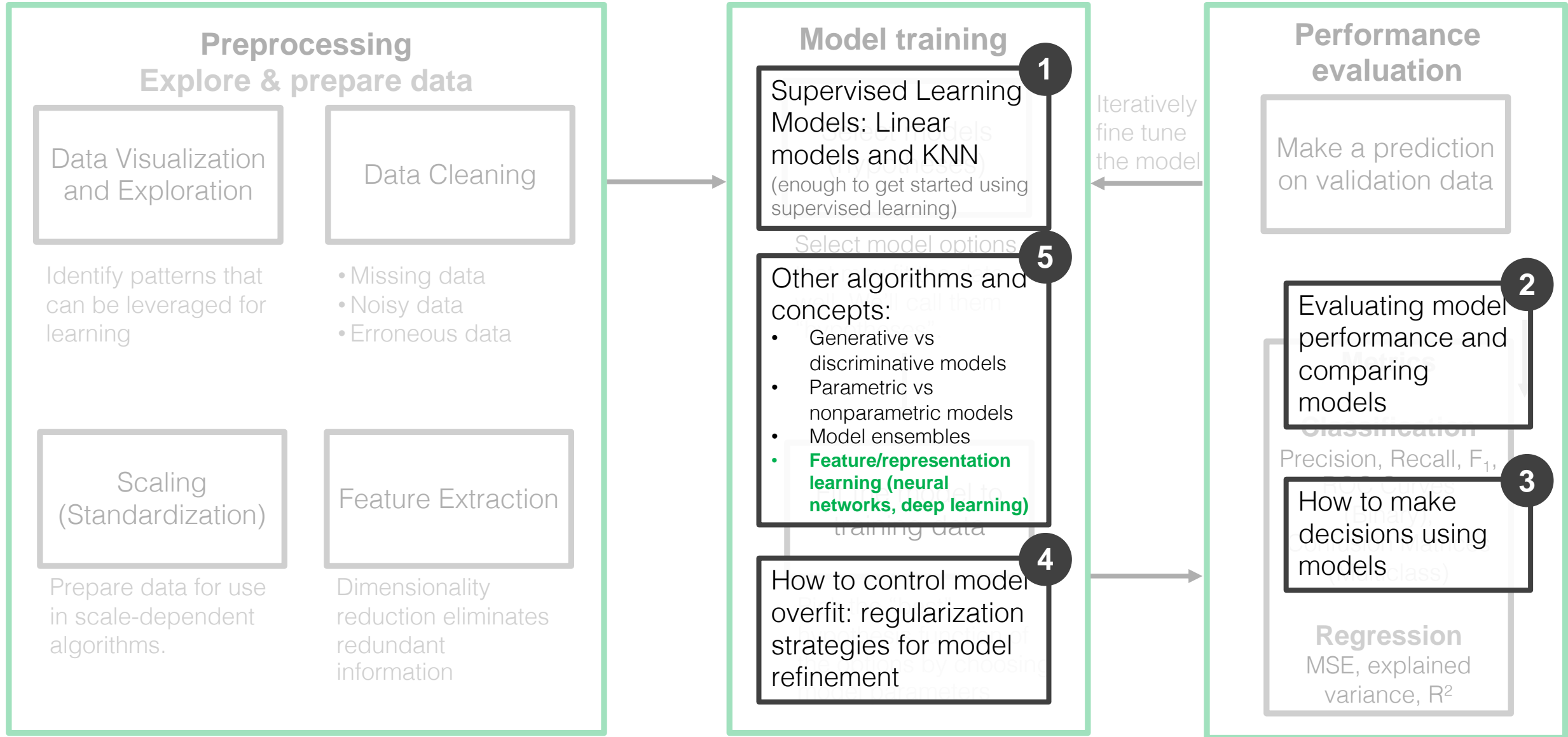


# Neural Networks I

# Supervised learning in practice



# What's the hype around neural networks?

Character/handwriting recognition

Self-driving cars

Natural language processing and translation

Medical devices, diagnosis, and treatment


Materials development


Automated financial trading systems


Industrial automation

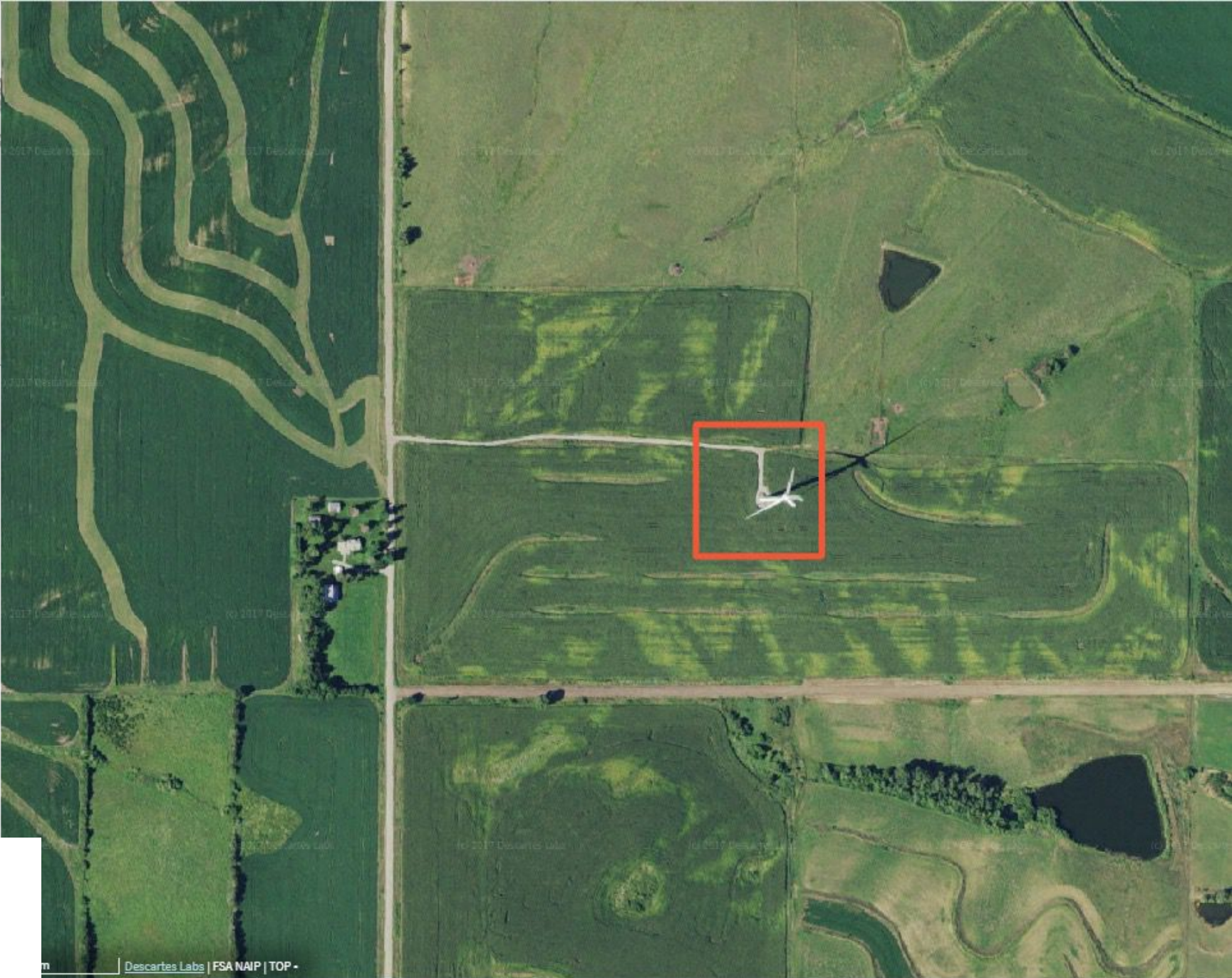

Computer vision applications...



Search Aerial Imagery (NAIP) 



Top 1000 Closest Matches:  [Clear Results](#)



# Geovisual Search

<https://search.descarteslabs.com/>

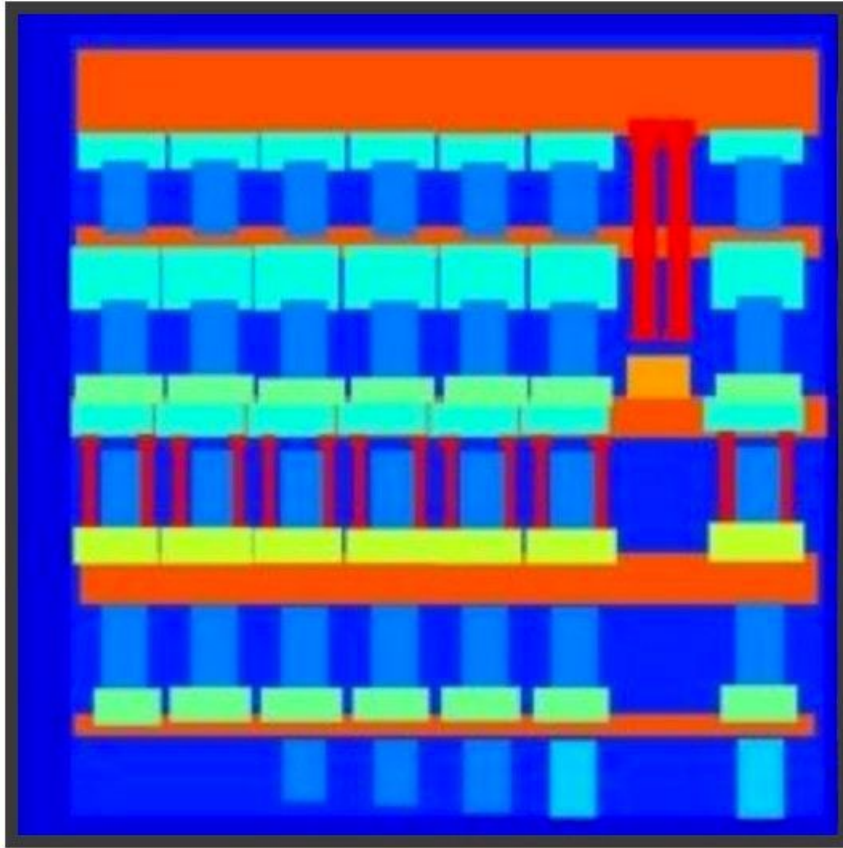


# Image-to-image translation

TOOL

- background
- wall
- door
- window**
- window sill
- window head
- shutter
- balcony
- trim
- cornice
- column
- entrance

INPUT



pix2pix

process

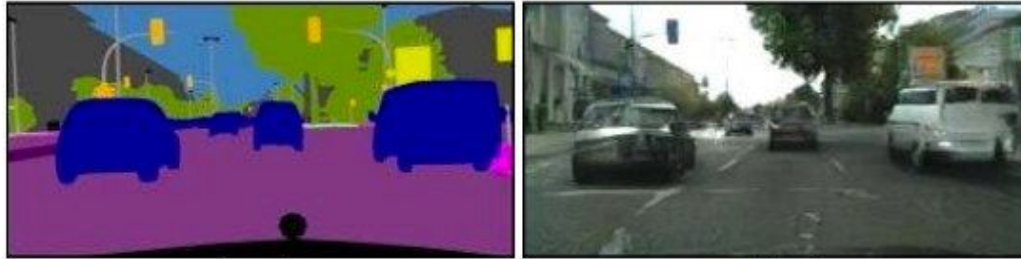
OUTPUT



Isola, Phillip, et al. "Image-to-image translation with conditional adversarial networks." arXiv preprint (2017).

# Image-to-image translation

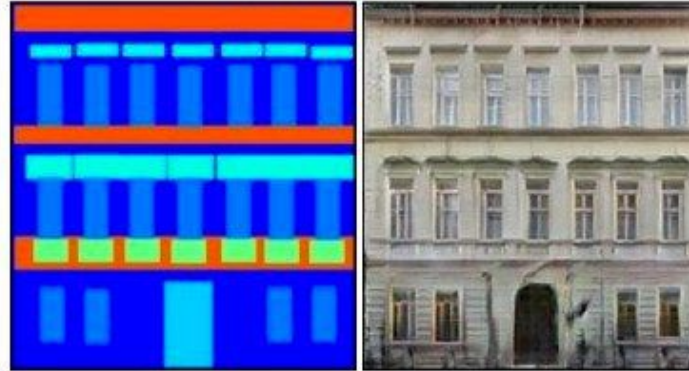
Labels to Street Scene



input

output

Labels to Facade



input

output

BW to Color



input

output

Aerial to Map



input

output

Day to Night



input

output

Edges to Photo



input

output

Isola, Phillip, et al. "Image-to-image translation with conditional adversarial networks." arXiv preprint (2017).



