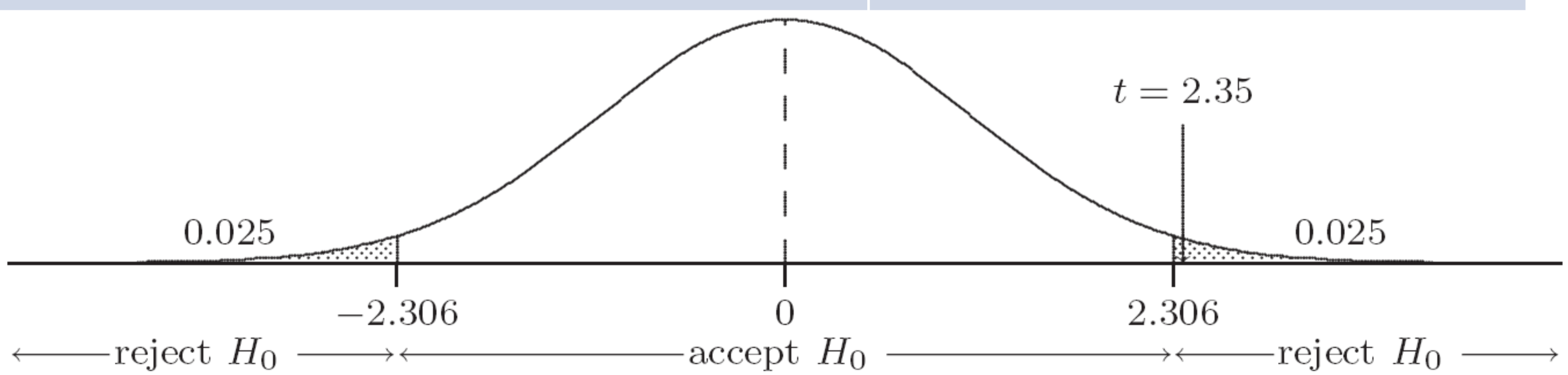

# Assumptions of *t*-test

◉ ***Parametric tests***

- Based on Normal distribution (from original population);
- Data are measured at the **interval scales**.

➡(*independent t*-test) Scores in different conditions are independent.

◉ ***Normality:***

➡(*independent t*-test) Sampling distribution should be normally distributed.

➡(*paired t*-test) Sampling distribution of the differences should be normal.

◉ ***Homogeneity of variance:***

➡(*independent t*-test) Theoretically, variance shall be equal.

- **When facing violation of assumptions**
  - If normality stands, but unequal variance happens (e.g., **unequal sample size**), then use *Welch's approximation (loss of DOF)*.
  - If sampling distributions are not normal, try using *nonparametric testing*!

# Hypothesis Tests: Two Sample Means

**Assumption**

2 samples

Independent

Dependent

Equal Variances

Unequal Variances

Equal Variance

**Testing**

**Independent, Equal Variances**

$$s_p^2 = \frac{(n_1-1)s_1^2 + (n_2-1)s_2^2}{n_1+n_2-2}$$

$$t = \frac{(\overline{X}_1 - \overline{X}_2) - (\mu_1 - \mu_2)}{\sqrt{s_p^2\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}}$$

$$\nu = n_1 + n_2 - 2.$$

**Independent, Unequal Variances**

$$t = \frac{(\overline{X}_1 - \overline{X}_2) - (\mu_1 - \mu_2)}{\sqrt{\left(\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}\right)}}$$

*Welch's approximation*

$$\nu = \frac{\left(\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}\right)^2}{\frac{\left(\frac{s_1^2}{n_1}\right)^2}{n_1-1} + \frac{\left(\frac{s_2^2}{n_2}\right)^2}{n_2-1}}.$$

**Dependent, Equal Variance**

$$t = \frac{\overline{X}_d - \mu_d}{\frac{s_d}{\sqrt{n_d}}}$$

$n_d - 1$ degrees of freedom

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# General Procedure for t test

① Hypothesis

1. **Enter data & setting null/alternative hypotheses**

② Assumption

2. **Explore the data:** check distributional assumptions
   - Normality check (*stat.desc*)
   - Homogeneity of variance (*leveneTest*)

③ Testing

3. **Compute the test**
   - Student's ***t*** test (***t.test***): compare two means

