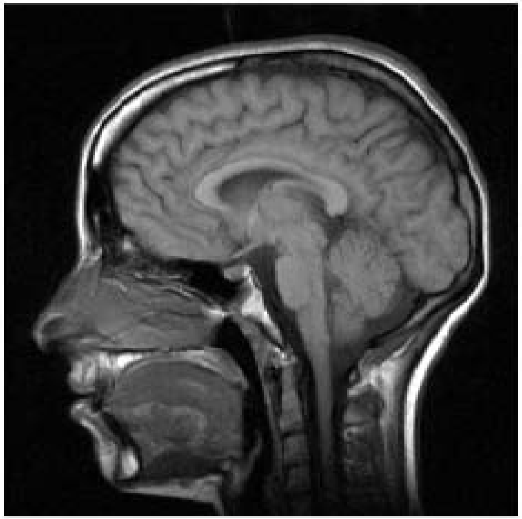
Image Segmentation

- Luminance-based segmentation
  - Optimal supervised thresholding
  - Maximum A Posteriori (MAP) detector
  - Unsupervised thresholding
- Color-based segmentation
  - Chroma keying
  - Multidimensional MAP detector
  - Linear discriminant function

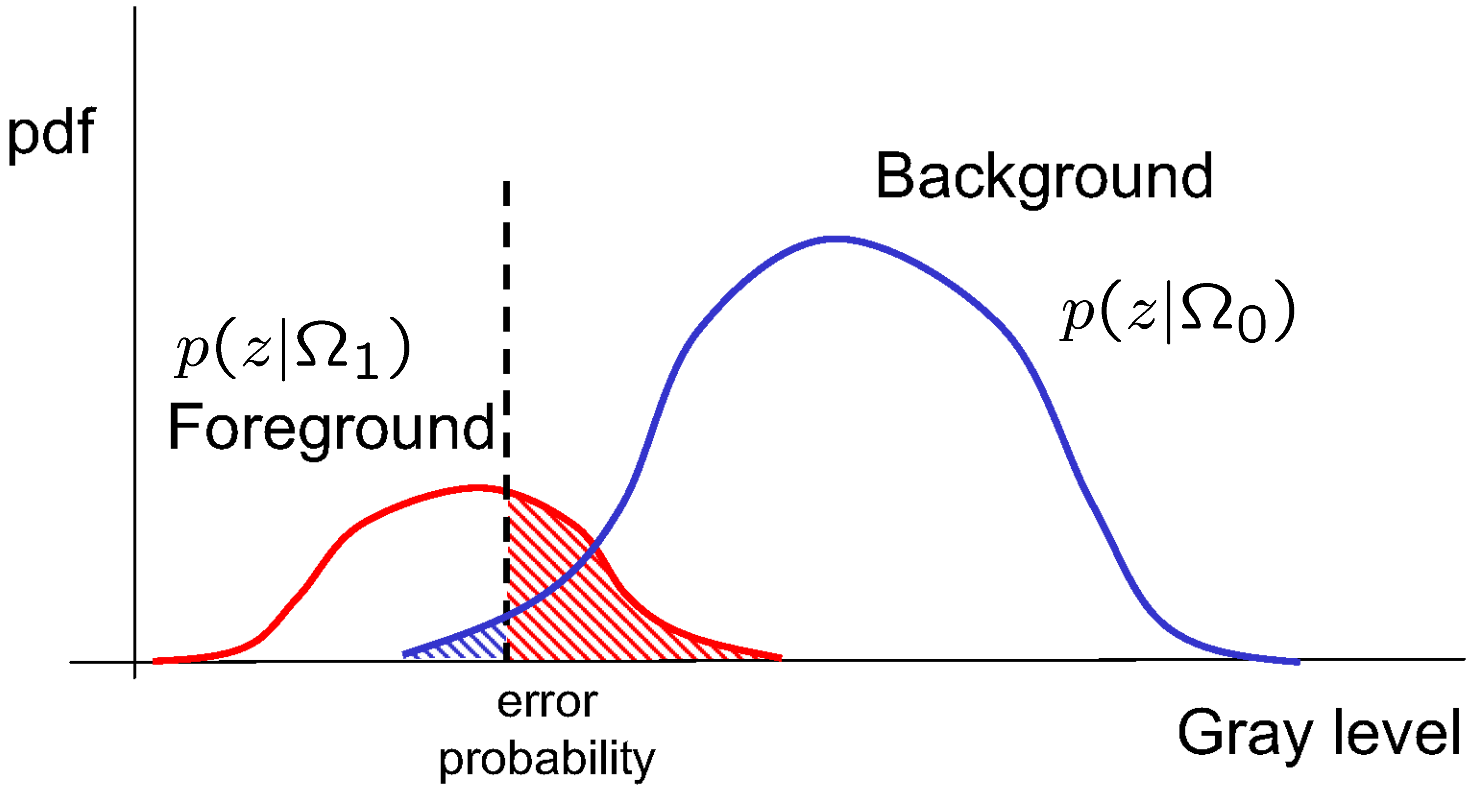
Luminance-Based Segmentation



original image f(x,y)

