Cipher

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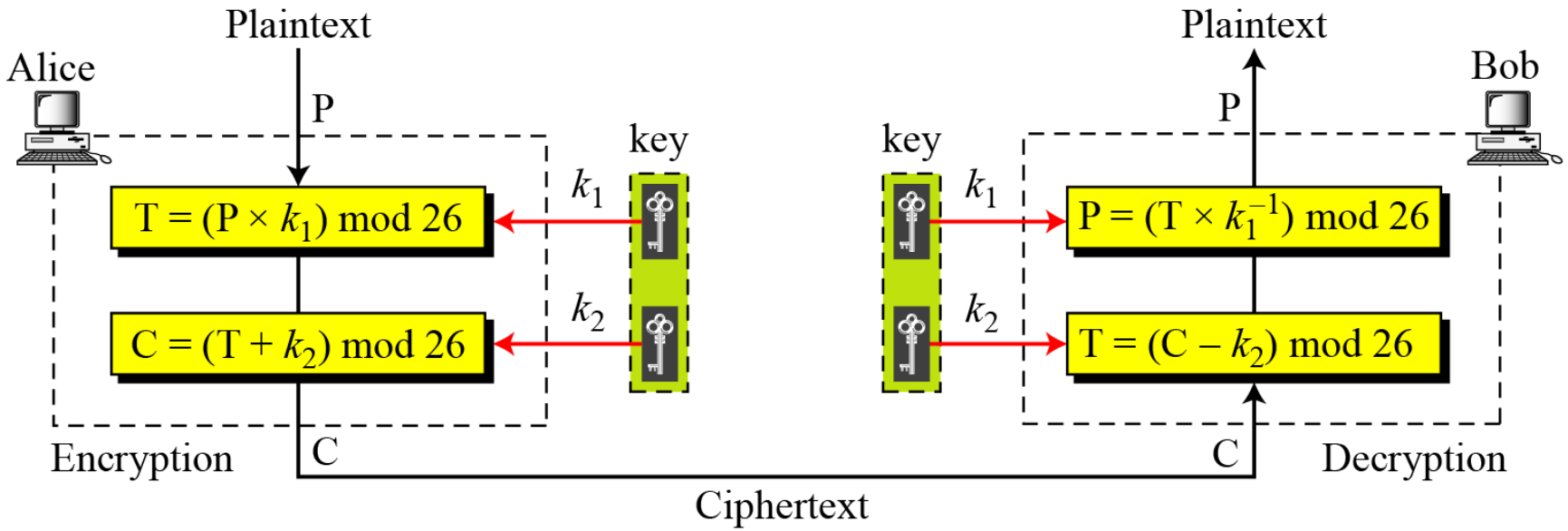
Affine Cipher

- Affine Ciphers



$$C = (P \times k_1 + k_2) \bmod 26 \qquad P = ((C - k_2) \times k_1^{-1}) \bmod 26$$

where k_1^{-1} is the multiplicative inverse of k_1 and $-k_2$ is the additive inverse of k_2

Affine Cipher

- In a multiplicative cipher,
  - the plaintext and ciphertext are integers in Z_{26};
  - the key is an integer in Z_{26^*}.

- decrypt the message “ZEBBW”.
- Message is encrypted with the key pair (7, 2).

Affine Cipher

- What is the key domain for any multiplicative cipher?
- The key needs to be in Z_{26}*. This set has only 12 members: 1, 3, 5, 7, 9, 11, 15, 17, 19, 21, 23, 25.

- We use a multiplicative cipher to encrypt the message “hello” with a key of 7. The ciphertext is “XCZZU”.

Plaintext: h $\rightarrow$ 07	Encryption: $(07 \times 07) \bmod 26$	ciphertext: 23 $\rightarrow$ X
Plaintext: e $\rightarrow$ 04	Encryption: $(04 \times 07) \bmod 26$	ciphertext: 02 $\rightarrow$ C
Plaintext: l $\rightarrow$ 11	Encryption: $(11 \times 07) \bmod 26$	ciphertext: 25 $\rightarrow$ Z
Plaintext: l $\rightarrow$ 11	Encryption: $(11 \times 07) \bmod 26$	ciphertext: 25 $\rightarrow$ Z
Plaintext: o $\rightarrow$ 14	Encryption: $(14 \times 07) \bmod 26$	ciphertext: 20 $\rightarrow$ U

Affine Cipher

- The affine cipher uses a pair of keys in which the first key is from Z_{26}^* and the second is from Z_{26}. The size of the key domain is $26 \times 12 = 312$.
- Use an affine cipher to encrypt the message “hello” with the key pair (7, 2).

