NYUAD Hackathon

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April 2024

1 Formulation of the problem as a QUBO

We are going to model the social media feed problem as a graph. Consider s the root user, who consults his feed. The closest users in the app will be those that he/she follows (depth 1). The next closest users will be those followed by the ones he/she follows (depth 2). We can model this as a binary tree:

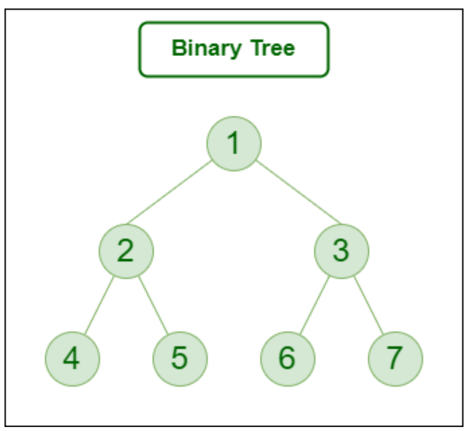

Figure 1: Representation of network from the point of view of the user (1)

With the Haq system we are able to compute a trustedness score for each user (I elaborate on that later). This allows us to assign a weight to each user, turning the binary tree into a weighted binary tree/graph. Now we consider that at every step we take from a node, each edge will carry a weight p, for step p. So the first step from the root node has weight 1, the second step has weight 2... etc. Now we would like to make a path that passes through each node only once, which represents the optimal order in which the feed will be displayed. This will obviously depend on the Haq score of the users, prioritising that the higher Haq and closer users come up first.

Problem: Find the path that starts at the user, and that crosses every node (only once), in a way such that it minimizes the sum of weight x step number.

Mathematically, considering that the network is a graph $G = (E, V)$, and that the total number of edges is N:

$$\min \sum_{v \in V} \sum_{p \geq 1}^{(N-1)} p h_v x_{v,p}$$

Where p is the step number (from the root node), h_v is the Haq score of the user corresponding to node v, and $x_{v,p}$ is a binary variable which is 1 if node v is in position p in the path from $x_{s,0}$.

Now, after a lot of thinking with Ahmad we wrote the following form for the QUBO hamiltonian, OMG!

$$\mathcal{H} = \mathcal{H}_A + \mathcal{H}_B + \mathcal{H}_C + \mathcal{H}_D$$

$$\mathcal{H}_D = D \sum_{v \in V} \sum_{p=1}^{N-1} p h_v x_{v,p}$$

$$\mathcal{H}_A = A_1 (1 - \sum_{v \in V} x_{v,0})^2 + A_2 \sum_{v \in V} (1 - \sum_{p=1}^{N-1} x_{v,p})^2$$

$$\mathcal{H}_{\mathcal{B}} = B_1 \sum_{v \in V} \sum_{p=1}^{(N-1)} (x_{v,p} - \sum_{u:(u,v) \in E} x'_{uv,i})^2$$

$$\mathcal{H}_C = C \sum_{(u,v) \in E} \sum_{p=1}^{N-1} \left[x'_{uv,p} (2 - x_{u,p-1} - x_{v,p}) + x'_{vu,p} (2 - x_{v,p-1} - x_{u,p}) \right]$$

Where we define a variable $x'_{uv,p}$ as 1 if edge (u, v) is part of the path and $x_{u,i-1}, x_{v,i} = 1$. The first term is what we want to minimize, and the others come from the constraints. The second and third terms enforce that there is only one root and that every node is accessed only once. The last terms enforce that the structure of the graph is respected, so we explore first the nearest neighbour terms.