

**2016 MCM  
Problem A  
A Hot Bath**

A person fills a bathtub with hot water from a single faucet and settles into the bathtub to cleanse and relax. Unfortunately, the bathtub is not a spa-style tub with a secondary heating system and circulating jets, but rather a simple water containment vessel. After a while, the bath gets noticeably cooler, so the person adds a constant trickle of hot water from the faucet to reheat the bathing water. The bathtub is designed in such a way that when the tub reaches its capacity, excess water escapes through an overflow drain.

Develop a model of the temperature of the bathtub water in space and time to determine the best strategy the person in the bathtub can adopt to keep the temperature even throughout the bathtub and as close as possible to the initial temperature without wasting too much water.

Use your model to determine the extent to which your strategy depends upon the shape and volume of the tub, the shape/volume/temperature of the person in the bathtub, and the motions made by the person in the bathtub. If the person used a bubble bath additive while initially filling the bathtub to assist in cleansing, how would this affect your model's results?

In addition to the required one-page summary for your MCM submission, your report must include a one-page non-technical explanation for users of the bathtub that describes your strategy while explaining why it is so difficult to get an evenly maintained temperature throughout the bath water.

**2016 MCM**  
**Problem B**  
**Space Junk**

The amount of small debris in orbit around earth has been a growing concern. It is estimated that more than 500,000 pieces of space debris, also called orbital debris, are currently being tracked as potential hazards to space craft. The issue itself became more widely discussed in the news media when the Russian satellite Kosmos-2251 and the USA satellite Iridium-33 collided on 10 February, 2009.

A number of methods to remove the debris have been proposed. These methods include small, space-based water jets and high energy lasers used to target specific pieces of debris and large satellites designed to sweep up the debris, among others. The debris ranges in size and mass from paint flakes to abandoned satellites. The debris' high velocity orbits make capture difficult.

Develop a time-dependent model to determine the best alternative or combination of alternatives that a private firm could adopt as a commercial opportunity to address the space debris problem. Your model should include quantitative and/or qualitative estimates of costs, risks, benefits, as well as other important factors. Your model should be able to assess independent alternatives as well as combinations of alternatives and be able to explore a variety of important "What if?" scenarios.

Using your model, determine whether an economically attractive opportunity exists or no such opportunity is possible. If a viable commercial opportunity exists as an alternative solution, provide a comparison of the different options for removing debris, and include a specific recommendation as to how the debris should be removed. If no such opportunity is possible, then provide innovative alternatives for avoiding collisions.

In addition to the required one-page summary for your MCM submission, your report must include a two-page Executive Summary that describes the options considered and major modeling results, and provides a recommendation for a particular action, combination of actions, or no action, as appropriate from your work. The Executive Summary should be written for high level policy makers and news media analysts who do not have a technical background.

### The Goodgrant Challenge

The Goodgrant Foundation is a charitable organization that wants to help improve educational performance of undergraduates attending colleges and universities in the United States. To do this, the foundation intends to donate a total of \$100,000,000 (US100 million) to an appropriate group of schools per year, for five years, starting July 2016. In doing so, they do not want to duplicate the investments and focus of other large grant organizations such as the Gates Foundation and Lumina Foundation.

Your team has been asked by the Goodgrant Foundation to develop a model to determine an optimal investment strategy that identifies the schools, the investment amount per school, the return on that investment, and the time duration that the organization's money should be provided to have the highest likelihood of producing a strong positive effect on student performance. This strategy should contain a 1 to N optimized and prioritized candidate list of schools you are recommending for investment based on each candidate school's demonstrated potential for effective use of private funding, and an estimated return on investment (ROI) defined in a manner appropriate for a charitable organization such as the Goodgrant Foundation.

To assist your effort, the attached data file (**ProblemCDATA.zip**) contains information extracted from the U.S. National Center on Education Statistics ([www.nces.ed.gov/ipeds](http://www.nces.ed.gov/ipeds)), which maintains an extensive database of survey information on nearly all post-secondary colleges and universities in the United States, and the College Scorecard data set (<https://collegescorecard.ed.gov>) which contains various institutional performance data. Your model and subsequent strategy must be based on some meaningful and defensible subset of these two data sets.