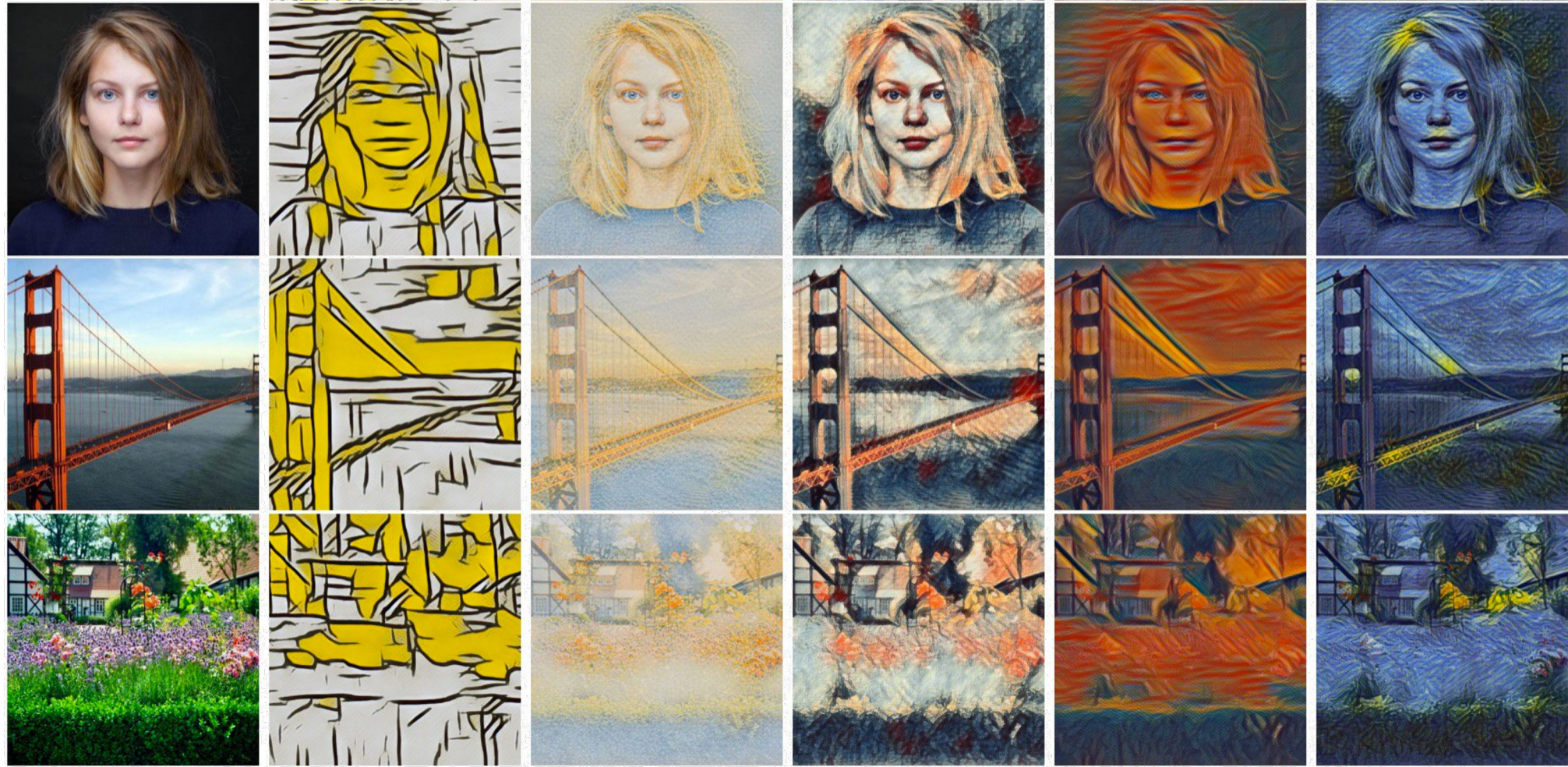
# Image-to-image translation



Isola, Phillip, et al. "Image-to-image translation with conditional adversarial networks." arXiv preprint (2017).



# Image Style Transfer

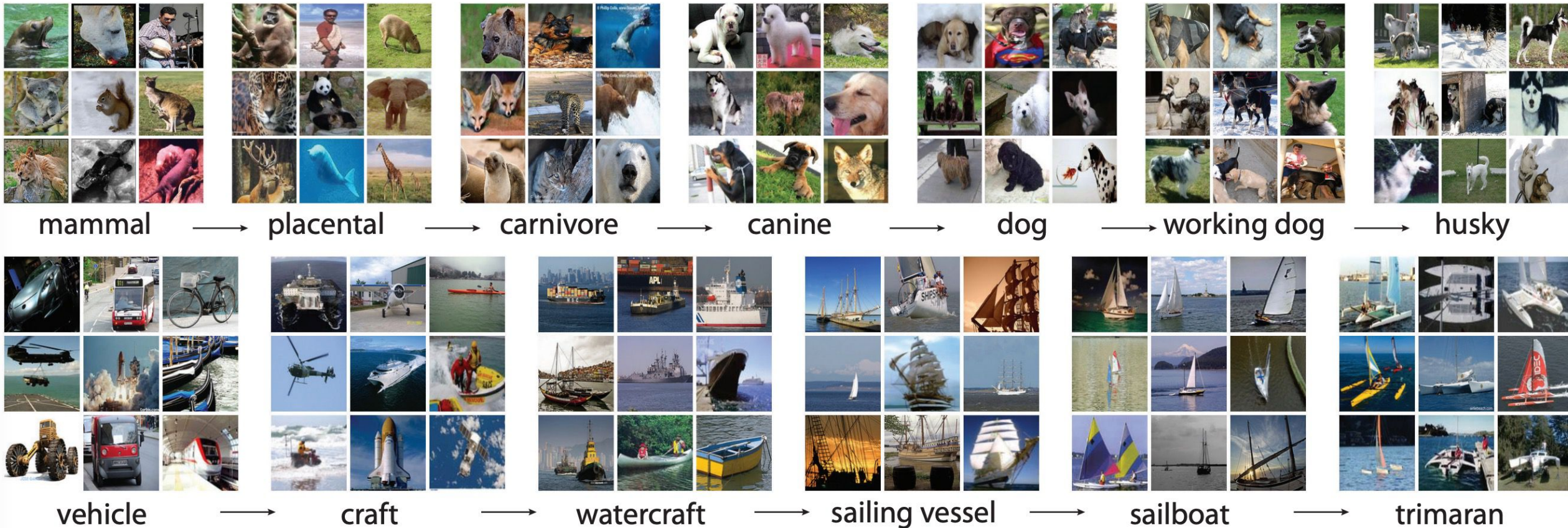


Dumoulin, Vincent, Jonathon Shlens, and Manjunath Kudlur. "A learned representation for artistic style." CoRR, abs/1610.07629 2.4 (2016): 5.



# ImageNet Competition

- Image classification challenge
- 14,197,122 annotated images
- 1,000 classes

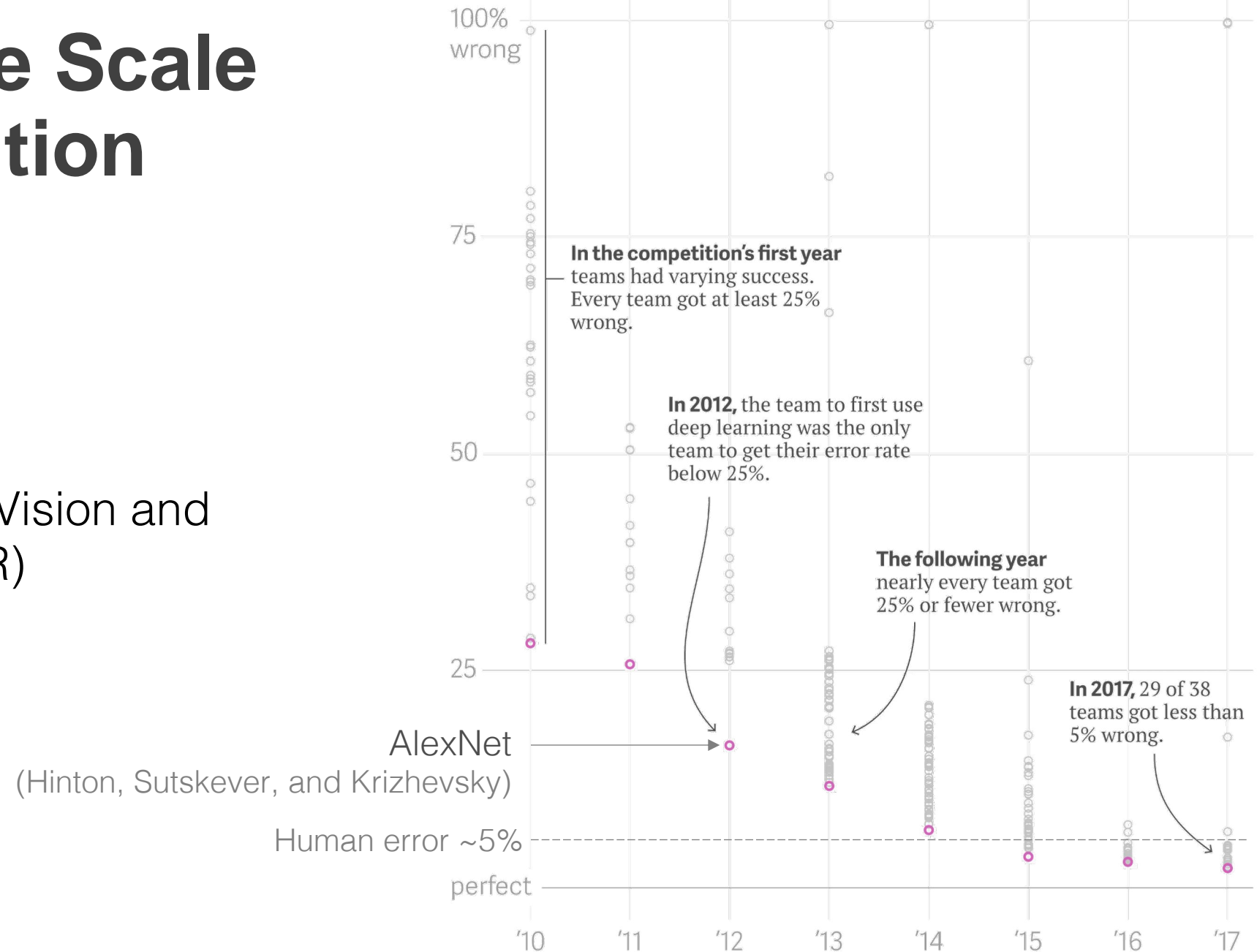


Deng, J., Dong, W., Socher, R., Li, L.J., Li, K. and Fei-Fei, L., 2009, June. Imagenet: A large-scale hierarchical image database. In 2009 IEEE conference on computer vision and pattern recognition (pp. 248-255). Ieee.

