SIG718 – Real World Analytics Endterm Assessment

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1. A garment factory produces shirts and pants for Kmart chain. The contract is such that quality control is done before shipping and all products supplied to Kmart satisfying the quality requirements would be accepted by the chain. The factory employs 20 workers in the cutting department, 50 workers in the sewing department, and 14 workers in the packaging department. The garment factory works 8 productive hours a day (no idle time during these 8 hours). There is a daily demand for at most 180 shirts. The demand for pants is unlimited. Each worker can participate only in one activity- the activity to which they are assigned. The table below gives the time requirements (in minutes) and profit per unit for the two garments.

	Amount (minutes) per operation			
	Cutting	Sewing	Packaging	Profit per unit($)
Shirts	40	40	20	10
Pants	20	100	20	8

a) Explain why a Linear Programming (LP) model would be suitable for this case study.

A Linear problem programming model is suited for this use case because it involves the optimization of a linear objective function with respect to various linear constraints. This problem in garment factory is all about allocating worker's time to maximize the profit while making shirts & pants.

Points which make this a Linear programming problem : -

Objective function & Constraints linearity- Here the objective function needs to be maximize in terms of profit & each shits & pant contributes to profit in linear terms. Profit per each shirt & pant is multiplied by the number of pants & shirts produced and this sum contribution constitutes the linear function. Now the constraints include the number of shirs/ pants produced and time spent on each activity, which are cutting, sewing & packaging are all related to number of workers assigned in each working department.

Here the goal is also to maximize the profit, which is our single objective .Linear programming helps in solving single objective problems

The number of pants / shirts produced cannot be negative. So, here the constraints are non-negative which are ideally handled by Linear programming model

The LP model involves decision variables, an objective function(profit), and a set of linear constraints that represent the resource limitations and demand requirements.

b) Formulate a LP model to help the factory management to determine the optimal daily production schedule that maximises the profit while satisfying all constraints.

In Order to maximize the profit for the garment factory while keeping the demand constraints, can be achieved using Linear programming problem.

For that , first we define

decision variables,

Objective function – needs to maximizer, here profit

Constraints –

Let

x - > number of shirts produced,   y -> number of pants produced

P -> Objective function (profit to be maximized)