④ Effect Size

4. **Calculate the effect size**
   - Cohen's ***d*** (***cohen.d***) *effsize*

⑤ Decision

5. **Report the test results**

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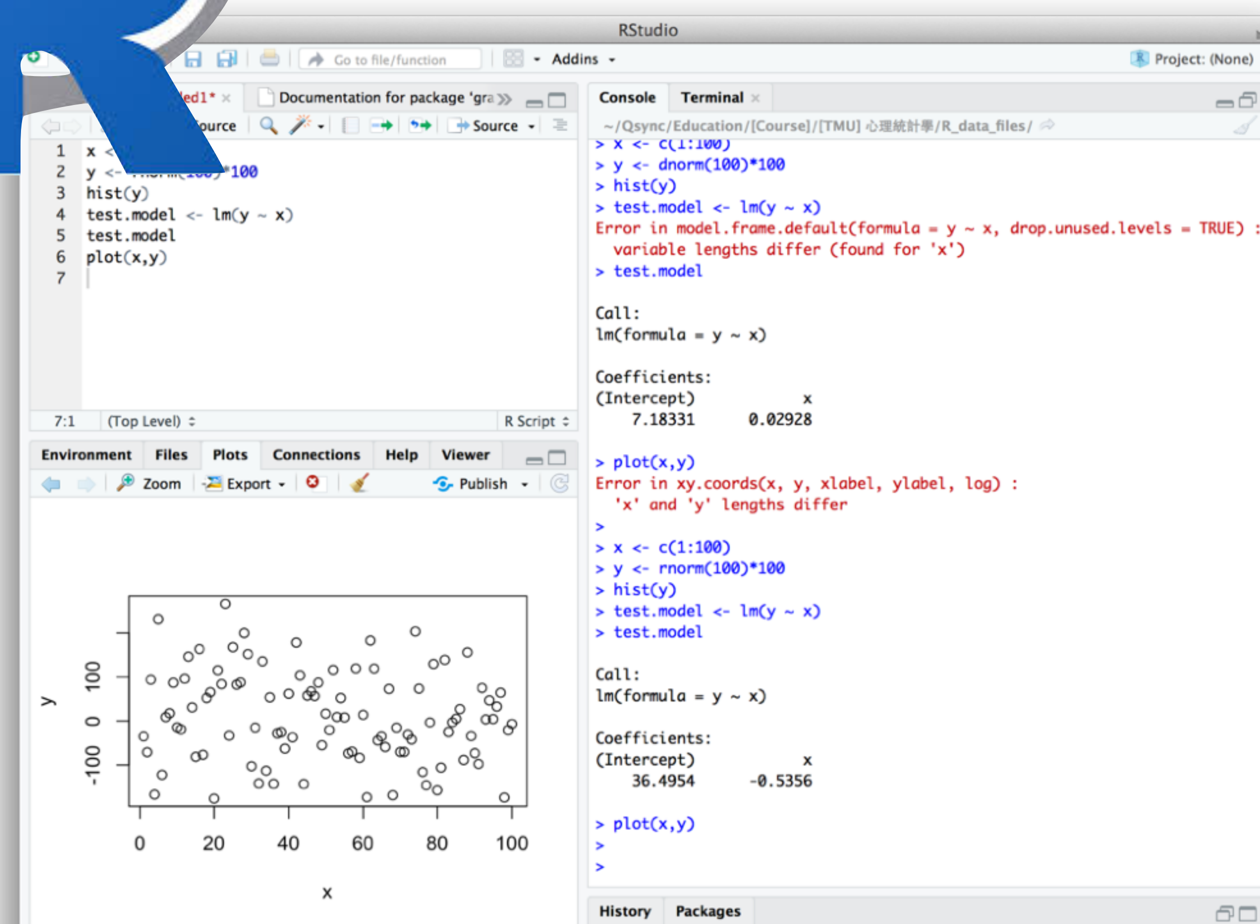
The simple case: Repeated measure

# ① PAIRED T-TEST

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# Dependent/Paired *t* Test

$$t = \frac{\overline{X}_d - \mu_d}{\frac{s_d}{\sqrt{n_d}}}$$

For repeated measures

- Comparing two means, when they came from the same entities (for example, the same participant attended two experimental conditions) or matched pairs (like twins).
- Under $\mathbf{H_0}$, we assume the experimental manipulation has no effect on the sampled data. ($\mathbf{H_0}$ = No effect)
- Compare the difference of sample means. We use the estimate of **standard error of the difference** as the gauge of <u>variability between sample means</u>.
- Use $\mu_d$ and S.E.D. to estimate "<u>whether the difference differs from zero.</u>"

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# Standard Error of the Difference

