Car

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Environment: Mac OS

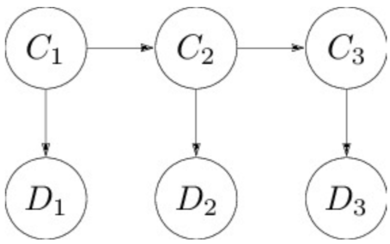
Problem 1: Warmup

1a. Question

Suppose we have a sensor reading for the second timestep, $D_2 = 0$. Compute the posterior distribution $P(C_2 = 1 \mid D_2 = 0)$.

1a. Answer

Here's what the Bayesian network (it's an HMM, in fact) looks like:



So the posterior distribution can be computed like following:

$$
\begin{aligned}
P(C_2 = 1 \mid D_2 = 0) &= \frac{P(C_2 = 1, D_2 = 0)}{P(D_2 = 0)} \\
&= \frac{P(D_2 = 0 \mid C_2 = 1) * P(C_2 = 1)}{P(D_2 = 0)} \\
&= \frac{P(D_2 = 0 \mid C_2 = 1) * P(C_2 = 1)}{\sum_{C_2} P(D_2 = 0 \mid C_2) P(C_2)} \\
&= \frac{P(D_2 = 0 \mid C_2 = 1) * \left[\sum_{C_1} P(C_2 = 1 \mid C_1) P(C_1)\right]}{\sum_{C_2} P(D_2 = 0 \mid C_2) P(C_2)} \\
&= \frac{\eta * \left[\epsilon * 0.5 + (1 - \epsilon) * 0.5\right]}{(1 - \eta) * \left[\epsilon * 0.5 + (1 - \epsilon) * 0.5\right] + \eta * \left[\epsilon * 0.5 + (1 - \epsilon) * 0.5\right]} \\
&= \frac{\eta * 0.5}{0.5} \\
&= \eta
\end{aligned}
$$

1b. Question

Suppose a time step has elapsed and we got another sensor reading $D_3 = 1$, but we are still interested in C_2. Compute the posterior distribution $P(C_2 = 1 \mid D_2 = 0, D_3 = 1)$.

1b. Answer

$$
\begin{aligned}
P(C_2 \mid D_2=0, D_3=1) &\propto \sum_{C_1, C_3} P(C_1)P(C_2 \mid C_1)P(D_2=0 \mid C_2)P(C_3 \mid C_2)P(D_3=1 \mid C_3) \\
&\propto P(D_2=0 \mid C_2)\Big[\sum_{C_1} P(C_1)P(C_2 \mid C_1)\Big]\Big[\sum_{C_3} P(C_3 \mid C_2)P(D_3=1 \mid C_3)\Big]
\end{aligned}
$$

$$
\begin{aligned}
P(C_2=1 \mid D_2=0, D_3=1) &\propto \eta\big[0.5(\epsilon+1-\epsilon)\big]\big[(1-\epsilon)(1-\eta)+\epsilon\eta\big] \\
P(C_2=0 \mid D_2=0, D_3=1) &\propto (1-\eta)\big[0.5(\epsilon+1-\epsilon)\big]\big[(1-\epsilon)\eta+\epsilon(1-\eta)\big]
\end{aligned}
$$

$$
\begin{aligned}
P(C_2=1 \mid D_2=0, D_3=1) &= \frac{P(C_2=1 \mid D_2=0, D_3=1)}{P(C_2=1 \mid D_2=0, D_3=1)+P(C_2=0 \mid D_2=0, D_3=1)} \\
&= \frac{\eta\big[(1-\epsilon)(1-\eta)+\epsilon\eta\big]}{\eta\big[(1-\epsilon)(1-\eta)+\epsilon\eta\big]+(1-\eta)\big[(1-\epsilon)\eta+\epsilon(1-\eta)\big]}
\end{aligned}
$$

1c. Question

c. [3 points] Suppose $\epsilon = 0.1$ and $\eta = 0.2$.

i. Compute and compare the probabilities $\mathbb{P}(C_2 = 1 \mid D_2 = 0)$ and $\mathbb{P}(C_2 = 1 \mid D_2 = 0, D_3 = 1)$. Give numbers, round your answer to 4 significant digits.

ii. How did adding the second sensor reading $D_3 = 1$ change the result? Explain your intuition in terms of the car positions with respect to the observations.

iii. What would you have to set ϵ while keeping $\eta = 0.2$ so that $\mathbb{P}(C_2 = 1 \mid D_2 = 0) = \mathbb{P}(C_2 = 1 \mid D_2 = 0, D_3 = 1)$? Explain your intuition in terms of the car positions with respect to the observations.

1c. Answer

i.

$$
\begin{aligned}
P(C_2=1 \mid D_2=0) &= \eta = 0.2000 \\
P(C_2=1 \mid D_2=0, D_3=1) &= 0.4157
\end{aligned}
$$

ii.

From our results, we know $P(C_2 = 1 \mid D_2 = 0) < P(C_2 = 1 \mid D_2 = 0, D_3 = 1)$. We can draw the conclusion that adding the second sensor $D_3 = 1$ increased the probability of $P(C_2) = 1$. The position of a car observed at time step t is also related to the position of the car at time step $t + 1$. So the observation of $D_3 = 1$ increased the probability of $C_2 = 1$.

iii.

Set $\epsilon = 0$. Hence $P(C_t \mid C_{t-1}) = 1, when\, C_t = C_{t-1}$. Thus the value of D_3 is only related to D_3 with parameter η, and we don't need to consider the transition probability of $P(C_3 \mid C_2)$.

You can also derive the above conclusion from the formula.

PROBLEM 2: Emission probabilities

2a. Check my Code.

PROBLEM 3: Transition probabilities

3a. Check my Code.

PROBLEM 4: LEARNING TO PLAY BLACKJACK

4a. Check my Code.