P: h → 07	Encryption: $(07 \times 7 + 2) \bmod 26$	C: 25 → Z
P: e → 04	Encryption: $(04 \times 7 + 2) \bmod 26$	C: 04 → E
P: l → 11	Encryption: $(11 \times 7 + 2) \bmod 26$	C: 01 → B
P: l → 11	Encryption: $(11 \times 7 + 2) \bmod 26$	C: 01 → B
P: o → 14	Encryption: $(14 \times 7 + 2) \bmod 26$	C: 22 → W

Affine Cipher

- Use the affine cipher to decrypt the message “ZEBBW” with the key pair (7, 2) in modulus 26.

C: Z $\rightarrow$ 25	Decryption: $((25 - 2) \times 7^{-1}) \bmod 26$	P:07 $\rightarrow$ h
C: E $\rightarrow$ 04	Decryption: $((04 - 2) \times 7^{-1}) \bmod 26$	P:04 $\rightarrow$ e
C: B $\rightarrow$ 01	Decryption: $((01 - 2) \times 7^{-1}) \bmod 26$	P:11 $\rightarrow$ l
C: B $\rightarrow$ 01	Decryption: $((01 - 2) \times 7^{-1}) \bmod 26$	P:11 $\rightarrow$ l
C: W $\rightarrow$ 22	Decryption: $((22 - 2) \times 7^{-1}) \bmod 26$	P:14 $\rightarrow$ o

Hill Cipher

- Invented by L. S. Hill in 1929.
- Inputs : String of English letters, A,B,…,Z.
  An $n \times n$ matrix **K**, with entries drawn from 0,1,…,25 (The matrix **K** serves as the secret key.)
- Divide the input string into blocks of size n.
- Identify A=0, B=1, C=2, …, Z=25.
- Encryption: Multiply each block by **K** and then reduce mod 26.
- Decryption: multiply each block by the inverse of **K**, and reduce mod 26

Hill Cipher

- Plaintext: ATTACK Key: CDDG

So $k = \begin{bmatrix} 2 & 3 \\ 3 & 6 \end{bmatrix}$ Since KEY is **2x2** matrix, plaintext should be converted

into vectors of length **2**

So $\begin{bmatrix} A \\ T \end{bmatrix} = \begin{bmatrix} 0 \\ 19 \end{bmatrix}, \begin{bmatrix} T \\ A \end{bmatrix} = \begin{bmatrix} 19 \\ 0 \end{bmatrix}, \begin{bmatrix} C \\ K \end{bmatrix} = \begin{bmatrix} 2 \\ 10 \end{bmatrix}$

Ciphertext C = K.P mod 26

$$= \begin{bmatrix} 2 & 3 \\ 3 & 6 \end{bmatrix} \text{X} \begin{bmatrix} A \\ T \end{bmatrix} \text{ mod } 26$$

$$= \begin{bmatrix} 2 & 3 \\ 3 & 6 \end{bmatrix} \text{X} \begin{bmatrix} 0 \\ 19 \end{bmatrix} \text{ mod } 26$$

$$= \begin{bmatrix} 2*0+ & 3*19 \\ 3*0+ & 6*19 \end{bmatrix} \text{ mod } 26$$

$$= \begin{bmatrix} 57 \\ 114 \end{bmatrix} \text{ mod } 26$$

For mod 26, we are dividing numbers by 26 and considering remainders.

$$\mathbf{C} = \begin{bmatrix} 5 \\ 10 \end{bmatrix}$$, So corresponding alphabets **FK.**

Ciphertext C = K.P mod 26

$$= \begin{bmatrix} 2 & 3 \\ 3 & 6 \end{bmatrix} \text{X} \begin{bmatrix} T \\ A \end{bmatrix} \bmod 26$$

$$= \begin{bmatrix} 2 & 3 \\ 3 & 6 \end{bmatrix} \text{X} \begin{bmatrix} 19 \\ 0 \end{bmatrix} \bmod 26$$

$$= \begin{bmatrix} 2*19+ & 3*0 \\ 3*19+ & 6*0 \end{bmatrix} \bmod 26$$

$$= \begin{bmatrix} 38 \\ 57 \end{bmatrix} \bmod 26$$

$$\mathbf{C} = \begin{bmatrix} 12 \\ 5 \end{bmatrix}$$, So corresponding alphabets **MF.**

