Kabul University
Faculty of CS
Dep of Software Engineering

A* (A star) Search

By Haqmal Noori

October 8 / 2024

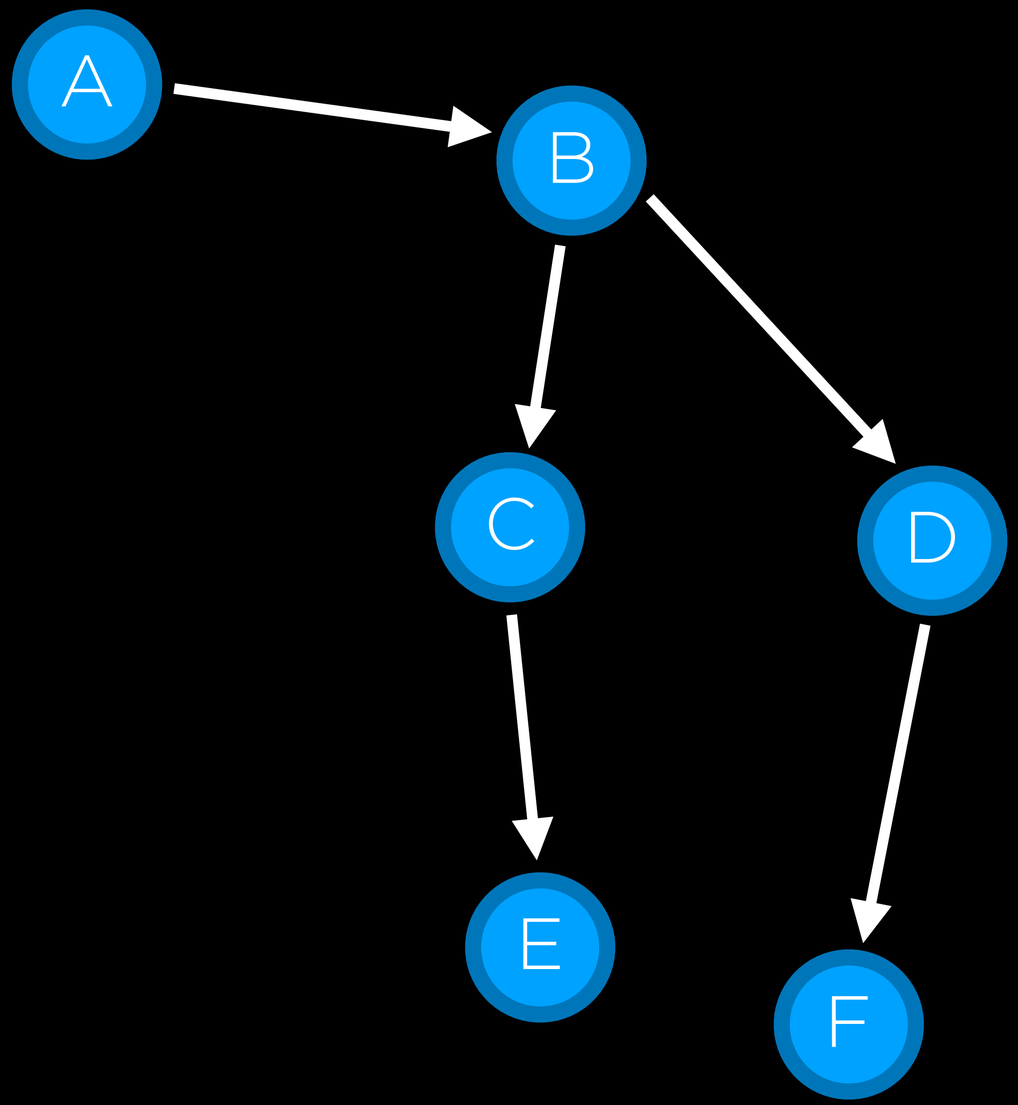
stack

last-in first-out data type

Find a path from A to E.

Frontier

Explored Set

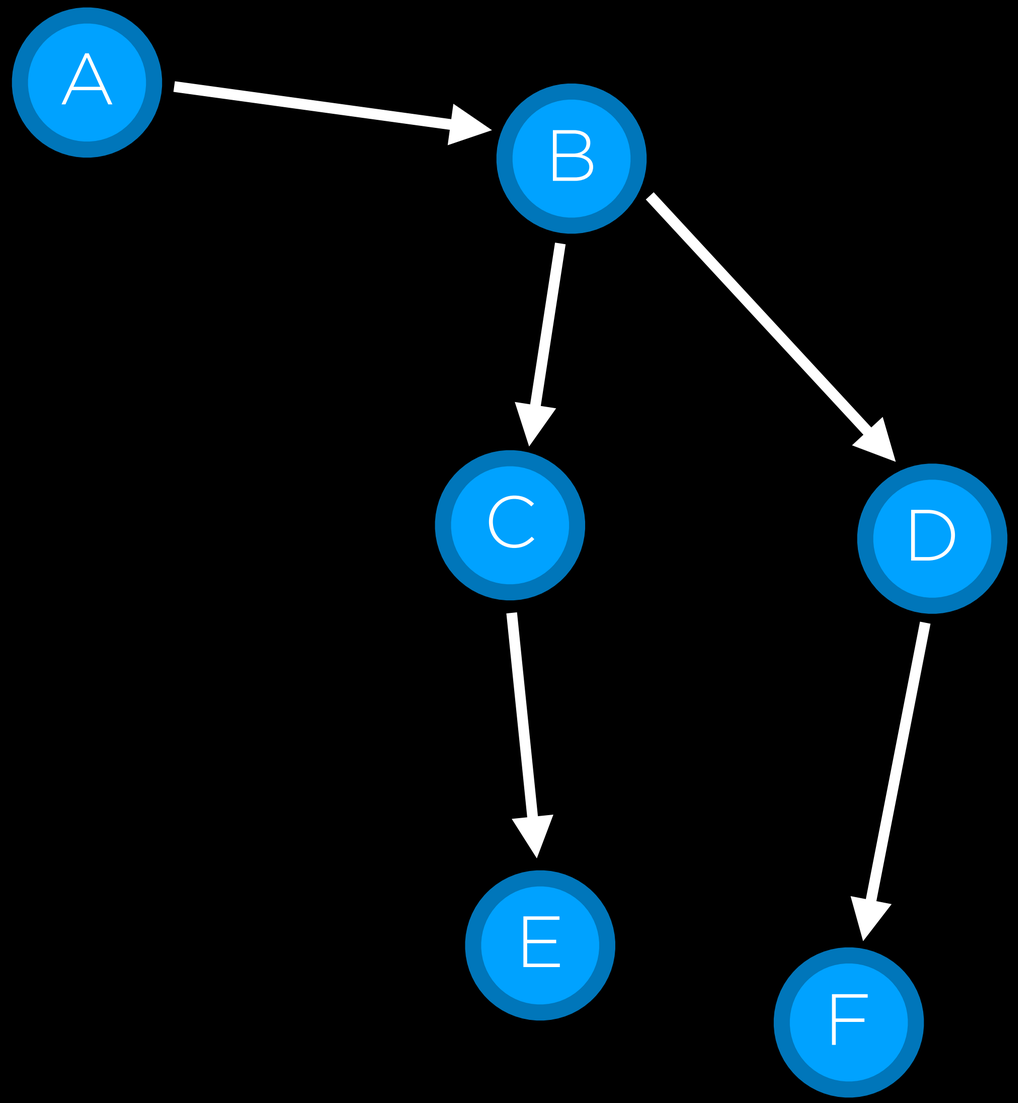


Find a path from A to E.

Frontier

A

Explored Set

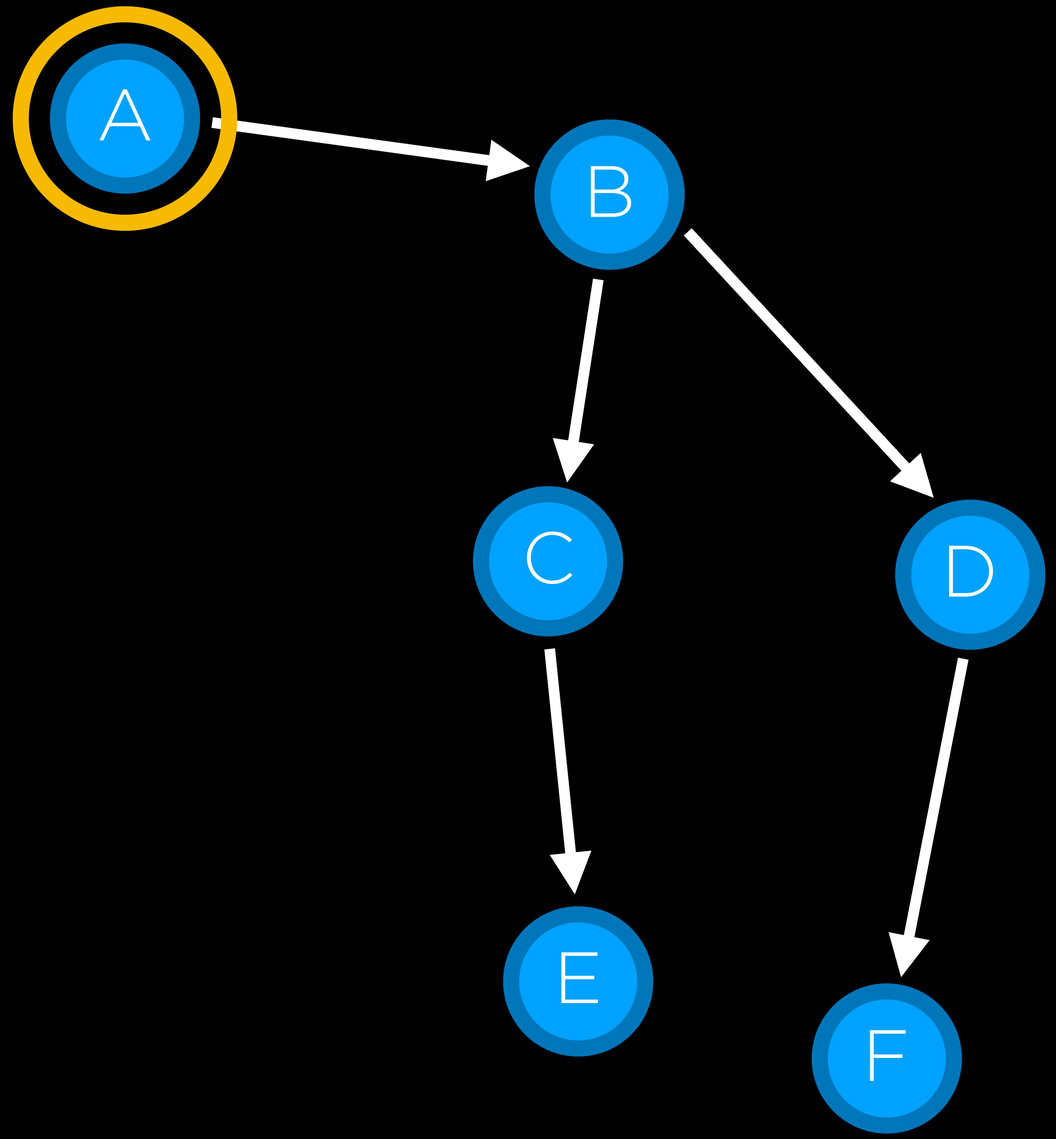


Find a path from A to E.

Frontier

Explored Set

A



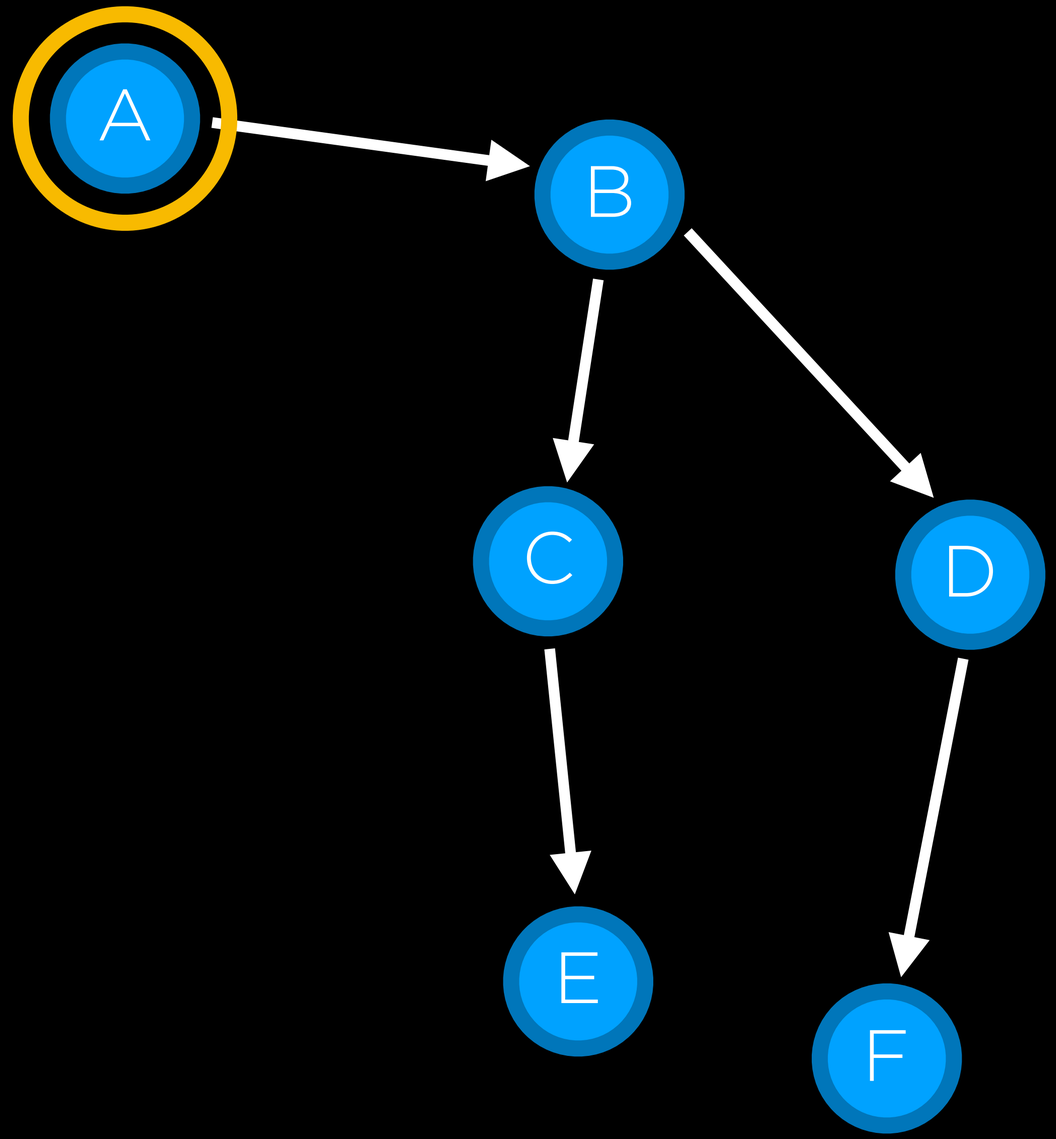
Find a path from A to E.

Frontier

B

Explored Set

A

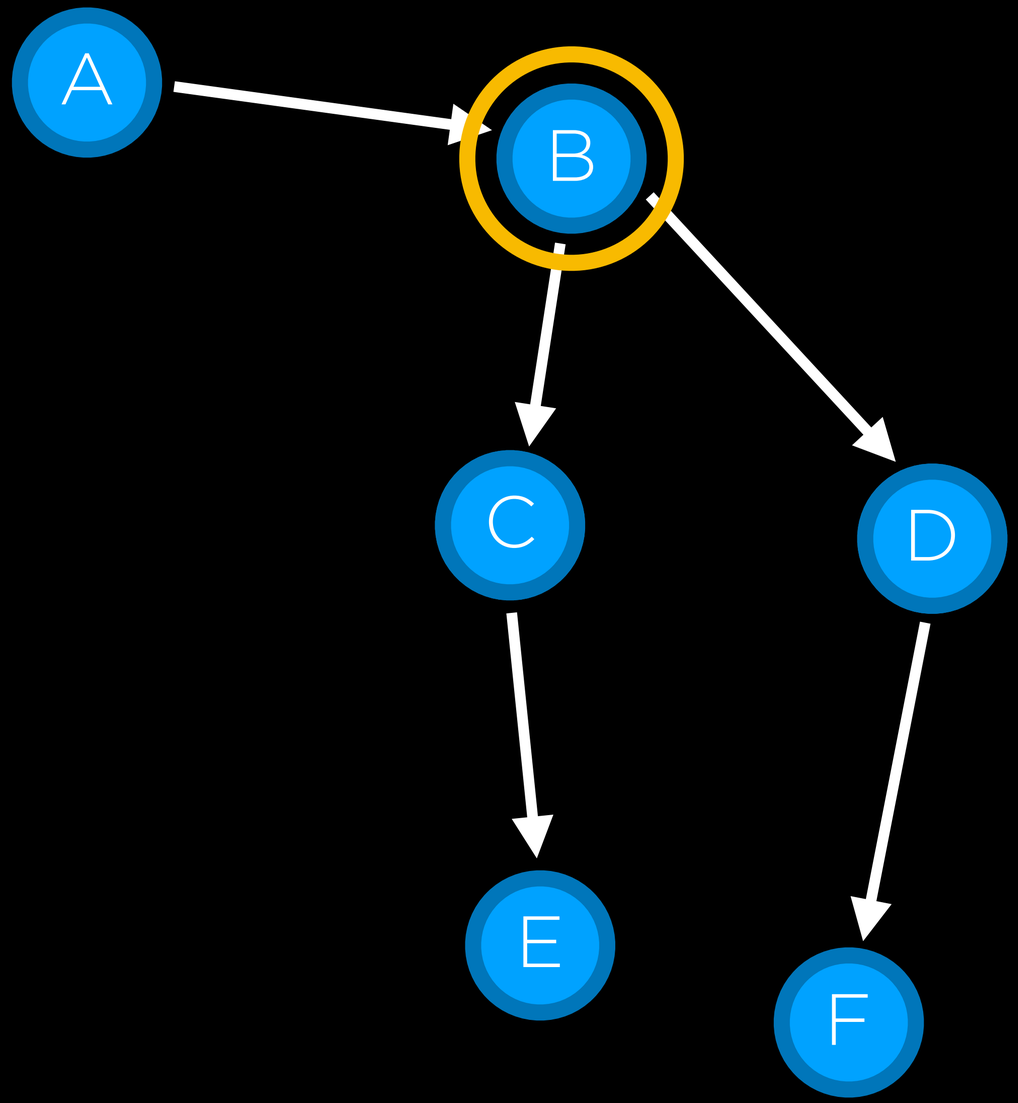


Find a path from A to E.

Frontier

Explored Set

A B



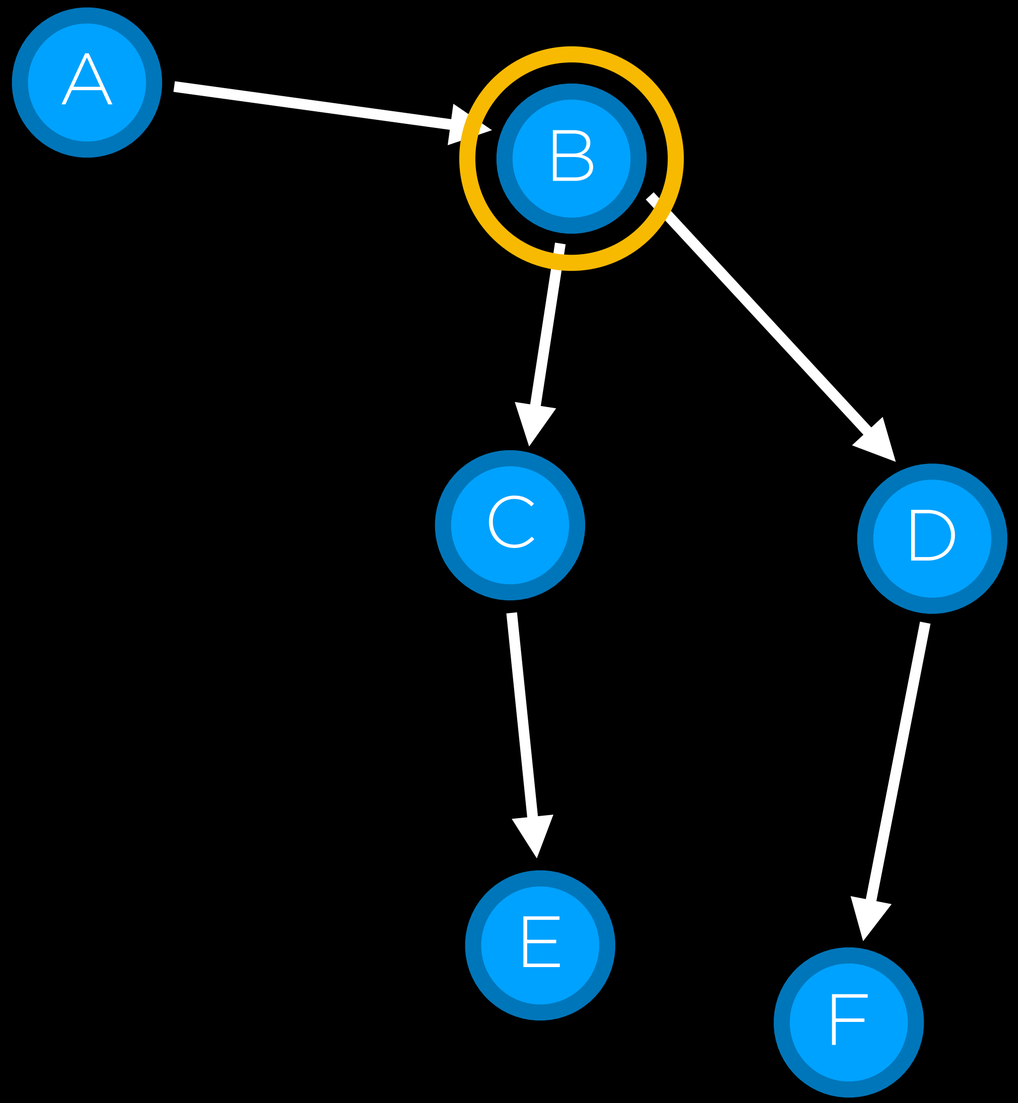
Find a path from A to E.

Frontier

C D

Explored Set

A B



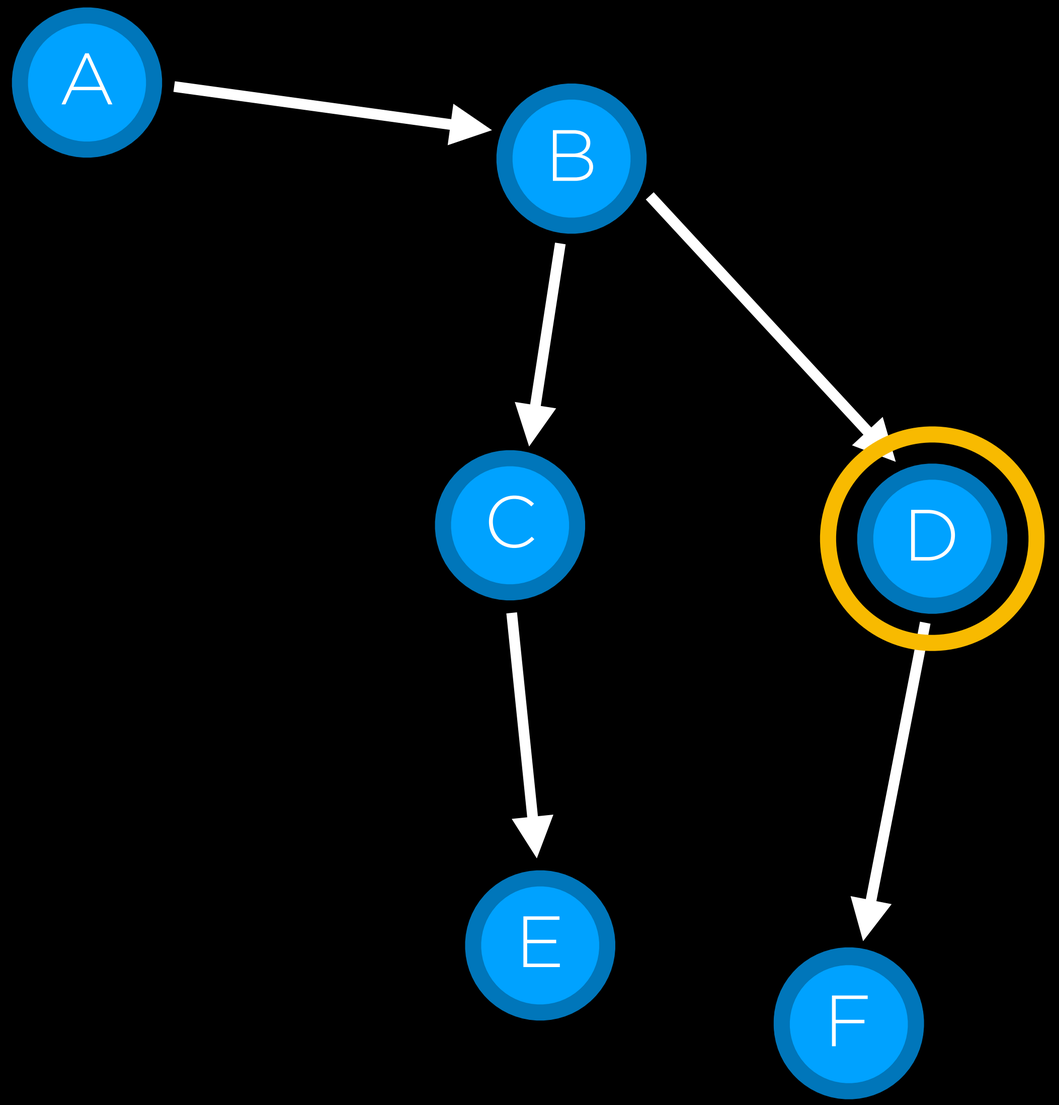
Find a path from A to E.

Frontier

C

Explored Set

A B D



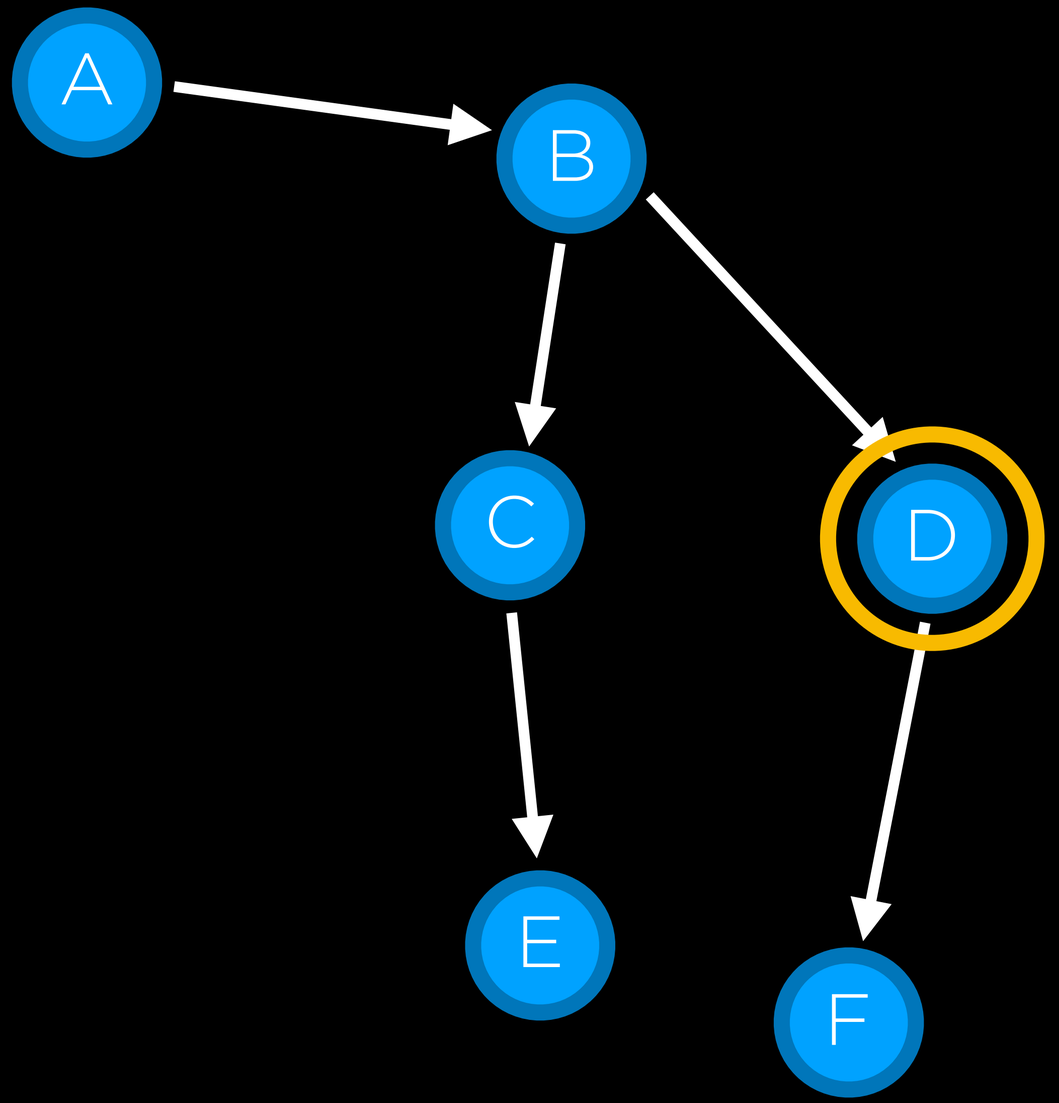
Find a path from A to E.

Frontier

C F

Explored Set

A B D



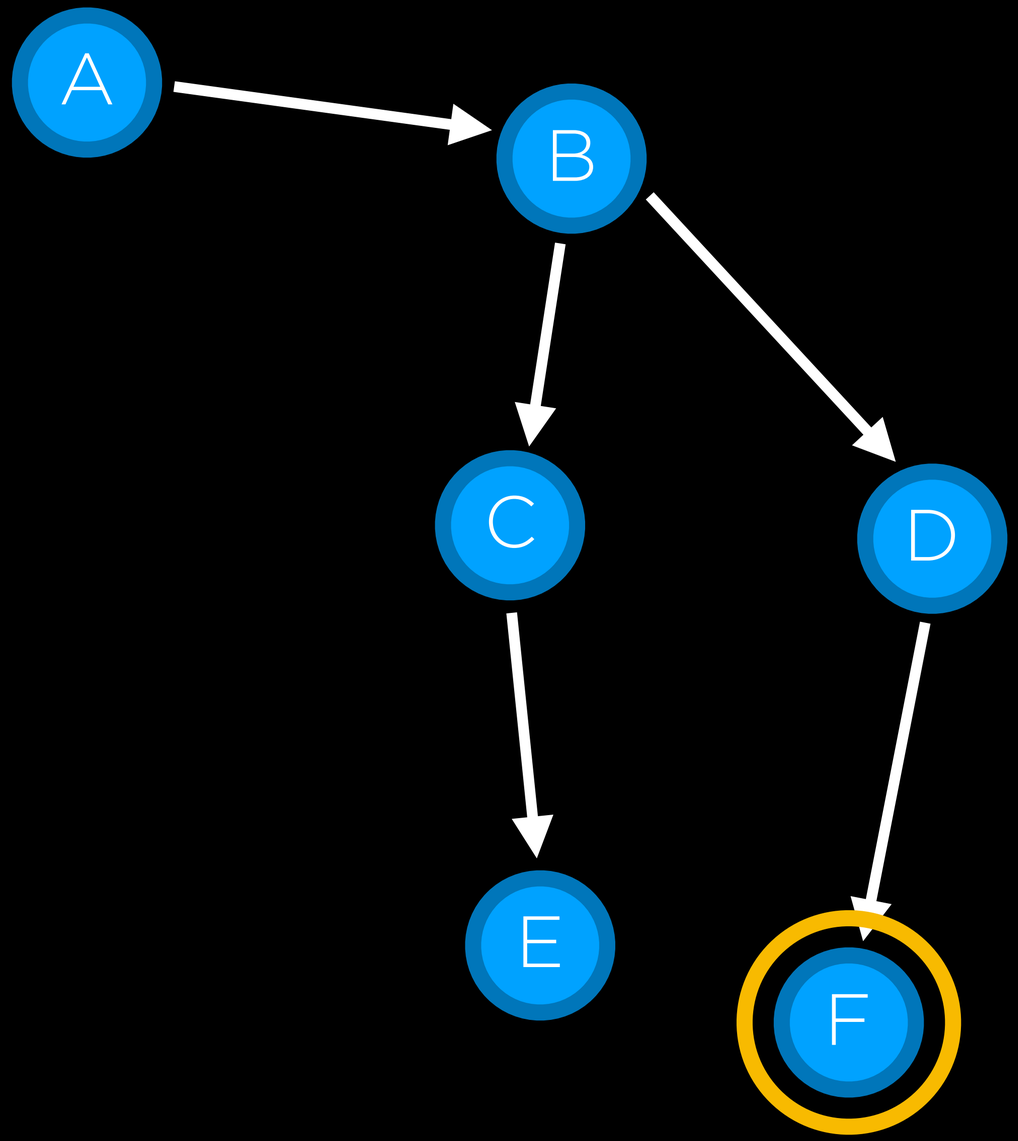
Find a path from A to E.

Frontier

C

Explored Set

A B D F

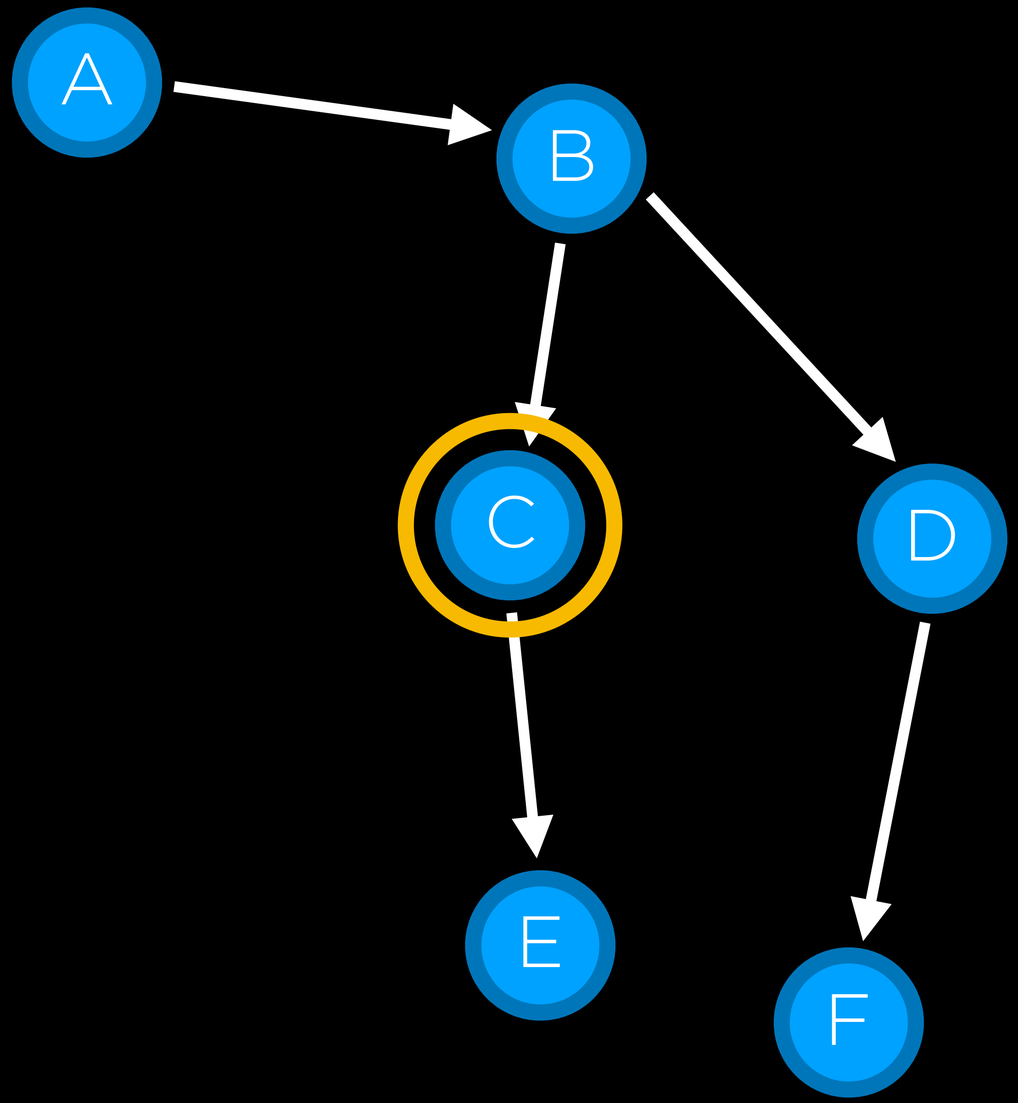


Find a path from A to E.

Frontier

Explored Set

A B D F C



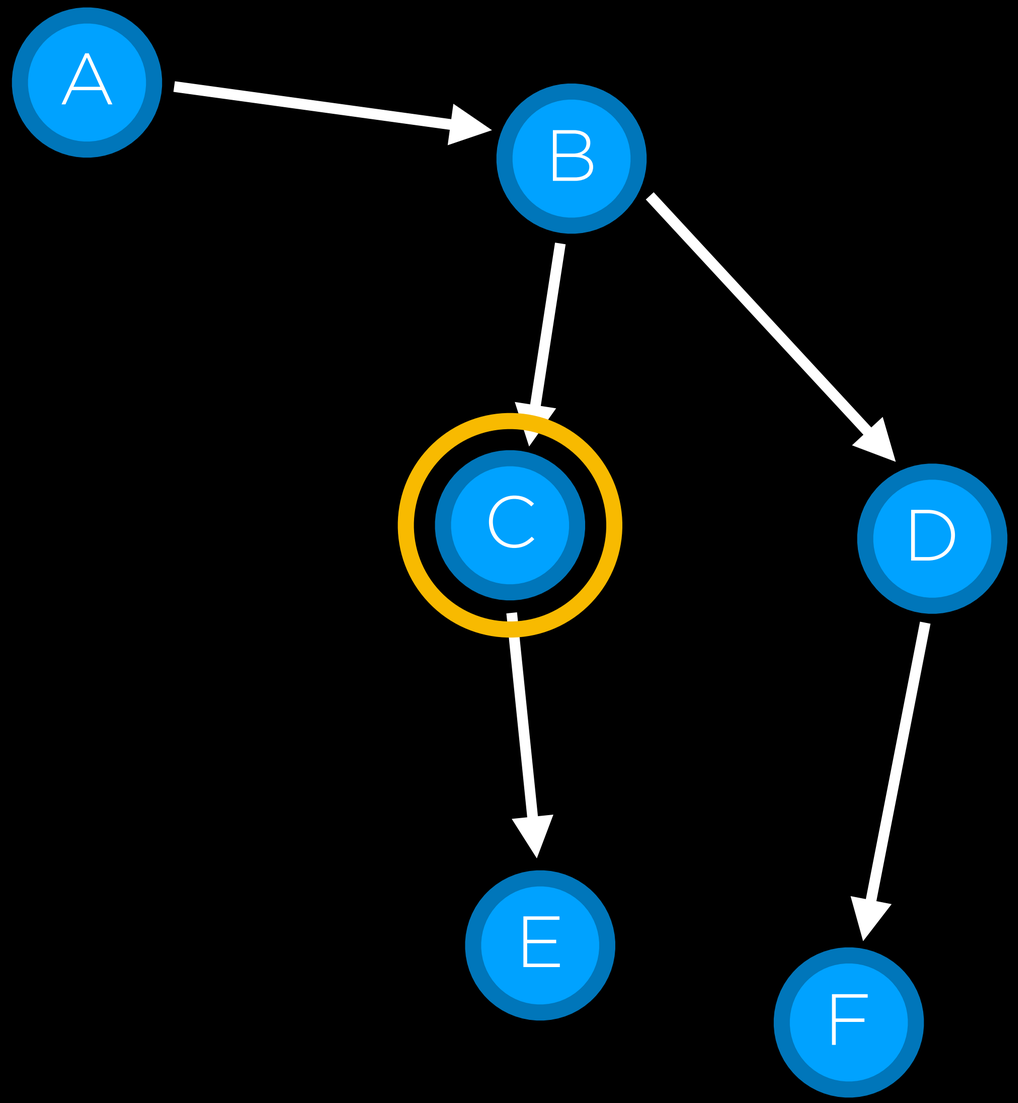
Find a path from A to E.

Frontier

E

Explored Set

A B D F C

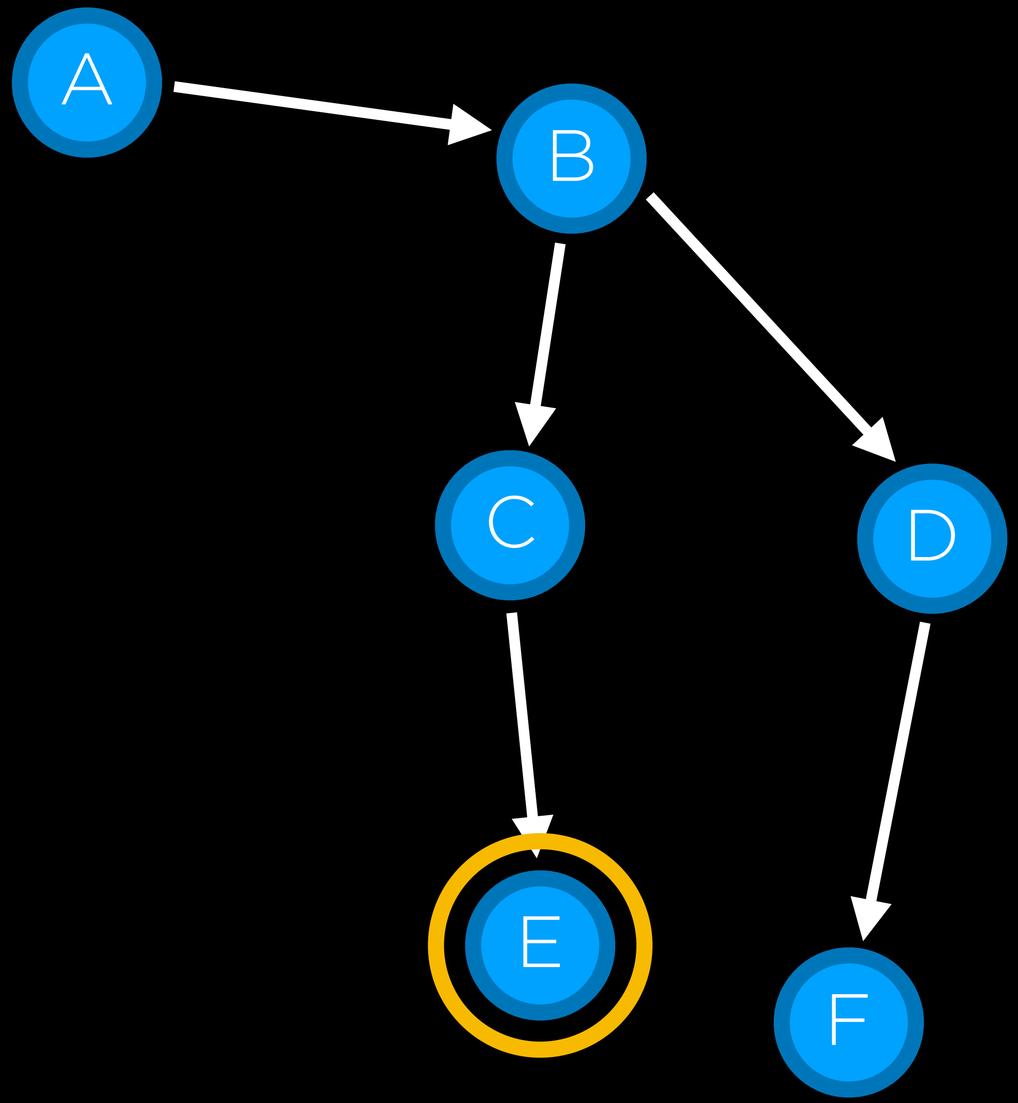


Find a path from A to E.

Frontier

Explored Set

A B D F C



Depth-First Search

depth-first search

search algorithm that always expands the deepest node in the frontier

Breadth-First Search

breadth-first search

search algorithm that always expands the shallowest node in the frontier

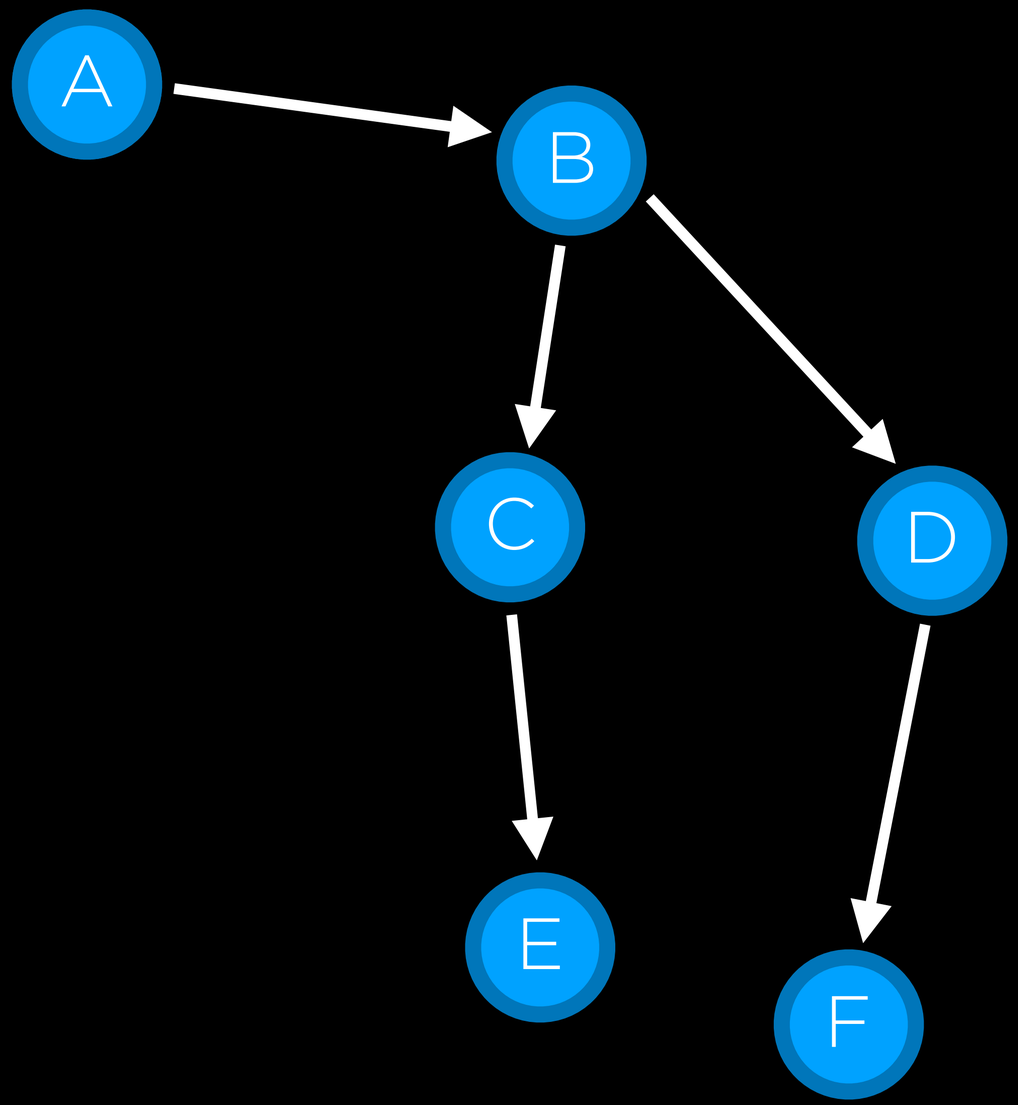
queue

first-in first-out data type

Find a path from A to E.

Frontier

Explored Set

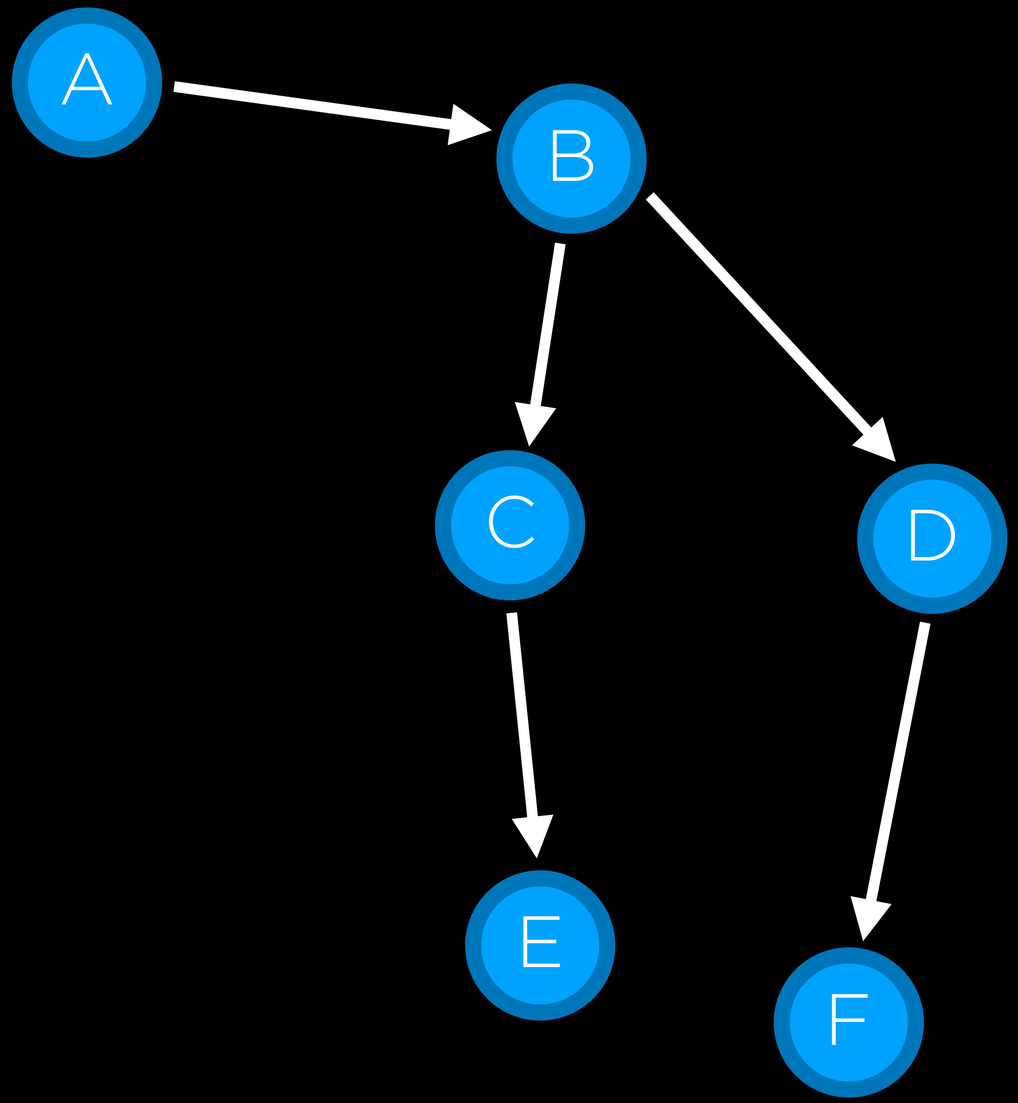


Find a path from A to E.

Frontier

A

Explored Set

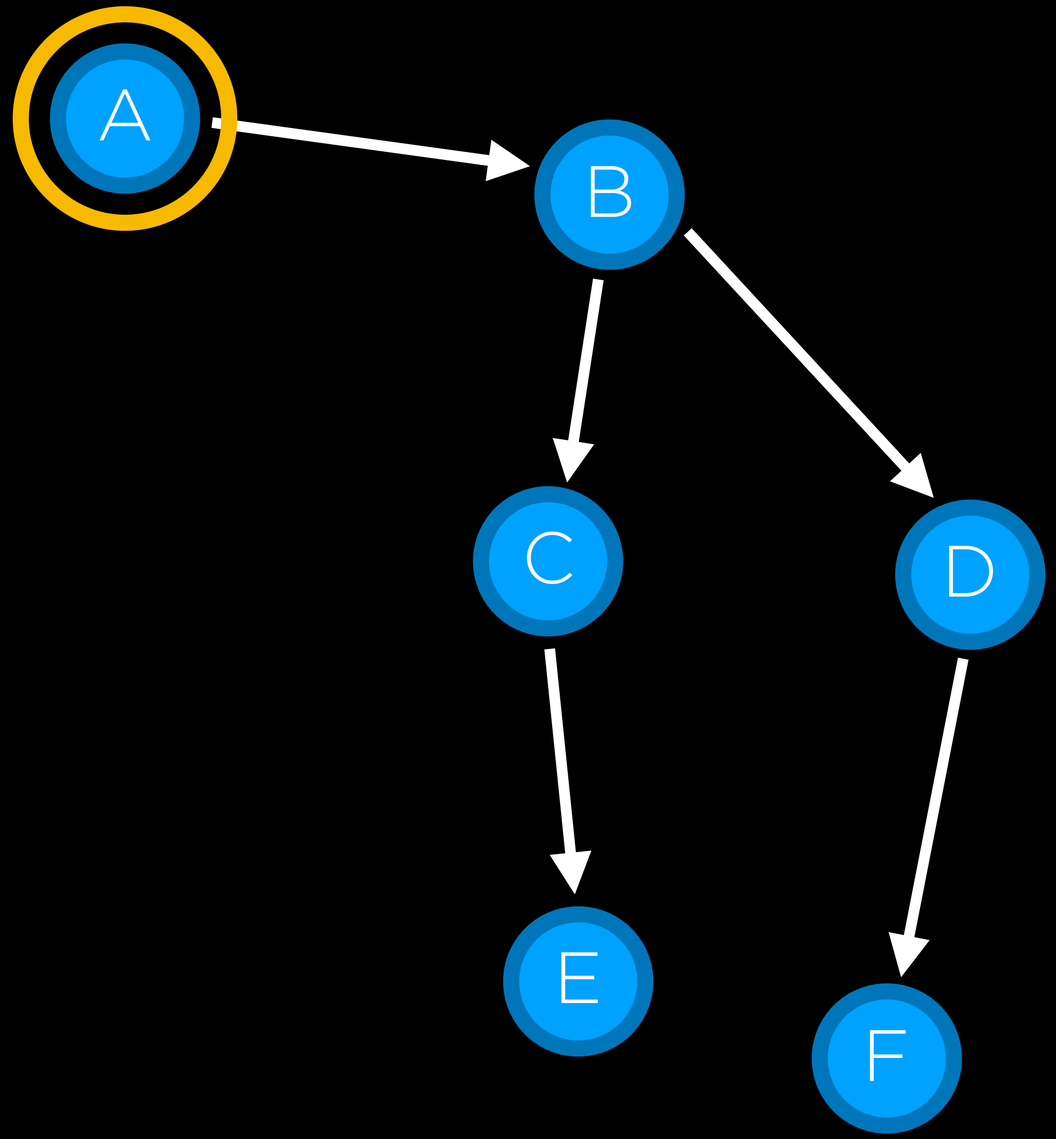


Find a path from A to E.

Frontier

Explored Set

A



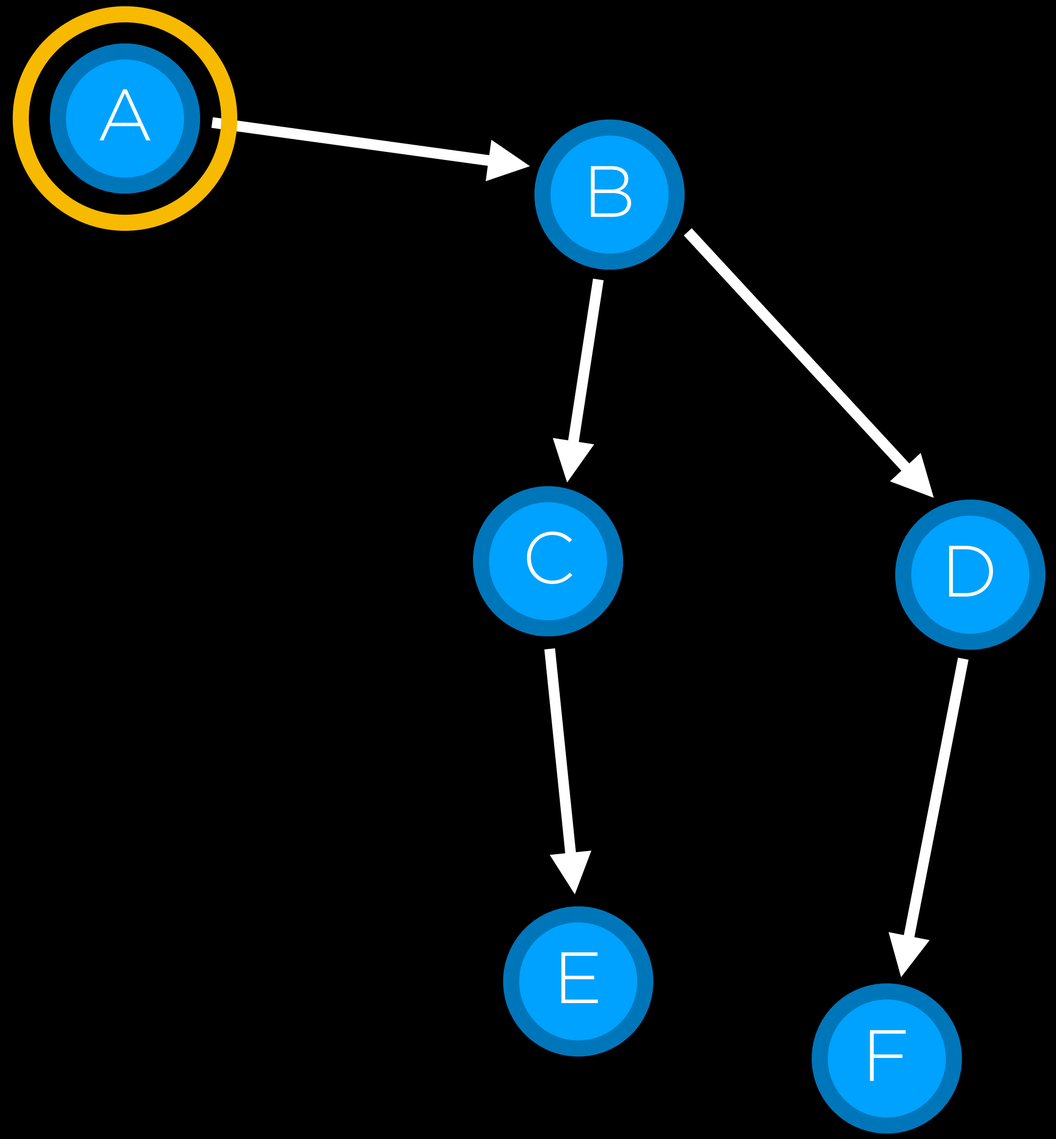
Find a path from A to E.

Frontier

B

Explored Set

A

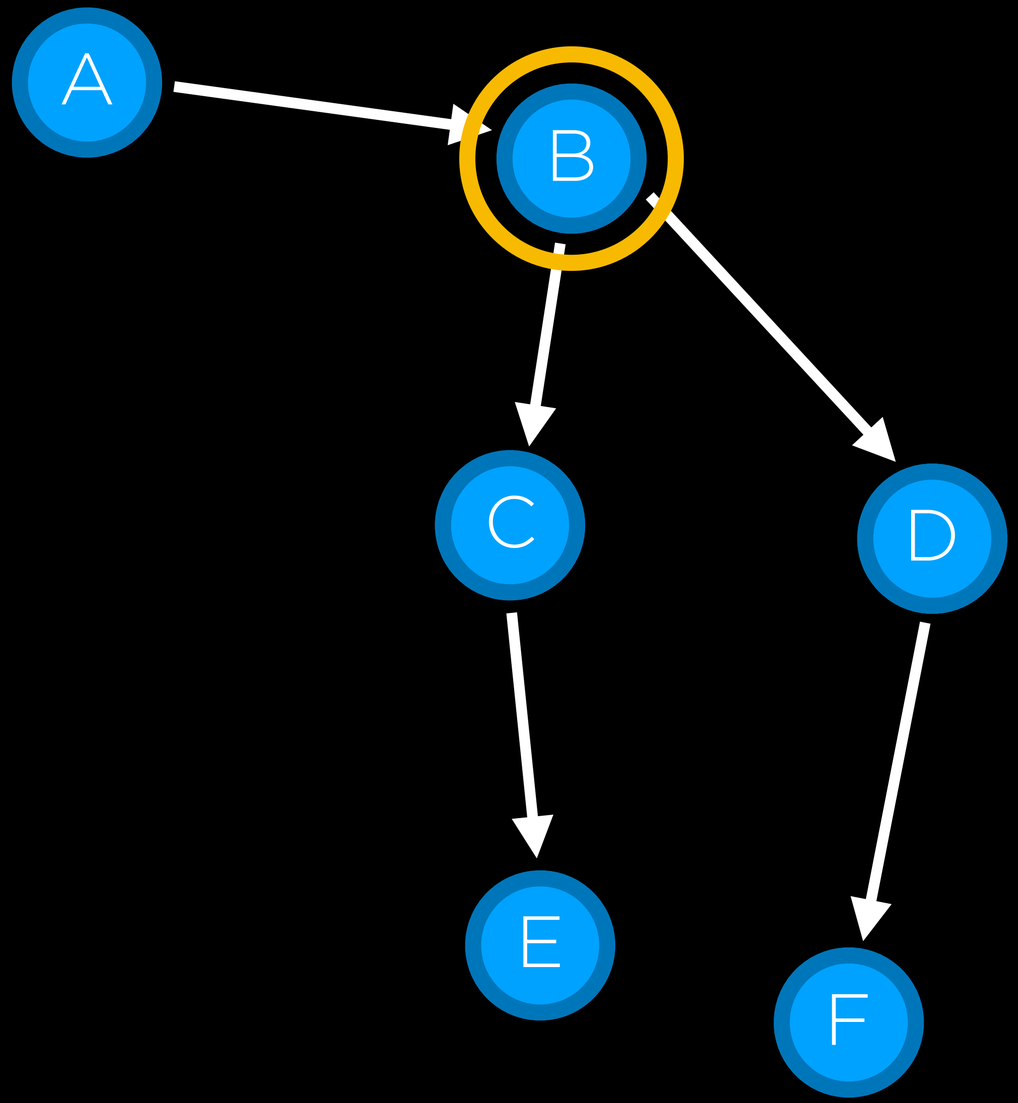


Find a path from A to E.

Frontier

Explored Set

A B



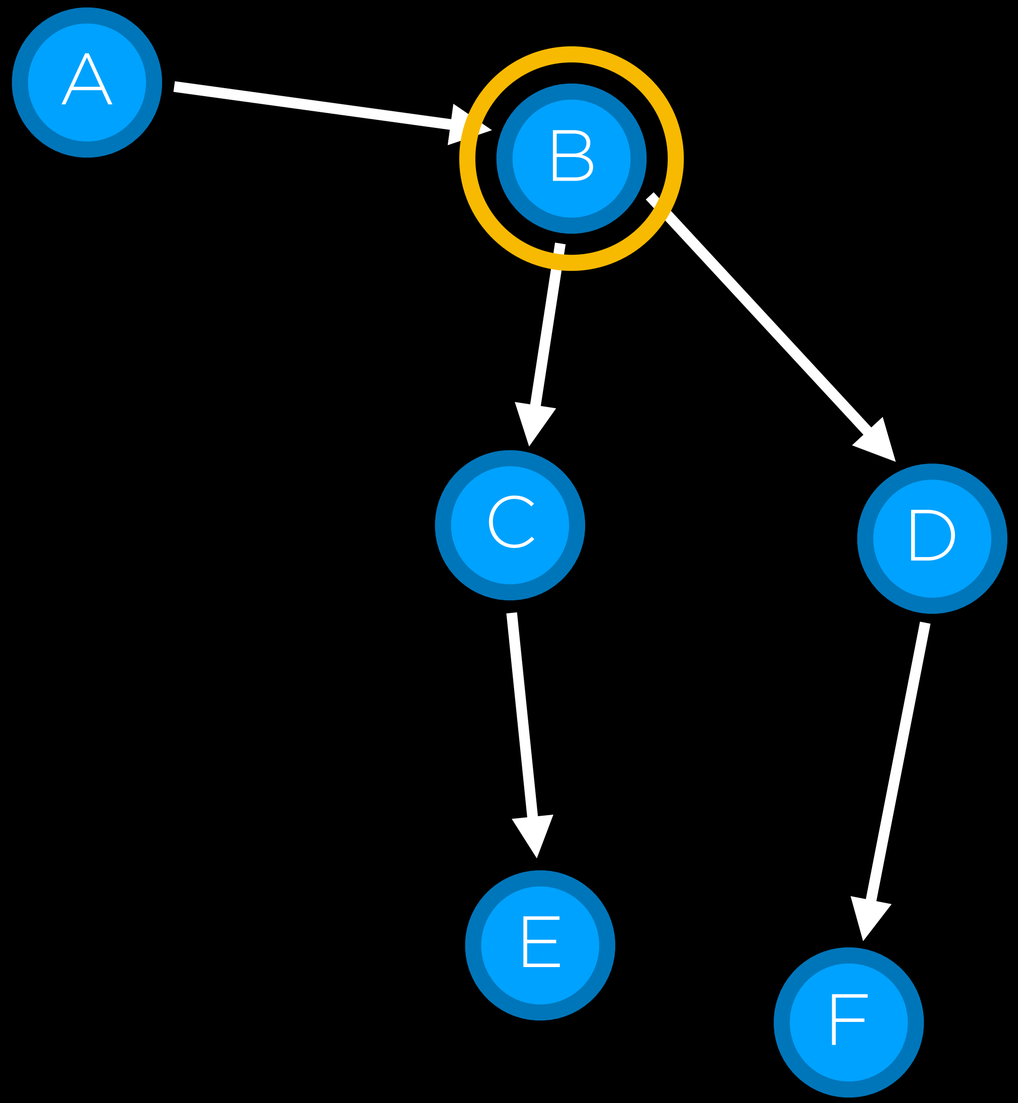
Find a path from A to E.

Frontier

C D

Explored Set

A B



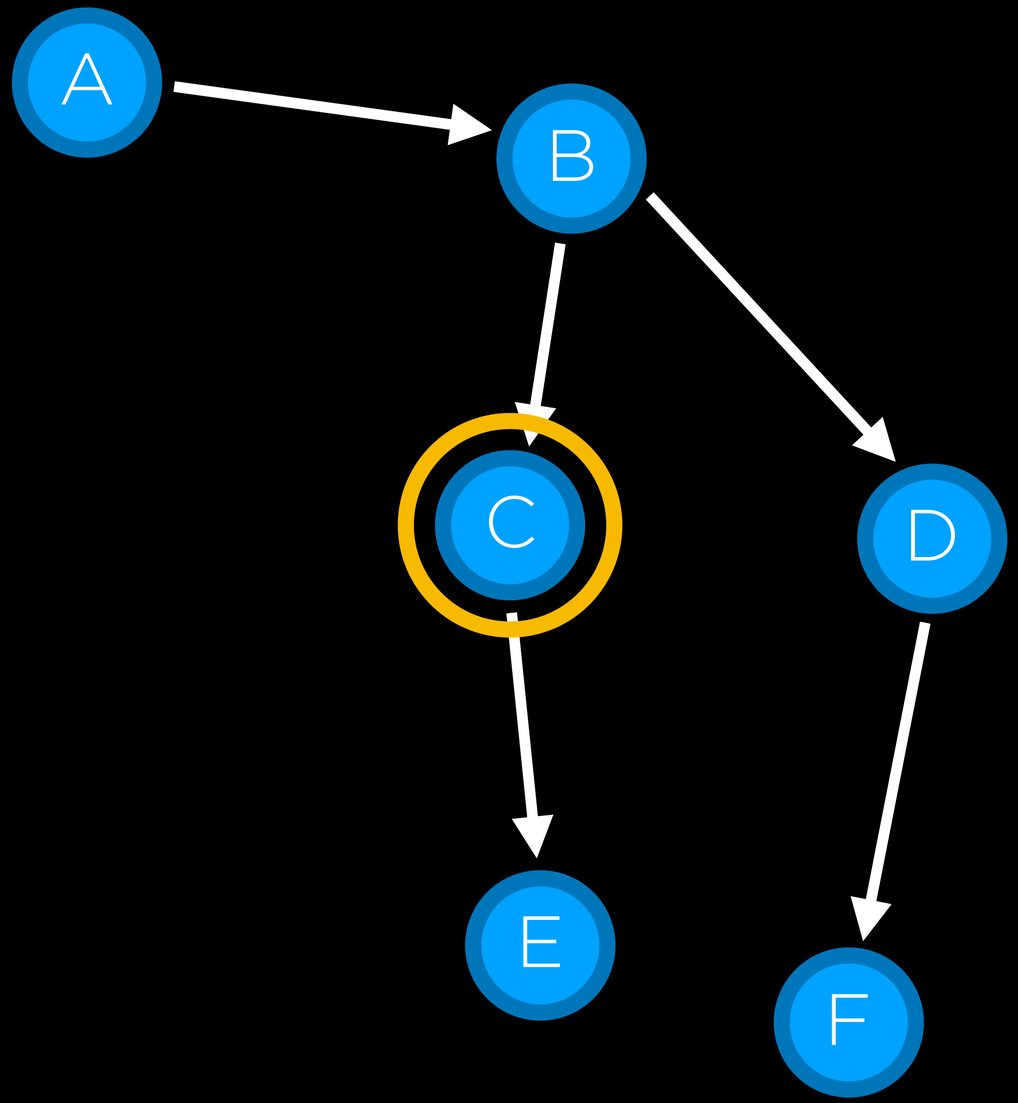
Find a path from A to E.

Frontier

D

Explored Set

A B C



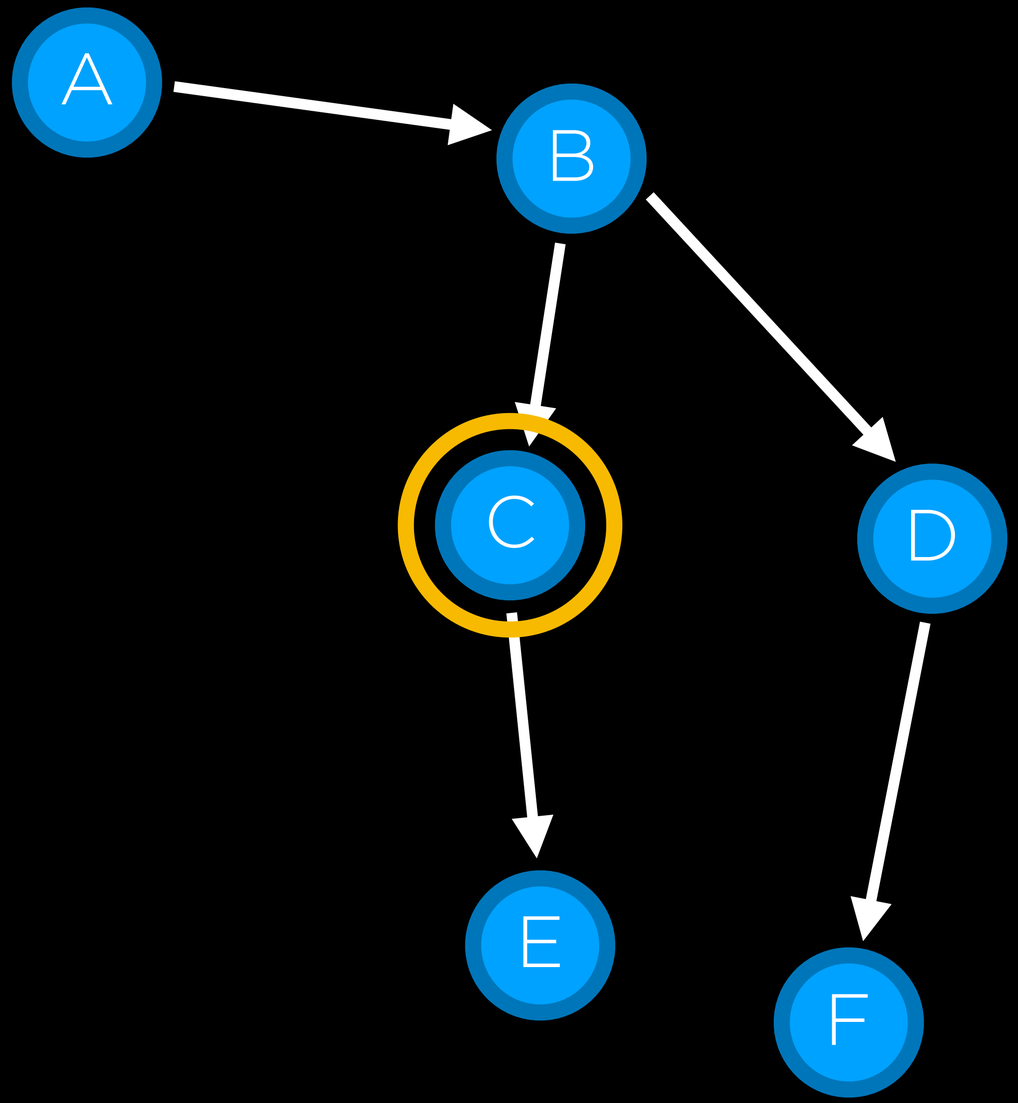
Find a path from A to E.

Frontier

D E

Explored Set

A B C



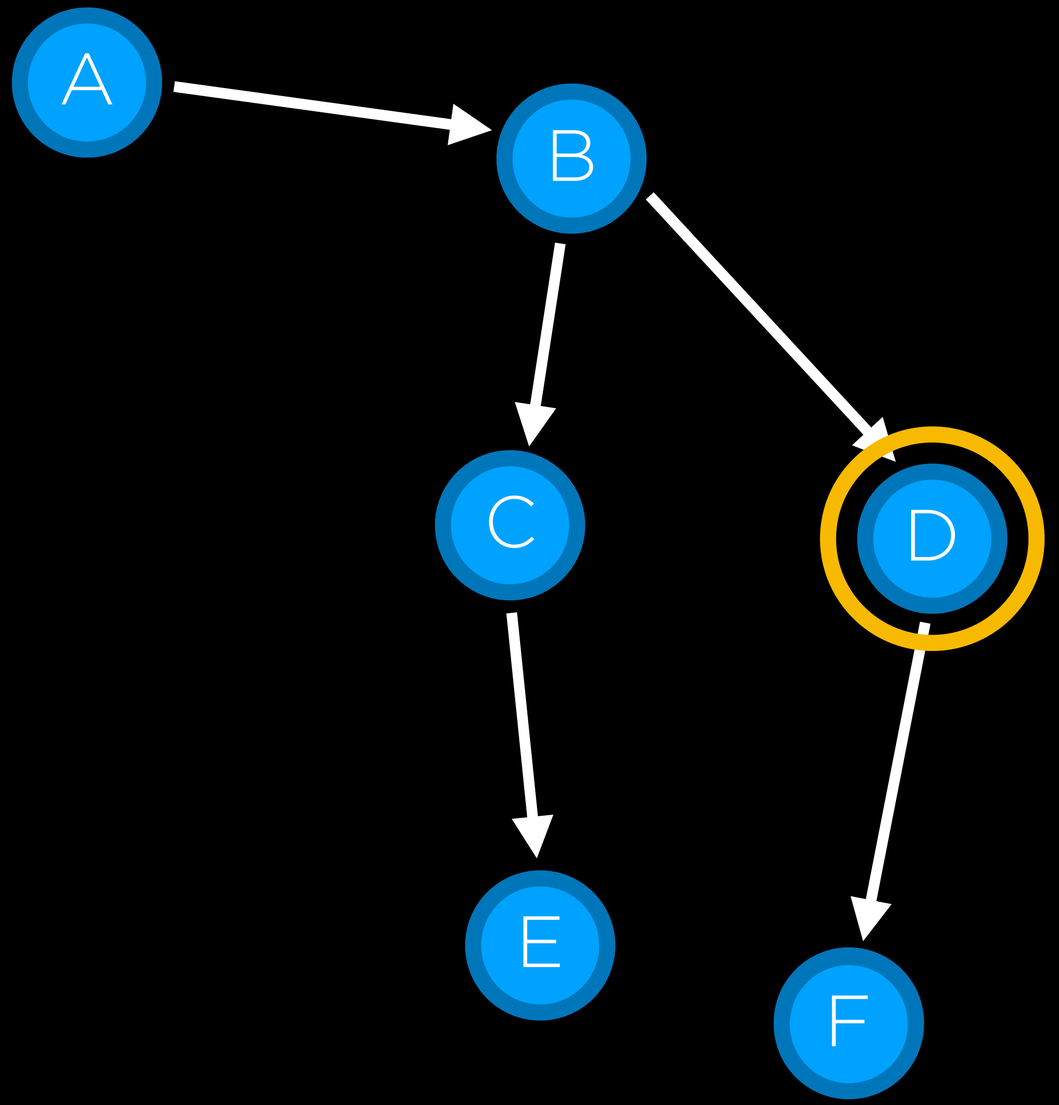
Find a path from A to E.

