EE 610: Image Processing

Assignment 1

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Fall 2023

August 23, 2023

Problem 1

You are preparing a report and have to insert an image of size 1024×1024. Assume that the printer has no constraints, what would the resolution in dpi of the printer be if the image is to fit in 2 × 2 inches ?

Solution.

This can be easily computed. We need to keep in mind that *dpi* is used as a measure of *spatial resolution*. The required *dpi* can be found as:

$$\begin{aligned}\text{Spatial Resolution}(dpi) &= \frac{\text{Image dimensions}}{\text{Print dimensions}}\\ &= \frac{\text{1024 pixels}}{\text{2 inches}}\\ &= 512\ dpi\end{aligned}$$

Hence, the printer should have a spatial resolution of 512 *dpi* in order to fit the given conditions.

Problem 2

Consider point-wise operation between two images.
(a) Is summation operation between two images a linear or non-linear operation ? Justify.
(b) Is multiplication operation between two images a linear or non-linear operation ? Justify.
You have to consider additivity and homogeneity properties to check linearity.

Solution.

(a) The point-wise summation operation between two images is a linear operation. This can be proven mathematically. An operation A is considered linear when for two inputs f and g:

$$A(f+g) = A(f) + A(g)$$

For point-wise summation between two images, P and Q, the resultant image, C, is given by:

$$C(x,y) = P(x,y) + Q(x,y)$$

Looking at additivity,

$$
\begin{aligned}
C(f+g) &= P(f+g) + Q(f+g) \\
&= P(f) + P(g) + Q(f) + Q(g) \\
&= (P(f) + Q(f)) + (P(g) + Q(g)) \\
&= C(f) + C(g)
\end{aligned}
$$

As for homogeneity for an operation A on two inputs f and g, we require:

$$A(c \cdot f) = c \cdot A(f)$$

Homogeneity can be proven for point-wise image summation as:

$$
\begin{aligned}
C(c \cdot f) &= P(c \cdot f) + Q(c \cdot f) \\
&= c \cdot P(f) + c \cdot Q(f) \\
&= c \cdot (P(f) + Q(f)) \\
&= c \cdot C(f)
\end{aligned}
$$

(b) Point-wise multiplication on two images is defined as:

$$C(x,y) = P(x,y) \cdot Q(x,y)$$

This operation is NOT a linear operation. This can be proven by taking a counter-example. Suppose for a pixel (i,j), we have $P(i,j) = 5$ and $Q(i,j) = 7$. Multiplying point-wise,

$$
\begin{aligned}
C(i,j) &= P(i,j) \cdot Q(i,j) \\
&= 5 \cdot 7 \\
&= 35
\end{aligned}
$$

Checking additivity,

$$
\begin{aligned}
D(i,j) &= (A+B)(i,j) \cdot (A+B)(i,j) \\
&= (5+7) \cdot (5+7) \\
&= 144
\end{aligned}
$$

As $D(i,j) \neq C(i,j)$, point-wise multiplication does NOT follow *additivity*. We do not need to test *homogeneity* as *both* additivity and homogeneity need to be followed for an operation to become linear. However, homogeneity can be easily looked into for point-wise multiplication.

Problem 3

Consider the geometrical transformation functions discussed in class. Provide a com-

posite transformation function to perform the following operations:
(a) Scaling and translation
(b) Does order of multiplication in obtaining the composition makes a difference ? Justify by giving an example.

Solution.
(a) Let us take the scaling function as $S_{s_x,s_y}(x,y)$ which scales an image by the scaling factors s_x and s_y along the x- and y-axes respectively.

$$S_{s_x,s_y}(x,y) = \begin{bmatrix} s_x & 0 & 0 \\ 0 & s_y & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

The translation function can be taken as $T_{t_x,t_y}(x,y)$:

$$T_{t_x,t_y}(x,y) = \begin{bmatrix} 1 & 0 & t_x \\ 0 & 1 & t_y \\ 0 & 0 & 1 \end{bmatrix}$$

Their composite effect, $C(x,y)$, can be obtained as:

$$\begin{aligned} C(x,y) &= T_{t_x,t_y}(x,y) \cdot S_{s_x,s_y}(x,y) \\ &= \begin{bmatrix} 1 & 0 & t_x \\ 0 & 1 & t_y \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} s_x & 0 & 0 \\ 0 & s_y & 0 \\ 0 & 0 & 1 \end{bmatrix} \\ &= \begin{bmatrix} s_x & 0 & t_x \\ 0 & s_y & t_y \\ 0 & 0 & 1 \end{bmatrix} \end{aligned}$$