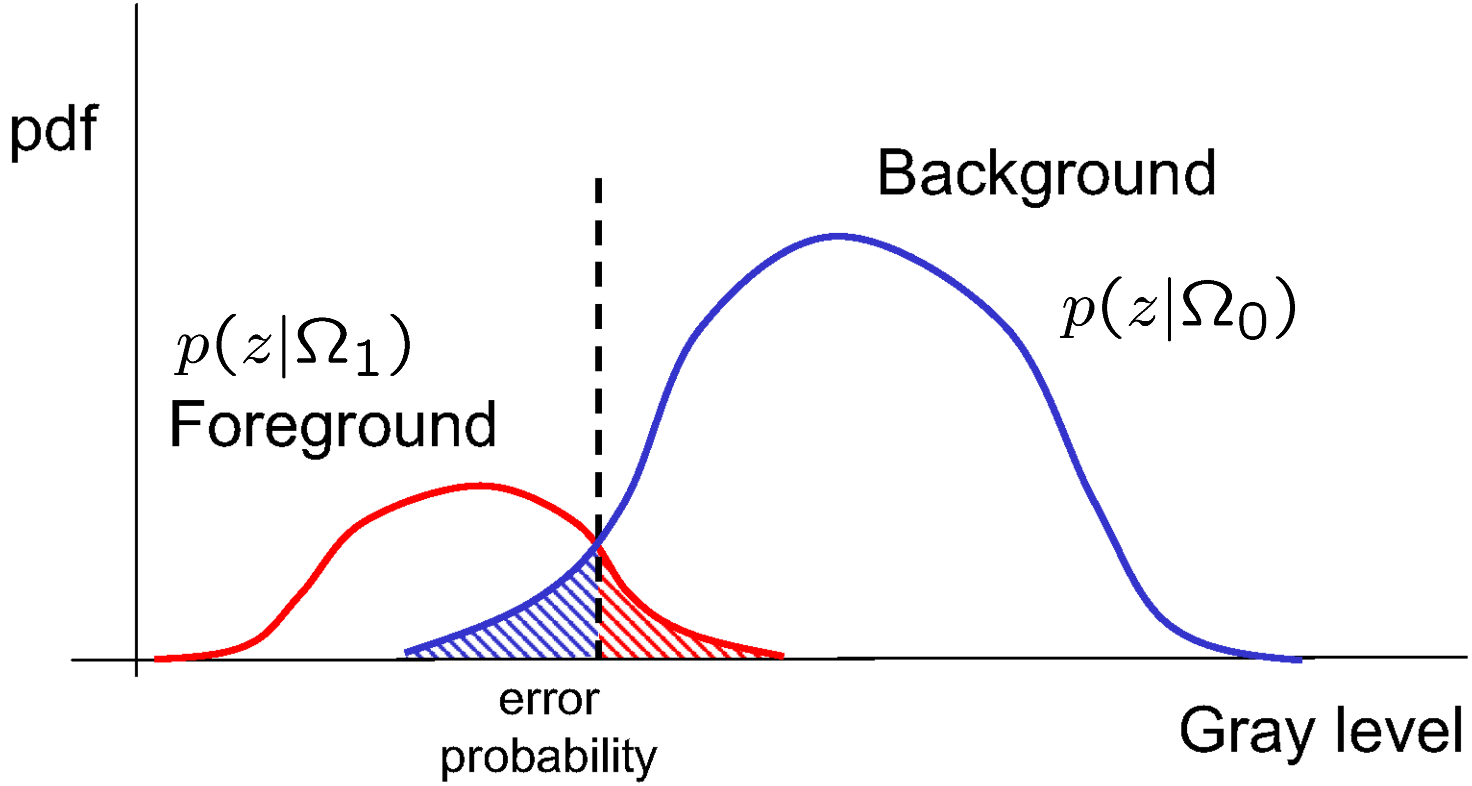
thresholded: f(x,y) > T

Error Probability for Thresholding



$$p_e(T) = \int_{-\infty}^{T} p(z|\Omega_0)dz \cdot p(\Omega_0) + \int_{T}^{\infty} p(z|\Omega_1)dz \cdot p(\Omega_1)$$

Optimal Supervised Thresholding



$$\frac{dp_e(T)}{dT} = 0 \quad \Rightarrow \quad p(T|\Omega_0)p(\Omega_0) = p(T|\Omega_1)p(\Omega_1)$$