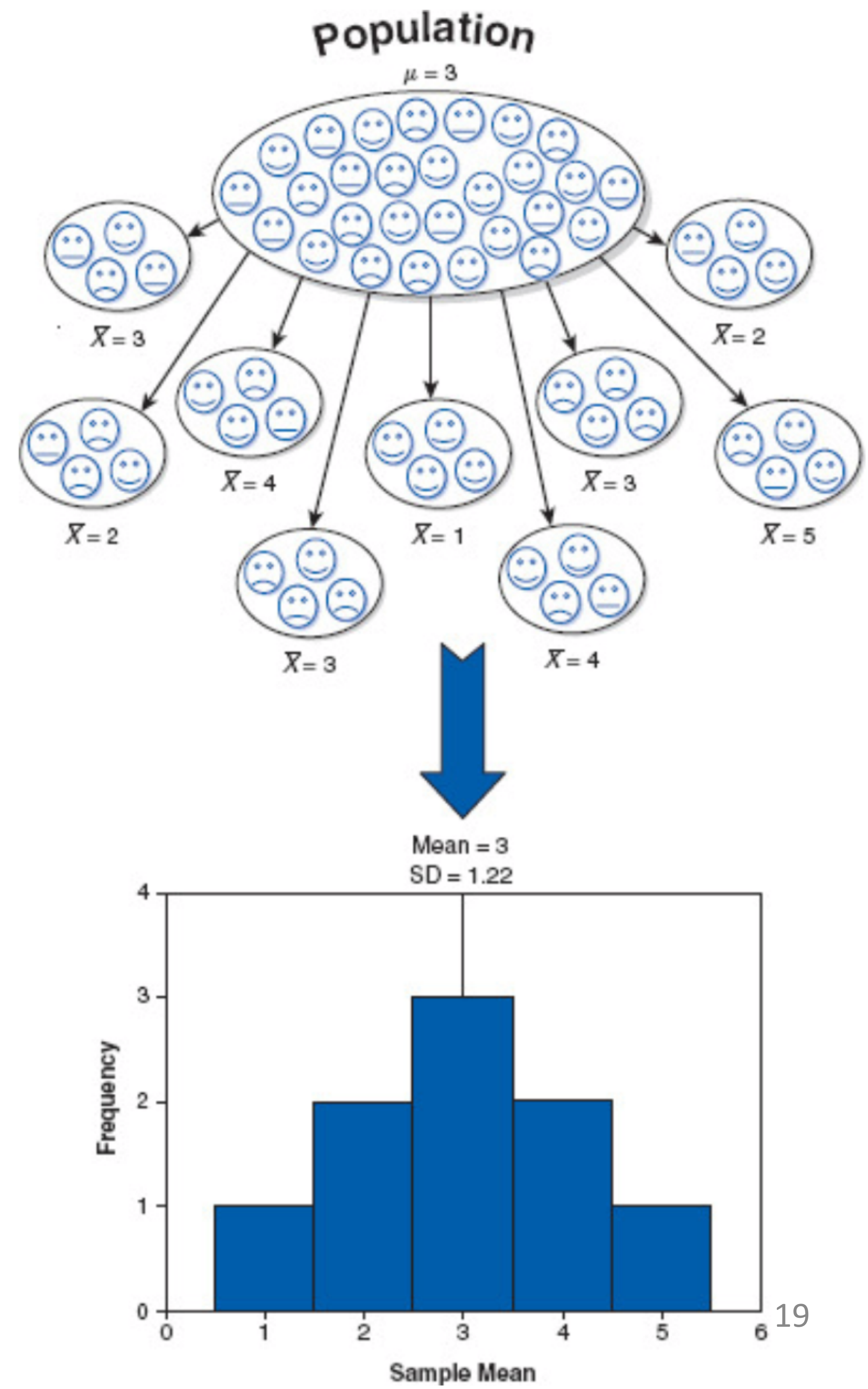
- If the same participants took part in different conditions of the experiment, we refer it as the ***matched pairs*** or ***paired samples***.

$$t = \frac{\text{difference between the two means}}{\text{SE of the difference}} = \frac{\bar{y}_{\mathrm{A}} - \bar{y}_{\mathrm{B}}}{SE_{\mathrm{diff}}}$$

- Standard error of the ***Difference***:

➡ You can also calculate variability across sample *differences* from the same population

➡ Treat it as One-sample t test

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Demo

# TESTING TWO SAMPLES IN REPEATED MEASURE

Demo

# Spider Anxiety

- Testing the Anxiety level using spider picture or real spider.
- Question: Does viewing real spider makes people more nervous?

① Hypothesis

**Dependent measures** for viewing spider picture or real spiders

**Measure: Anxiety score**

1.**Set up hypothesis & import the dataset:**

**$H_0$: No difference btw Real & Picture**

**$H_a$: Viewing Real spider differs from viewing Picture**

Load:
**SpiderLong.dat** &
**SpiderWide.dat**

---

② Assumption

2.**Assumption check:** fulfil assumptions?