Profit per shirt - > 10, profit on all shirts - > 10x

Profit per pant - > 8, profit on all pants - >8y

Total profit , Maximize P = 10x + 8y

Lets define constraints

Cutting time per shirt - > 40 mins, Cutting time on all shirts -> 40x

Cutting time per pant -> 20 mins, Cutting time on all pants -> 20y

Workers in Cutting department -> 20, total production time in factory -> 8 *60

40x + 20y <= 8 * 60 * 20

Similarly. For sewing & Packaging

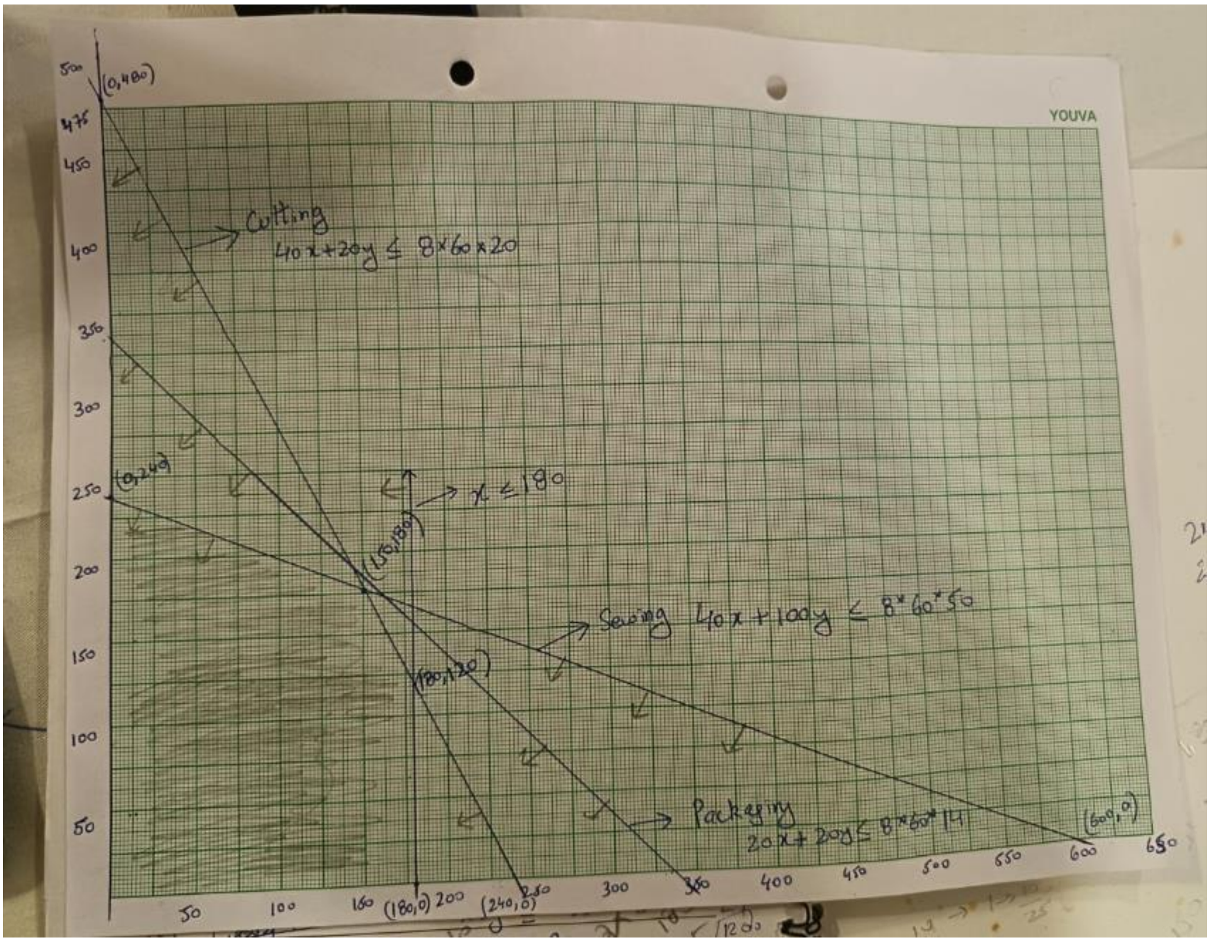
40x + 100y <= 8 * 60 * 50

20x + 20y <= 8 * 60 * 14

X <= 180 (for daily shirts demand)

x>=0, y>=0 (number of shirts / pants cannot be negative)

c) Use the graphical method to find the optimal solution. Show the feasible region and the optimal solution on the graph. Annotate all lines on your graph. What is the optimal daily profit for the factory?



Profit at (0,240) = 10 *0 + 8 * 240 = 0 + 1920 = 1920

Profit at (150,180) = 10*150 + 8*180 = 1500 + 1440 = 2940

Profit at (180,120) = 10*180 + 8 *120 = 1800 + 960 = 2760

Profit at (180,0) = 10*180 + 8 *0 = 1800 + 0 = 1800

So, the Optimal daily maximum profit is 2940 when 150 shirts and 180 pants are produced

d) Find the range for the profit ($) per shirt (if any) that can be obtained without affecting the optimal point of part (c).

In order to find the range for profit per shirt that does not affect optimal solution, sensitivity analysis should be done. Sensitivity analysis involves changing the profit per shirt while keeping the rest of constraints parameters constant and checking the optimal solution.