Maximum A Posteriori (MAP) Detector

- Choose the class with maximum *a posteriori* probability

$$\Omega_{\mathrm{MAP}} = \arg\max_{\Omega_i} p(\Omega_i|z)$$

- Bayes' rule:

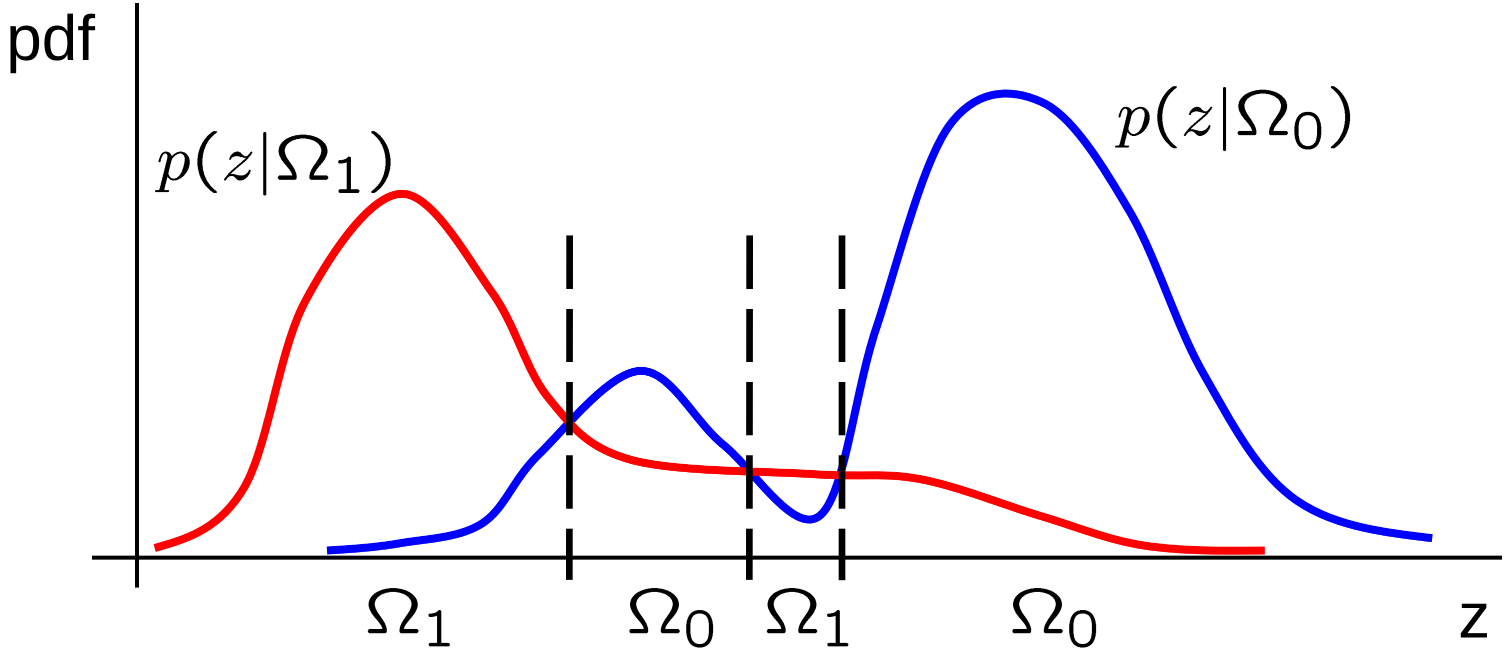
$$p(\Omega_i|z) = \frac{p(z|\Omega_i)p(\Omega_i)}{p(z)}$$