Frontier

E

Explored Set

A B C D



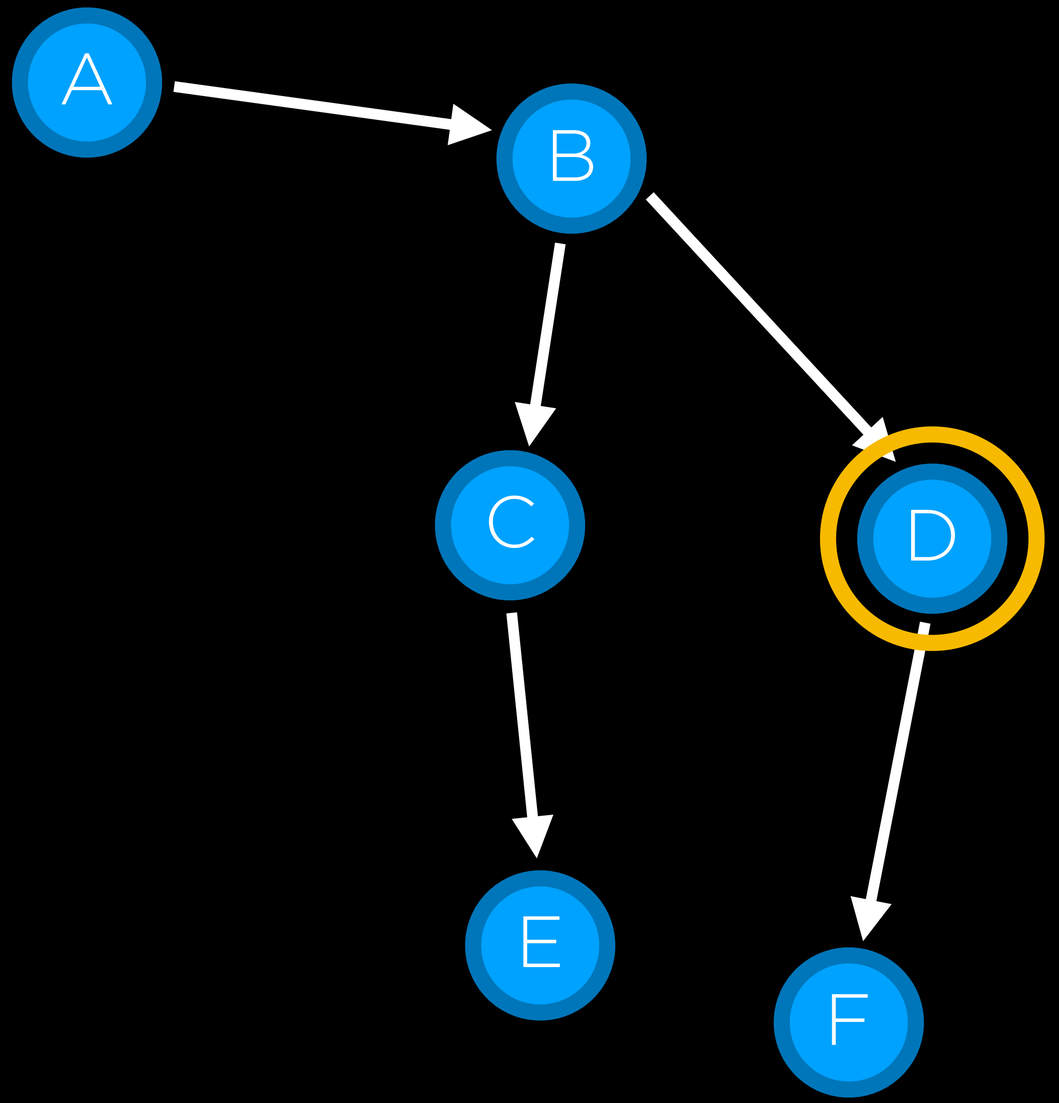
Find a path from A to E.

Frontier

E F

Explored Set

A B C D



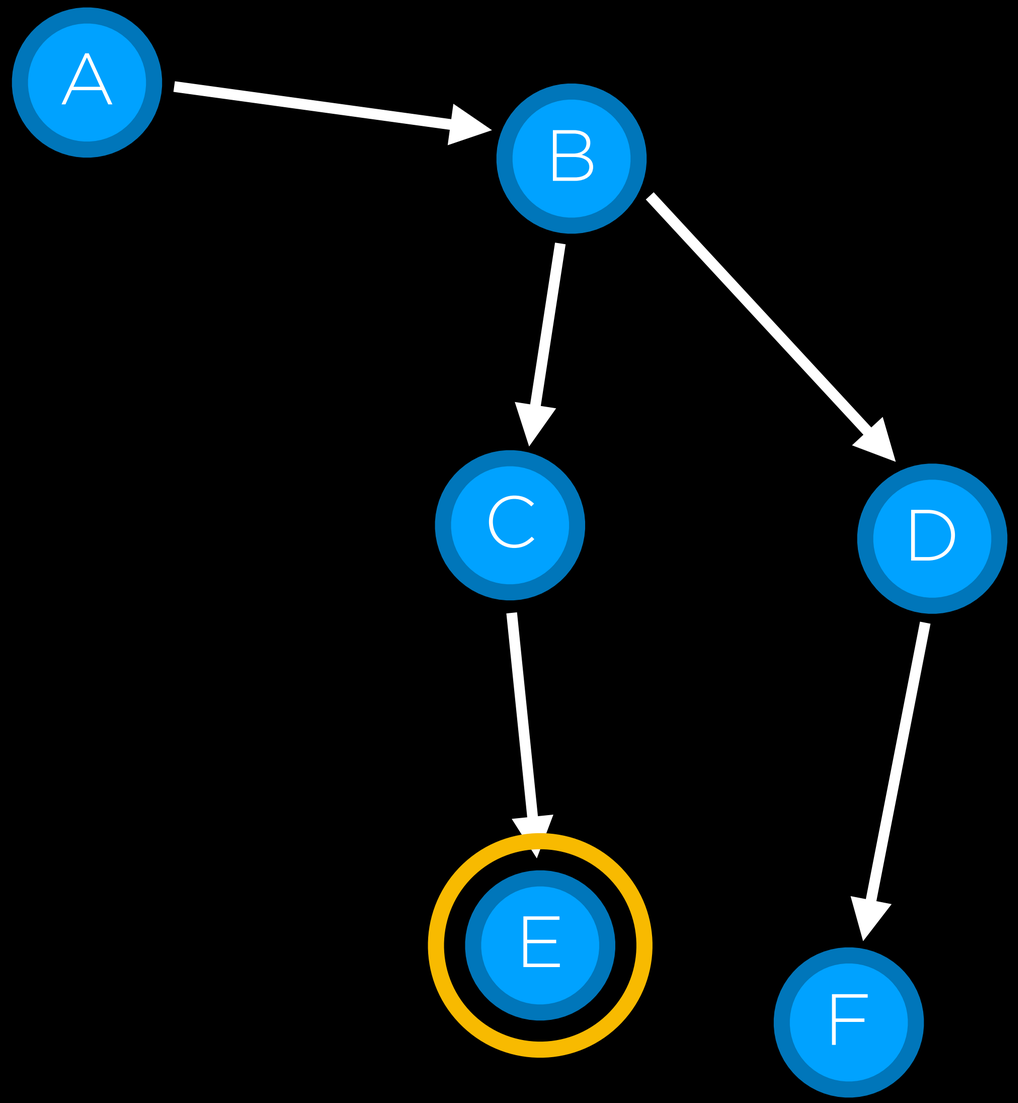
Find a path from A to E.

Frontier

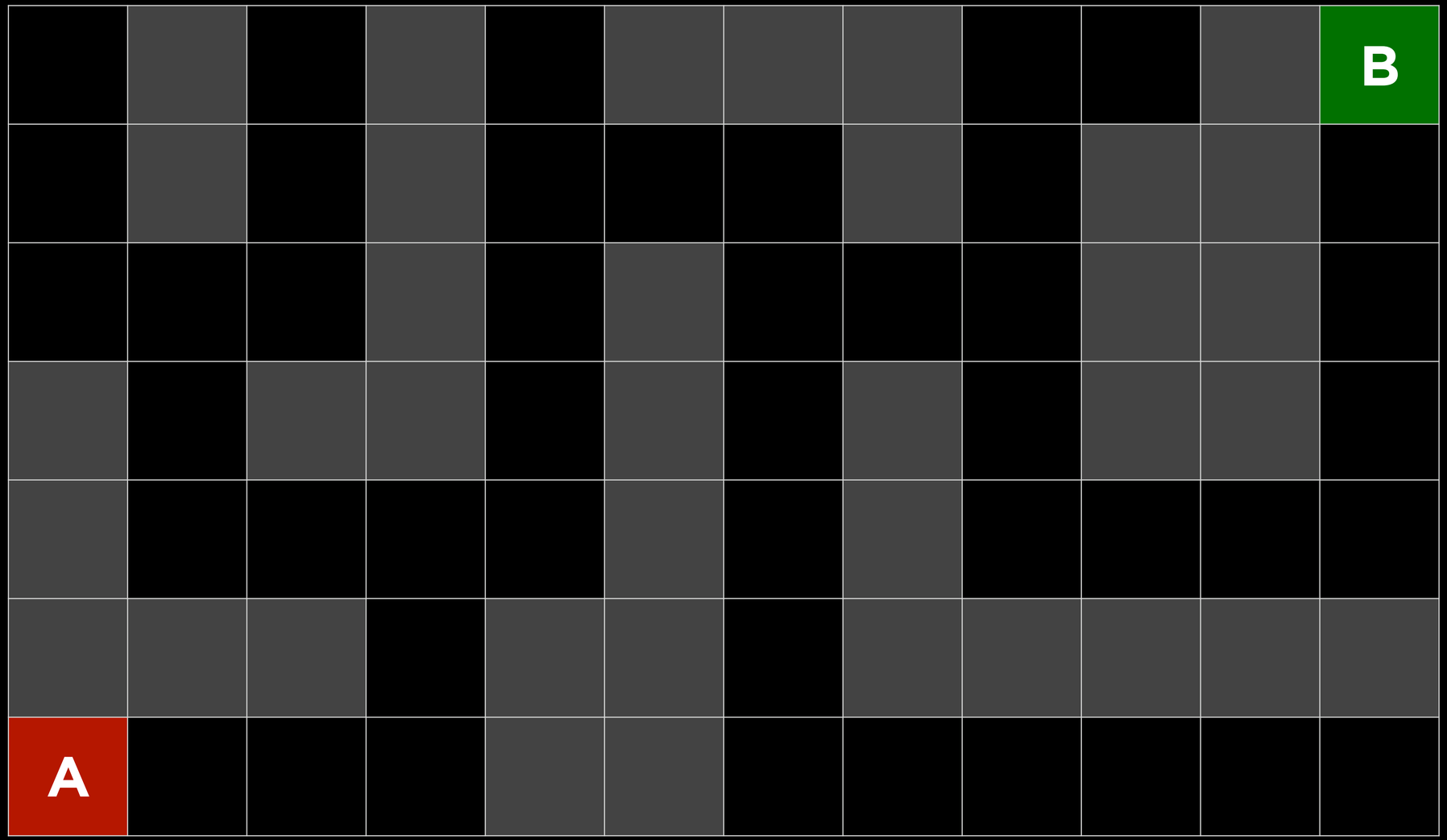
F

Explored Set

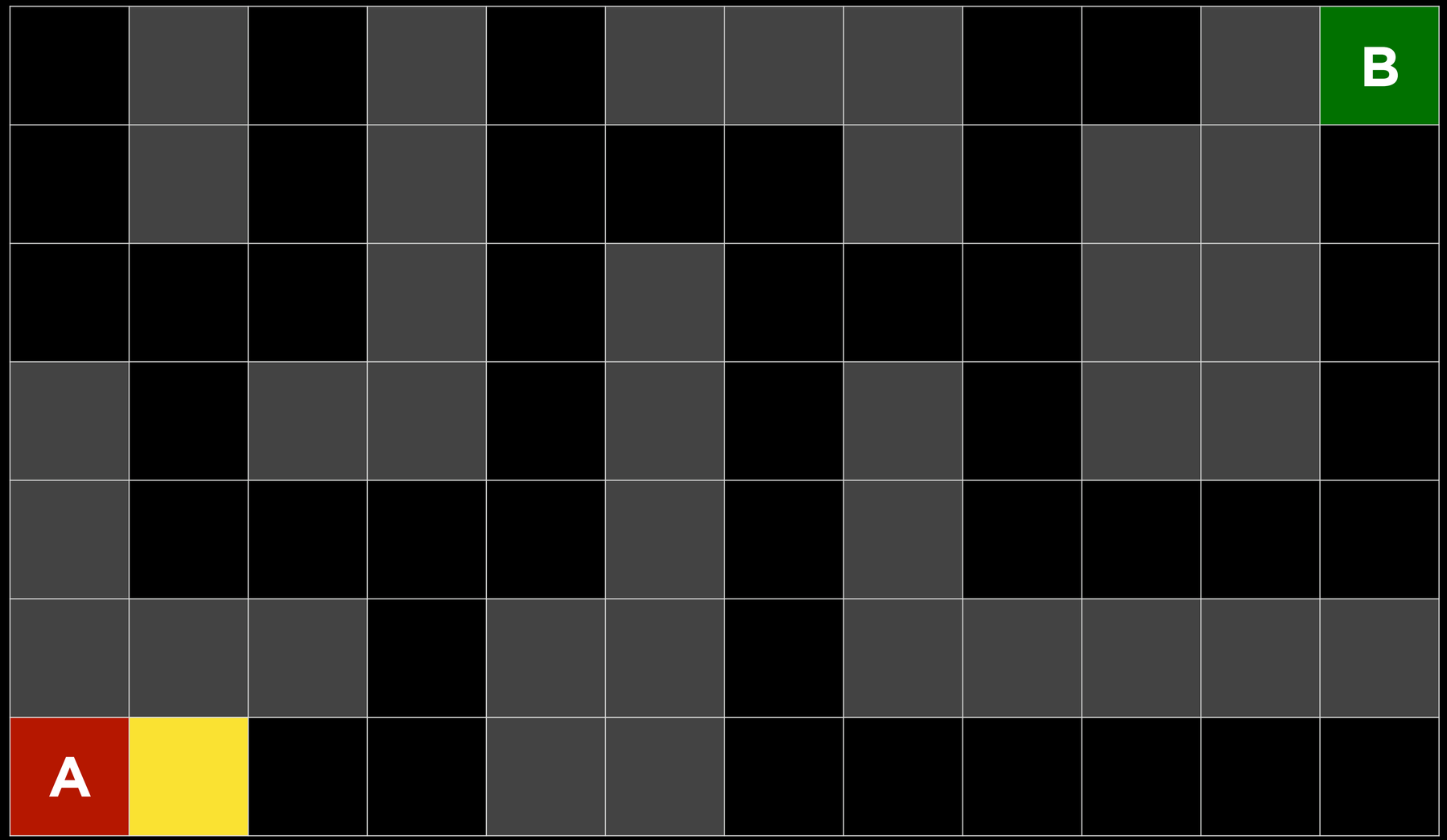
A B C D



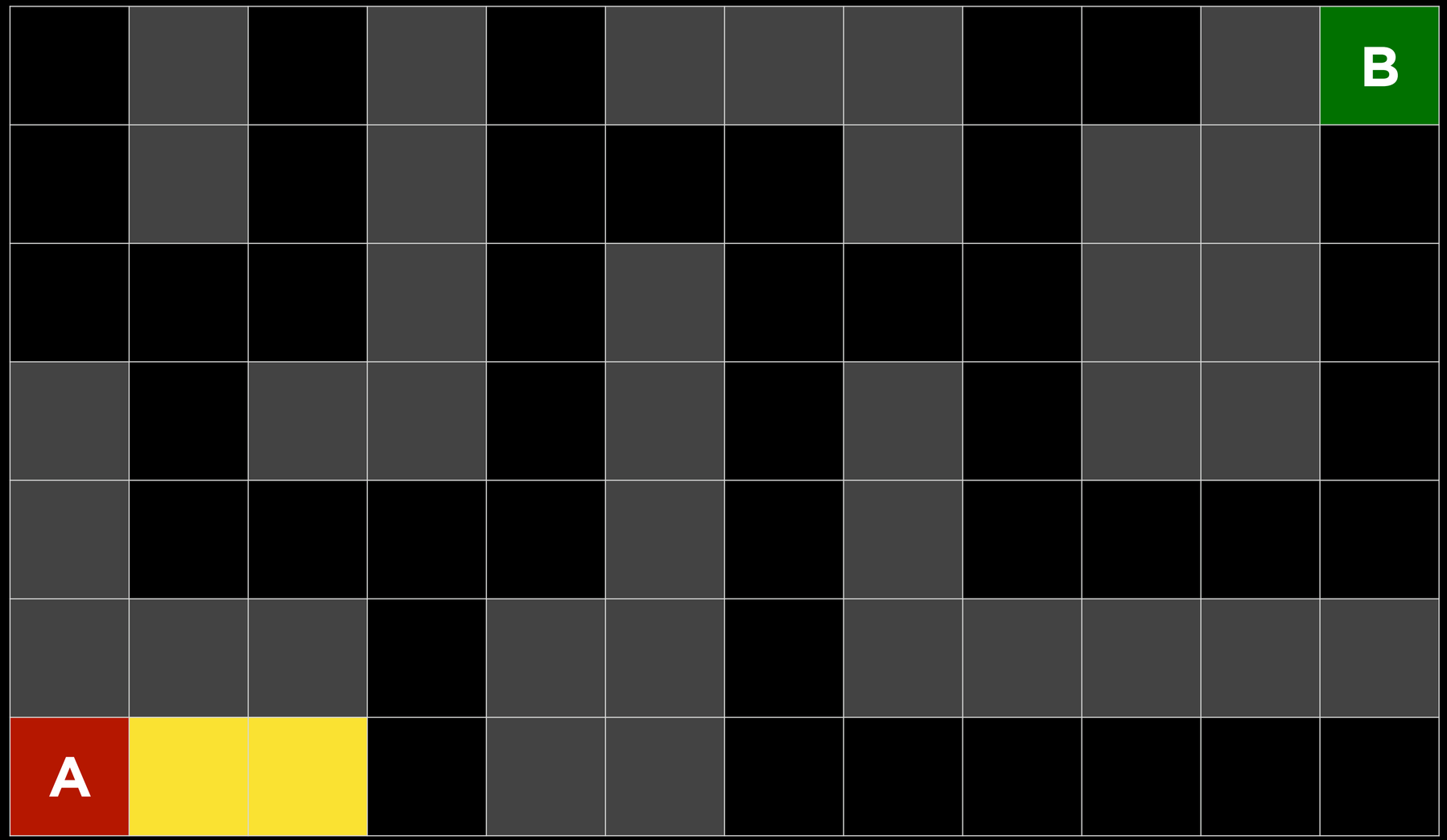
Depth-First Search



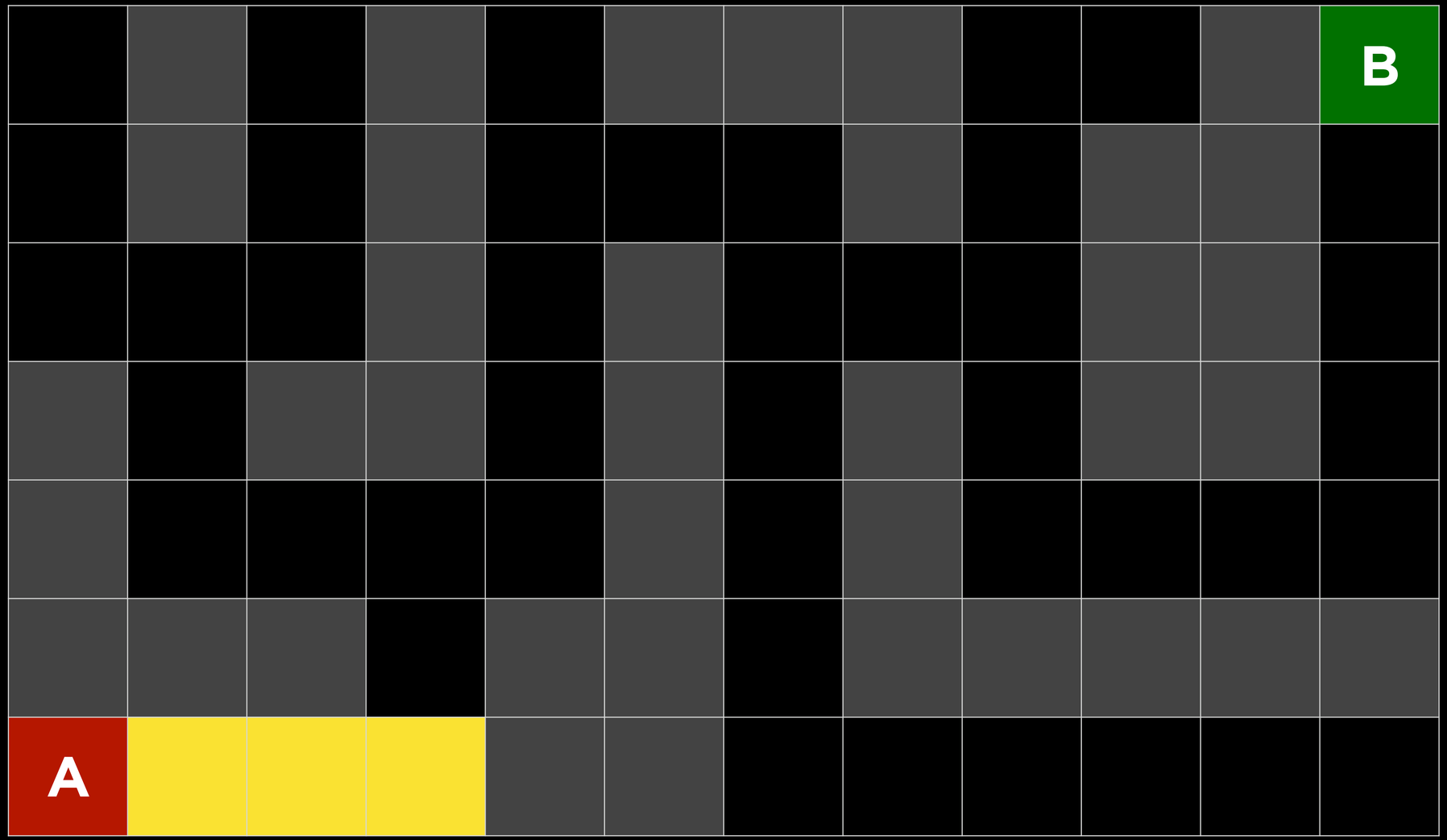
Depth-First Search



Depth-First Search



Depth-First Search



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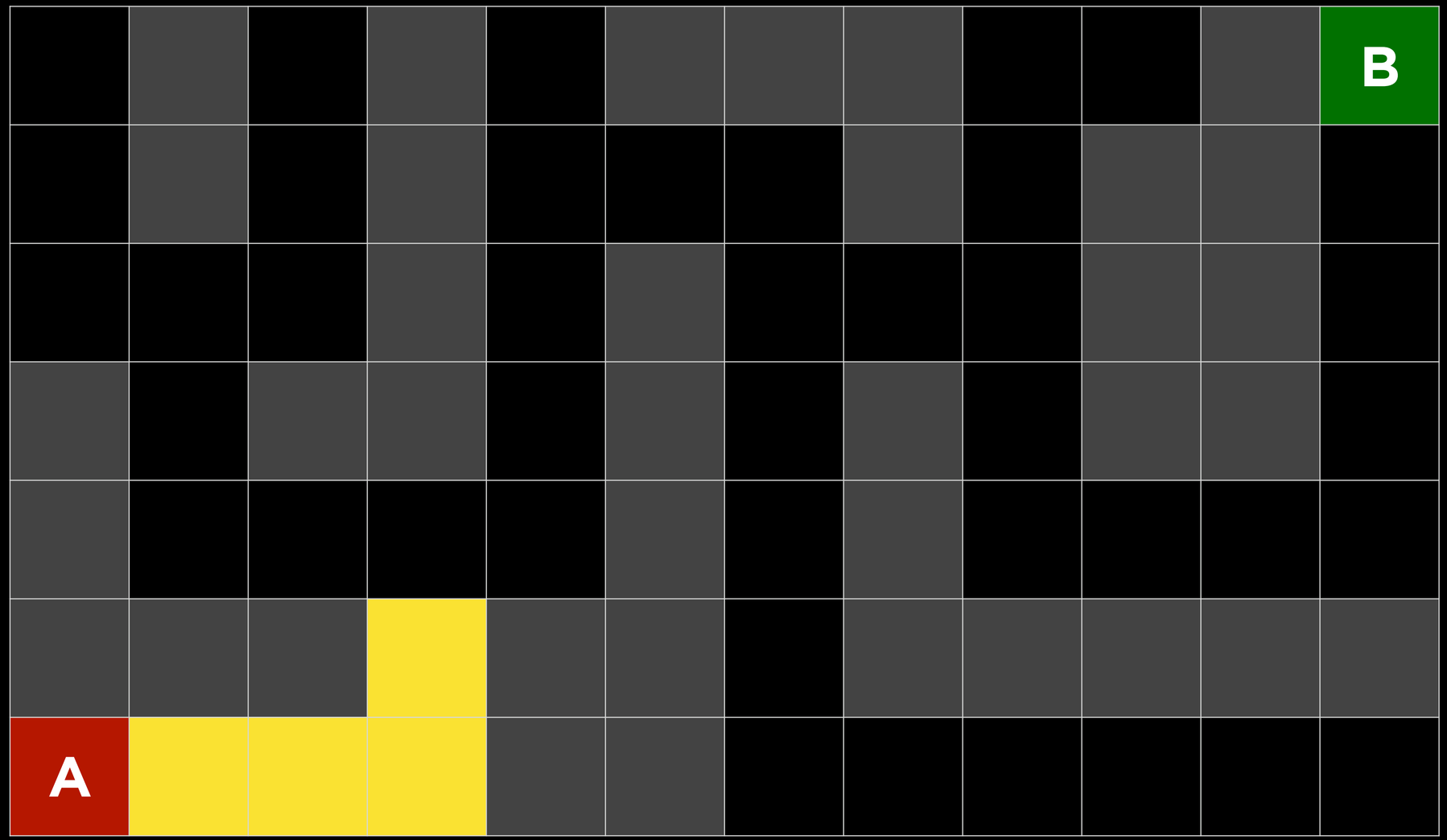

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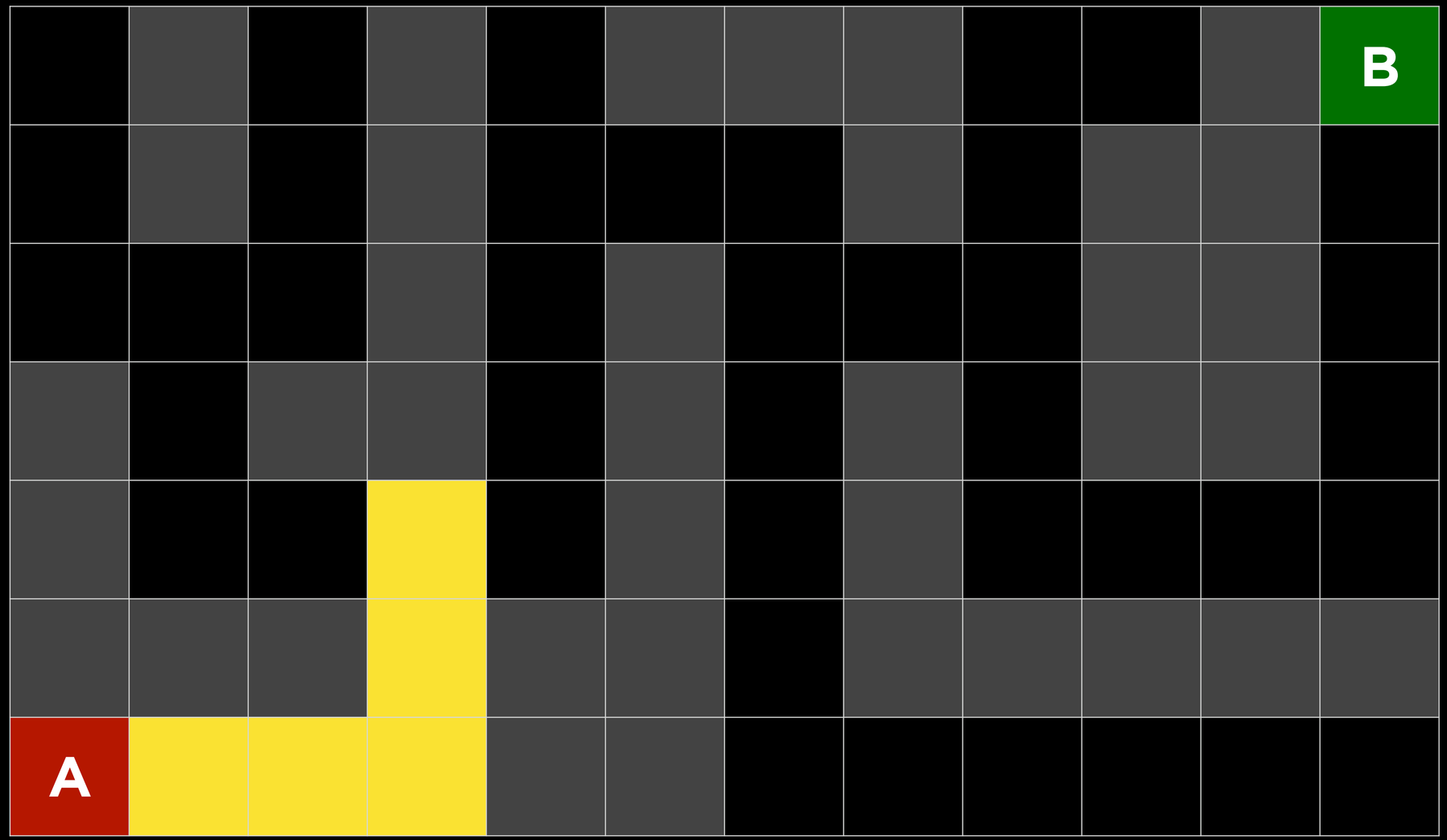

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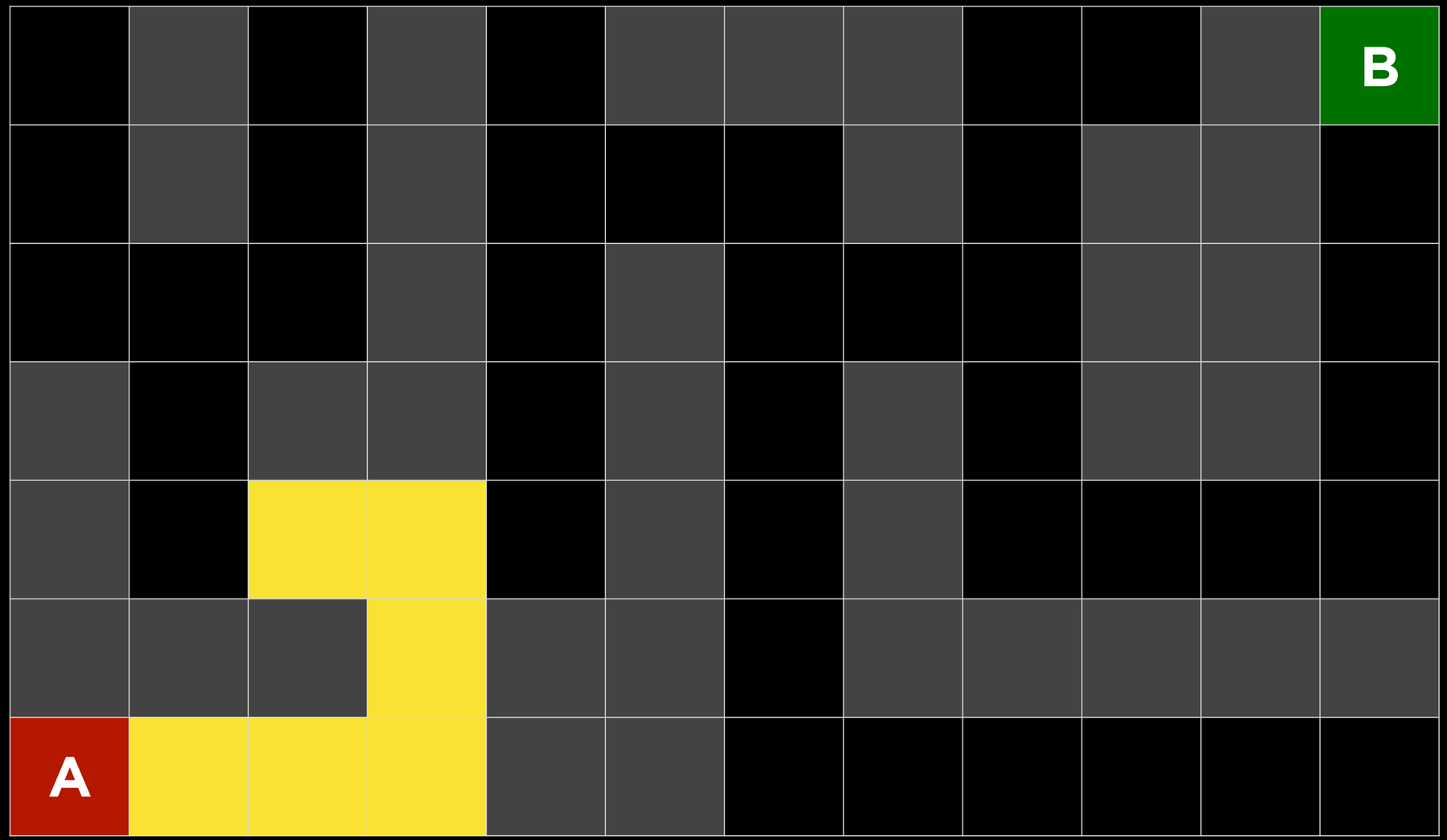

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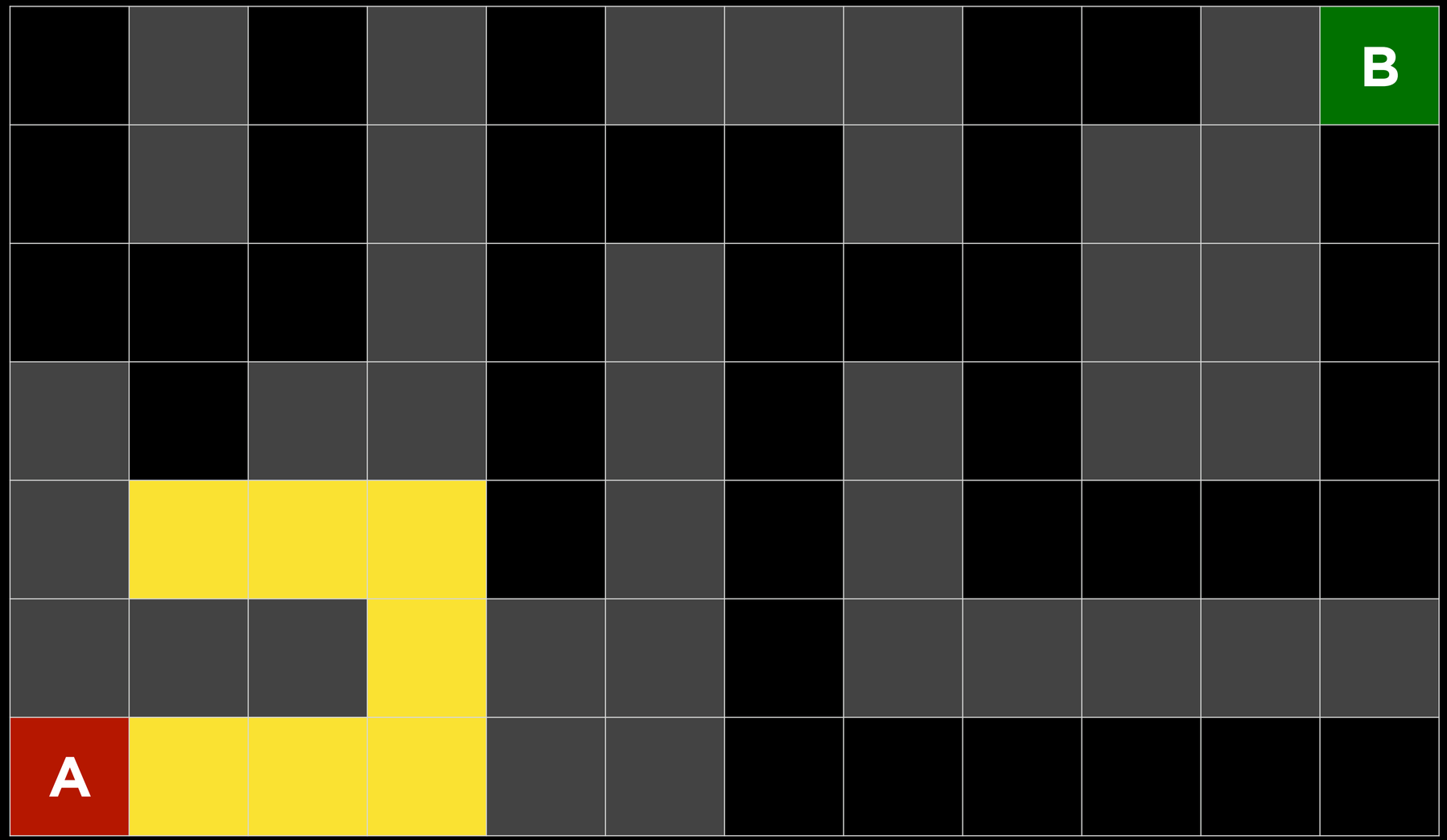
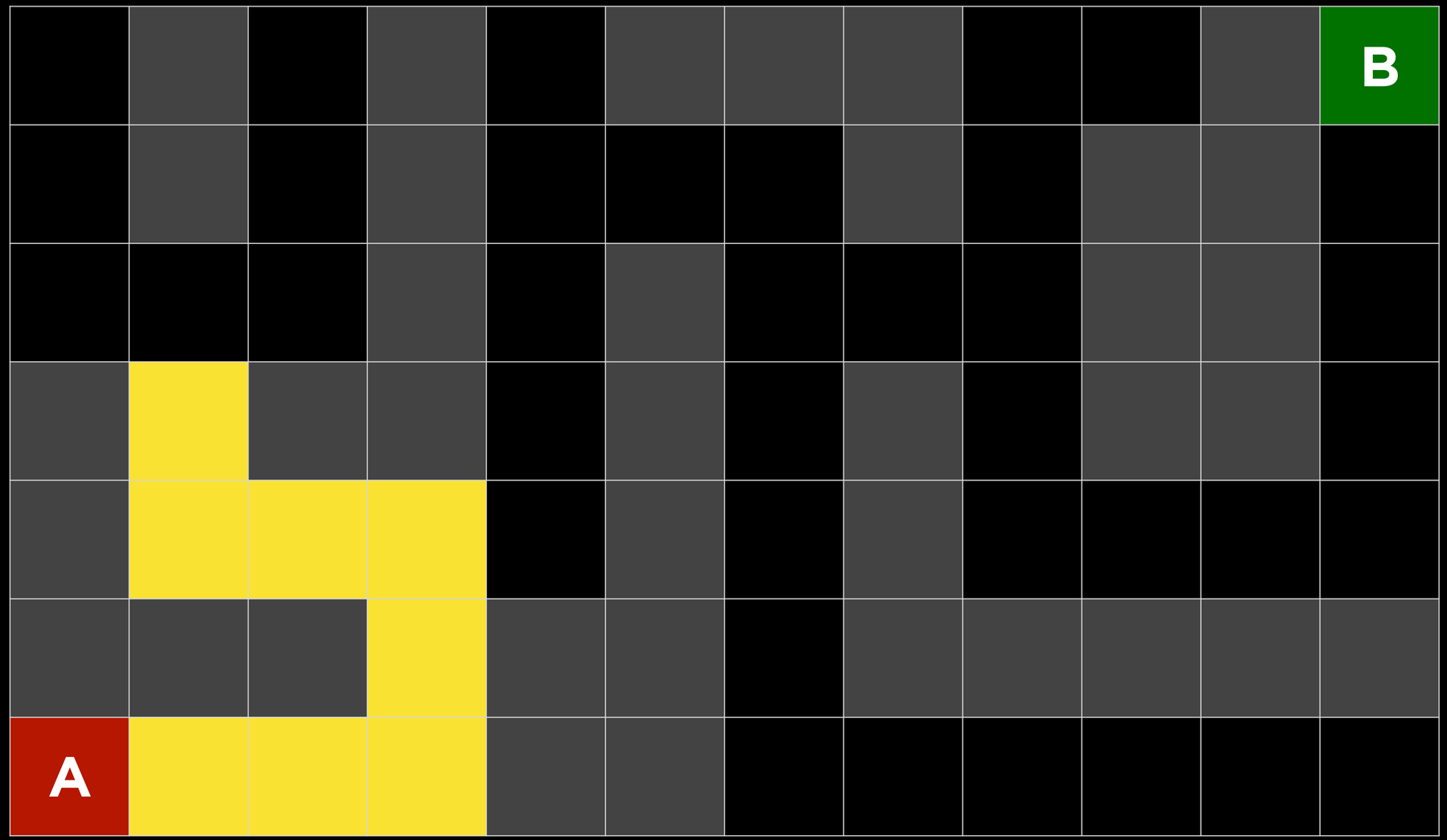
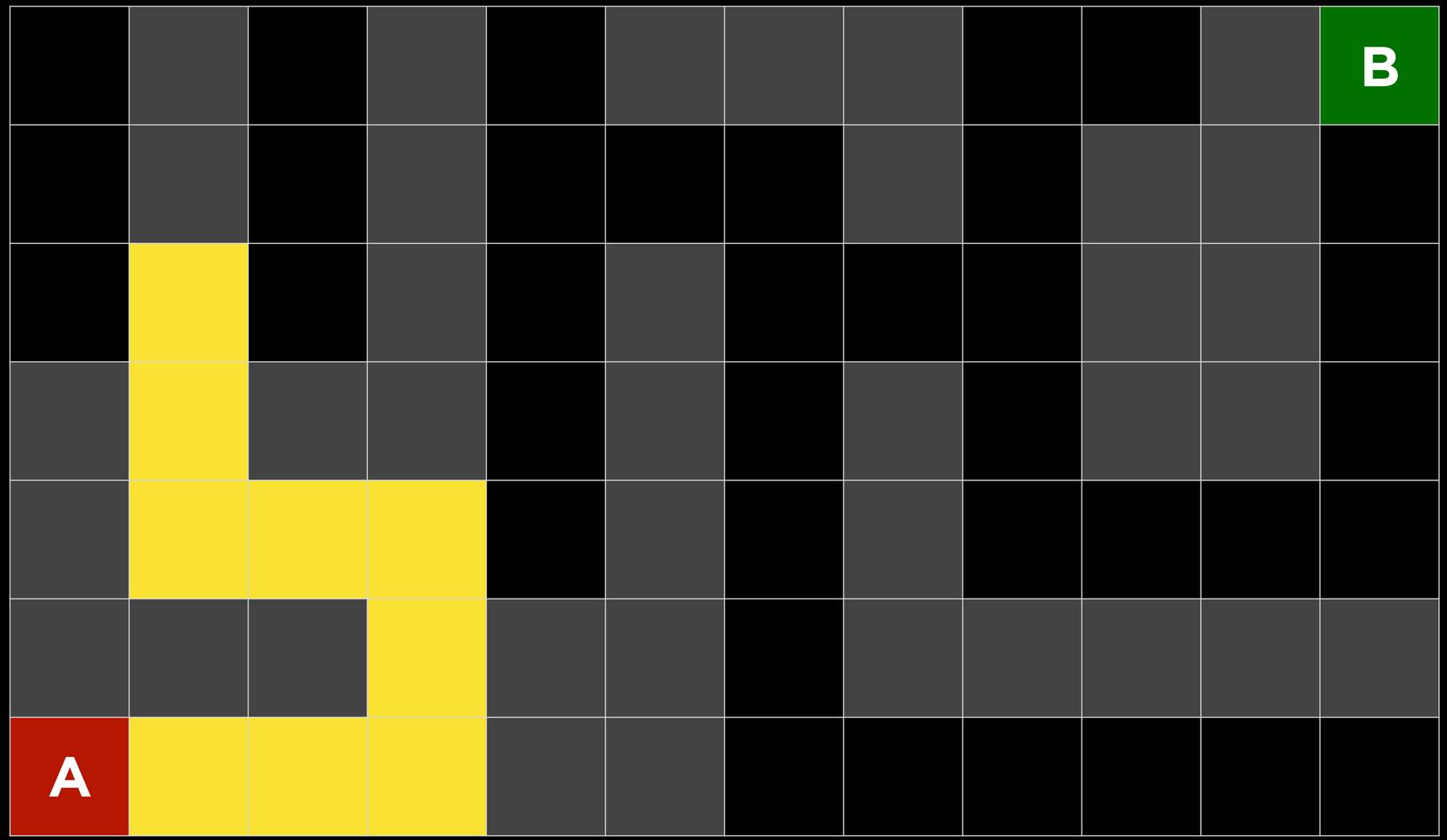


Depth-First Search



Depth-First Search



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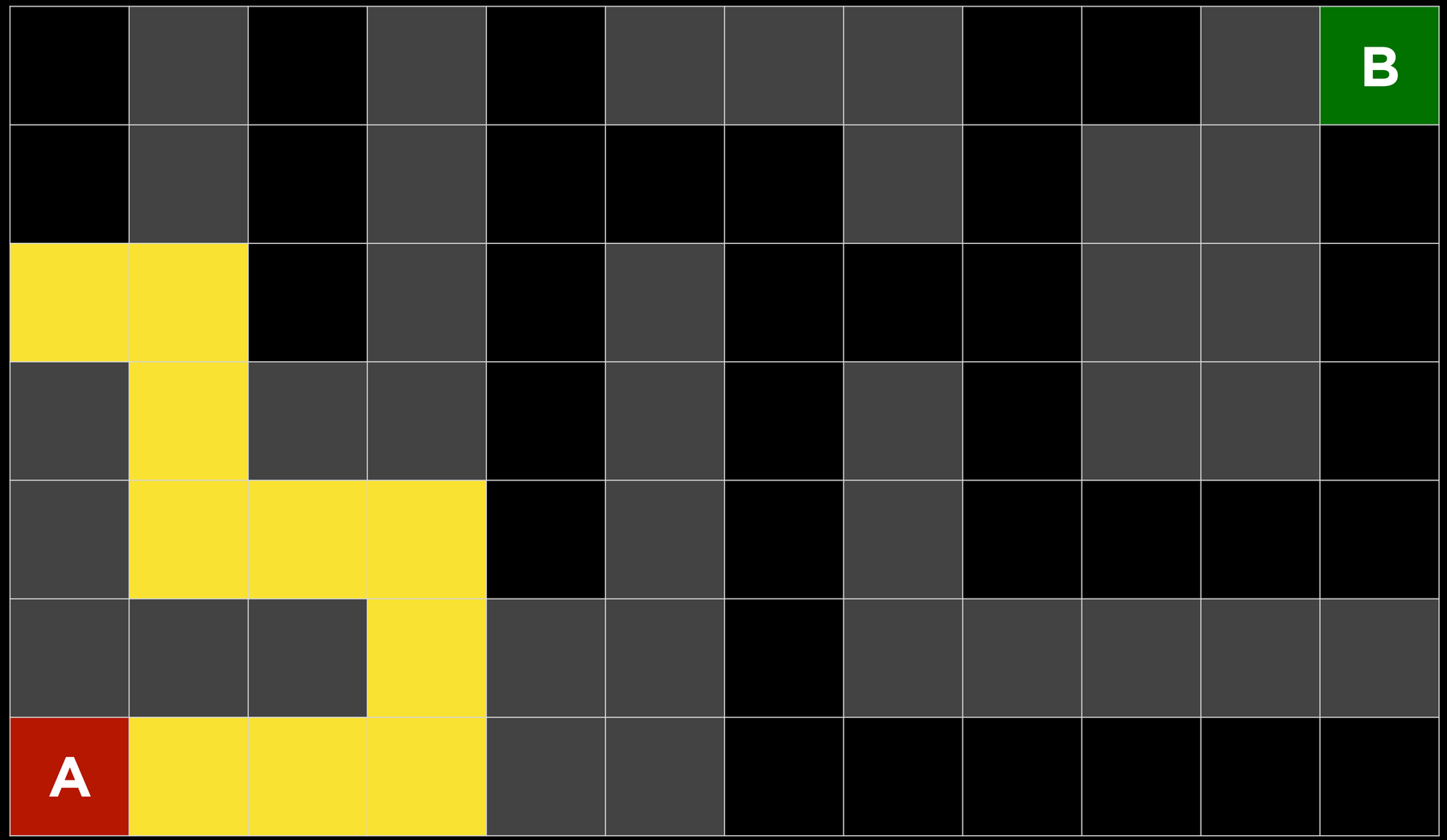

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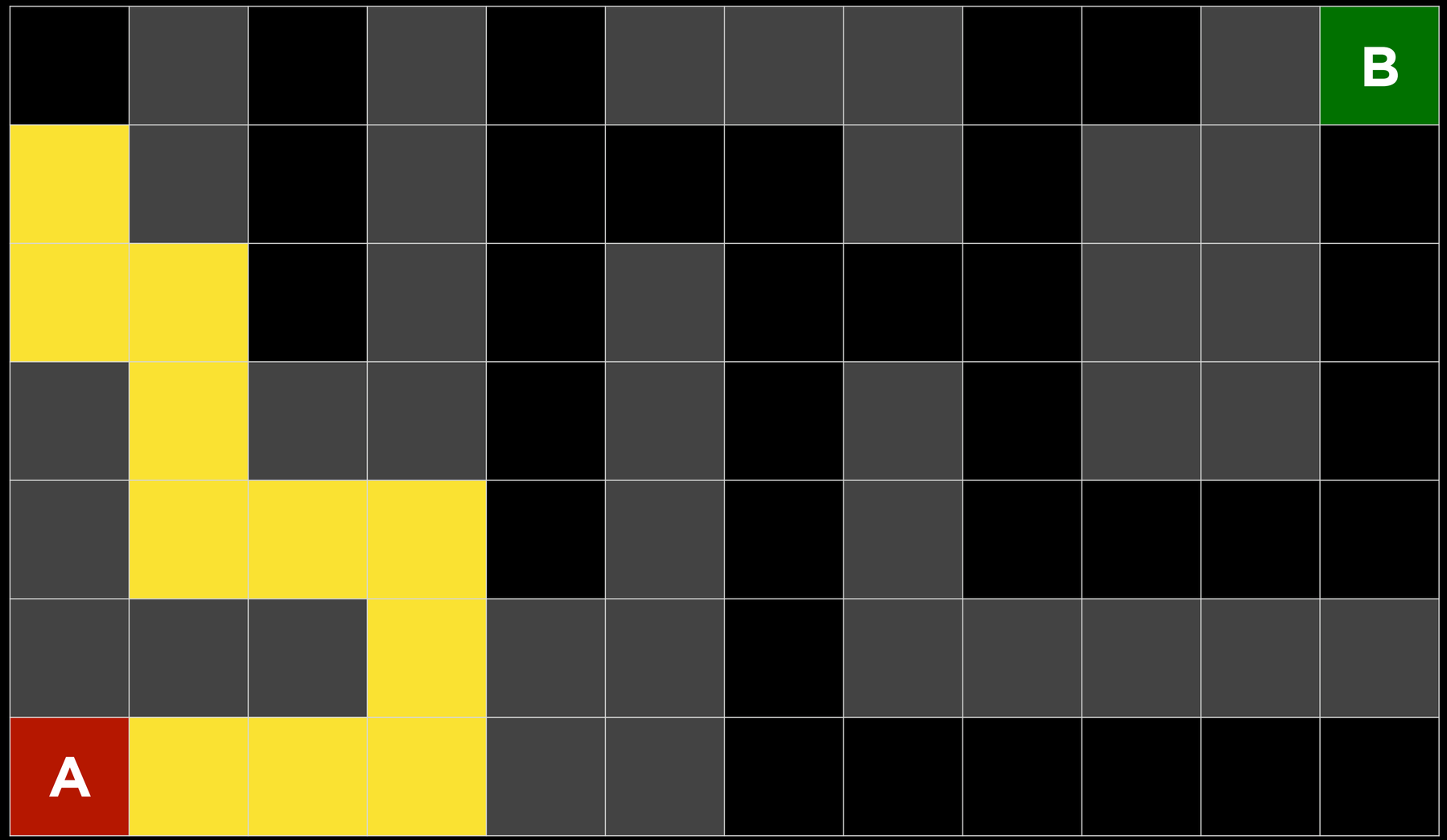
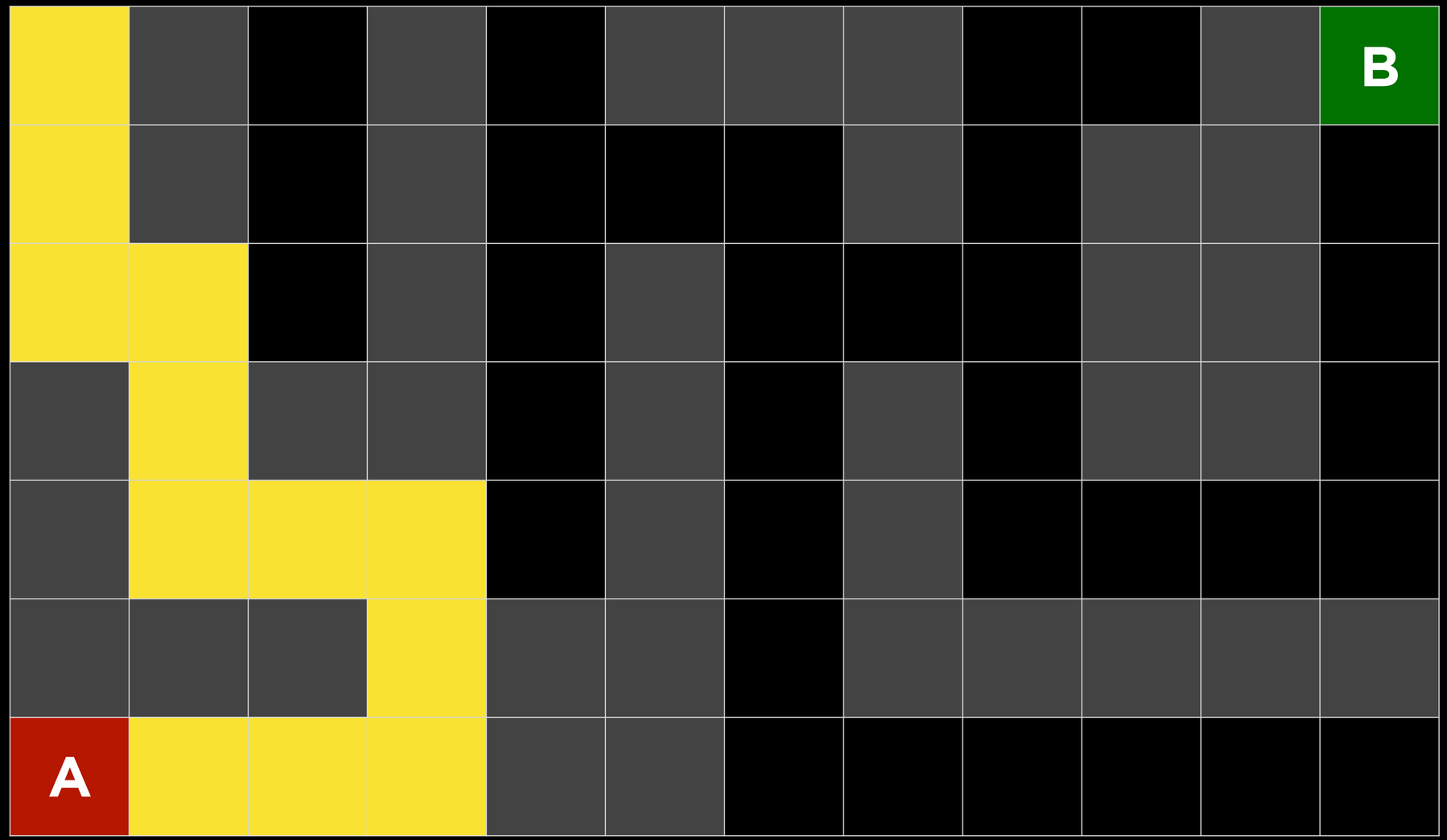


Depth-First Search



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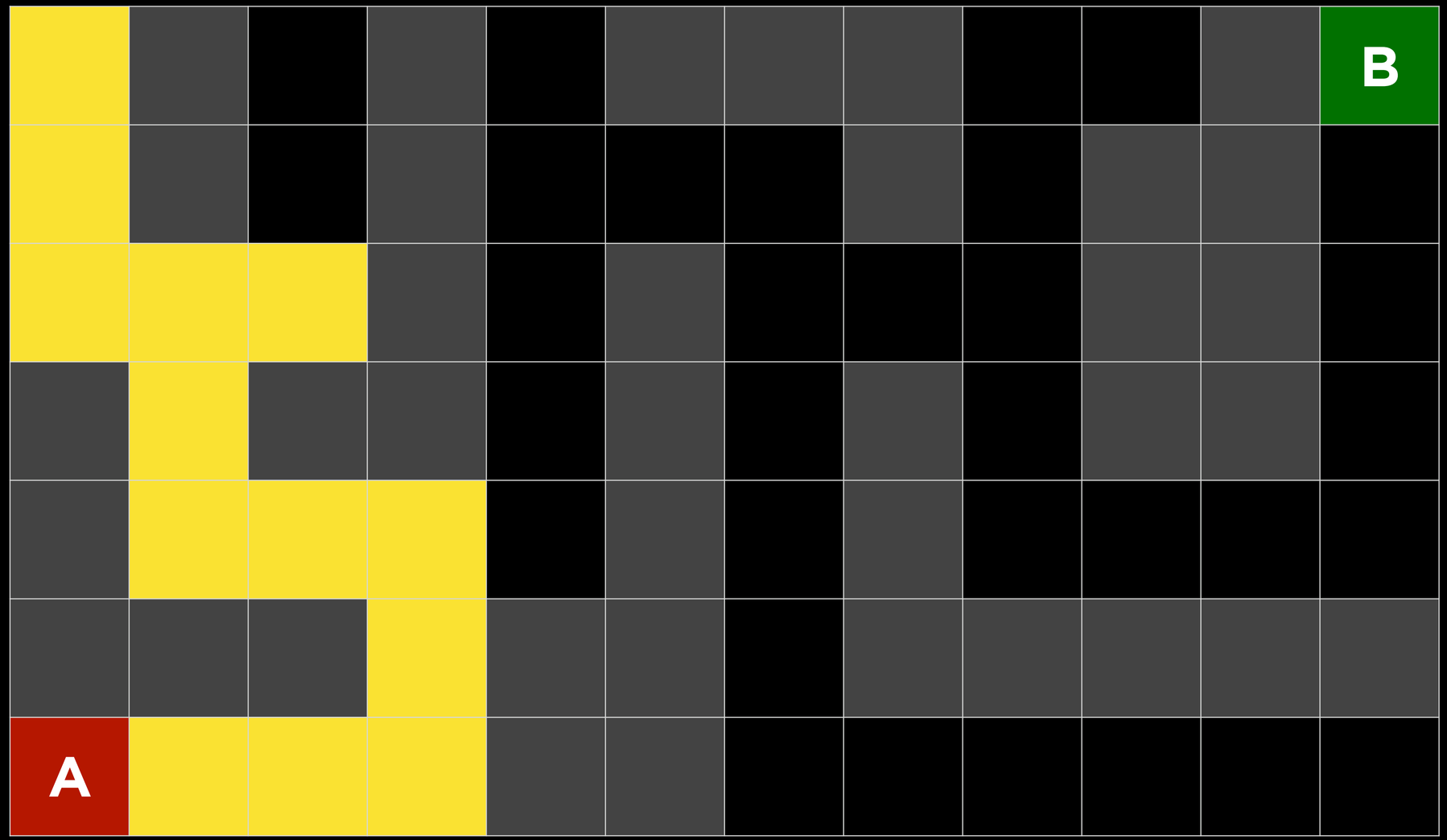

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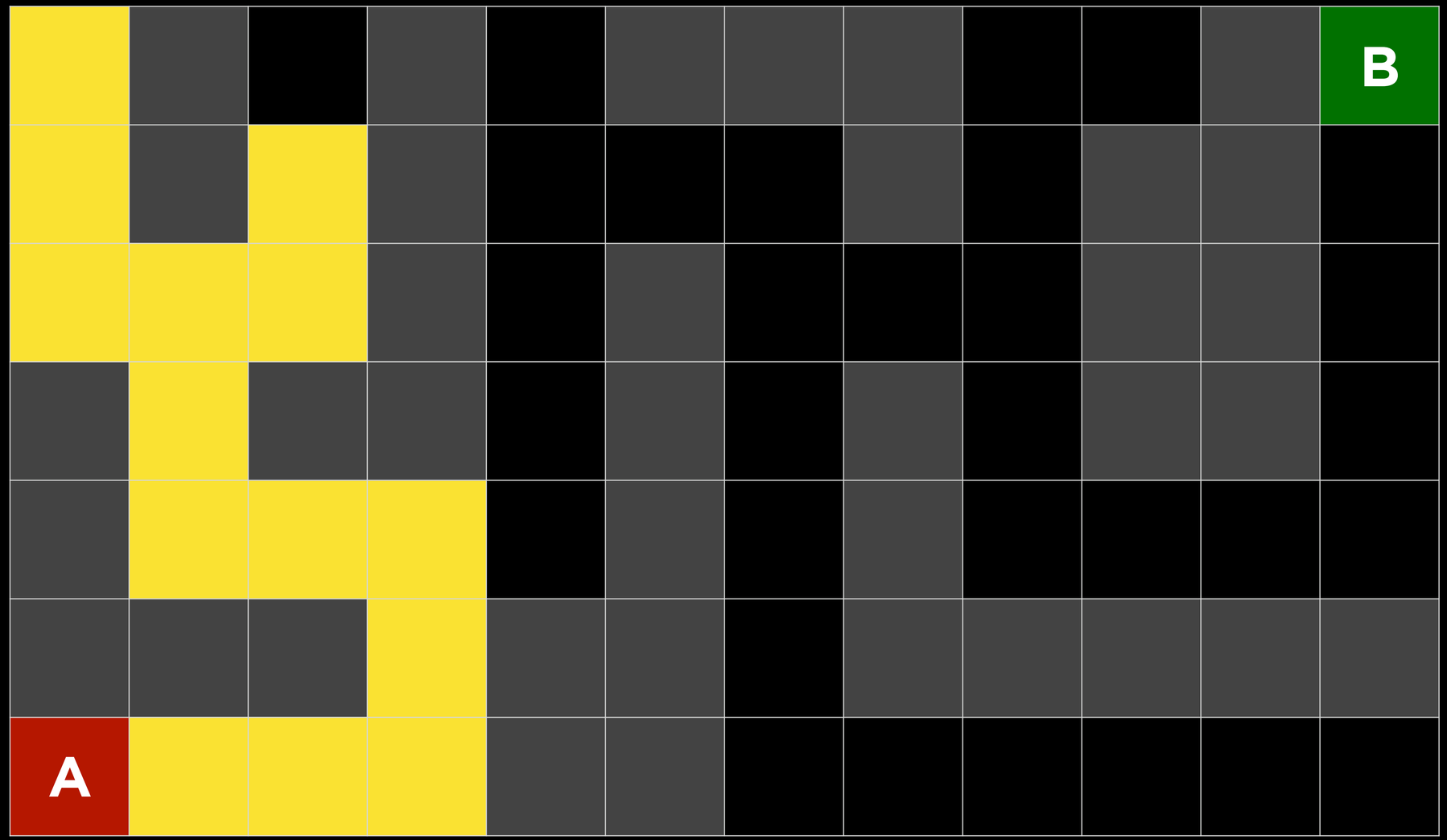

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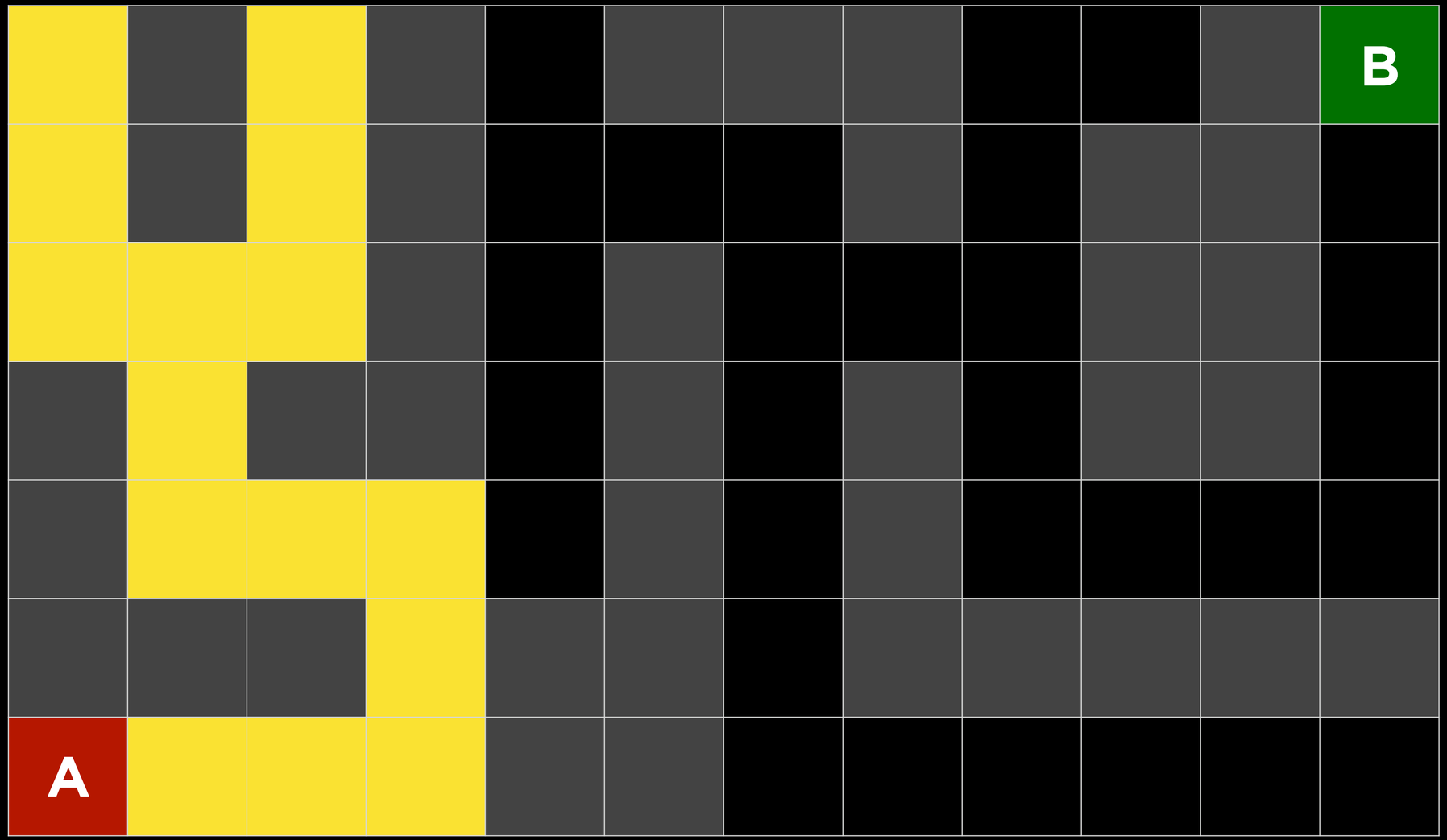

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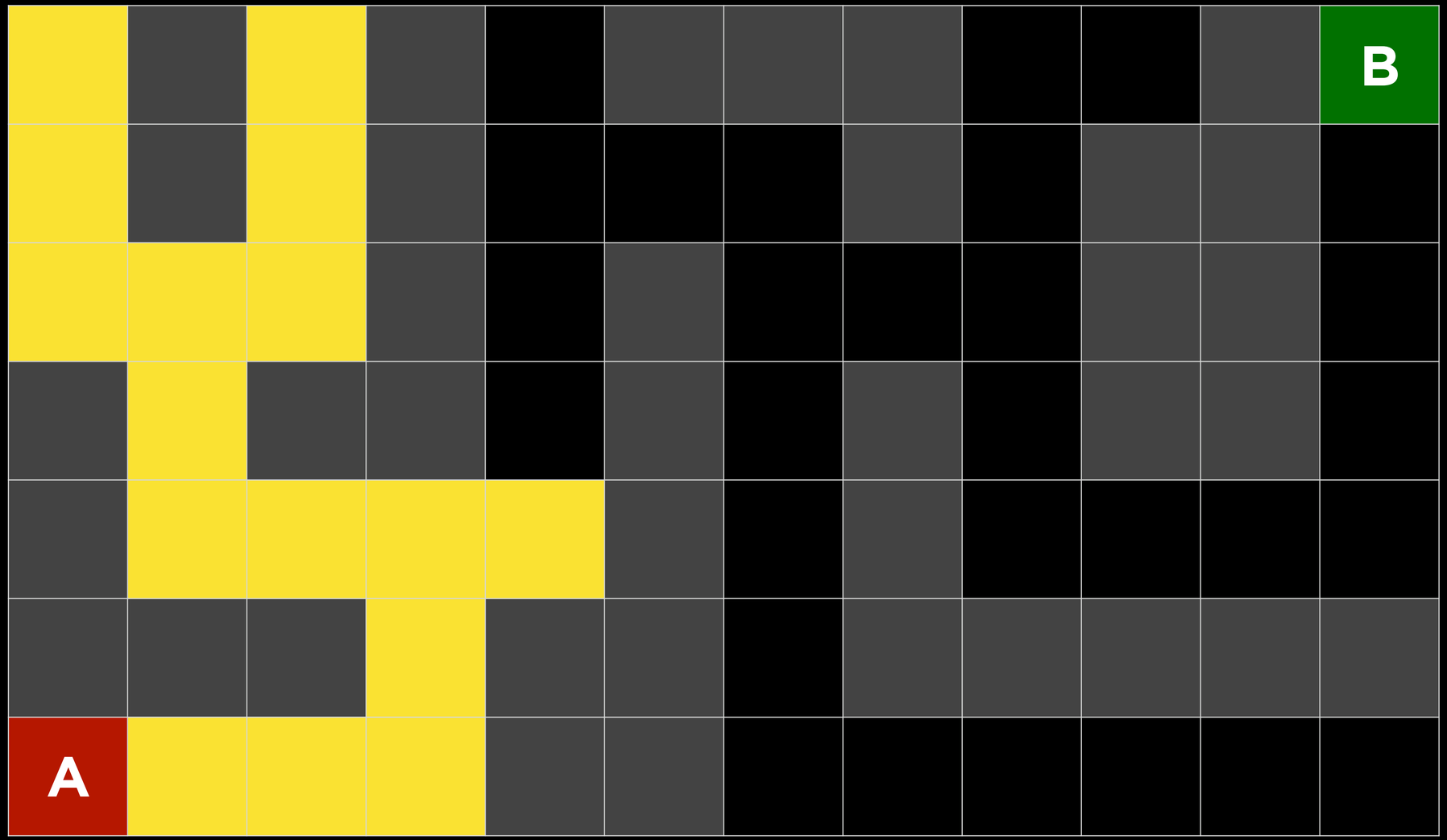