Let P be the profit per shirt.,

Profit becomes Z = P*x + 8*y

Consider a new profit per shirt P'. The new objective is Z' = P'*x + 8*y

The sensitivity range for P' can be found by solving the following linear programming problems:

Find the optimal solution for P = 10

Find the optimal solution for P' where P' <=10,observe changes

Find the optima solution for P' where P' >= 10,observe changes

If the optimal solution does not change , then the range for the profit per shirt that does not affect the optimal solution is -infinity < P' < + infinity

If there is a change in the optimal solution, then there is a specific range for P' within which the optimal solution remains the same

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2. A factory makes three products called Bloom, Amber, and Leaf, from three materials containing Cotton, Wool and Nylon. The following table provides details on the sales price, production cost and purchase cost per ton of products and materials respectively.

	Sales price	Production cost		Purchase price
Bloom	\$60	\$5	Cotton	\$40
Amber	\$55	\$4	Wool	\$45
Leaf	\$60	\$5	Nylon	\$30

The maximal demand (in tons) for each product, the minimum cotton and wool proportion in each product are as follows:

	Demand	min Cotton proportion	min Wool proportion
Bloom	4200	50%	40%
Amber	3200	60%	40%
Leaf	3500	50%	30%

a) Formulate an LP model for the factory that maximises the profit, while satisfying the demand and the cotton and wool proportion constraints.

Let

x be the amount of Bloom produced in tons

y be the amount of Amber produced in tons

z be the amount of Leaf produced in tons

Sales price for Bloom,Amber and Leaf be 60x,55y and 60z respectively

Production cost for Bloom, Amber and Leaf be 5x,4y and 5z respectively

The objective is to maximize the total profit Z : sales – production

Z = (60x + 55y + 60z) – (5x + 4y + 5z)

Demand Constraints:

x <= 4200

y <=3200

z <= 3500

Materials constraints (wool & Cotton)

0.5x + 0.6y + 0.5z >= 0.4*(x + y + z)

0.4x + 0.4y + 0.3z >= 0.4* (x + y +z)

0.5x >= 0.5*(x + y + z)

0.6y >= 0.4*(x + y + z)

0.4x + 0.4y >= 0.4* (x + y +z)

x >=0, y >=0, z >=0

This LP model maximizes the profit while ensuring that the production quantities meet the demand and adhere to the specified cotton and wool proportion constraints.