# ImageNet Large Scale Visual Recognition Challenge

Fei-Fei Li et al. 2010 ([link](#))

Competition at:  
Conference on Computer Vision and  
Pattern Recognition (CVPR)



Source: Quartz, [link](#) David Yanofsky | Quartz

Data: ImageNet



# What makes neural networks special?

# Neural network learning is representation learning

Previous ML algorithms we discussed required us to manually determine feature transformations

Neural networks **learn** feature transformations

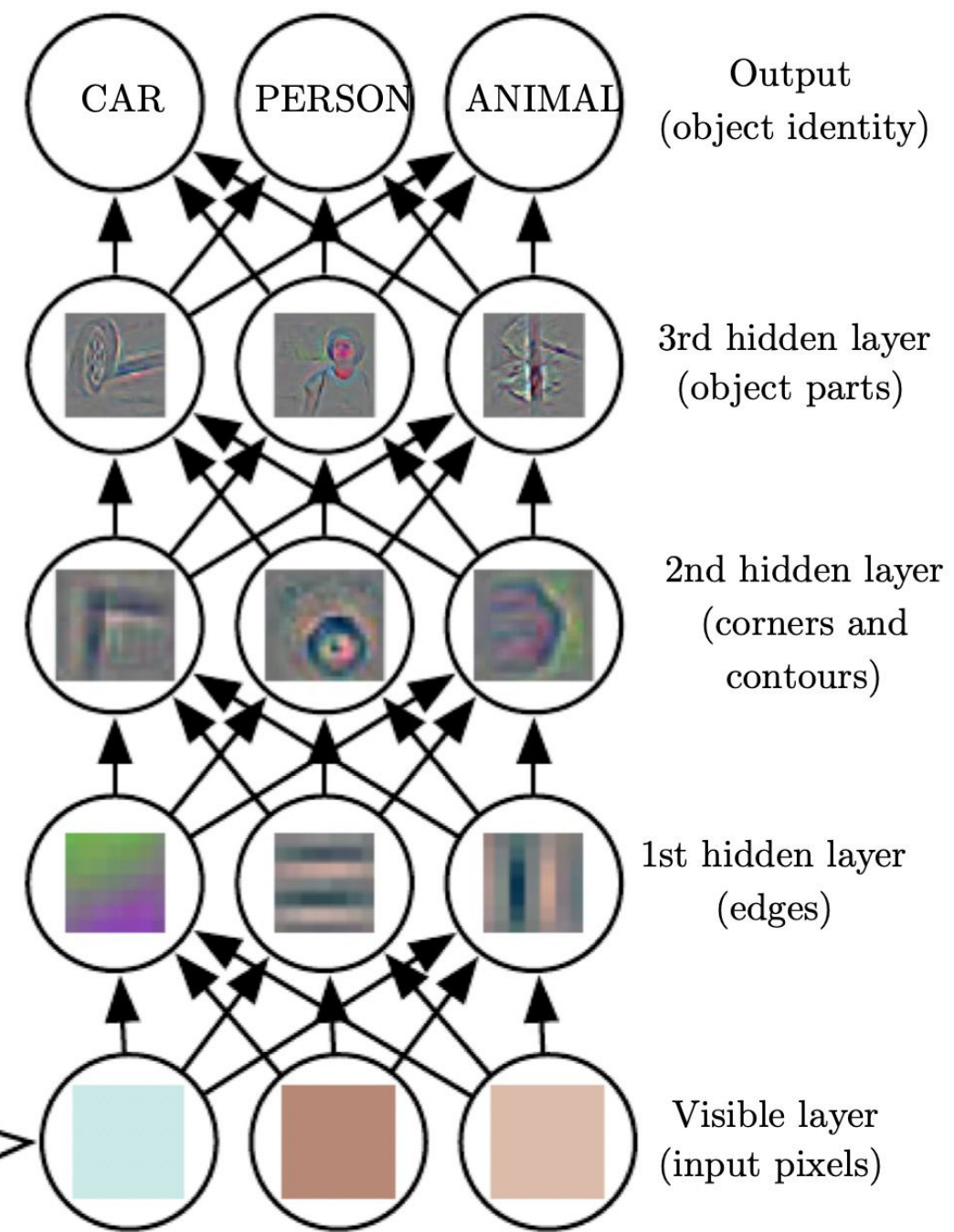


Image from Goodfellow, I., Bengio, Y., Courville, A. and Bengio, Y., 2016.  
Deep learning (Vol. 1, No. 2). Cambridge: MIT press.

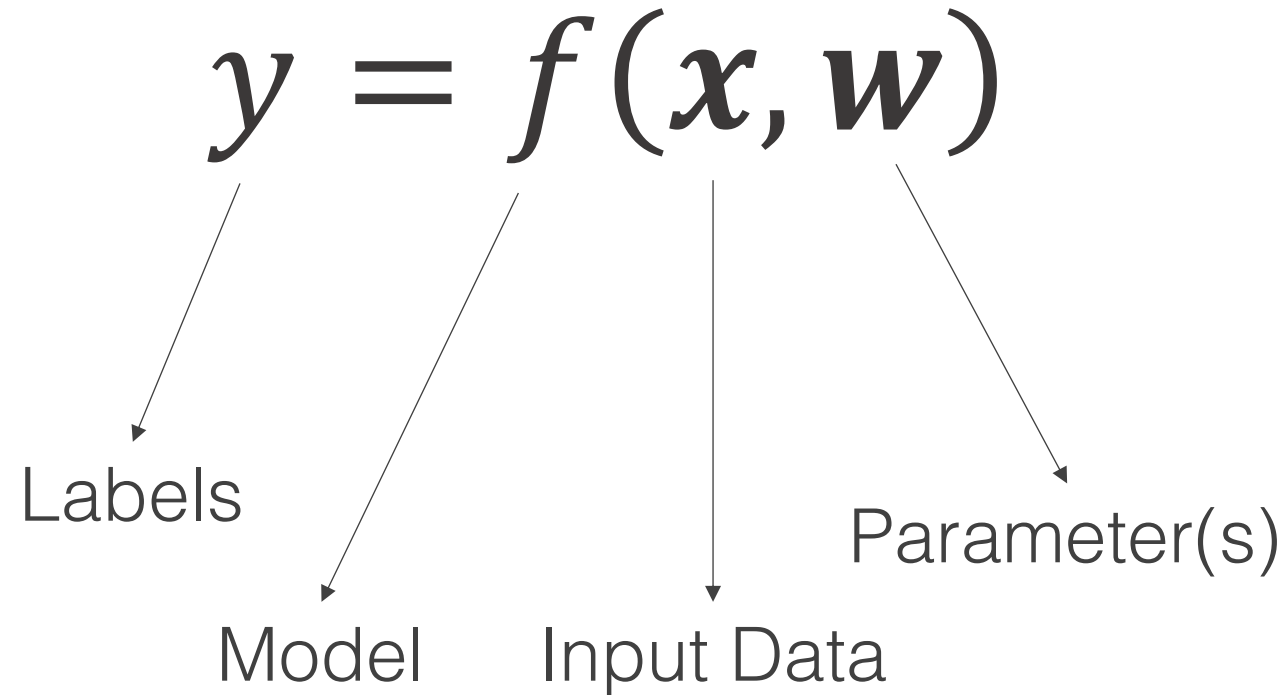


What is a neural network and **how does it work?**

How do we **optimize model weights?**  
(i.e. how do we fit our model to data)

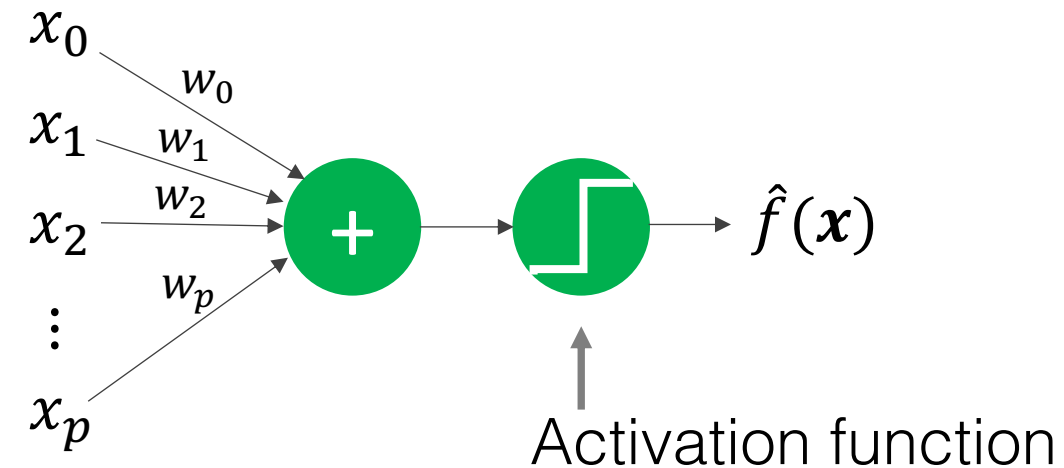
What are the challenges of using neural networks?