In addition to the required one-page summary for your MCM submission, your report must include a letter to the Chief Financial Officer (CFO) of the Goodgrant Foundation, Mr. Alpha Chiang, that describes the optimal investment strategy, your modeling approach and major results, and a brief discussion of your proposed concept of a return-on-investment (ROI) that the Goodgrant Foundation should adopt for assessing the 2016 donation(s) and future philanthropic educational investments within the United States. This letter should be no more than two pages in length.

Note: When submitting your final electronic solution **DO NOT** include any database files. The only thing that should be submitted is your electronic (Word or PDF) solution.

The ProblemCDATA.zip data file contains:

- Problem C - IPEDS UID for Potential Candidate Schools.xlsx
- Problem C - Most Recent Cohorts Data (Scorecard Elements).xlsx
- Problem C - CollegeScorecardDataDictionary-09-08-2015.xlsx
- IPEDS Variables for Data Selection.pdf

You can download the data (ProblemCDATA.zip) on the following websites:

<http://www.comap-math.com/mcm/ProblemCDATA.zip>  
<http://www.mathismore.net/mcm/ProblemCDATA.zip>  
<http://www.mathportals.com/mcm/ProblemCDATA.zip>  
<http://www.immchallenge.org/mcm/ProblemCDATA.zip>

**2016 ICM**

**Problem D**

**Measuring the Evolution and Influence in Society's Information Networks**

Information is spread quickly in today's tech-connected communications network; sometimes it is due to the inherent value of the information itself, and other times it is due to the information finding its way to influential or central network nodes that accelerate its spread through social media. While content has varied -- in the 1800s, news was more about local events (e.g., weddings, storms, deaths) rather than viral videos of cats or social lives of entertainers -- the prevailing premise is that this cultural characteristic to share information (both serious and trivial) has always been there. However, the flow of information has never been as easy or wide-ranging as it is today, allowing news of various levels of importance to spread quickly across the globe in our tech connected world. By taking a historical perspective of flow of information relative to inherent value of information, the Institute of Communication Media (ICM) seeks to understand the evolution of the methodology, purpose, and functionality of society's networks. Specifically, your team, as part of ICM's Information Analytics Division, has been assigned to analyze the relationship between speed/flow of information vs inherent value of information based on consideration of 5 periods: in the 1870s, when newspapers were delivered by trains and stories were passed by telegraph; in the 1920s, when radios became a more common household item; in the 1970s, when televisions were in most homes; in the 1990s, when households began connecting to the early internet; in the 2010s, when we can carry a connection to the world on our phones. Your supervisor reminds you to be sure to report the assumptions you make and the data you use to build your models.

Your specific tasks are:

- (a) Develop one or more model(s) that allow(s) you to explore the flow of information and filter or find what qualifies as news.
- (b) Validate your model's reliability by using data from the past and the prediction capability of your model to predict the information communication situation for today and compare that with today's reality.
- (c) Use your model to predict the communication networks' relationships and capacities around the year 2050.
- (d) Use the theories and concepts of information influence on networks to model how public interest and opinion can be changed through information networks in today's connected world.
- (e) Determine how information value, people's initial opinion and bias, form of the message or its source, and the topology or strength of the information network in a region, country, or worldwide could be used to spread information and influence public opinion.

**Possible Data Sources:**

As you develop your model and prepare to test it, you will need to assemble a collection of data. Below are just some examples of the types of data you may find useful in this project. Depending on your exact model, some types of data may be very important and others may be entirely irrelevant. In addition to the sample sources provided below, you might want to consider a few important world events throughout history -- if some recent big news events, such as the rumors of country-turned-pop singer Taylor Swift's possible engagement had instead happened in 1860, what percentage of the population would know about it and how quickly; likewise, if an important person was assassinated today, how would that news spread? How might that compare to the news of US President Abraham Lincoln's assassination?