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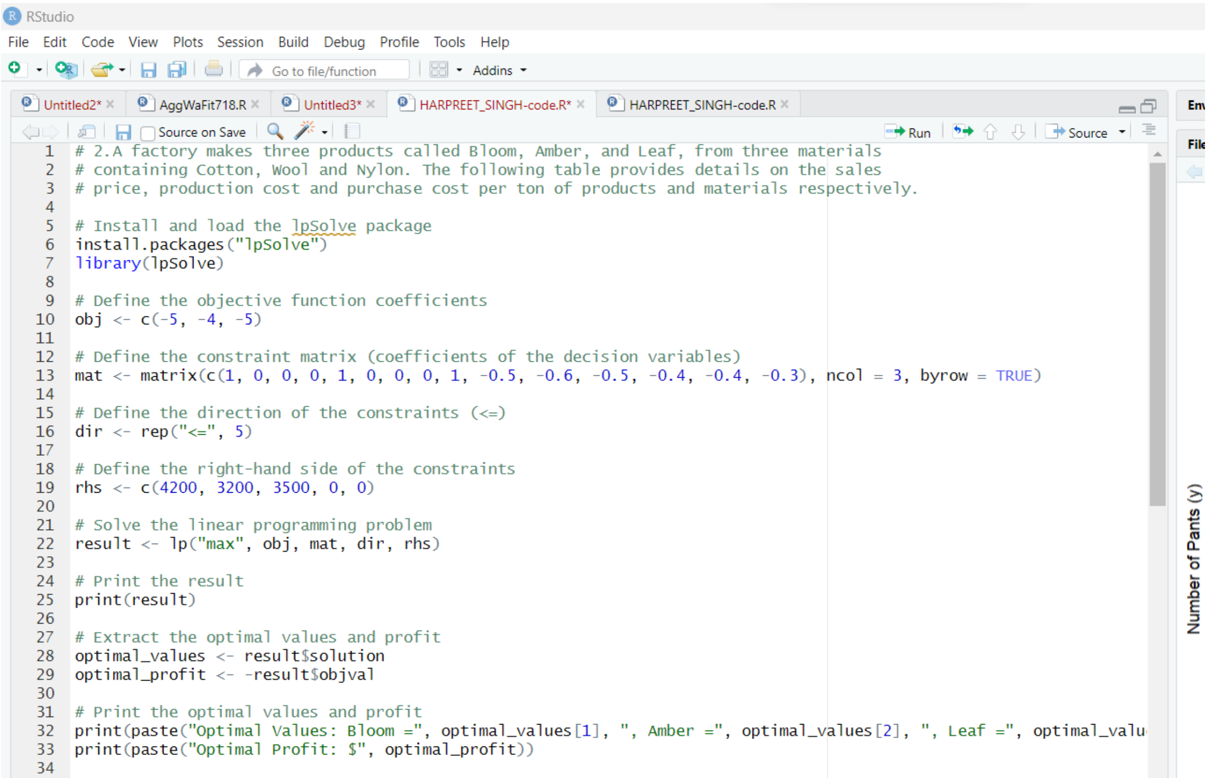
b) Solve the model using R/R Studio. Find the optimal profit and optimal values of the decision variables.



```r
# 2.A factory makes three products called Bloom, Amber, and Leaf, from three materials
# containing Cotton, Wool and Nylon. The following table provides details on the sales
# price, production cost and purchase cost per ton of products and materials respectively.

# Install and load the lpSolve package
install.packages("lpSolve")
library(lpSolve)

# Define the objective function coefficients
obj <- c(-5, -4, -5)

# Define the constraint matrix (coefficients of the decision variables)
mat <- matrix(c(1, 0, 0, 0, 1, 0, 0, 0, 1, -0.5, -0.6, -0.5, -0.4, -0.4, -0.3), ncol = 3, byrow = TRUE)

# Define the direction of the constraints (<=)
dir <- rep("<=", 5)

# Define the right-hand side of the constraints
rhs <- c(4200, 3200, 3500, 0, 0)

# Solve the linear programming problem
result <- lp("max", obj, mat, dir, rhs)

# Print the result
print(result)

# Extract the optimal values and profit
optimal_values <- result$solution
optimal_profit <- -result$objval

# Print the optimal values and profit
print(paste("Optimal Values: Bloom =", optimal_values[1], ", Amber =", optimal_values[2], ", Leaf =", optimal_valu
print(paste("Optimal Profit: $", optimal_profit))
```

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3. Two construction companies, Giant and Sky, bid for the right to build in a field. The possible bids are \$ 10 Million, \$ 20 Million, \$ 30 Million, \$ 35 Million and \$ 40 Million. The winner is the company with the higher bid.

The two companies decide that in the case of a tie (equal bids), Giant is the winner and will get the field.

Giant has ordered a survey and, based on the report from the survey, concludes that getting the field for more than \$ 35 Million is as bad as not getting it (assume loss), except in case of a tie (assume win). Sky is not aware of this survey.

(a) State reasons why/how this game can be described as a two-players-zero-sum game

This game can be described as a two-player zero-sum game because the total gain or loss in the game is fixed. In a zero-sum game, the interests of the players are directly opposed to each other, and any gain by one player is exactly balanced by an equivalent loss by the other player.

The players are two players :

Giant & Sky – the two construction companies bidding for the right to build in the field.

The total value is fixed. The possible bids are \$10 million, \$20 million, \$30 million, \$35 million, and \$40 million. The outcome of the game is the allocation of the field, and the gain of one player (the winning bid) is exactly offset by the loss of the other player (the losing bid). The total monetary value remains constant, making it a zero-sum game.