# Recall our goal in supervised learning



# Perceptron

$$\hat{f}(x) = \text{sign} \left( \sum_{i=0}^p w_i x_i \right)$$

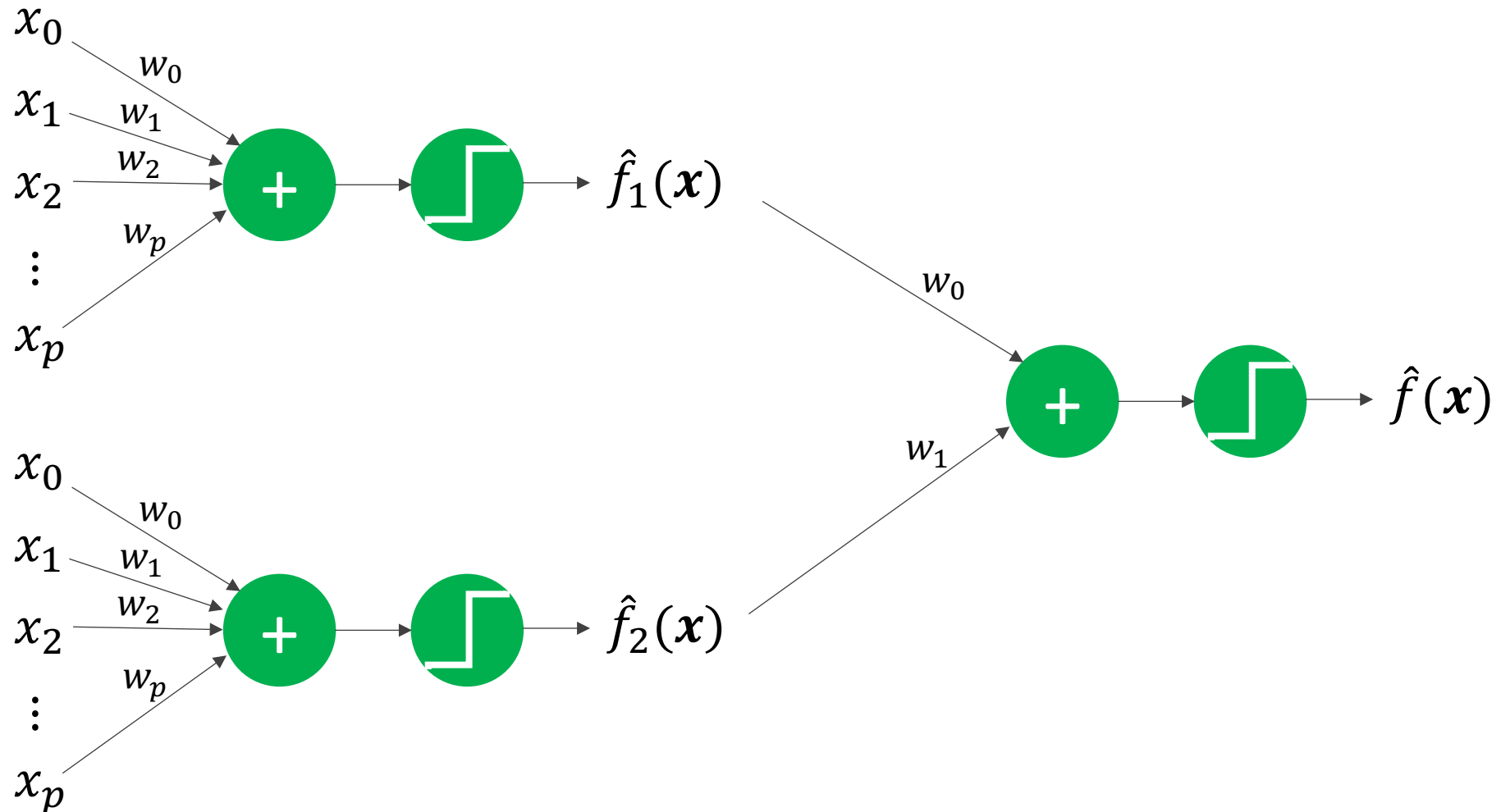


Source: Abu-Mostafa, Learning from Data, Caltech

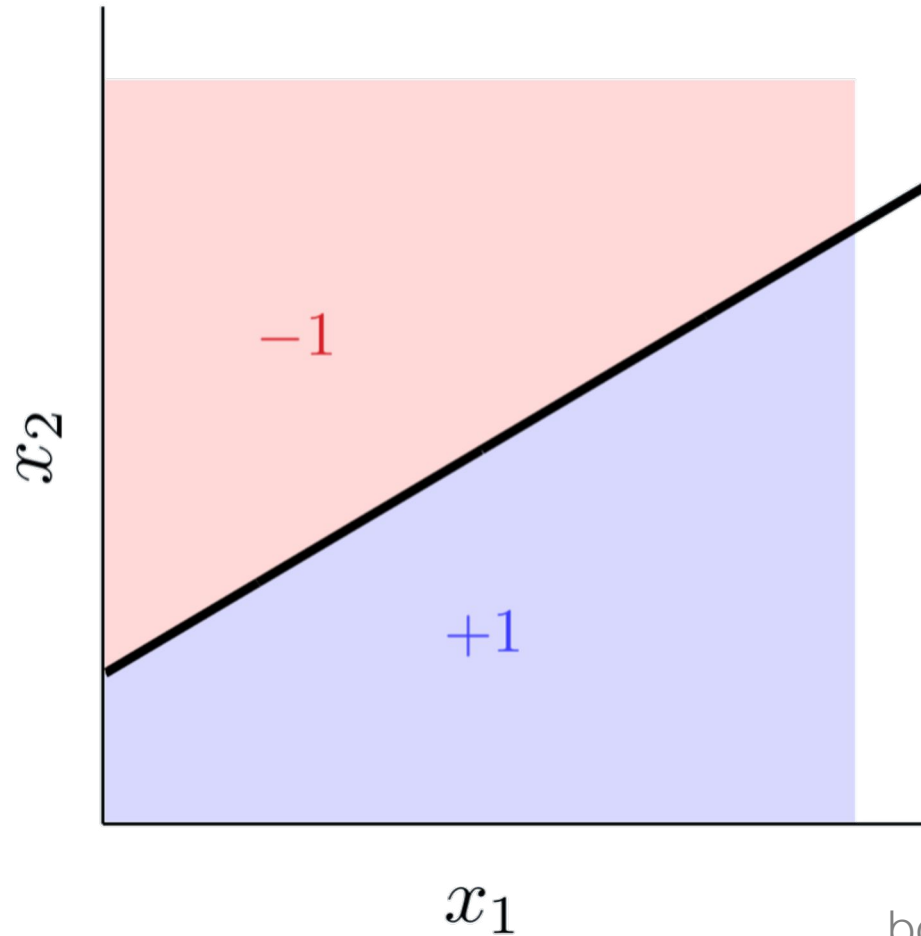


# Multilayer Perceptron

What if we stuck multiple perceptrons together?



# Perceptron #1

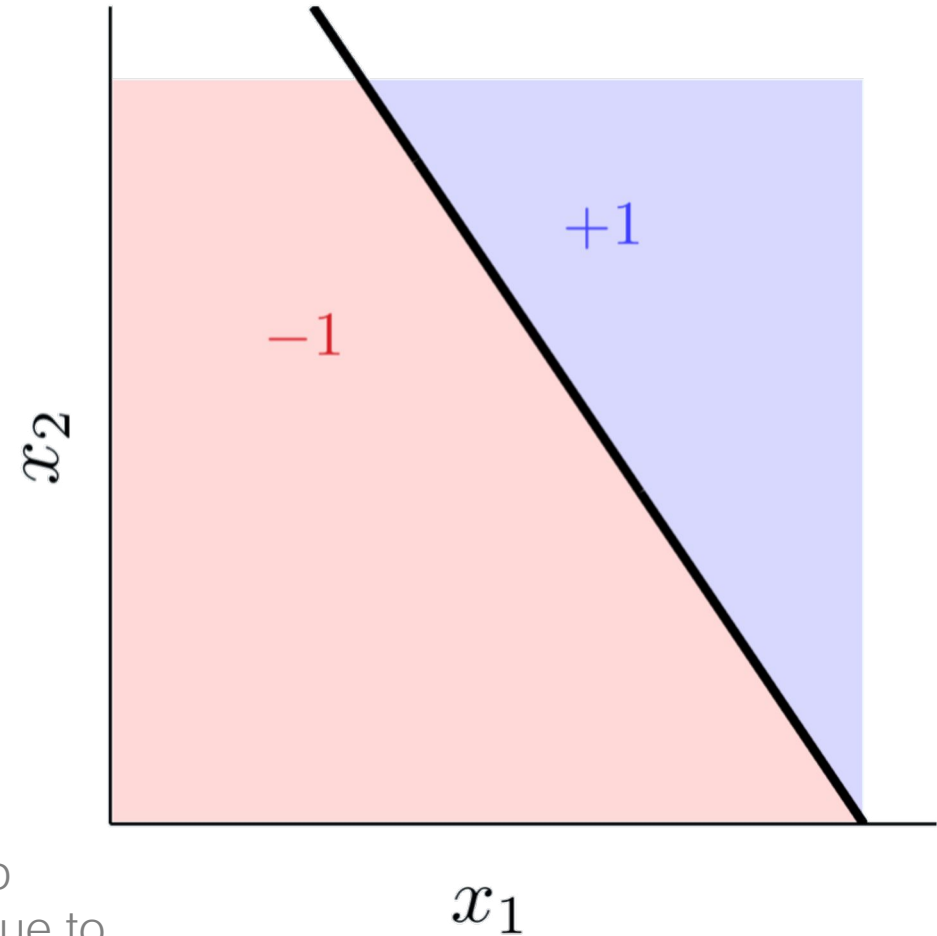


$$\hat{f}_1(\mathbf{x}) = \text{sign}(\mathbf{w}_1^T \mathbf{x})$$

The sharp  
boundary is due to  
our sign function



# Perceptron #2



$$\hat{f}_2(\mathbf{x}) = \text{sign}(\mathbf{w}_2^T \mathbf{x})$$

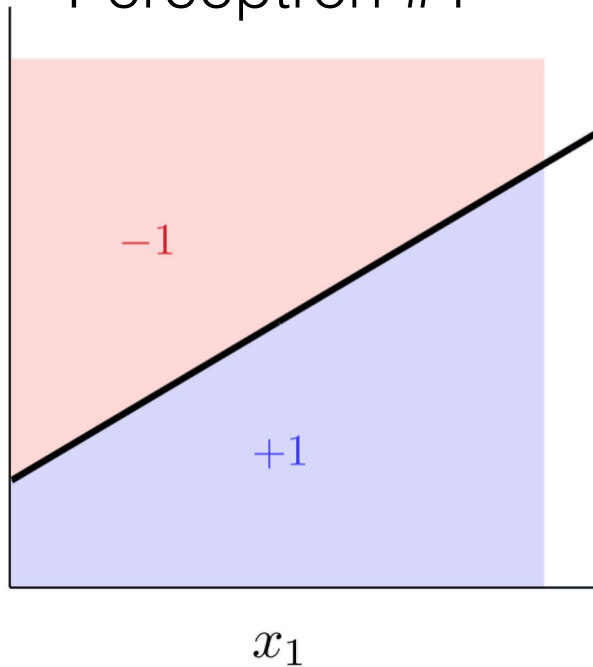
Source: Abu-Mostafa, Learning from Data, Caltech



# Multilayer perceptron:

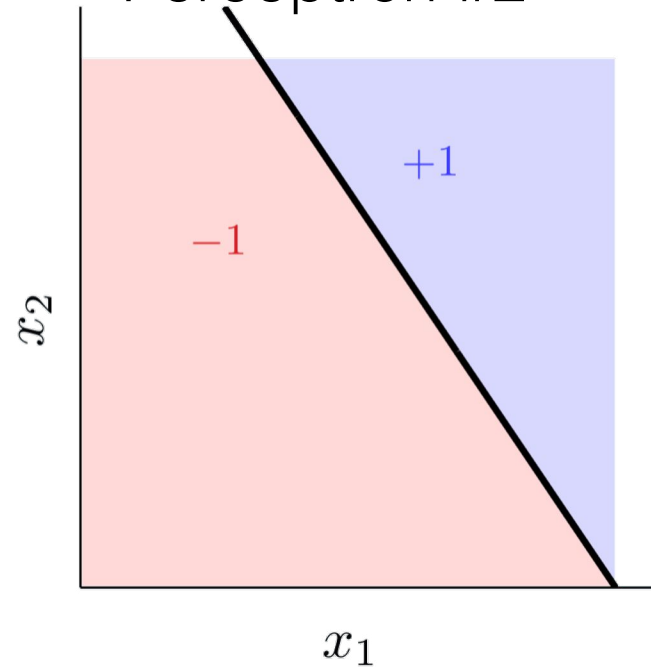
$$\hat{f}(\mathbf{x}) = \begin{cases} +1, & \hat{f}_1(\mathbf{x}) \neq \hat{f}_2(\mathbf{x}) \\ -1, & \hat{f}_1(\mathbf{x}) = \hat{f}_2(\mathbf{x}) \end{cases}$$