- Normality check (*stat.desc*)

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Demo

# Spider Anxiety

③ Testing

*General form:*

- ***t.test(data1, data2, paired=TRUE/FALSE)***
- ***t.test(data1 ~ data2, paired=TRUE/FALSE)***

- **Parameters**:
  - alternative=(two.sided / less / greater)
  - mu=0: difference between means = 0 is default null hypothesis
  - var.equal=FALSE: by default it assumes unequal variances
  - conf.level=0.95: by default setting confidence interval = 95%
  - na.action: exclude the missing data
- **Usage:**
  - ***t.test(spiderWide$real, spiderWide$picture, paired = TRUE)***

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# Spider Anxiety

④ Effect Size

## 4. Effect size (quantitative measure of the magnitude):

- Cohen's ***d*** $(0 \le d \le \infty)$

*General form:*

effsize

- ***cohen.d(data1, data2, paired=TRUE)***
- ***cohen.d(data ~ group)***

$$d = \frac{M_E - M_C}{\text{Sample } SD \text{ pooled}}$$

$$\text{where sample SD pooled} = \sqrt{\frac{[(SD_E)^2 + (SD_C)^2]}{2}}$$

```
> cohen.d(Anxiety ~ Group, paired=T)

Cohen's d

d estimate: -0.6864065 (medium)
95 percent confidence interval:
     lower      upper
-1.5576365  0.1848236
```

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# Spider Anxiety

⑤ Decision

- **People were more scared of real spiders (t=2.47).**
  - more scared than what?
  - where are the degree of freedom?
  - is it significant?
  - what was the effect size?

- **Participants experienced significantly greater anxiety from real spiders** (mean±SD = 47.0 ± 3.2) **than from pictures of spiders** (mean±SD = 40.0 ± 2.7), **$t_{(11)}$ = 2.47, *p* < .05, Cohen's d = .69**

✓

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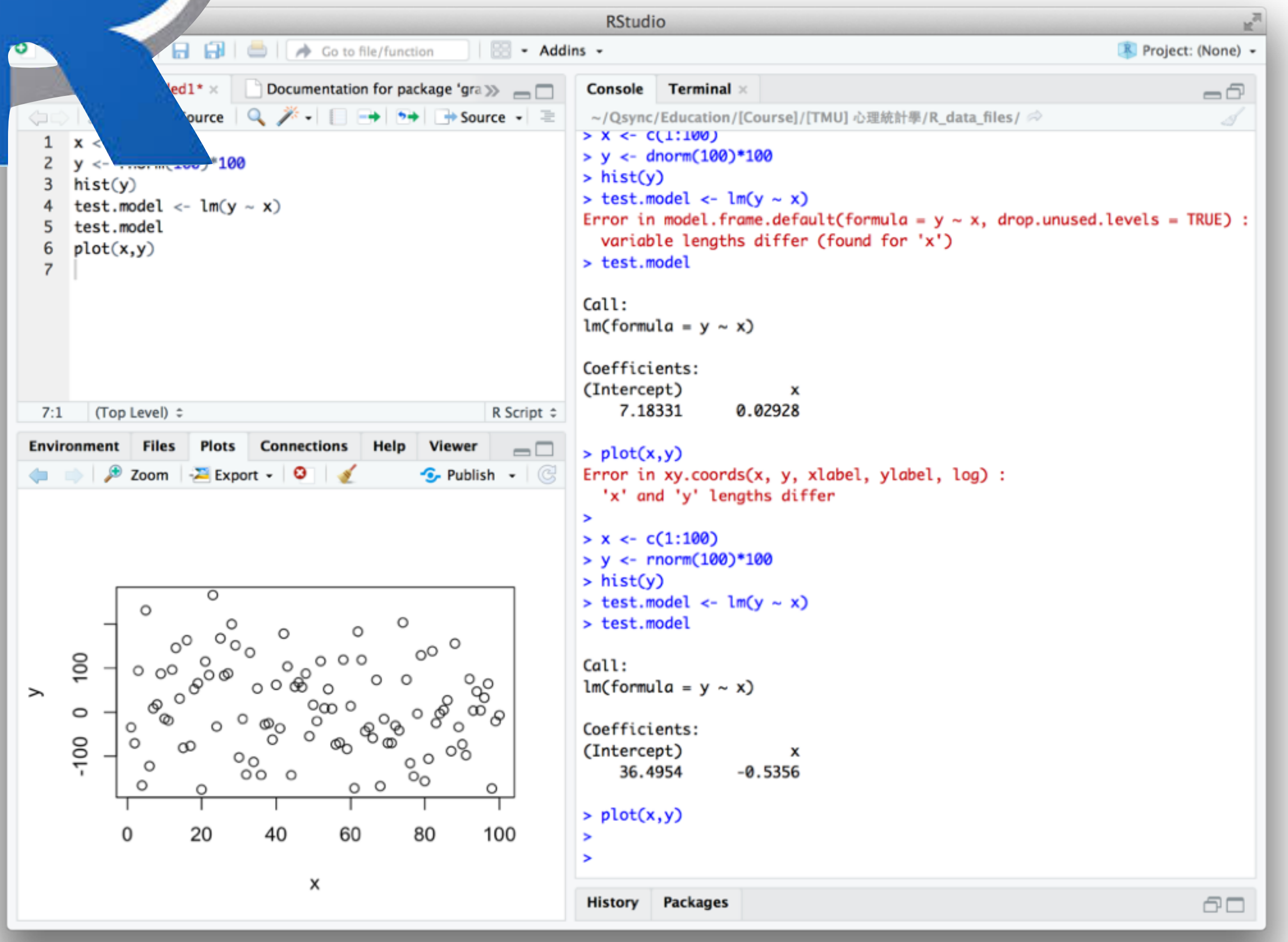
Comparison between 2 Independent samples