更多竞赛信息参见微信公众号：考研竞赛数学(ID:xwmath)

**Sample Circulation Data and Media Availability:**

[http://media-cmi.com/downloads/Sixty\\_Years\\_Daily\\_Newspaper\\_Circulation\\_Trends\\_050611.pdf](http://media-cmi.com/downloads/Sixty_Years_Daily_Newspaper_Circulation_Trends_050611.pdf)

<http://news.bbc.co.uk/2/hi/technology/8552410.stm>

<http://www.gov.scot/Publications/2006/01/12104731/6>

<http://www.technologyreview.com/news/427787/are-smart-phones-spreading-faster-than-any-technology-in-human-history/>

<http://newsroom.fb.com/content/default.aspx?NewsAreaId=22>

<http://www.poynter.org/news/mediawire/189819/pew-tv-viewing-habit-grays-as-digital-news-consumption-tops-print-radio/>

<http://www.people-press.org/2012/09/27/section-1-watching-reading-and-listening-to-the-news-3/>

<http://theconversation.com/hard-evidence-how-does-false-information-spread-online-25567>

**Historical Perspectives of News and Media:**

<https://www.quora.com/How-did-news-get-around-the-world-before-the-invention-of-newspapers-and-other-media>

<http://2012books.lardbucket.org/books/a-primer-on-communication-studies/s15-media-technology-and-communicat.html>

<http://firstmonday.org/article/view/885/794>

Richard Campbell, Christopher R. Martin, and Bettina Fabos, *Media & Culture: An Introduction to Mass Communication*, 5th ed. (Boston, MA: Bedford St. Martin's, 2007)

Marshall T. Poe, *A History of Communications: Media and Society from the Evolution of Speech to the Internet* (New York: Cambridge, 2011)

Shirley Biagi, *Media/Impact: An Introduction to Mass Media* (Boston, MA: Wadsworth, 2007)

**Your ICM submission should consist of a 1 page Summary Sheet and your solution cannot exceed 20 pages for a maximum of 21 pages. Note: The appendix and references do not count toward the 20 page limit.**

**2016 ICM**

**Problem E**

**Are we heading towards a thirsty planet?**

Will the world run out of clean water? According to the United Nations, 1.6 billion people (one quarter of the world's population) experience water scarcity. Water use has been growing at twice the rate of population over the last century. Humans require water resources for industrial, agricultural, and residential purposes. There are two primary causes for water scarcity: physical scarcity and economic scarcity. Physical scarcity is where there is inadequate water in a region to meet demand. Economic scarcity is where water exists but poor management and lack of infrastructure limits the availability of clean water. Many scientists see this water scarcity problem becoming exacerbated with climate change and population increase. The fact that water use is increasing at twice the rate of population suggests that there is another cause of scarcity – is it increasing rates of personal consumption, or increasing rates of industrial consumption, or increasing pollution which depletes the supply of fresh water, or what? \*\*

Is it possible to provide clean fresh water to all? The supply of water must take into account the physical availability of water (e.g., natural water source, technological advances such as desalination plants or rainwater harvesting techniques). Understanding water availability is an inherently interdisciplinary problem. One must not only understand the environmental constraints on water supply, but also how social factors influence availability and distribution of clean water. For example, lack of adequate sanitation can cause a decrease in water quality. Human population increase also places increased burden on the water supply within a region. When analyzing issues of water scarcity, the following types of questions must be considered. How have humans historically exacerbated or alleviated water scarcity? What are the geological, topographical, and ecological reasons for water scarcity, and how can we accurately predict future water availability? What is the potential for new or alternate sources of water (for example, desalinization plants, water harvesting techniques or undiscovered aquifers)? What are the demographic and health related problems tied to water scarcity?

**Problem Statement**

The International Clean water Movement (ICM) wants your team to help them solve the world's water problems. Can you help improve access to clean, fresh water?

Task 1: Develop a model that provides a measure of the ability of a region to provide clean water to meet the needs of its population. You may need to consider the dynamic nature of the factors that affect both supply and demand in your modeling process.

Task 2: Using the UN water scarcity map (<http://www.unep.org/dewa/vitalwater/jpg/0222-waterstress-overuse-EN.jpg>) pick one country or region where water is either heavily or moderately overloaded. Explain why and how water is scarce in that region. Make sure to explain both the social and environmental drivers by addressing physical and/or economic scarcity.

Task 3: In your chosen region from Task 2, use your model from Task 1 to show what the water situation will be in 15 years. How does this situation impact the lives of citizens of this region? Be sure to incorporate the environmental drivers' effects on the model components.