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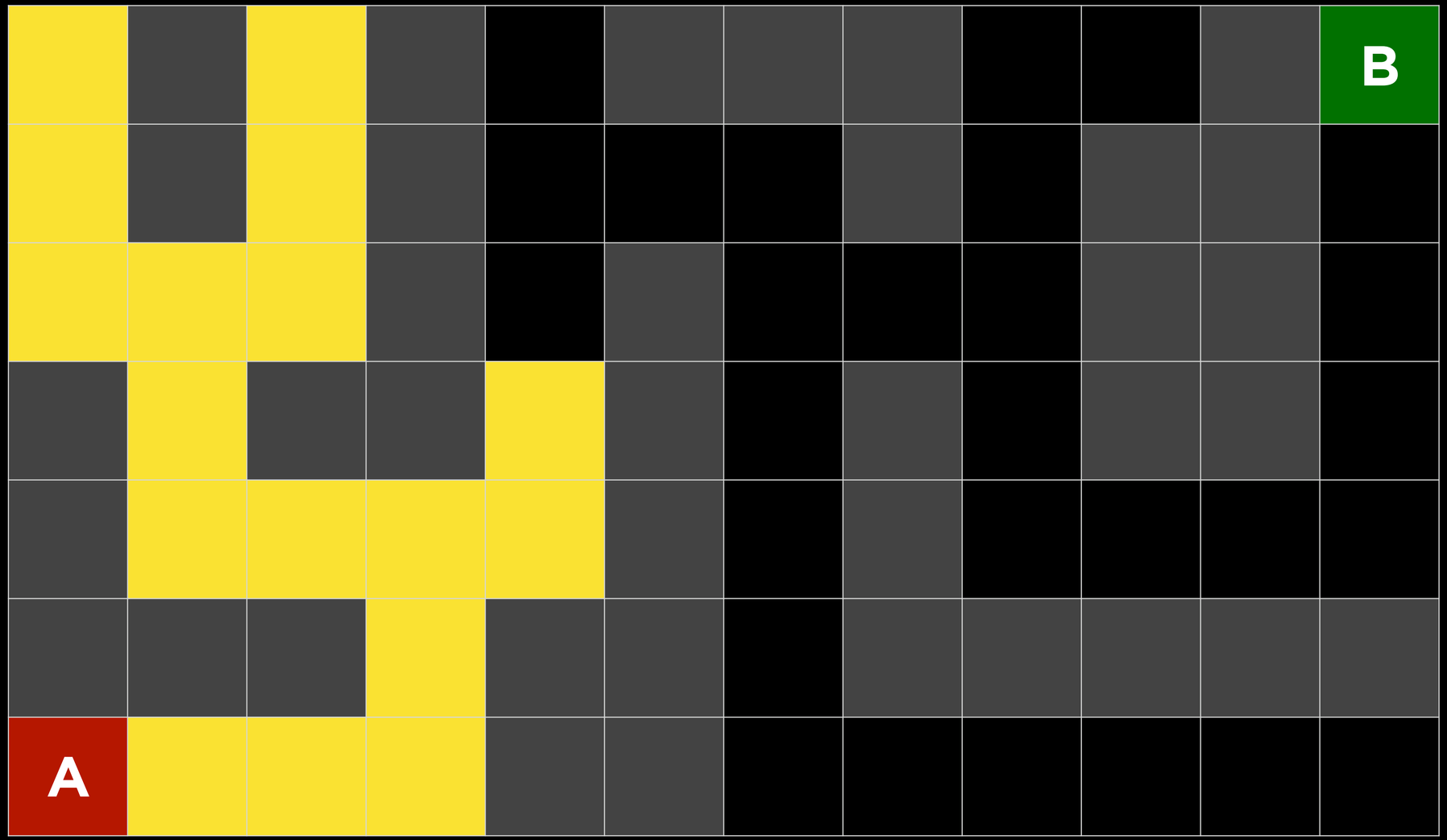
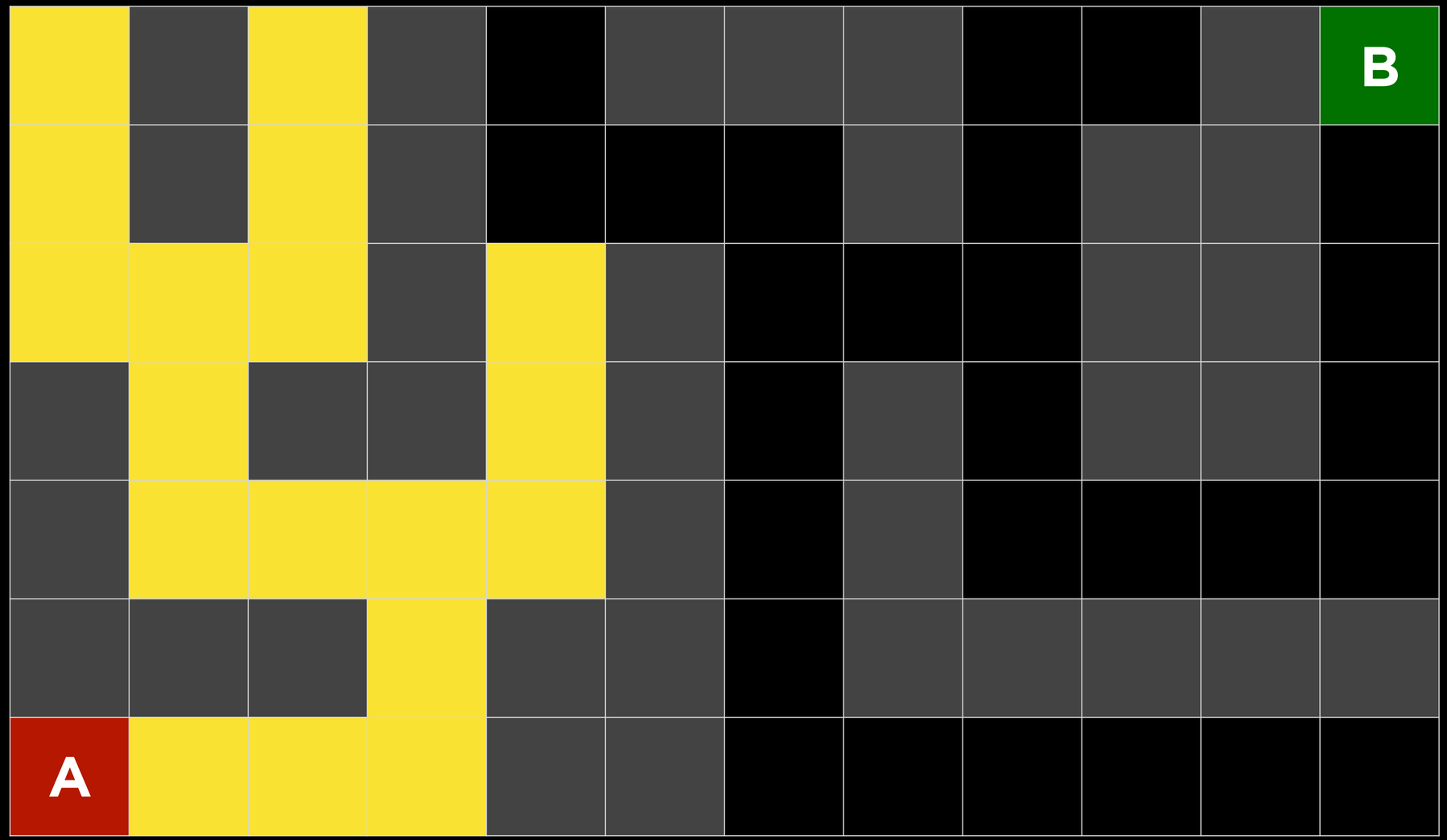
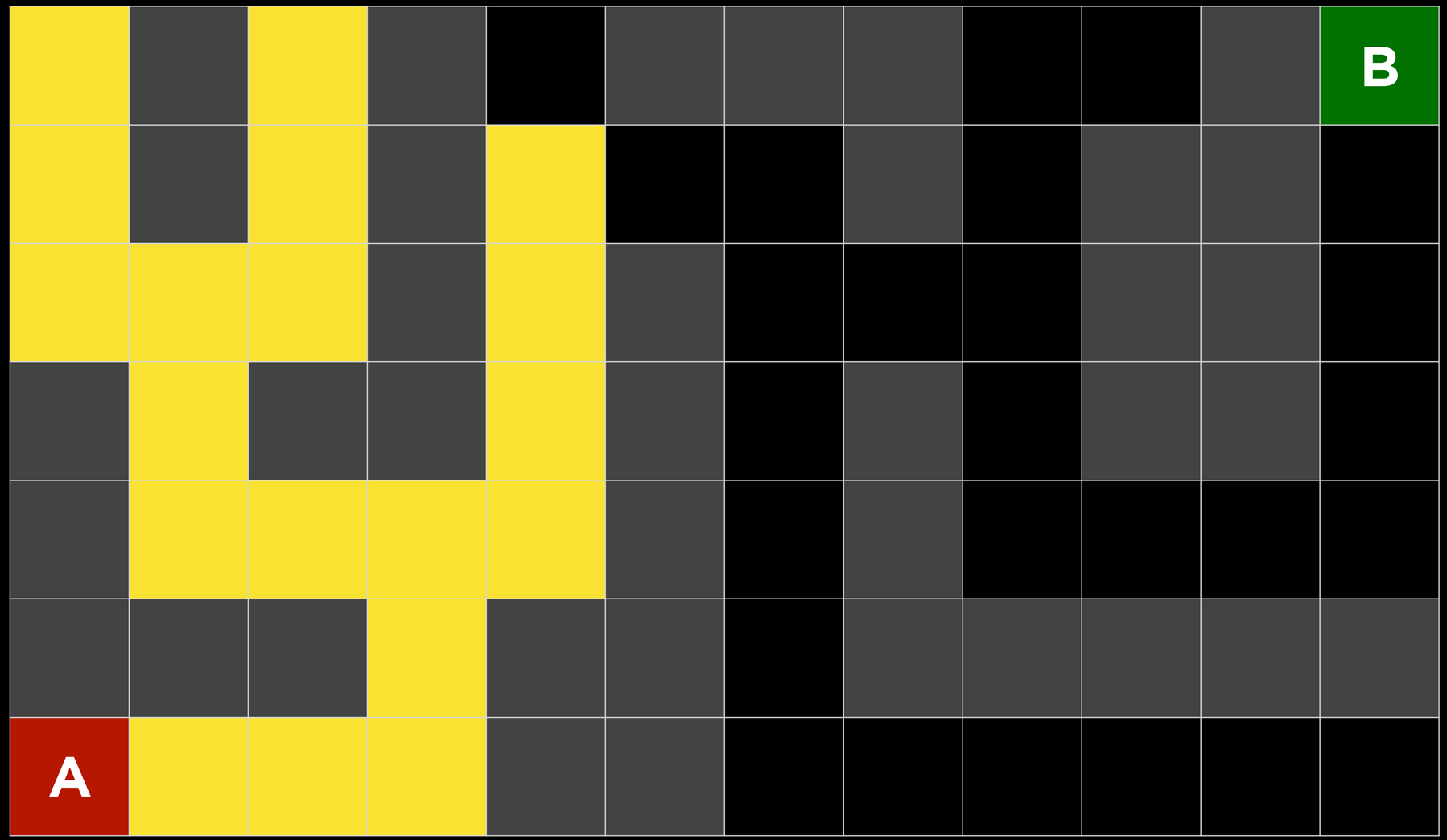


Depth-First Search



Depth-First Search



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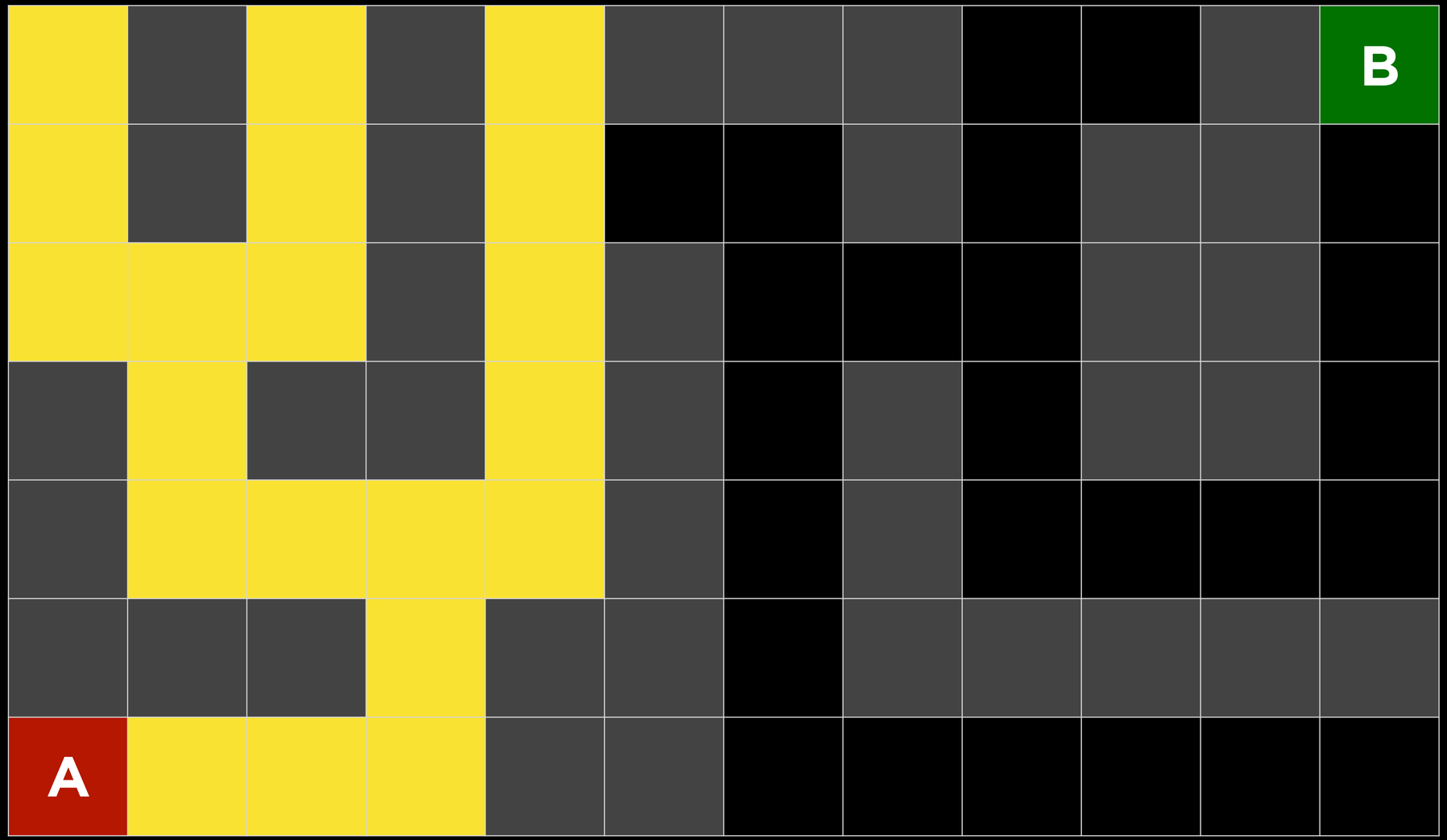

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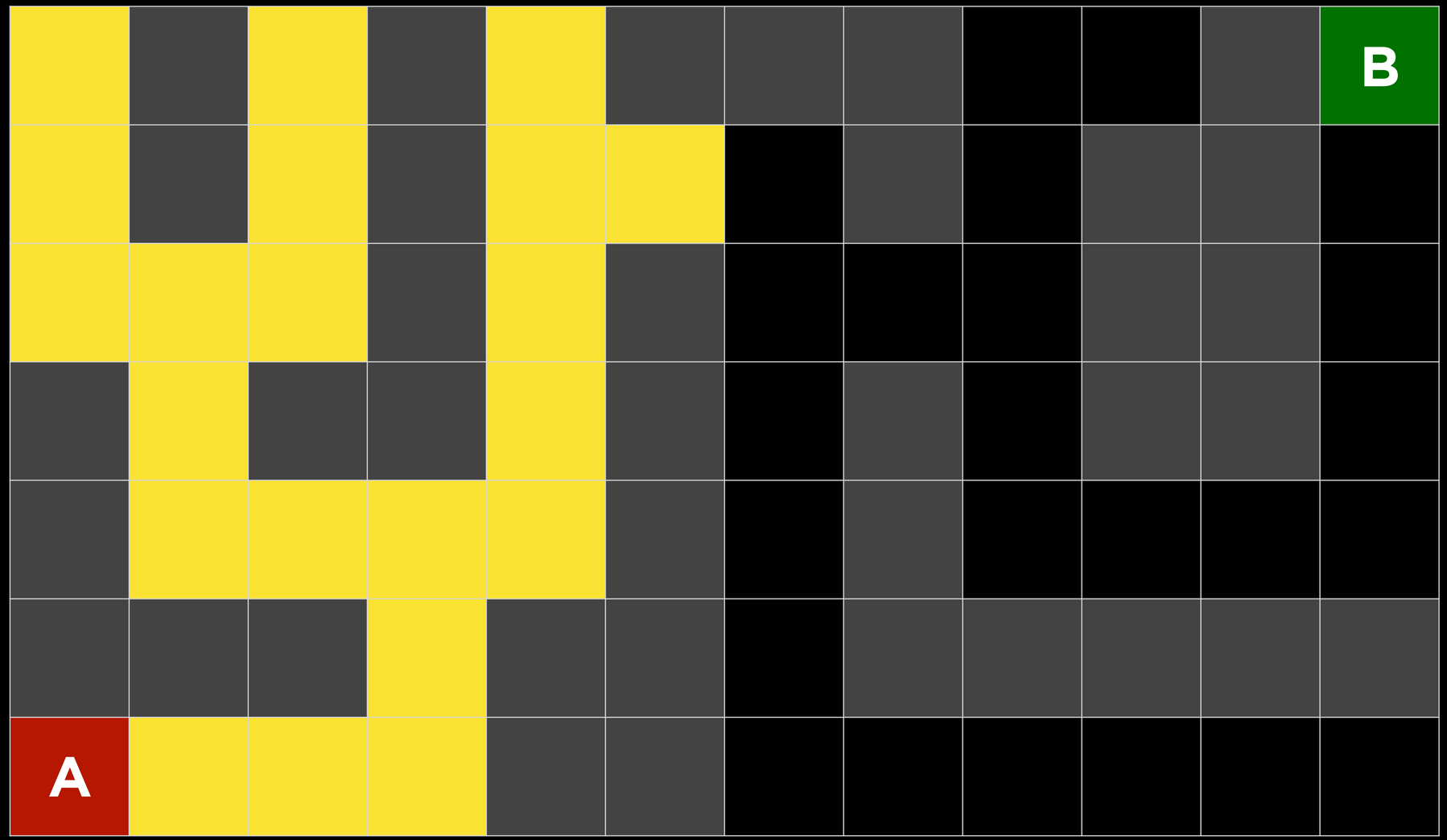

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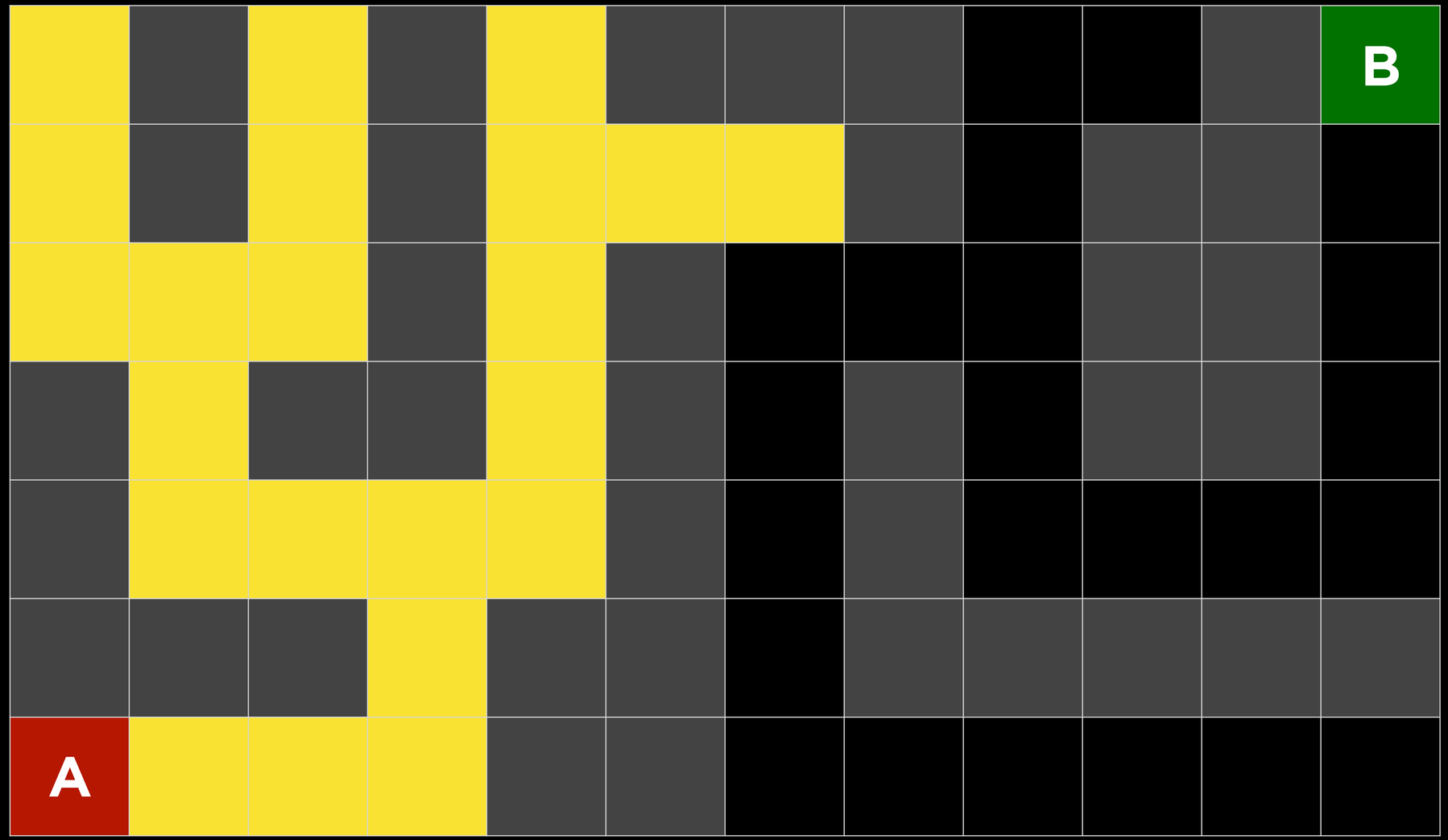
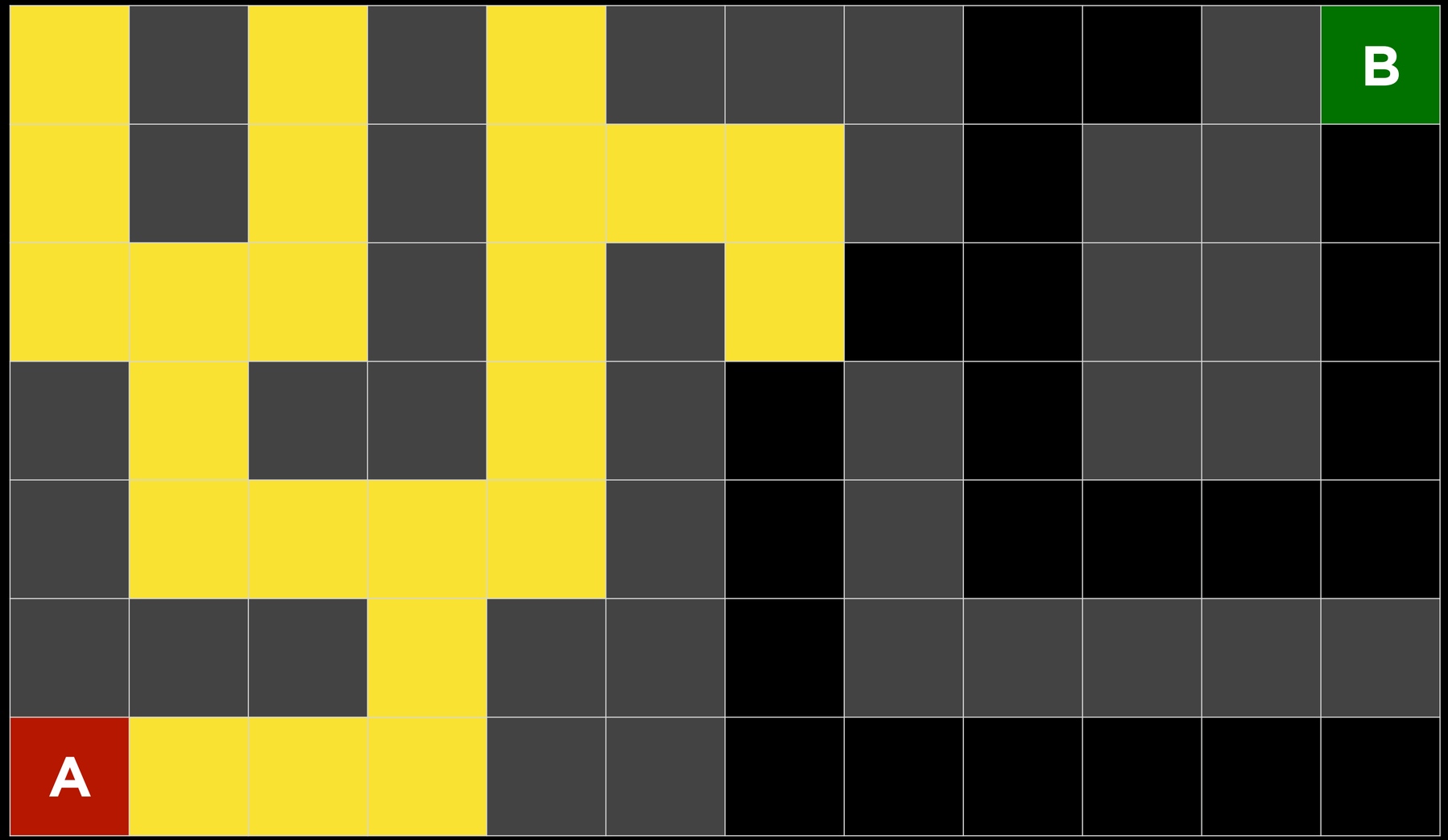
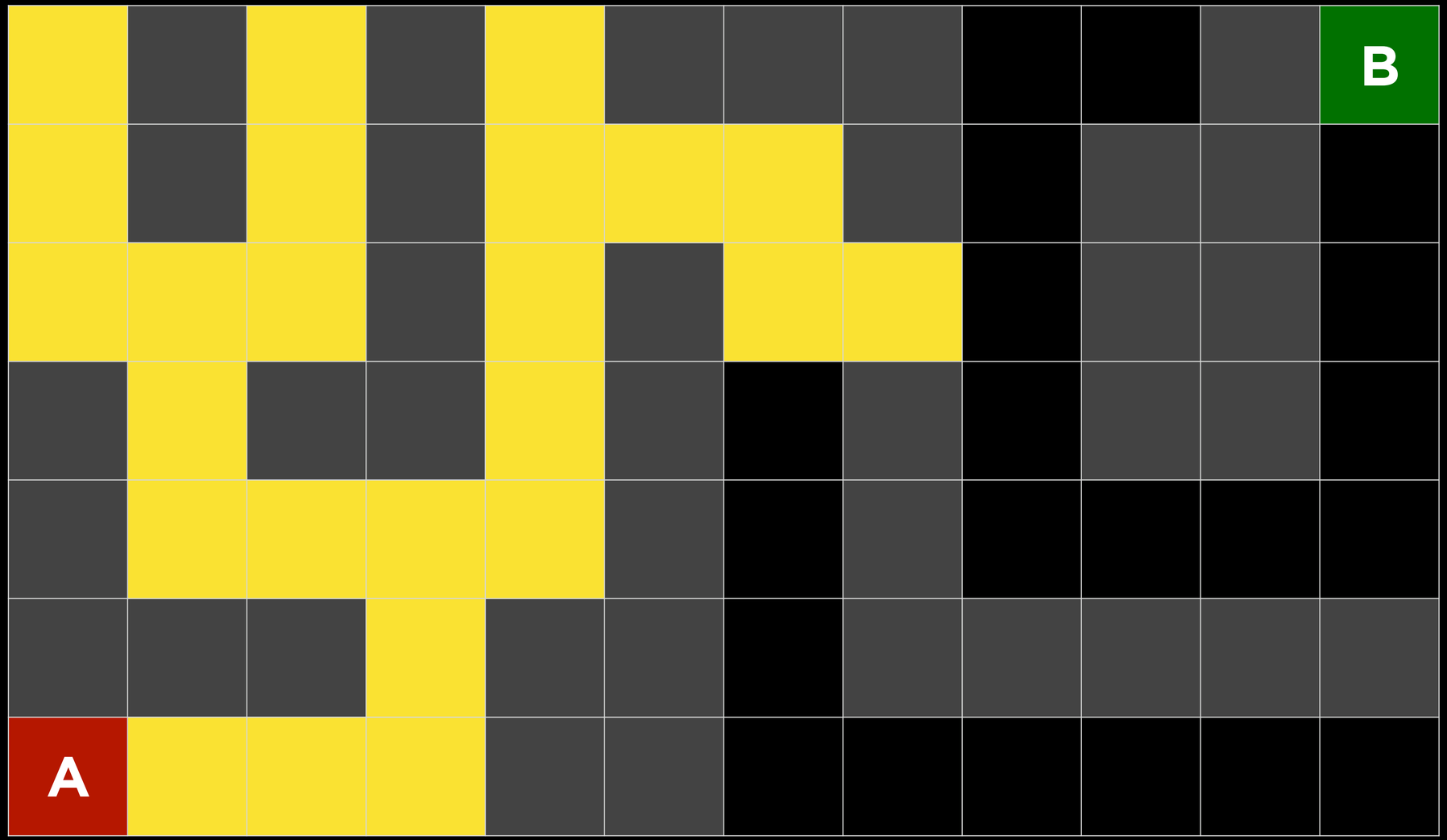


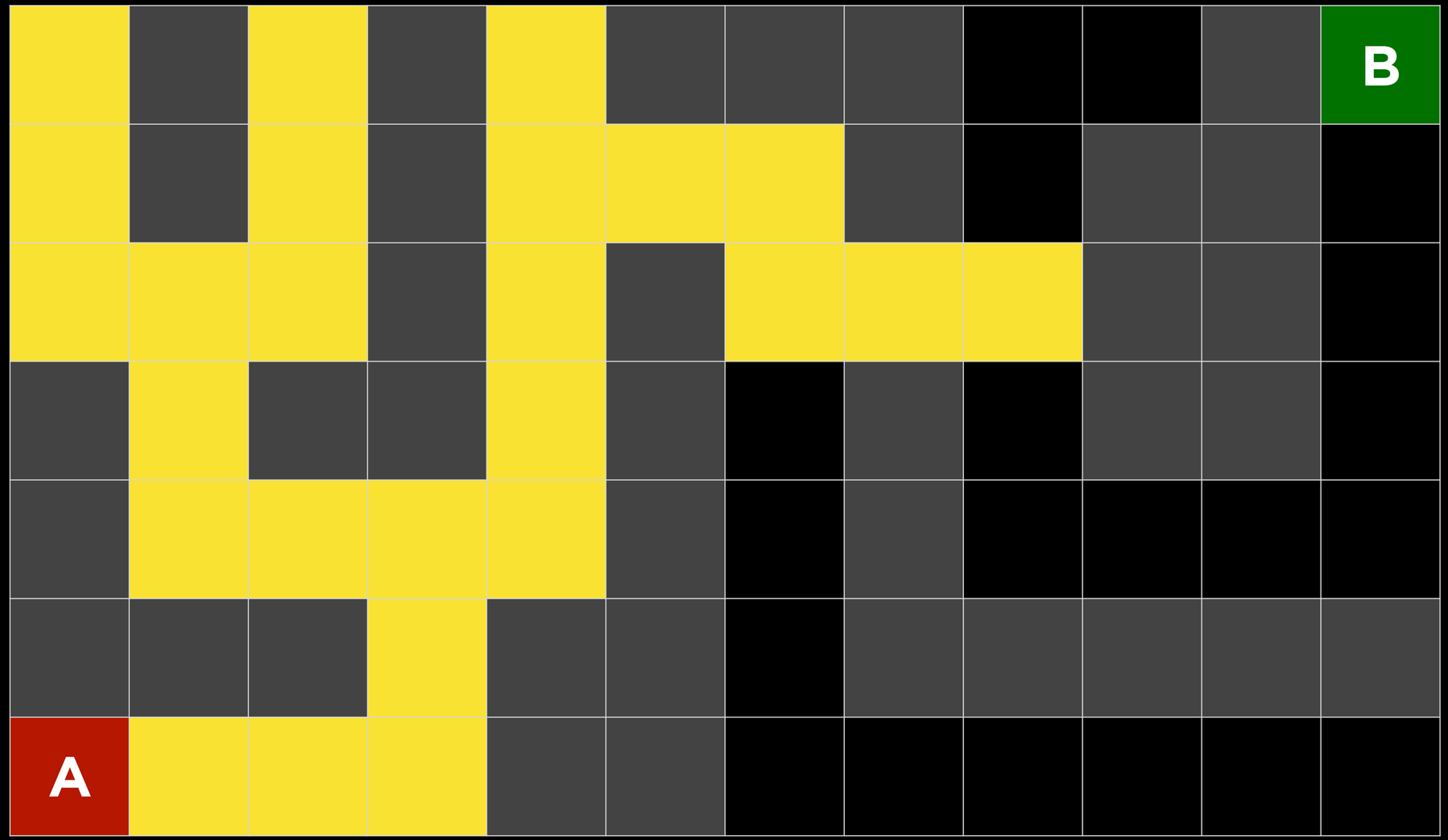
Depth-First Search



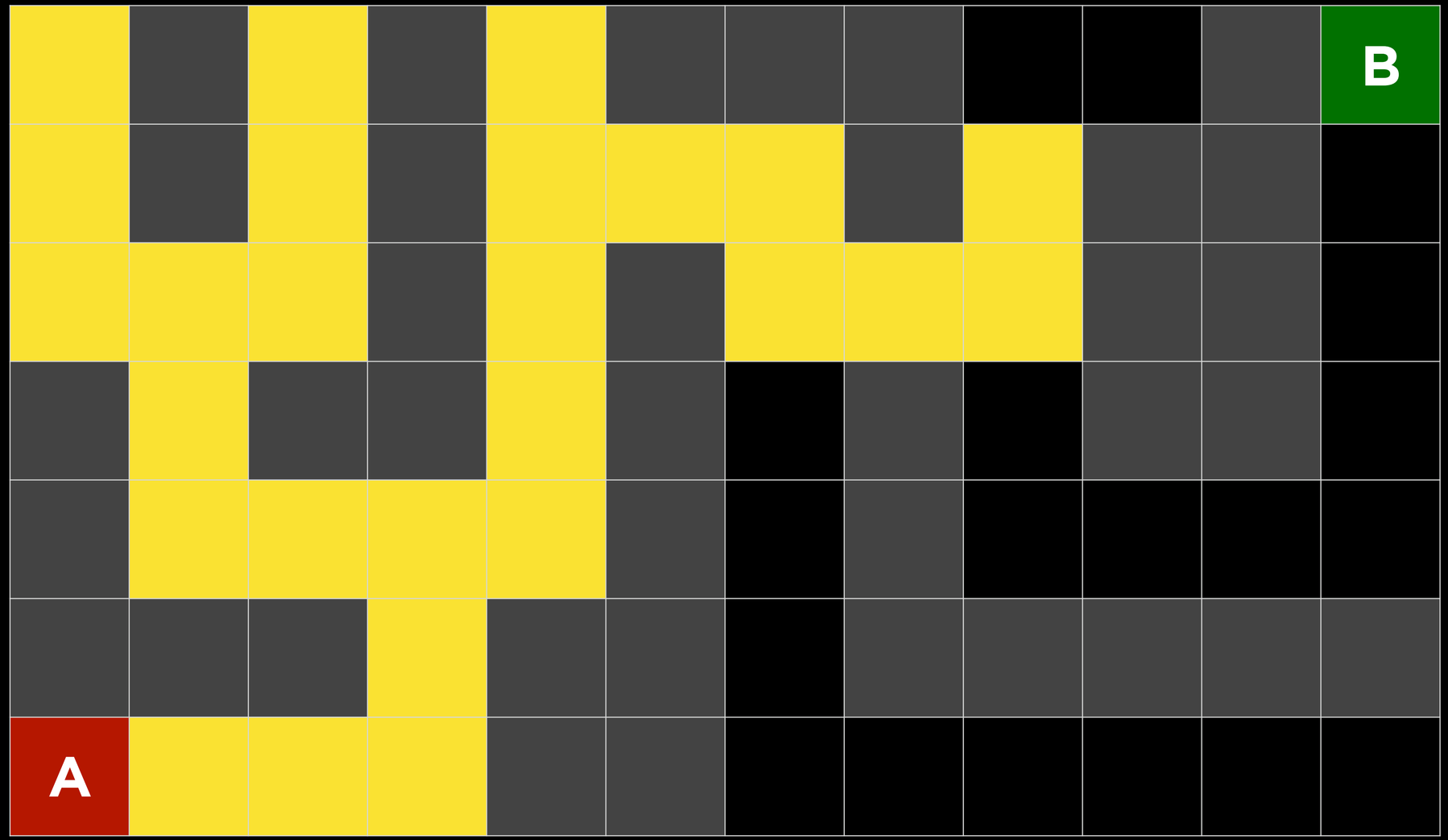
Depth-First Search



Depth-First Search



Depth-First Search



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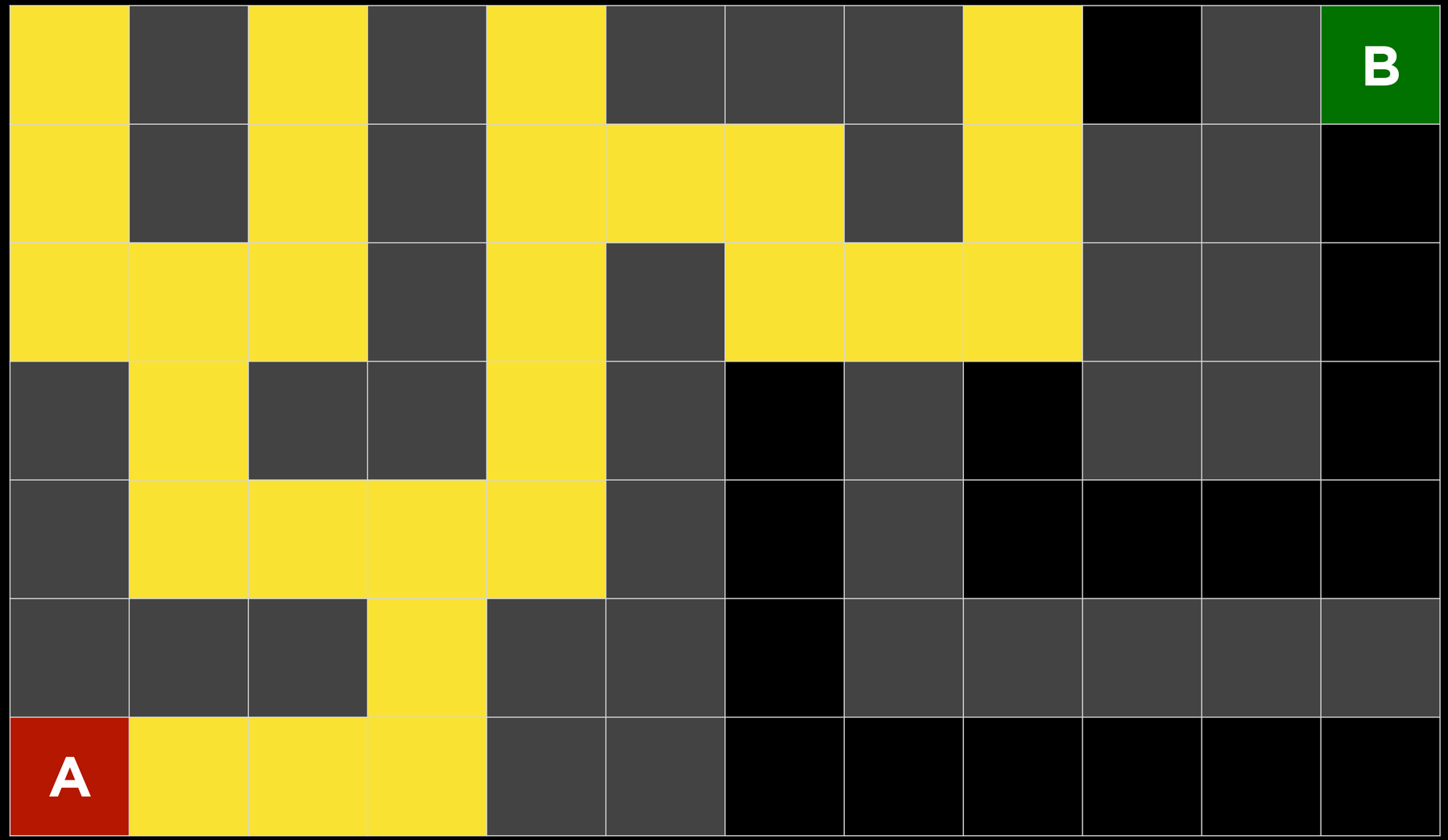
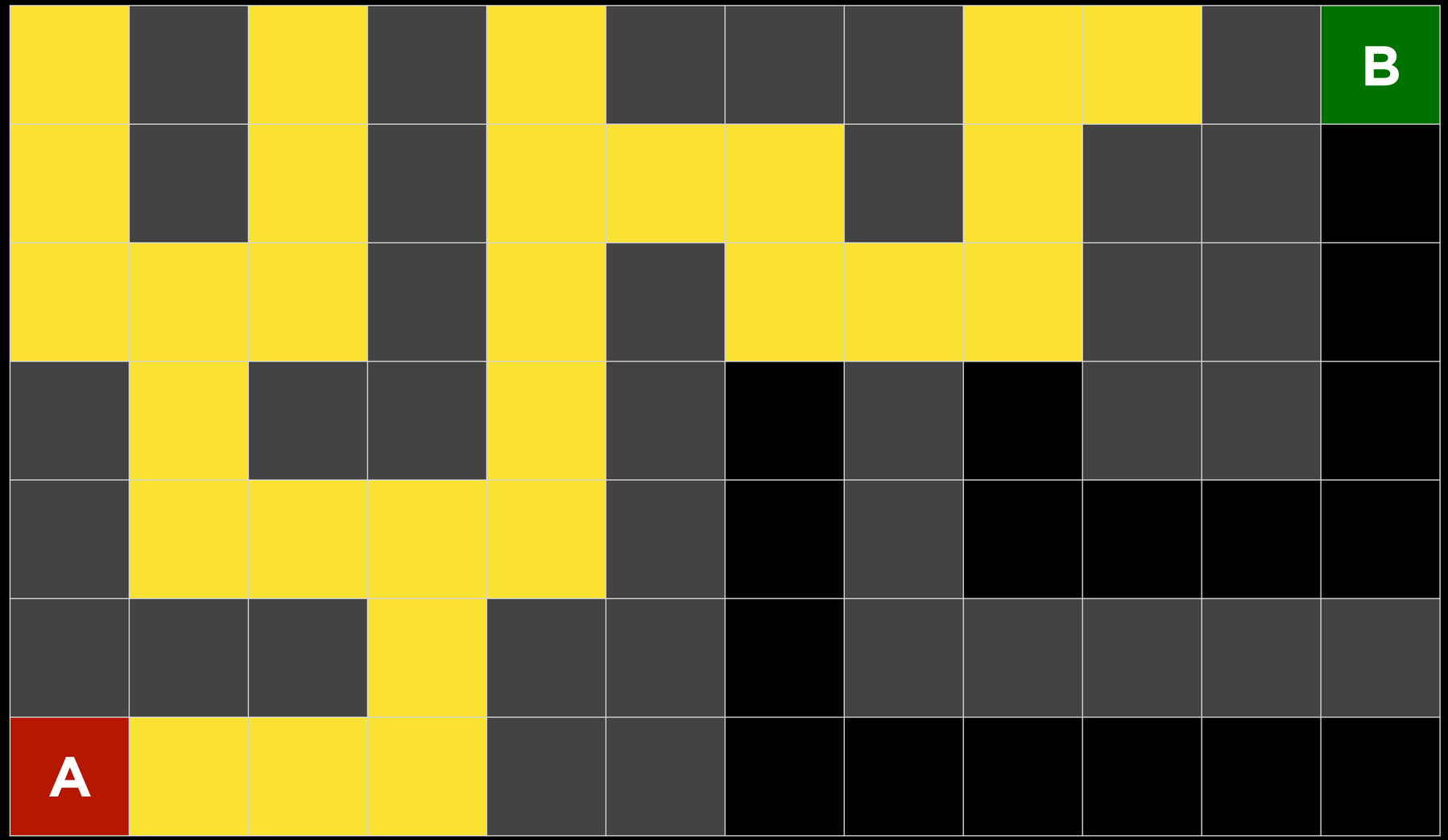


Depth-First Search



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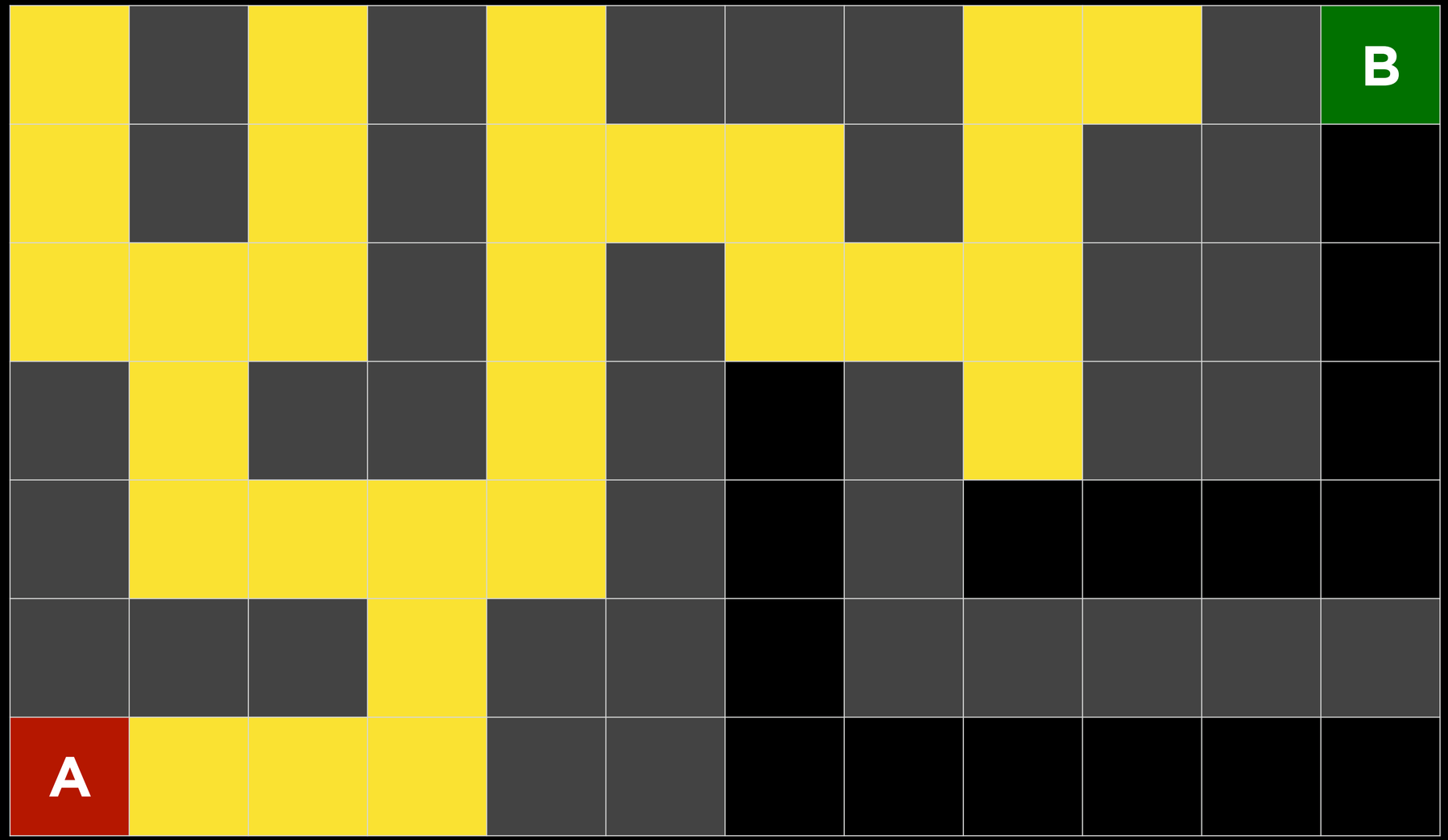

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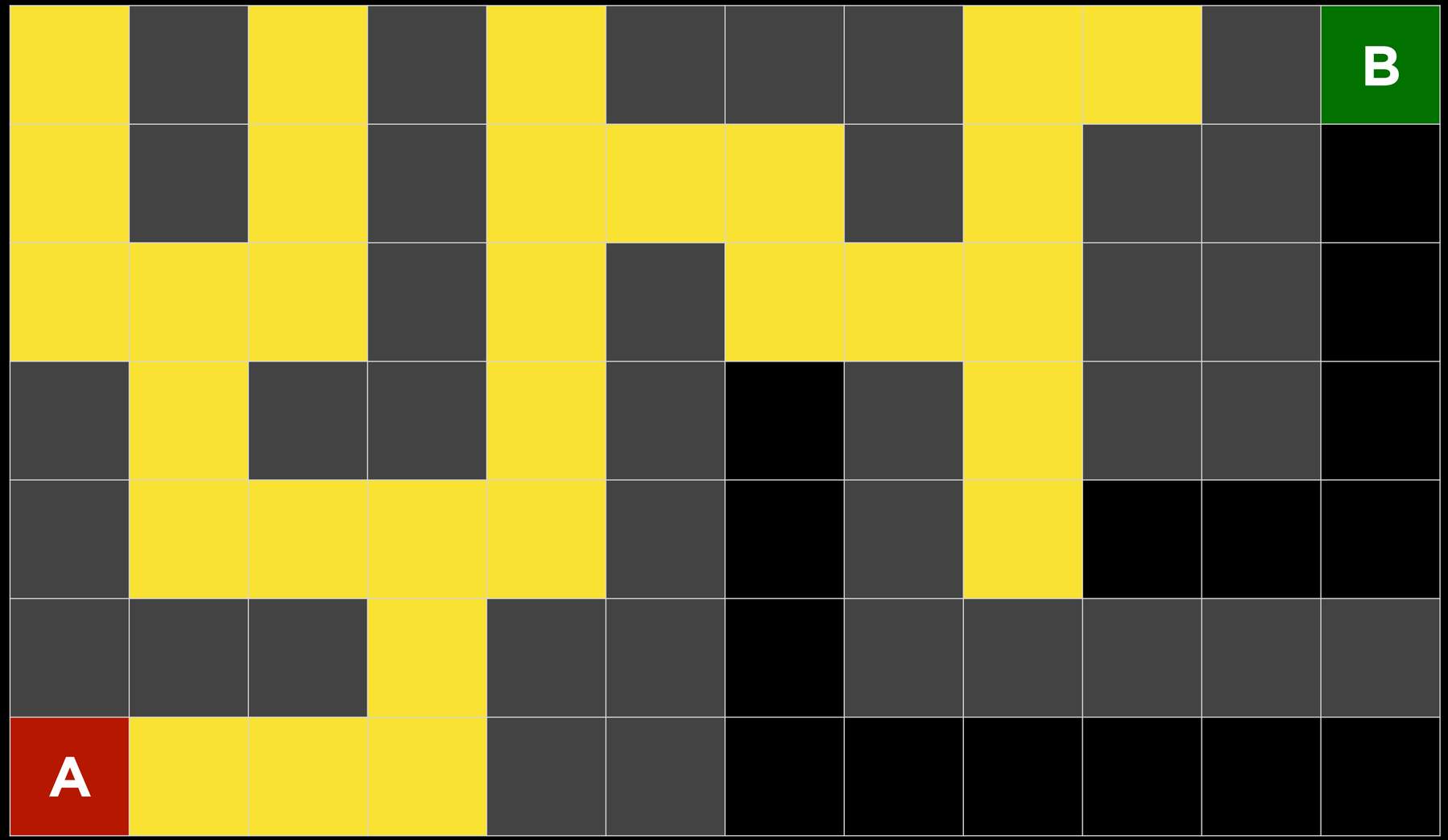

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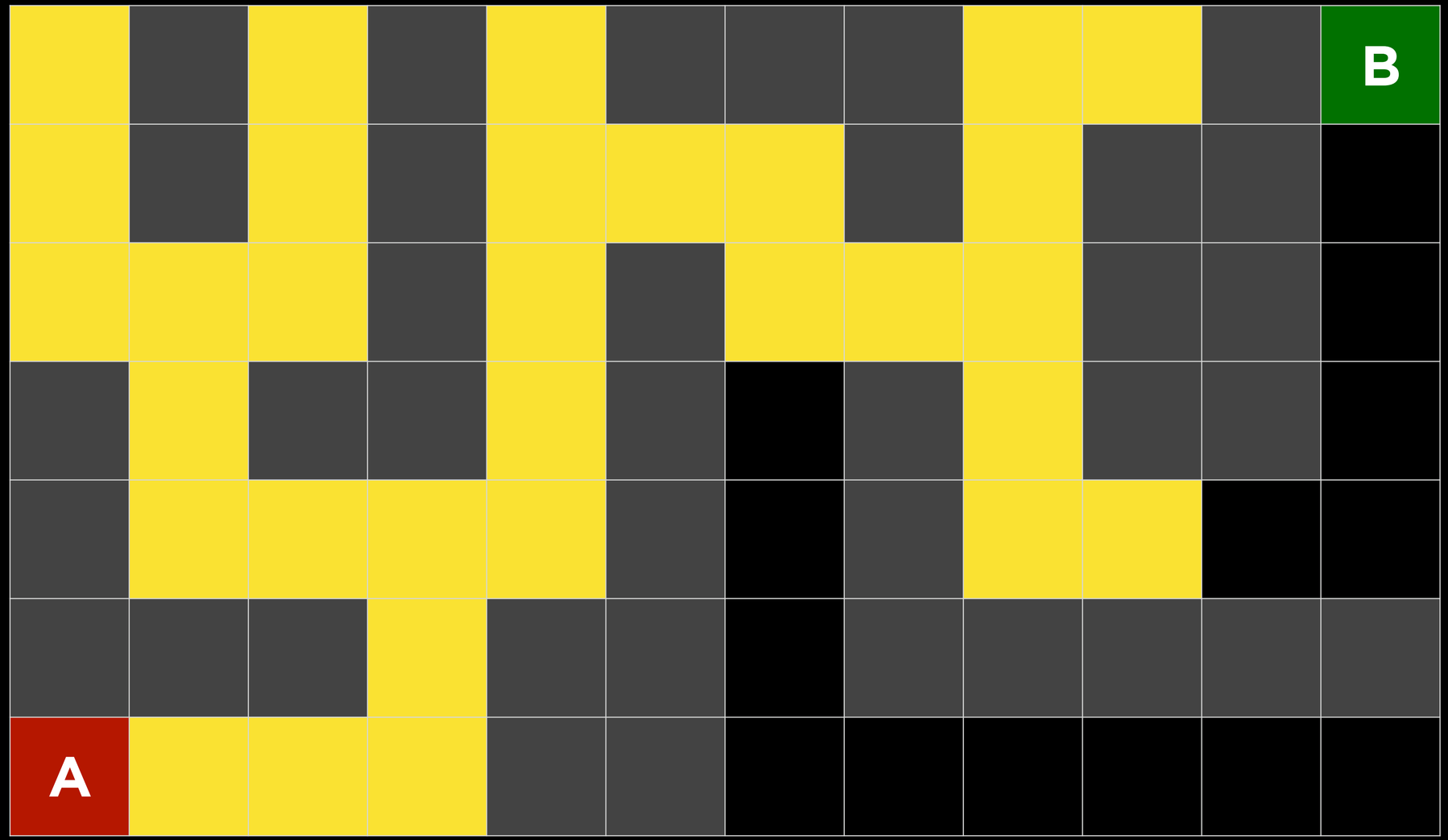

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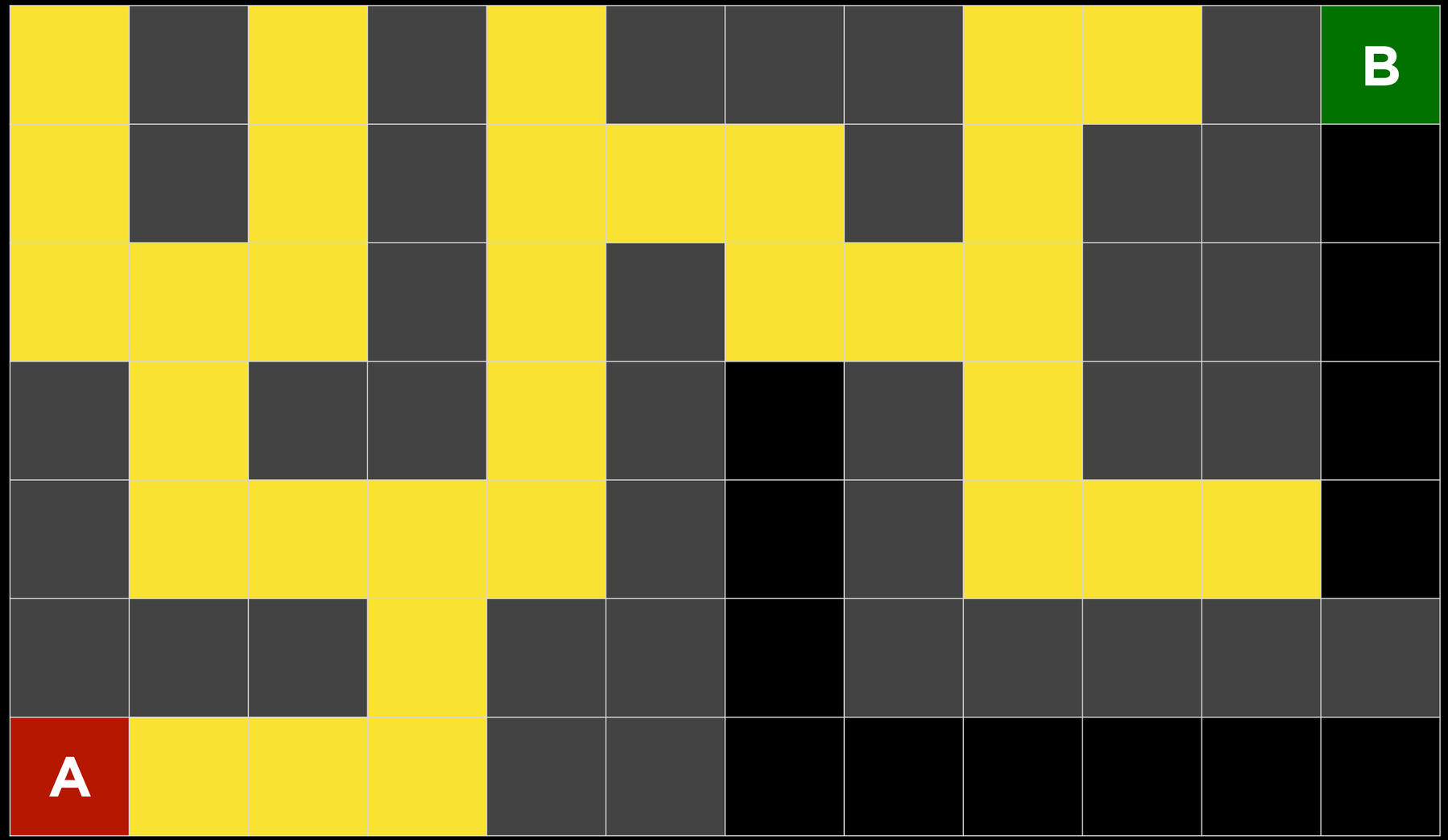

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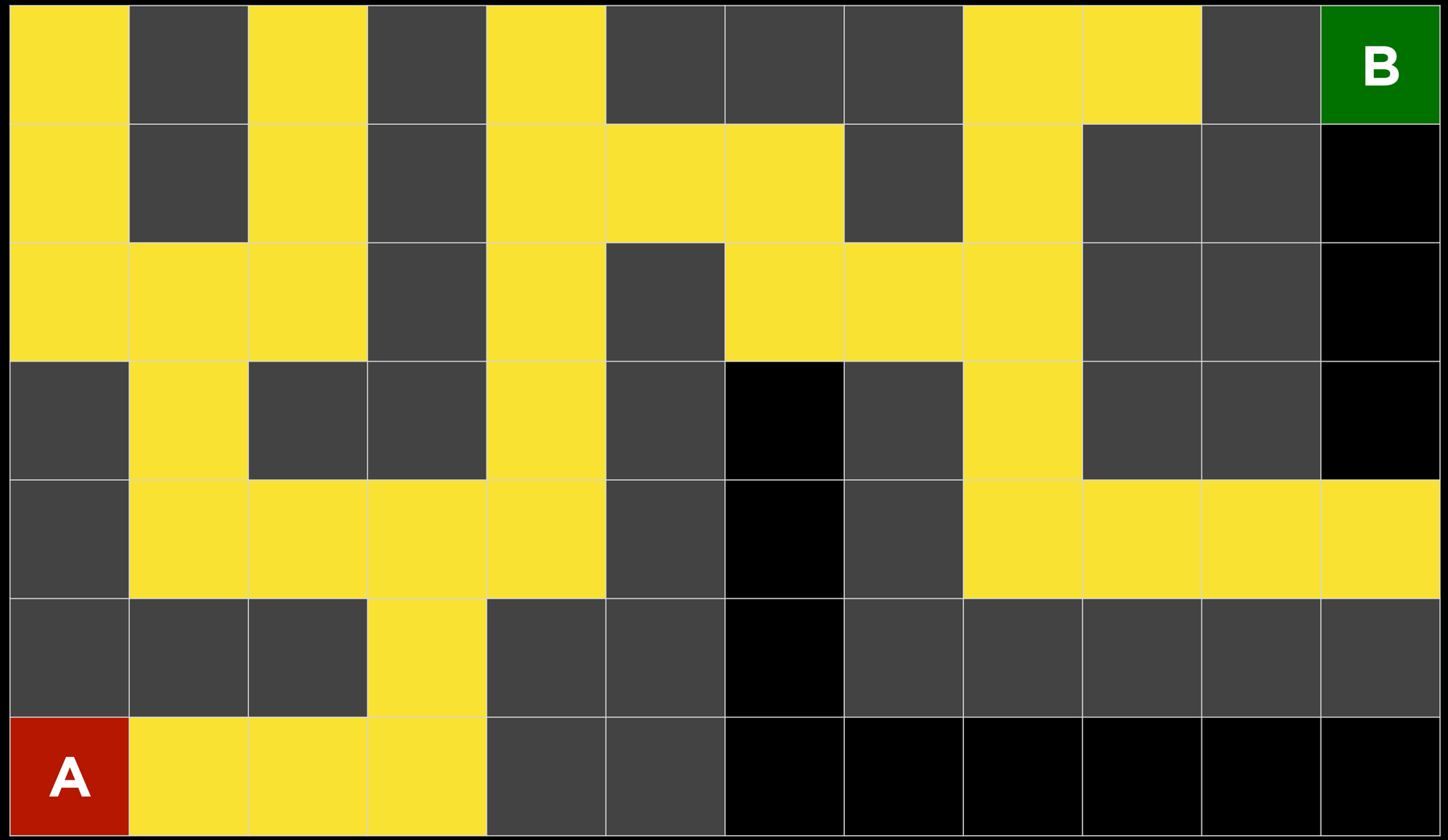

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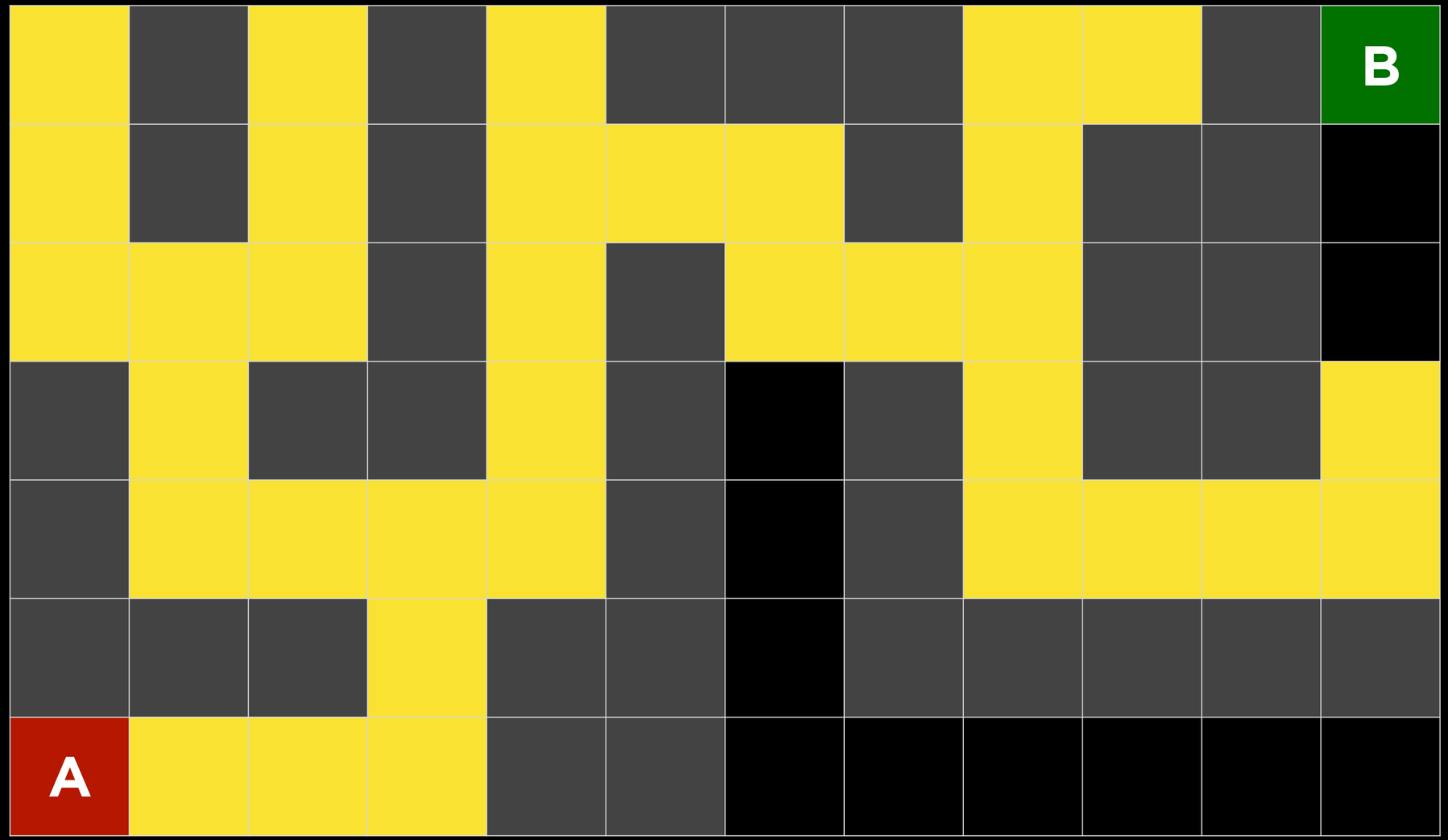

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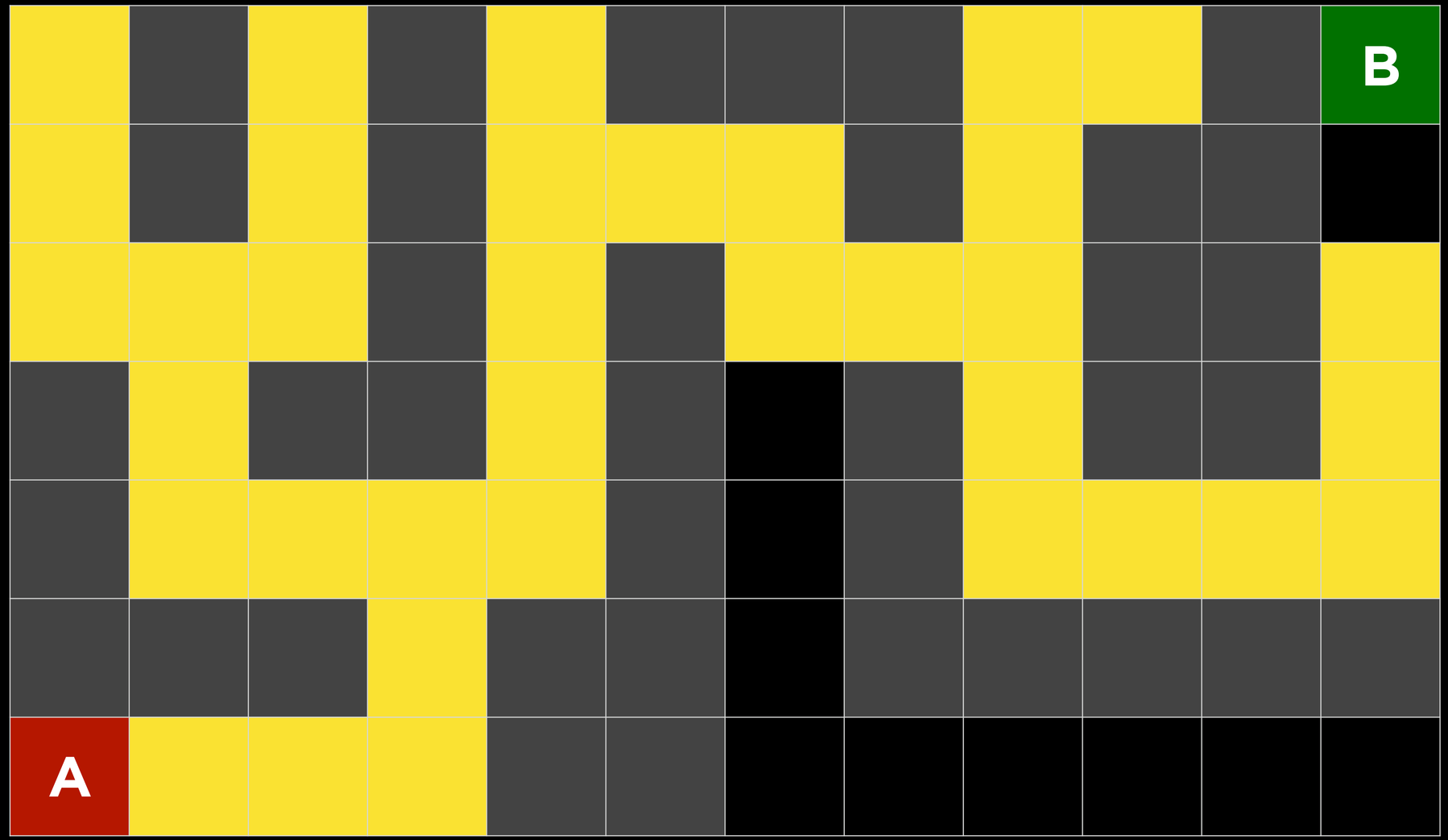

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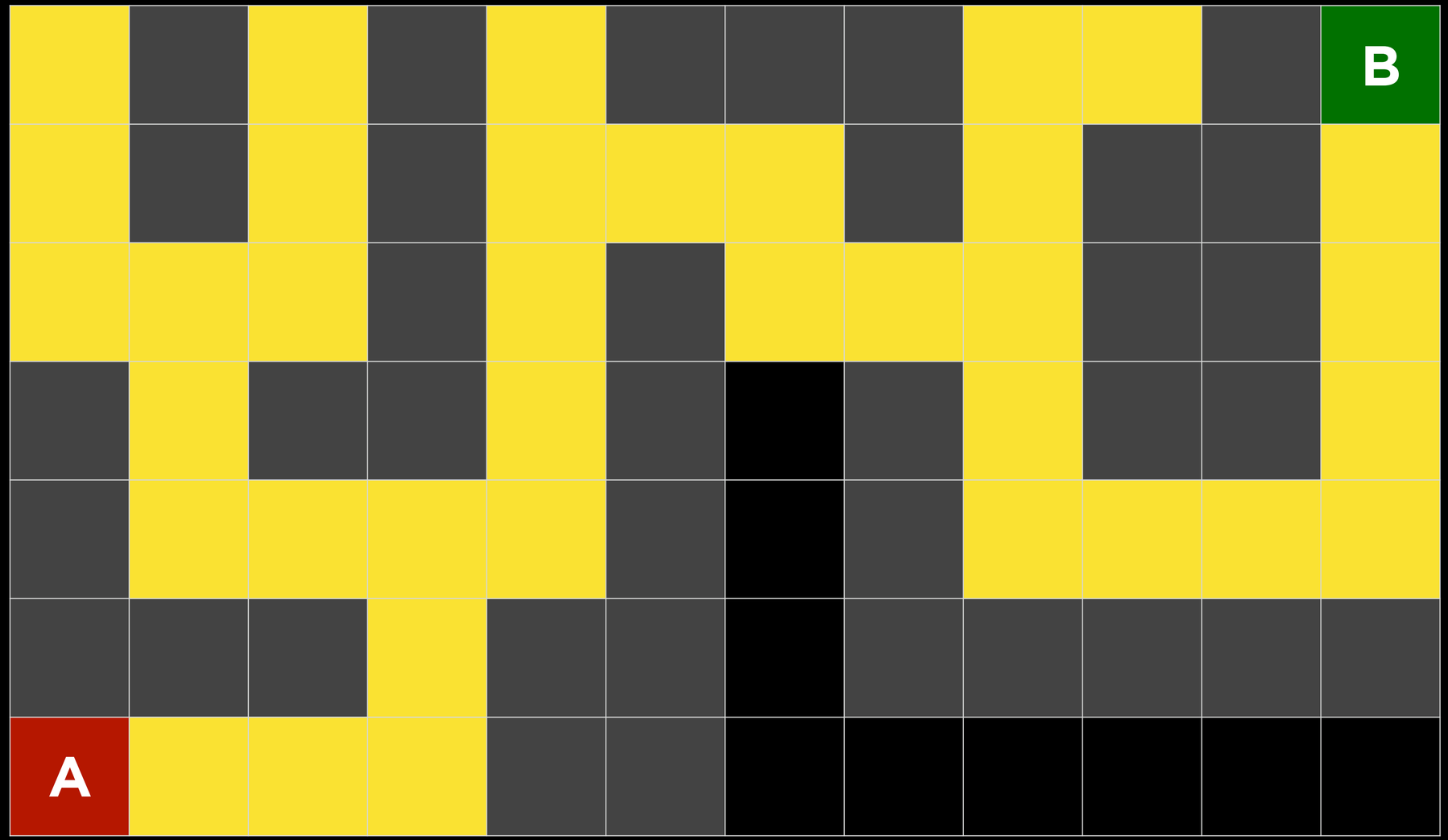

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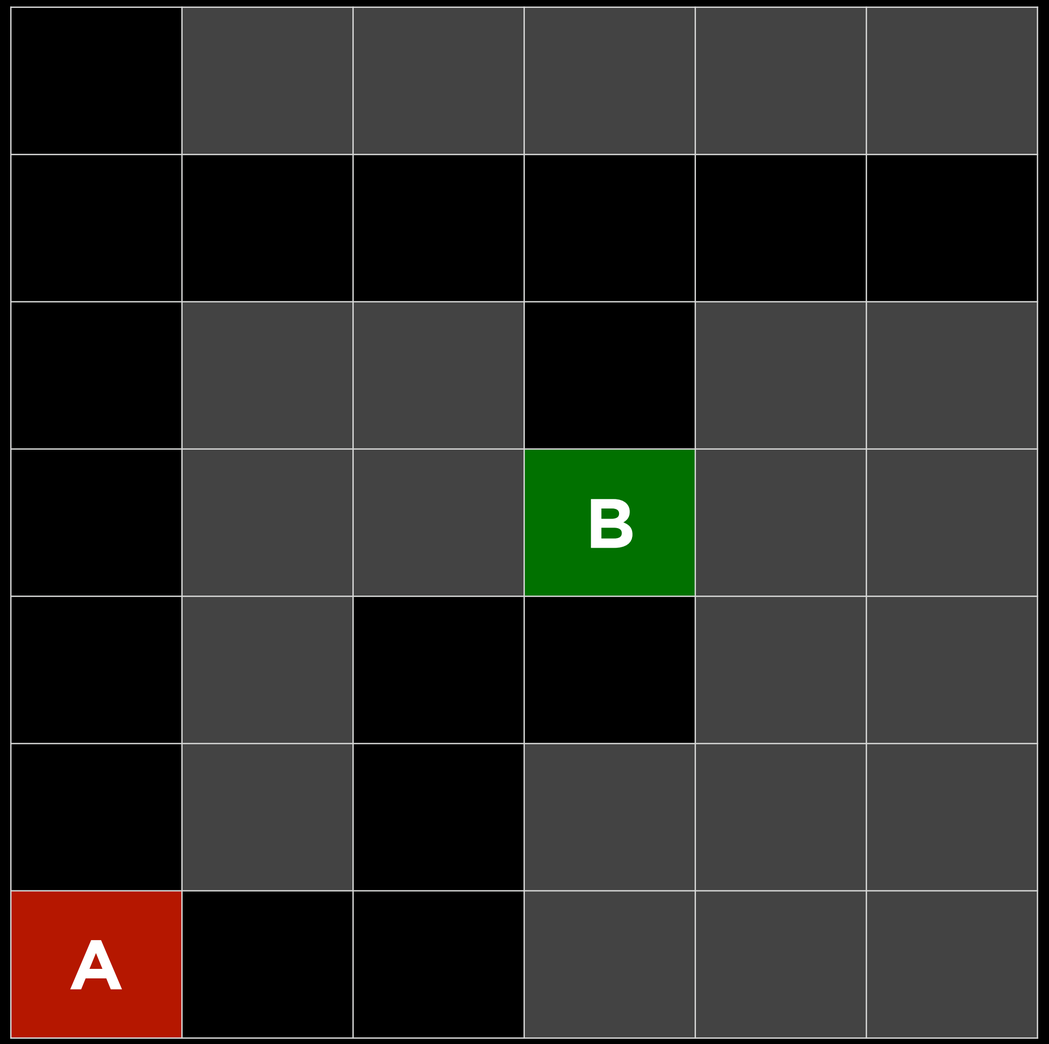

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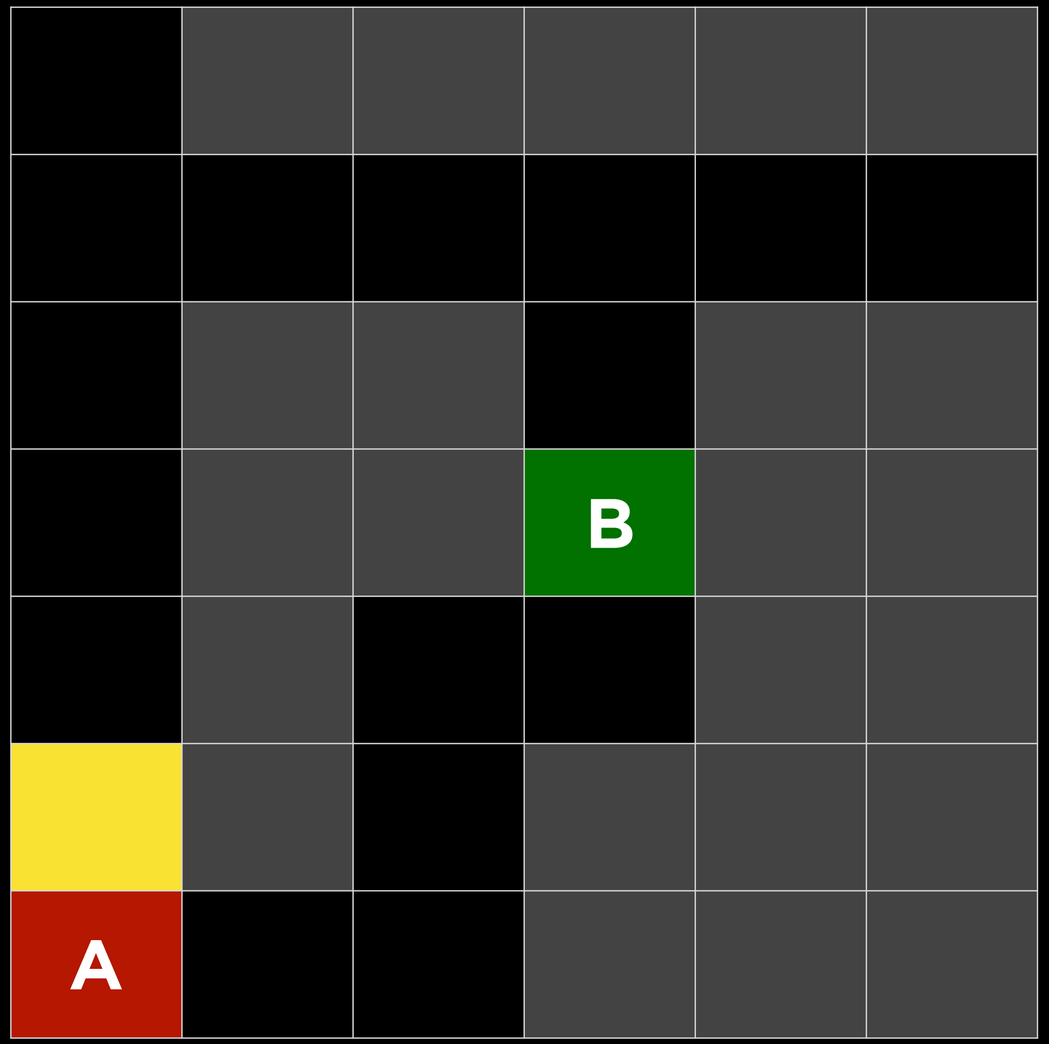

