MCM: The Mathematical Contest in Modeling
ICM: The Interdisciplinary Contest in Modeling

2013 MCM Problems

PROBLEM A: The Ultimate Brownie Pan

When baking in a rectangular pan heat is concentrated in the 4 corners and the product gets overcooked at the corners (and to a lesser extent at the edges). In a round pan the heat is distributed evenly over the entire outer edge and the product is not overcooked at the edges. However, since most ovens are rectangular in shape using round pans is not efficient with respect to using the space in an oven. Develop a model to show the distribution of heat across the outer edge of a pan for pans of different shapes - rectangular to circular and other shapes in between.

Assume

1. A width to length ratio of W/L for the oven which is rectangular in shape.
2. Each pan must have an area of A.
3. Initially two racks in the oven, evenly spaced.

Develop a model that can be used to select the best type of pan (shape) under the following conditions:

1. Maximize number of pans that can fit in the oven (N)
2. Maximize even distribution of heat (H) for the pan
3. Optimize a combination of conditions (1) and (2) where weights p and (1- p) are assigned to illustrate how the results vary with different values of W/L and p.

In addition to your MCM formatted solution, prepare a one to two page advertising sheet for the new Brownie Gourmet Magazine highlighting your design and results.

PROBLEM B: Water, Water, Everywhere

Fresh water is the limiting constraint for development in much of the world. Build a mathematical model for determining an effective, feasible, and cost-efficient water strategy for 2013 to meet the projected water needs of [pick one country from the list below] in 2025, and identify the best water strategy. In particular, your mathematical model must address storage and movement; de-salinization; and conservation. If possible, use your model to discuss the economic, physical, and environmental implications of your strategy. Provide a non-technical position paper to governmental leadership outlining your approach, its feasibility and costs, and why it is the “best water strategy choice.”

Countries: United States, China, Russia, Egypt, or Saudi Arabia

2013 ICM Problem
Network Modeling of Earth's Health

Background: Society is interested in developing and using models to forecast the biological and environmental health conditions of our planet. Many scientific studies have concluded that there is growing stress on Earth's environmental and biological systems, but there are very few global models to test those claims. The UN-backed *Millennium Ecosystem Assessment Synthesis Report* found that nearly two-thirds of Earth's life-supporting ecosystems— including clean water, pure air, and stable climate— are being degraded by unsustainable use. Humans are blamed for much of this damage. Soaring demands for food, fresh water, fuel, and timber have contributed to dramatic environmental changes; from deforestation to air, land, and water pollution. Despite the considerable research being conducted on local habitats and regional factors, current models do not adequately inform decision makers how their provincial polices may impact the overall health of the planet. Many models ignore complex global factors and are unable to determine the long-range impacts of potential policies. While scientists realize that the complex relationships and cross-effects in myriad environmental and biological systems impact Earth's biosphere, current models often ignore these relationships or limit the systems' connections. The system complexities manifest in multiple interactions, feedback loops, emergent behaviors, and impending state changes or tipping points. The recent *Nature* article written by 22 internationally known scientists entitled "Approaching a state shift in Earth's biosphere" outlines many of the issues associated with the need for scientific models and the importance of predicting potential state changes of the planetary health systems. The article provides two specific quantitative modeling challenges in their call for better predictive models:

1) To improve bio-forecasting through global models that embrace the complexity of Earth's interrelated systems and include the effects of local conditions on the global system and vice versa.
2) To identify factors that could produce unhealthy global state-shifts and to show how to use effective ecosystem management to prevent or limit these impending state changes.

The resulting research question is whether we can build global models using local or regional components of the Earth's health that predict potential state changes and help decision makers design effective policies based on their potential impact on Earth's health. Although many warning signs are appearing, no one knows if Planet Earth is truly nearing a global tipping point or if such an extreme state is inevitable.