- Probability p(z) is constant:

$$\Omega_{\mathrm{MAP}} = \arg\max_{\Omega_i} p(z|\Omega_i)p(\Omega_i)$$

Maximum A Posteriori (MAP) Detector

- Assuming p(z) = const. and $p(\Omega_i)$ = const.



- Recall: Necessary condition for optimal supervised thresholding . . .

Unsupervised Thresholding

- Idea: Find threshold T that minimizes **within-class variance** of both foreground F and background B

$$\sigma^2_{within}(T) = \frac{N_F(T)}{N}\sigma^2_F(T) + \frac{N_B(T)}{N}\sigma^2_B(T)$$

- Equivalently, maximize **between-class variance**

$$\begin{aligned}
\sigma^2_{between}(T) &= \sigma^2 - \sigma^2_{within}(T) \\
&= \left[\frac{1}{N}\sum_{x,y} f^2(x,y) - \mu^2\right] - \frac{N_F}{N}\left[\frac{1}{N_F}\sum_{x,y\in F} f^2(x,y) - \mu_F^2\right] - \frac{N_B}{N}\left[\frac{1}{N_B}\sum_{x,y\in B} f^2(x,y) - \mu_B^2\right] \\
&= -\mu^2 + \frac{N_F}{N}\mu_F^2 + \frac{N_B}{N}\mu_B^2 = \frac{N_F}{N}(\mu_F - \mu)^2 + \frac{N_B}{N}(\mu_B - \mu)^2 \\
&= \frac{N_F(T)N_B(T)}{N^2}\left[\mu_F(T) - \mu_B(T)\right]^2
\end{aligned}$$

[Otsu, 1979]

Unsupervised Thresholding

- Algorithm: Search for threshold T to maximize

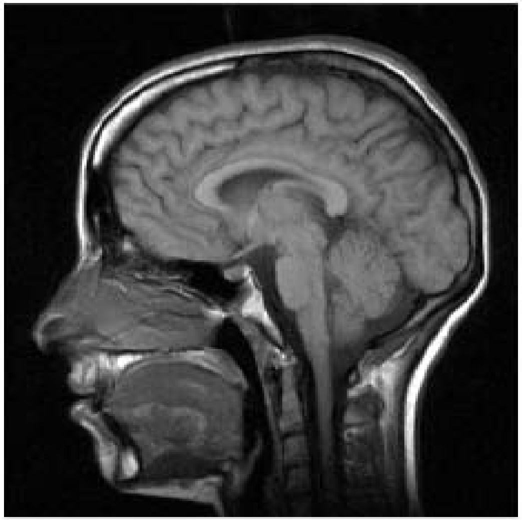
$$\sigma^2_{between}(T) = \frac{N_F(T)N_B(T)}{N^2}\left[\mu_F(T) - \mu_B(T)\right]^2$$