# ② INDEPENDENT T-TEST

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# T Tests for Independent Means

- The t test for dependent means: compare two sets of scores from *a single group of people*.
  - ➢ Before vs. After; Pre-test vs. Post-test
  - ➢ "Within subject design" or "Repeated measures design".

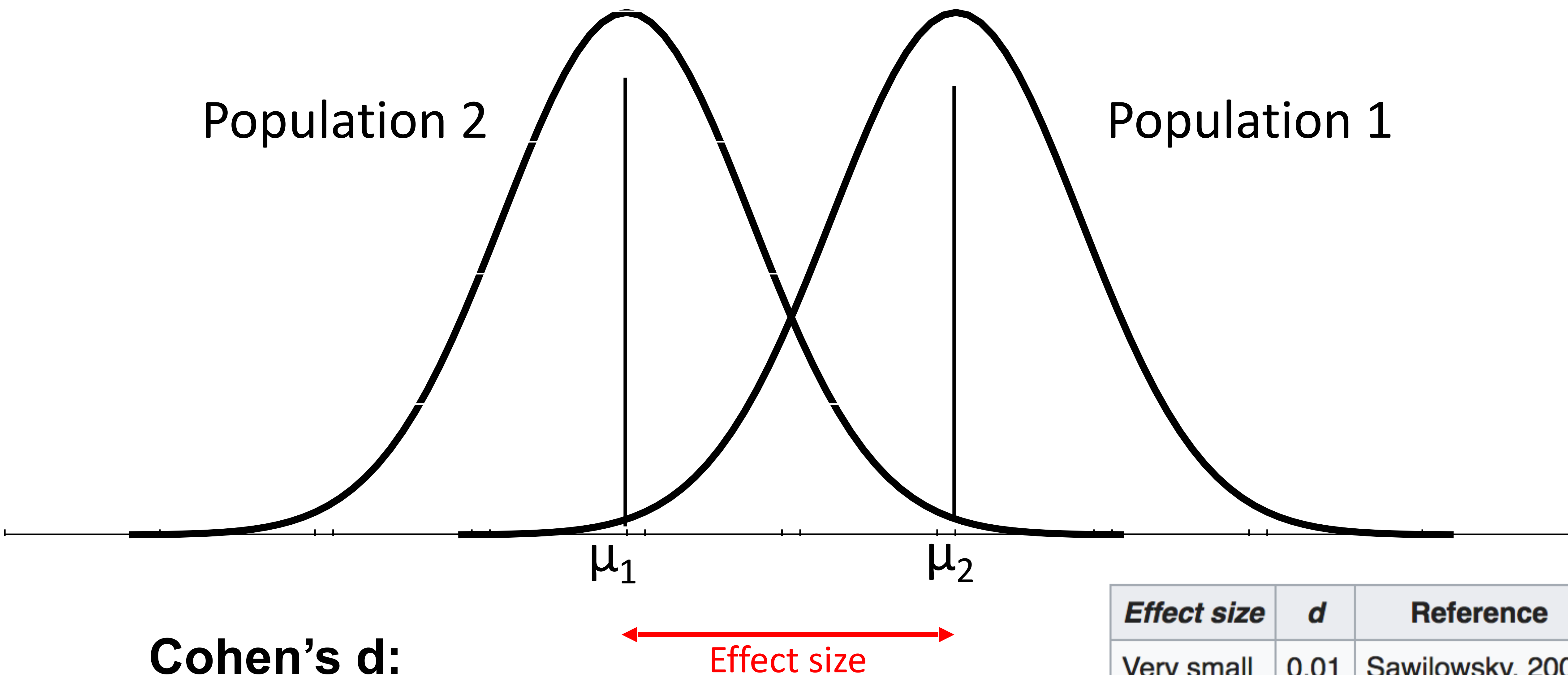
- The t test for independent means: compare two sets of scores, *one from each of two entirely separate groups of people*:
  - ➢ Experimental group vs. Control group; A group vs. B group.
  - ➢ "Between subject design"

Examples:

- The cognitive control ability in Schizophrenia patients and normal controls.
- The working memory capacity in young adults vs. older adults.
- The achievement in special job program vs. ordinary job program.