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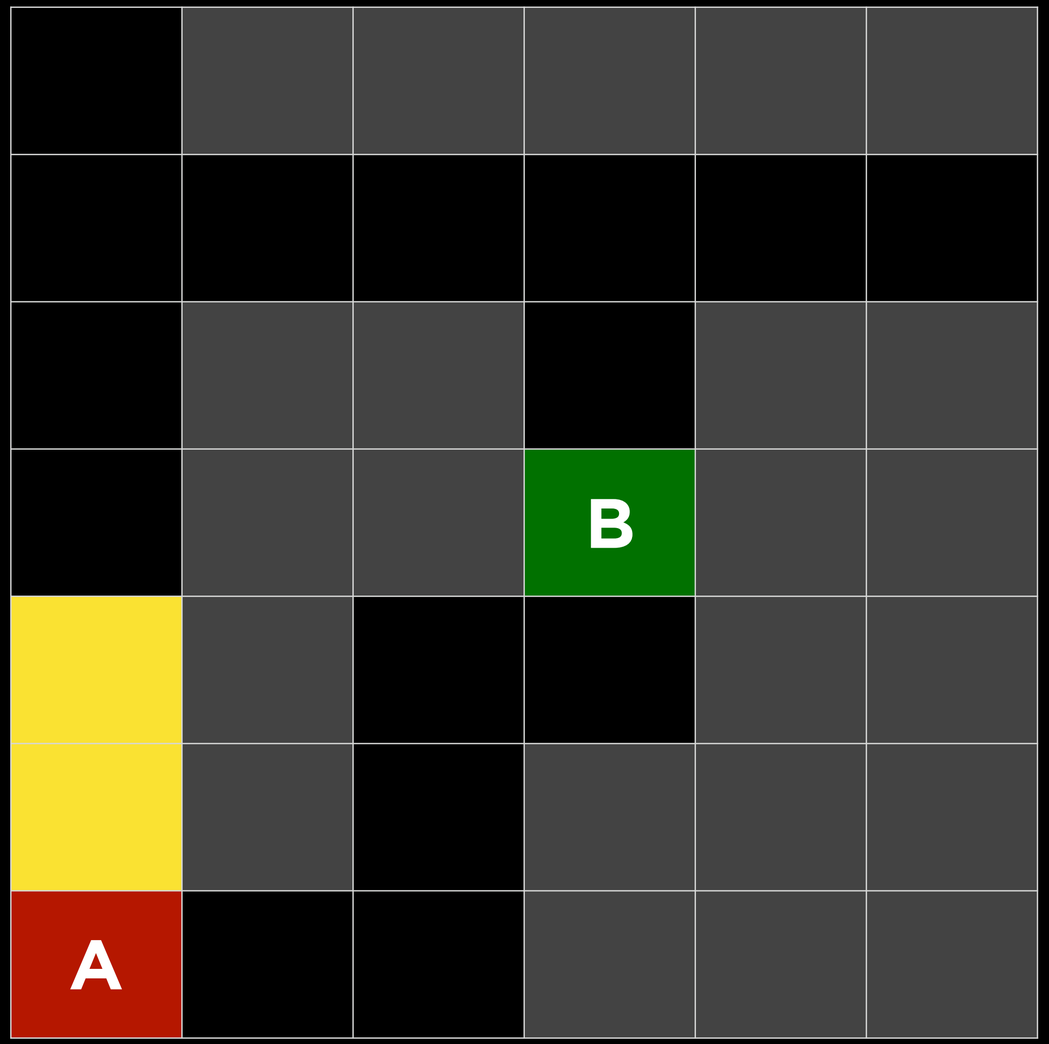

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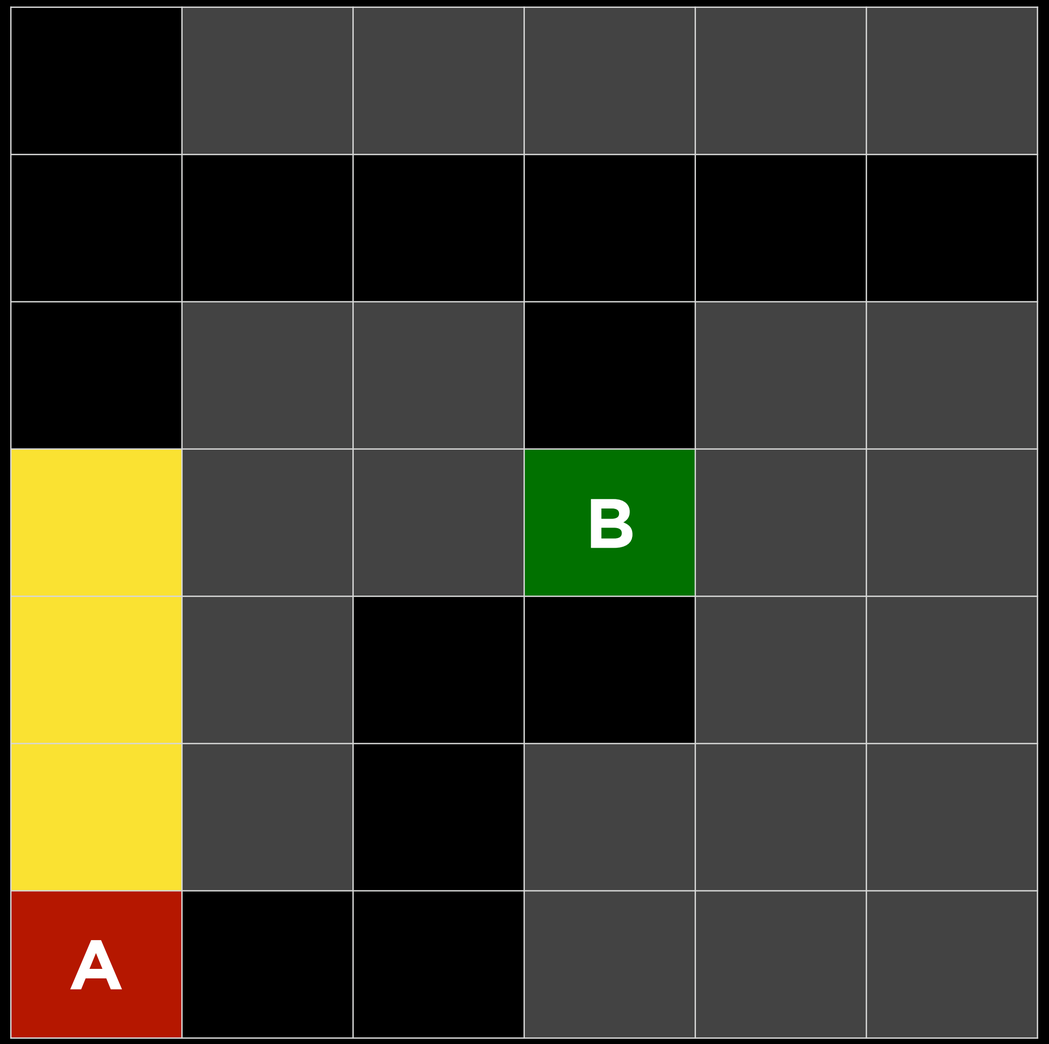

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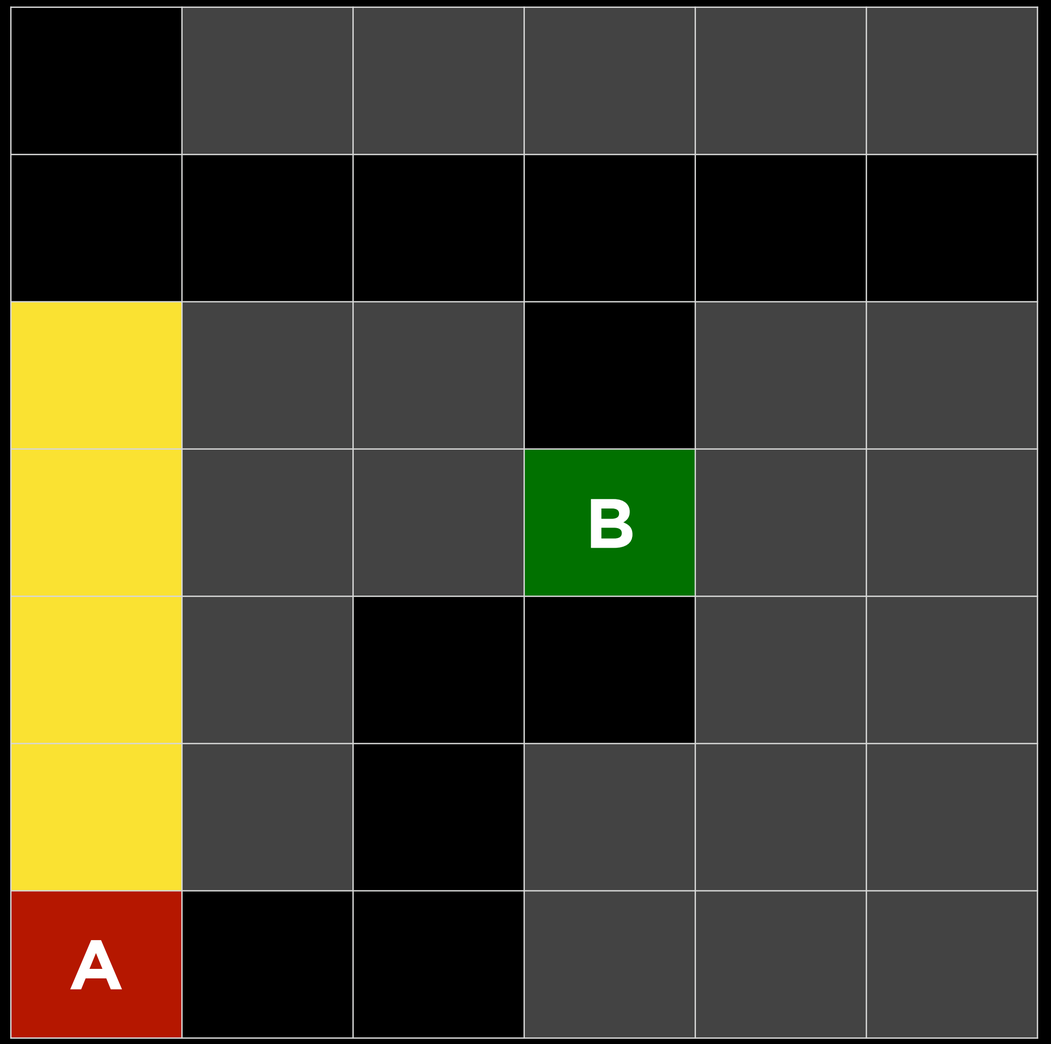

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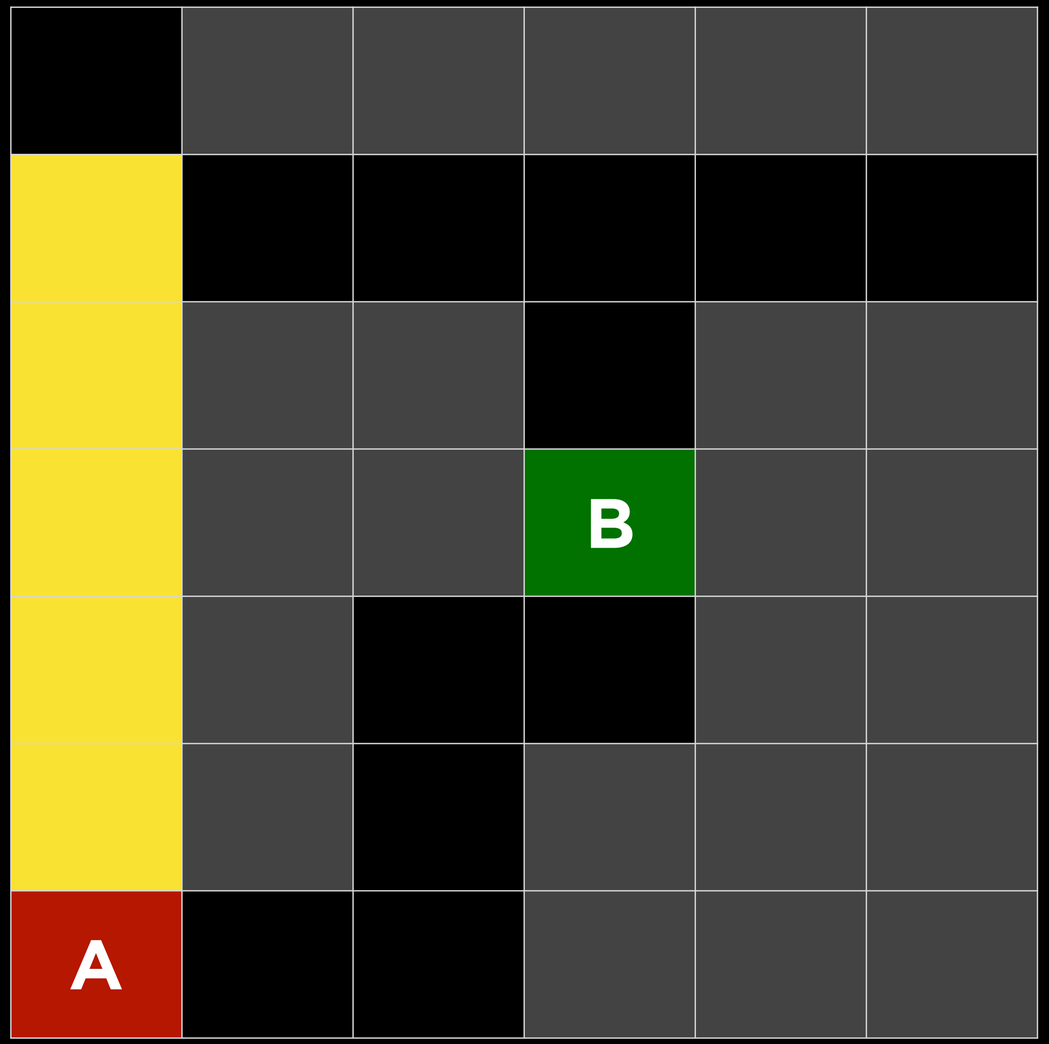

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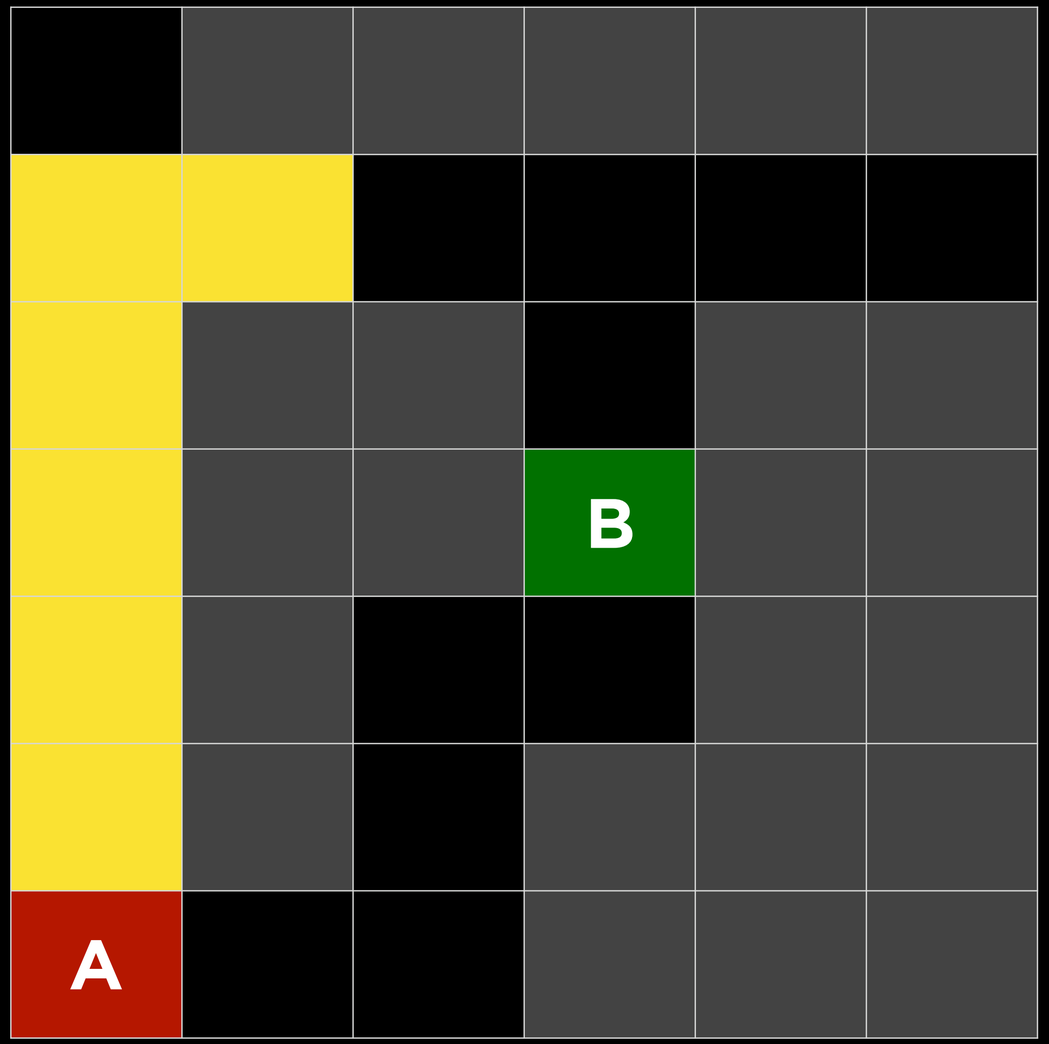

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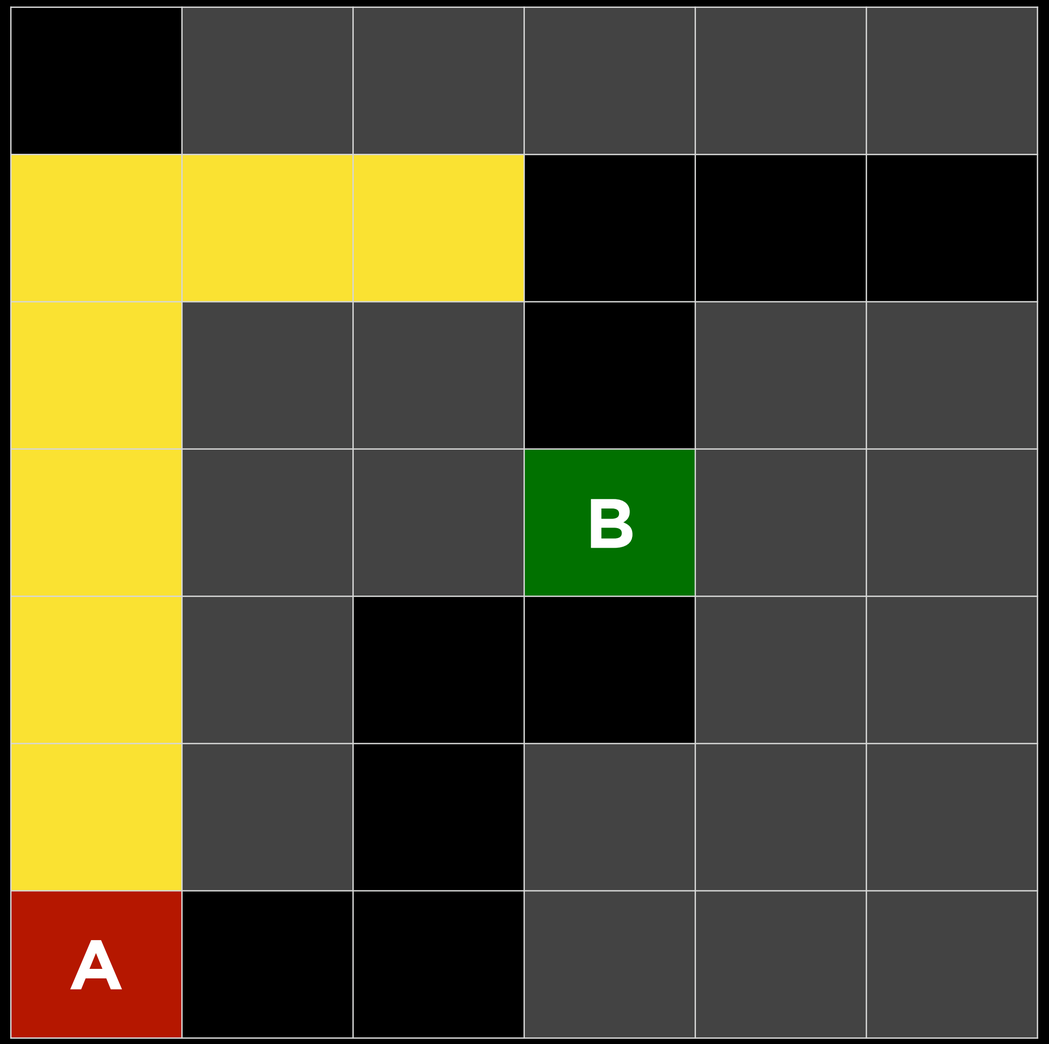

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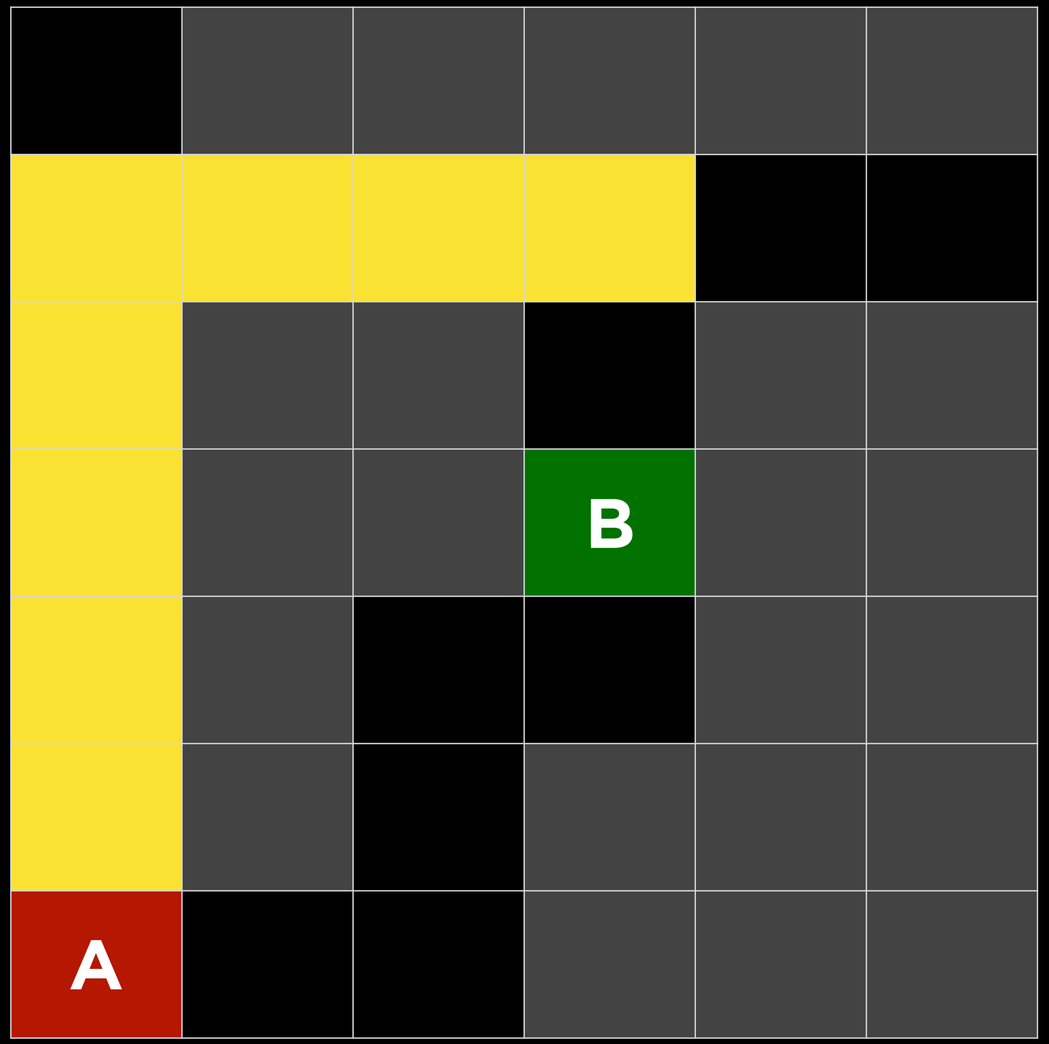

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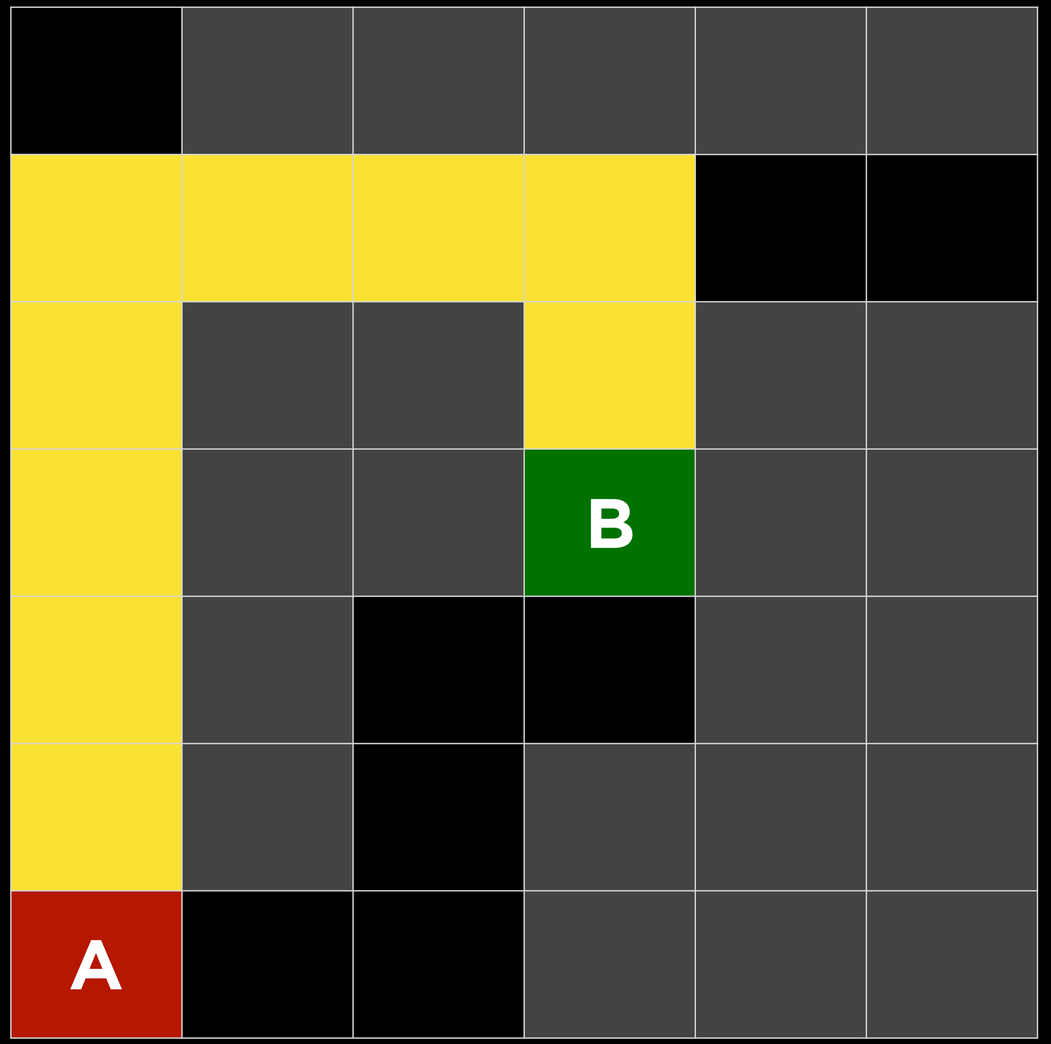

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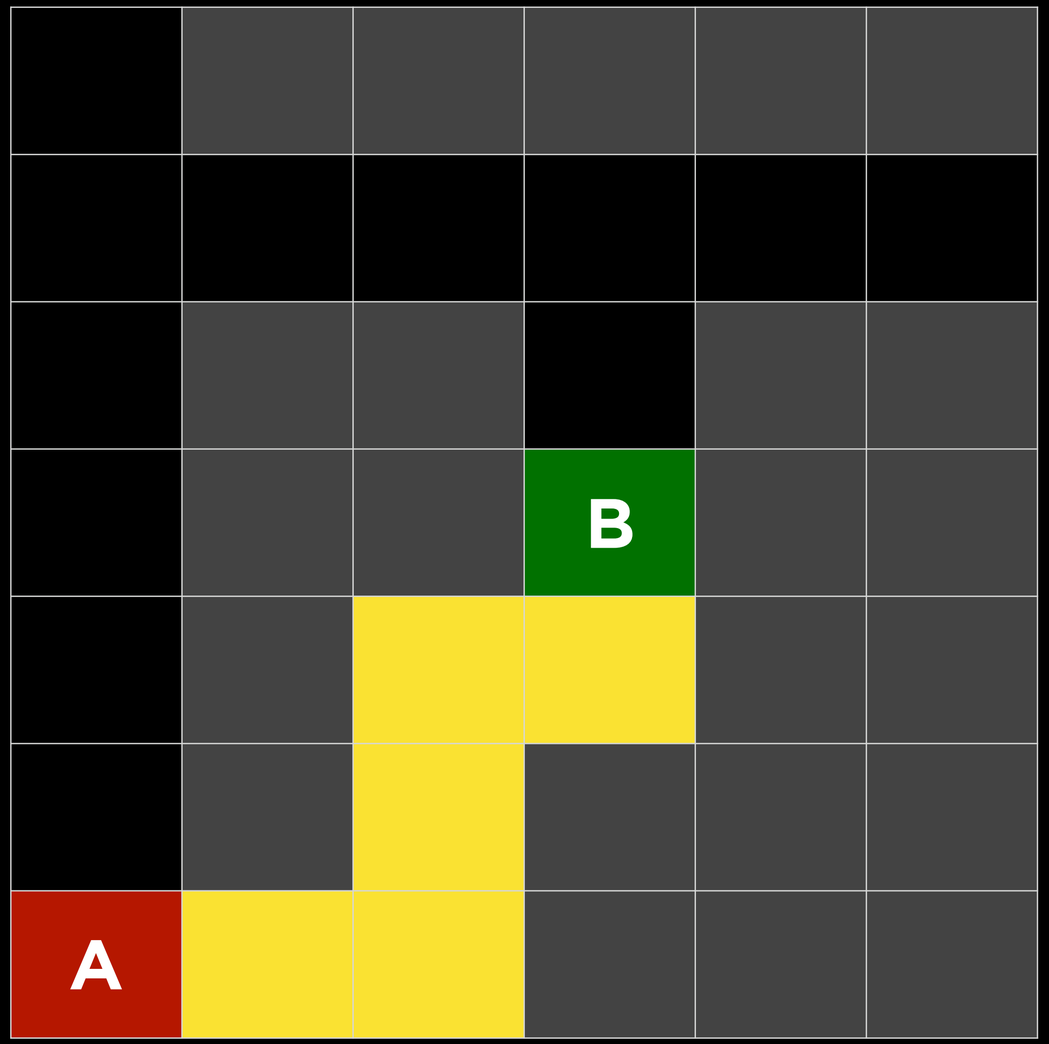

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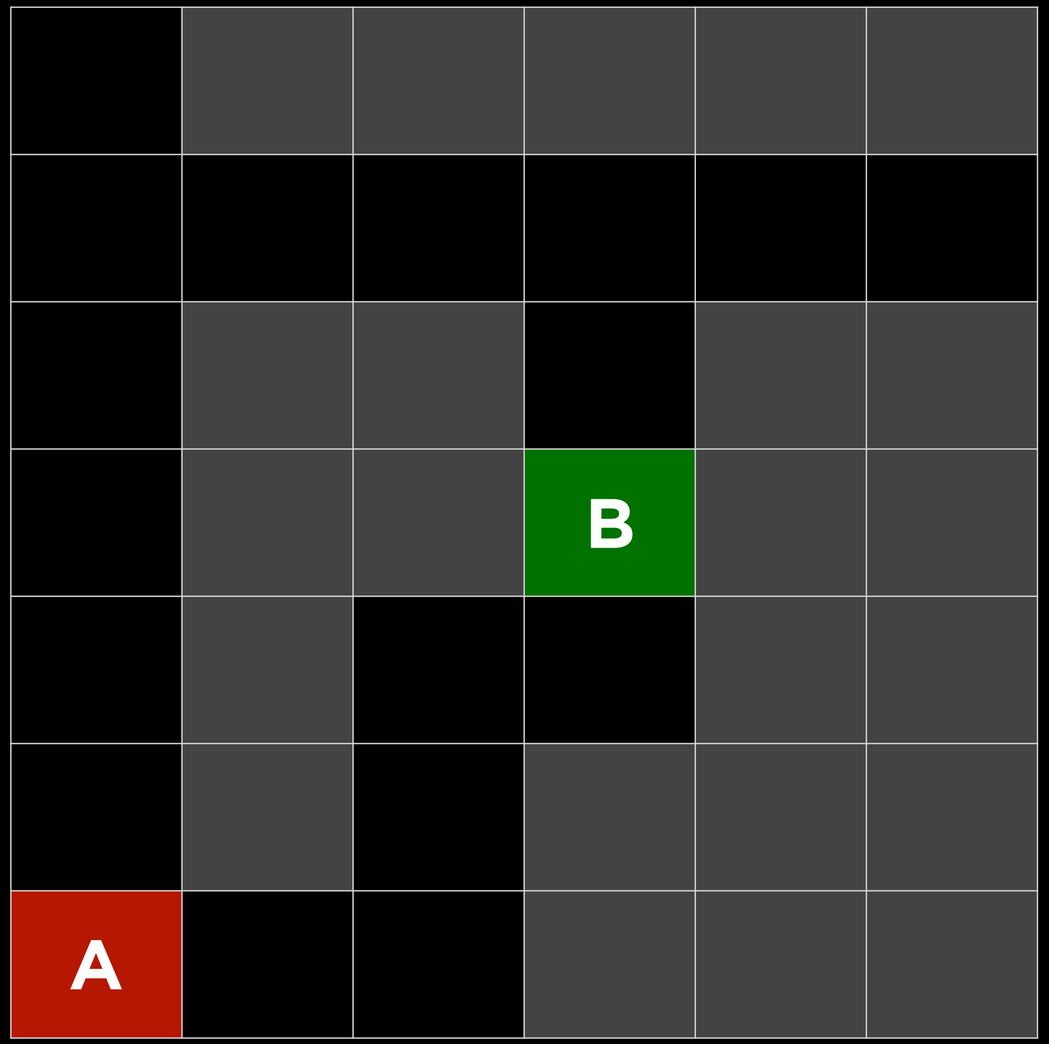

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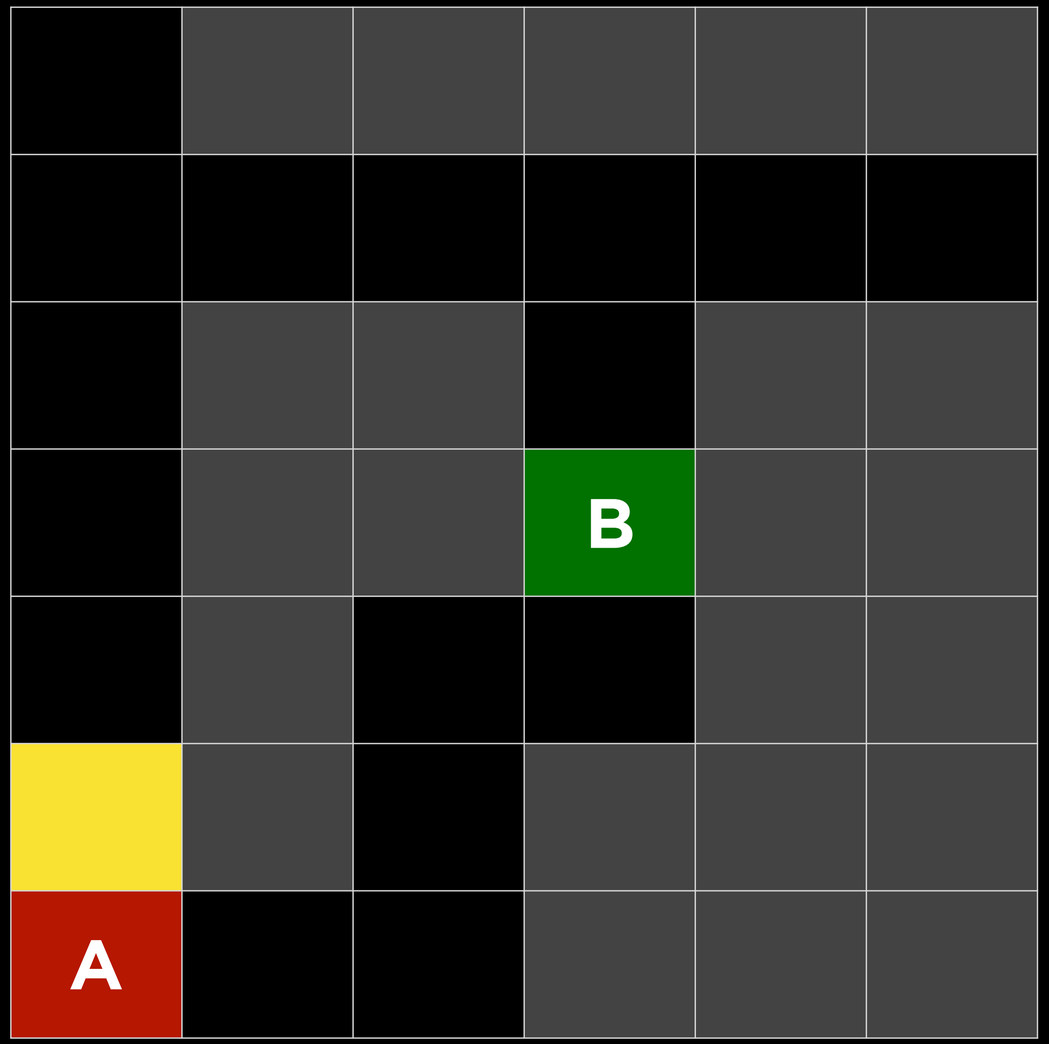

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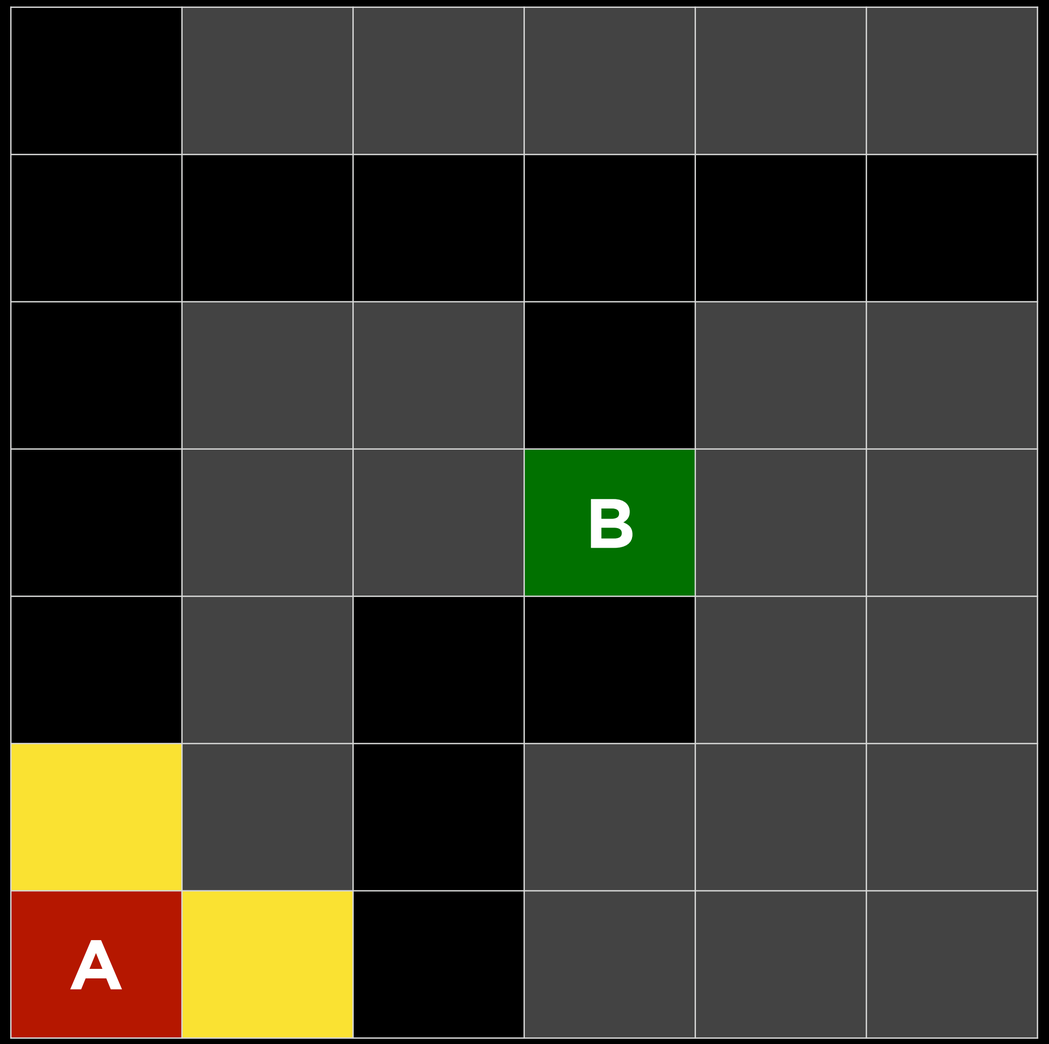

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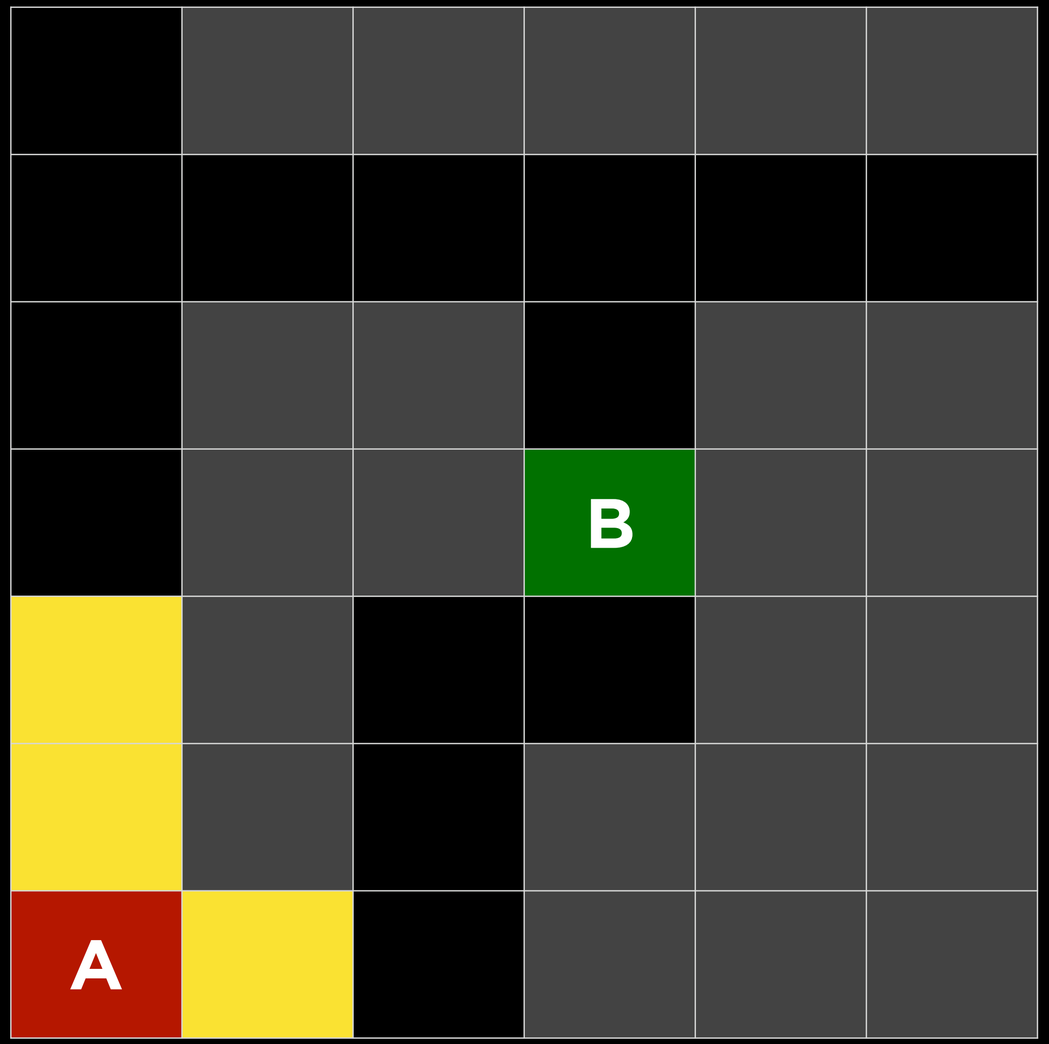

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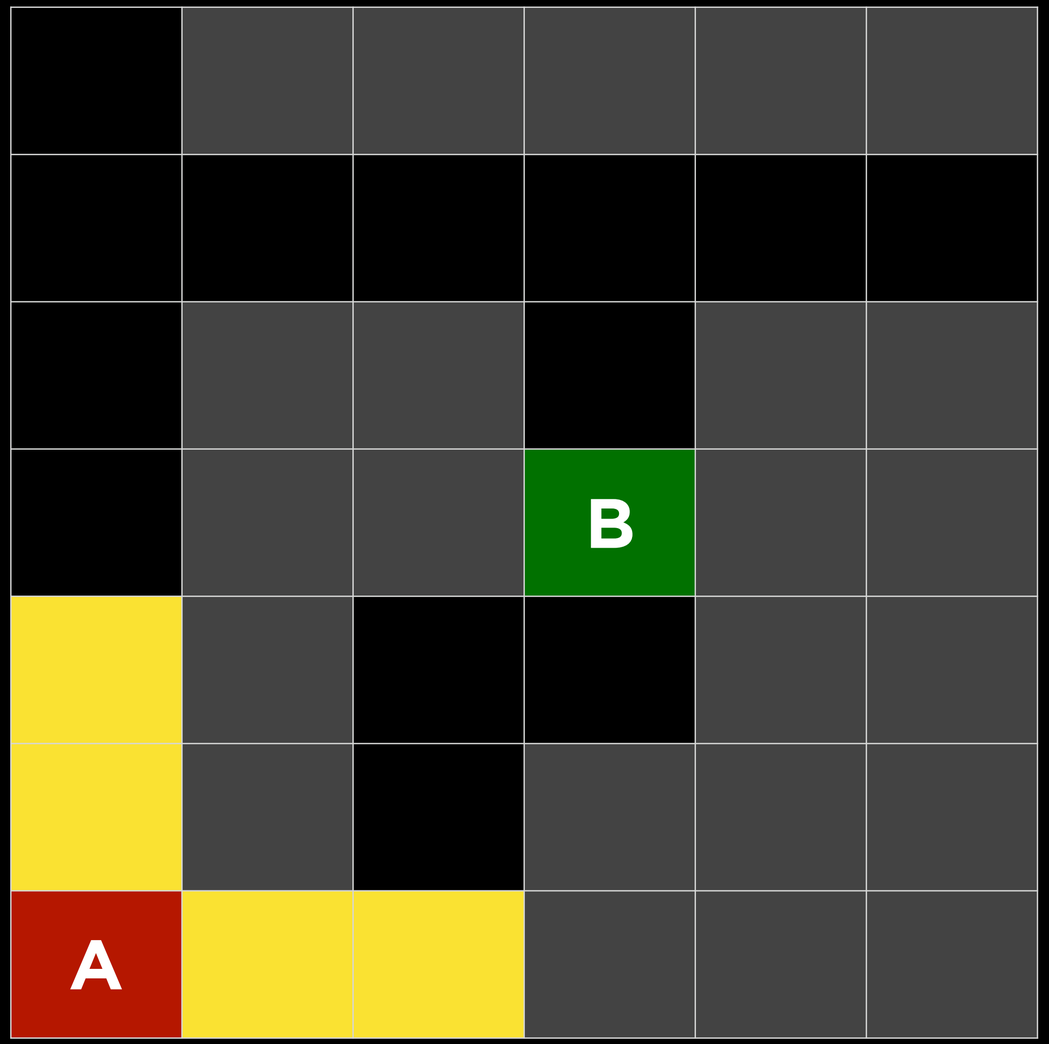

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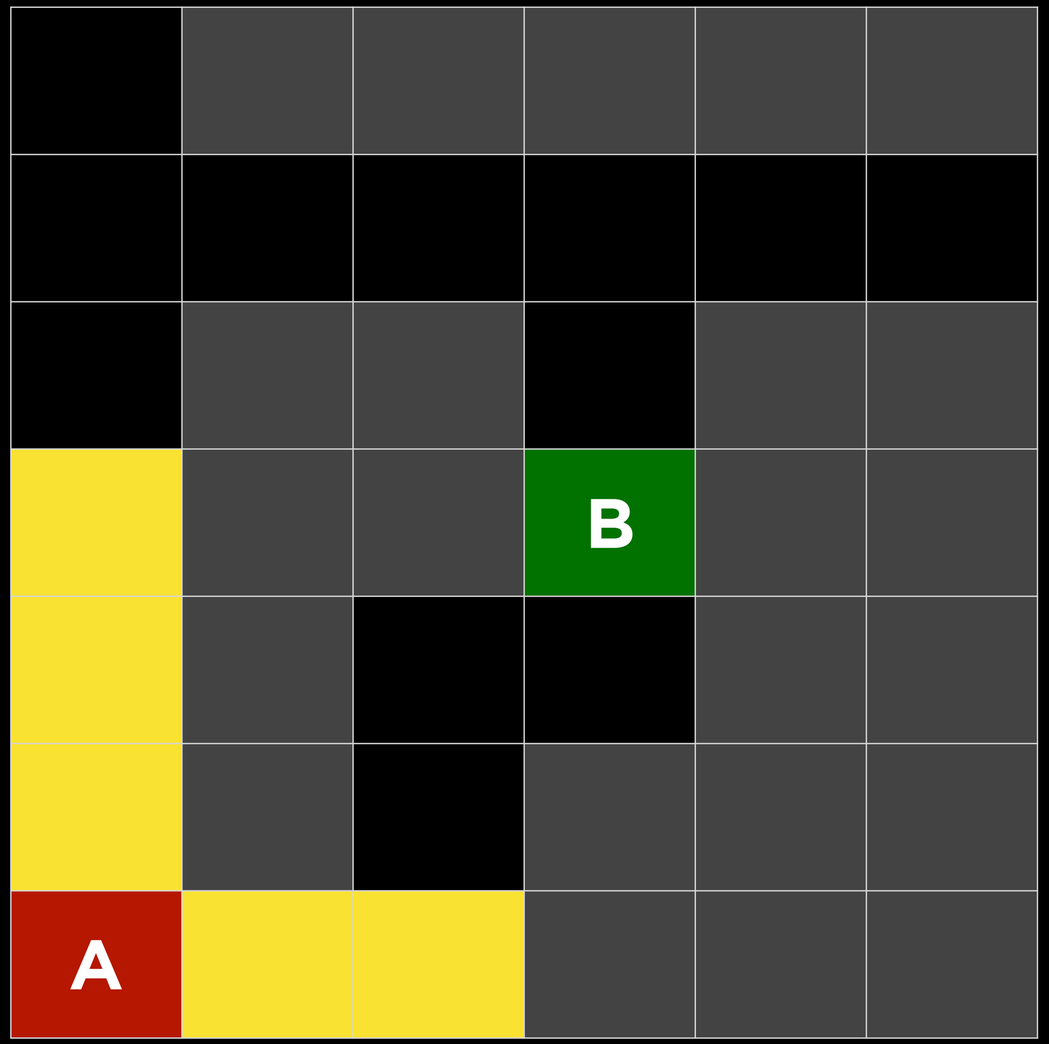

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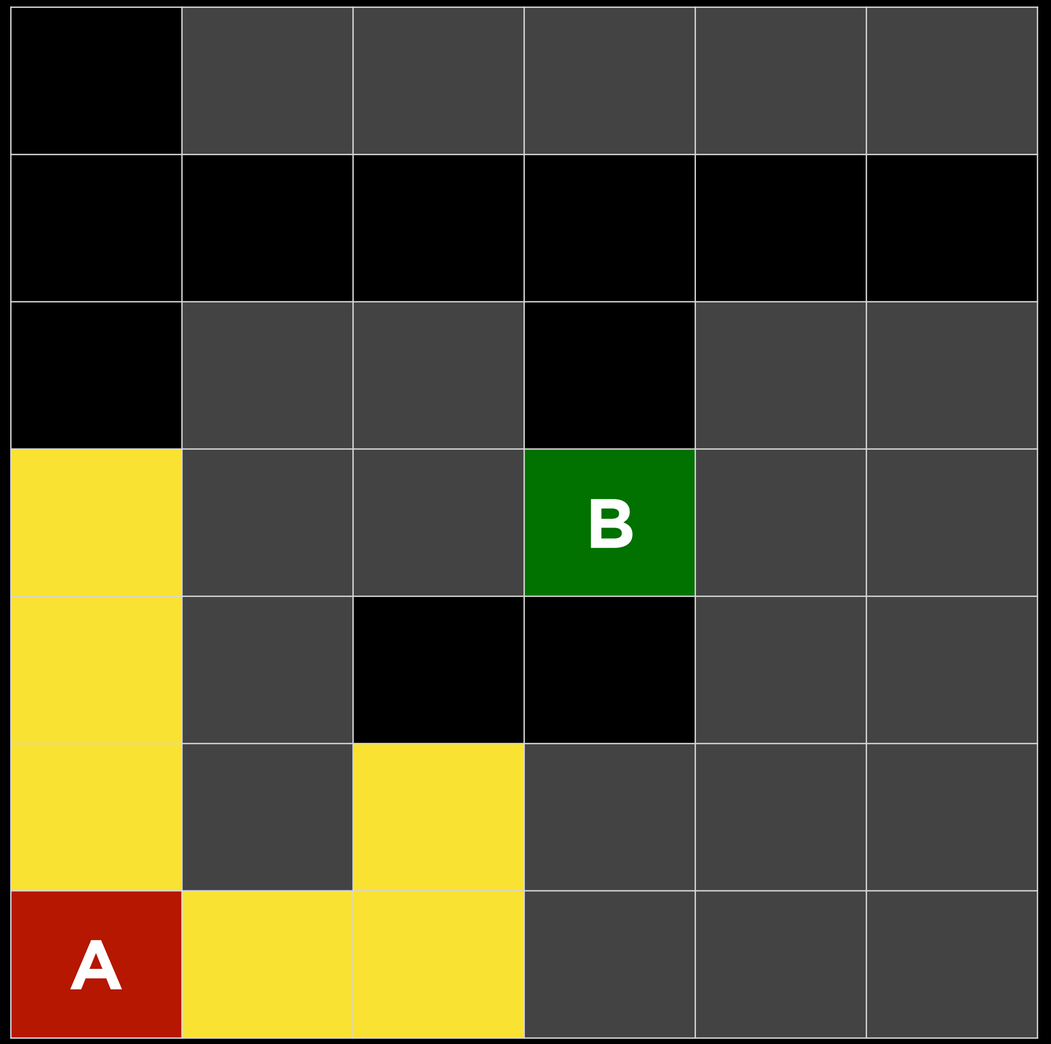

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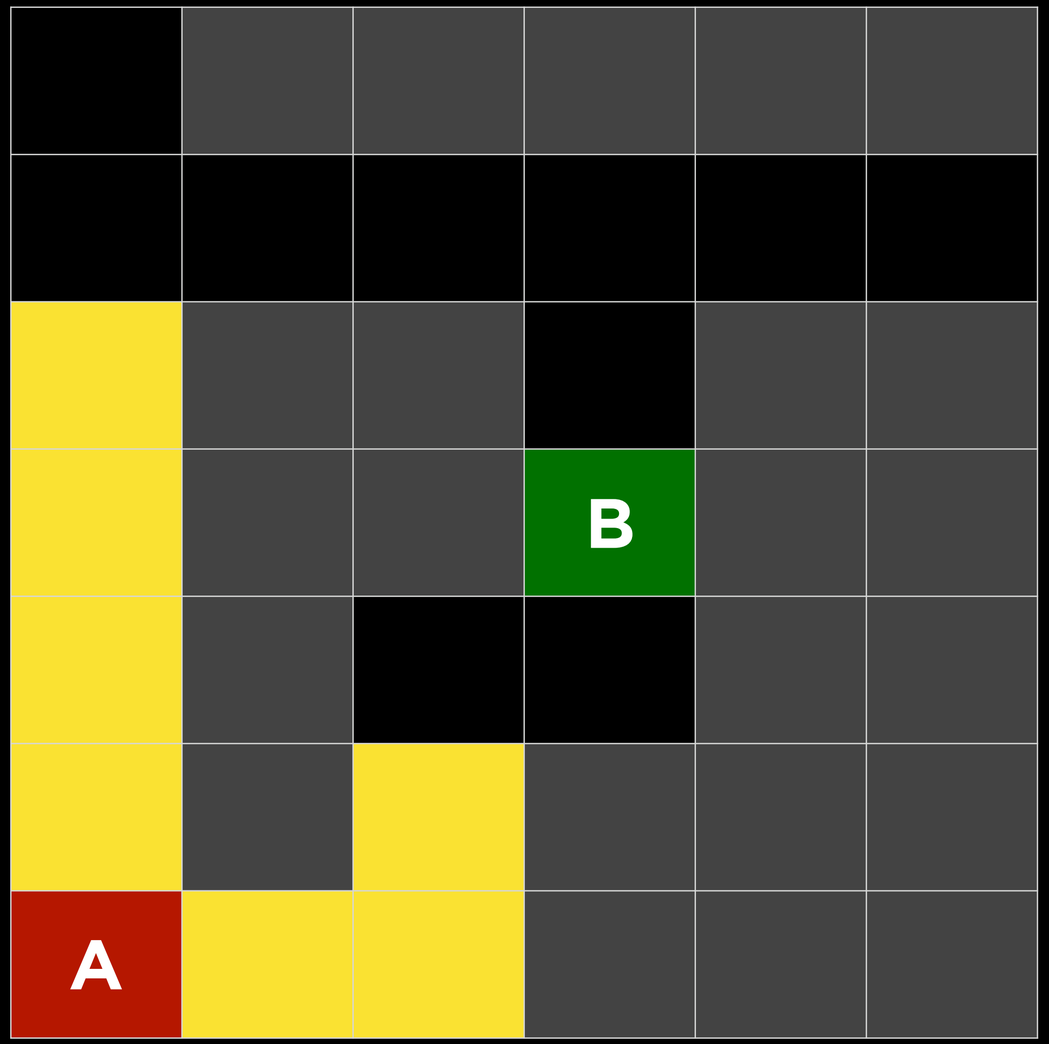

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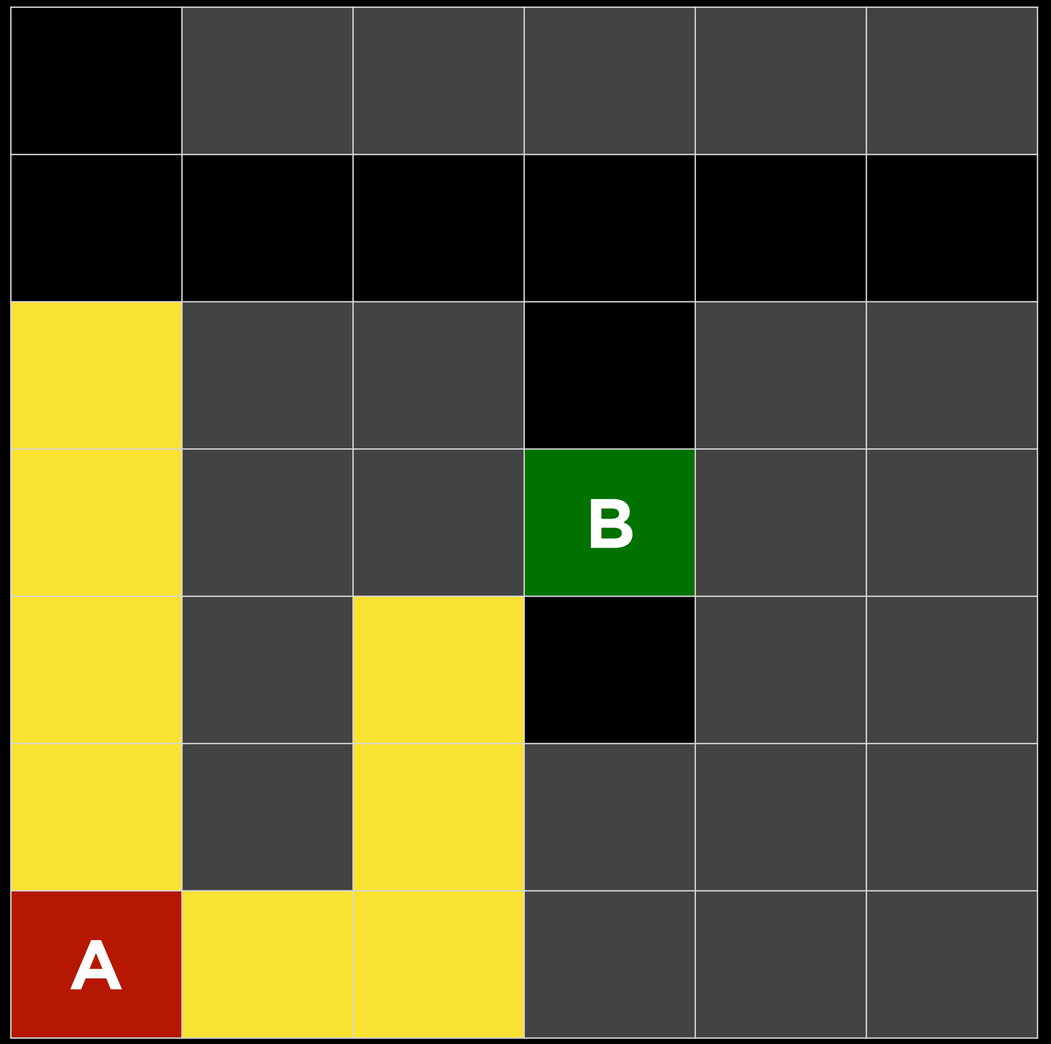

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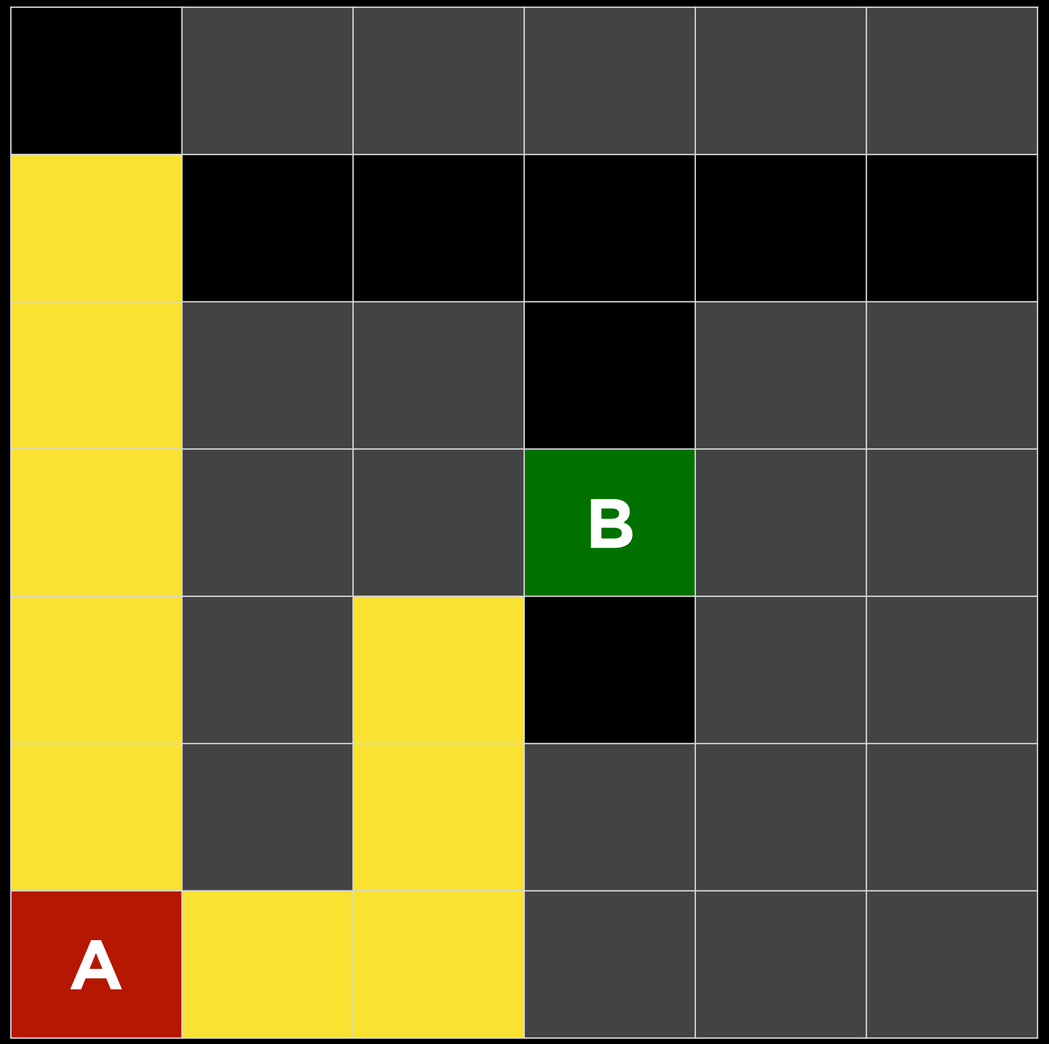

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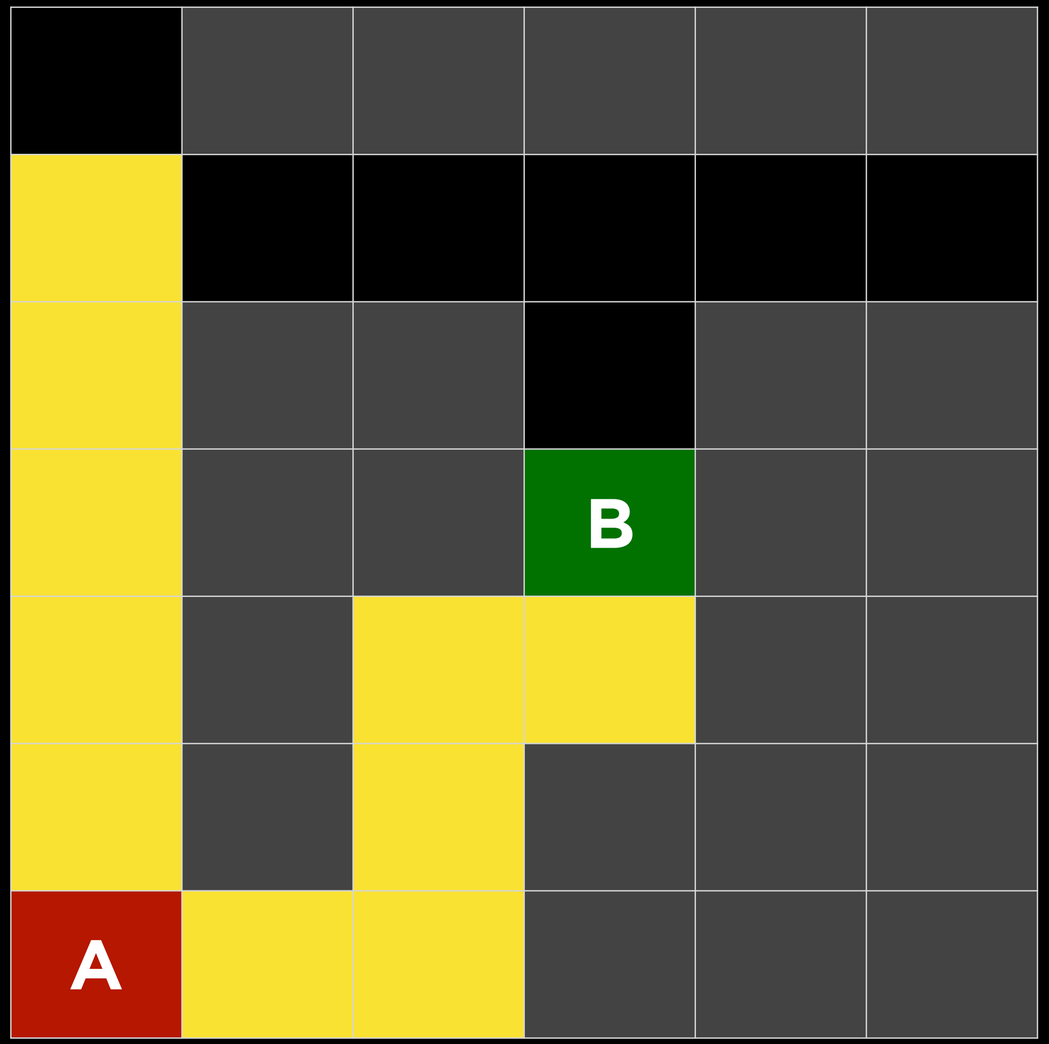

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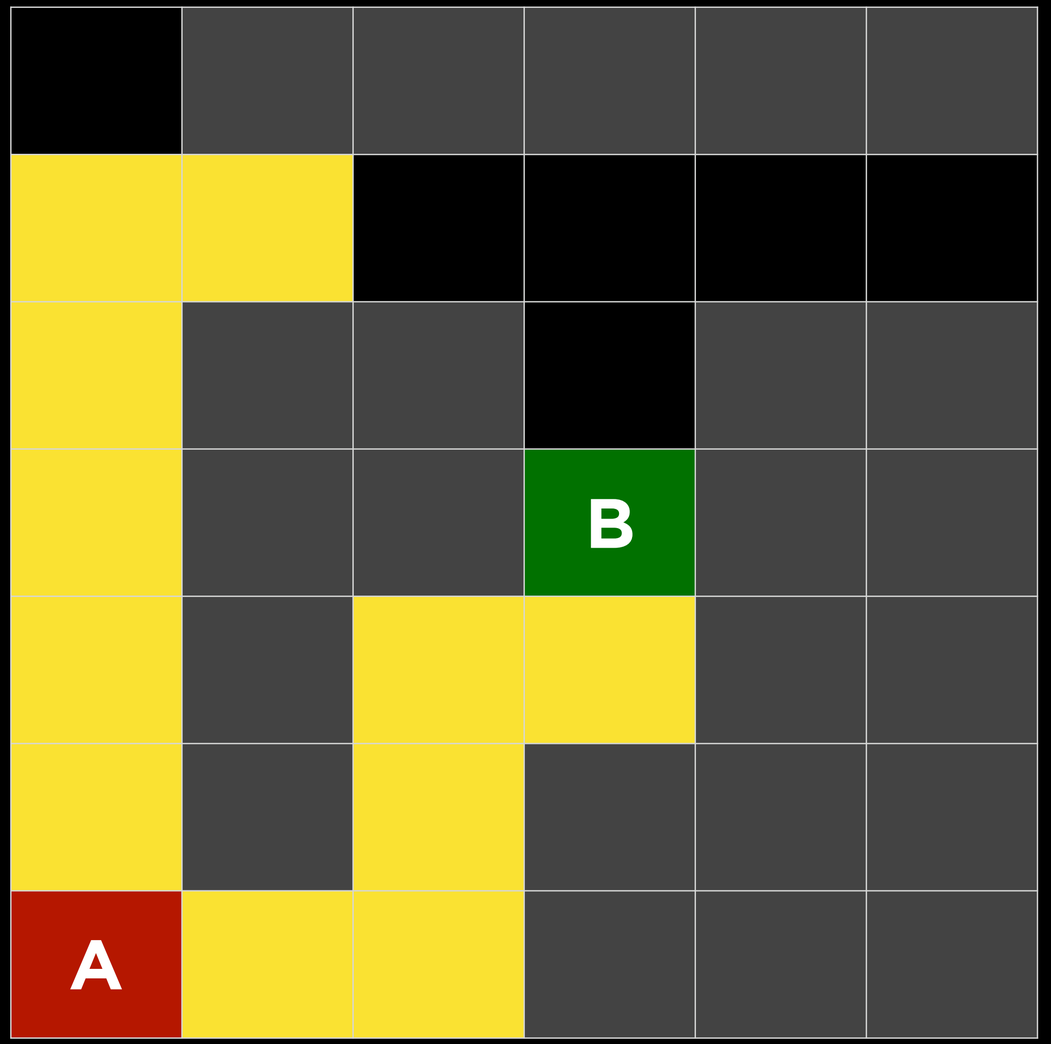

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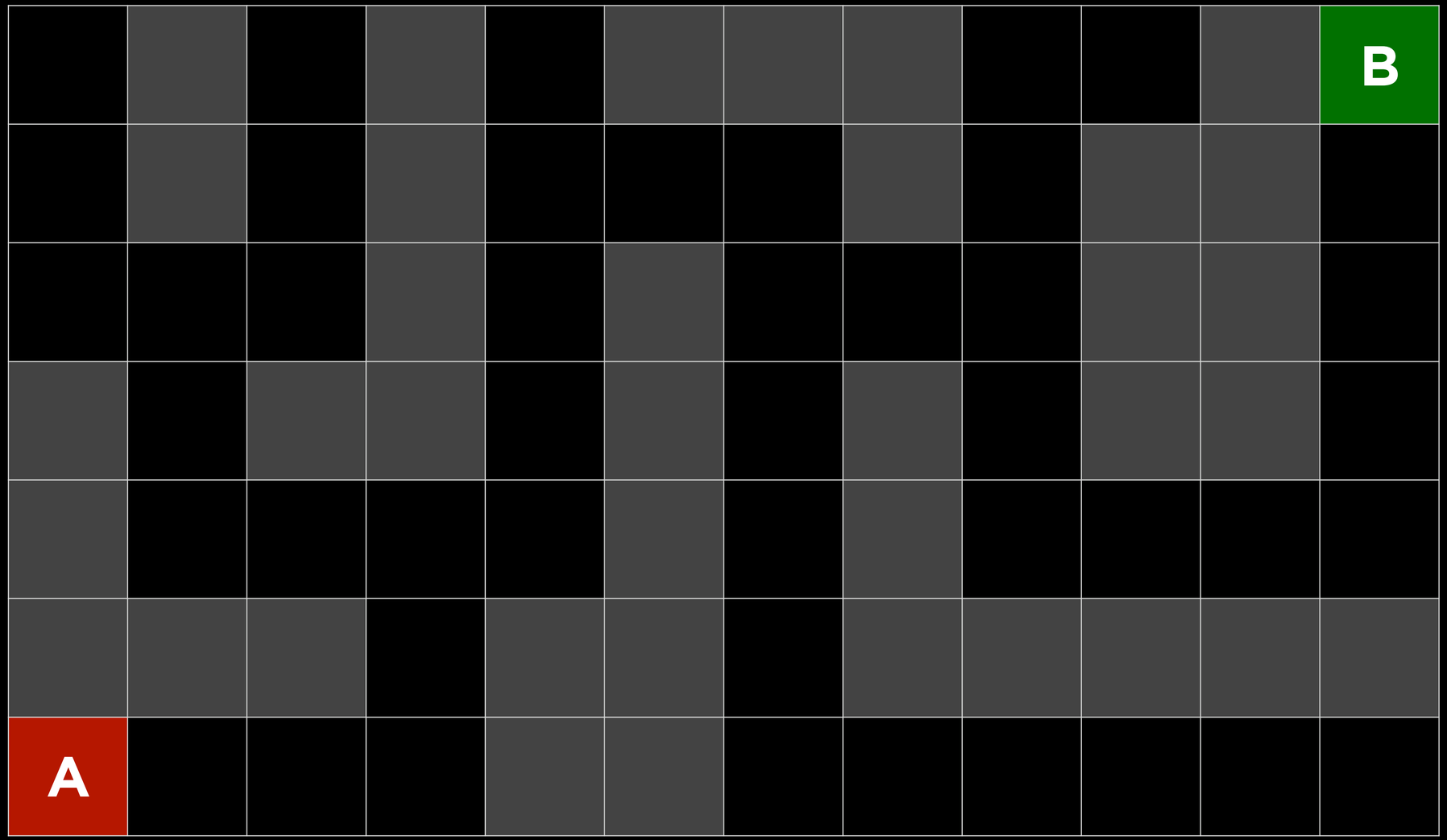