The interests of Giant and Sky are directly opposed to each other. Each player aims to win the field, and the winner takes the entire value of the bid, while the loser gets nothing.

The gains and losses are balanced and advantage of one player is at the disadvantage of the other player

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(b) Considering all possible combinations of bids, formulate the payoff matrix for the game.

Below is the payoff matrix based on below conditions provided.

1. Giant and sky bid below 35 million, the payoff is the difference between the bids
2. Giant and Sky bid the same amount, Giant wins, payoff is positive
3. Giant ids more than sky, but bids are 35M or higher , Giant loses and payoff is negative

Rows represent Giant’s possible bids

Columns represent sky’s possible bids

	Sky: 10M	20M	30 M	35M	40M
Giant: 10M ($)	0	-10	-20	-25	-30
20M	10	0	-10	-15	-20
30M	20	10	0	-5	-10
35M	-25	-15	-5	0	5
40M	-30	-20	-10	-5	0

(c) Explain what is a saddle point. Verify: does the game have a saddle point?

A saddle point in the context of game theory refers to a specific cell in the payoff matrix of a two-player zero-sum game. It is a point where the maximum value (among minimum values in a row) coincides with the minimum value(among the maximum values in a column)in the corresponding column.

Date: ___/___/___

	Sky: 10M	20M	30 M	35M	40M	Min rows
Giant: 10M ($)	0	-10	-20	-25	-30	-30
20M	10	0	-10	-15	-20	-20
30M	20	10	0	-5	-10	-10
35M	-25	-15	-5	0	5	-25
40M	-30	-20	-10	-5	0	-30
Max columns	20	10	0	0	5	

Here we can see that there is no cell where maximum value in rows is equal to the minimum value in its column. Therefore there is no saddle point in the game

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(d) Construct a linear programming model for Company Sky in this game.

Here, Let us define S for Sky’s bid

Decision variable S: bid amount of Sky

Sky wants to minimize the amount it bids, so objective function is the bid amount,

Minimize Z = S

Sky’s bid amount should be less than or equal to 40M, the maximum bid amount:

S <= 40

Sky's bid amount should be greater than or equal to 10M, the minimum bid amount:

S >= 10

This model represents Sky's objective to minimize its bid amount while ensuring it is within the allowed bidding range. The optimal solution to this linear programming model will provide the optimal bid amount for Sky.

(e) Produce an appropriate code to solve the linear programming model in part (d).

```r
# 3.Two construction companies, Giant and Sky, bid for the right to build in a field. The
# possible bids are $ 10 Million, $ 20 Million, $ 30 Million, $ 35 Million and $ 40 Million.
# The winner is the company with the higher bid.
# The two companies decide that in the case of a tie (equal bids), Giant is the winner and
# will get the field.
# Giant has ordered a survey and, based on the report from the survey, concludes that
# getting the field for more than $ 35 Million is as bad as not getting it (assume loss),
# except in case of a tie (assume win). Sky is not aware of this survey.

# Install the lpSolve package if not installed
if (!requireNamespace("lpSolve", quietly = TRUE)) {
  install.packages("lpSolve")
}
# Load the lpSolve library
library(lpSolve)

# Objective function coefficients
obj_coef <- c(1)

# Inequality constraint matrix
mat <- matrix(c(1), nrow = 1)

# Inequality constraint right-hand side
rhs <- c(40)

# Set the direction of inequalities (1 for <=)
direction <- "L"

# Solve the linear programming problem
lp_solution <- lp("min", obj_coef, mat, direction, rhs)

# Display the results
print(lp_solution)

# Extract the optimal bid amount for Sky
optimal_bid <- lp_solution$solution

cat("Optimal Bid Amount for Sky:", optimal_bid, "\n")
```

References

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