# Effect size

Please see appendix




**Cohen's d:**

A measure of the difference between sample means

$$d = \frac{\mu_1 - \mu_2}{\sigma}$$

| *Effect size* | *d* | Reference |
|---|---|---|
| Very small | 0.01 | Sawilowsky, 2009 |
| Small | 0.20 | Cohen, 1988 |
| Medium | 0.50 | Cohen, 1988 |
| Large | 0.80 | Cohen, 1988 |
| Very large | 1.20 | Sawilowsky, 2009 |
| Huge | 2.0 | Sawilowsky, 2009 |



https://rpsychologist.com/d3/cohend/

# T Tests for Independent Means

- [Estimate of the population variance: **Pooled sample variance**]().
  - Assume both samples have ‘equal’ variance.
  - Generating a new SE by the weightings of *degree of freedom*.

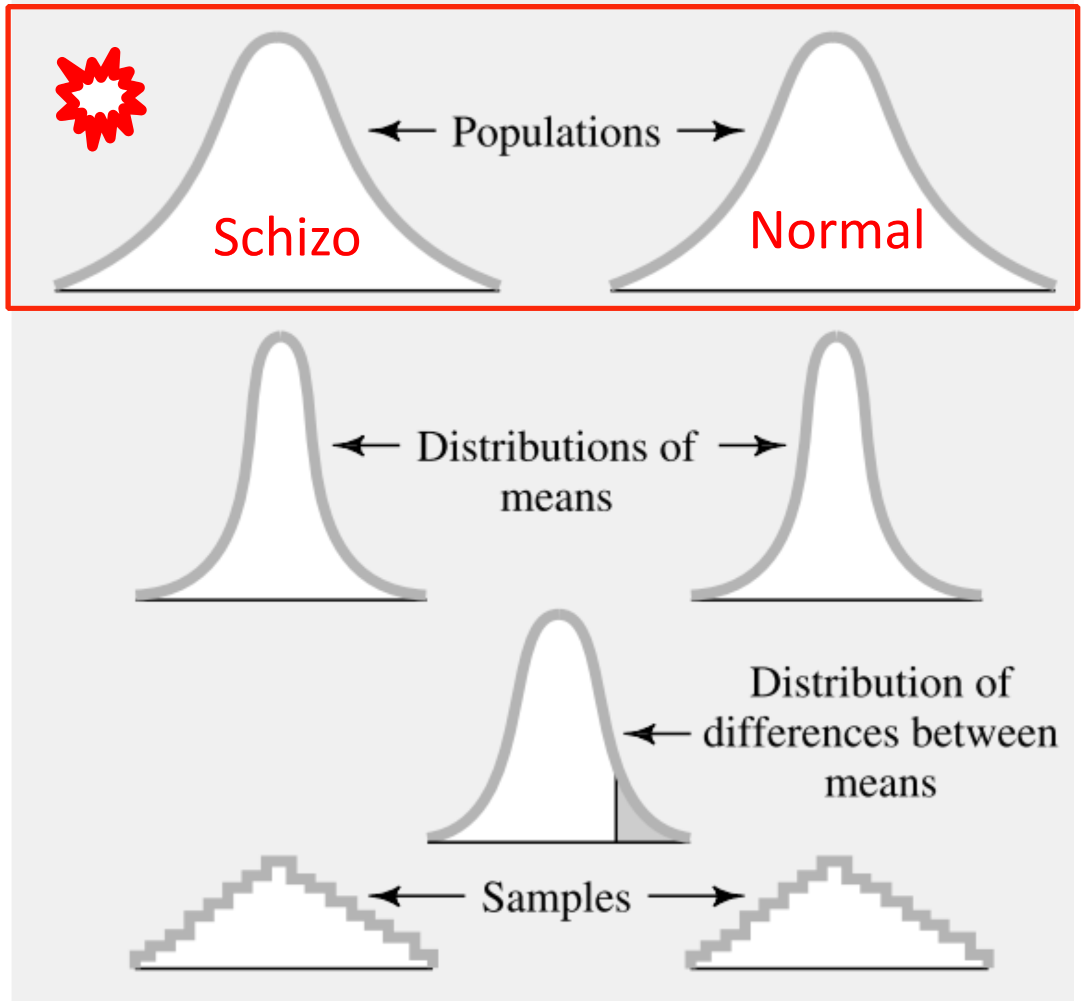
$$S_1^2 = \frac{SS_1}{df_1}$$

Var(Schizo)

$$S_2^2 = \frac{SS_2}{df_2}$$

Var(Normal)

$$S^2_{Pooled} = \frac{df_1}{df_{Total}}(S_1^2) + \frac{df_2}{df_{Total}}(S_2^2)$$