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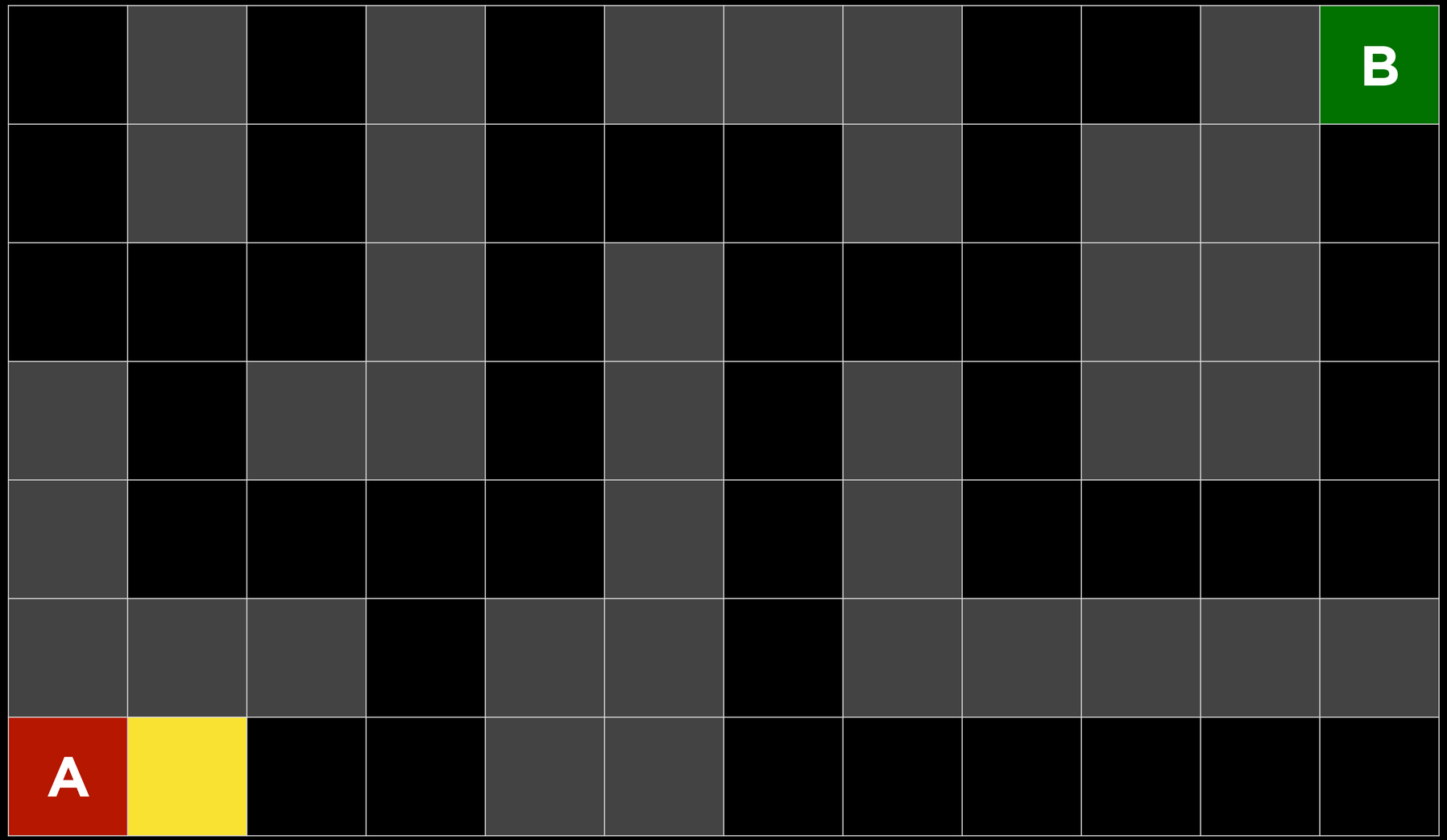

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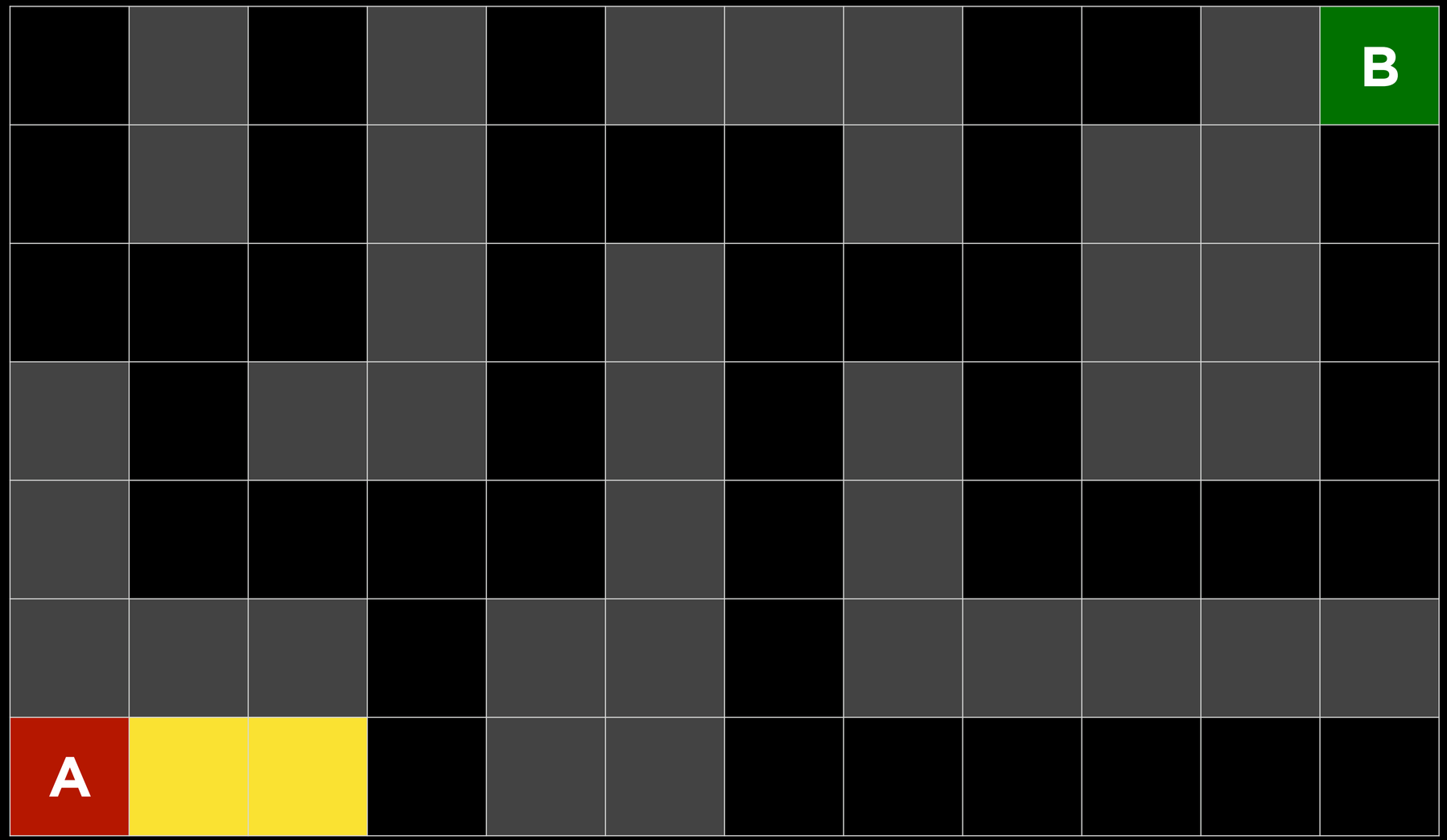

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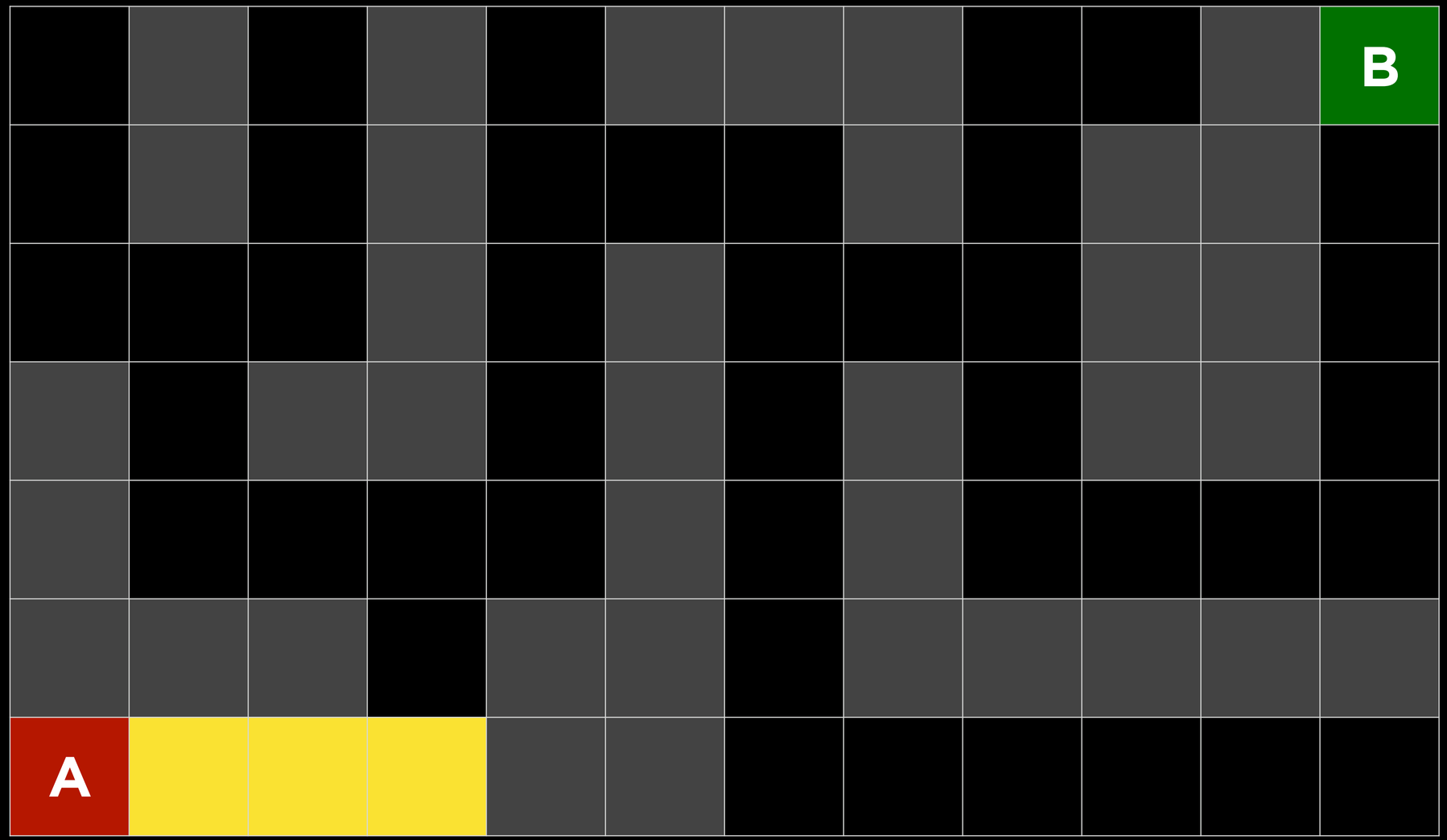

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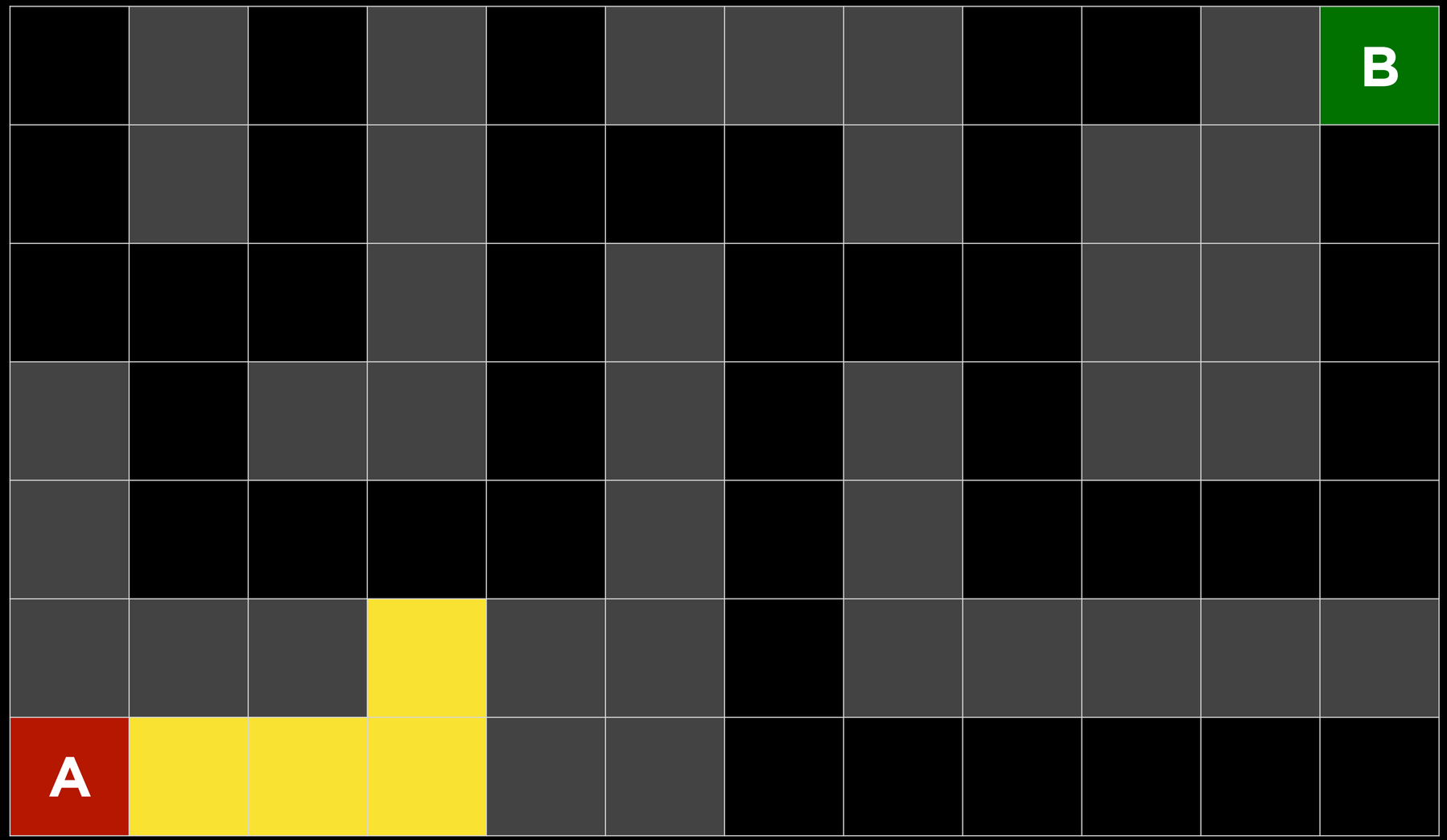

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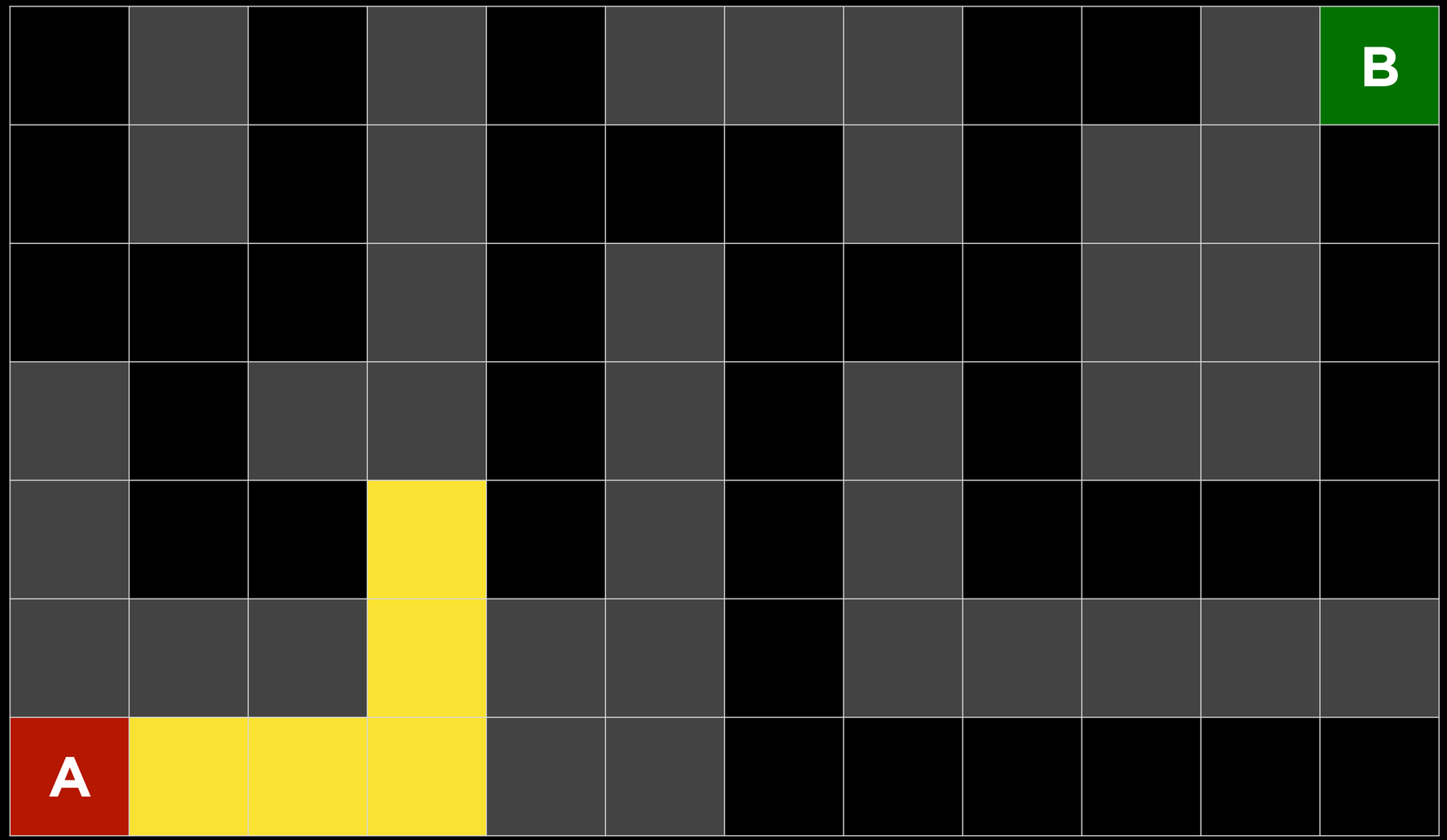

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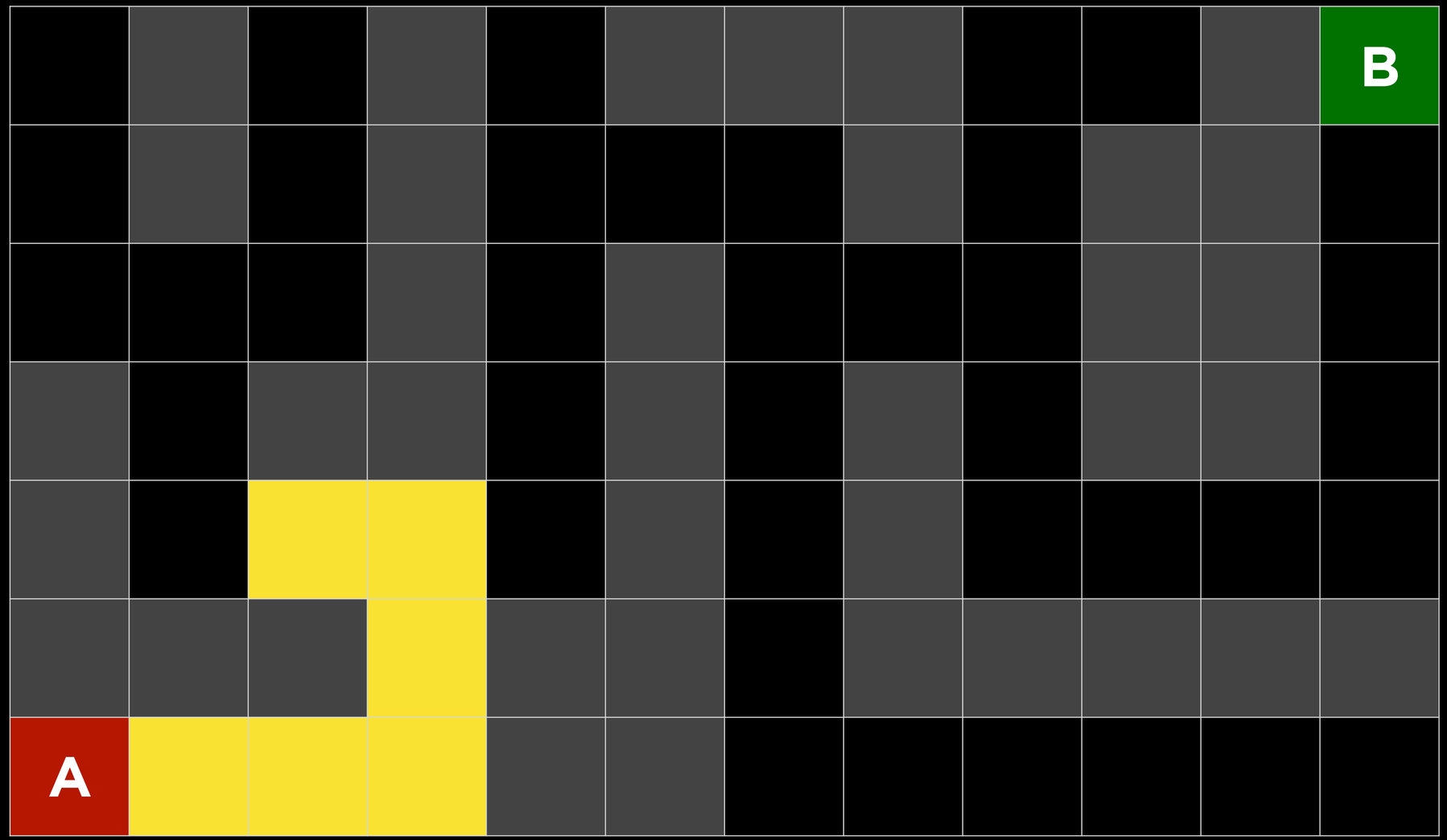

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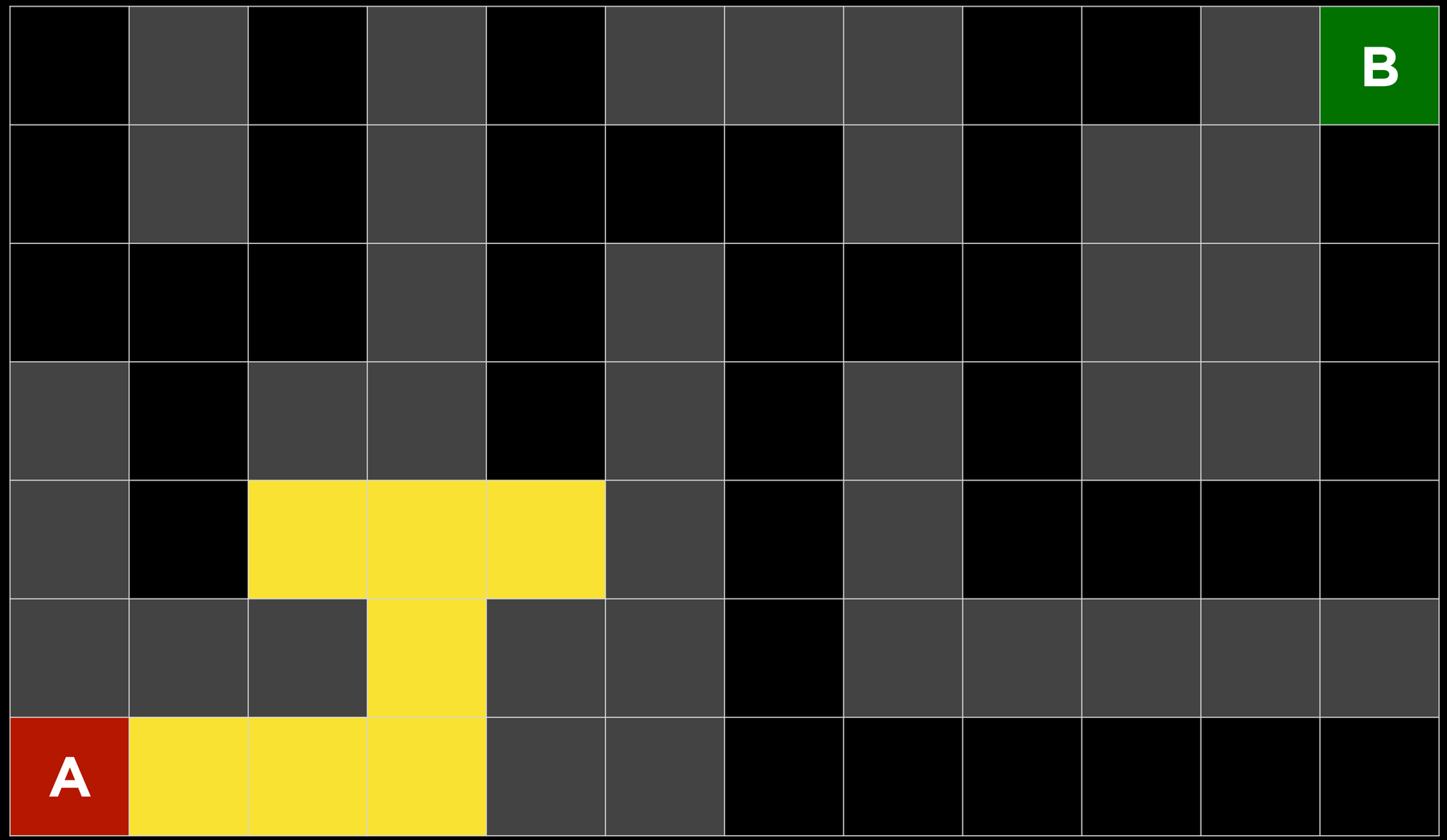

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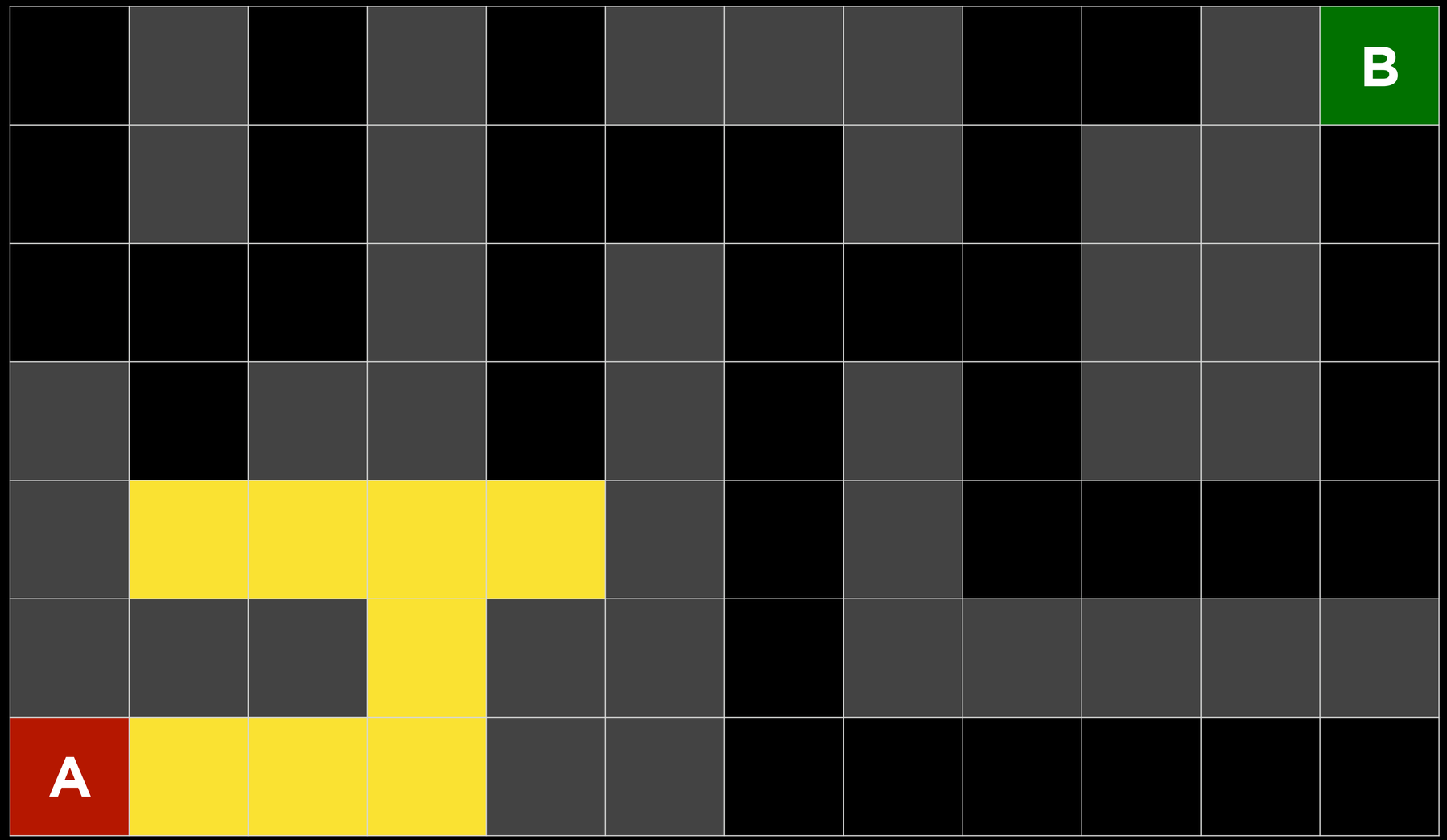

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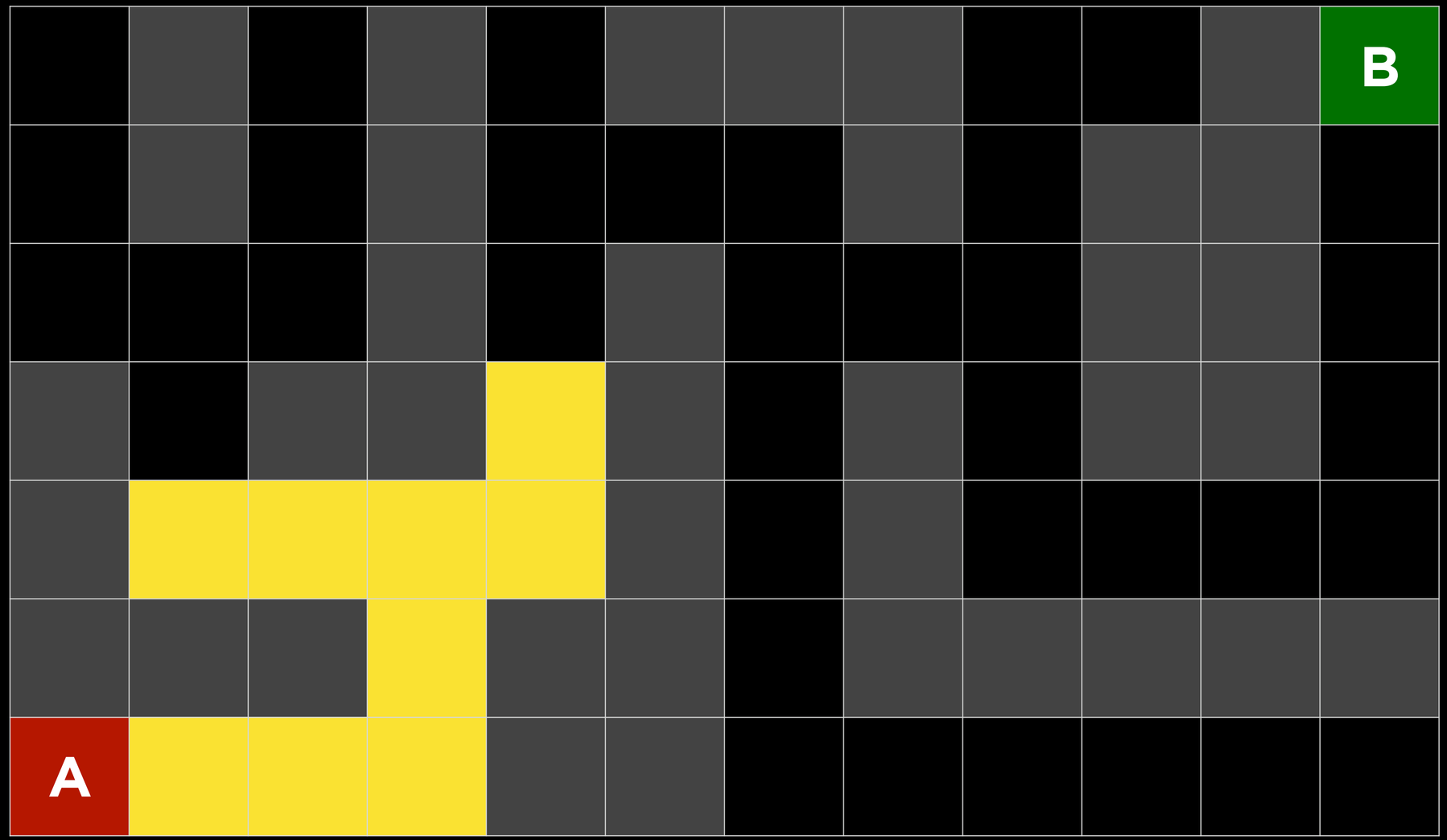

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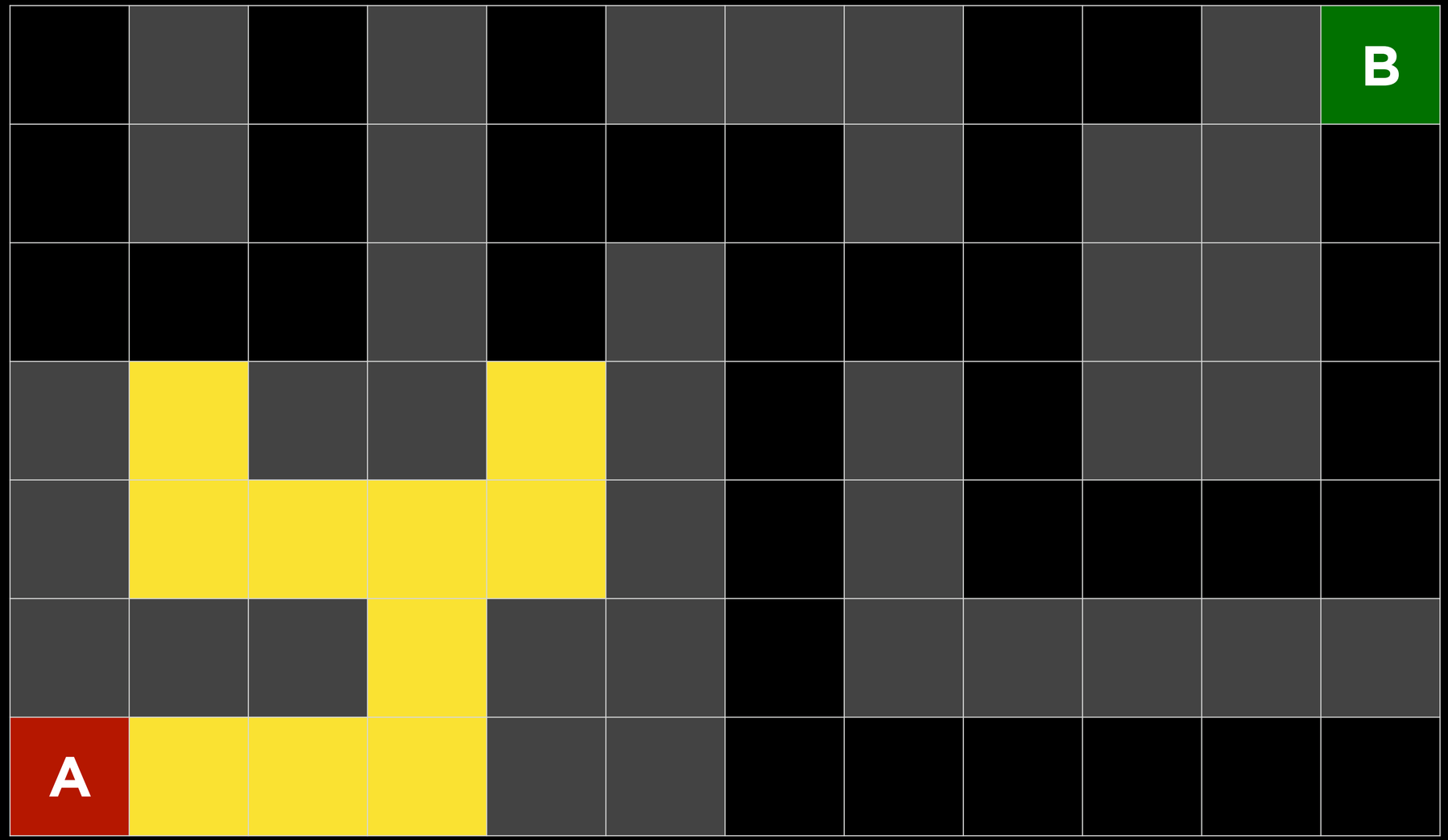

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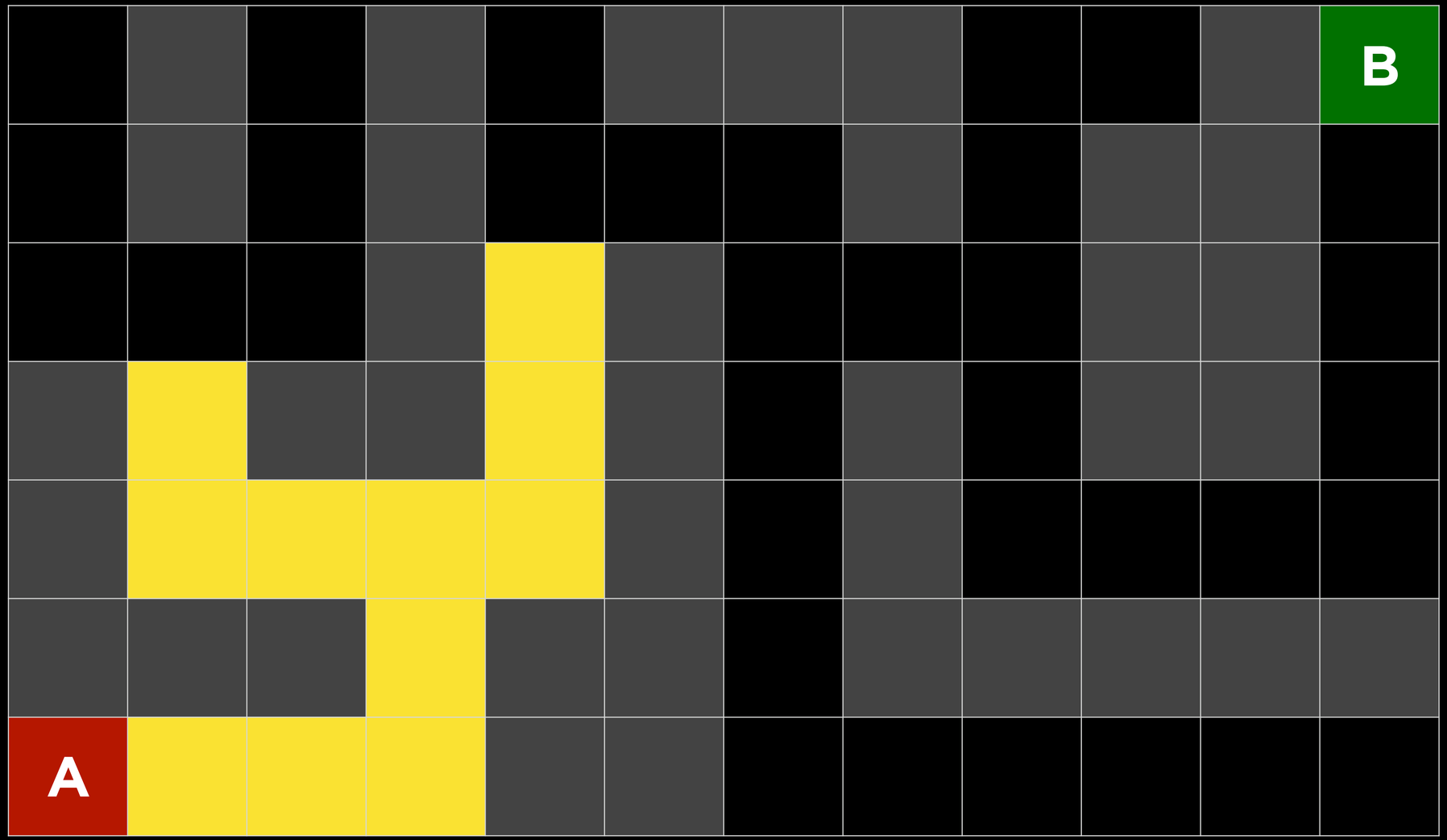

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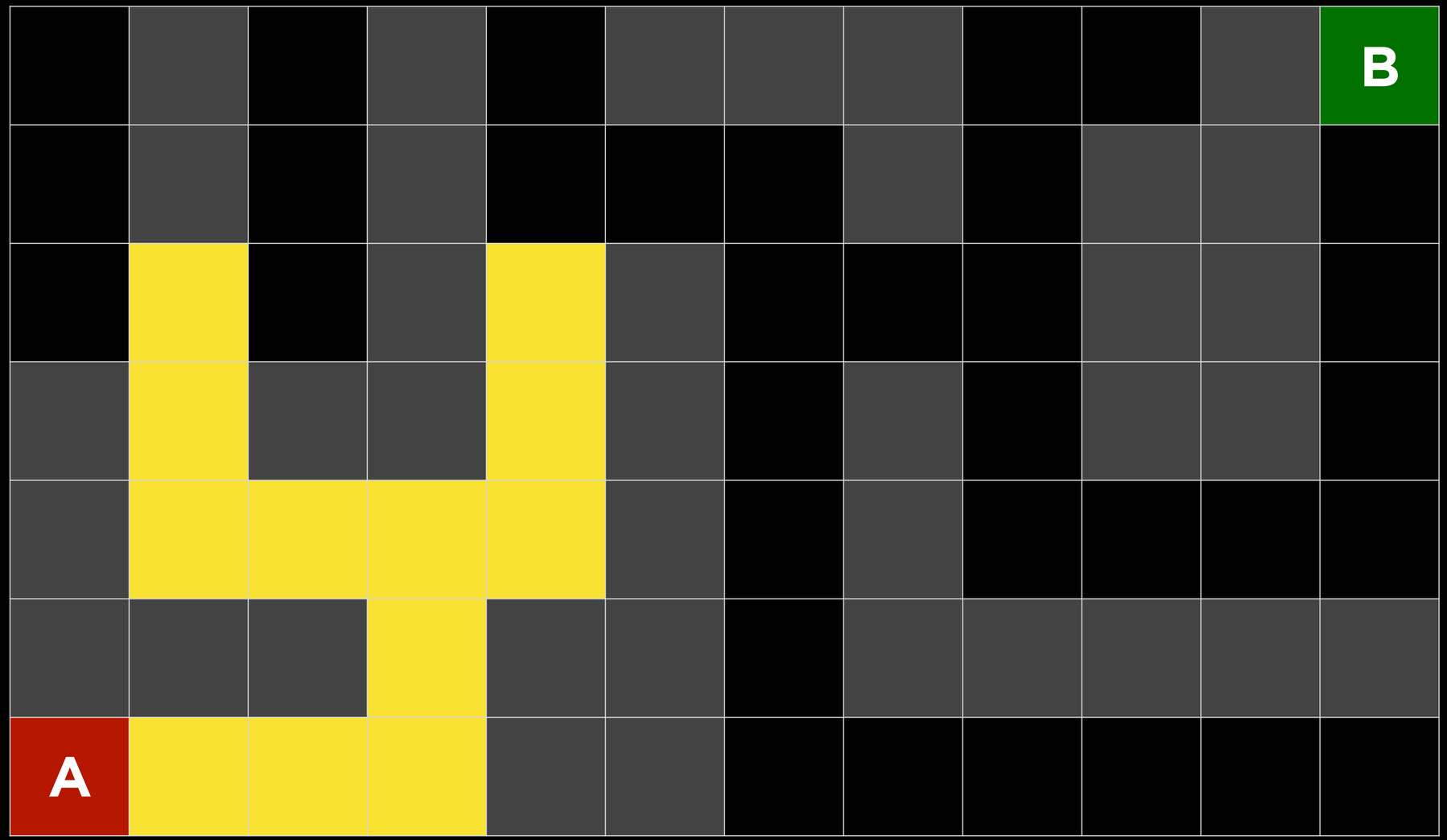

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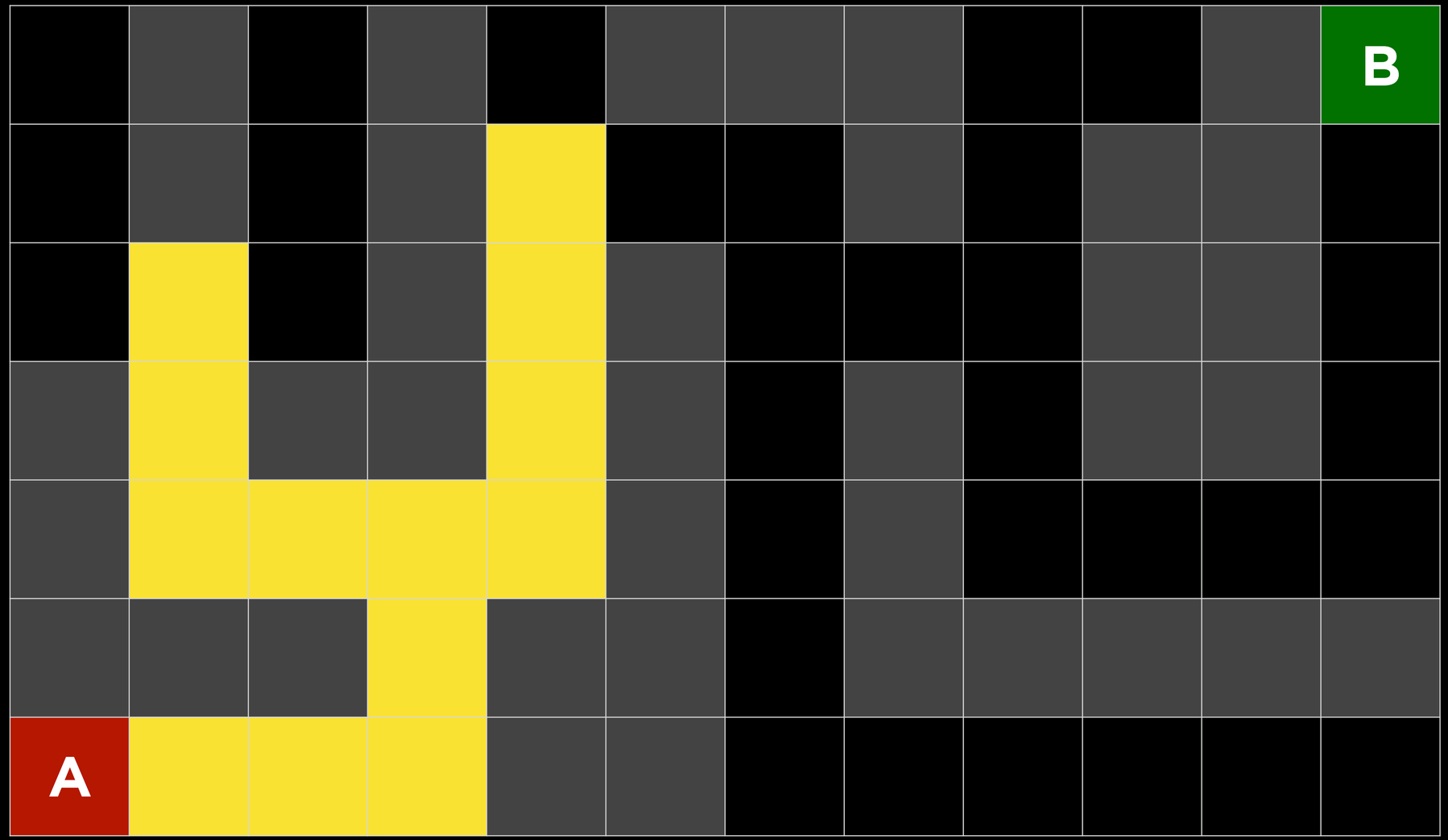

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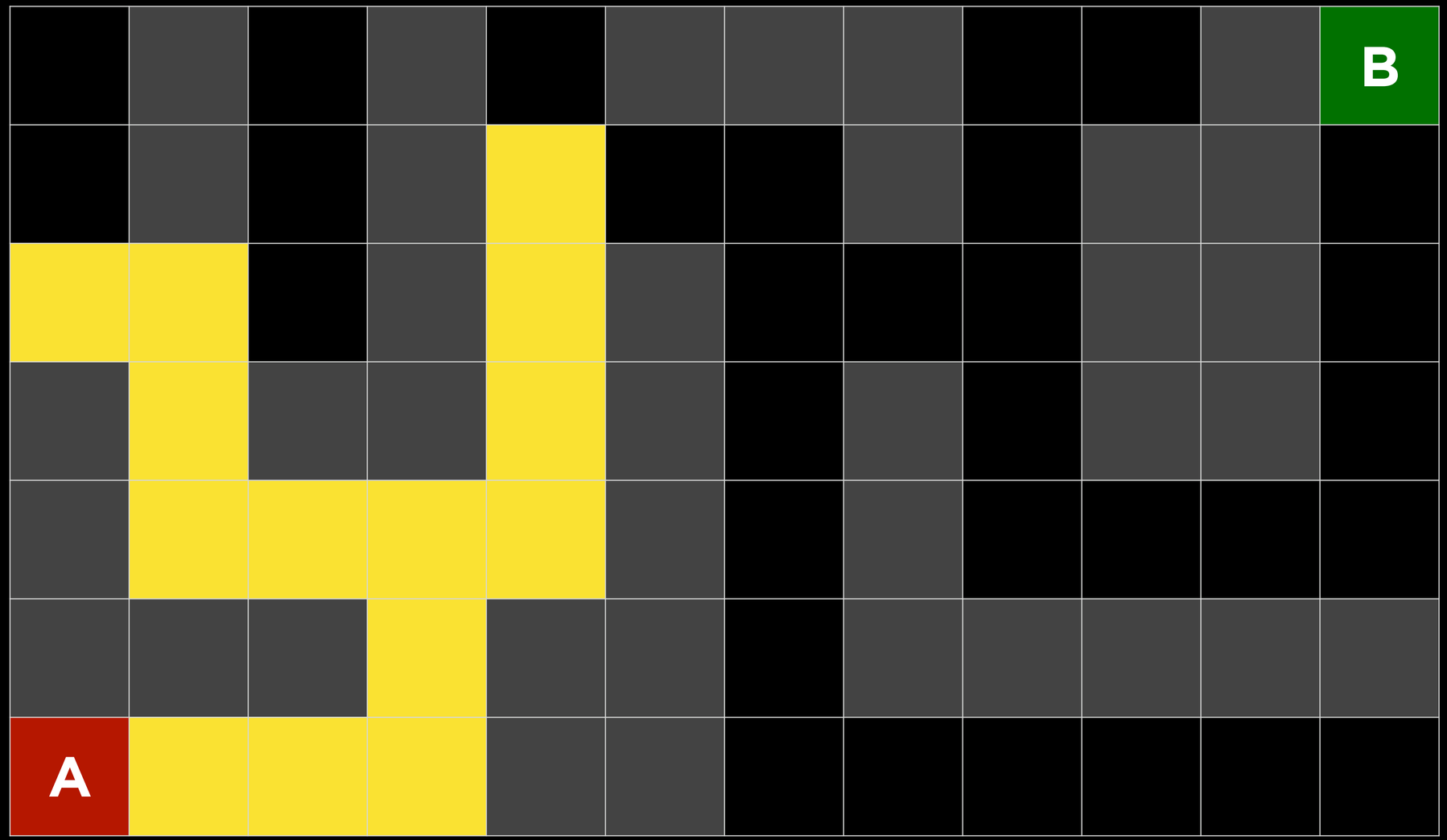

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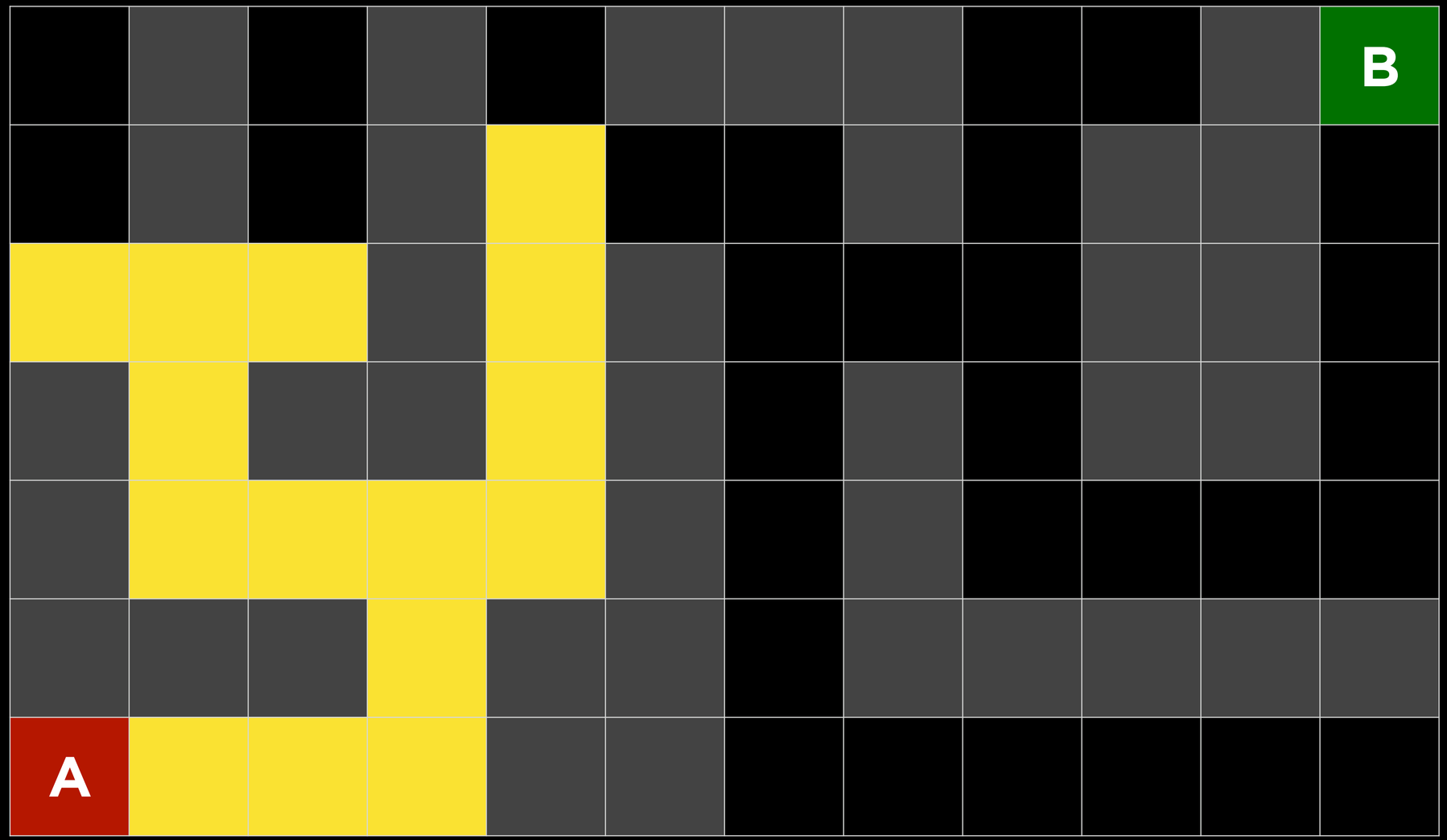

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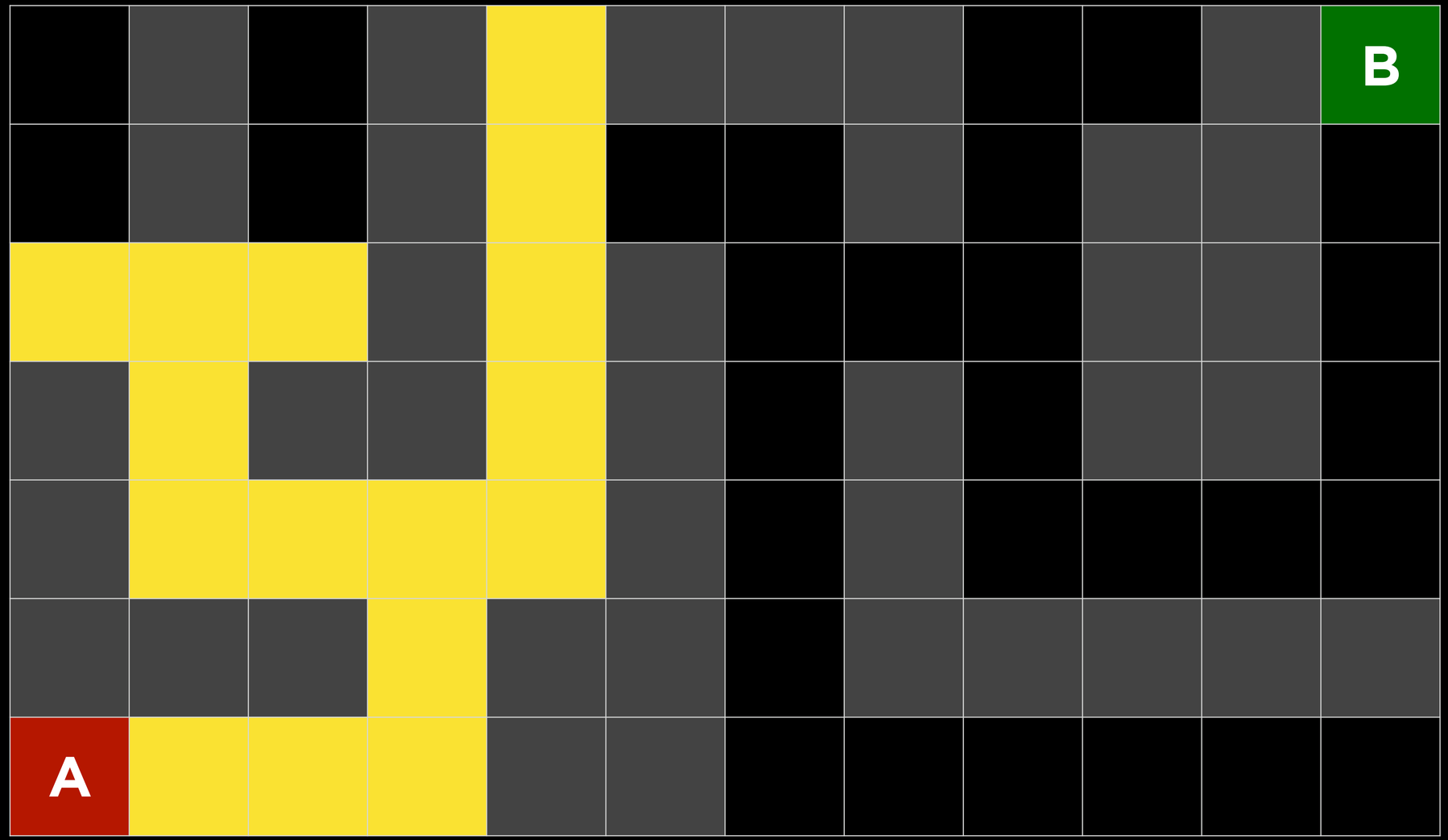

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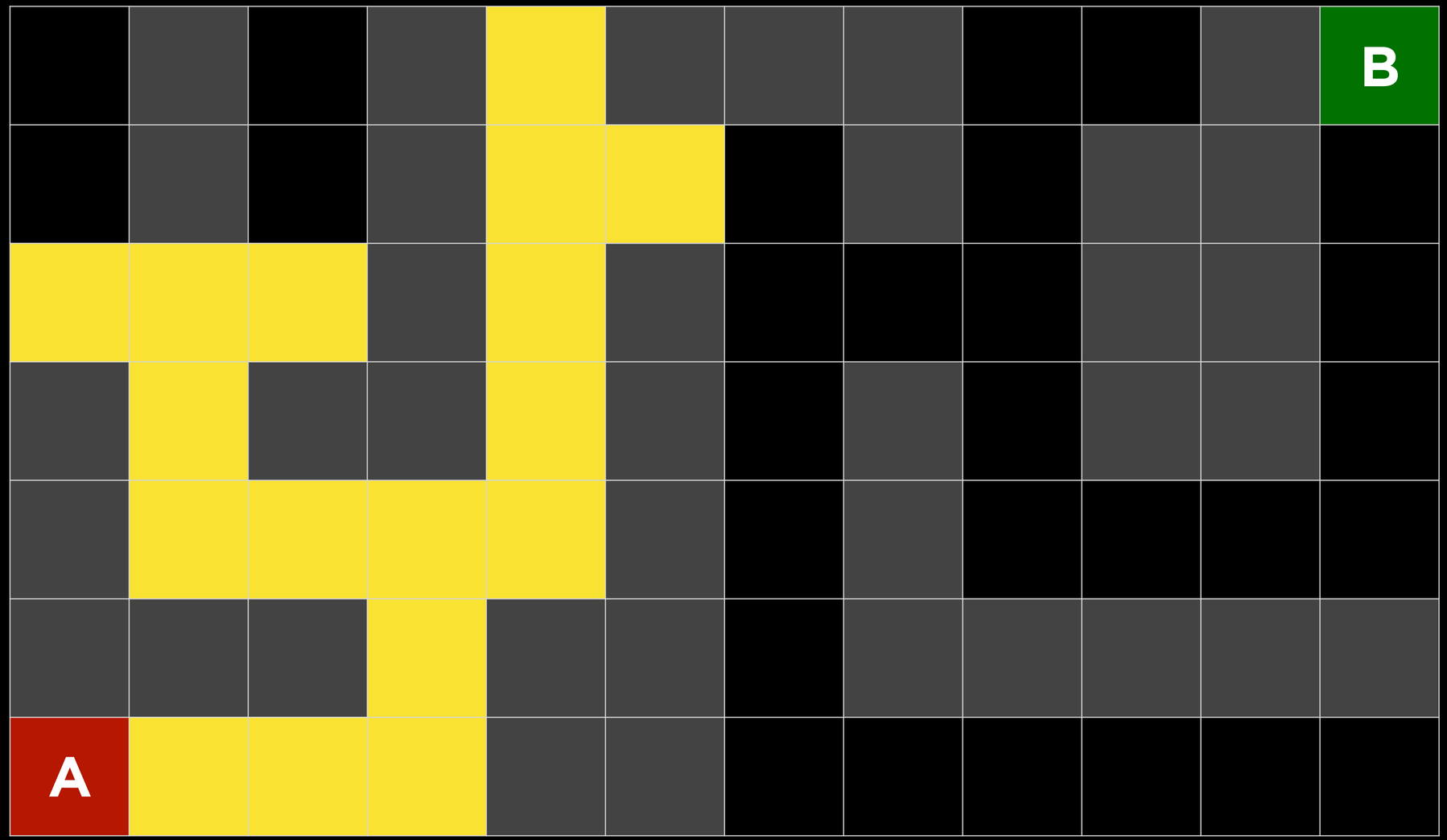

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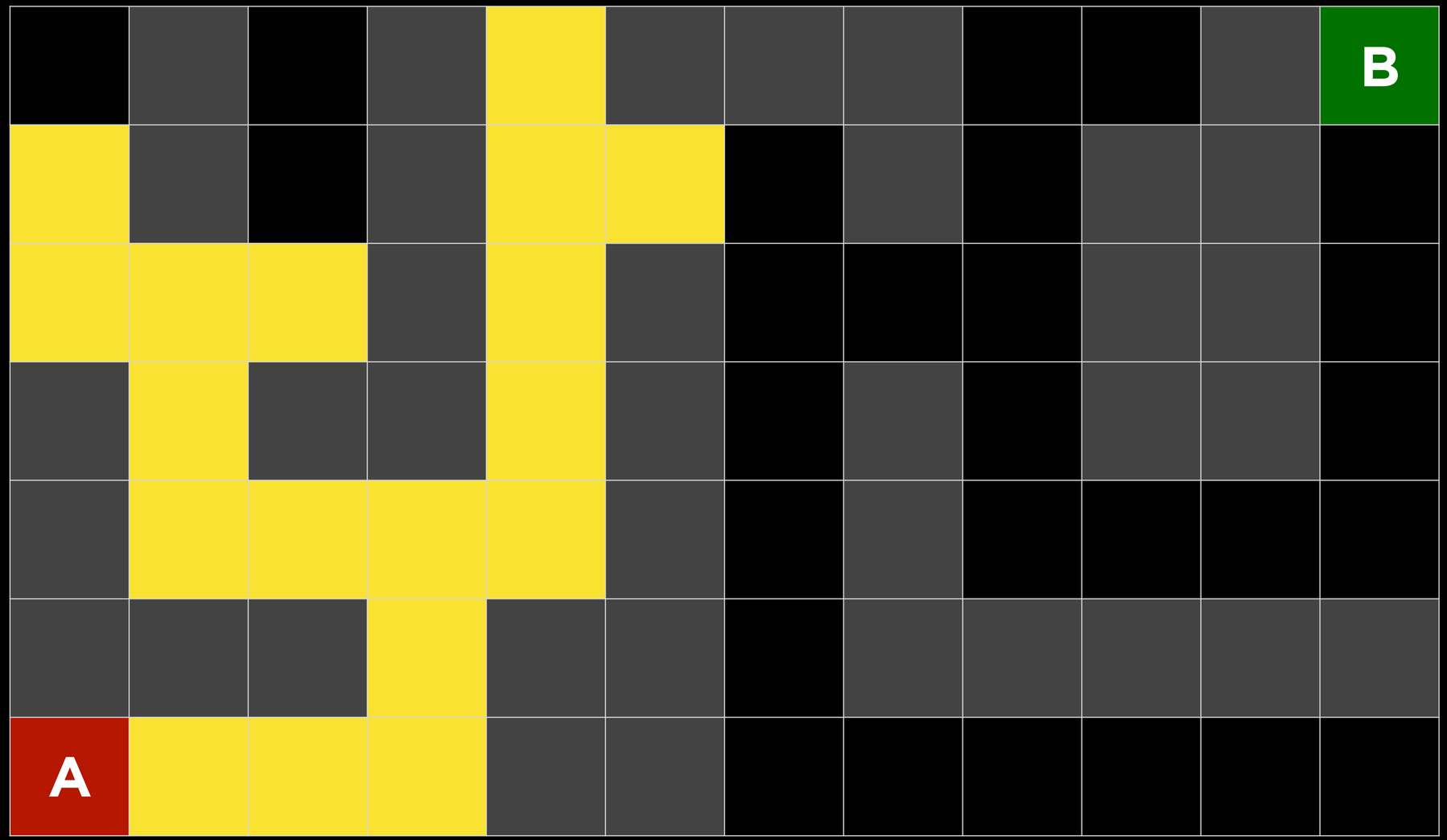

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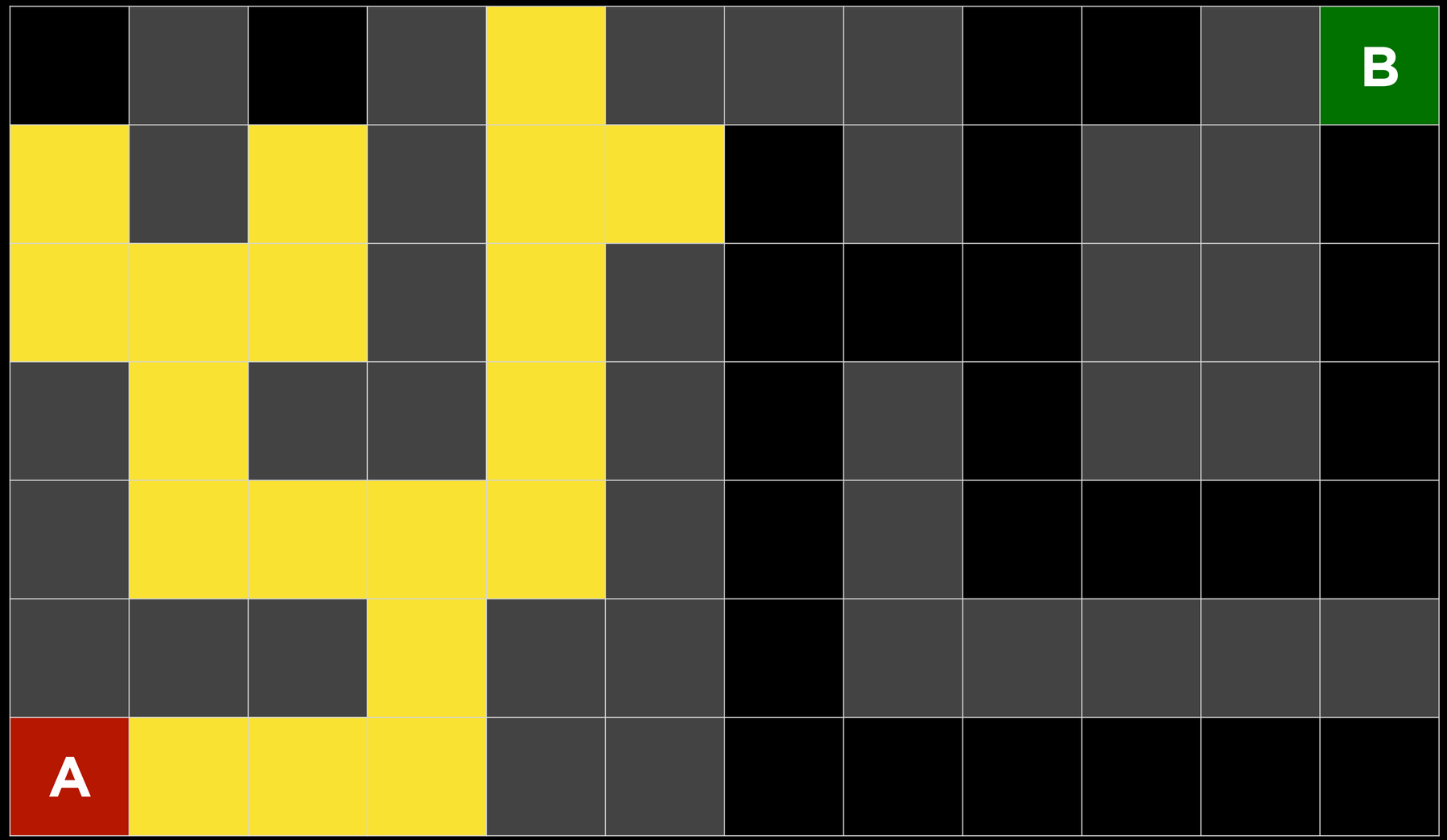

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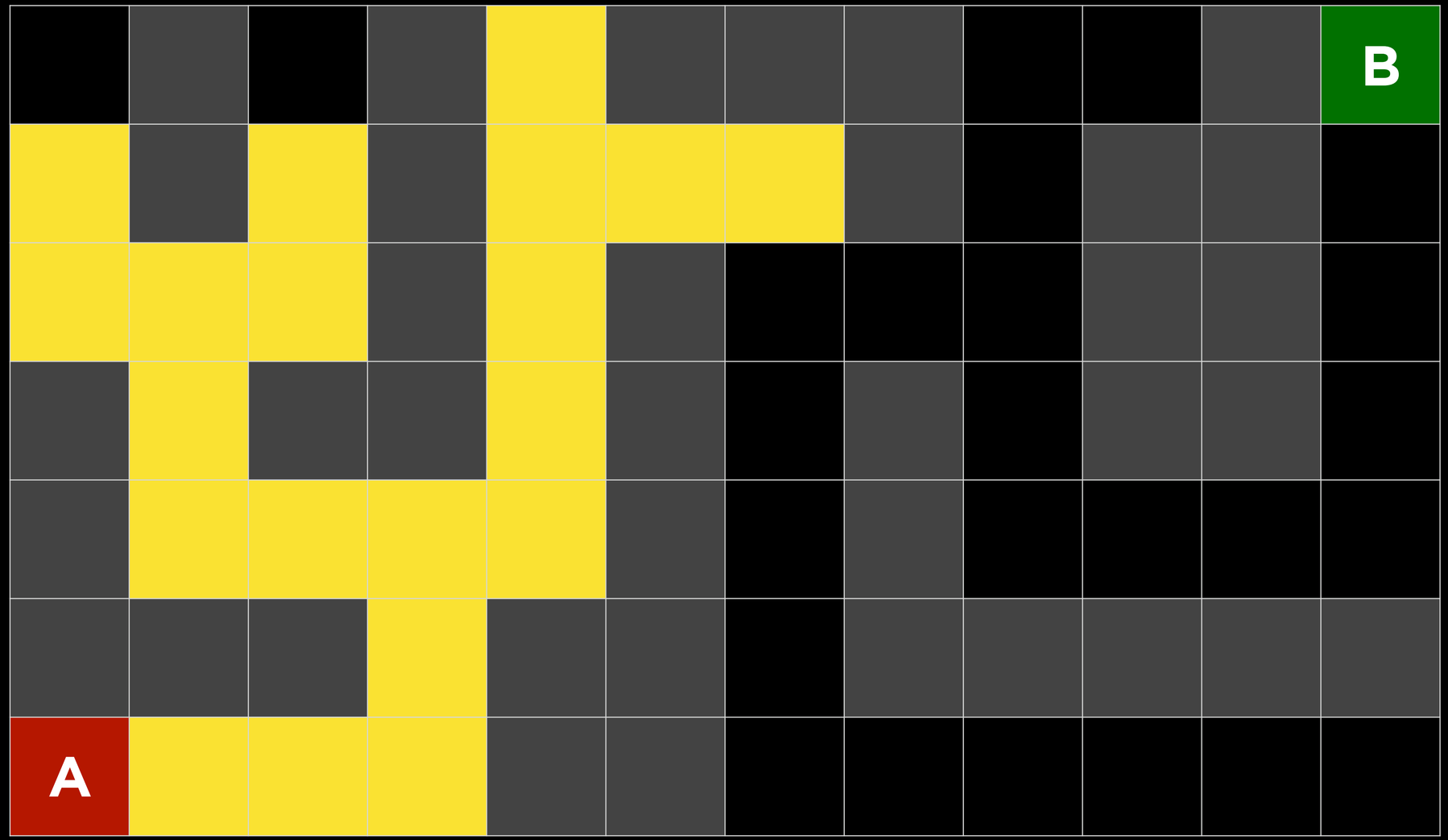