Perceptron #1

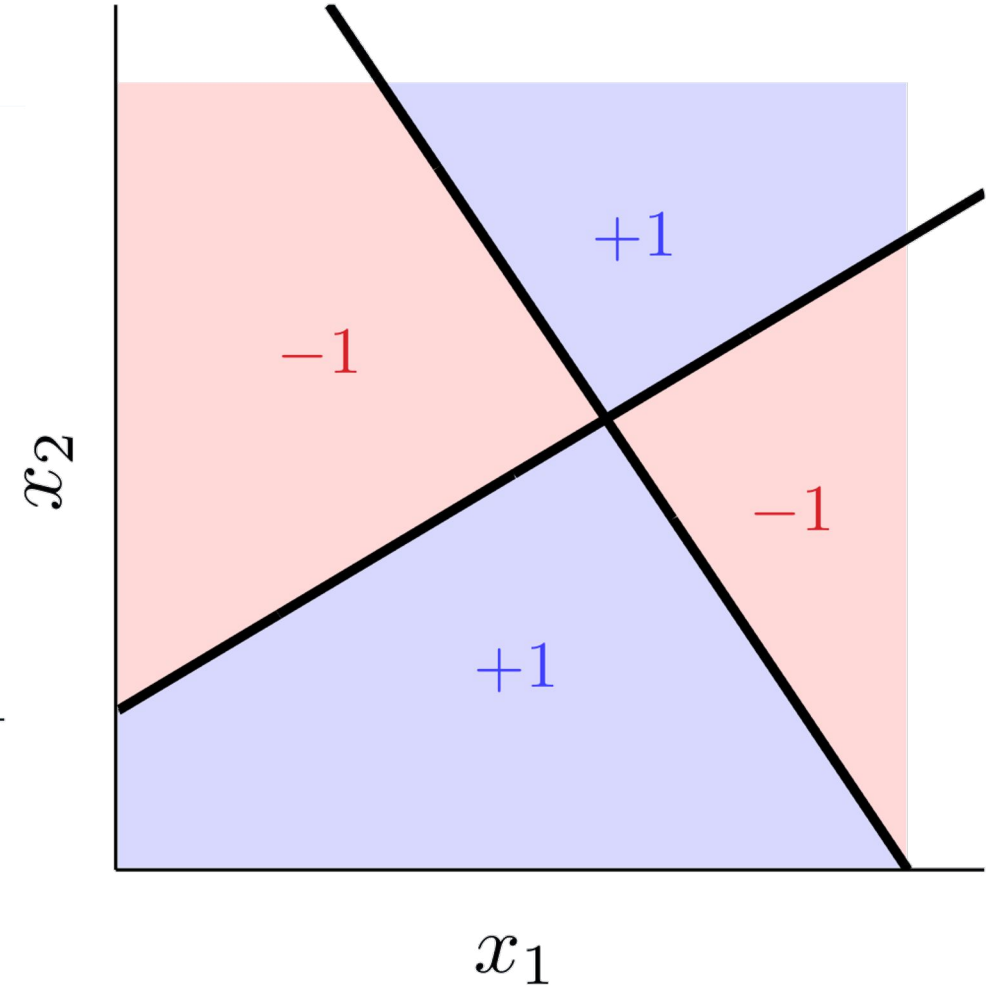


$$\hat{f}_1(\mathbf{x}) = \text{sign}(\mathbf{w}_1^T \mathbf{x})$$

Perceptron #2

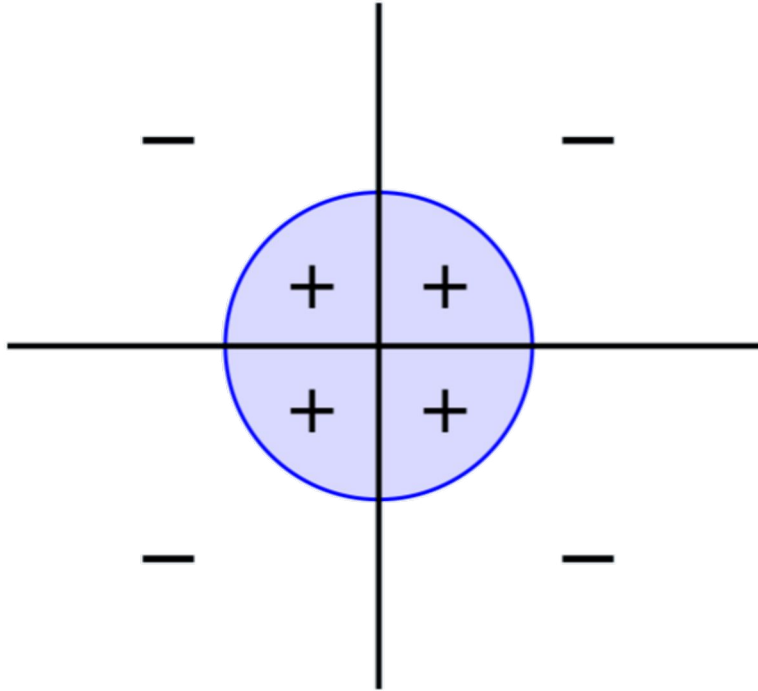


$$\hat{f}_2(\mathbf{x}) = \text{sign}(\mathbf{w}_2^T \mathbf{x})$$

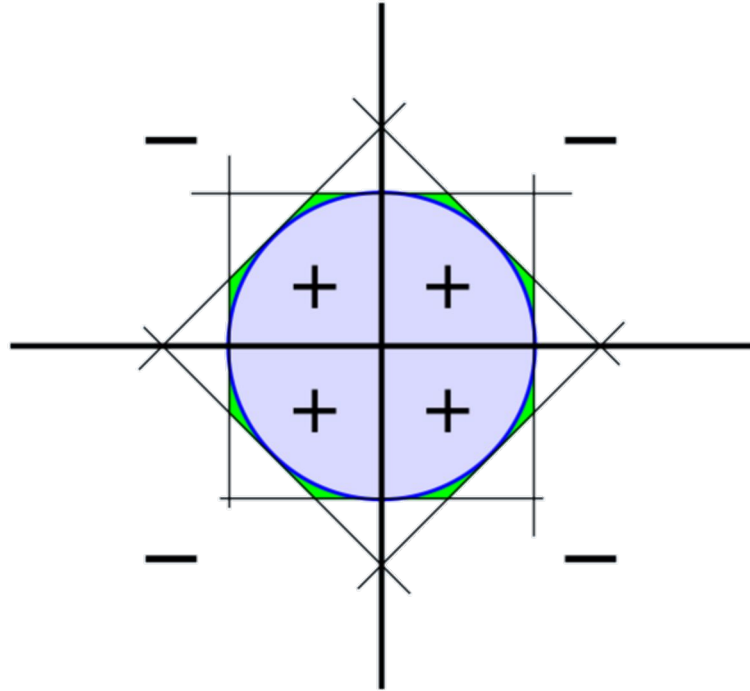


Source: Abu-Mostafa, Learning from Data, Caltech

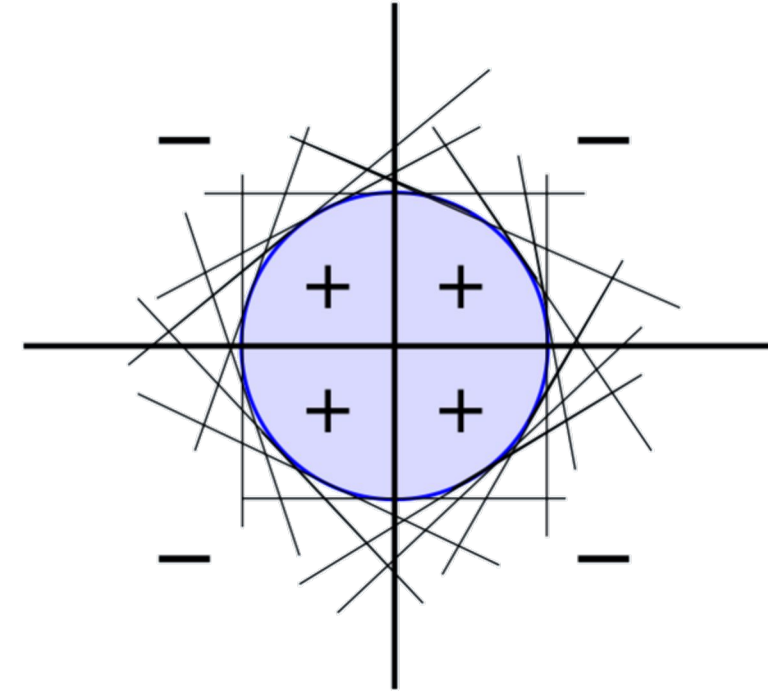
# Multilayer Perceptron



Target



8 perceptrons



16 perceptrons

The more nodes/neurons, the more flexible is the model

Source: Abu-Mostafa, Learning from Data, Caltech

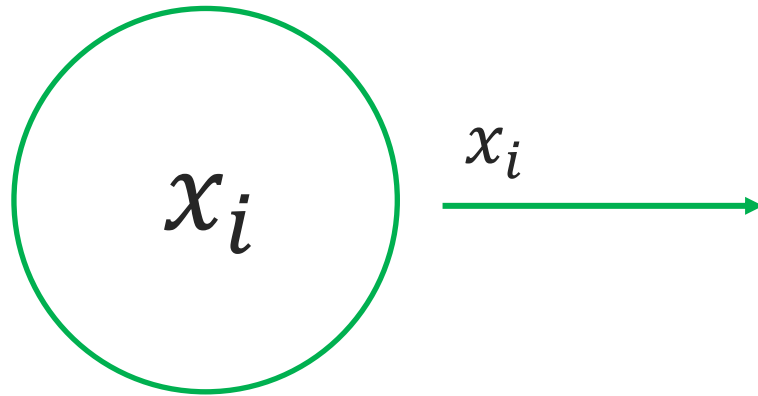


# Universal function approximation

“A **feedforward network** with a single layer is sufficient to represent **any function**, but the layer may be infeasibly large and may fail to learn and generalize correctly.”

Ian Goodfellow, Deep Learning  
Creator of generative adversarial networks

# Input nodes / neurons



Simply passes the input value to the next layer

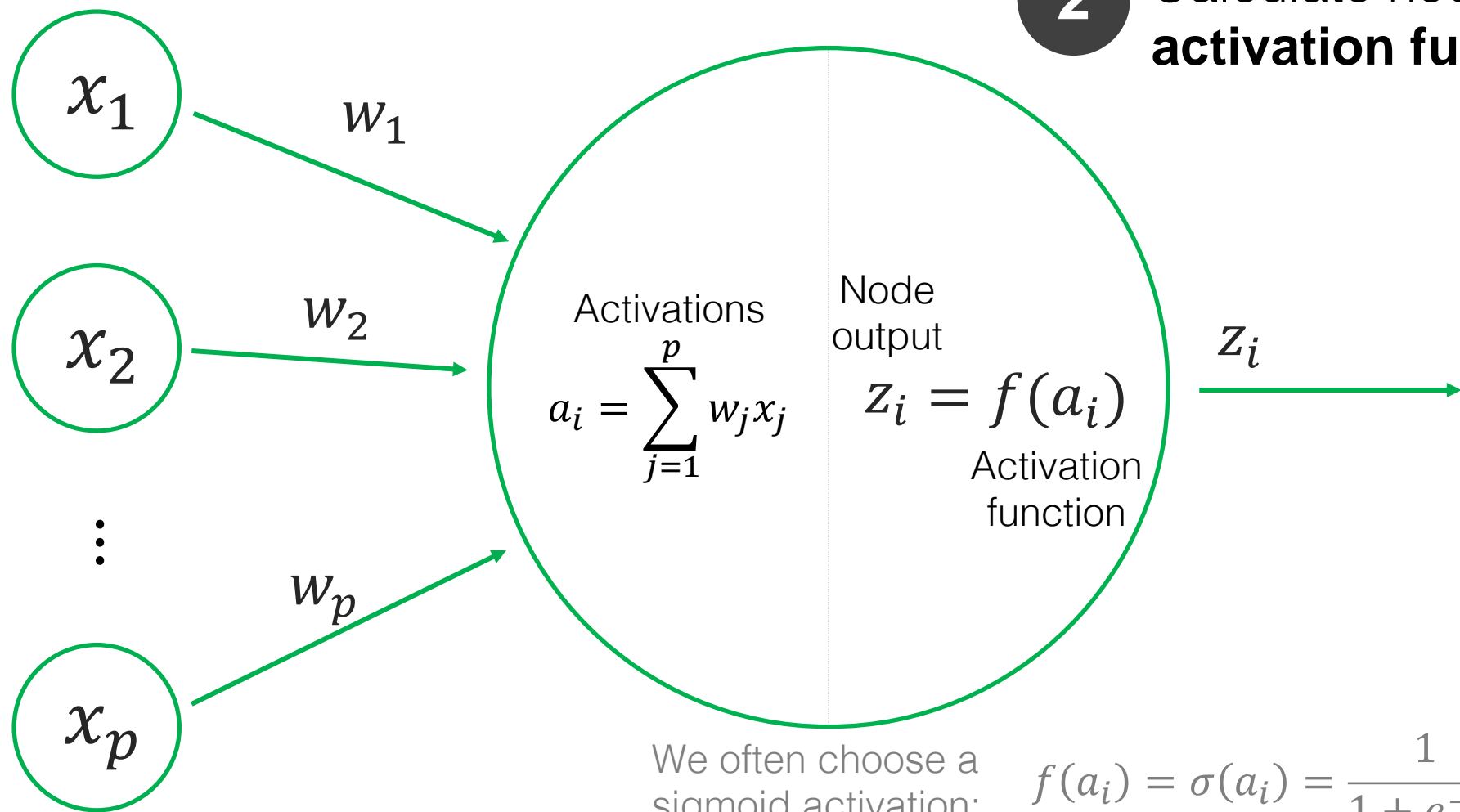
# Hidden & output nodes

1

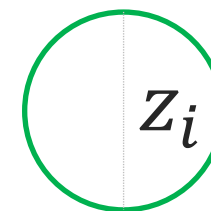
Calculate the **activations**: linear combinations of weights and the last layer's output

2

Calculate node output: apply the **activation function** to the activations



Represented as:

