Assigned Date: 13/9/2023 | Assignment 2 | Due Date : 29/9/2023
IIT Bombay | EE610 - Image Processing | Total marks : 12

- Reading: Digital Image Processing, Gonzalez and Woods, Ed. 4; Sections 3.4 to 3.4, and Chapter 4.
- Show your work to get credits and state any assumptions you make. You can use/state results derived in class.
- Submit only the **starred** * problems.

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1. [2 marks] *You are given the following kernel and image:

$$w = \begin{bmatrix} 1 & 2 & 1 \\ 2 & 4 & 2 \\ 1 & 2 & 1 \end{bmatrix}$$
$$f = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

   (a) Write the matrices that will be used in computation, when the kernel w is centered at the point $(2, 3)$ of the image f. Note: the left most entry in f corresponds to $(0, 0)$. Give the output of convolution corresponding to this point.

   (b) Give the convolution output for $f * w$, assuming zero-padding.

2. An image is filtered with a kernel whose coefficients sum to 1. Show that sum of the pixel values in the original and filtered images is the same.

3. [2 marks] *In the original image used to generate the three blurred images shown, the vertical bars are 5 pixels wide, 100 pixels high, and their separation is 20 pixels. The image was blurred using square box kernels of sizes 23, 25, and 45 elements on the side, respectively. The vertical bars on the left, lower part of (a) and (c) are blurred, but a clear separation exists between them. However, the bars have merged in image (b), despite the fact that the kernel used to generate this image is much smaller than the kernel that produced image (c). Explain the reason for this.

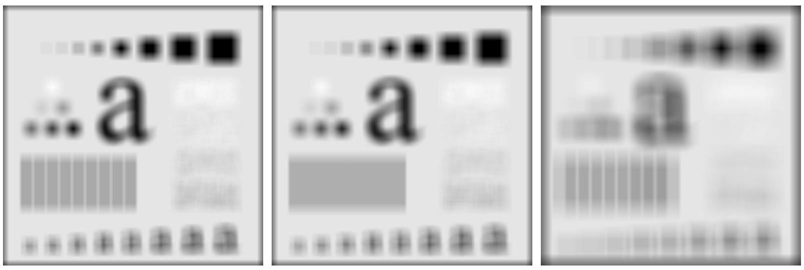

Figure 1: Two images with similar histogram

4. [3 marks] *Consider the function $f(t) = \sin(2\pi nt)$, where n is an integer. Its Fourier transform, $F(\mu)$, is purely imaginary. Because the transform, $\overline{F}(\mu)$, of sampled data consists of periodic copies of $F(\mu)$, it follows that $\overline{F}(\mu)$ will also be purely imaginary. Draw a time-domain plot of the signal $f(t)$ and answer the following questions.

   (a) What is the period of $f(t)$?

   (b) What is the frequency of $f(t)$?

   (c) What would the sampled function and its Fourier transform look like in general if $f(t)$ is sampled at a rate ΔT higher than the Nyquist rate ?

   (d) What would the sampled function look like in general if $f(t)$ is sampled at a rate lower than the Nyquist rate?

(e) What would the sampled function look like if $f(t)$ is sampled at the Nyquist rate, with samples taken at $t = 0, \pm\Delta T, \pm 2\Delta T, \dots$?

5. [2 marks] *The image on the left in the figure below consists of alternating stripes of black/white, each stripe being two pixels wide. The image on the right is the Fourier spectrum of the image on the left, showing the dc term and the frequency terms cor- responding to the stripes. (Remember, the spectrum is symmetric so all components, other than the dc term, appear in two symmetric locations.)
   (a) Suppose that the stripes of an image of the same size are four pixels wide. Sketch what the spectrum of the image would look like, including only the dc term and the two highest-value frequency terms, which correspond to the two spikes in the spectrum above.
   (b) Why are the components of the spectrum limited to the horizontal axis?
   (c) What would the spectrum look like for an image of the same size but having stripes that are one pixel wide? Explain the reason for your answer.
   (d) Are the dc terms in (a) and (c) the same, or are they different? Explain.

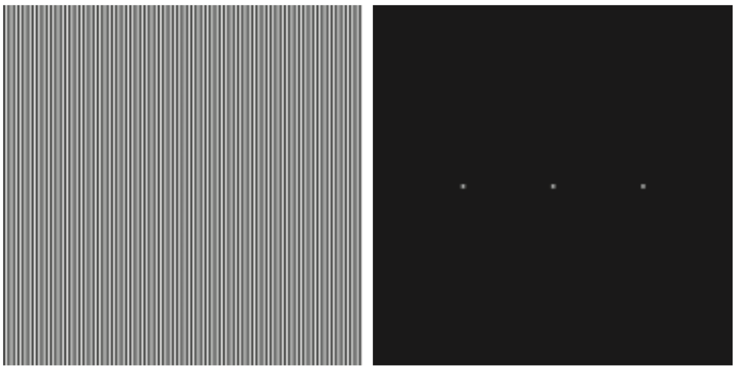

Figure 2: (Left) Image with two pixel wide stripes and (Right) corresponding Fourier spectrum.

6. [1 mark] *We are interested in finding resolution of the sensor that we need in an industry. They have cloth patterns with repeating black-and-white vertical stripes, with each having a width of 2 cm. Each image captured of a cloth piece will have 250 such stripes. Ignore any other distortions caused due to image capture. You are to choose the imaging sensor so that you appropriately sample the image and capture it without any loss. What should be minimum resolution of the sensor that need to be used to capture this image ? Following parts will help you answer this.
   (a) What is the frequency of the vertical stripe (pair) in cycles/cm ?
   (b) What is the required minimum sampling frequency in cycles/cm ?
   (c) What is the required resolution for capturing a cloth piece ?

7. Show the validity of the following 2-D discrete Fourier transform pairs.
   (a) $\delta(x, y) \Longleftrightarrow 1$
   (b) $\delta(x - x_0, y - y_0) \Longleftrightarrow e^{-j2\pi(u\frac{x_0}{M} + \frac{vy_0}{N})}$
   (c) $cos(2\pi u_0 \frac{x}{M} + 2\pi v_0 \frac{y}{N}) \Longleftrightarrow \frac{MN}{2}\left[\delta\left(u - u_0, v - v_0\right) + \delta\left(u + u_0, v + v_0\right)\right]$

8. Consider a 3×3 spatial kernel that averages the four closest neighbors of a point (x, y), but excludes the point itself from the average.
   (a) Find the equivalent filter transfer function, $H(u, v)$, in the frequency domain.
   (b) Show that your result is a lowpass filter transfer function.

9. [2 marks] *First-order derivatives can be approximated by the spatial differences $g_x = \frac{\partial f x,y}{\partial x} = f(x + 1, y) - f(x, y)$ and $g_y = \frac{\partial f x,y}{\partial y} = f(x, y + 1) - f(x, y)$.
   (a) Find the equivalent filter transfer functions $H_x(u, v)$ and $H_y(u, v)$ in the frequency domain.

(b) Show that these are highpass filter transfer functions.

10. Combining high-frequency emphasis and histogram equalization is an effective method for achieving edge sharpening and contrast enhancement.

    (a) Show whether or not it matters which process is applied first.

    (b) If the order does matter, give a rationale for using one or the other method first.