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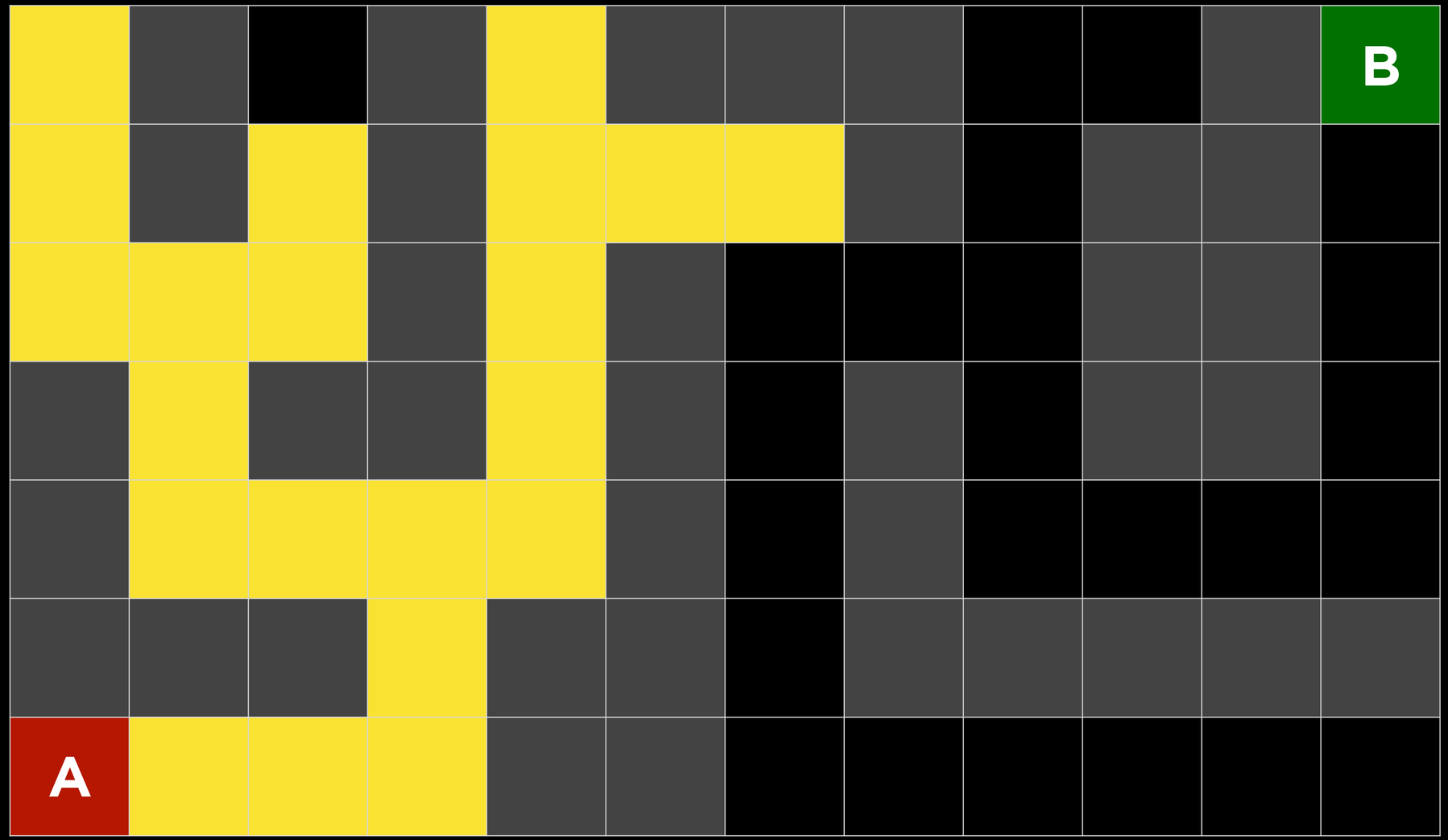

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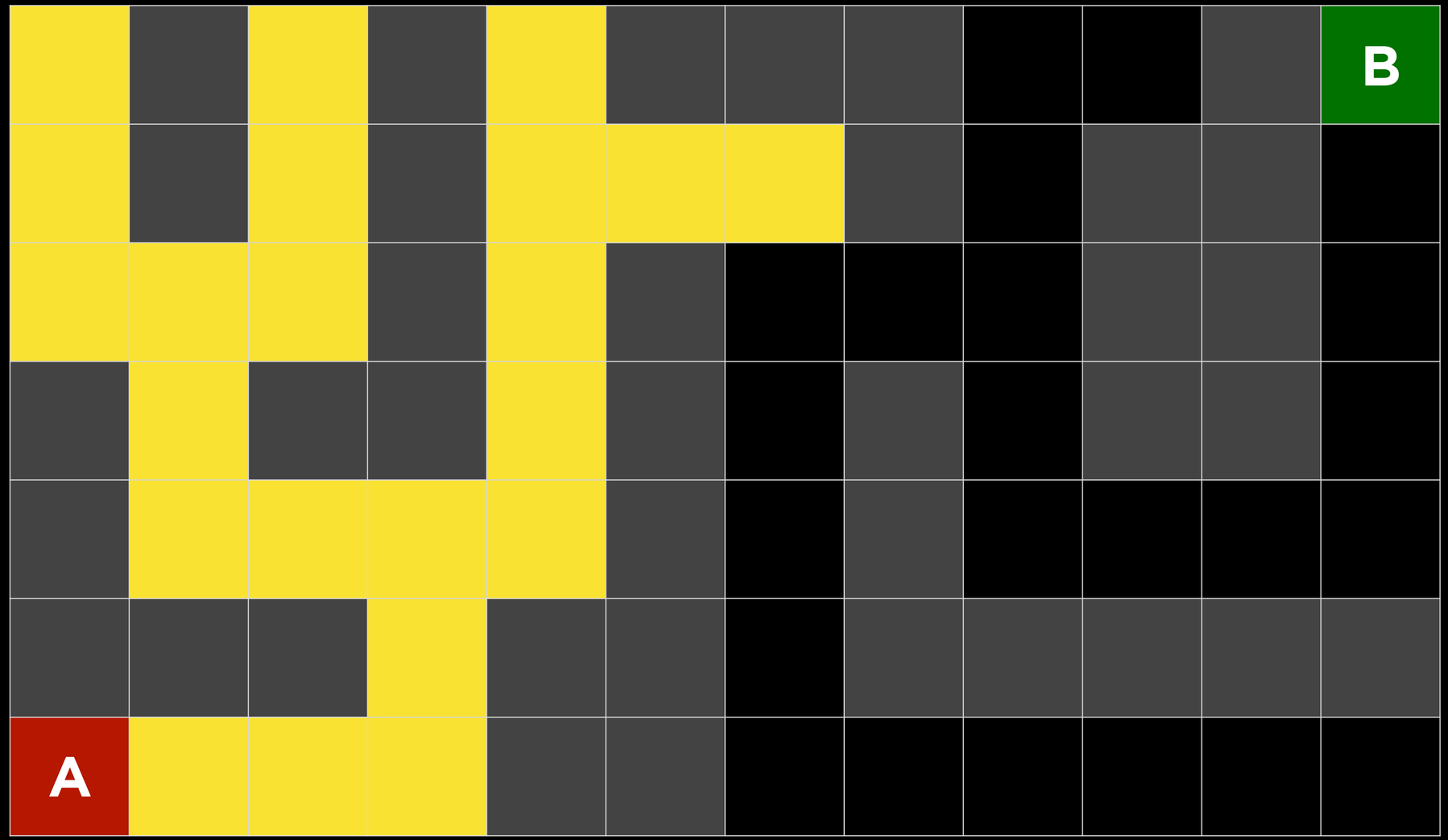

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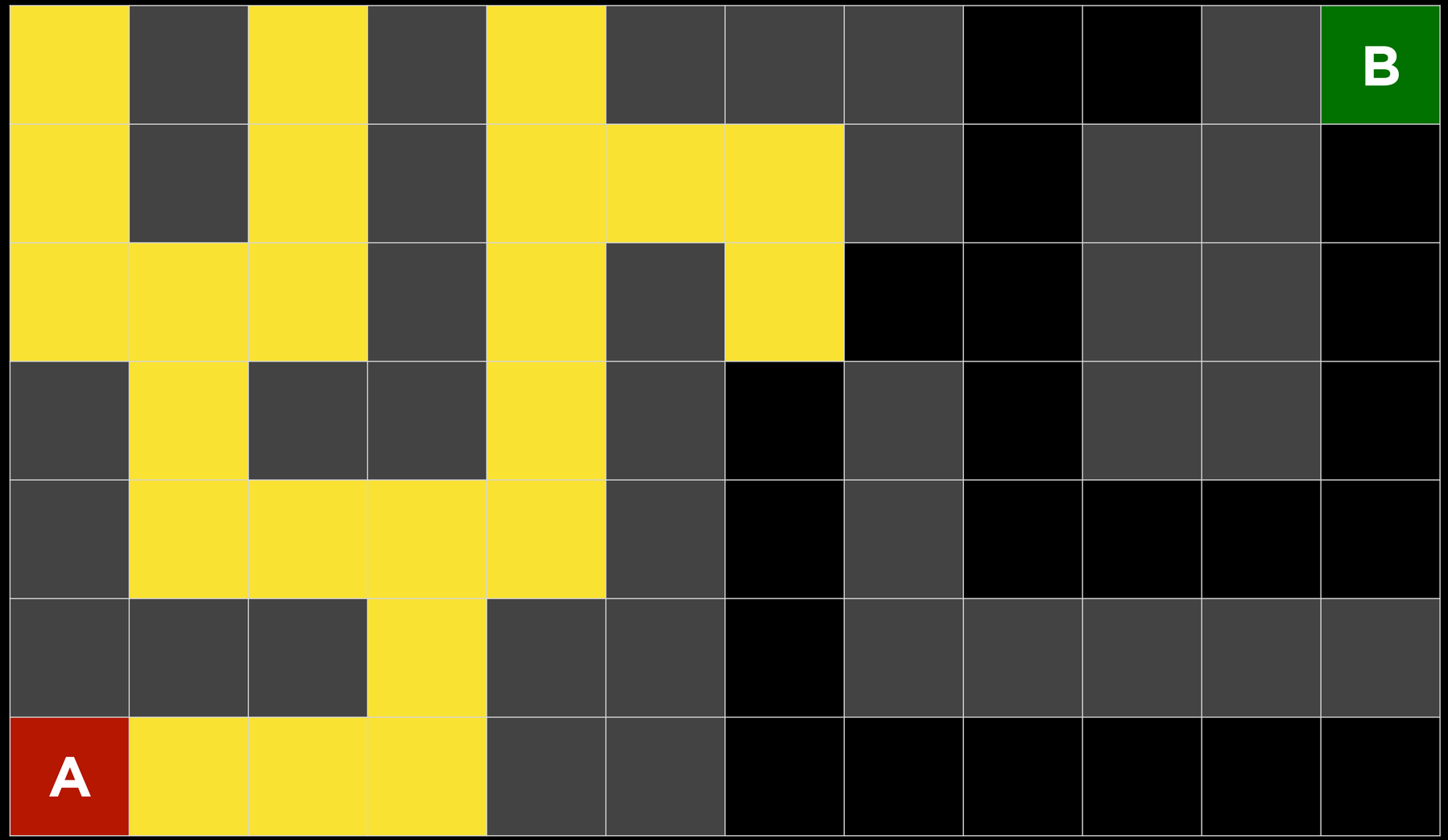

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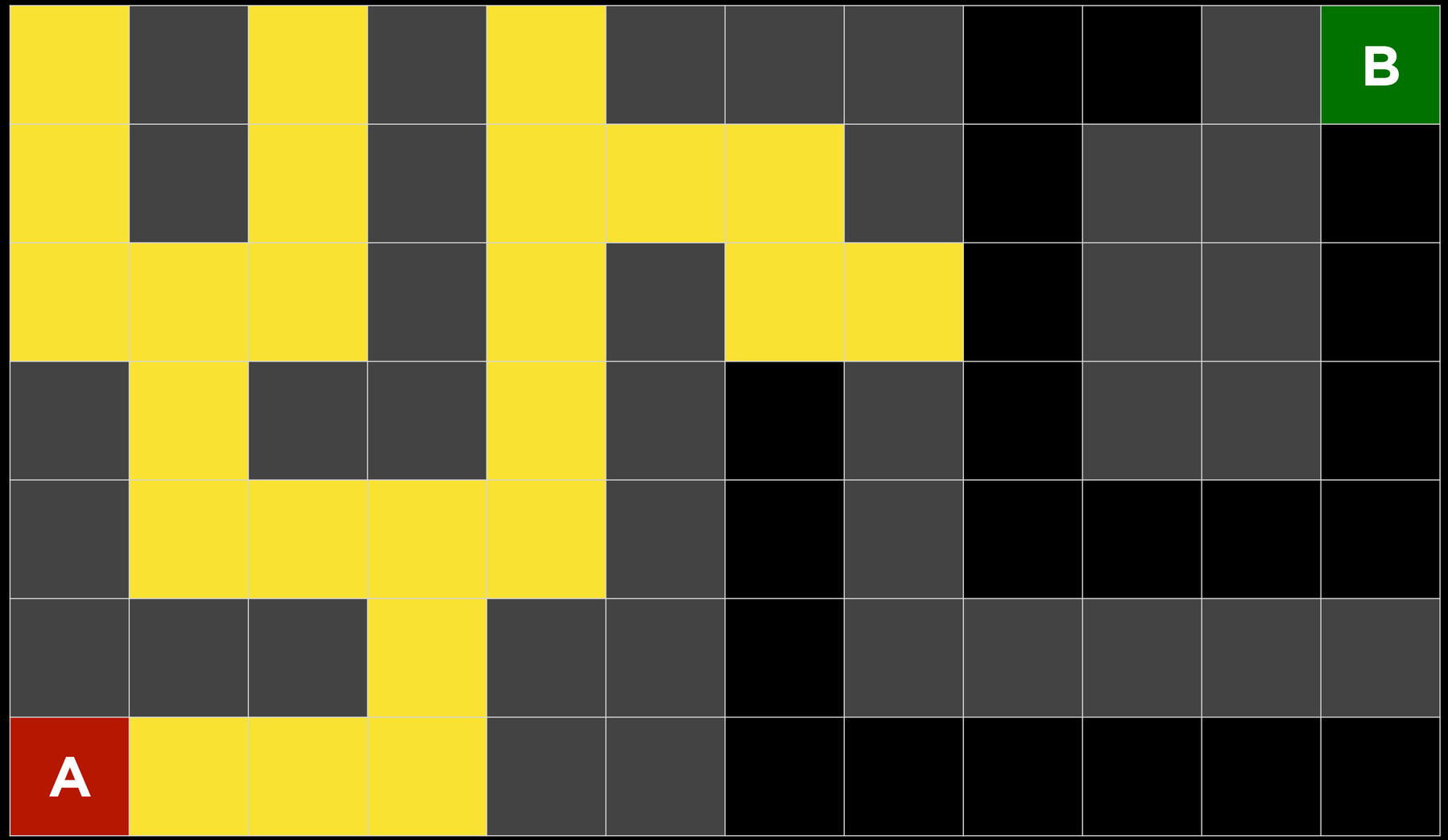

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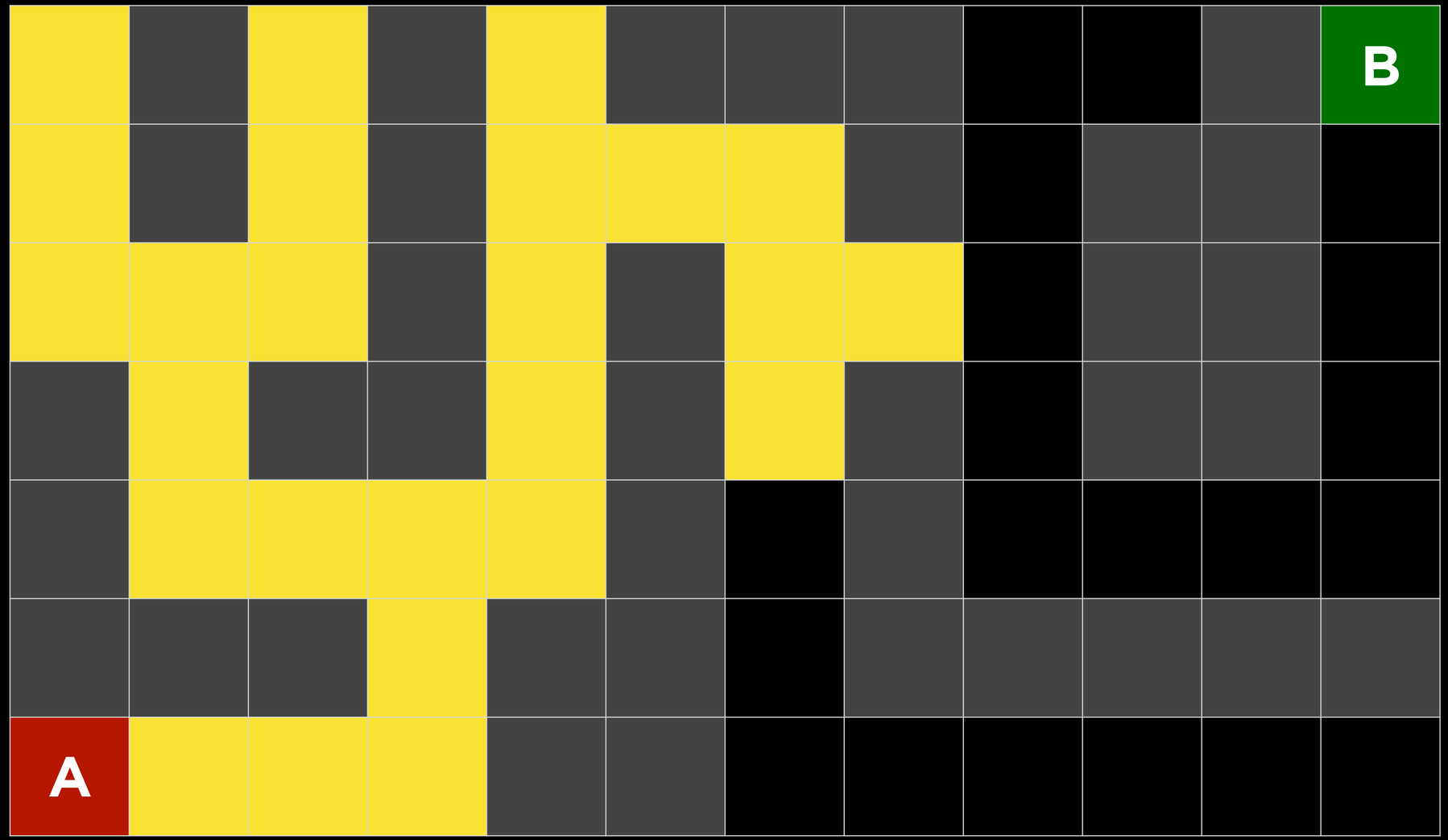

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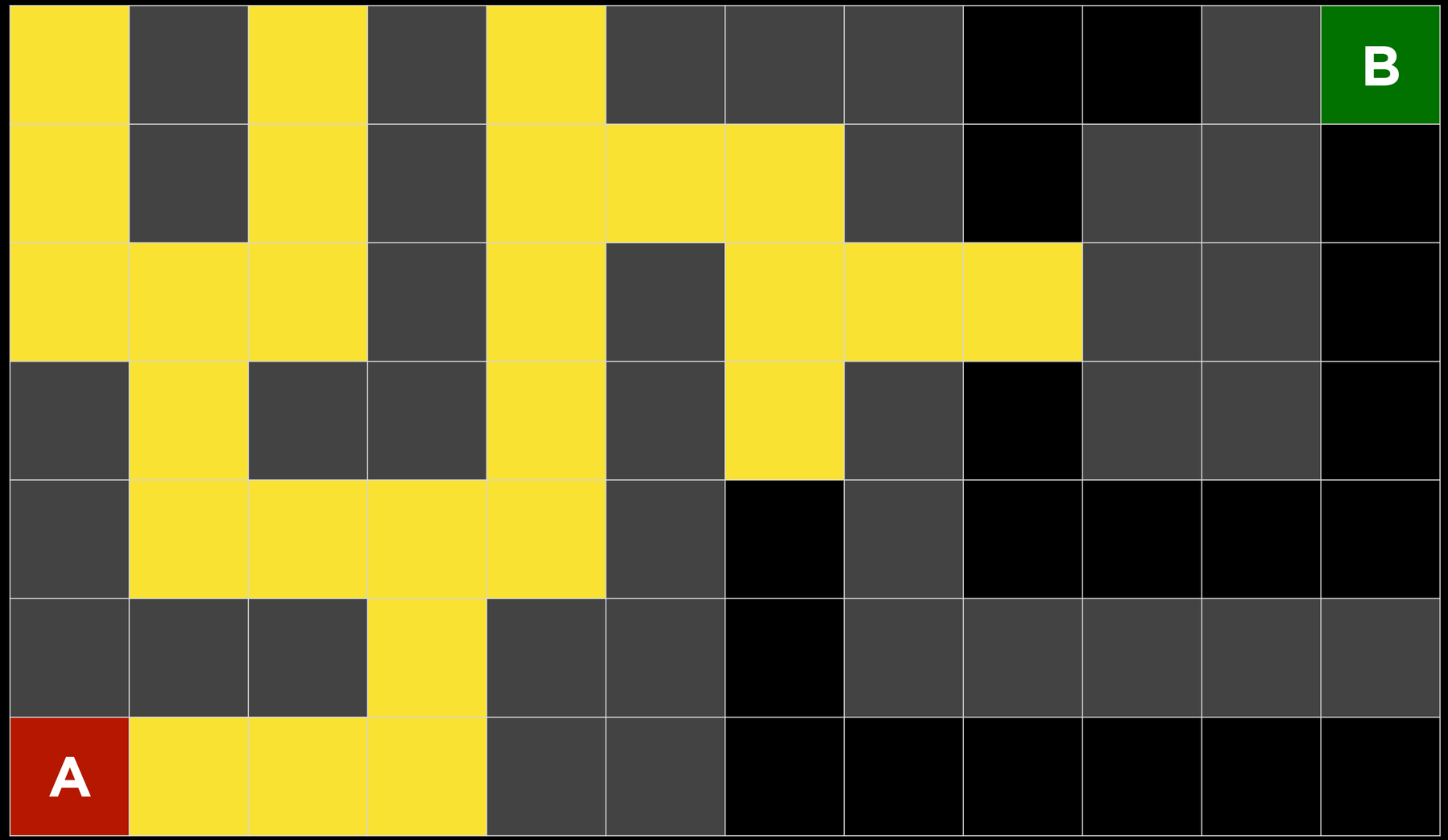

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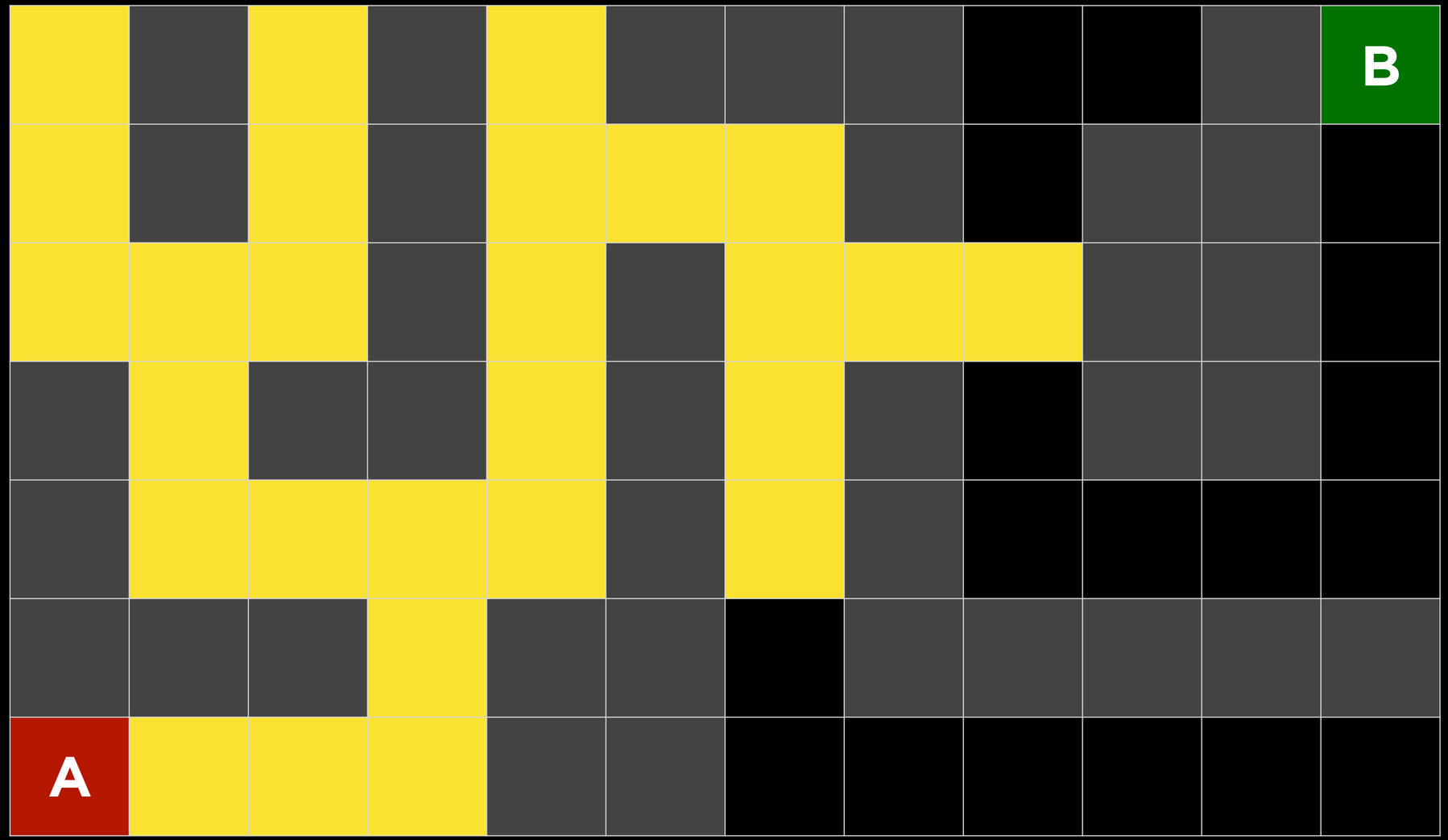

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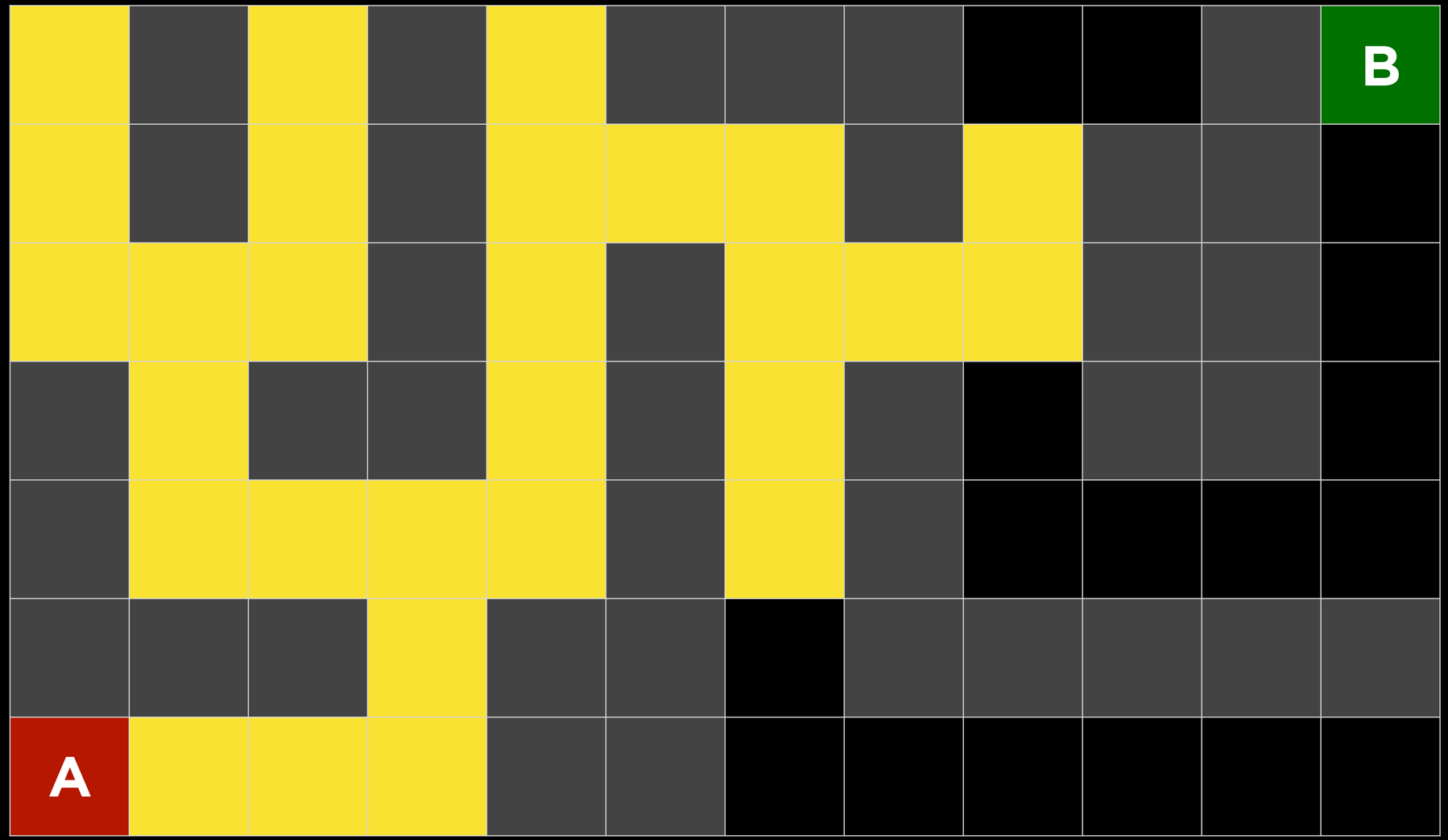

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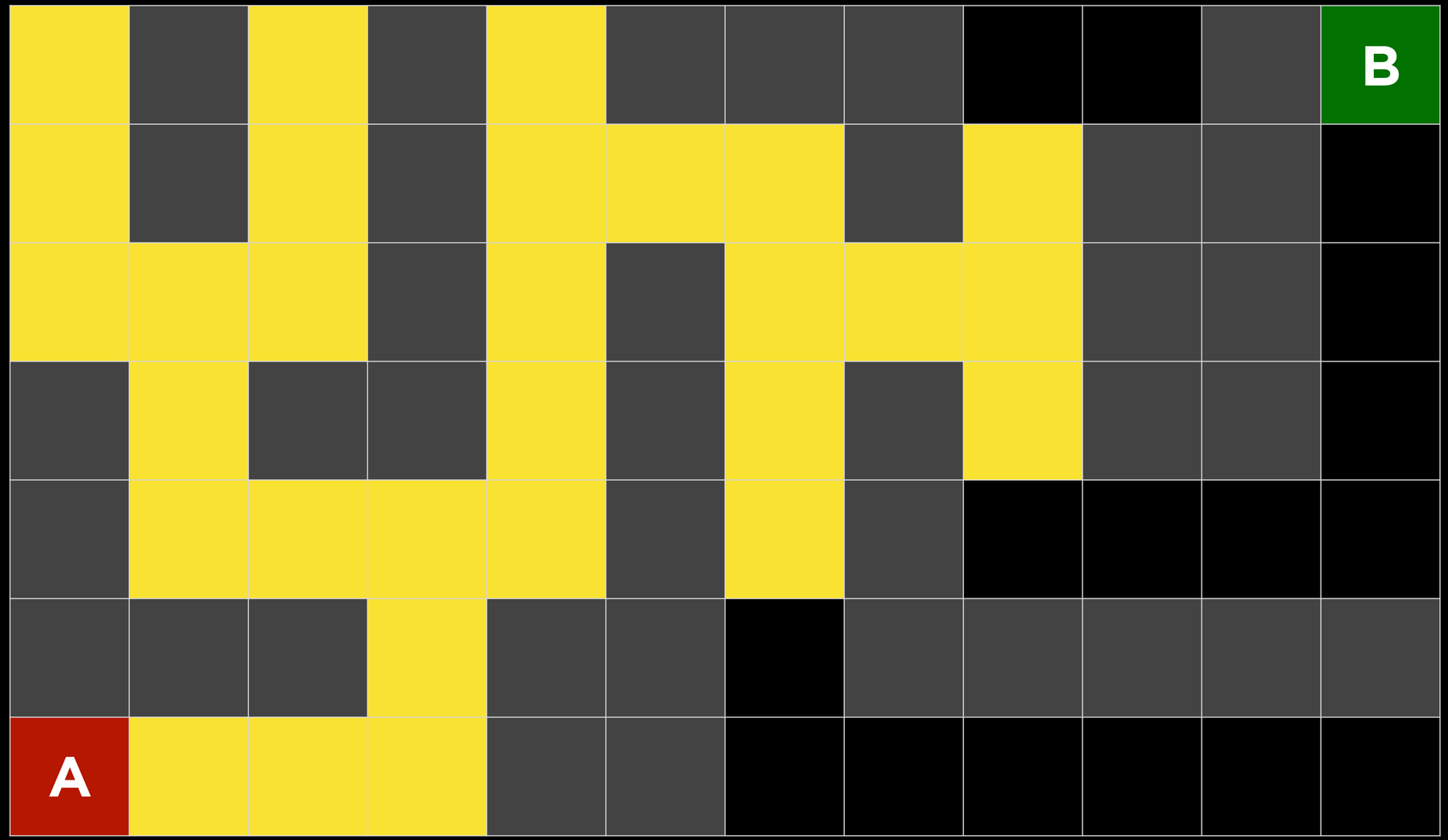

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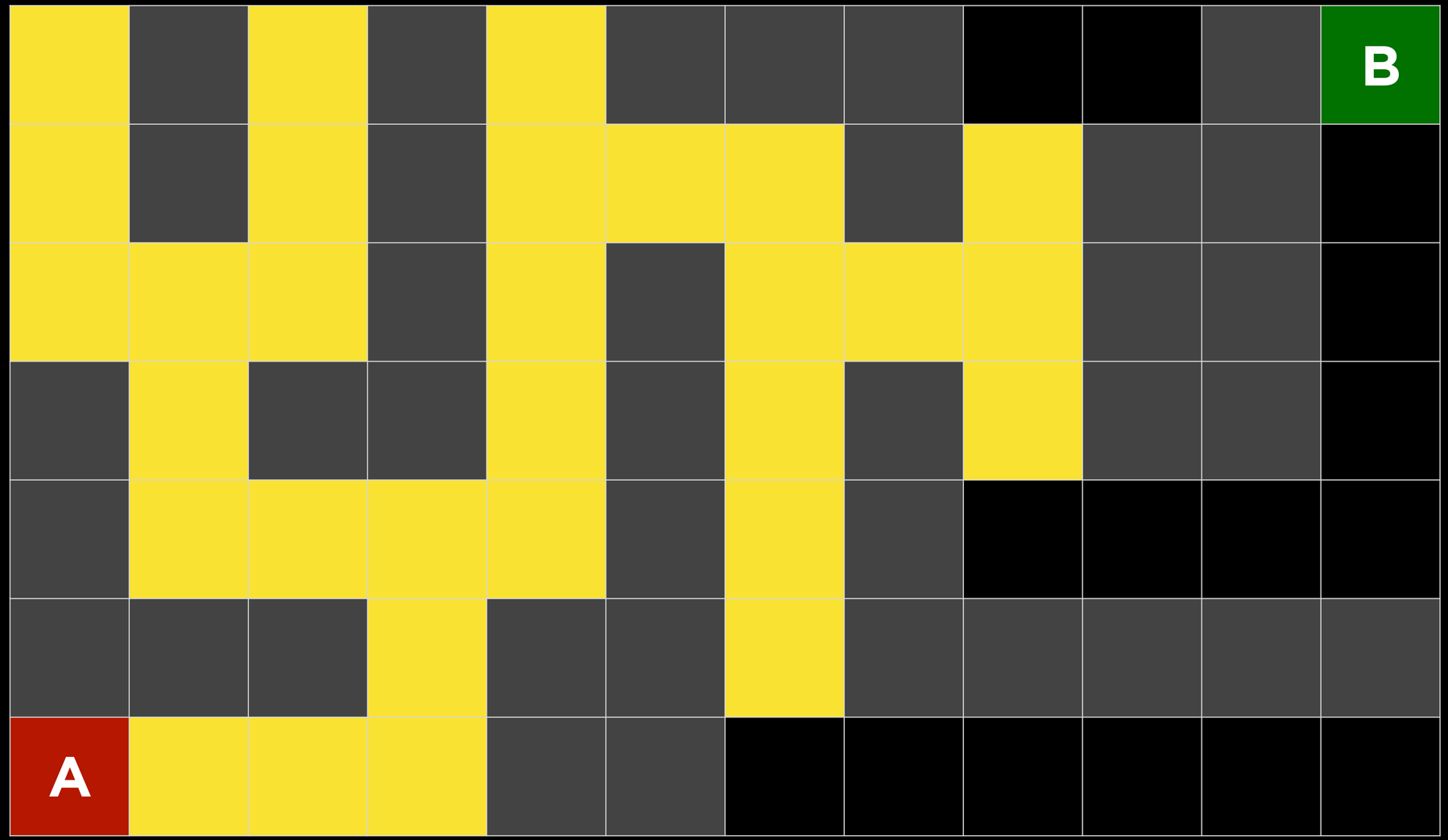

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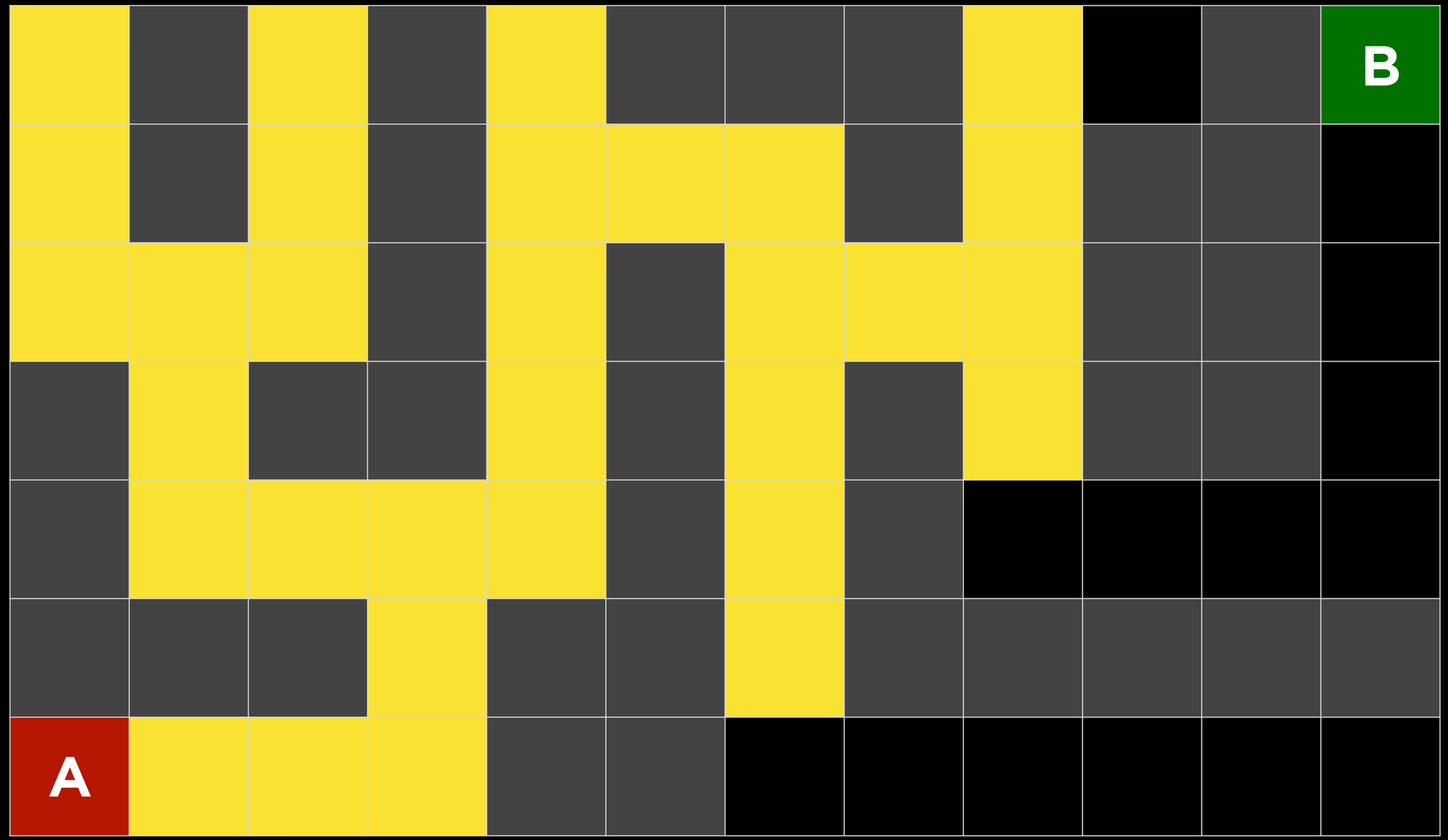

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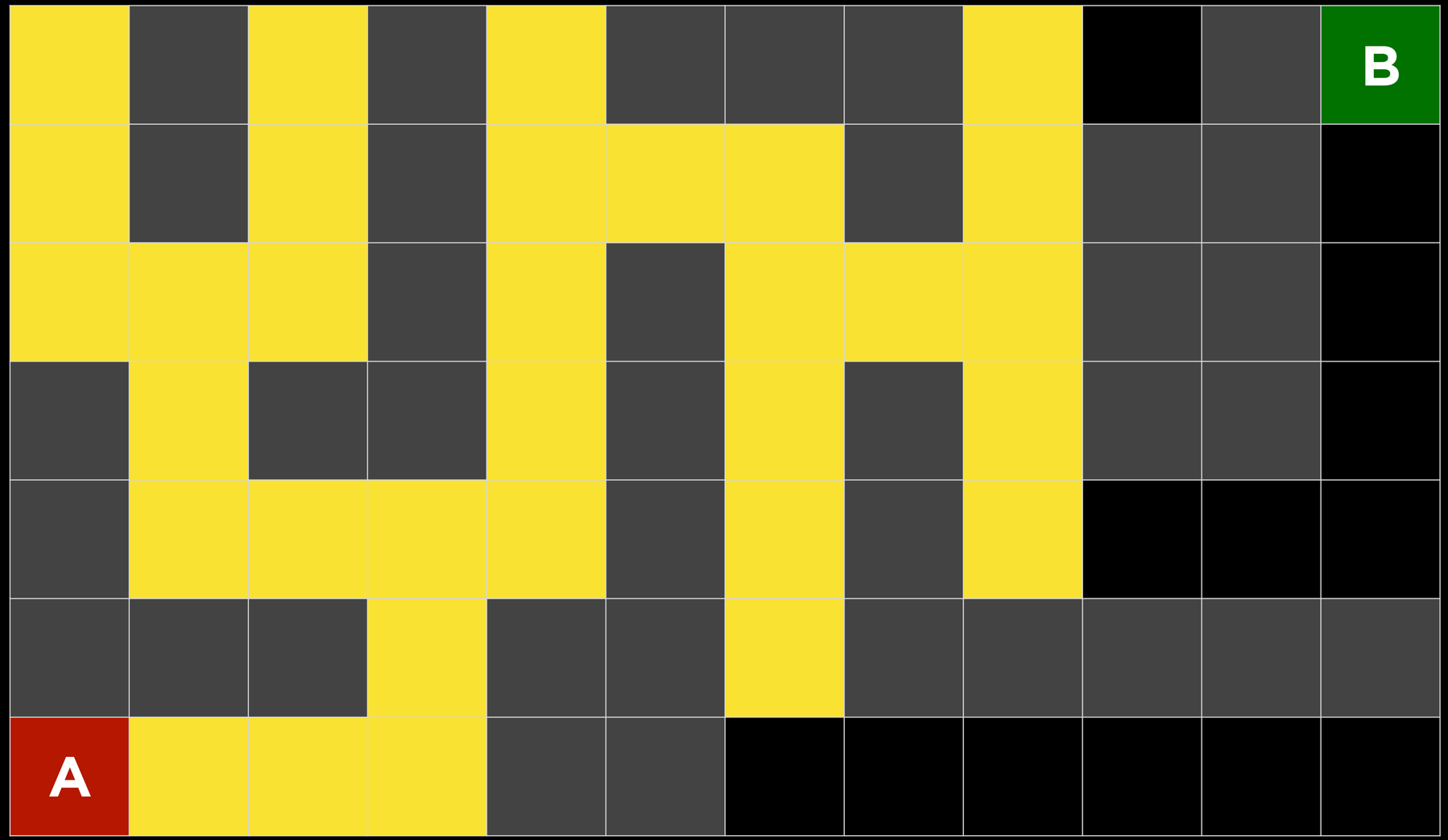

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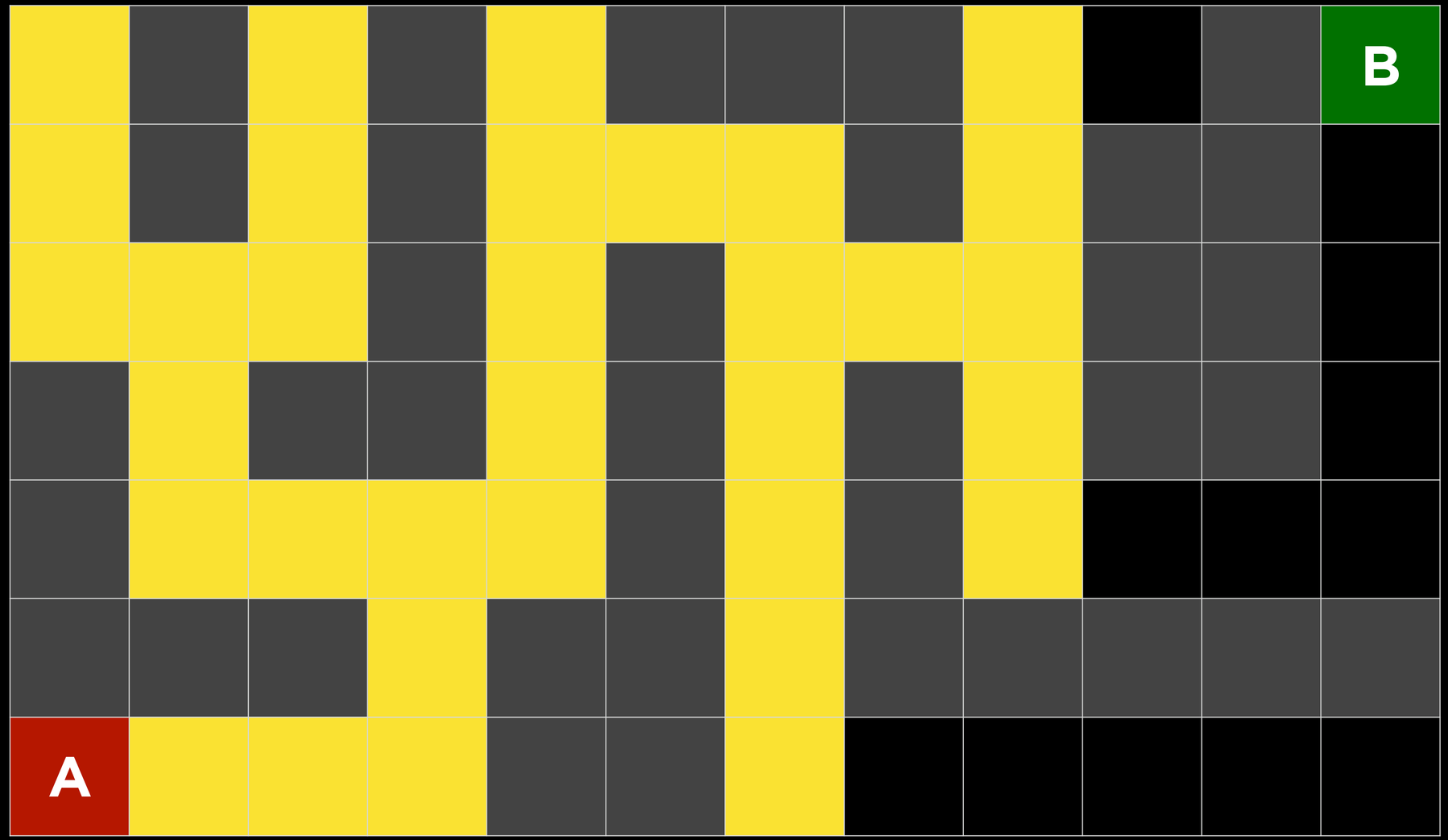

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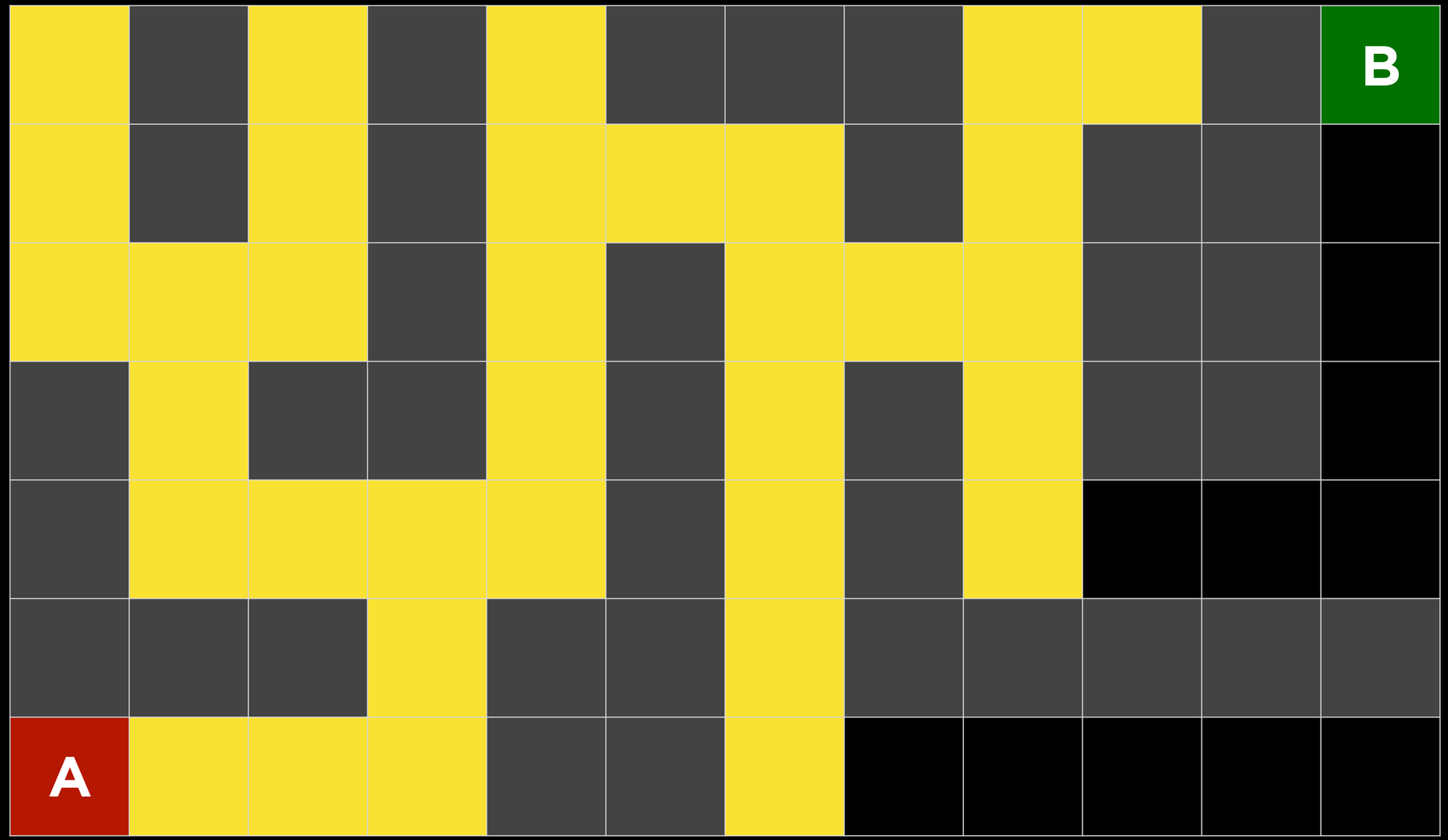

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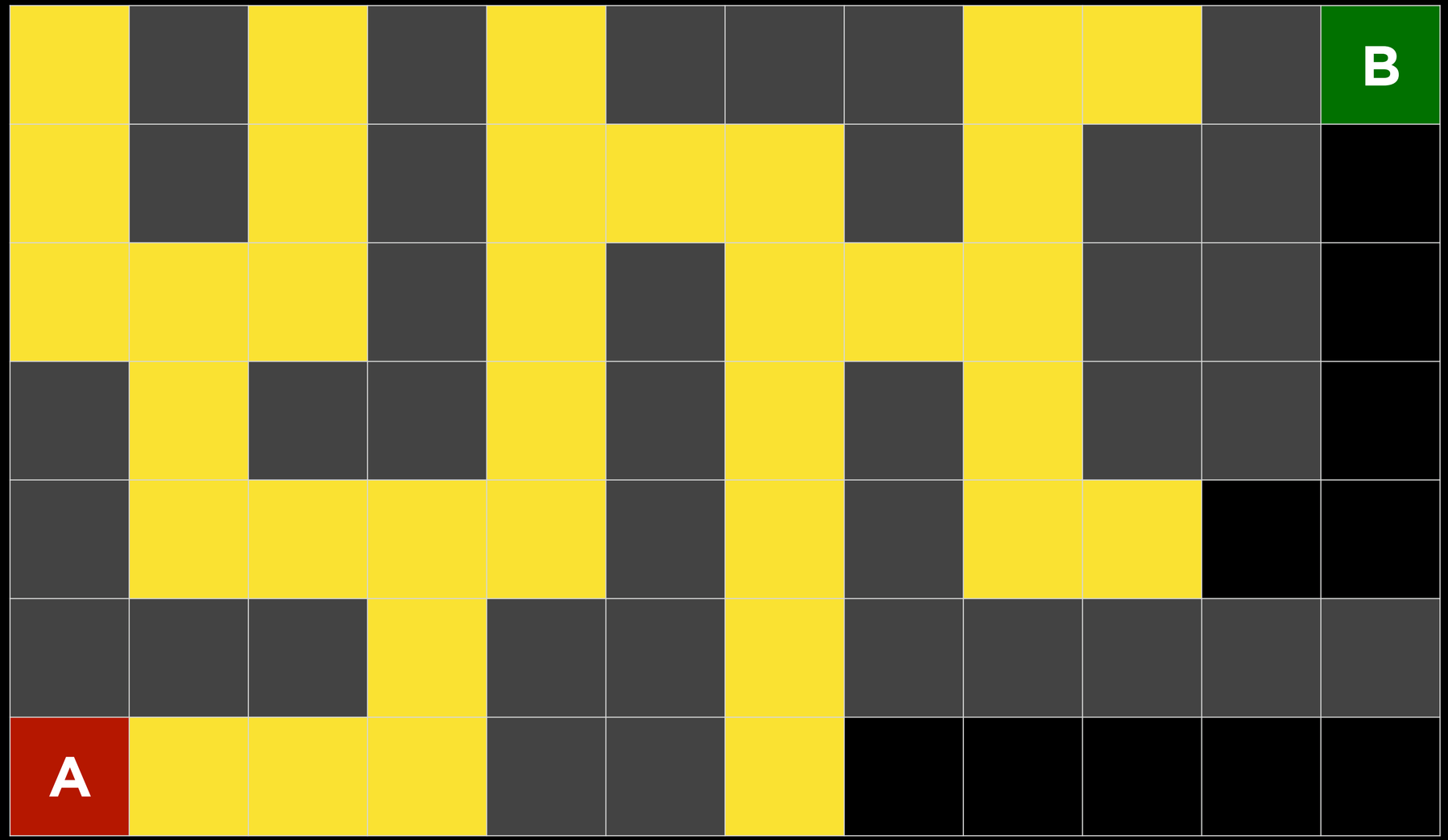

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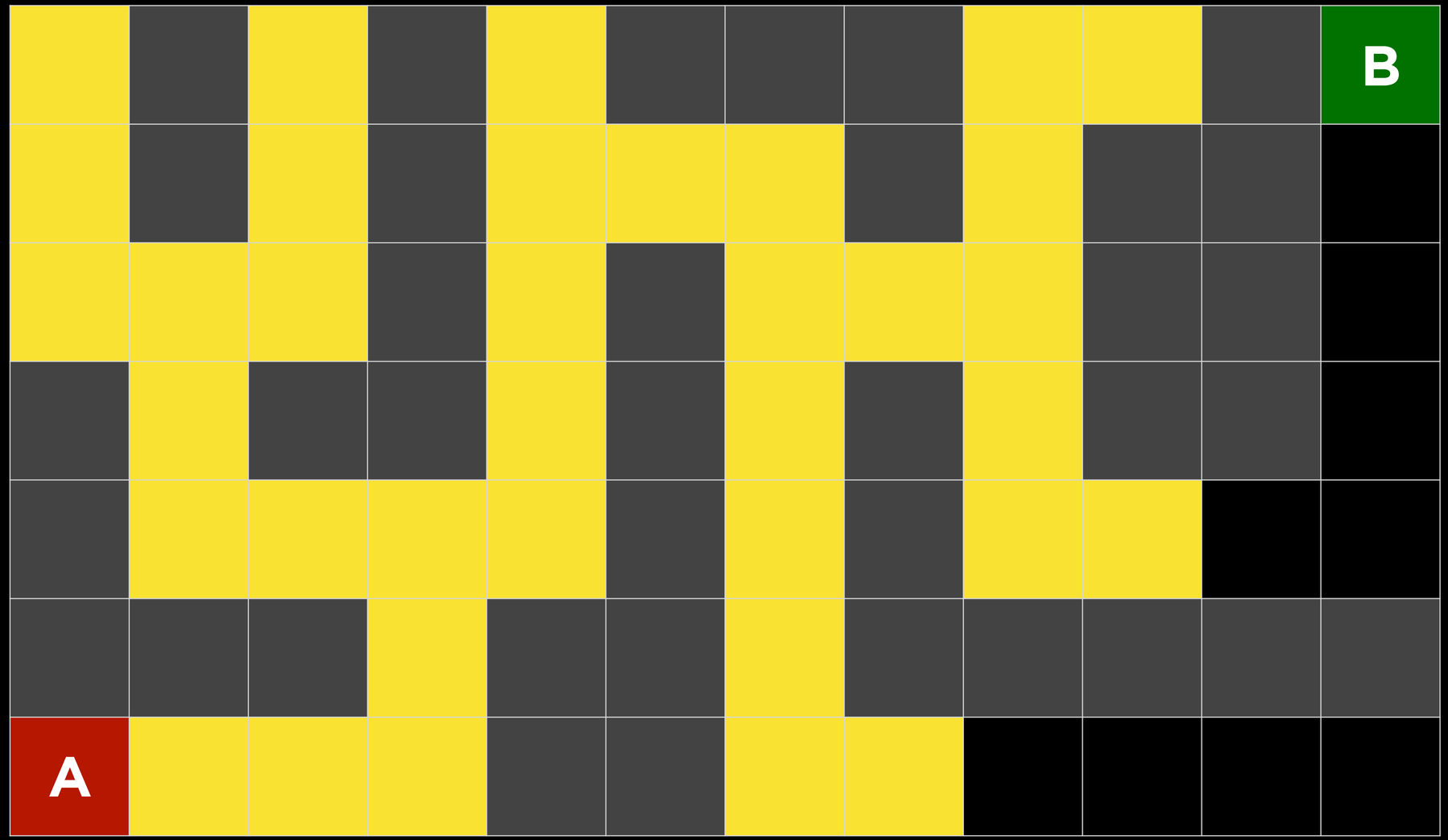

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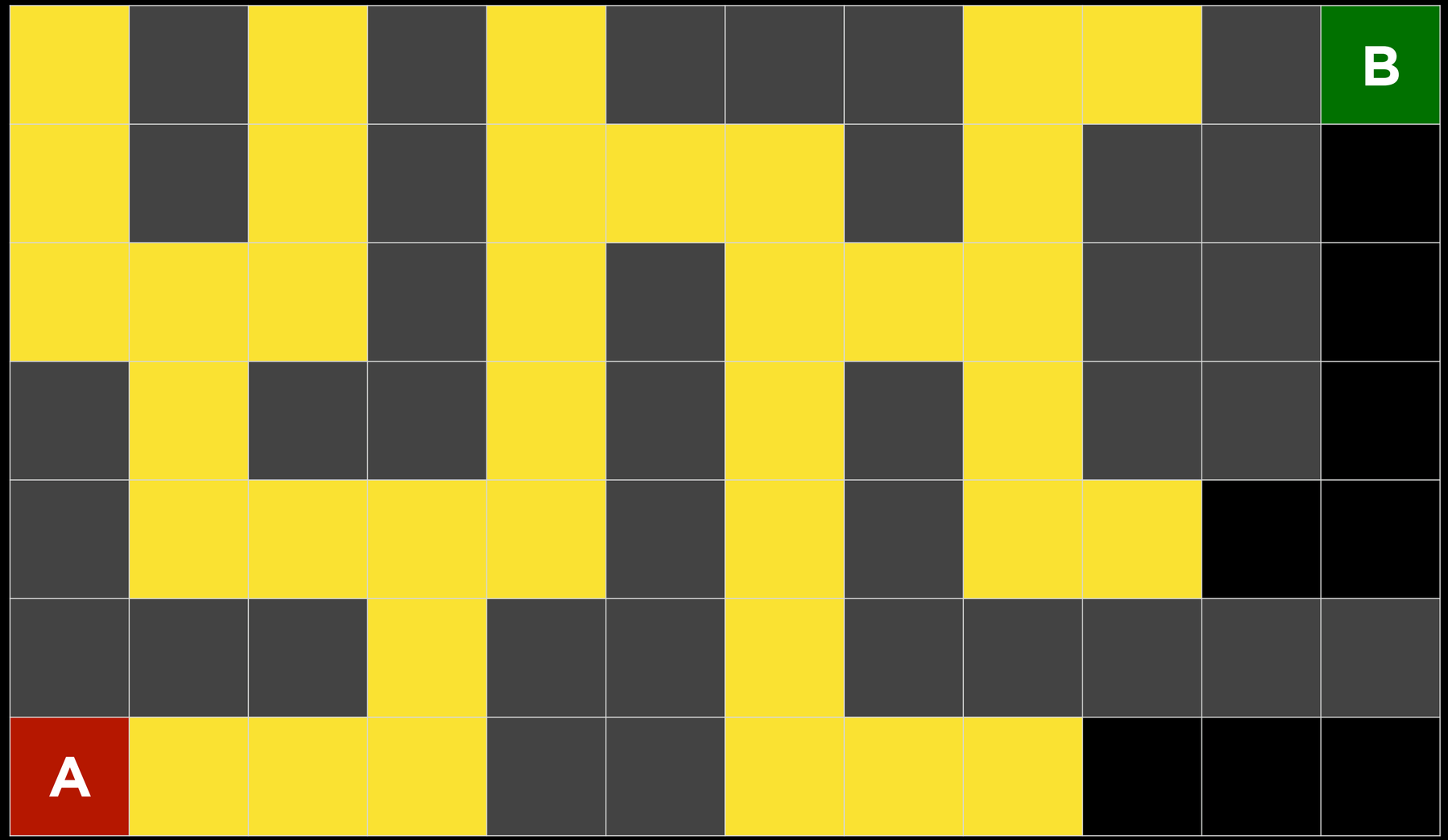

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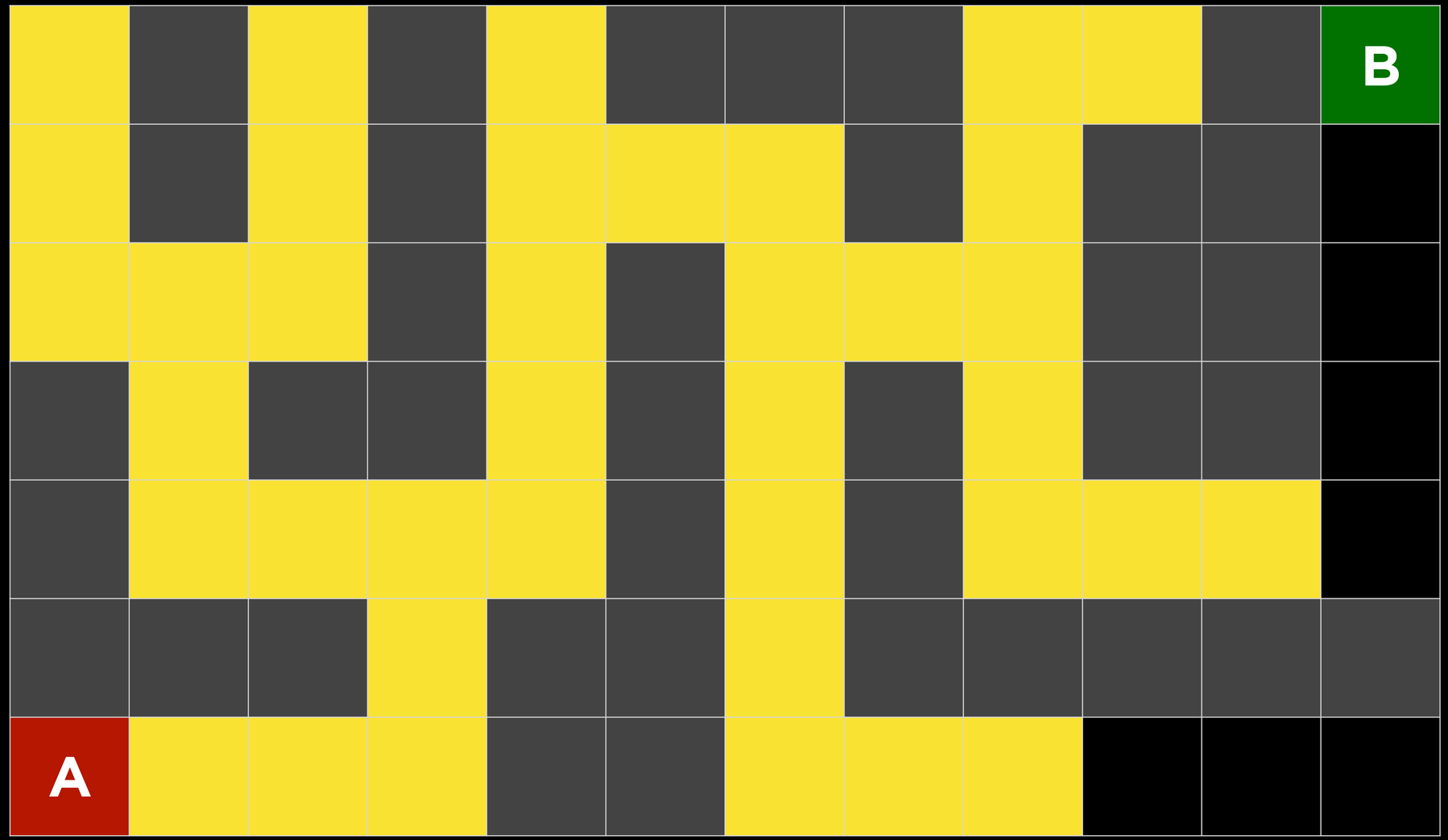

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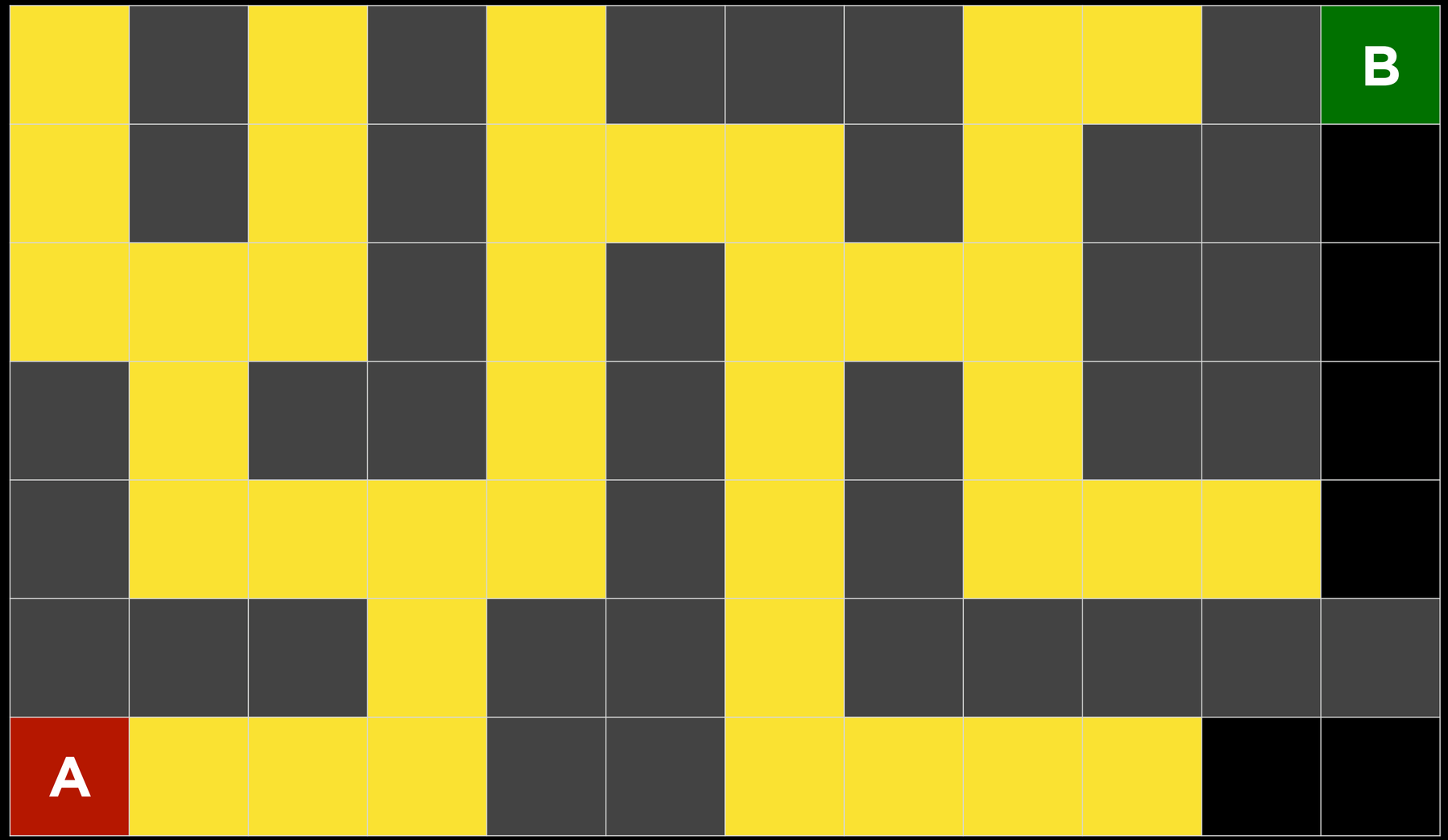

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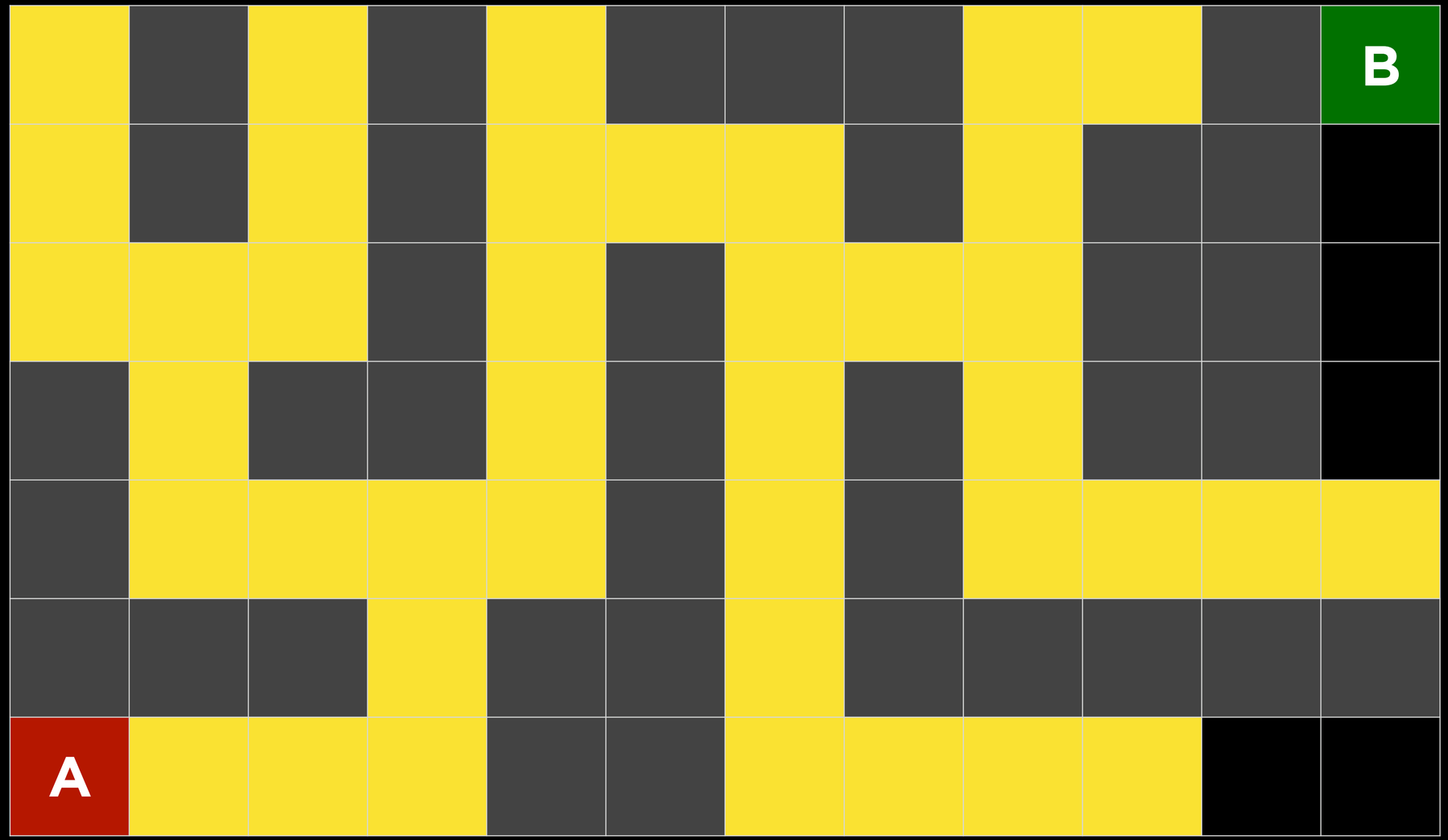