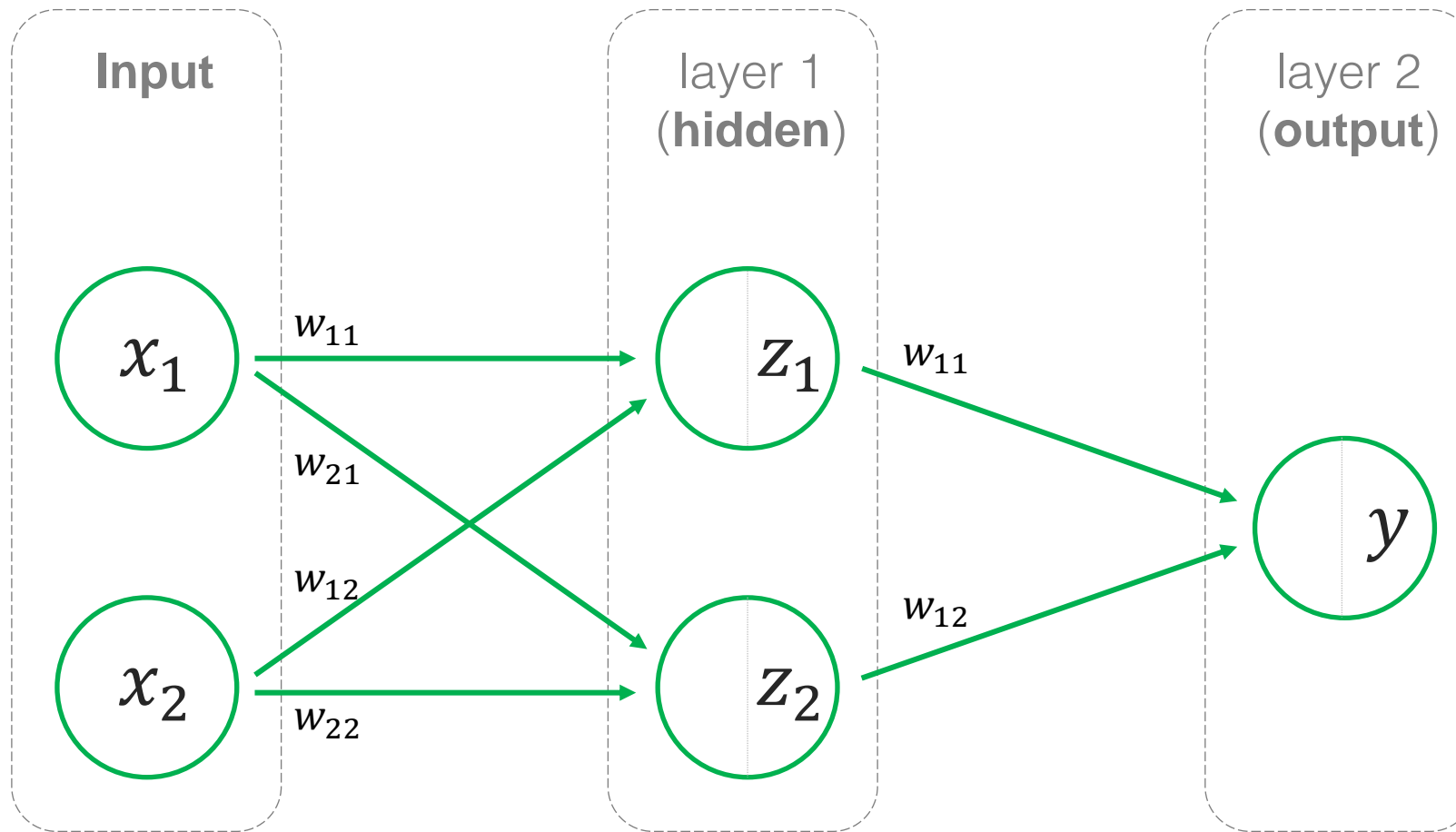


We often choose a sigmoid activation:

$$f(a_i) = \sigma(a_i) = \frac{1}{1 + e^{-a_i}}$$



# Simple Neural Network



**Notational shorthand:**  
(a more precise  
alternative notation)

$$w_{ij} = w_{ij}^{(1)}$$

$$z_i = z_i^{(1)}$$

$$w_{ij} = w_{ij}^{(2)}$$

$$y = z_1^{(2)}$$

$w_{ij}^{(k)}$

- Layer  $k$
- From node  $j$   
(in the last layer)
- to node  $i$   
(in the next layer)

# Forward Propagation

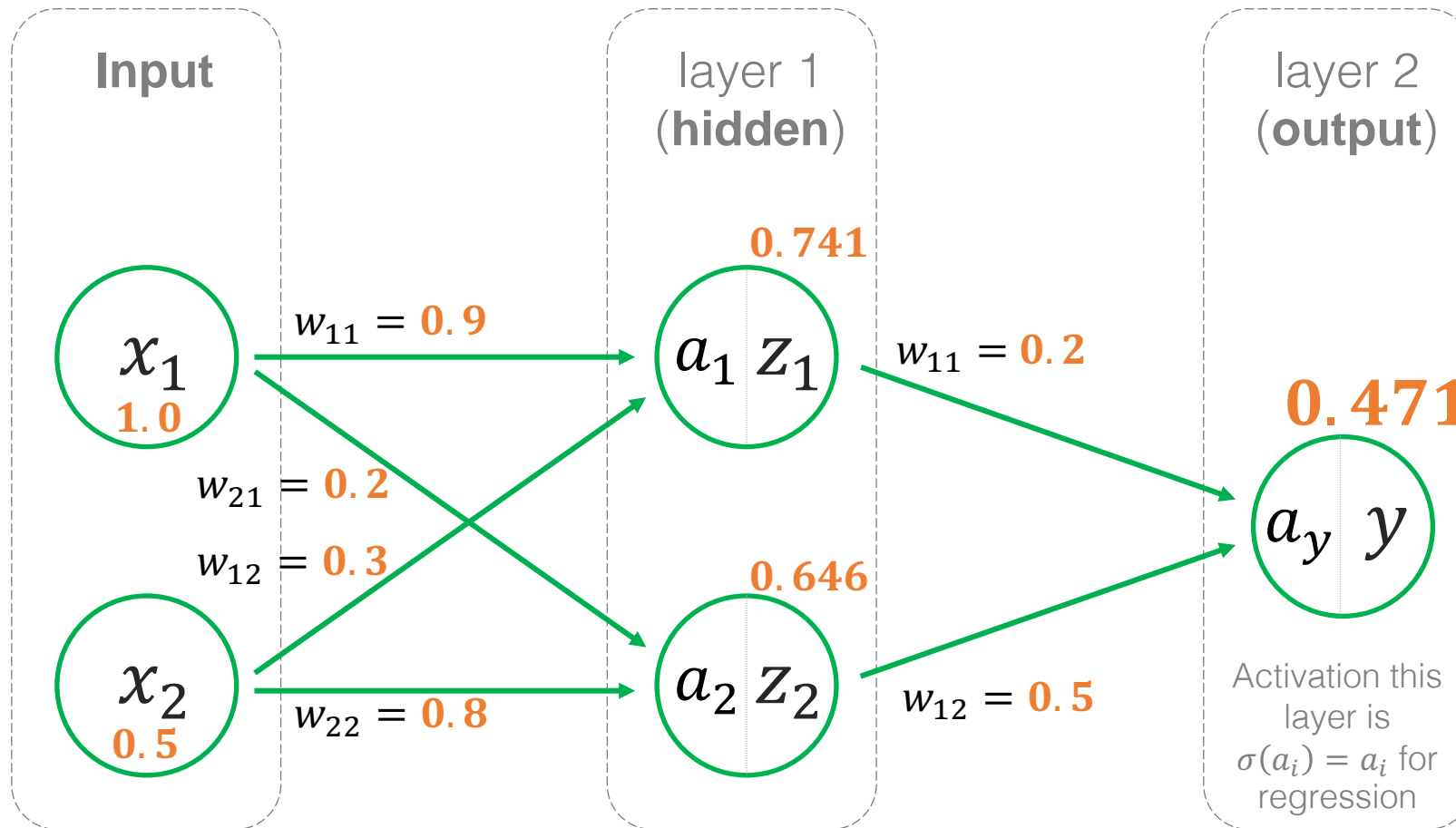
Calculating the output from input

$$a_1 = (0.9)(1.0) + (0.3)(0.5) = 1.05 \quad \text{Hidden layer calculations}$$

$$a_2 = (0.2)(1.0) + (0.8)(0.5) = 0.6$$

$$z_1 = \sigma(a_1) = \sigma(1.05) = 0.741$$

$$z_2 = \sigma(a_2) = \sigma(0.6) = 0.646$$



Output layer calculations

$$a_y = (0.2)(0.741) + (0.5)(0.646) = 0.471$$

$$y = a_y = 0.471 \quad \text{Regression}$$

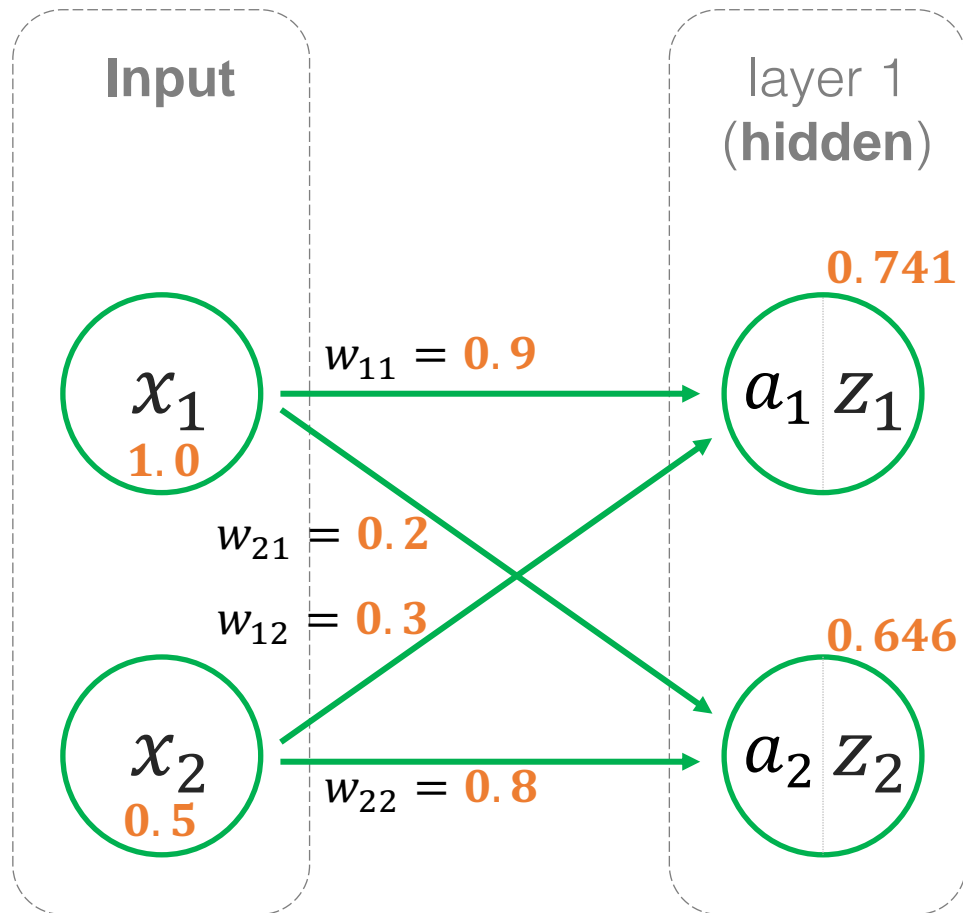
Alternatively...

$$y = \sigma(a_y) = \sigma(0.471) = 0.616 \quad \text{Classification}$$

$$\sigma(a_i) = \frac{1}{1 + e^{-a_i}}$$

# Forward Propagation

Calculating the output from input



## Hidden layer matrix calculations

$$\mathbf{x} = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \quad \mathbf{a} = \begin{bmatrix} a_1 \\ a_2 \end{bmatrix} \quad \mathbf{z} = \begin{bmatrix} z_1 \\ z_2 \end{bmatrix}$$

$$\mathbf{W} = \begin{bmatrix} w_{11} & w_{12} \\ w_{21} & w_{22} \end{bmatrix} \begin{array}{l} \longrightarrow \text{The weights INTO node } z_1 \\ \longrightarrow \text{The weights INTO node } z_2 \end{array}$$

$$\mathbf{a} = \mathbf{W}\mathbf{x} = \begin{bmatrix} w_{11} & w_{12} \\ w_{21} & w_{22} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