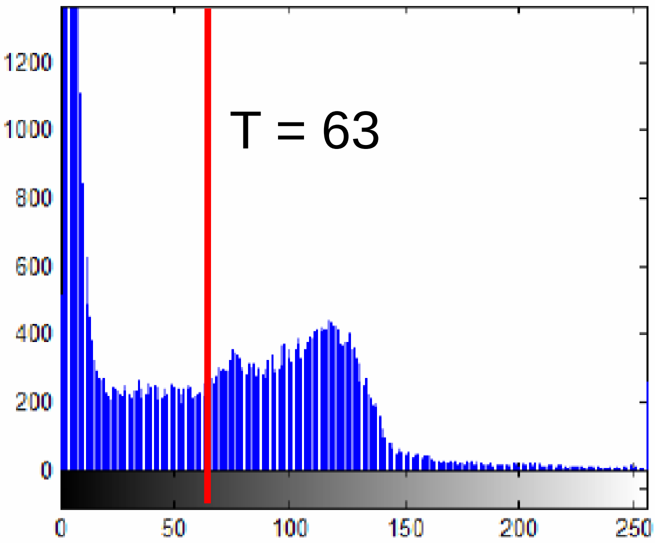
- Efficient recursive computation:

$$\begin{aligned}
N_F(T+1) &= N_F(T) + n_T \\
N_B(T+1) &= N_B(T) - n_T \\
\mu_F(T+1) &= \frac{\mu_F(T)N_F(T) + n_T T}{N_F(T+1)} \\
\mu_B(T+1) &= \frac{\mu_B(T)N_B(T) - n_T T}{N_B(T+1)}
\end{aligned}$$

[Otsu, 1979]