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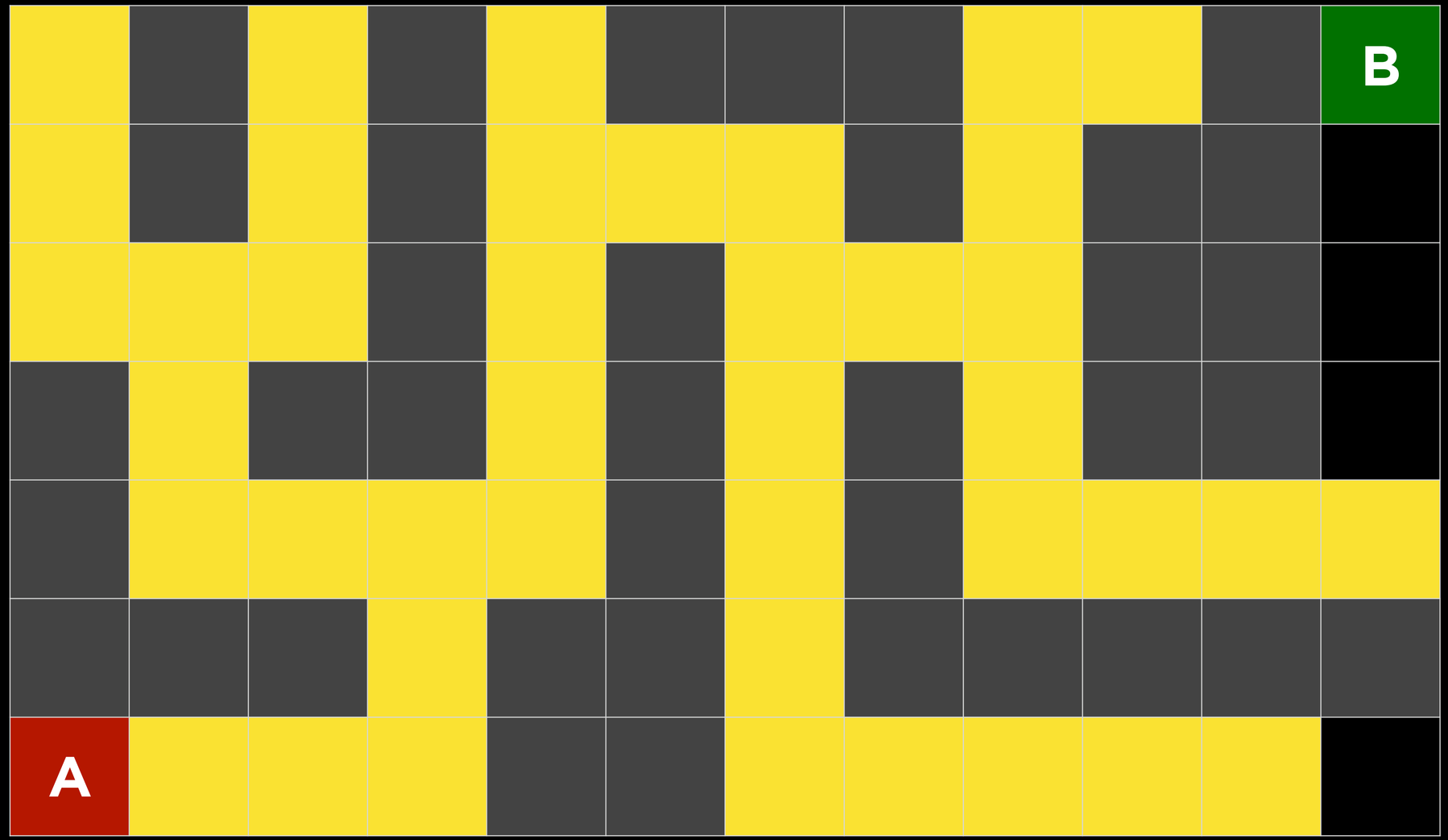

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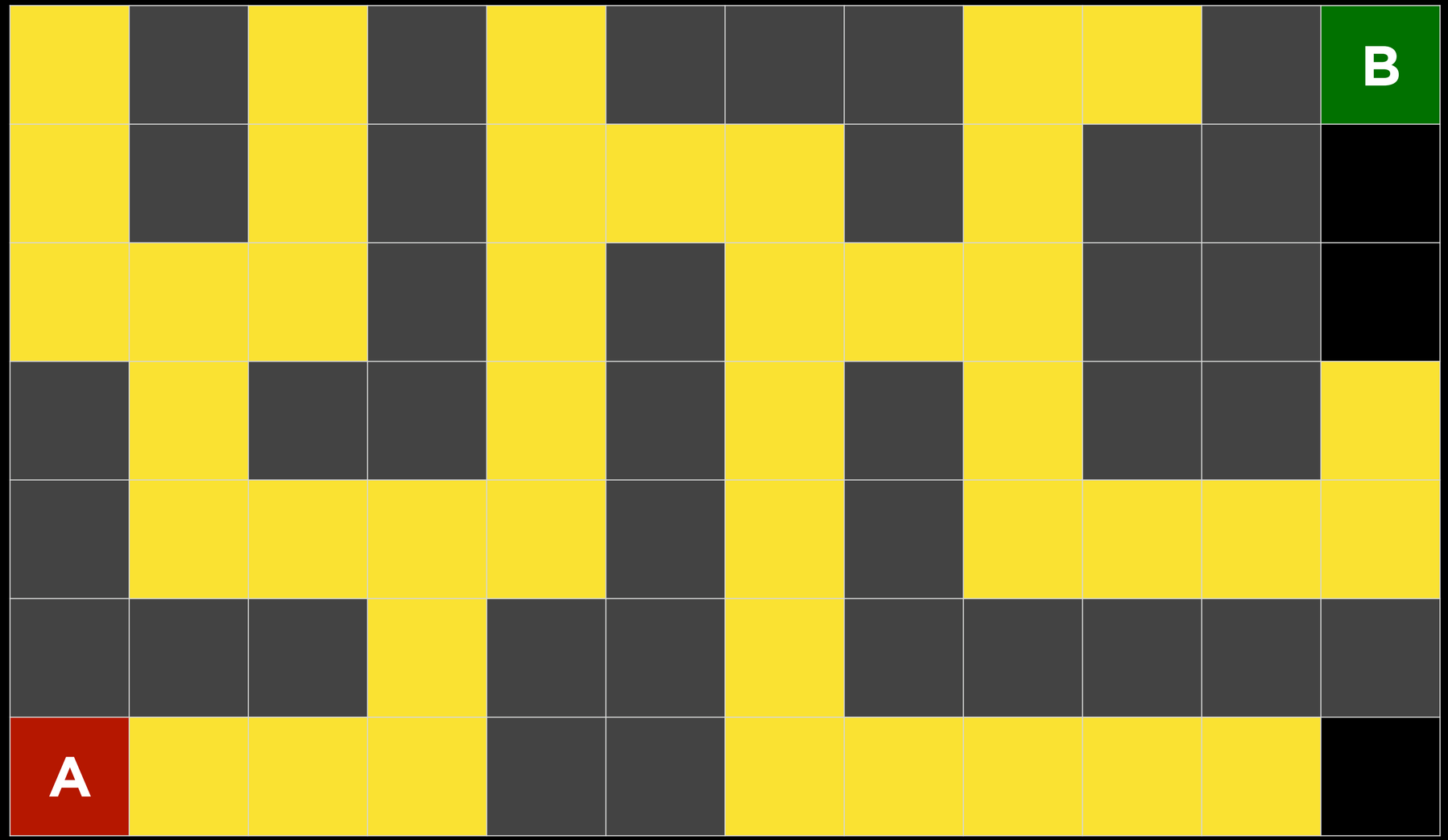

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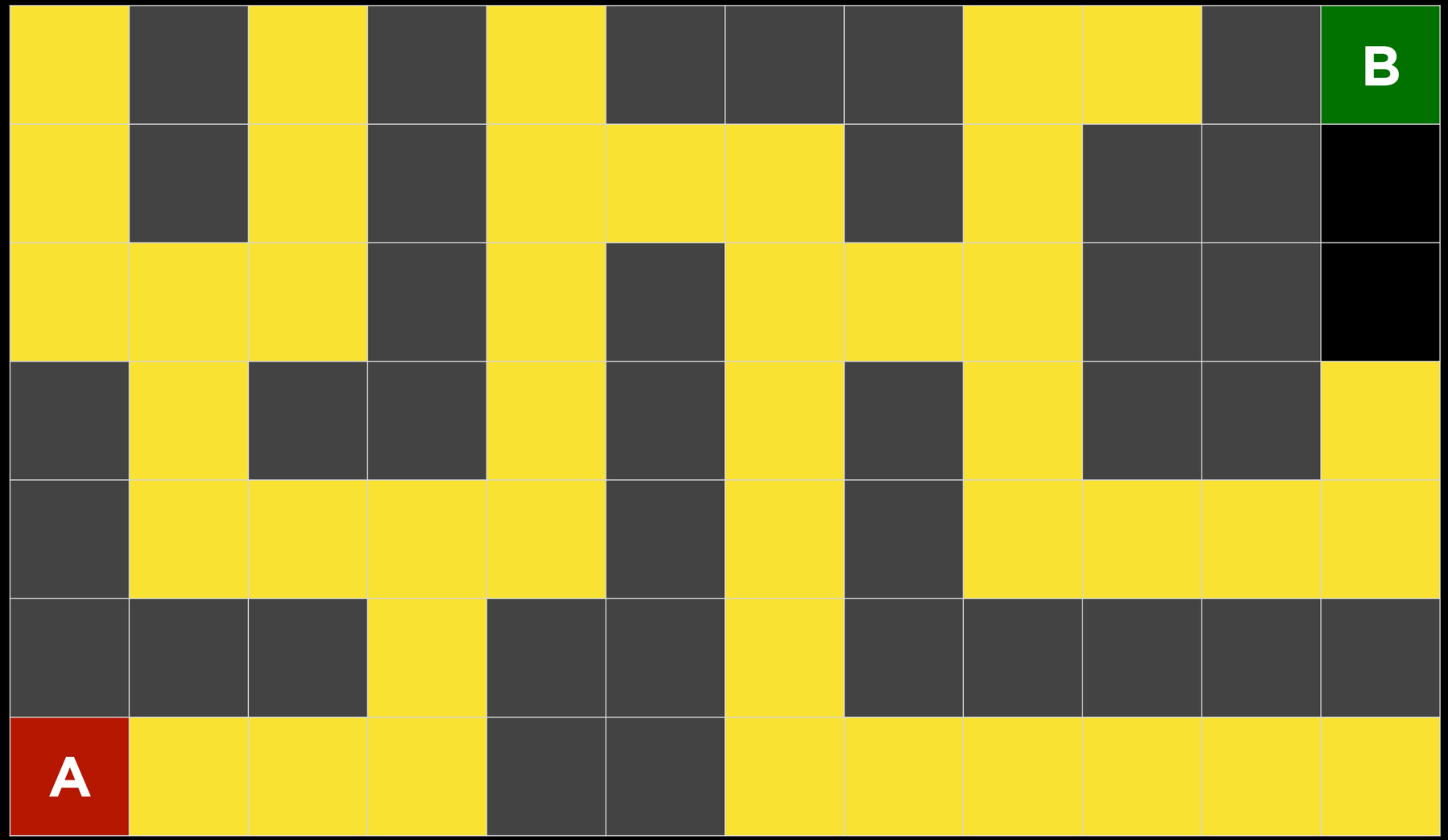

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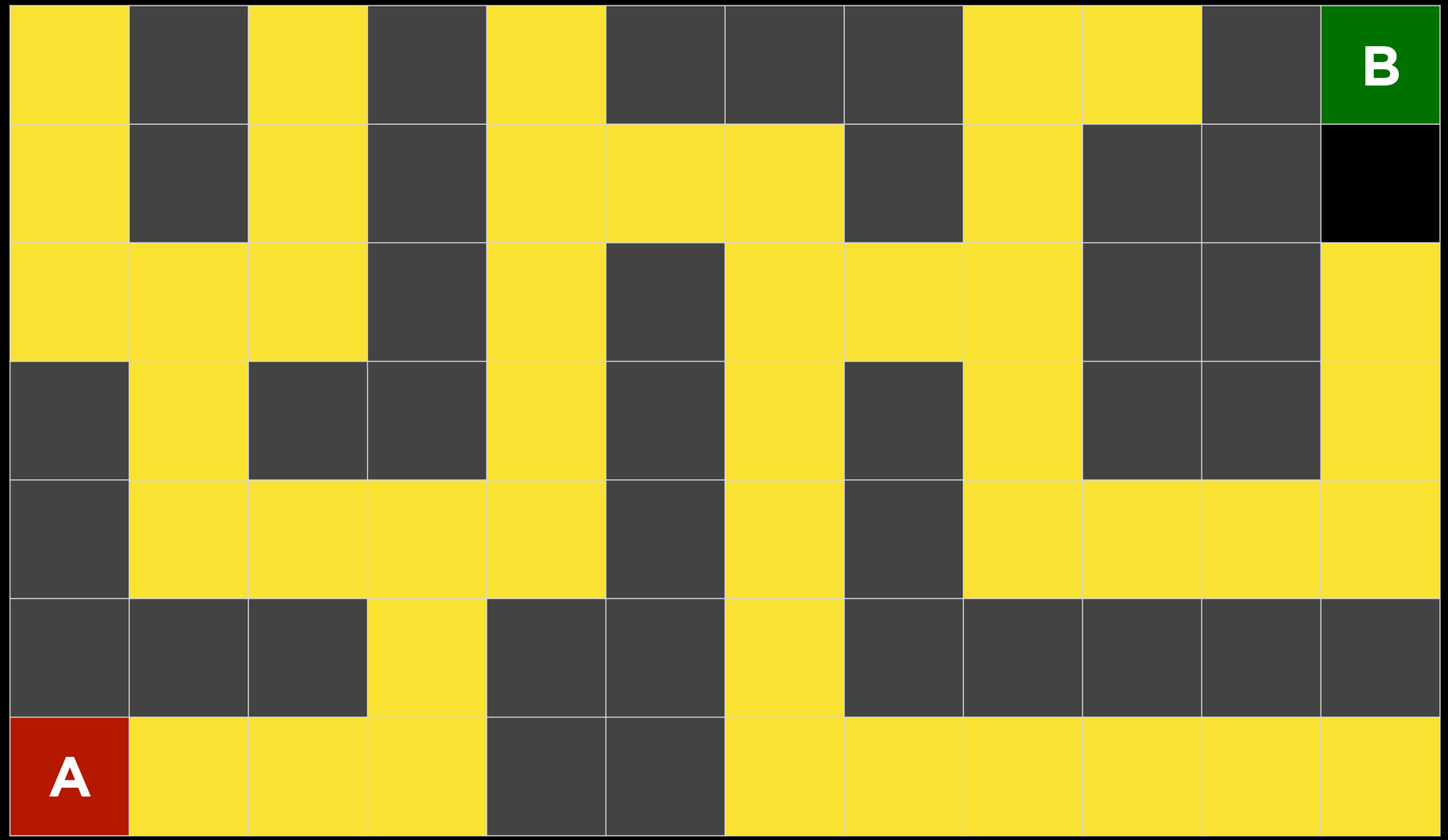

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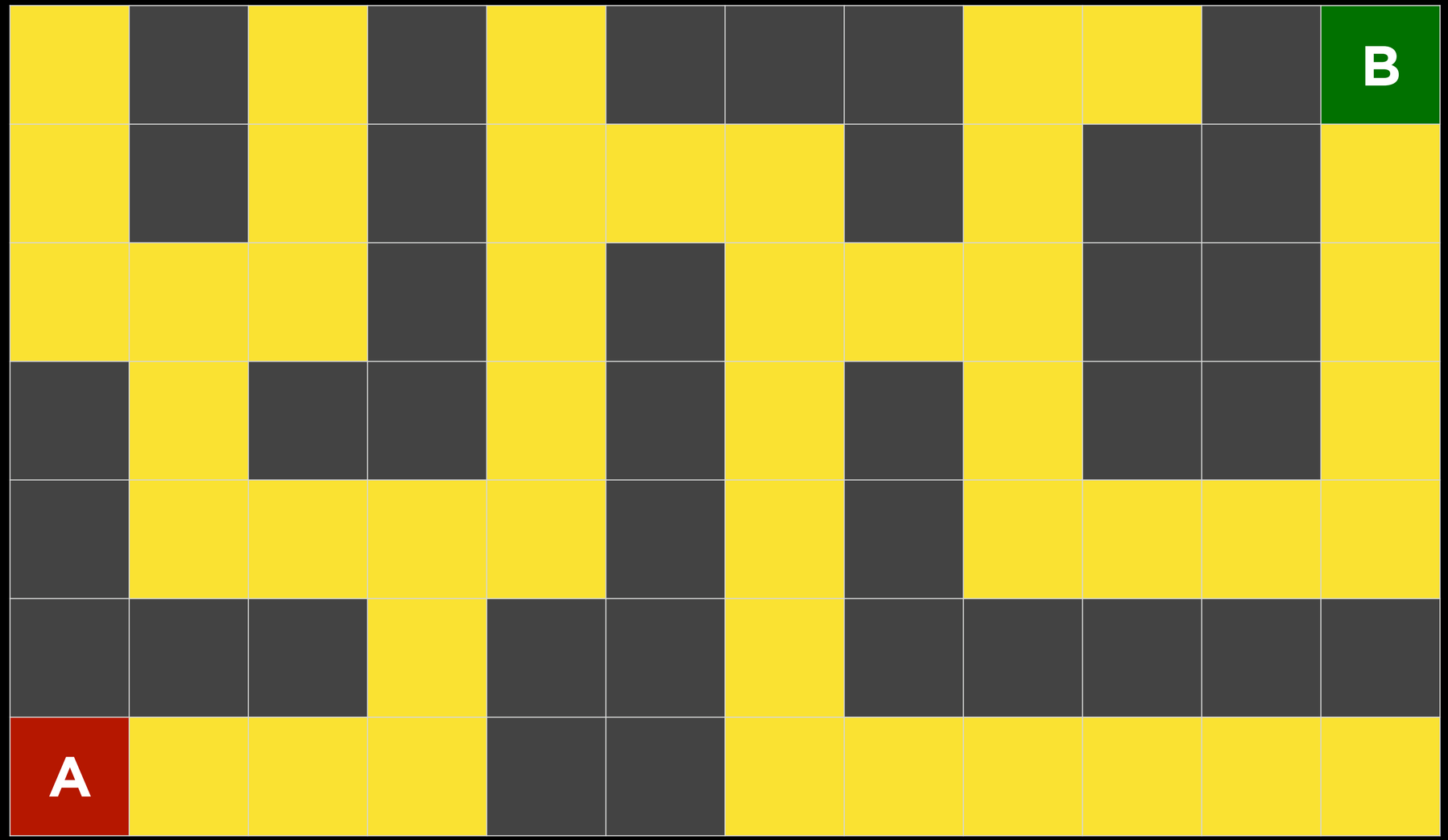


uninformed search

search strategy that uses no problem-specific knowledge

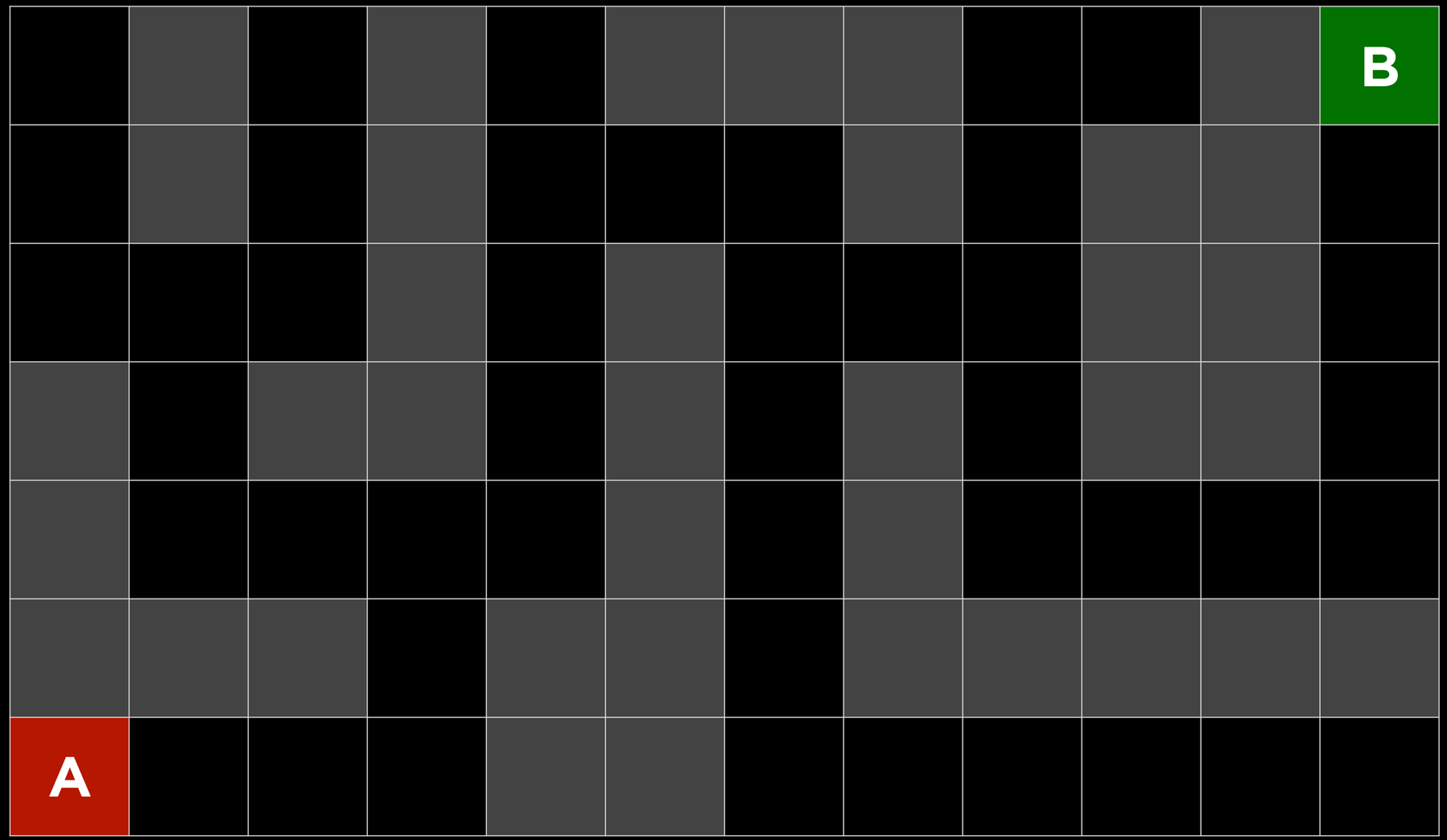
informed search

search strategy that uses problem-specific knowledge to find solutions more efficiently

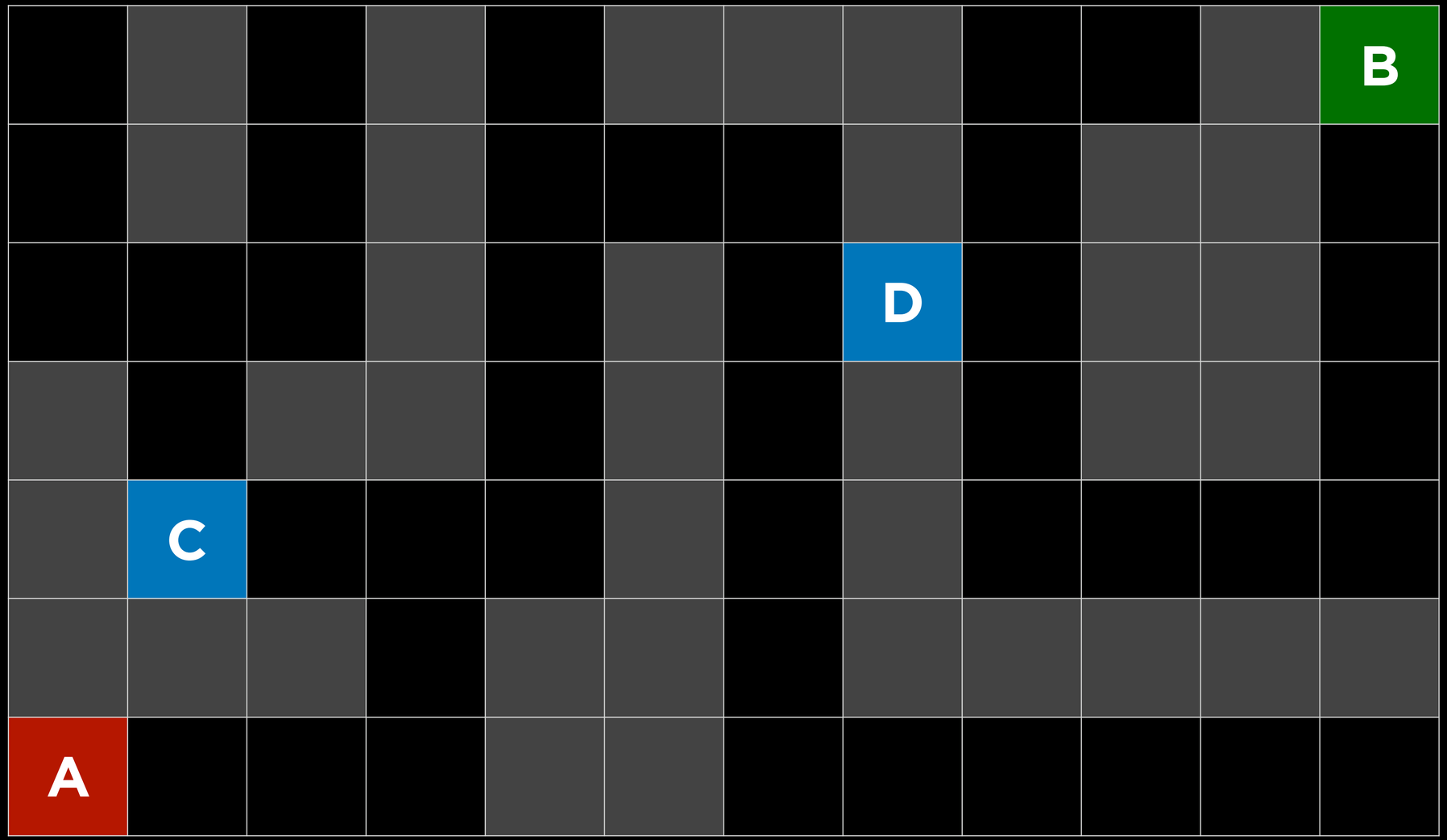
greedy best-first search

search algorithm that expands the node that is closest to the goal, as estimated by a heuristic function $h(n)$

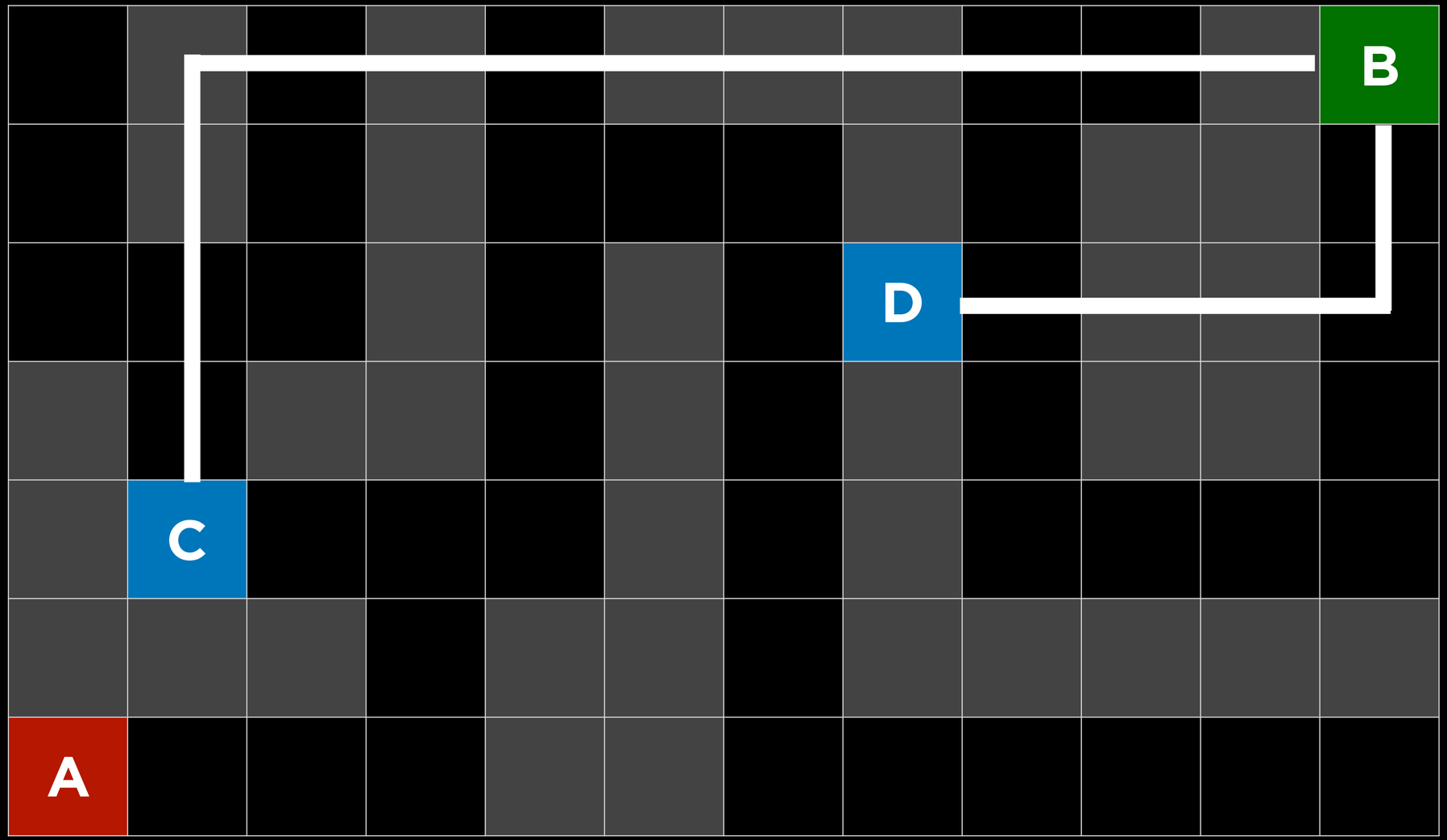
Heuristic function?



Heuristic function?



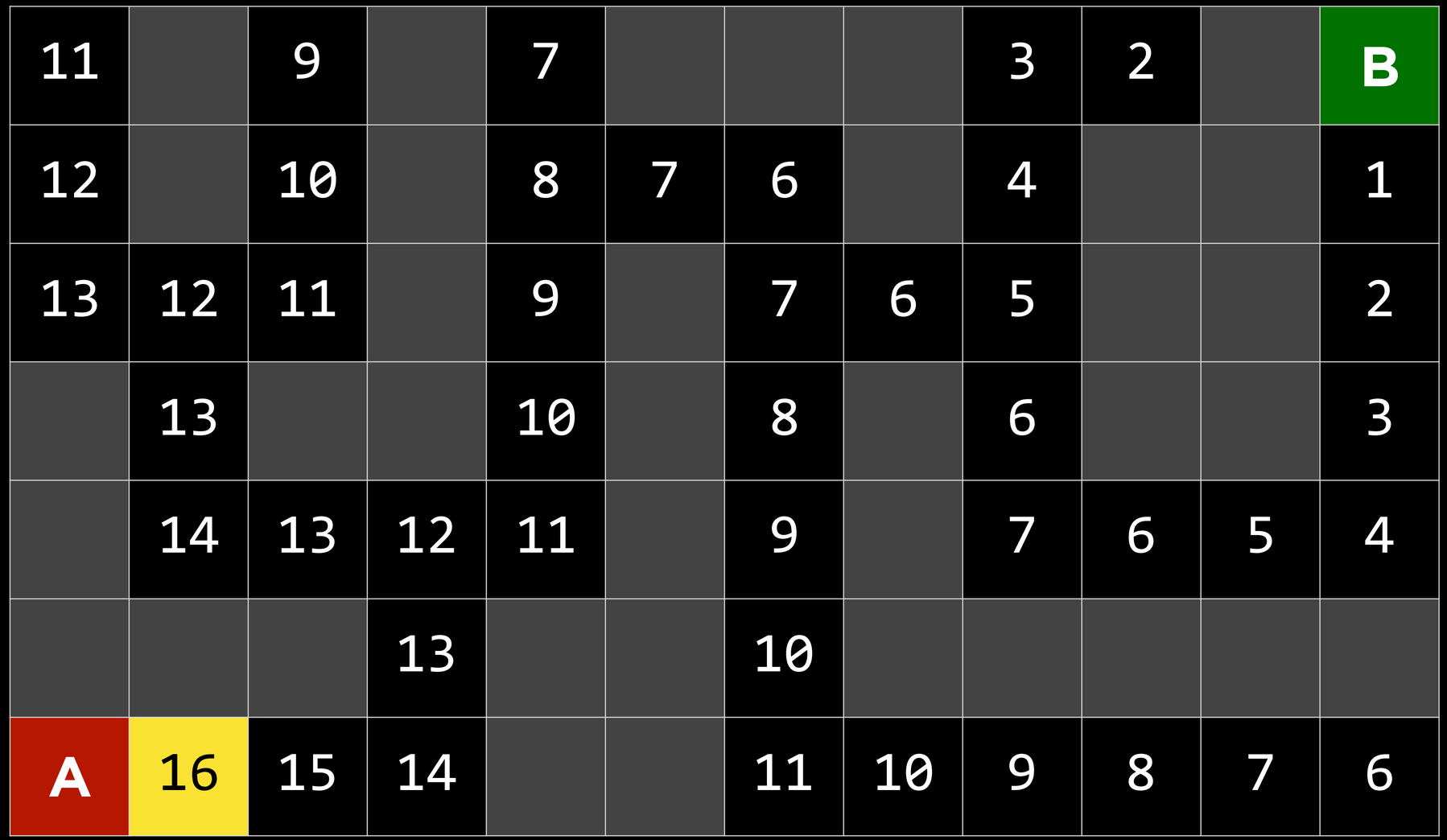
Heuristic function? Manhattan distance.



Greedy Best-First Search

11		9		7				3	2		B
12		10		8	7	6		4			1
13	12	11		9		7	6	5			2
	13			10		8		6			3
	14	13	12	11		9		7	6	5	4
			13			10					
A	16	15	14			11	10	9	8	7	6

Greedy Best-First Search



Greedy Best-First Search

11		9		7				3	2		**B**
12		10		8	7	6		4			1
13	12	11		9		7	6	5			2
	13			10		8		6			3
	14	13	12	11		9		7	6	5	4
			13			10					
A	16	15	14			11	10	9	8	7	6

Greedy Best-First Search

11		9		7				3	2		**B**
12		10		8	7	6		4			1
13	12	11		9		7	6	5			2
	13			10		8		6			3
	14	13	12	11		9		7	6	5	4
			13			10					
A	16	15	14			11	10	9	8	7	6

Greedy Best-First Search

11		9		7				3	2		B
12		10		8	7	6		4			1
13	12	11		9		7	6	5			2
	13			10		8		6			3
	14	13	12	11		9		7	6	5	4
			13			10					
A	16	15	14			11	10	9	8	7	6

Greedy Best-First Search

11		9		7				3	2		B
12		10		8	7	6		4			1
13	12	11		9		7	6	5			2
	13			10		8		6			3
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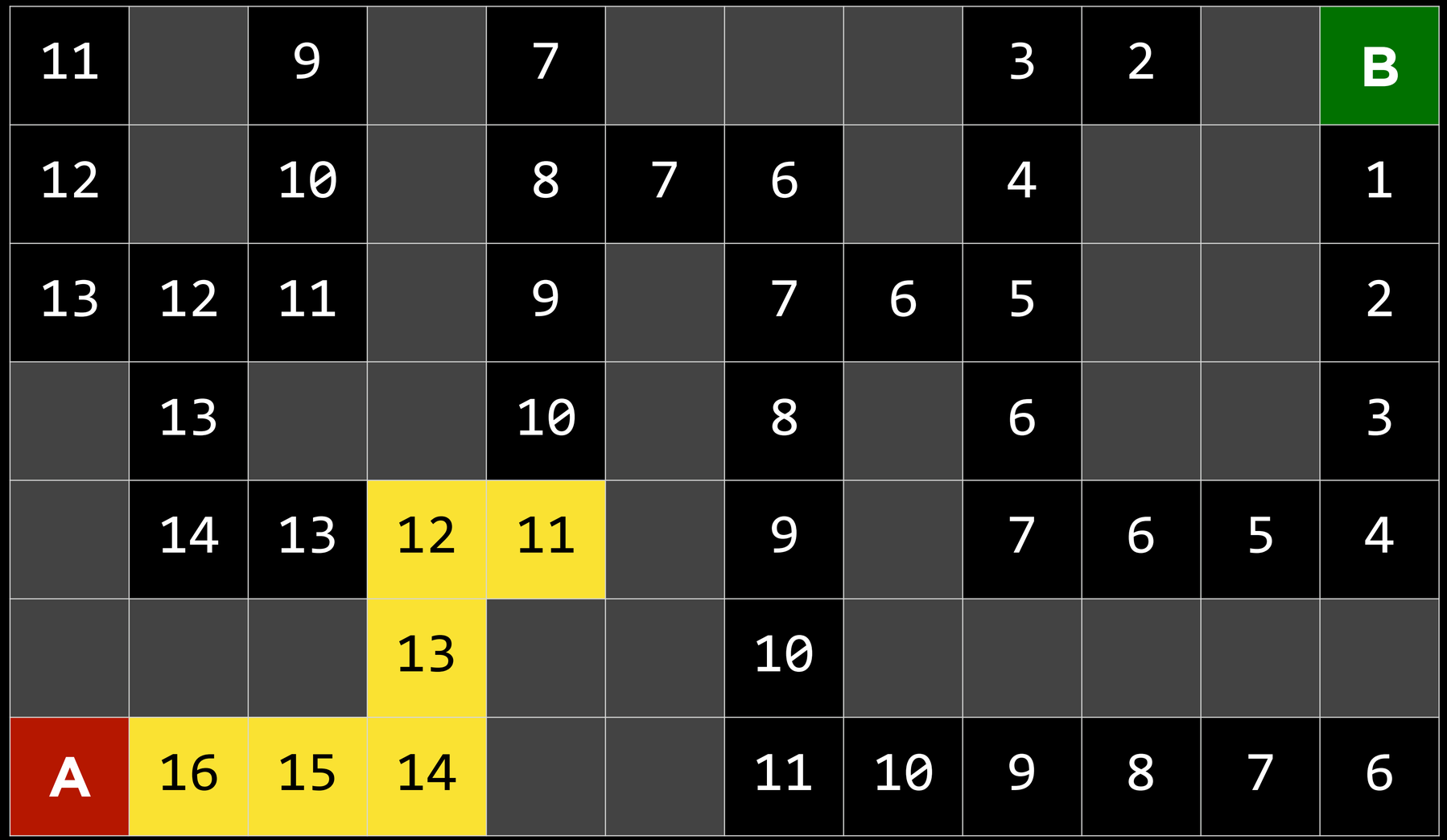
			13			10					
A	16	15	14			11	10	9	8	7	6

Greedy Best-First Search



Greedy Best-First Search

11		9		7				3	2		B
12		10		8	7	6		4			1
13	12	11		9		7	6	5			2
	13			10		8		6			3
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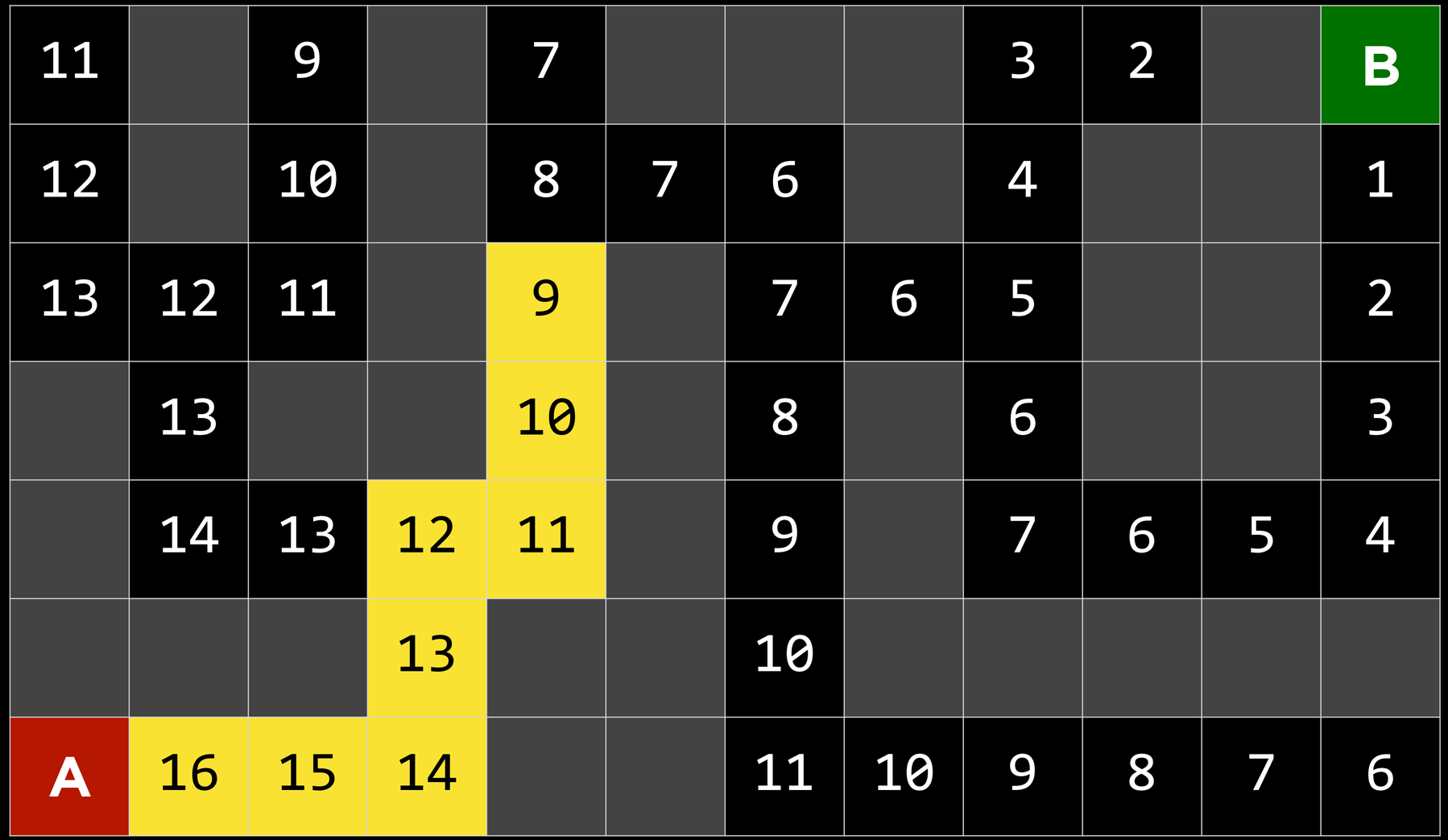
			13			10					
A	16	15	14			11	10	9	8	7	6

Greedy Best-First Search



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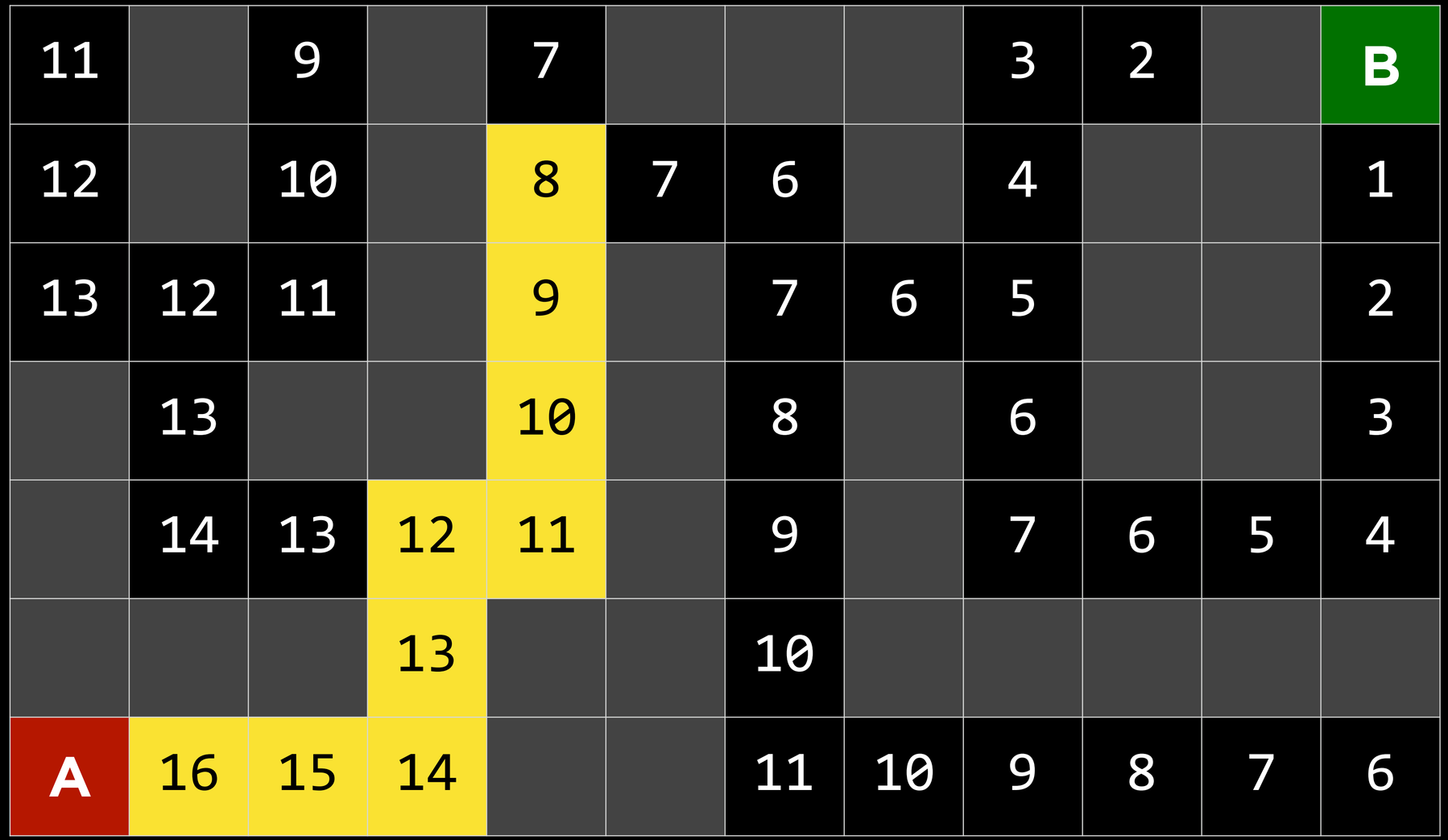

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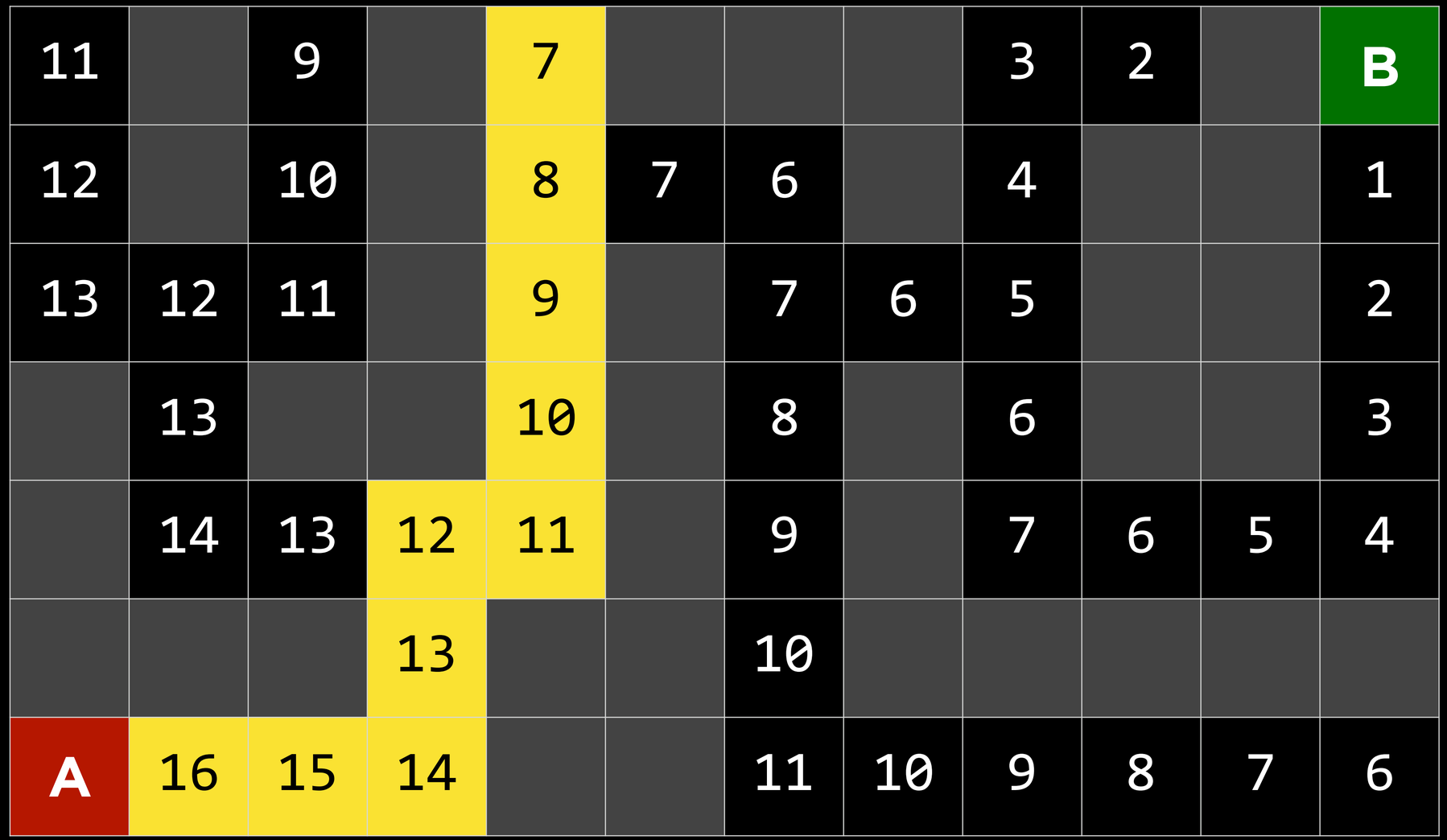

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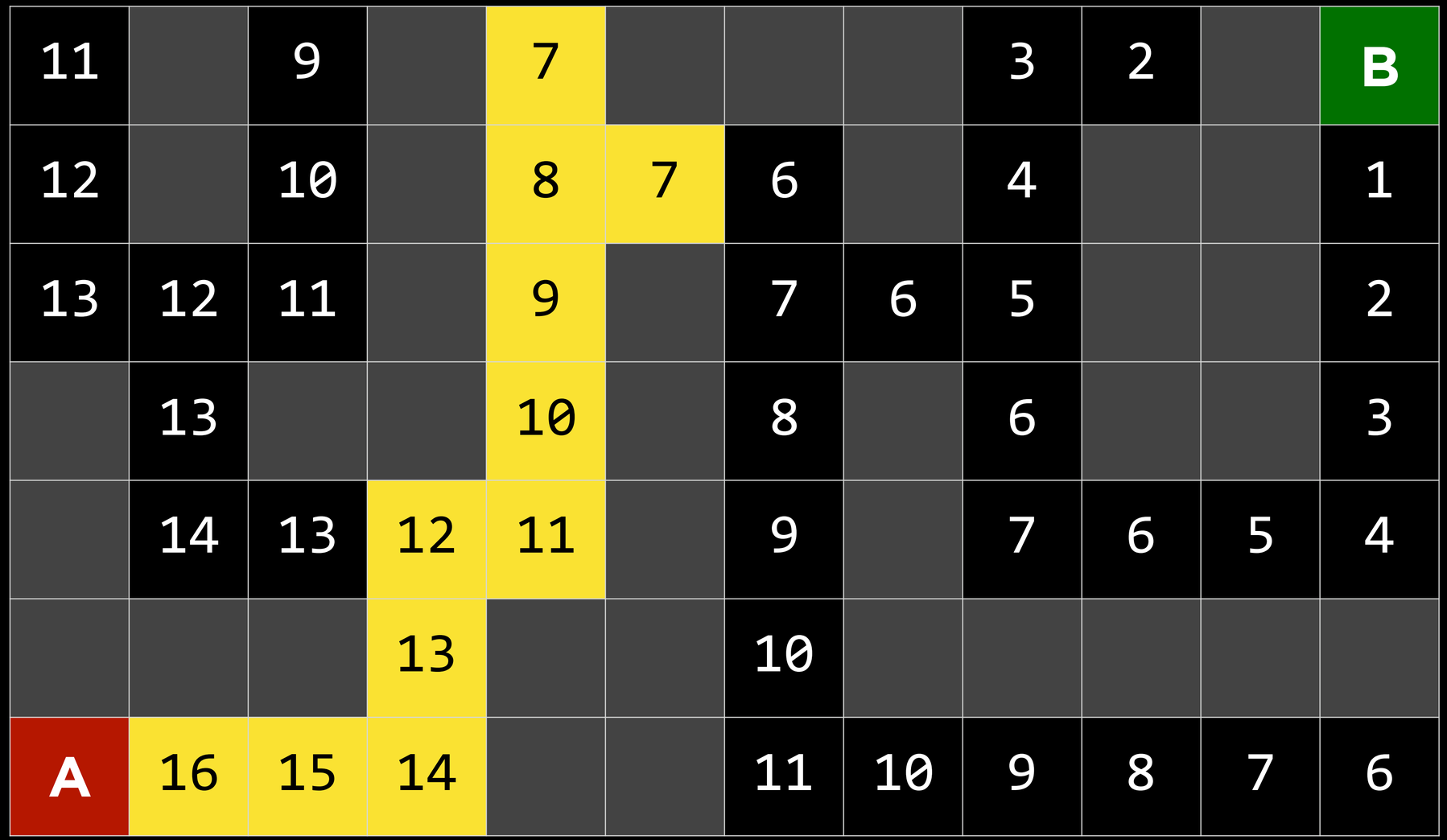

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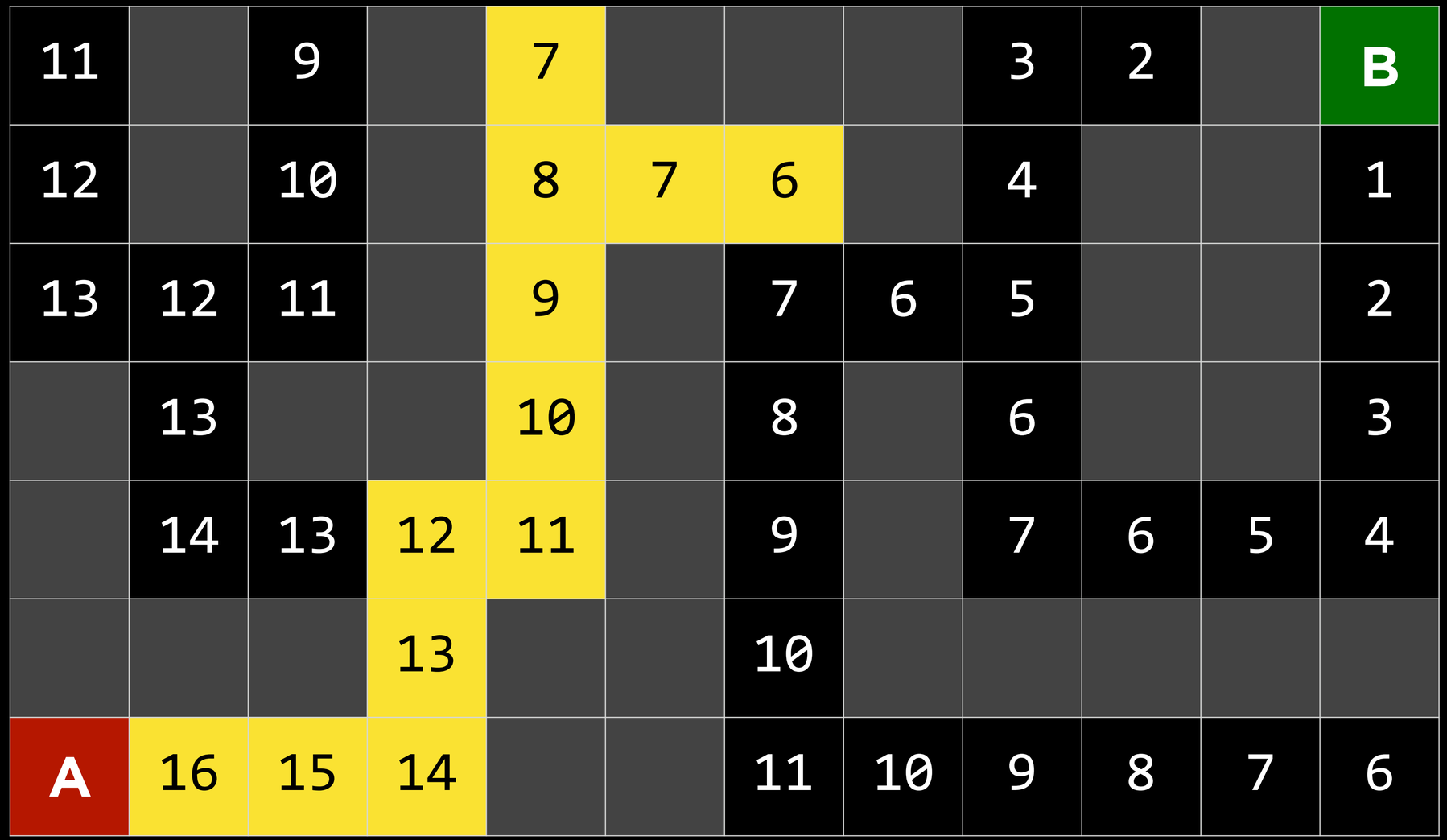

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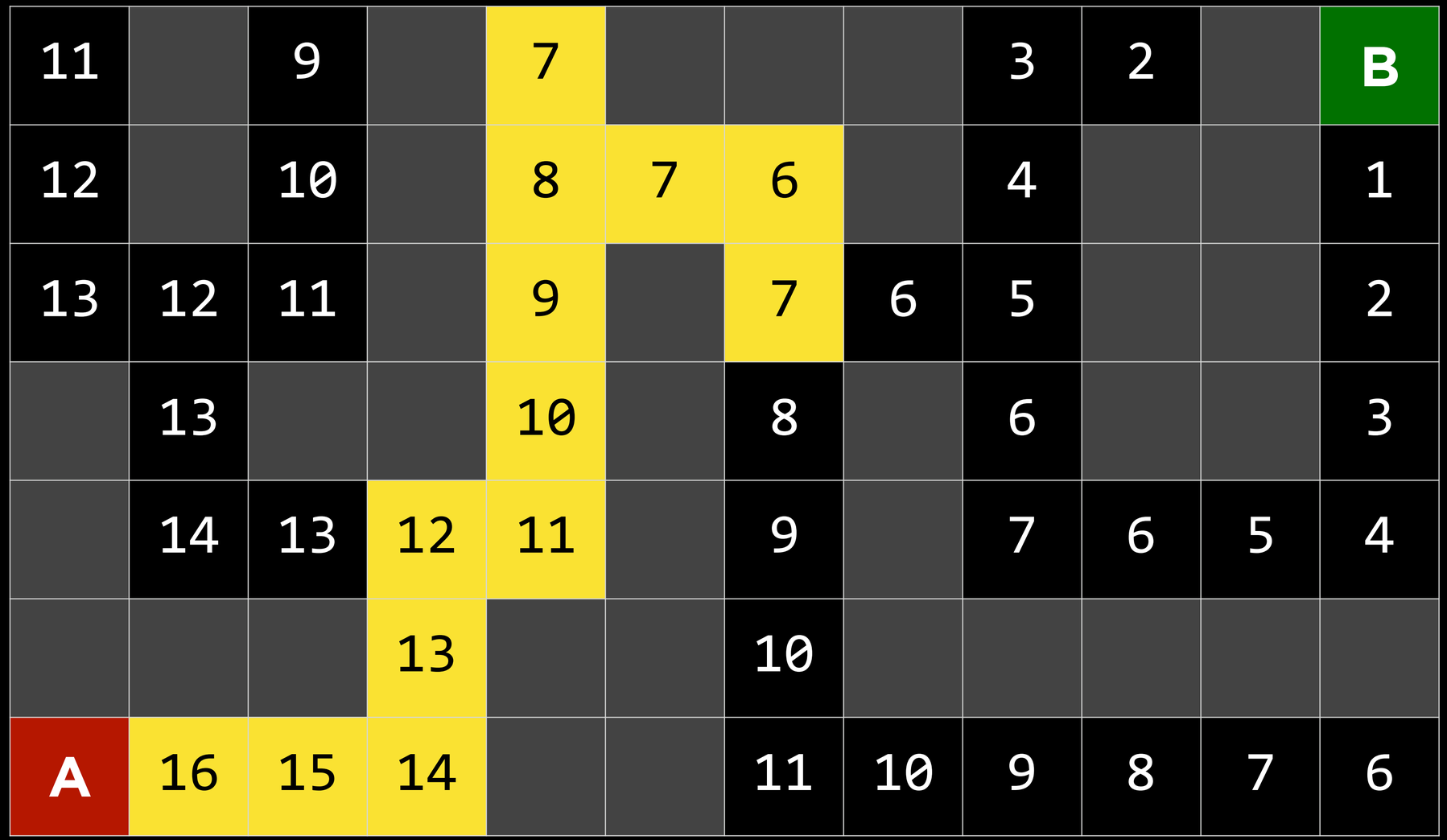
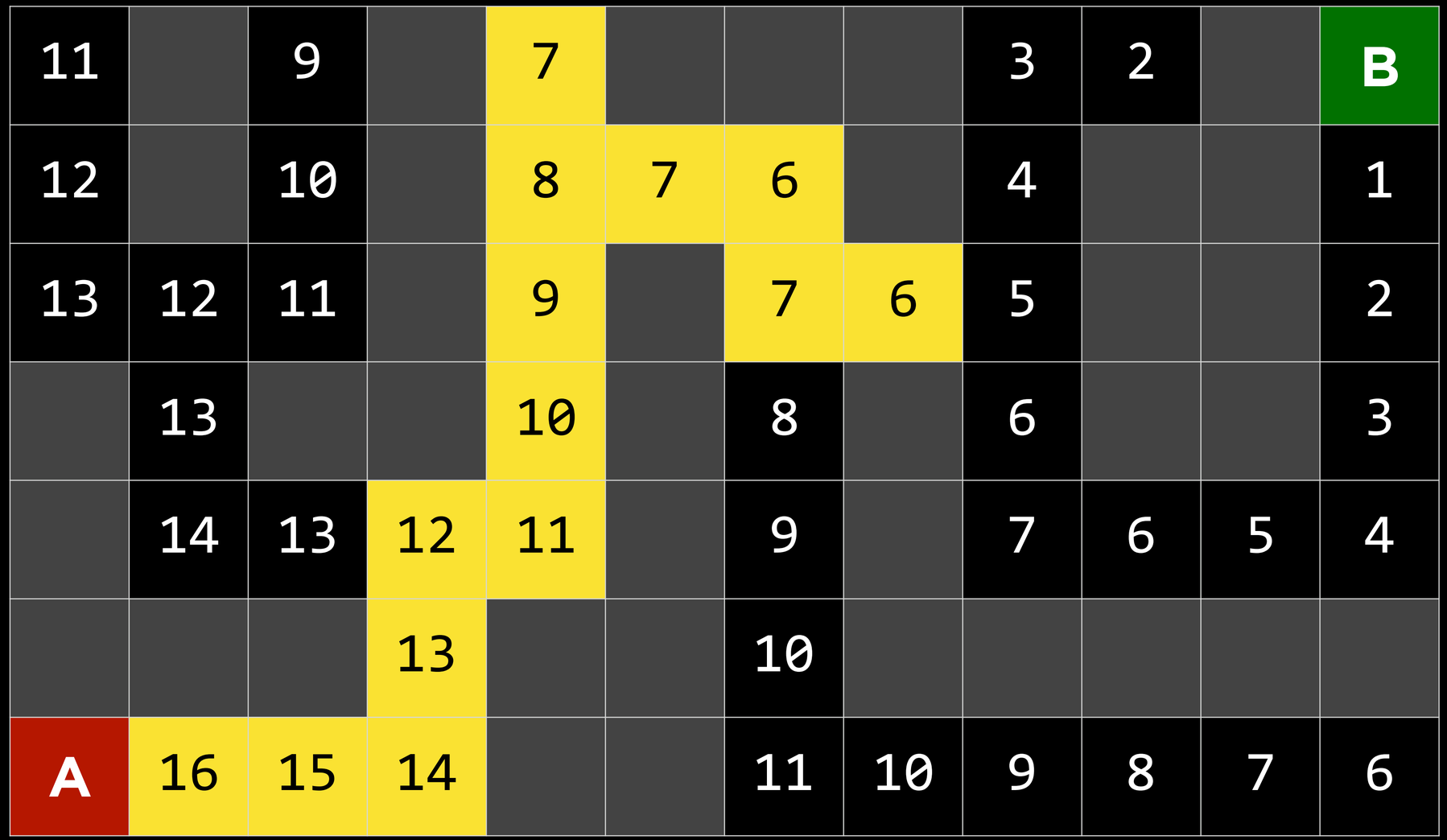


Greedy Best-First Search



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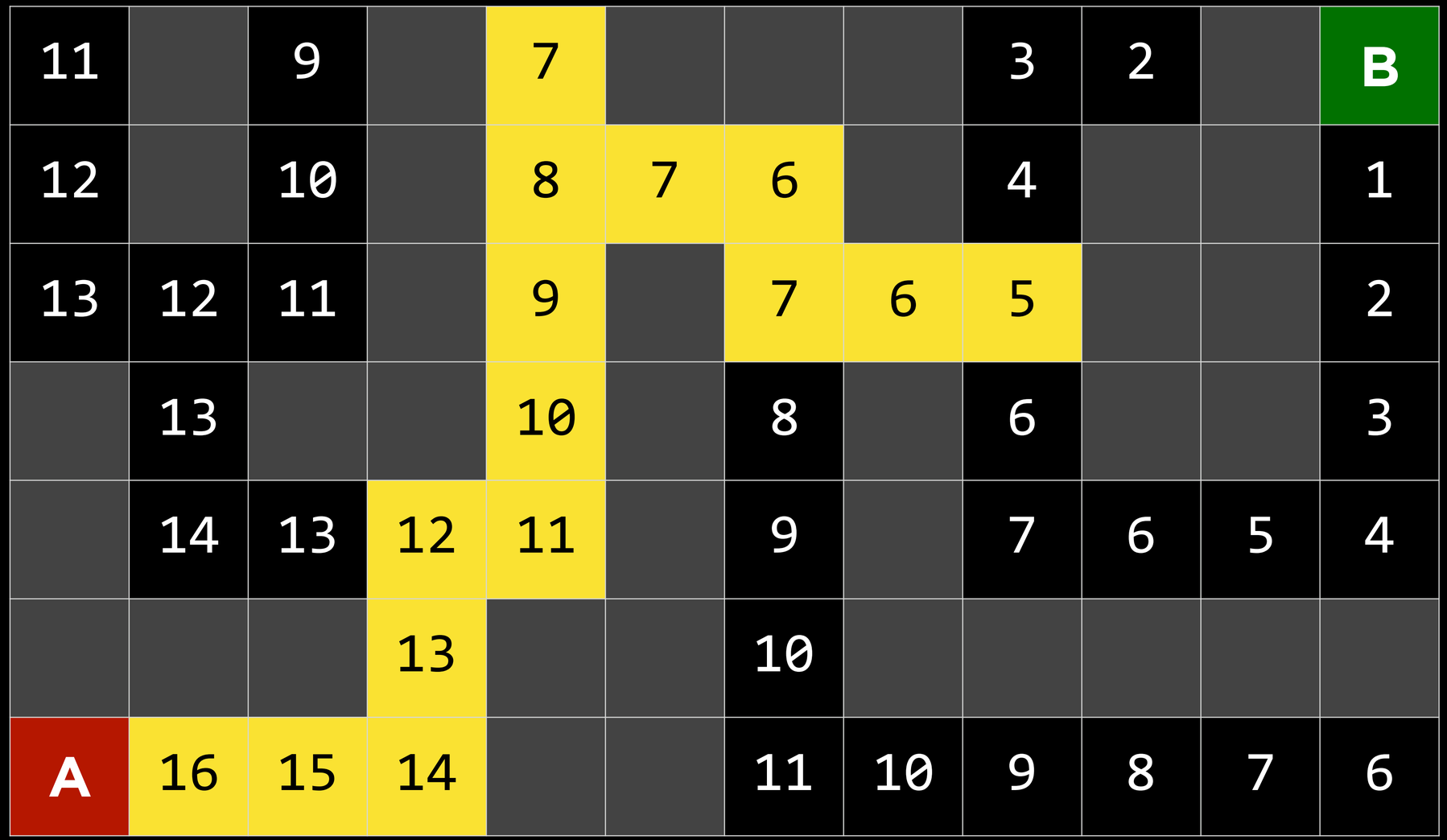

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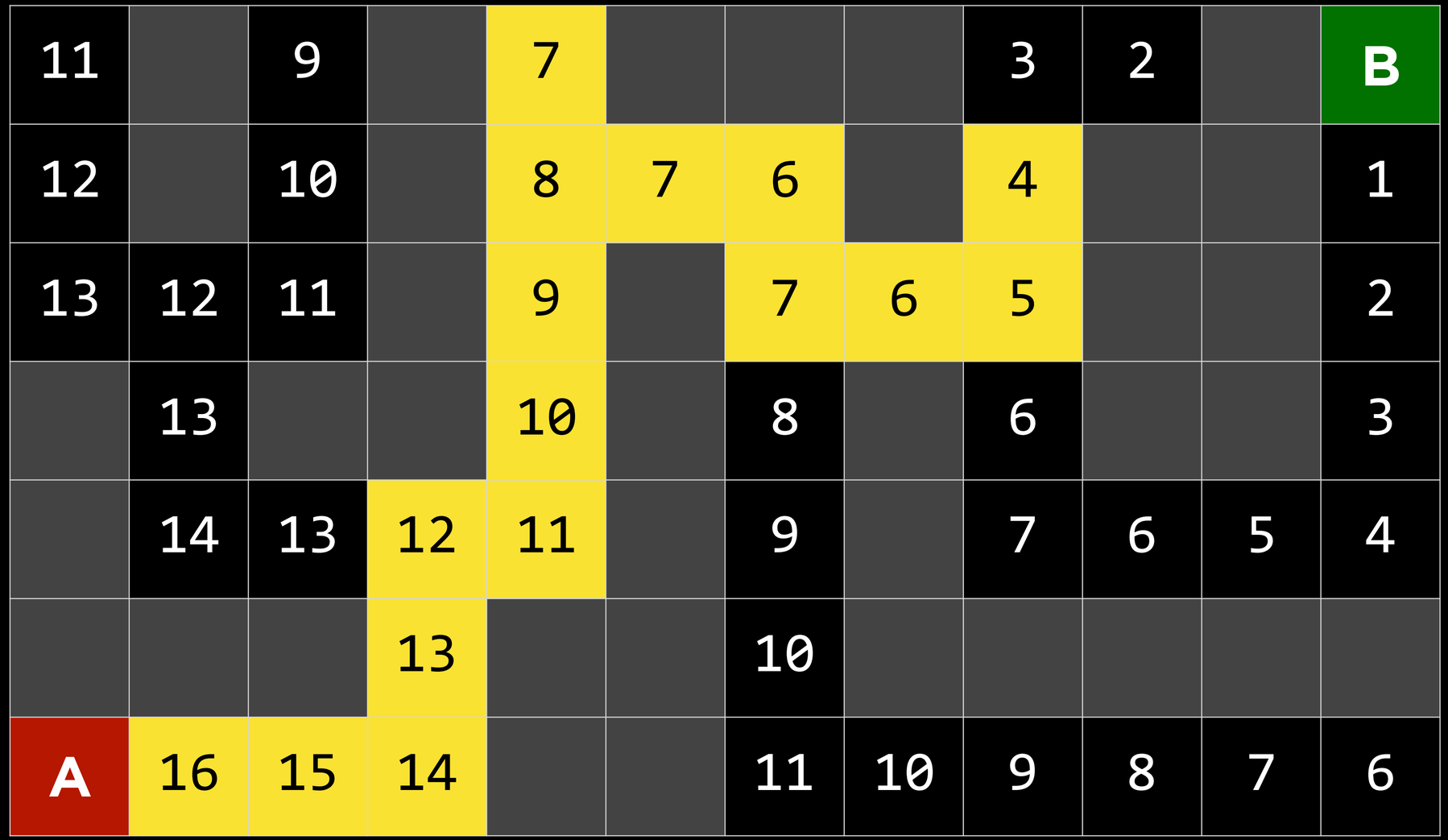

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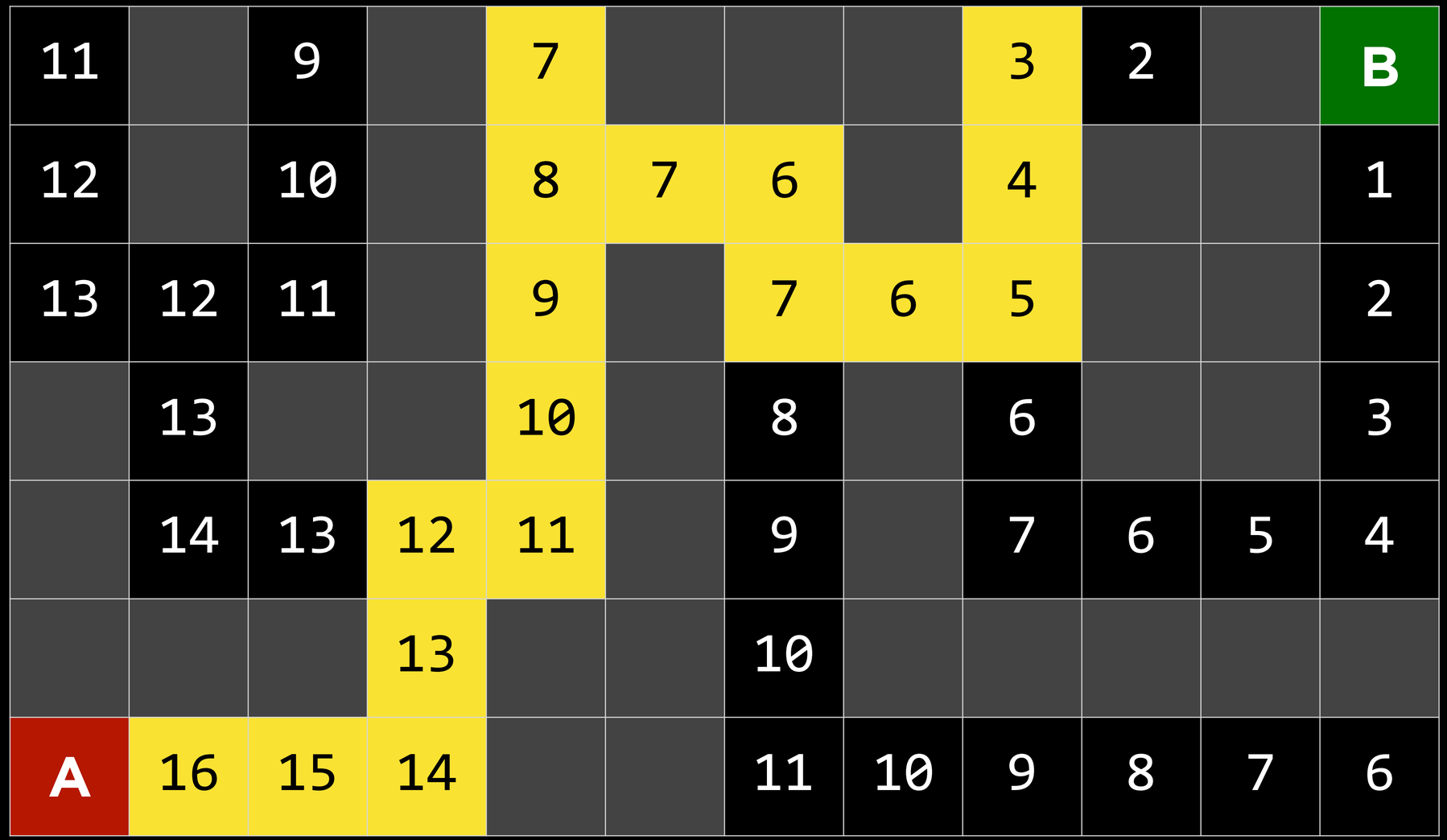