The *Nature* article and many others point out that there are several important elements at work in the Earth's ecosystem (e.g., local factors, global impacts, multi-dimensional factors and relationships, varying time and spatial scales). There are also many other factors that can be included in a predictive model — human population, resource and habitat stress, habitat transformation, energy consumption, climate change, land use patterns, pollution, atmospheric chemistry, ocean chemistry, bio diversity, and political patterns such as social unrest and economic instability. Paleontologists have studied and modeled ecosystem behavior and response during previous cataclysmic state shifts and thus historic-based qualitative and quantitative information can provide background for future predictive models. However, it should be noted that human effects have increased significantly in our current biosphere situation.

Requirements:

You are members of the International Coalition of Modelers (ICM) which will soon be hosting a workshop entitled "Networks and Health of Planet Earth" and your research leader has asked you to perform modeling and analysis in advance of the workshop. He requires your team to do the following:

Requirement 1: Build a dynamic global network model of some aspect of Earth's health (you develop the measure) by identifying local elements of this condition (network nodes) and appropriately connecting them (network links) to track relationship and attribute effects. Since the dynamic nature of these effects is important, this network model must include a dynamic time element that allows the model to predict future states of this health measure. For example, your nodes could be nations, continents, oceans, habitats, or any combination of these or other elements which together constitute a global model. Your links could represent nodal or environmental influences, or the flow or propagation of physical elements (such as pollution) over time. Your health measure could be any element of Earth's condition to include demographic, biological, environmental, social, political, physical, and/or chemical conditions. Be sure to define all the elements of your model and explain the scientific bases for your modeling decisions about network measures, nodal entities, and link properties. Determine a methodology to set any parameters and explain how you could test your model if sufficient data were available. What kinds of data could be used to validate or verify the efficacy of your model? (Note: If you do not have the necessary data to determine parameters or perform verification, do not throw out the model. Your supervisor realizes that, at this stage, good creative ideas and theories are as important as verified data-based models.) Make sure you include the human element in your model and explain where human behavior and government policies could affect the results of your model.

Requirement 2: Run your model to see how it predicts future Earth health. You may need to estimate parameters that you would normally determine from data. (Remember, this is just to test and understand the elements of your model, not to use it for prediction or decision making.) What kinds of factors will your model produce? Could it predict state change or tipping points in Earth's condition? Could it provide warning about global consequences of changing local conditions? Could it inform decision makers on important policies? Do you take into account the human elements in your measures and network properties?

Requirement 3: One of the powerful elements of using network modeling is the ability to analyze the network structure. Can network properties help identify critical nodes or relationships in your model? If so, perform such analysis. How sensitive is your model to missing links or changing relationships? Does your model use feedback loops or take into account uncertainties? What are the data collection issues? Does your model react to various government policies and could it thus help inform planning?

Requirement 4: Write a 20-page report (summary sheet does not count in the 20 pages) that explains your model and its potential. Be sure to detail the strengths and weaknesses of the model. Your supervisor will use your report as a major theme in the upcoming workshop and, if it is appropriate and insightful to planetary health modeling, will ask you to present at the upcoming workshop. Good luck in your network modeling work!

Potentially helpful references include:

Anthony D. Barnosky, Elizabeth A. Hadly, Jordi Bascompte, Eric L. Berlow, James H. Brown, Mikael Fortelius, Wayne M. Getz, John Harte, Alan Hastings, Pablo A. Marquet, Neo D. Martinez, Arne Mooers, Peter Roopnarine, Geerat Vermeij, John W. Williams, Rosemary Gillespie, Justin Kitzes, Charles Marshall, Nicholas Matzke, David P. Mindell, Eloy Revilla, Adam B. Smith. "Approaching a state shift in Earth's biosphere,". *Nature*, 2012; 486 (7401): 52 DOI: 10.1038/nature11018

Donella Meadows, Jorgen Randers, and Dennis Meadows. *Limits to Growth: The 30-year update*, 2004.

Robert Watson and A.Hamid Zakri. *UN Millennium Ecosystem Assessment Synthesis Report*, United Nations Report, 2005**.**

University of California - Berkeley. "Evidence of impending tipping point for Earth." *ScienceDaily*, 6 Jun. 2012. Web. 22 Oct. 2012.