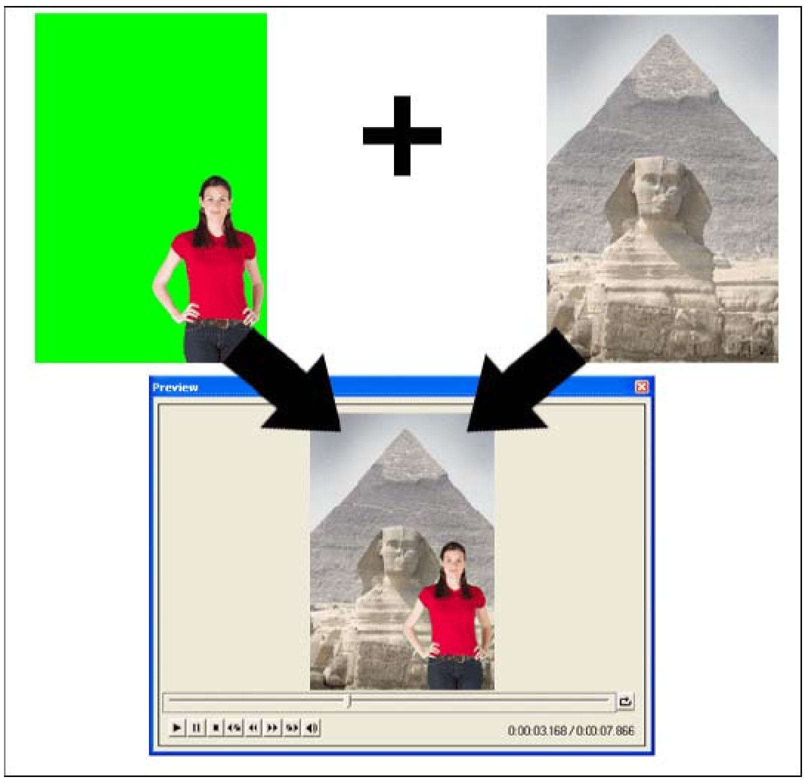
Unsupervised Thresholding Example

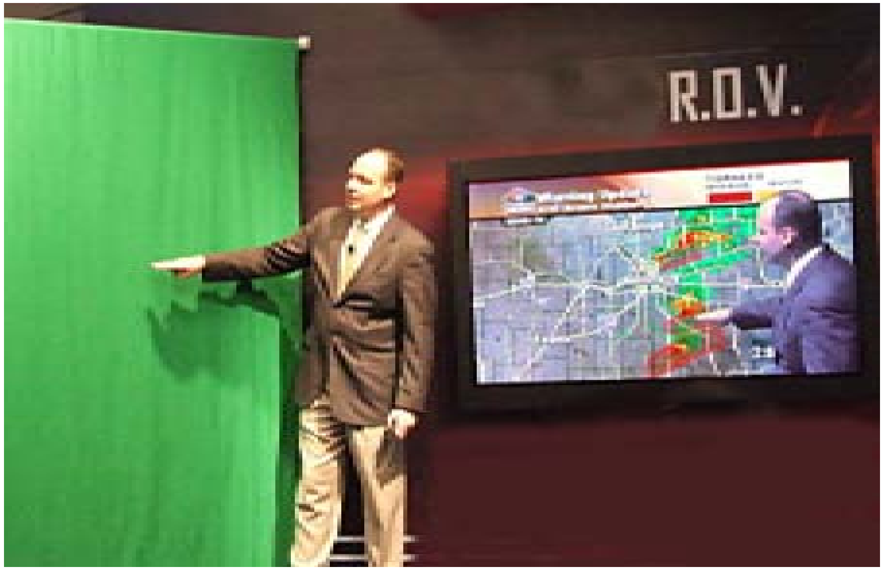




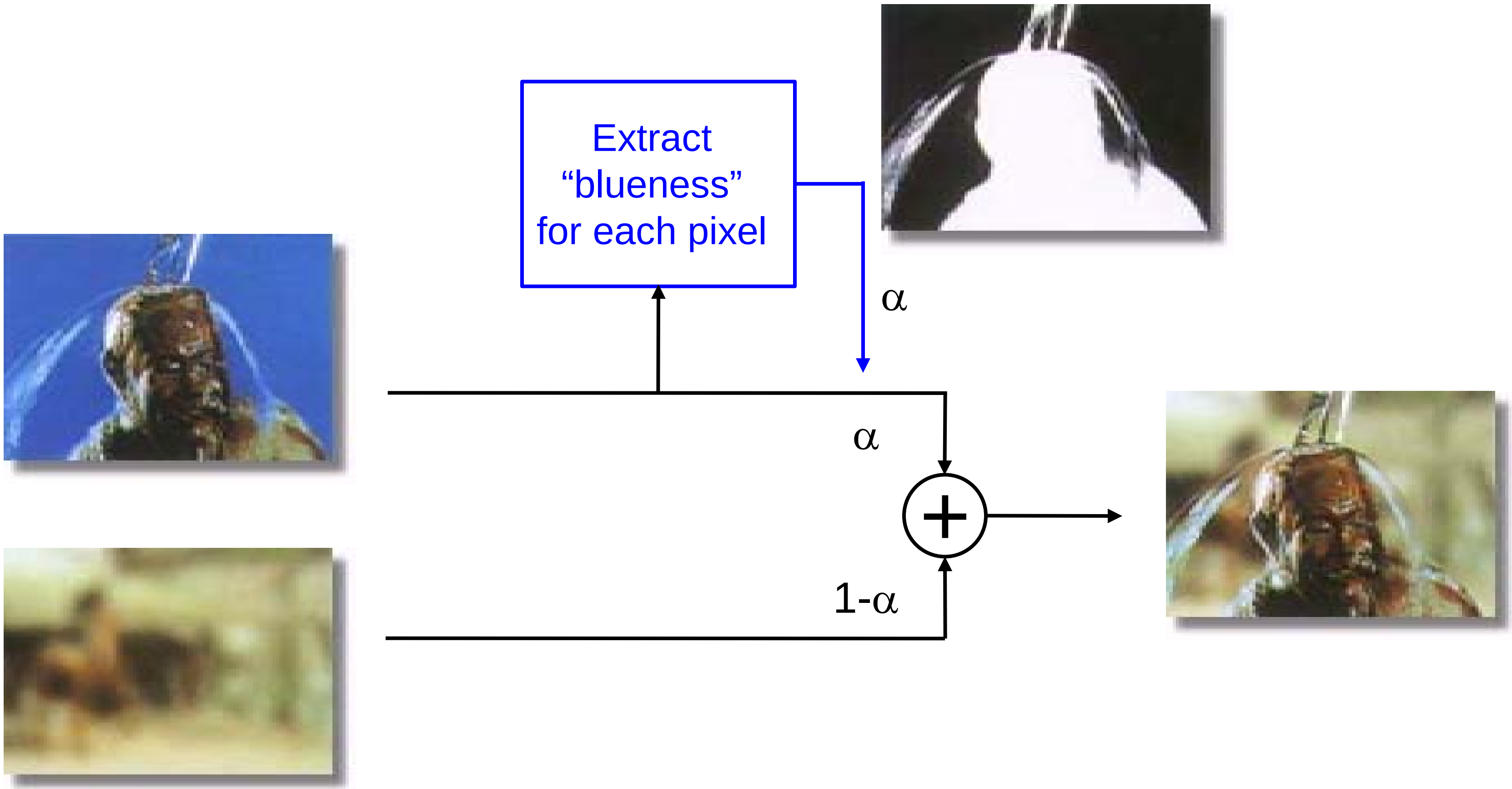
Chroma Keying

- Color is more powerful for pixel-wise segmentation: 3-d vs. 1-d space
- Take picture in front of a blue screen (or green, or orange)