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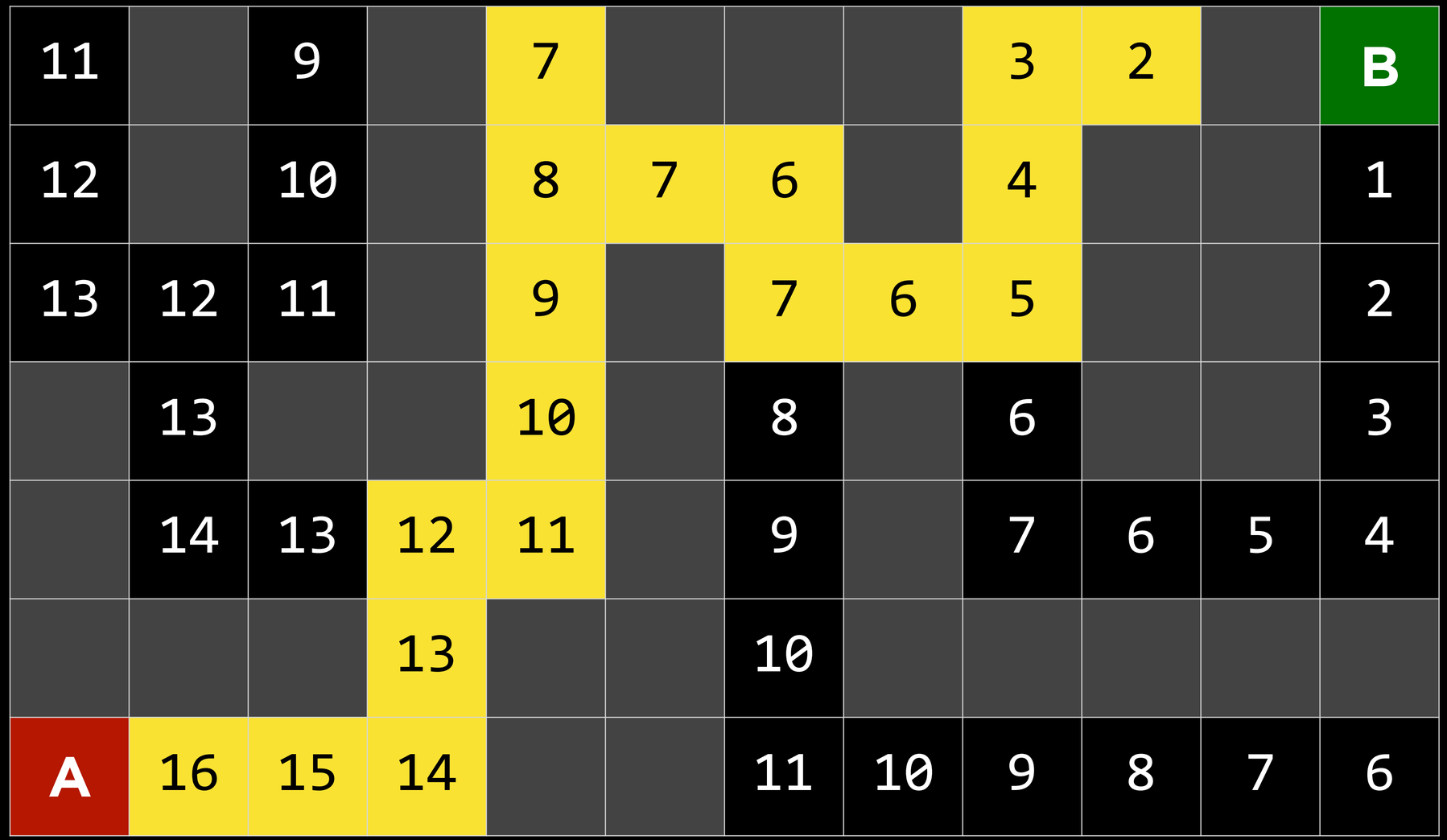

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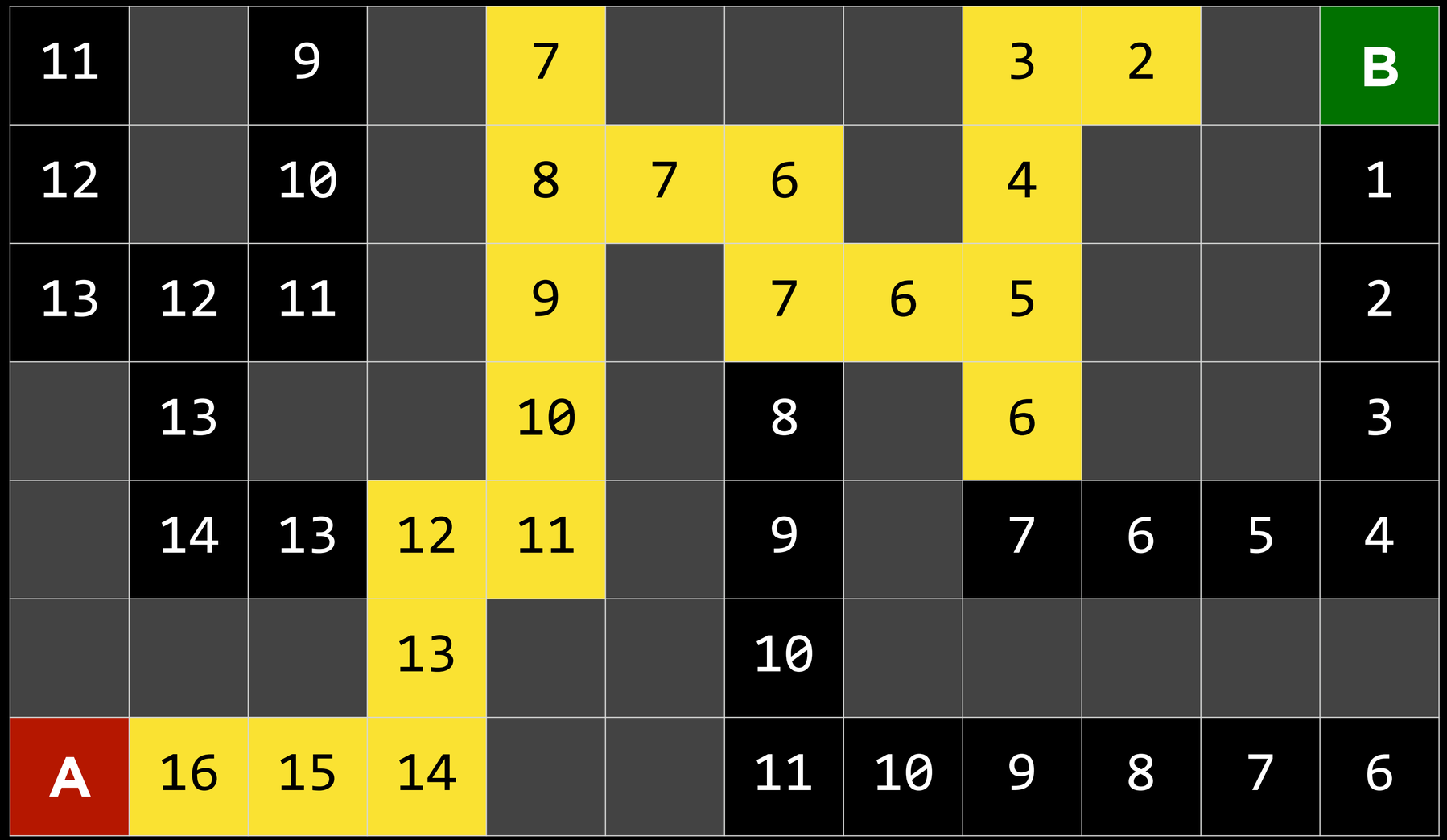

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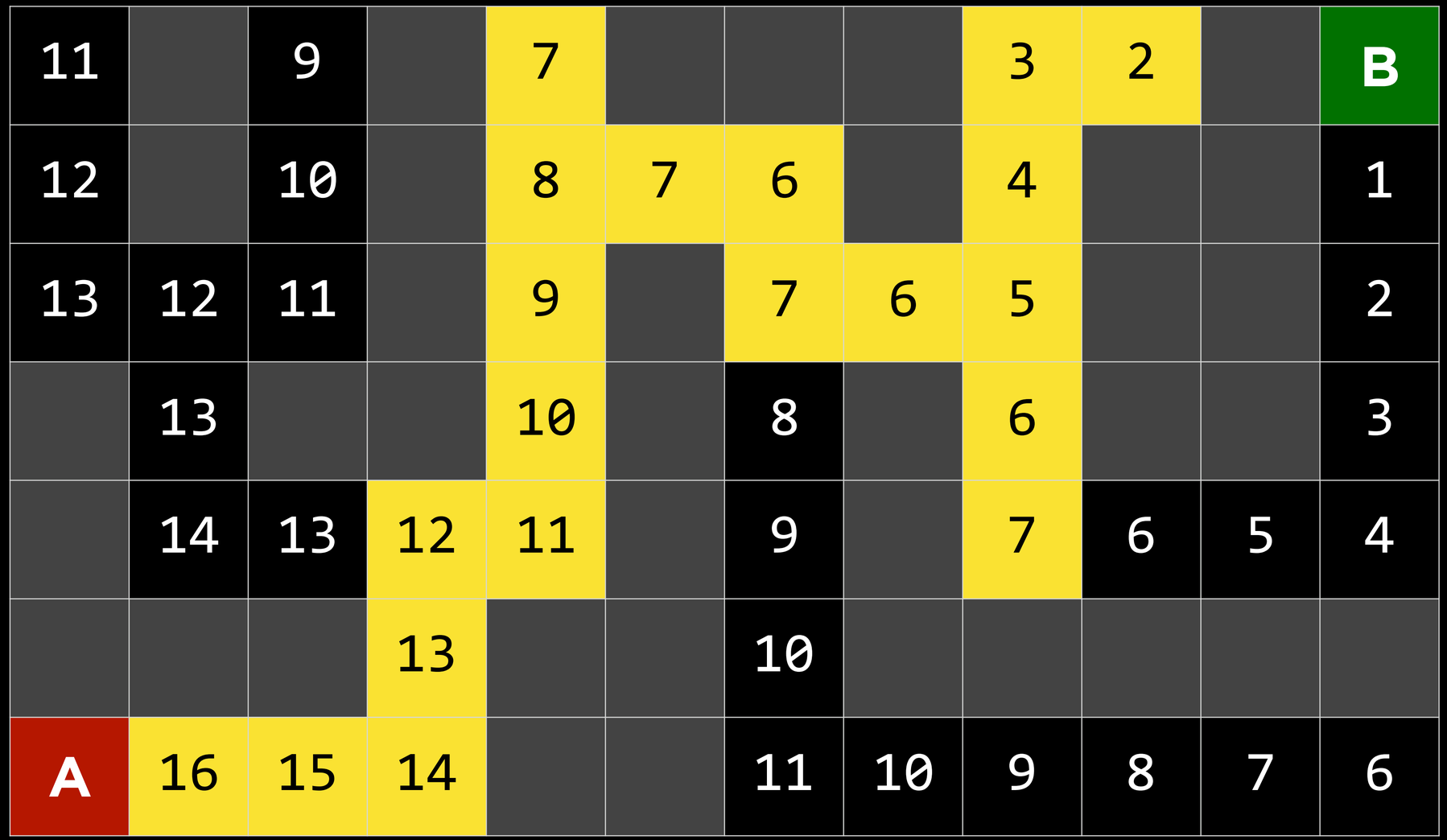

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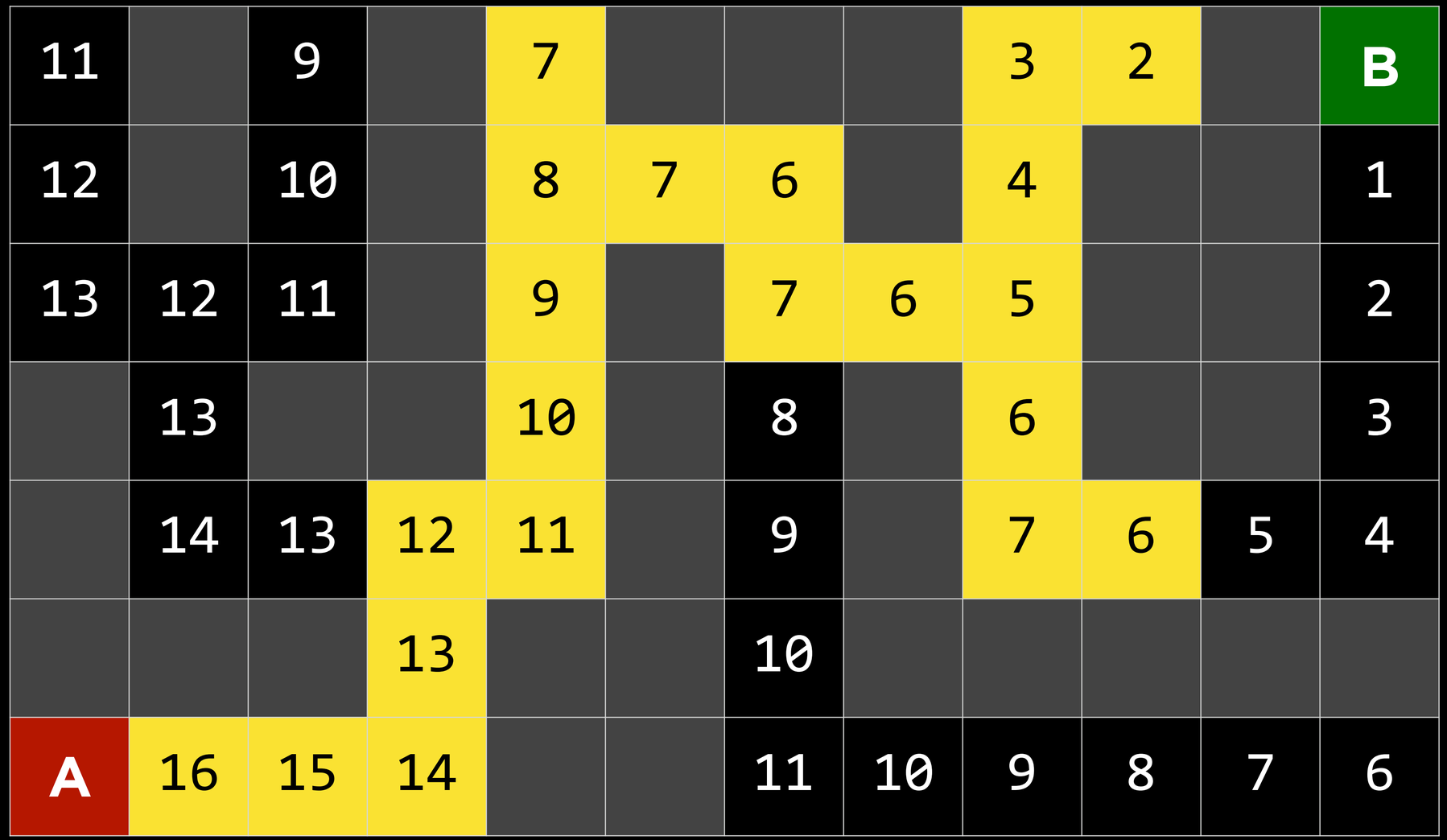

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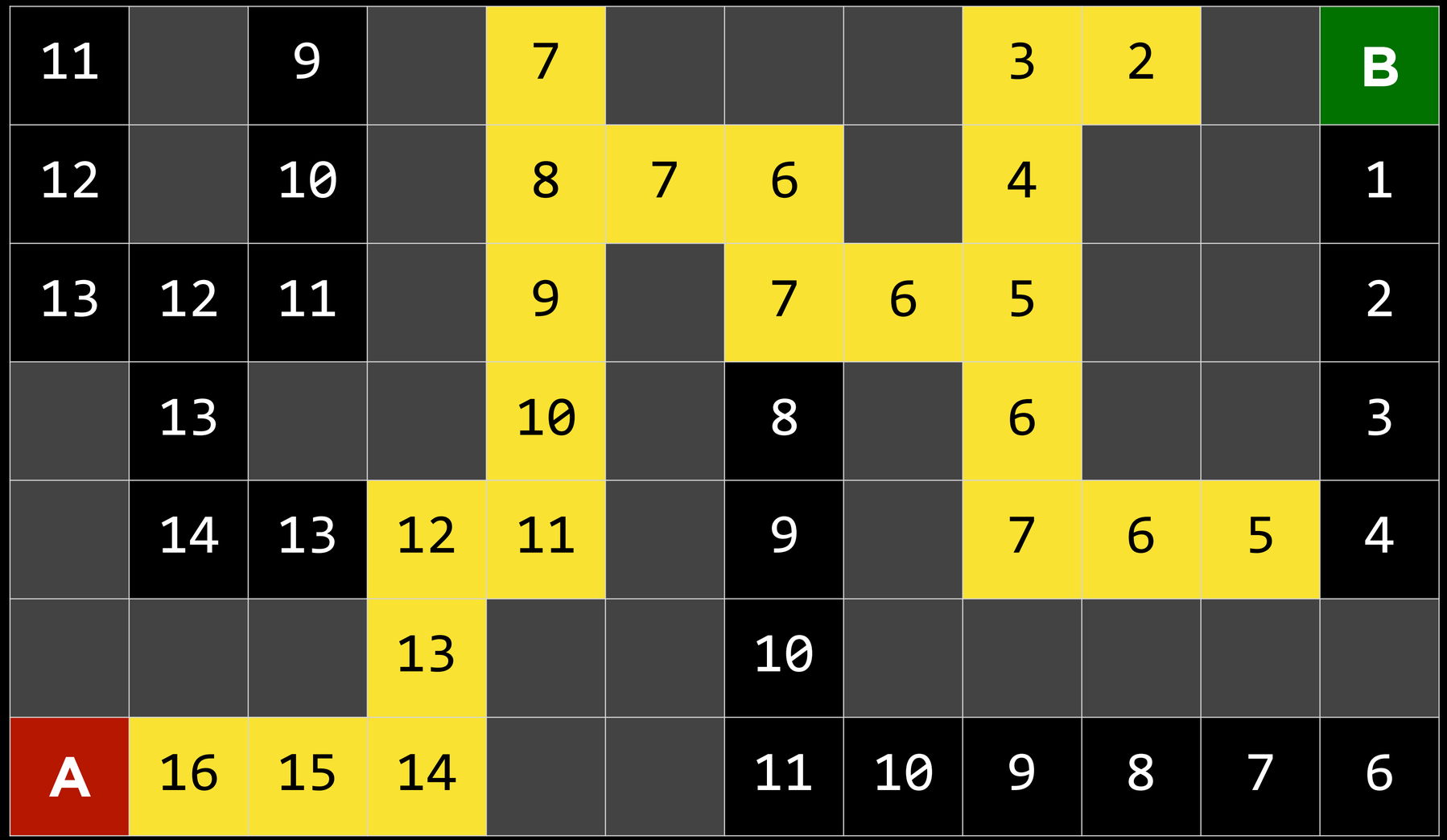

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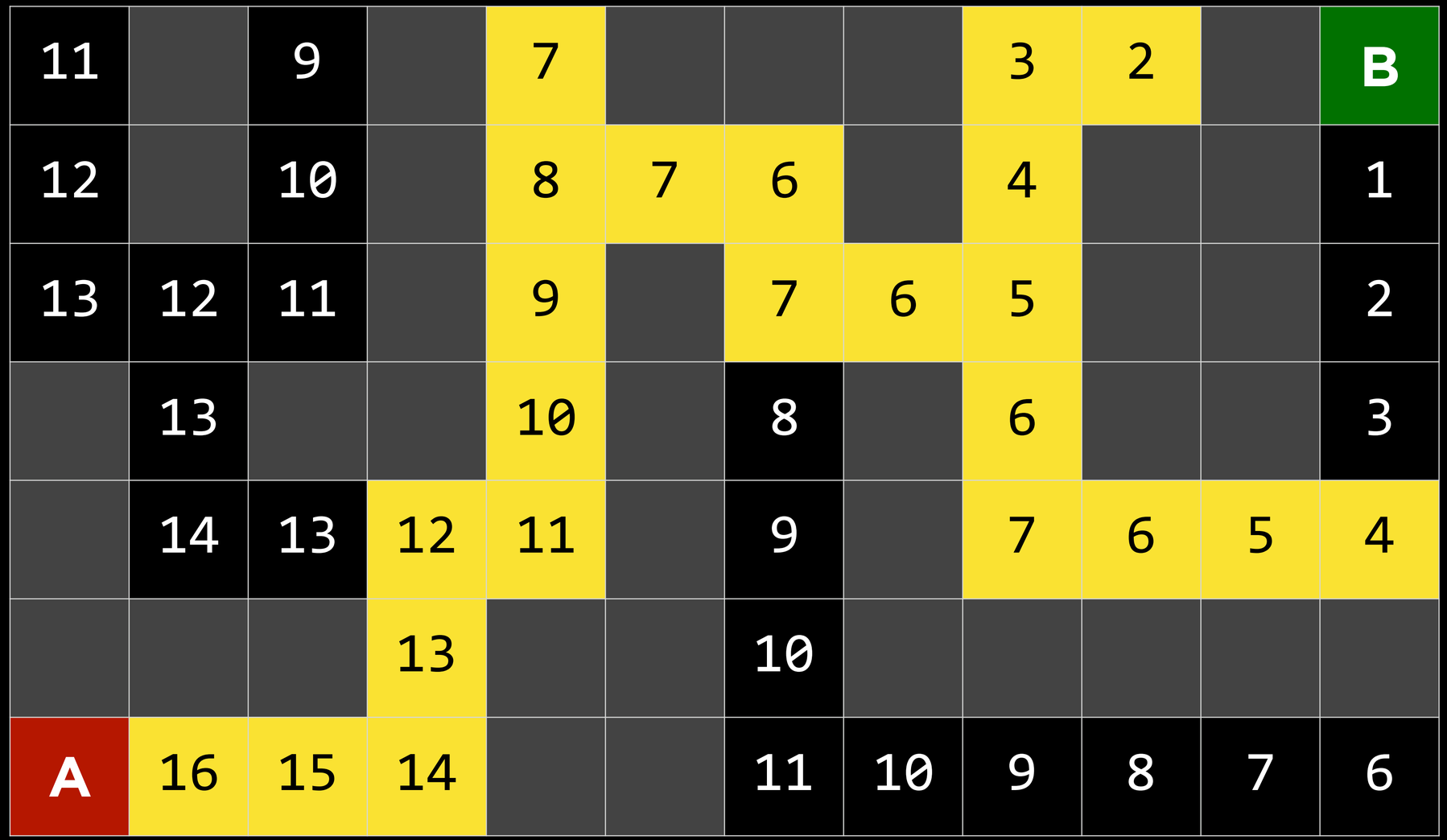

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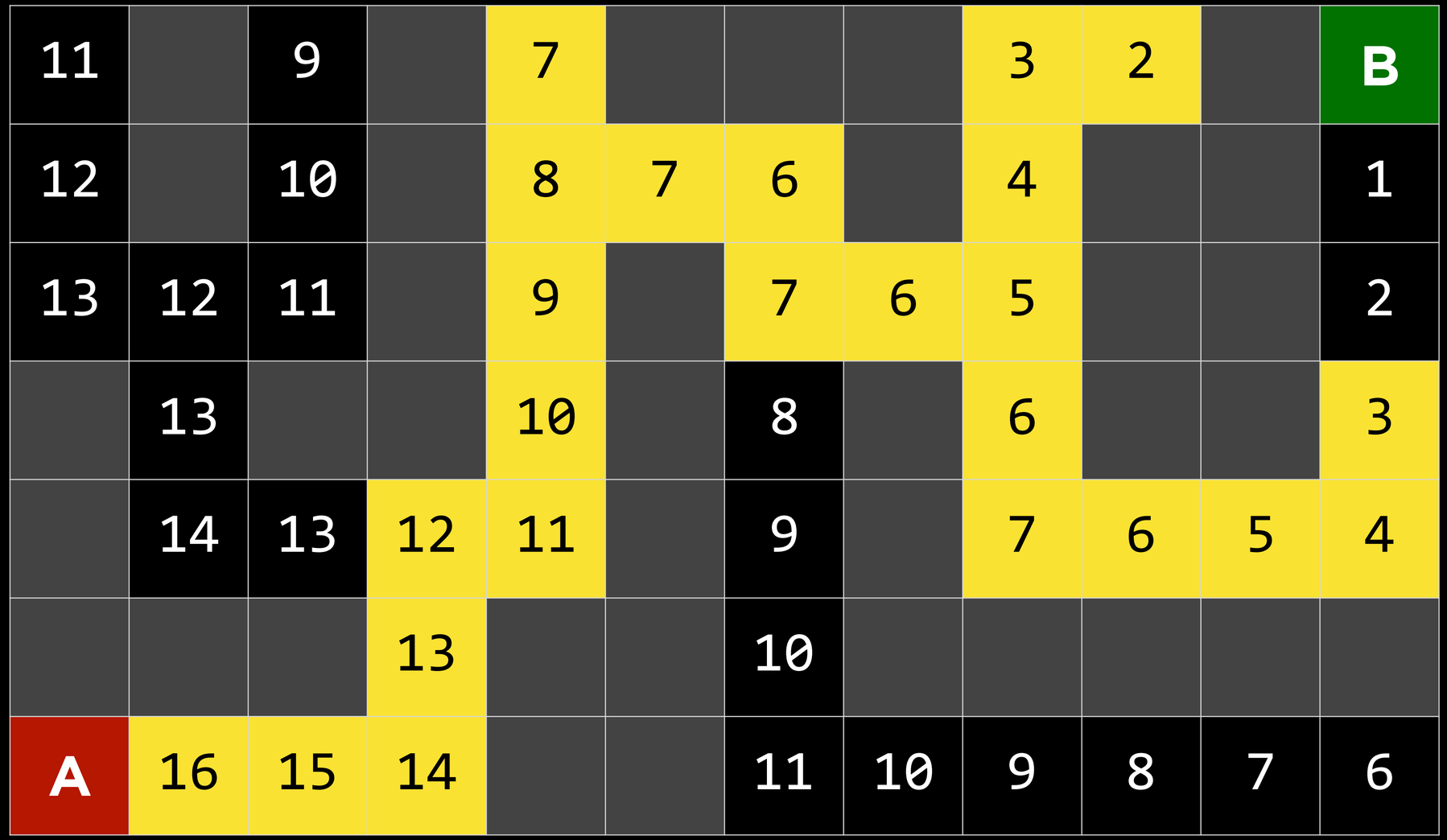

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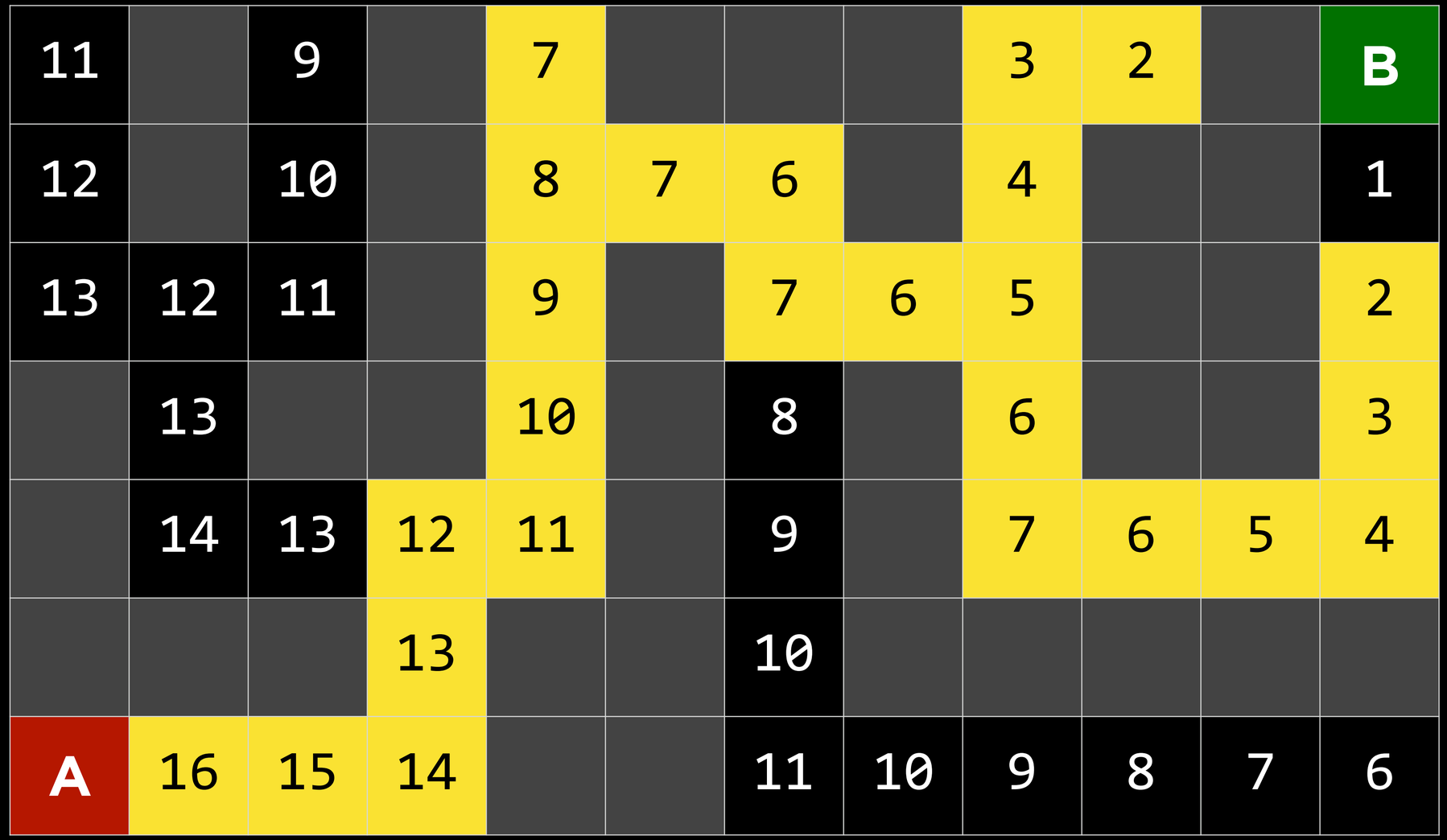

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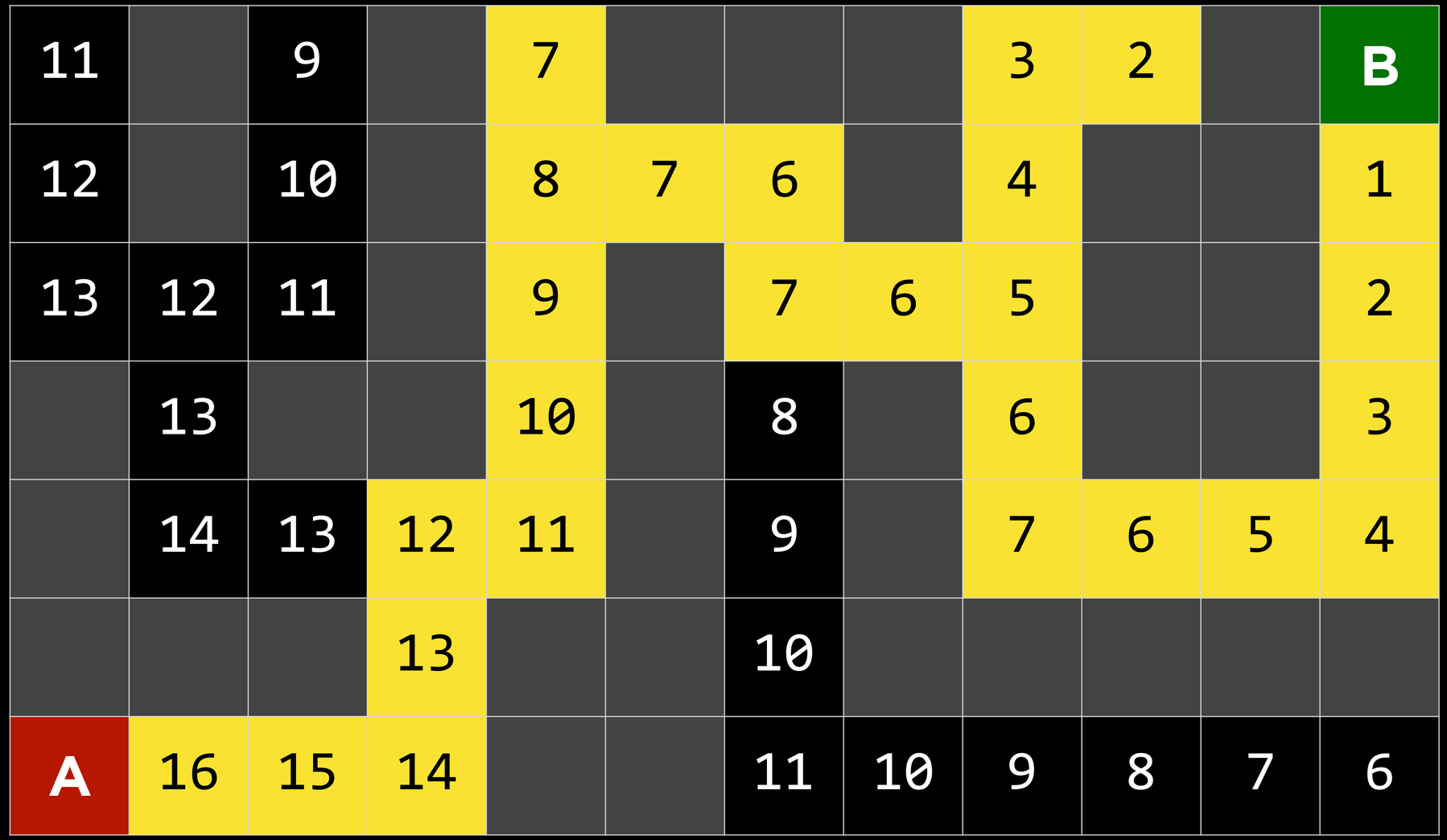


Greedy Best-First Search

	10	9	8	7	6	5	4	3	2	1	**B**
	11										1
	12		10	9	8	7	6	5	4		2
	13		11						5		3
	14	13	12		10	9	8	7	6		4
			13		11						5
A	16	15	14		12	11	10	9	8	7	6

Greedy Best-First Search

	10	9	8	7	6	5	4	3	2	1	**B**
	11										1
	12		10	9	8	7	6	5	4		2
	13		11						5		3
	14	13	12		10	9	8	7	6		4
			13		11						5
A	16	15	14		12	11	10	9	8	7	6

Greedy Best-First Search

	10	9	8	7	6	5	4	3	2	1	**B**
	11										1
	12		10	9	8	7	6	5	4		2
	13		11						5		3
	14	13	12		10	9	8	7	6		4
			13		11						5
A	16	15	14		12	11	10	9	8	7	6

A* search

search algorithm that expands node with lowest value of $g(n) + h(n)$

$g(n)$ = cost to reach node
$h(n)$ = estimated cost to goal

A* Search

	10	9	8	7	6	5	4	3	2	1	**B**
	11										1
	12		10	9	8	7	6	5	4		2
	13		11						5		3
	14	13	12		10	9	8	7	6		4
			13		11						5
A	16	15	14		12	11	10	9	8	7	6

A* Search

	10	9	8	7	6	5	4	3	2	1	**B**
	11										1
	12		10	9	8	7	6	5	4		2
	13		11						5		3
	14	13	12		10	9	8	7	6		4
			13		11						5
A	1+16	15	14		12	11	10	9	8	7	6

A* Search

	10	9	8	7	6	5	4	3	2	1	**B**
	11										1
	12		10	9	8	7	6	5	4		2
	13		11						5		3
	14	13	12		10	9	8	7	6		4
			13		11						5
A	1+16	2+15	14		12	11	10	9	8	7	6

A* Search

	10	9	8	7	6	5	4	3	2	1	**B**
	11										1
	12		10	9	8	7	6	5	4		2
	13		11						5		3
	14	13	12		10	9	8	7	6		4
			13		11						5
A	1+16	2+15	3+14		12	11	10	9	8	7	6

A* Search

	10	9	8	7	6	5	4	3	2	1	**B**
	11										1
	12		10	9	8	7	6	5	4		2
	13		11						5		3
	14	13	12		10	9	8	7	6		4
			4+13		11						5
A	1+16	2+15	3+14		12	11	10	9	8	7	6

A* Search

	10	9	8	7	6	5	4	3	2	1	**B**
	11										1
	12		10	9	8	7	6	5	4		2
	13		11						5		3
	14	13	5+12		10	9	8	7	6		4
			4+13		11						5
A	1+16	2+15	3+14		12	11	10	9	8	7	6

A* Search

	10	9	8	7	6	5	4	3	2	1	**B**
	11										1
	12		10	9	8	7	6	5	4		2
	13		6+11						5		3
	14	13	5+12		10	9	8	7	6		4
			4+13		11						5
A	1+16	2+15	3+14		12	11	10	9	8	7	6

A* Search

	10	9	8	7	6	5	4	3	2	1	**B**
	11										1
	12		7+10	9	8	7	6	5	4		2
	13		6+11						5		3
	14	13	5+12		10	9	8	7	6		4
			4+13		11						5
A	1+16	2+15	3+14		12	11	10	9	8	7	6

A* Search

	10	9	8	7	6	5	4	3	2	1	**B**
	11										1
	12		7+10	8+9	8	7	6	5	4		2
	13		6+11						5		3
	14	13	5+12		10	9	8	7	6		4
			4+13		11						5
A	1+16	2+15	3+14		12	11	10	9	8	7	6

A* Search

	10	9	8	7	6	5	4	3	2	1	**B**
	11										1
	12		7+10	8+9	9+8	7	6	5	4		2
	13		6+11						5		3
	14	13	5+12		10	9	8	7	6		4
			4+13		11						5
A	1+16	2+15	3+14		12	11	10	9	8	7	6

A* Search

	10	9	8	7	6	5	4	3	2	1	**B**
	11										1
	12		7+10	8+9	9+8	10+7	6	5	4		2
	13		6+11						5		3
	14	13	5+12		10	9	8	7	6		4
			4+13		11						5
A	1+16	2+15	3+14		12	11	10	9	8	7	6

A* Search

	10	9	8	7	6	5	4	3	2	1	**B**
	11										1
	12		7+10	8+9	9+8	10+7	11+6	5	4		2
	13		6+11						5		3
	14	13	5+12		10	9	8	7	6		4
			4+13		11						5
A	1+16	2+15	3+14		12	11	10	9	8	7	6

A* Search

	10	9	8	7	6	5	4	3	2	1	**B**
	11										1
	12		7+10	8+9	9+8	10+7	11+6	12+5	4		2
	13		6+11						5		3
	14	13	5+12		10	9	8	7	6		4
			4+13		11						5
A	1+16	2+15	3+14		12	11	10	9	8	7	6

A* Search

	10	9	8	7	6	5	4	3	2	1	**B**
	11										1
	12		7+10	8+9	9+8	10+7	11+6	12+5	13+4		2
	13		6+11						5		3
	14	13	5+12		10	9	8	7	6		4
			4+13		11						5
A	1+16	2+15	3+14		12	11	10	9	8	7	6

A* Search

	10	9	8	7	6	5	4	3	2	1	**B**
	11										1
	12		7+10	8+9	9+8	10+7	11+6	12+5	13+4		2
	13		6+11						14+5		3
	14	13	5+12		10	9	8	7	6		4
			4+13		11						5
A	1+16	2+15	3+14		12	11	10	9	8	7	6

A* Search

	10	9	8	7	6	5	4	3	2	1	**B**
	11										1
	12		7+10	8+9	9+8	10+7	11+6	12+5	13+4		2
	13		6+11						14+5		3
	14	6+13	5+12		10	9	8	7	6		4
			4+13		11						5
A	1+16	2+15	3+14		12	11	10	9	8	7	6

A* Search

	10	9	8	7	6	5	4	3	2	1	**B**
	11										1
	12		7+10	8+9	9+8	10+7	11+6	12+5	13+4		2
	13		6+11						14+5		3
	14	6+13	5+12		10	9	8	7	15+6		4
			4+13		11						5
A	1+16	2+15	3+14		12	11	10	9	8	7	6

A* Search

	10	9	8	7	6	5	4	3	2	1	**B**
	11										1
	12		7+10	8+9	9+8	10+7	11+6	12+5	13+4		2
	13		6+11						14+5		3
	7+14	6+13	5+12		10	9	8	7	15+6		4
			4+13		11						5
A	1+16	2+15	3+14		12	11	10	9	8	7	6

A* Search

	10	9	8	7	6	5	4	3	2	1	**B**
	11										1
	12		7+10	8+9	9+8	10+7	11+6	12+5	13+4		2
	8+13		6+11						14+5		3
	7+14	6+13	5+12		10	9	8	7	15+6		4
			4+13		11						5
A	1+16	2+15	3+14		12	11	10	9	8	7	6

A* Search

	10	9	8	7	6	5	4	3	2	1	**B**
	11										1
	9+12		7+10	8+9	9+8	10+7	11+6	12+5	13+4		2
	8+13		6+11						14+5		3
	7+14	6+13	5+12		10	9	8	7	15+6		4
			4+13		11						5
A	1+16	2+15	3+14		12	11	10	9	8	7	6

A* Search

	10	9	8	7	6	5	4	3	2	1	B
	10+11										1
	9+12		7+10	8+9	9+8	10+7	11+6	12+5	13+4		2
	8+13		6+11						14+5		3
	7+14	6+13	5+12		10	9	8	7	15+6		4
			4+13		11						5
A	1+16	2+15	3+14		12	11	10	9	8	7	6

A* Search

	11+10	9	8	7	6	5	4	3	2	1	**B**
	10+11										1
	9+12		7+10	8+9	9+8	10+7	11+6	12+5	13+4		2
	8+13		6+11						14+5		3
	7+14	6+13	5+12		10	9	8	7	15+6		4
			4+13		11						5
A	1+16	2+15	3+14		12	11	10	9	8	7	6

A* Search

	11+10	12+9	8	7	6	5	4	3	2	1	**B**
	10+11										1
	9+12		7+10	8+9	9+8	10+7	11+6	12+5	13+4		2
	8+13		6+11						14+5		3
	7+14	6+13	5+12		10	9	8	7	15+6		4
			4+13		11						5
A	1+16	2+15	3+14		12	11	10	9	8	7	6

A* Search

	11+10	12+9	13+8	7	6	5	4	3	2	1	**B**
	10+11										1
	9+12		7+10	8+9	9+8	10+7	11+6	12+5	13+4		2
	8+13		6+11						14+5		3
	7+14	6+13	5+12		10	9	8	7	15+6		4
			4+13		11						5
A	1+16	2+15	3+14		12	11	10	9	8	7	6

A* Search

	11+10	12+9	13+8	14+7	6	5	4	3	2	1	**B**
	10+11										1
	9+12		7+10	8+9	9+8	10+7	11+6	12+5	13+4		2
	8+13		6+11						14+5		3
	7+14	6+13	5+12		10	9	8	7	15+6		4
			4+13		11						5
A	1+16	2+15	3+14		12	11	10	9	8	7	6

A* Search

	11+10	12+9	13+8	14+7	15+6	5	4	3	2	1	**B**
	10+11										1
	9+12		7+10	8+9	9+8	10+7	11+6	12+5	13+4		2
	8+13		6+11						14+5		3
	7+14	6+13	5+12		10	9	8	7	15+6		4
			4+13		11						5
A	1+16	2+15	3+14		12	11	10	9	8	7	6

A* Search

	11+10	12+9	13+8	14+7	15+6	16+5	4	3	2	1	**B**
	10+11										1
	9+12		7+10	8+9	9+8	10+7	11+6	12+5	13+4		2
	8+13		6+11						14+5		3
	7+14	6+13	5+12		10	9	8	7	15+6		4
			4+13		11						5
A	1+16	2+15	3+14		12	11	10	9	8	7	6

A* Search

	11+10	12+9	13+8	14+7	15+6	16+5	17+4	3	2	1	B
	10+11										1
	9+12		7+10	8+9	9+8	10+7	11+6	12+5	13+4		2
	8+13		6+11						14+5		3
	7+14	6+13	5+12		10	9	8	7	15+6		4
			4+13		11						5
A	1+16	2+15	3+14		12	11	10	9	8	7	6

A* Search

	11+10	12+9	13+8	14+7	15+6	16+5	17+4	18+3	2	1	B
	10+11										1
	9+12		7+10	8+9	9+8	10+7	11+6	12+5	13+4		2
	8+13		6+11						14+5		3
	7+14	6+13	5+12		10	9	8	7	15+6		4
			4+13		11						5
A	1+16	2+15	3+14		12	11	10	9	8	7	6

A* Search

	11+10	12+9	13+8	14+7	15+6	16+5	17+4	18+3	19+2	1	B
	10+11										1
	9+12		7+10	8+9	9+8	10+7	11+6	12+5	13+4		2
	8+13		6+11						14+5		3
	7+14	6+13	5+12		10	9	8	7	15+6		4
			4+13		11						5
A	1+16	2+15	3+14		12	11	10	9	8	7	6

A* Search

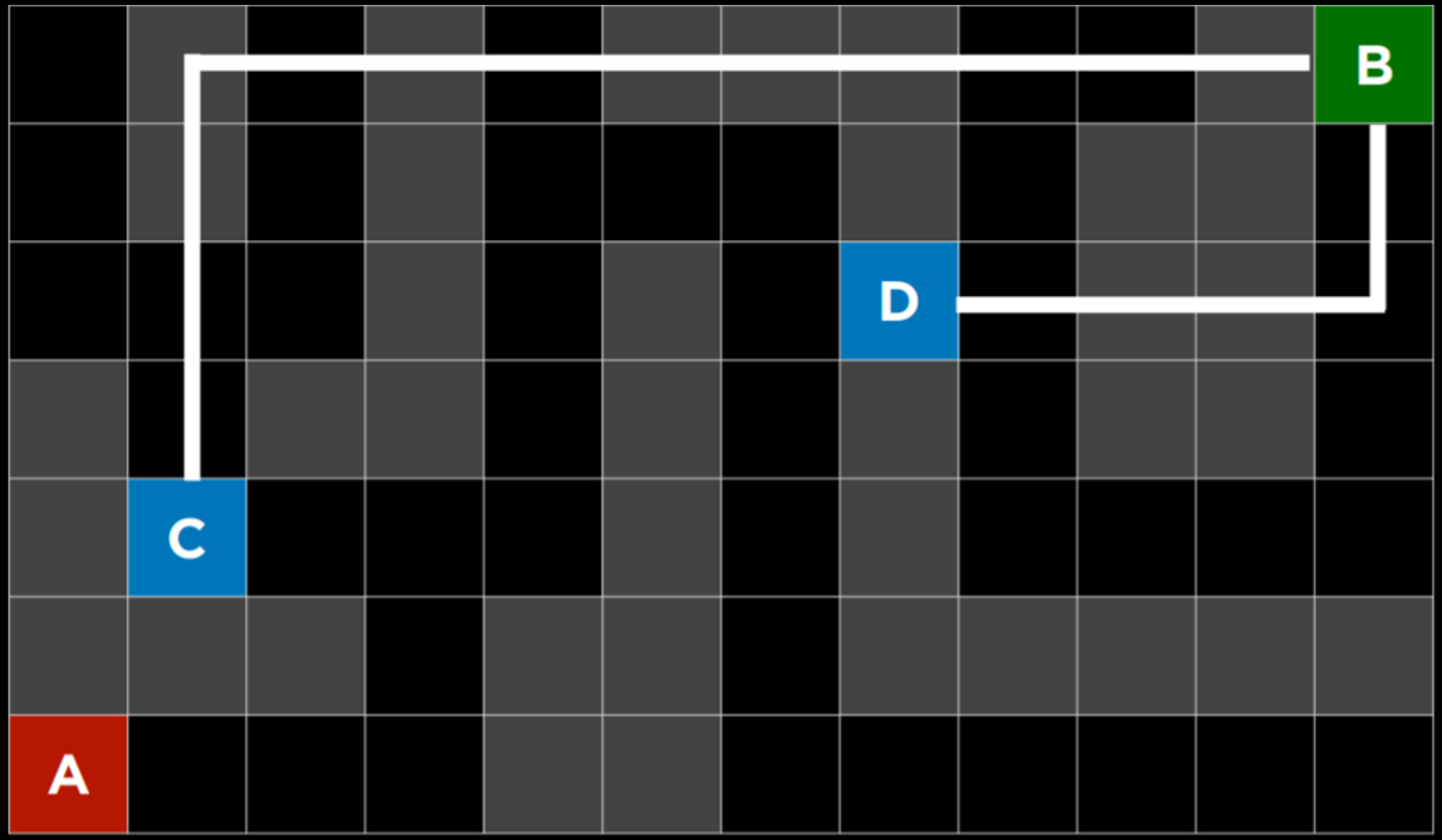
	11+10	12+9	13+8	14+7	15+6	16+5	17+4	18+3	19+2	20+1	B
	10+11										1
	9+12		7+10	8+9	9+8	10+7	11+6	12+5	13+4		2
	8+13		6+11						14+5		3
	7+14	6+13	5+12		10	9	8	7	15+6		4
			4+13		11						5
A	1+16	2+15	3+14		12	11	10	9	8	7	6

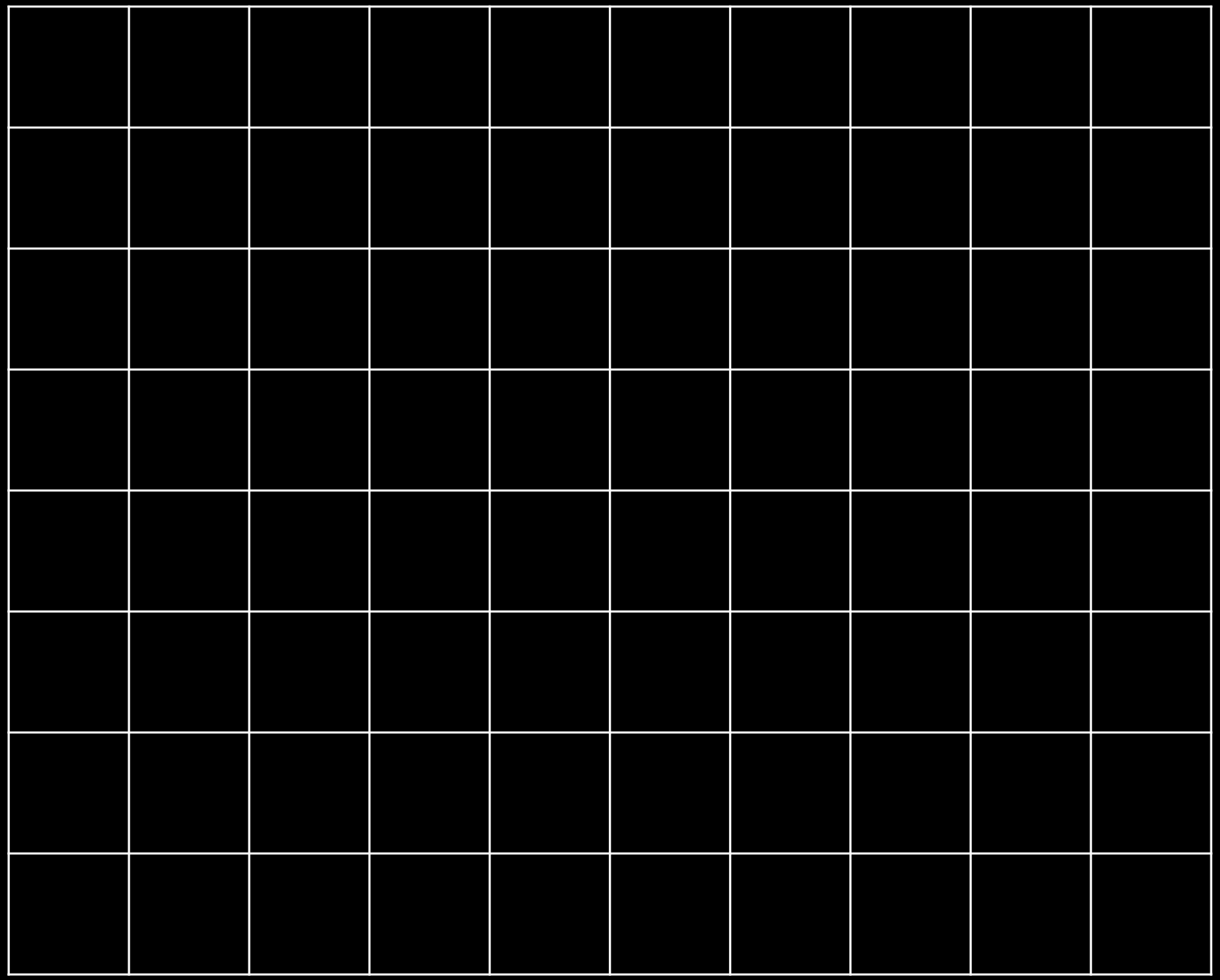
Heuristic Function h(n)

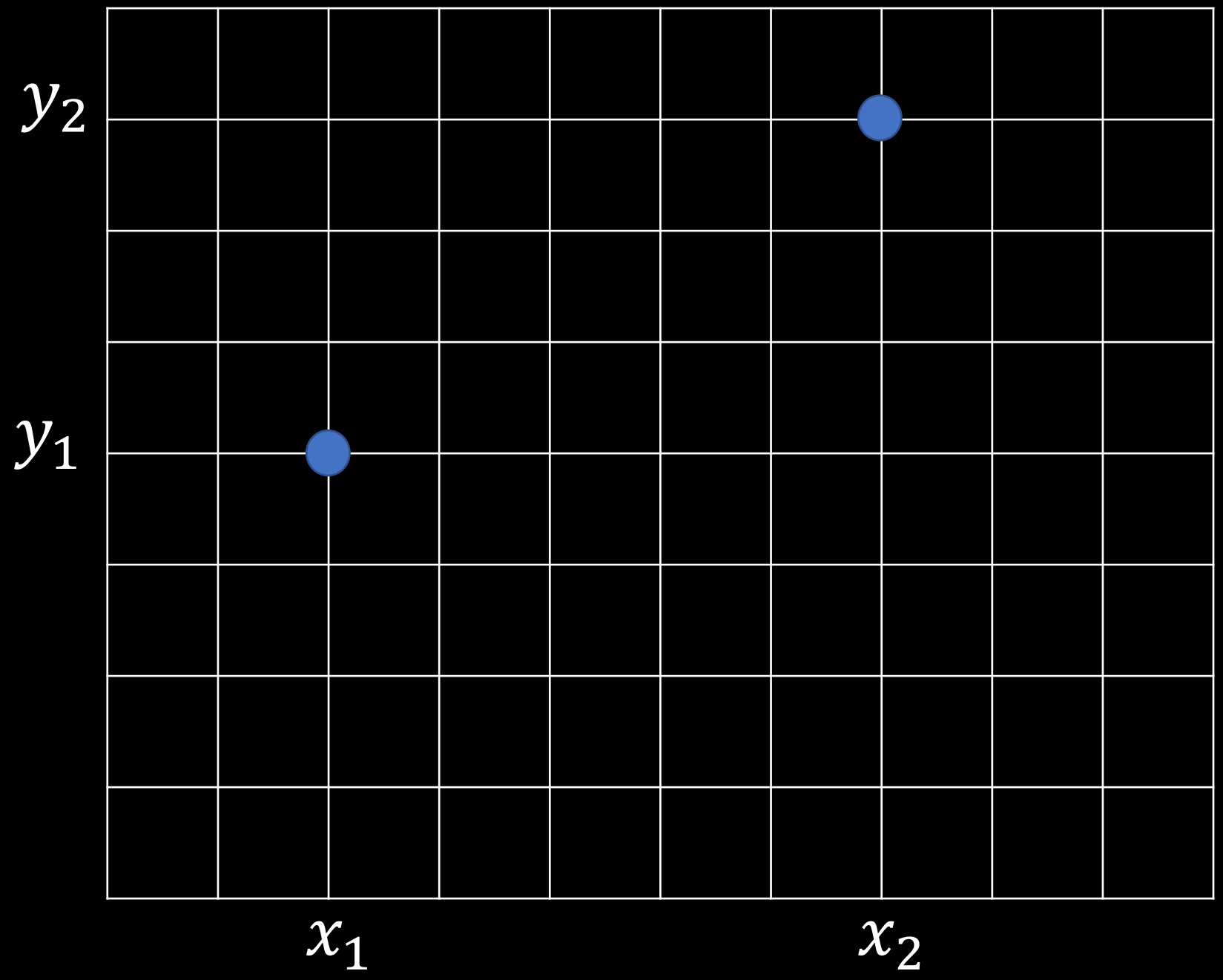
The heuristic function h(n) tells A* an estimate of the minimum cost from any vertex n to the goal. It’s important to choose a good heuristic function.

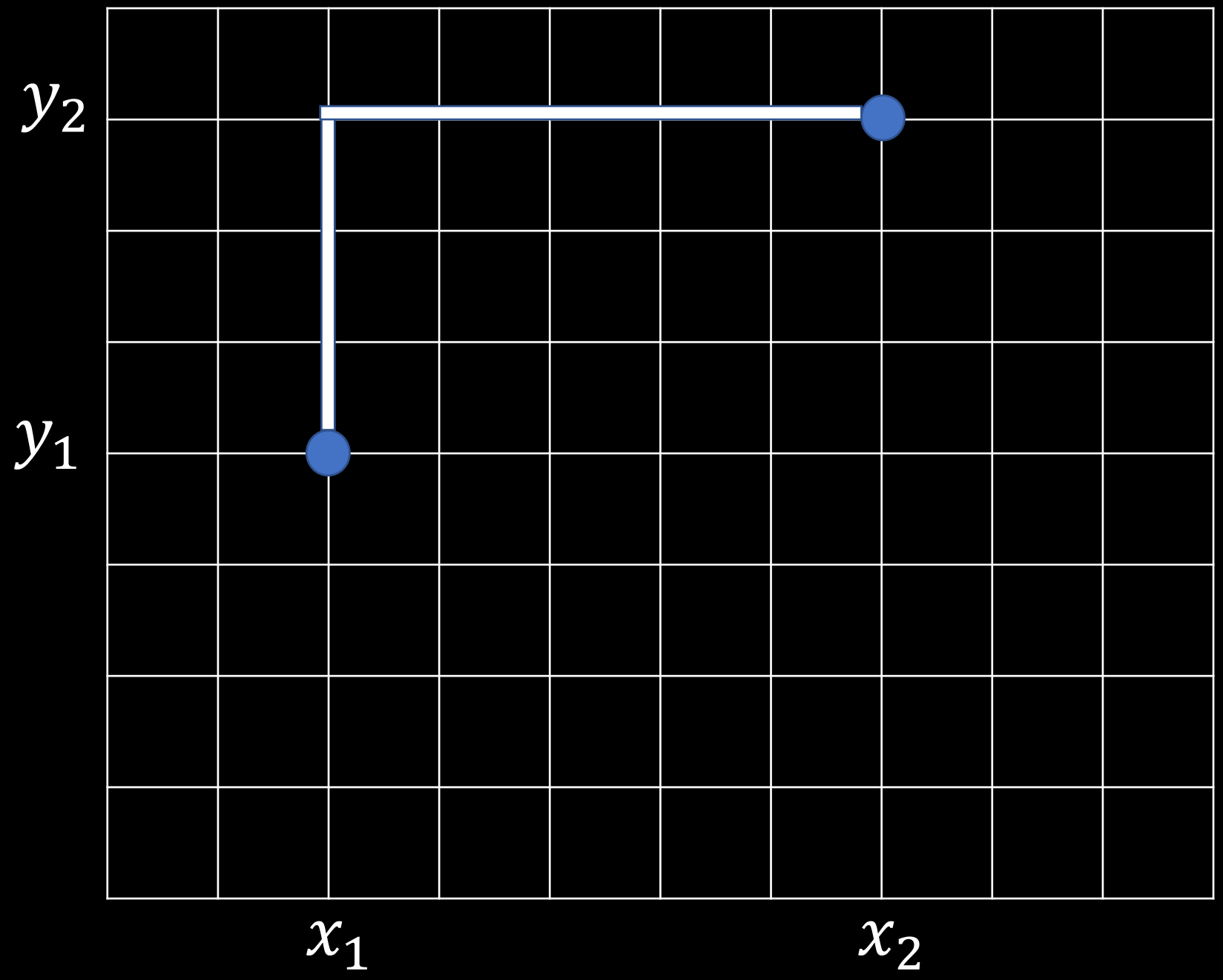
Manhattan method

This method of computing $h(n)$ is called the **Manhattan** method because it is computed by calculating the total number of squares moved horizontally and vertically to reach the target square from the current square. We ignore diagonal movement and any obstacles that might be in the way.


B
D
C
A



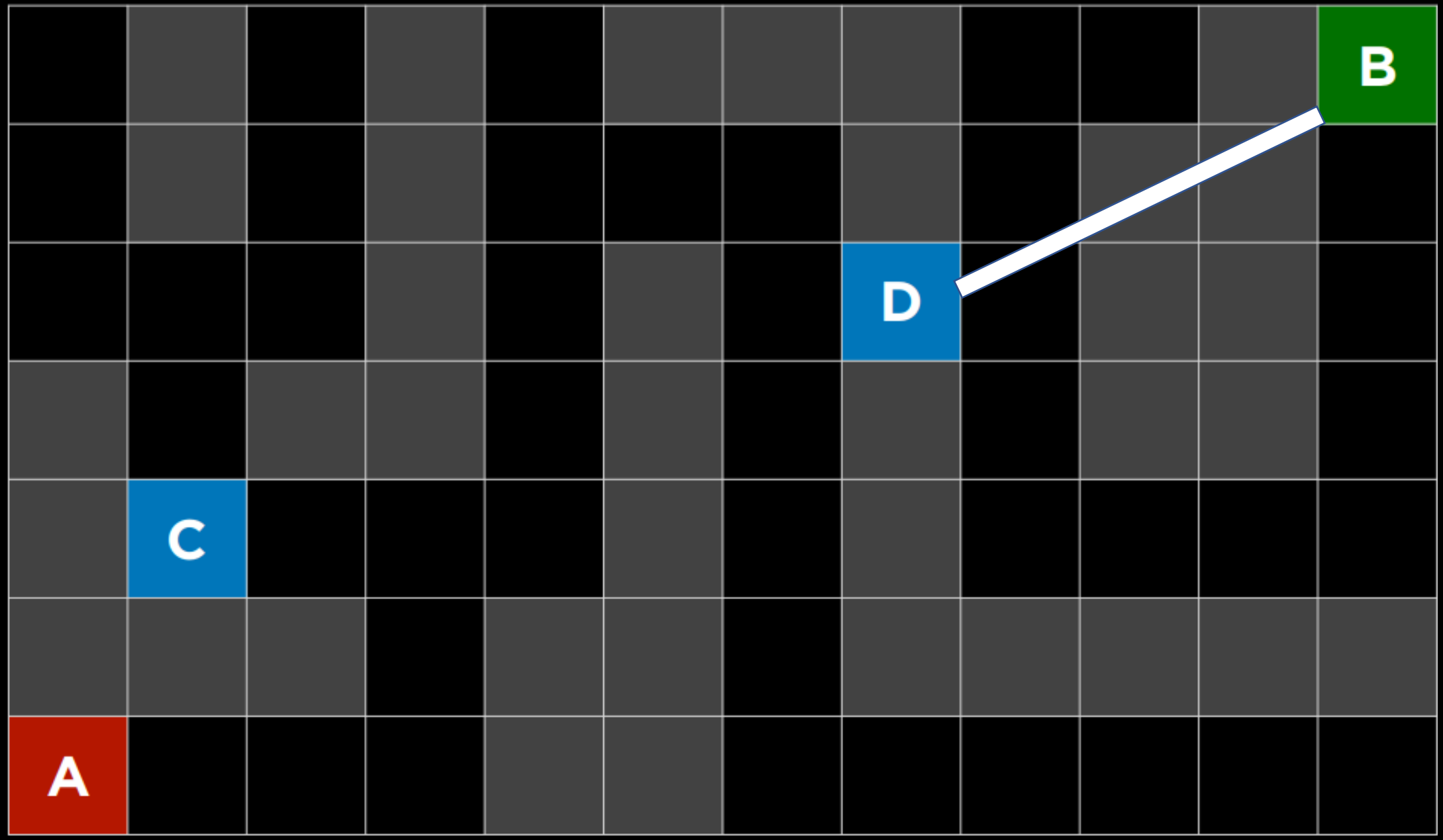

y_2
y_1
x_1
x_2

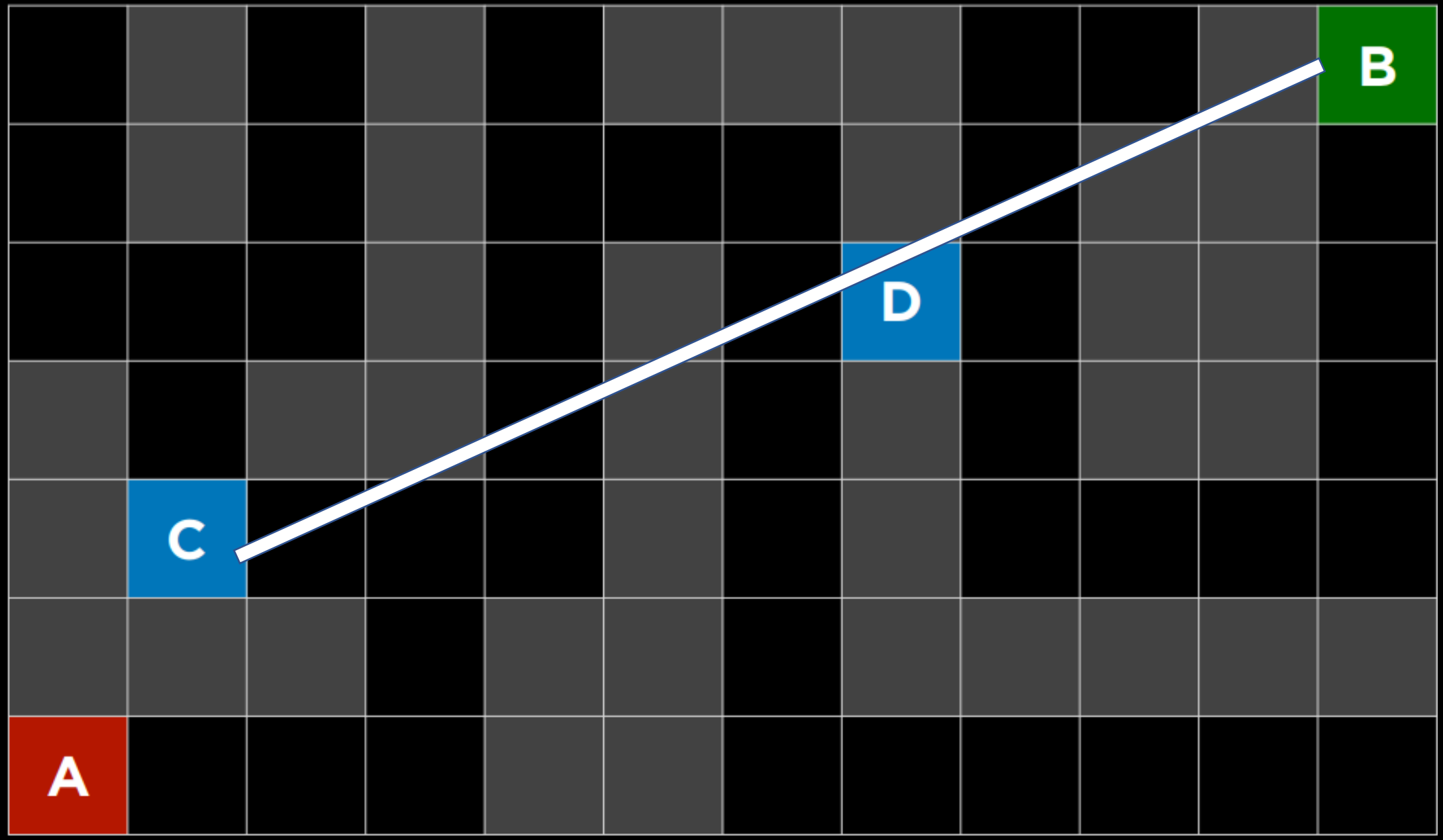

y_2
y_1
x_1
x_2

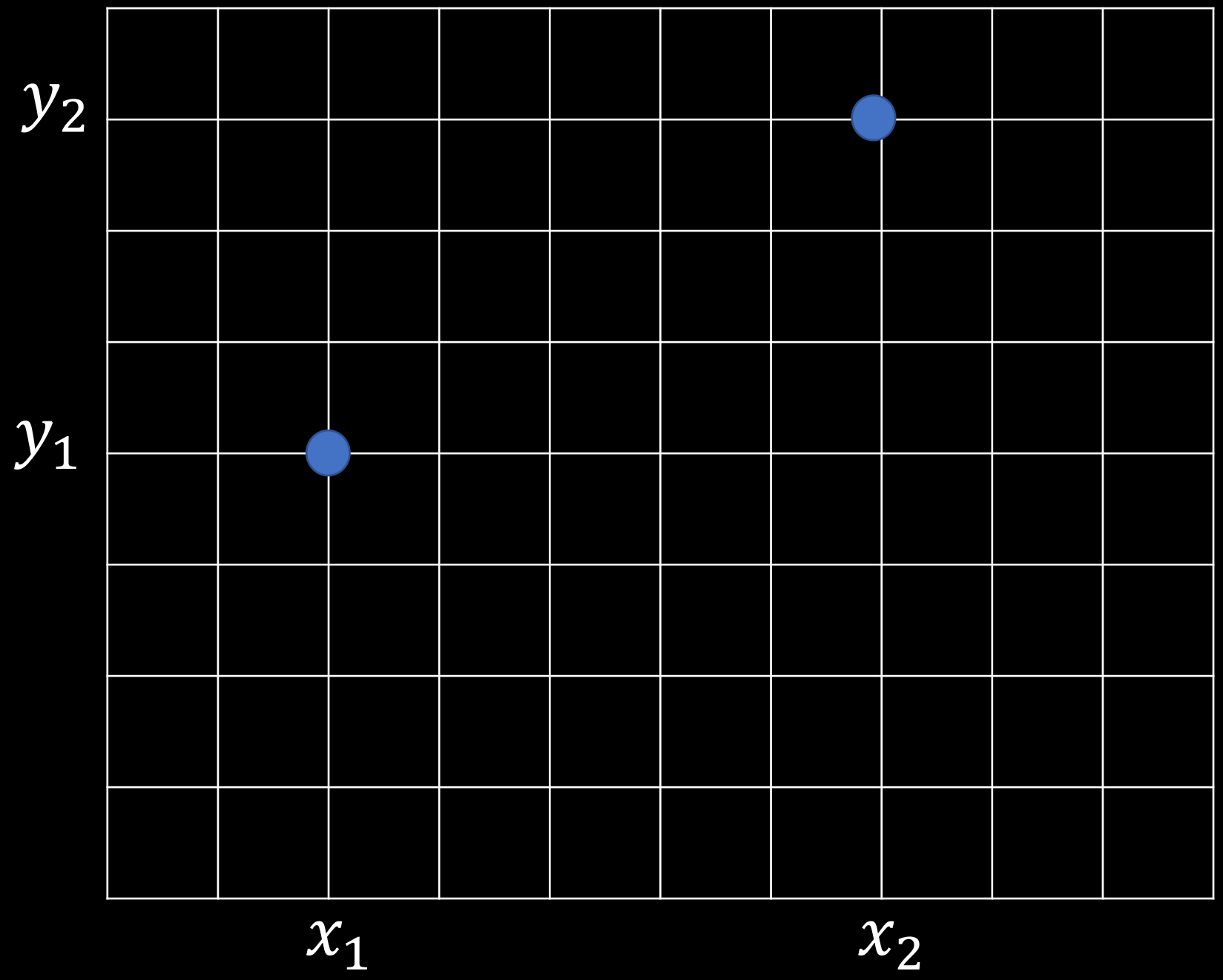
$$h = | x_2 - x_1 | + | y_2 - y_1 |$$

The Euclidean Distance Heuristic:

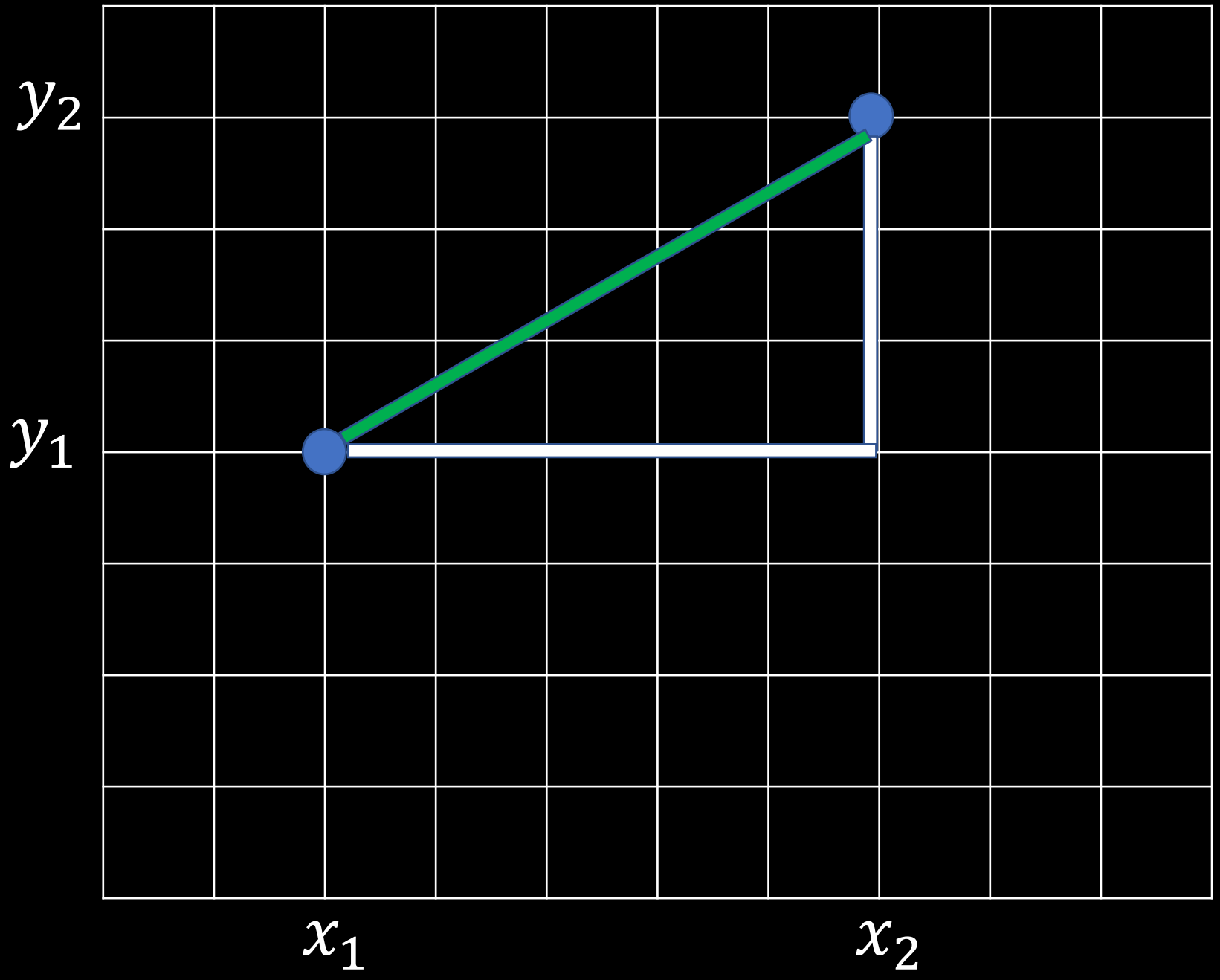
Sometimes we might prefer a path that tends to follow a straight line directly to our destination. This heuristic is slightly more accurate than its Manhattan counterpart. If we try run both simultaneously on the same maze, the **Euclidean** path finder favors a path along a straight line. This is more accurate but it is also slower because it has to explore a larger area to find the path.


B
D
C
A


B
D
C
A


y_2
y_1
x_1
x_2

y_2
y_1
x_1
x_2


y_2
y_1
x_1
x_2

$$h = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

```
Make an open list containing only the starting node
Make an empty closed list

While (the destination node has not been reached and open list is not empty):
    Consider the node with the lowest f score in the open list (call it current_node)
    If (this node is the destination node):
        We are finished (reconstruct the path from start to destination)
    Else:
        Put current_node in the closed list
        Look at all of its neighbors:
            For each neighbor of current_node:
                If (neighbor is not within the grid bounds or is an obstacle):
                    Skip this neighbor
                If (neighbor is in the closed list):
                    Skip this neighbor
```

```
        Calculate the cost (g) to reach the neighbor from current_node:
            Set h as the heuristic estimate from the neighbor to the goal
            Calculate f = g + h
            If (neighbor is in the open list and the new g value is lower than the old g value):
                Update the neighbor with the new, lower g value
                Change the neighbor's parent to current_node
            Else if (neighbor is not in both lists):
                Add the neighbor to the open list and set its g, h, and f values
                Set current_node as the parent of the neighbor

If open list is empty:
    No path is found, return None
```

References:

https://theory.stanford.edu/~amitp/GameProgramming/Heuristics.html
https://brilliant.org/wiki/a-star-search/