Ciphertext C = K.P mod 26

$$= \begin{bmatrix} 2 & 3 \\ 3 & 6 \end{bmatrix} \mathbf{X} \begin{bmatrix} C \\ K \end{bmatrix} \text{ mod } 26$$

$$= \begin{bmatrix} 2 & 3 \\ 3 & 6 \end{bmatrix} \mathbf{X} \begin{bmatrix} 2 \\ 10 \end{bmatrix} \text{ mod } 26$$

$$= \begin{bmatrix} 2*2+ & 3*10 \\ 3*2+ & 6*10 \end{bmatrix} \text{ mod } 26$$

$$= \begin{bmatrix} 34 \\ 66 \end{bmatrix} \text{ mod } 26$$

$$\mathbf{C} = \begin{bmatrix} 8 \\ 14 \end{bmatrix}$$, So corresponding alphabets **IO.**

So word **ATTACK** became **FKMFIO.**

Decryption

- Find Inverse of Given Matrix
- Multiply Inverse Matrix with ciphertext against mod 26

Example

- Plaintext: ”retreat now”
- Key Matrix: “BACKUPABC”

$$\begin{pmatrix} 1 & 0 & 2 \\ 10 & 20 & 15 \\ 0 & 1 & 2 \end{pmatrix}$$

- Plaintext in group of three:

$$\begin{pmatrix} r \\ e \\ t \end{pmatrix} \begin{pmatrix} r \\ e \\ a \end{pmatrix} \begin{pmatrix} t \\ n \\ o \end{pmatrix} \begin{pmatrix} w \\ x \\ x \end{pmatrix}$$

$$\begin{pmatrix} 17 \\ 4 \\ 19 \end{pmatrix} \begin{pmatrix} 17 \\ 4 \\ 0 \end{pmatrix} \begin{pmatrix} 19 \\ 13 \\ 14 \end{pmatrix} \begin{pmatrix} 22 \\ 23 \\ 23 \end{pmatrix}$$

$$\begin{pmatrix} B & A & C \\ K & U & P \\ A & B & C \end{pmatrix} \begin{pmatrix} r \\ e \\ t \end{pmatrix} = \begin{pmatrix} 1 & 0 & 2 \\ 10 & 20 & 15 \\ 0 & 1 & 2 \end{pmatrix} \begin{pmatrix} 17 \\ 4 \\ 19 \end{pmatrix}$$

$$= \begin{pmatrix} 1 \times 17 + 0 \times 4 + 2 \times 19 \\ 10 \times 17 + 20 \times 4 + 15 \times 19 \\ 0 \times 17 + 1 \times 4 + 2 \times 19 \end{pmatrix}$$

$$= \begin{pmatrix} 55 \\ 535 \\ 42 \end{pmatrix}$$

$$= \begin{pmatrix} 3 \\ 15 \\ 16 \end{pmatrix} mod\ 26$$

$$= \begin{pmatrix} D \\ P \\ Q \end{pmatrix}$$

$$\begin{pmatrix} B & A & C \\ K & U & P \\ A & B & C \end{pmatrix} \begin{pmatrix} r \\ e \\ a \end{pmatrix} = \begin{pmatrix} 1 & 0 & 2 \\ 10 & 20 & 15 \\ 0 & 1 & 2 \end{pmatrix} \begin{pmatrix} 17 \\ 4 \\ 0 \end{pmatrix}$$

$$= \begin{pmatrix} 1 \times 17 + 0 \times 4 + 2 \times 0 \\ 10 \times 17 + 20 \times 4 + 15 \times 0 \\ 0 \times 17 + 1 \times 4 + 2 \times 0 \end{pmatrix}$$

$$= \begin{pmatrix} 17 \\ 250 \\ 4 \end{pmatrix}$$

$$= \begin{pmatrix} 17 \\ 16 \\ 4 \end{pmatrix} mod\ 26$$

$$= \begin{pmatrix} R \\ Q \\ E \end{pmatrix}$$

$$\begin{pmatrix} B & A & C \\ K & U & P \\ A & B & C \end{pmatrix} \begin{pmatrix} t \\ n \\ o \end{pmatrix} = \begin{pmatrix} 1 & 0 & 2 \\ 10 & 20 & 15 \\ 0 & 1 & 2 \end{pmatrix} \begin{pmatrix} 19 \\ 13 \\ 14 \end{pmatrix}$$

$$= \begin{pmatrix} 1 \times 19 + 0 \times 13 + 2 \times 14 \\ 10 \times 19 + 20 \times 13 + 15 \times 14 \\ 0 \times 19 + 1 \times 13 + 2 \times 14 \end{pmatrix}$$