Task 4: For your chosen region, design an intervention plan taking all the drivers of water scarcity into account. Any intervention plan will inevitably impact the surrounding areas, as well

as the entire water ecosystem. Discuss this impact and the overall strengths and weaknesses of the plan in this larger context. How does your plan mitigate water scarcity?

Task 5: Use the intervention you designed in Task 4 and your model to project water availability into the future. Can your chosen region become less susceptible to water scarcity? Will water become a critical issue in the future? If so, when will this scarcity occur?

Task 6: Write a 20-page report (the one-page summary sheet does not count in the 20 pages) that explains your model, water scarcity in your region with no intervention, your intervention, and the effect of your intervention on your region's and the surrounding area's water availability. Be sure to detail the strengths and weaknesses of your model. The ICM will use your report to help with its mission to produce plans to provide access to clean water for all citizens of the world. Good luck in your modeling work!

### Possible Resources

An Overview of the State of the World's Fresh and Marine Waters. 2nd Edition, 2008.  
(<http://www.unep.org/dewa/vitalwater/index.html>).

The World's Water: Information on the World's Freshwater Resources.  
(<http://worldwater.org>).

AQUASTAT. Food and Agriculture Organization of the United Nations. FAO Water Resources. ([http://www.fao.org/nr/water/aquastat/water\\_res/index.stm](http://www.fao.org/nr/water/aquastat/water_res/index.stm)).

The State of the World's Land and Water Resources for food and agriculture. 2011.  
(<http://www.fao.org/docrep/017/i1688e/i1688e00.htm>).

GrowingBlue: Water. Economics. Life. (<http://growingblue.com>).

World Resources Institute. [www.wri.org](http://www.wri.org).

\*\*Note that the 2013 Mathematical Competition in Modeling (Problem B) and the 2009 High School Modeling Competition in Modeling (Problem A) were related to modeling different aspects of water scarcity.

**Your ICM submission should consist of a 1 page Summary Sheet and your solution cannot exceed 20 pages for a maximum of 21 pages. Note: The appendix and references do not count toward the 20 page limit.**

**2016 ICM**  
**Problem F**  
**Modeling Refugee Immigration Policies**

With hundreds of thousands of refugees moving across Europe and more arriving each day, considerable attention has been given to refugee integration policies and practices in many countries and regions. History has shown us that mass fleeing of populations occur as a result of major political and social unrest and warfare. These crises bring a set of unique challenges that must be managed carefully through effective policies. Events in the Middle East have caused a massive surge of refugees emigrating from the Middle East into safe haven countries in Europe and parts of Asia, often moving through the Mediterranean and into countries such as Turkey, Hungary, Germany, France, and UK. By the end of October 2015, European countries had received over 715,000 asylum applications from refugees. Hungary topped the charts with nearly 1,450 applications per 100,000 inhabitants, but with only a small percentage of those requests granted (32% in 2014), leaving close to a thousand refugees homeless per every 100K residents of the country. Europe has established a quota system where each country has agreed to take in a particular number of refugees, with the majority of the resettlement burden lying with France and Germany.

The refugees travel multiple routes – from the Middle East through (1) West Mediterranean, (2) Central Mediterranean, (3) Eastern Mediterranean, (4) West Balkans, (5) Eastern Borders, and (6) Albania to Greece (See these routes mapped out in <http://www.bbc.com/news/world-europe-34131911>). Each route has different levels of safety and accessibility, with the most popular route being Eastern Mediterranean and the most dangerous, Central Mediterranean. Countries that have been burdened the most are concerned about their capacity to provide resources for the refugees such as food, water, shelter, and healthcare. There are numerous factors that determine how the refugees decide to move through the region. Transportation availability, safety of routes and access to basic needs at destination are considered by each individual or family in this enormous migration.

The UN has asked your team, the ICM-RUN (**RefU**gee **aN**alytics) to help develop a better understanding of the factors involved with facilitating the movement of refugees from their countries of origin into safe-haven countries.