- $df_{total} = df_1 + df_2 = (n_1 - 1) + (n_2 - 1)$

# T Tests for Independent Means

- The estimate of the population variances:
  - ➢Average of the estimates of the population variance from two samples, weighted by degree of freedom.

$$s_p^2 = \frac{(n_1 - 1)s_1^2 + (n_2 - 1)s_2^2}{n_1 + n_2 - 2}$$

Pooled estimate of the population variance

$$t = \frac{\overline{X}_1 - \overline{X}_2}{\sqrt{\frac{s_p^2}{n1} + \frac{s_p^2}{n2}}}$$

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# TESTING TWO INDEPENDENT SAMPLES?

Demo

# Spider Anxiety

⑤ Decision

- **People were more scared of real spiders.**
  - more scared than what?
  - where are the degree of freedom?
  - is it significant?
  - what was the effect size?

➡ **If you treated the data as independent measure:**

- **Participants did not experience significant difference of anxiety between viewing real spiders** (mean ± SD = 47.0 ± 3.2) **and pictures of spiders** (mean ± SD = 40.0 ± 2.7), $t_{(21.4)} = -1.68$, $p > .05$, **Cohen's** $d = -.69$

**Independent measure**

➡ **Error variance increased dramatically in the 2-sample t, which is harder to get significance.**

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# Tuatara of New Zealand

**EXAMPLE 7.1.** Among the few reptilian lineages to emerge from the Mesozoic extinction are today's reptiles. One of these, the tuatara, *Sphenodon punctatum*, of New Zealand, is the sole survivor of a group that otherwise disappeared 100 million years ago. The mass (in g) of random samples of adult male tuatara from two islets in the Cook Strait are given below.

Does the average mass of adult males differ between Location A and Location B?

| Location A | | Location B |
|---|---|---|
| 510 | 790 | 650 |
| 773 | 440 | 600 |
| 836 | 435 | 600 |
| 505 | 815 | 575 |
| 765 | 460 | 452 |
| 780 | 690 | 320 |
| 235 | | 660 |

① Hypothesis

$$H_0\colon \mu_A = \mu_B$$
$$H_a\colon \mu_A \neq \mu_B.$$

② Assumption

Assumption check: normality/variance

$$t = \frac{(\overline{X}_1 - \overline{X}_2) - (\mu_1 - \mu_2)}{\sqrt{s_p^2\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}},$$

| Location A | Location B |
|---|---|
| $n_A = 13$ | $n_B = 7$ |
| $\overline{X}_A = 618$ | $\overline{X}_B = 551$ |
| $s_A^2 = 37{,}853.17$ | $s_B^2 = 15{,}037.00$ |

If we assume equal variance…

③ Testing

$$t = \frac{(618 - 551) - 0}{\sqrt{30{,}247.78\left(\frac{1}{13} + \frac{1}{7}\right)}} = \frac{67}{\sqrt{6{,}647.86}} = \frac{67}{81.5} = 0.82.$$

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Demo

# Tuatara of New Zealand

**EXAMPLE 7.1.** Among the few reptilian lineages to emerge from the Mesozoic extinction are today's reptiles. One of these, the tuatara, *Sphenodon punctatum*, of New Zealand, is the sole survivor of a group that otherwise disappeared 100 million years ago. The mass (in g) of random samples of adult male tuatara from two islets in the Cook Strait are given below.

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① Hypothesis

$H_0$: $\mu_A = \mu_B$

$H_a$: $\mu_A \neq \mu_B$.

③ Testing

$$t = \frac{(618 - 551) - 0}{\sqrt{30{,}247.78\left(\frac{1}{13} + \frac{1}{7}\right)}} = \frac{67}{\sqrt{6{,}647.86}} = \frac{67}{81.5} = 0.82.$$

⑤ Decision

➡ Try to use R for this testing

At $\alpha = 0.05$ and $\nu = 18$ in Table C.4 we find the critical values to be $\pm 2.101$. Since $-2.101 < 0.82 < 2.101$, we cannot reject $H_0$. The mean masses for Locations A and B are not significantly different.