$$= \begin{pmatrix} 47 \\ 660 \\ 41 \end{pmatrix}$$

$$= \begin{pmatrix} 21 \\ 10 \\ 15 \end{pmatrix} mod\ 26$$

$$= \begin{pmatrix} V \\ K \\ P \end{pmatrix}$$

$$\begin{pmatrix} B & A & C \\ K & U & P \\ A & B & C \end{pmatrix}\begin{pmatrix} w \\ x \\ x \end{pmatrix} = \begin{pmatrix} 1 & 0 & 2 \\ 10 & 20 & 15 \\ 0 & 1 & 2 \end{pmatrix}\begin{pmatrix} 22 \\ 23 \\ 23 \end{pmatrix}$$

$$= \begin{pmatrix} 1 \times 22 + 0 \times 23 + 2 \times 23 \\ 10 \times 22 + 20 \times 23 + 15 \times 23 \\ 0 \times 22 + 1 \times 23 + 2 \times 23 \end{pmatrix}$$

$$= \begin{pmatrix} 68 \\ 1025 \\ 69 \end{pmatrix}$$

$$= \begin{pmatrix} 16 \\ 11 \\ 17 \end{pmatrix} \ mod\ 26$$

$$= \begin{pmatrix} Q \\ L \\ R \end{pmatrix}$$

Decryption

- Find Inverse Key Matrix

$$\begin{pmatrix} 0 & 11 & 15 \\ 7 & 0 & 1 \\ 4 & 19 & 0 \end{pmatrix}$$

- **find the determinant**

$$\begin{vmatrix} 0 & 11 & 15 \\ 7 & 0 & 1 \\ 4 & 19 & 0 \end{vmatrix} = 0\begin{vmatrix} 0 & 1 \\ 19 & 0 \end{vmatrix} - 11\begin{vmatrix} 7 & 1 \\ 4 & 0 \end{vmatrix} + 15\begin{vmatrix} 7 & 0 \\ 4 & 19 \end{vmatrix}$$

$$= 0(0 - 19) - 11(0 - 4) + 15(133 - 0)$$

$$= 0 + 44 + 1995$$

$$= 2039$$

$$= 11 \; mod \; 26$$

Decryption

- Inverse of determinant

$$dd^{-1} = 1 \, mod \, 26$$

$$11 \times 19 = 209 = 1 \, mod \, 26$$

Decryption

- **find the adjugate matrix**

$$adj\begin{pmatrix}0 & 11 & 15\\ 7 & 0 & 1\\ 4 & 19 & 0\end{pmatrix} = \begin{pmatrix}+\begin{vmatrix}0 & 1\\ 19 & 0\end{vmatrix} & -\begin{vmatrix}11 & 15\\ 19 & 0\end{vmatrix} & +\begin{vmatrix}11 & 15\\ 0 & 1\end{vmatrix}\\ -\begin{vmatrix}7 & 1\\ 4 & 0\end{vmatrix} & +\begin{vmatrix}0 & 15\\ 4 & 0\end{vmatrix} & -\begin{vmatrix}0 & 15\\ 7 & 1\end{vmatrix}\\ +\begin{vmatrix}7 & 0\\ 4 & 19\end{vmatrix} & -\begin{vmatrix}0 & 11\\ 4 & 19\end{vmatrix} & +\begin{vmatrix}0 & 11\\ 7 & 0\end{vmatrix}\end{pmatrix}$$

$$= \begin{pmatrix}-19 & 285 & 11\\ 4 & -60 & 105\\ 133 & 44 & -77\end{pmatrix}$$

$$= \begin{pmatrix}7 & 25 & 11\\ 4 & 18 & 1\\ 3 & 18 & 1\end{pmatrix} mod\ 26$$

Decryption

- **We need to multiply the inverse determinate (19) by each of the numbers in this new matrix**

$$19 \times \begin{pmatrix} 7 & 25 & 11 \\ 4 & 18 & 1 \\ 3 & 18 & 1 \end{pmatrix} = \begin{pmatrix} 133 & 475 & 209 \\ 76 & 342 & 19 \\ 57 & 342 & 19 \end{pmatrix} = \begin{pmatrix} 3 & 7 & 1 \\ 24 & 4 & 19 \\ 5 & 4 & 19 \end{pmatrix} mod\ 26$$

$$if\ K = \begin{pmatrix} 0 & 11 & 15 \\ 7 & 0 & 1 \\ 4 & 19 & 0 \end{pmatrix}, then\ K^{-1} = \begin{pmatrix} 3 & 7 & 1 \\ 24 & 4 & 19 \\ 5 & 4 & 19 \end{pmatrix}$$

Decryption

- For decryption, multiply ciphertext with inverse key matrix mod 26.

Columnar Transposition Cipher

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