**Your Specific Tasks:**

1. **Metrics of refugee crises.** Determine the specific factors which can either enable or inhibit the safe and efficient movement of refugees. There are attributes of the individuals themselves, the routes they must take, the types of transportation, the countries' capacity, including number of entry points and resources available to refugee population. This first task requires ICM-RUN to develop a set of measures and parameters and justify why they should be included in the analysis of this crisis.
2. **Flow of refugees.** Create a model of optimal refugee movement that would incorporate projected flows of refugees across the six travel routes mentioned in the problem, with consideration of transportation routes/accessibility, safety of route and countries' resource capacities. You can include different routes, different entry points, single or multiple entry points, and even different countries. Use the metrics that you established in Task 1 to determine the number of refugees, as well as the rate and point of entry necessary to accommodate their movement. Be sure to justify



any new elements you have added to the migration and explain the sensitivities of your model to these dynamics.

3. **Dynamics of the crisis.** Refugee conditions can change rapidly. Refugees seek basic necessities for themselves and their families in the midst of continuously changing political and cultural landscapes. In addition, the capacity to house, protect, and feed this moving population is dynamic in that the most desired destinations will reach maximum capacity the quickest, creating a cascade effect altering the parameters for the patterns of movement. Identify the environmental factors that change over time; and show how capacity can be incorporated into the model to account for these dynamic elements. What resources can be prepositioned and how should they be allocated in light of these dynamics? What resources need priority and how do you incorporate resource availability and flow in your model? Consider the role and resources of both government and non-government agencies (NGOs). How does the inclusion of NGO's change your model and strategy? Also consider the inclusion of other refugee destinations such as Canada, China, and the United States. Does your model work for these regions as well?
4. **Policy to support refugee model.** Now that you have a working model, ICM-RUN has been asked to attend a policy strategy meeting where your team is asked to write a report on your model and propose a set of policies that will support the optimal set of conditions ensuring the optimal migration pattern. Your UN commission has asked you to consider and prioritize the health and safety of refugees and of the local populations. You can include as many parameters and considerations as you see fit to help to inform the strategic policy plan, keeping in mind the laws and cultural constraints of the effected countries. Consider also the role and actions of non-governmental organizations (NGOs).
5. **Exogenous events.** In addition to endogenous systemic dynamics, exogenous events are also highly likely to occur and alter the situation parameters in these volatile environments, For example, a major terrorist attack in Paris, France has been linked to the Syrian refuge crisis, and has resulted in substantial shifts in the attitudes and policies of many European countries with respect to refugees. The event has also raised concerns among local populations. For example, Brussels, Belgium was placed in a lockdown after the Paris raids in attempts to capture possible terrorists.
  - a) What parameters of the model would likely shift or change completely in a major exogenous event?
  - b) What would be the cascading effects on the movement of refugees in neighboring countries?
  - c) How will the immigration policies that you recommend be designed to be resilient to these types of events?
6. **Scalability.** Using your model, expand the crisis to a larger scale – by a factor of 10. Are there features of your model that are not scalable to larger populations? What parameters in your model change or become irrelevant when the scope of the crisis increases dramatically? Do new parameters need to be added? How does this increase the time required to resolve refugee placement? If resolution of the refugee integration is significantly prolonged, what new issues might arise in maintaining the health and safety of the refugee and local populations? What is the threshold of time where these new considerations are in play? For example, what policies need to be in place to manage issues such as disease control, childbirth, and education?

**The Report:** The UN Commission on Refugees has asked your ICM-RUN team to provide them a 20-page report that considers the factors given in your tasks. Each team should also write a 1 page policy recommendation letter which will be read by the UN Secretary General and the Chief of Migration.

**Your ICM submission should consist of a 1 page Summary Sheet, a 1 page letter to the UN, and your solution (not to exceed 20 pages) for a maximum of 22 pages. Note: The appendix and references do not count toward the 22 page limit.**

The Commission has also provided you with some on-line references that may be helpful:

<http://www.bbc.com/news/world-europe-34131911>

<http://www.iom.int/>

<http://iusp2009.princeton.edu/papers/90854>

<http://www.unhcr.org/pages/49c3646c4d6.html>

[http://www.nytimes.com/2015/08/28/world/migrants-refugees-europe-syria.html?\\_r=0](http://www.nytimes.com/2015/08/28/world/migrants-refugees-europe-syria.html?_r=0)

<http://www.who.int/features/qa/88/en/>

<http://www.euro.who.int/en/health-topics/health-determinants/migration-and-health/migrant-health-in-the-european-region/migration-and-health-key-issues>

<https://www.icrc.org/en/war-and-law/protected-persons/refugees-displaced-persons>