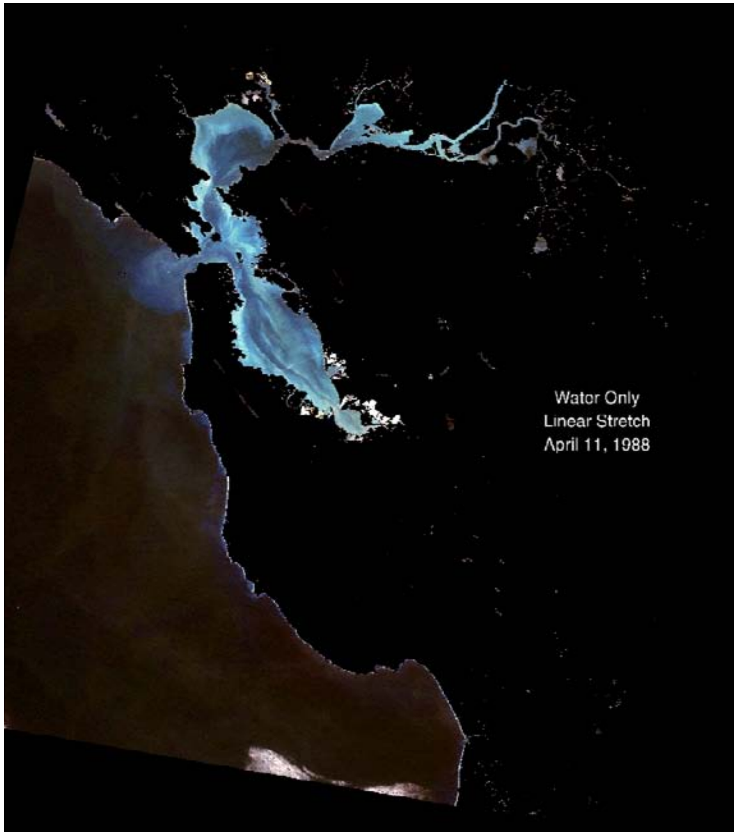


Soft Chroma Keying



Landsat Image Processing

original Landsat image
false color picture out of bands 4,5,6

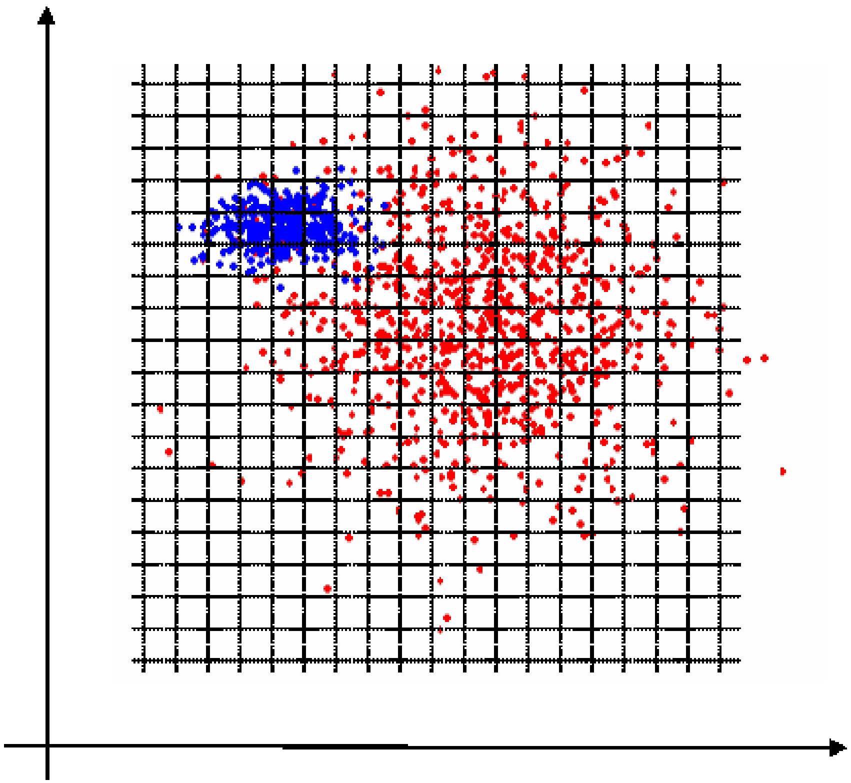


water area segmented and
enhanced to show sediments

Source: US Geological Survey USGS, http://sfbay.wr.usgs.gov/

Multidimensional MAP Detector

- Label categories in training set by hand
- Subdivide n-dimensional space into small bins
- Count frequency of occurrence for each bin and class in training set
- For test data:
  - identify bin
  - detect the more probable category



MAP Detector in RGB-Space

original image

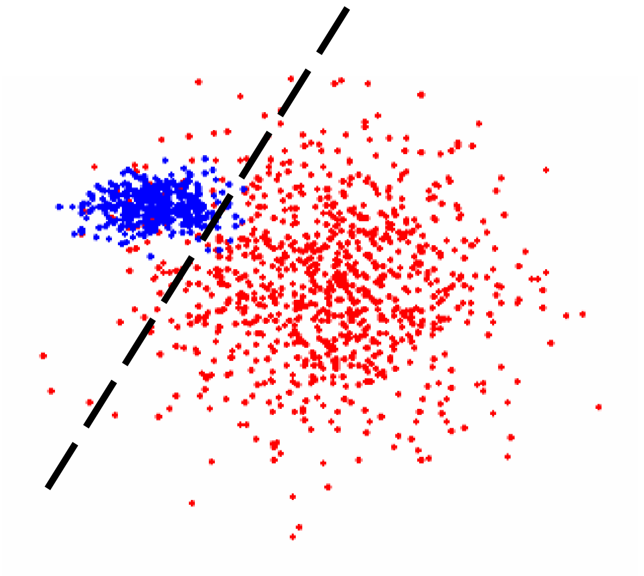
skin color detector

Source: Class project, Stanford University

Linear Discriminant Function

- To segment an image with n components f_i, i=1,2,…,n, into two categories, perform test

$$\sum_{i=1}^{n} w_i f_i + w_0 \geq 0$$



- Categories are separated by hyperplane in n-space
- Numerous techniques to determine weights w_i, i=0,1,2,…,n. See, e.g., [Duda, Hart, Stork, 2001]
- Can be extended to the intersection of several linear discriminant functions
- Can be extended to multiple categories