(b) Yes, the order of multiplication DOES MATTER. Let us take an example to verify this. Let $(x,y) = (2,3)$, $s_x = 2$, $s_y = 1$, $t_x = 5$, and $t_y = 0$. Performing scaling followed by translation, we get the modified location (x', y') as:

$$\begin{aligned} \begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} &= C(\left(\begin{bmatrix} x \\ y \\ 1 \end{bmatrix}\right)) \\ &= \begin{bmatrix} 2 & 0 & 5 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 2 \\ 3 \\ 1 \end{bmatrix} \\ &= \begin{bmatrix} 9 \\ 3 \\ 1 \end{bmatrix} \end{aligned}$$

Upon changing the order of the operations as: $D(x, y) = S_{s_x,s_y}(x, y) \cdot T_{t_x,t_y}(x, y)$, we get:

$$\begin{aligned}
\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} &= D\left(\begin{bmatrix} x \\ y \\ 1 \end{bmatrix}\right) \\
&= \begin{bmatrix} 2 & 0 & 10 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 2 \\ 3 \\ 1 \end{bmatrix} \\
&= \begin{bmatrix} 14 \\ 3 \\ 1 \end{bmatrix}
\end{aligned}$$

Hence, we have verified that the order of multiplication of the transformation matrices matter with an example.

Problem 4

Give a single intensity transformation function for spreading the intensities of an image so the lowest intensity is 0 and the highest is L − 1.

Solution.
We can define such an intensity transformation function, $T(r)$ as:

$$T(r) = \frac{L - 1}{I_{\text{max}} - I_{\text{min}}}(r - I_{\text{min}})$$

Here,

- r is the pixel intensity
- I_{max} is the maximum pixel intensity in the original image
- I_{min} is the minimum pixel intensity in the original image

Problem 5

This is related to bit-plane slicing.
(a) Propose a method for extracting the bit planes of an image based on converting the value of its pixels to binary. Consider a 4-bit image and provide a set of transformation functions ($T(r)$) to achieve individual bit-planes.

(b) Using the approach in (a) find all the bit planes of the following 4-bit image:

$$\begin{bmatrix} 0 & 1 & 8 & 6 \\ 2 & 2 & 1 & 1 \\ 1 & 15 & 14 & 12 \\ 3 & 6 & 9 & 10 \end{bmatrix}$$

Solution.

(a) Bit-plane slicing can be achieved by following these steps:

- Convert all the pixel intensity values to 4-bit binary values.
- Since we are dealing with 4-bit values, we will get 4 bit-planes. These can be extracted by isolating the bit values at each bit location for each pixel coordinate.

We can use the following transformation functions, $T_i(r)$, for computing each bit-plane at bit-level i:

$$T_i(r) = \begin{cases} 0, & \text{if } r = 0 \\ 1, & \text{if } r = 1 \end{cases}$$

where, $i = 0, 1, 2, 3$.

(b) For the given image, upon using the transformation functions found in (a), we get the following bit-planes:

$$B_0 = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

$$B_1 = \begin{bmatrix} 0 & 0 & 0 & 1 \\ 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 1 & 1 & 0 & 1 \end{bmatrix}$$

$$B_2 = \begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 1 & 1 & 1 \\ 0 & 1 & 0 & 0 \end{bmatrix}$$

$$B_3 = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

These bit-planes can be constructed into individual plane images by multiplying each bit in the i-th bit-plane by 2^{i-1}.

Problem 6

In general:
(a) What effect would setting to zero the lower-order bit planes have on the histogram of an image ?
(b) What would be the effect on the histogram if we set to zero the higher-order bit planes instead ?

Solution.
(a) If we set the lower-order bit-planes to zero, the histogram will get heavily affected. Since we will be effectively lowering the number of intensities that can occur in the image, it will result in a reduction in the number of bins in the histogram, leading to a narrower histogram. Also, setting the lower-order bit-planes to zero will lead to the image losing some of the finer details. Lastly, the statistical variables of the image like mean, median, etc. will get affected.
(b) Setting the higher-order bit-planes will have a different set of effects on the image histogram. In addition to histogram compression and shifting in the statistical variables of the image, we will also observe a *loss in contrast*. This can lead to a concentration of histogram peaks in a central region. Moreover, the image will get more homogenous. Note that this is not the same thing as blurring an image. Due to this homogenising, the histogram will have a lower number of distinct peaks. The image will overall experience a darkening effect due to a major lowering of intensity values due to vanishing of bits near the MSB.