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# Discussion

1. **Five Steps of Hypothesis Testing**
   - Hypothesis/effect size/decision
2. **Comparing two samples**
   - Pooled sample variance (equal / unequal)
3. **[R] Independent & paired t-test**
   - Examples for practices

# Suggested Reading for Effect Size

## How to Select, Calculate, and Interpret Effect Sizes

Joseph A. Durlak
*Loyola University Chicago*

*Durlak J.A.* ***Journal of Pediatric Psychology*** *2009;34(9) p.917-928*

**Table I.** Guidelines for Calculating, Reporting, and Interpreting ESs

1. Choose the most suitable type of effect based on the purpose, design, and outcome(s) of a research study.
2. Provide the basic essential data for the major variables[a]
   (a) for group designs, present means, standard deviations, and sample size for all groups on all outcomes at all time points of measurement
   (b) for correlational studies, provide a complete correlation matrix at all time points of measurement
   (c) for dichotomous outcomes, present the cell frequencies or proportions and the sample sizes for all groups
3. Be explicit about the type of ES that is used.
4. Present the effects for all outcomes regardless of whether or not statistically significant findings have been obtained.
5. Specify exactly how effects were calculated by giving a specific reference or providing the algebraic equation used.
6. Interpret effects in the context of other research
   (a) the best comparisons occur when the designs, types of outcomes, and methods of calculating effects are the same across studies.
   (b) evaluate the magnitude of effect based on the research context and its practical or clinical value.
   (c) if effects from previous studies are not presented, strive to calculate some using the procedures described here and in the additional references.
   (d) use Cohen's (1988) benchmarks, only if comparisons to other relevant research are impossible

[a]These data have consistently been recommended as essential information in any report, but they also can serve a useful purpose in subsequent research if readers need to make any adjustments to your calculations based on new analytic strategies or want to conduct more sophisticated analyses. For example, the data from a complete correlation matrix is needed for conducting meta-analytic mediational analyses.

# Suggested Reading for Effect Size

***ESs for Group Designs*** ▸ ***Group difference*** *(continuous interval)*

(1) Calculating $\boxed{\text{Hedges' g}}$ from means, standard deviations and *ns*

$$g = \frac{M_E - M_C}{SD \text{ pooled}} \times \left(\frac{N-3}{N-2.25}\right) \times \sqrt{\frac{N-2}{N}}$$

$$SD \text{ pooled} = \sqrt{\frac{[(SD_E)^2(n_E - 1)] + [(SD_C)^2(n_C -}{(n_E + n_C) - 2}}$$

(2) Calculating $\boxed{\text{Cohen's } d}$ from means and standard deviations

$$d = \frac{M_E - M_C}{\text{Sample } SD \text{ pooled}} \times \left(\frac{N-3}{N-2.25}\right) \times \sqrt{\frac{N-2}{N}}$$

$$\text{where sample SD pooled} = \sqrt{\frac{[(SD_E)^2 + (SD_C)^2]}{2}}$$

(4) Calculating an $\boxed{\text{OR}}$ ▸ ***Dichotomous outcomes***

$$\text{OR} = \frac{ad}{bc}$$

where *a* and *c* are the number of favorable or desired outcomes in the intervention and control groups respectively and *b* and *d* are the number of failures or undesirable outcomes in these two respective groups

(5) Alternative calculation formula for OR

$$\text{OR} = \frac{[P_E/(1 - P_E)]}{[P_C - (1 - P_C)]}$$

where *PE* is the proportional success for the Experimental group, and *PC* the proportional success for the Control group.

***Effects for Correlational Designs*** ▸ ***Correlation/Regression***

(6) Computing $\boxed{r}$ from an independent *t*-test

$$r = \sqrt{\frac{t^2}{(t^2 + df)}}$$

# Transforming Effect Size

***Transforming One ES Into Another***

(7) Hedges' g computed from $r$

$$g = \frac{r/\sqrt{1-r^2}}{\sqrt{df(n_1+n_2)/n_1 n_2}}$$

(8) Transforming Hedge's g to $r$

$$r = \sqrt{\frac{g^2 n_1 n_2}{g^2 n_1 n_2 + (n_1+n_2)df}}$$

(9) Hedges' g computed from Cohen's $d$

$$g = \frac{d}{\sqrt{N/df}}$$

(10) Cohen's $d$ calculated from Hedges' g

$$d = g\sqrt{N/df}$$

(11) Transforming Cohen's $d$ to $r$

$$r = \frac{d}{\sqrt{d^2+4}}$$

(12) Transforming Hedge's g to $r$

$$r = \sqrt{\frac{g^2 n_1 n_2}{g2 n_1 n_2 + (n_1+n_2)df}}$$

# THANK YOU FOR YOUR ATTENTION

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