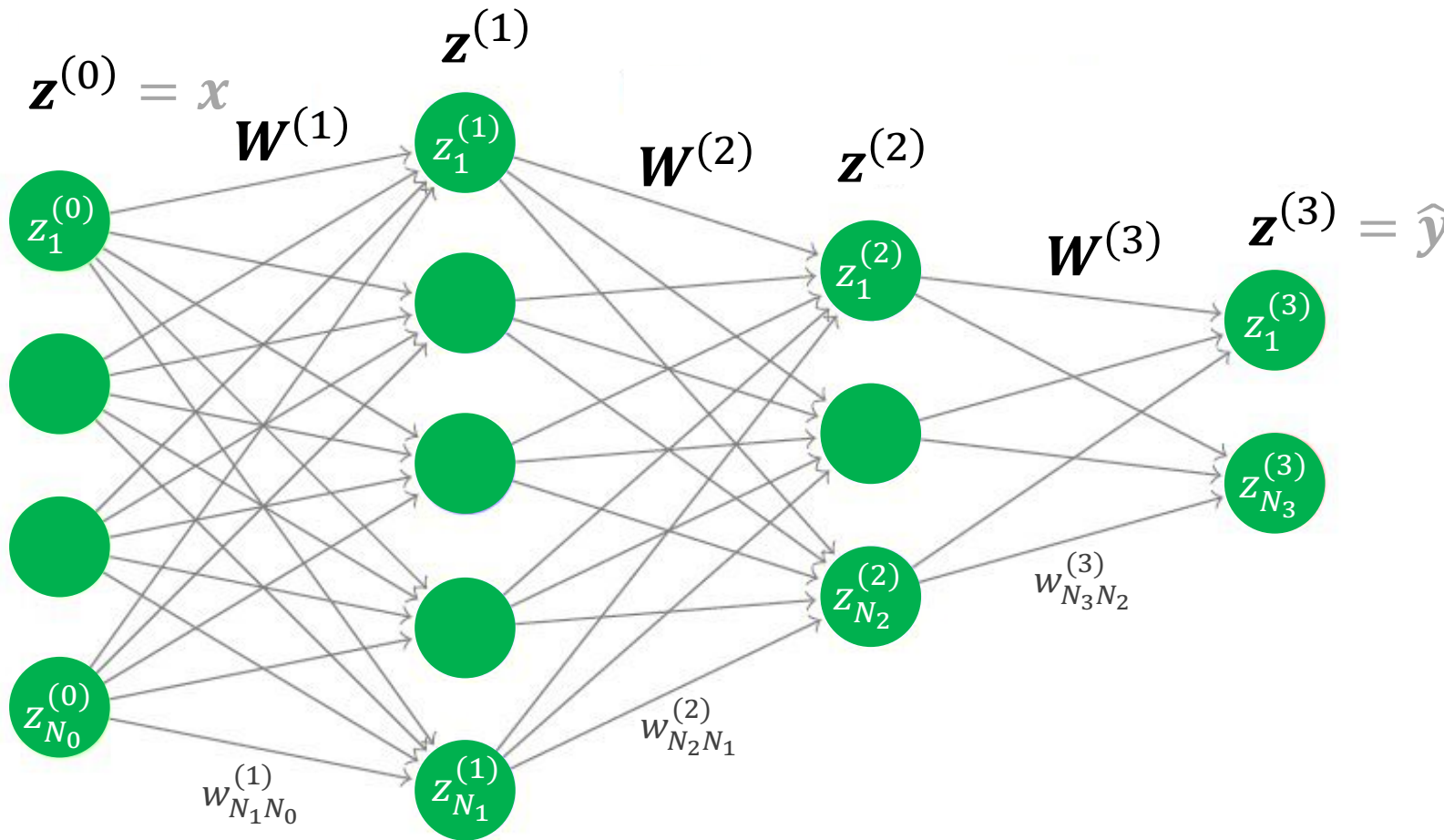
$$= \begin{bmatrix} w_{11}x_1 + w_{12}x_2 \\ w_{21}x_1 + w_{22}x_2 \end{bmatrix}$$

$$\mathbf{z} = \sigma(\mathbf{a}) = \begin{bmatrix} \sigma(w_{11}x_1 + w_{12}x_2) \\ \sigma(w_{21}x_1 + w_{22}x_2) \end{bmatrix}$$



# Forward Propagation

Example neural network with  $L = 3$  layers and the  $i$ th layer has  $N_i$  nodes



Simple steps for forward propagation:

$$\text{For } i = 1 \text{ to } L - 1:$$
$$\mathbf{z}^{(i)} = \sigma(\mathbf{W}^{(i)} \mathbf{z}^{(i-1)})$$

Where:

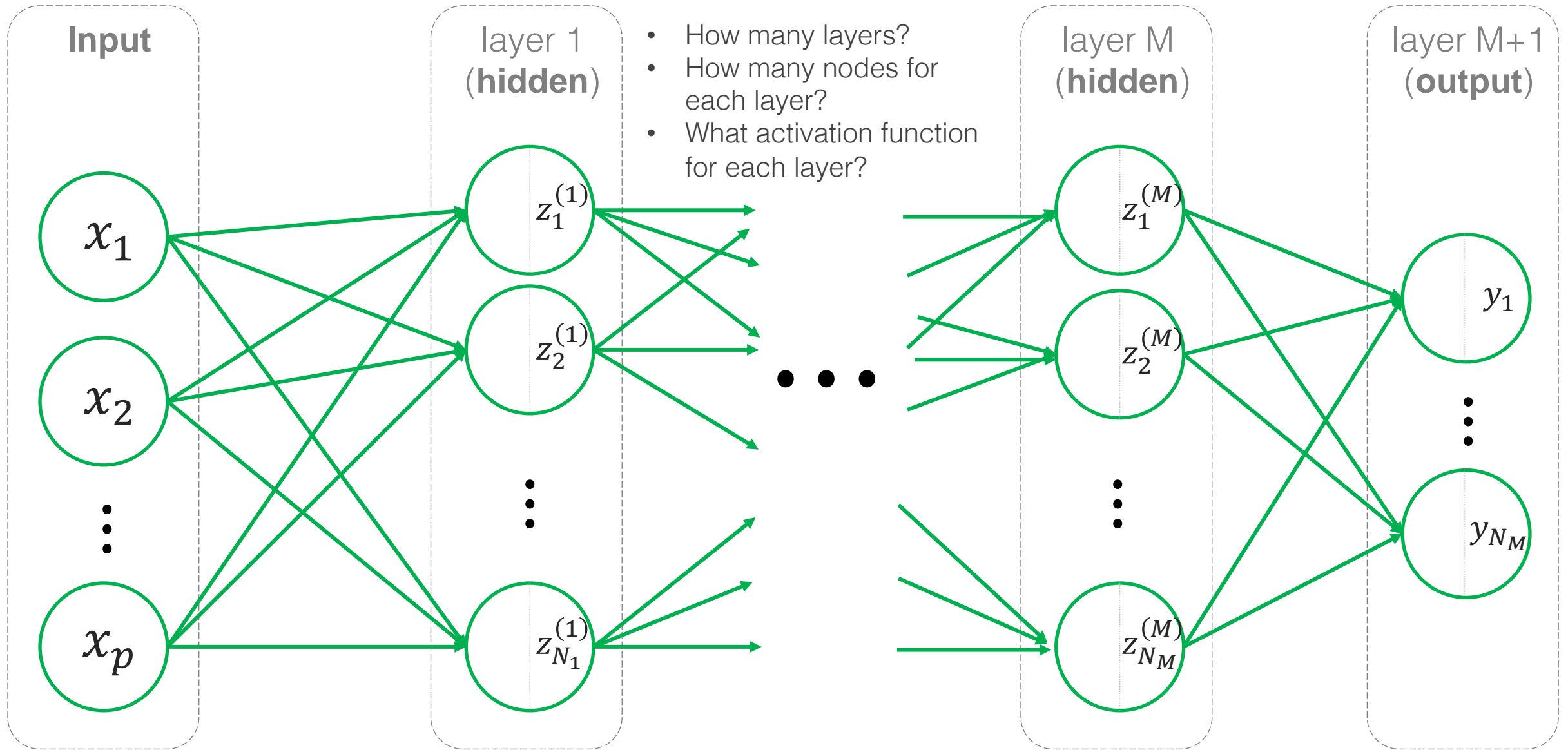
$$\mathbf{z}^{(0)} = \mathbf{x}$$

$$\hat{\mathbf{y}} = \mathbf{z}^{(L)}$$

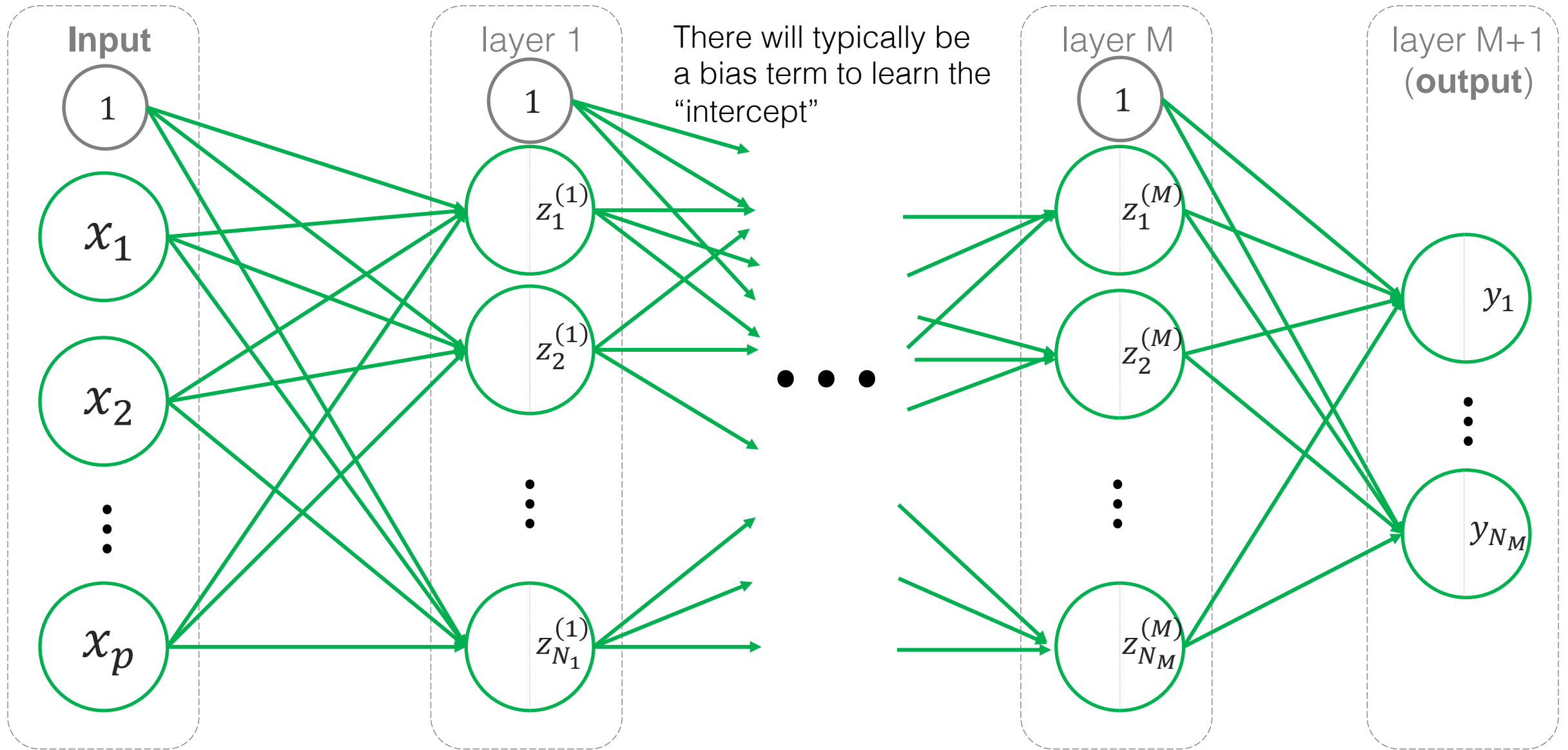
Prediction error is measured:

$$E_n = \frac{1}{2} (\hat{y}_n - y_n)^2$$

# Neural networks can be customized



# Neural networks can be customized



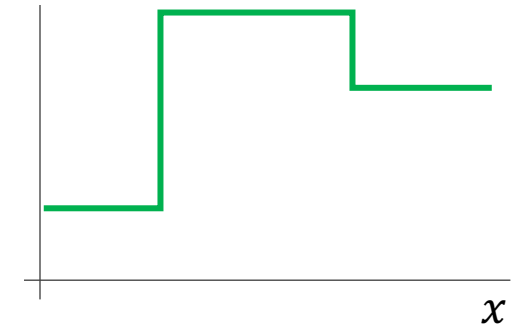
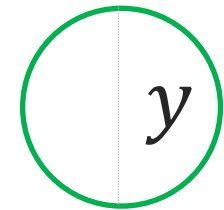
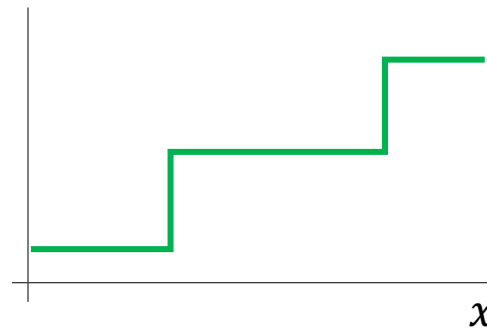
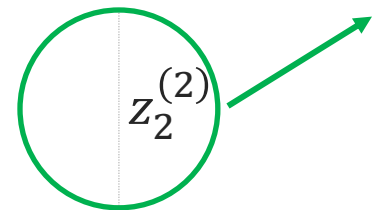
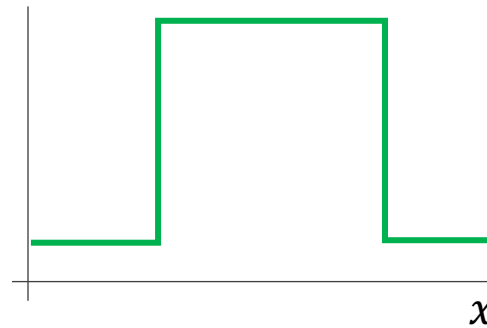
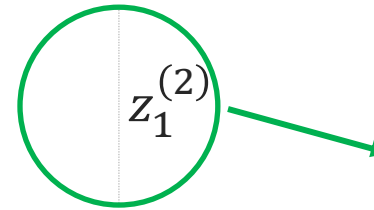
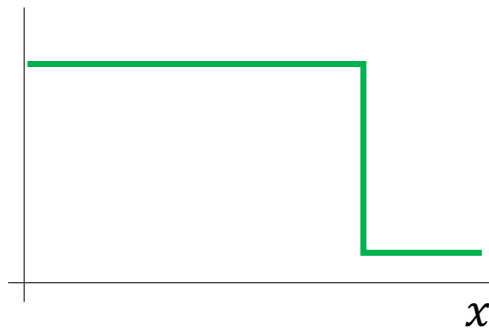
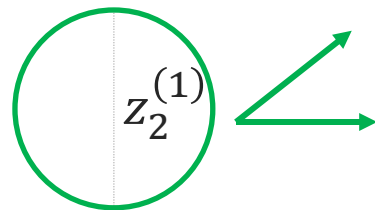
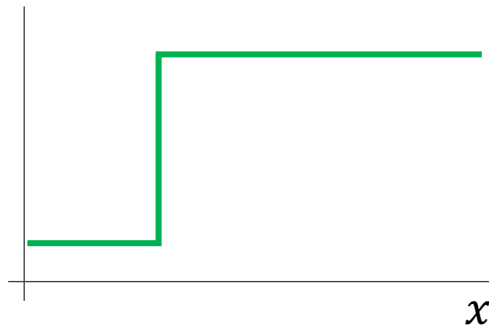
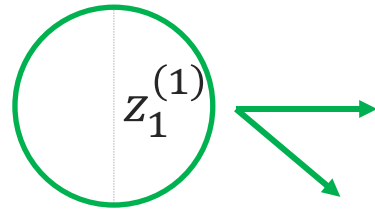
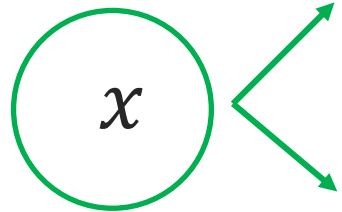


# Input

# Hidden 1

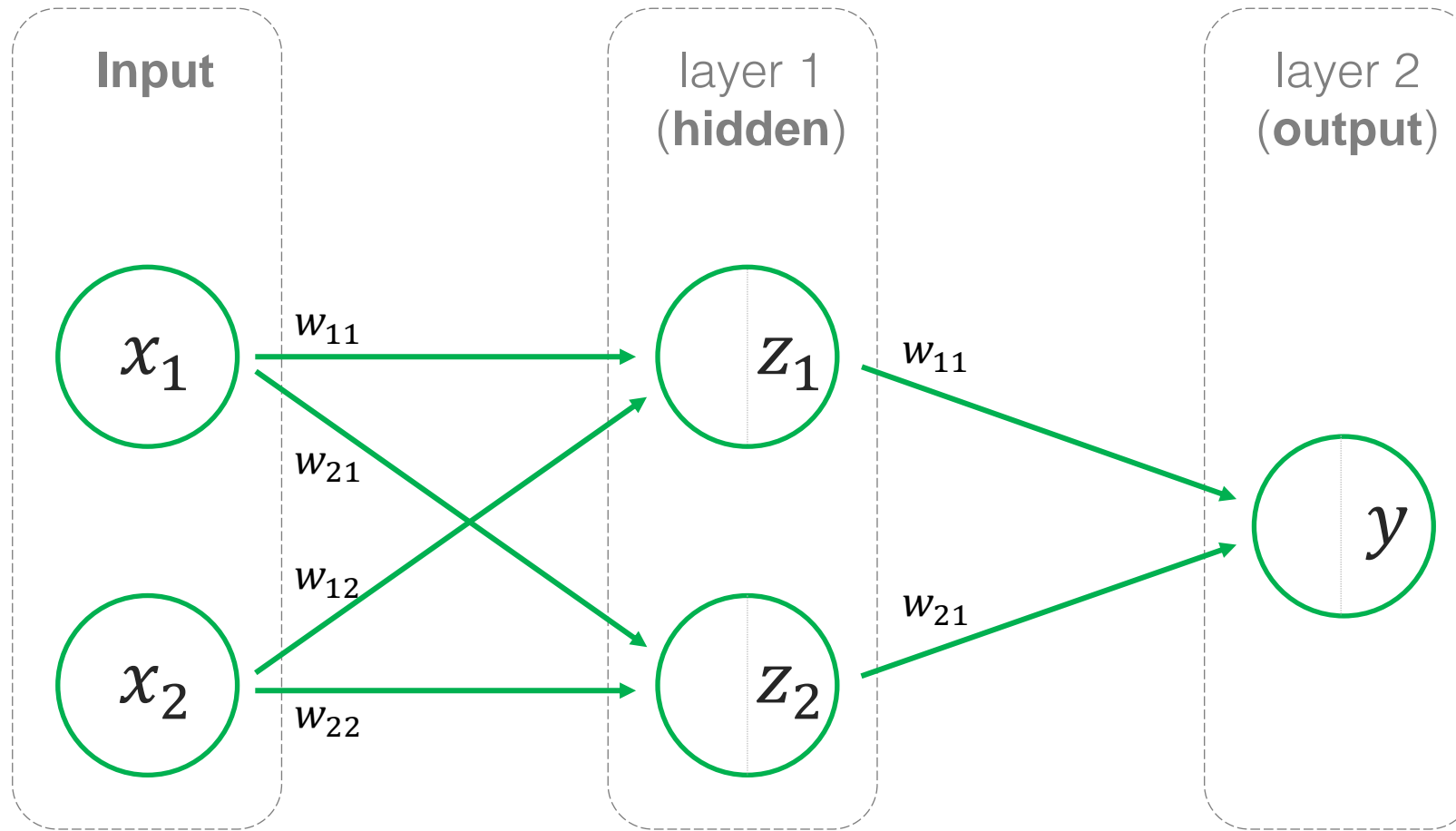
# Hidden 2

# Output



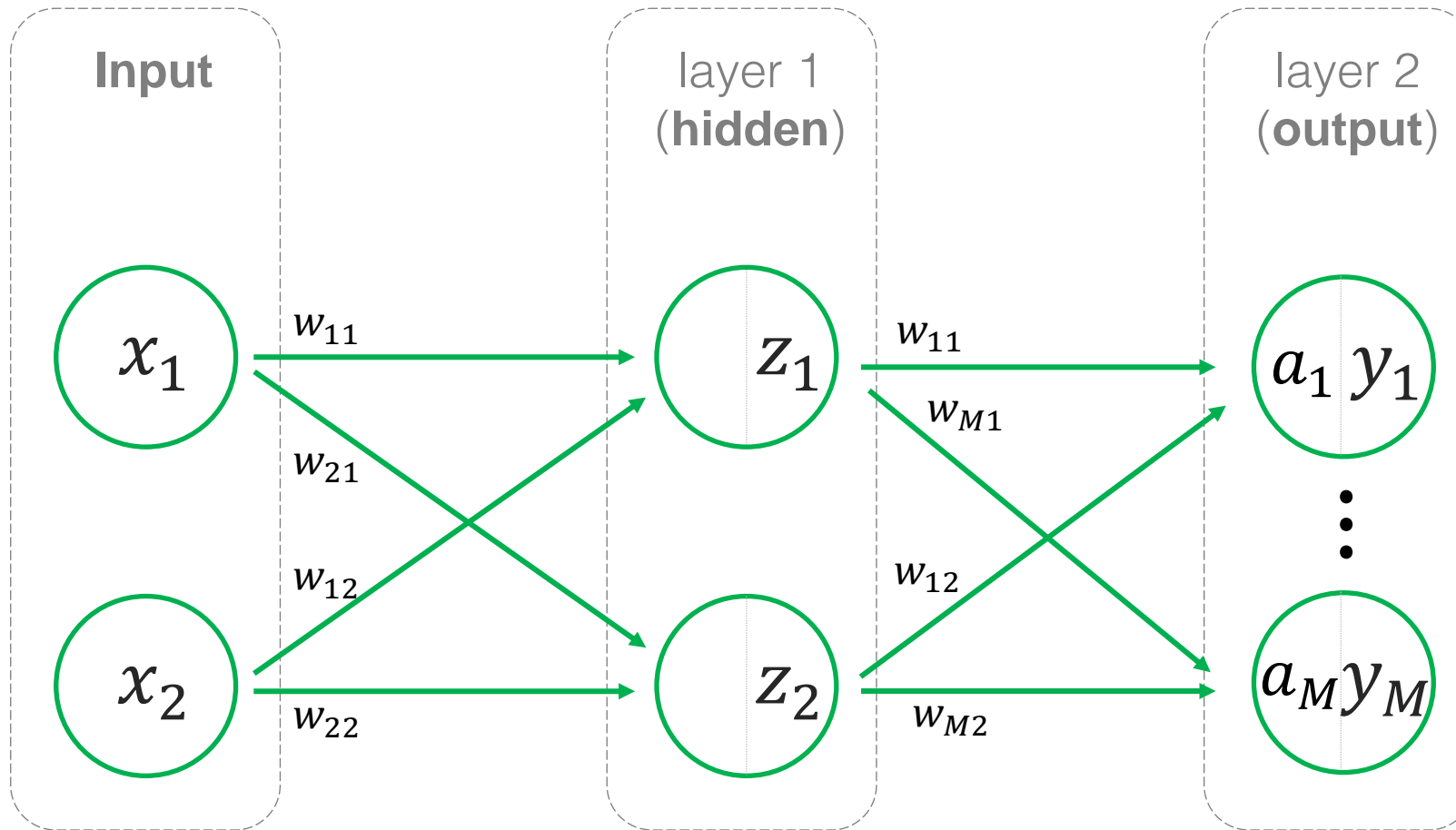
Multilayer neural nets for **regression** can build up from basic building blocks to more complex structures

# From **binary** to multiclass classification



For **binary classification** with a sigmoid activation function, the output is between zero and one, so threshold this value to assign the class

# From binary to **multiclass** classification



For **multiclass problems**, we can have multiple outputs and use a softmax function:

(a generalization of the sigmoid / logistic function)

$$y_i = g(a_i) = \frac{e^{a_i}}{\sum_{n=1}^M e^{a_n}}$$

Choose the largest  $y$  value as the predicted class



# Softmax

Generalization of the logistic function to multiple dimensions

Output activations

$a_i$

$$\begin{bmatrix} 5.1 \\ 4.2 \\ -3.1 \\ 0.7 \end{bmatrix}$$

Activation Function

$\text{softmax}(a_i)$

$$\text{Softmax} \\ \frac{e^{a_i}}{\sum_{n=1}^M e^{a_n}}$$

Output

$y_i$

$$\begin{bmatrix} 0.7046 \\ 0.2865 \\ 0.0002 \\ 0.0087 \end{bmatrix}$$

Always sums to 1  
(normalizes to be a  
probability distribution)

# Next time...

What is a neural network and **how does it work?**

How do we **optimize model weights?**

(i.e. how do we fit our model to data)

What are the challenges of using neural networks?