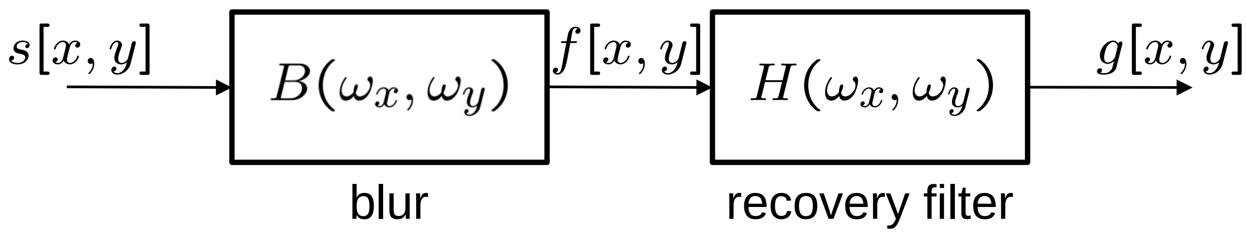
Restoration

- Image deconvolution
- Wiener filtering
- Wiener filtering example
- Nonlinear noise reduction/sharpening

Image Deconvolution

- Given an image f[x,y] that is a blurred version of the original image s[x,y], recover the original image.
- Assume linear shift-invariant blur, transfer function B(ω_x, ω_y)



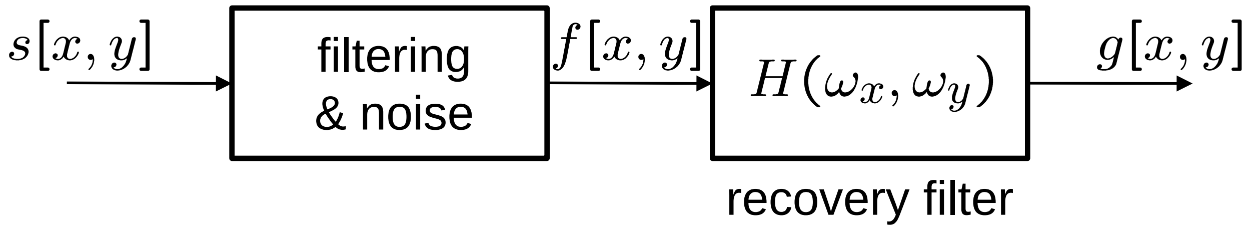
- Naïve solution: inverse filter

$$H(\omega_x,\omega_y) = \frac{1}{B(\omega_x,\omega_y)}$$

- Problem: B(ω_x, ω_y) might be zero, noise amplification

Wiener Filtering

- Model



- Minimize mean squared estimation error

$$E\left\{e^2[x,y]\right\} = E\left\{[g[x,y]-s[x,y]]^2\right\} \xrightarrow{H(\omega_x,\omega_y)} \min$$

Power Spectrum and Cross Spectrum

- 2-d discrete-space cross correlation function for ergodic, stationary signals

$$\phi_{fg}[m,n] = E\left\{f[x+m, y+n]g^*[x,y]\right\}$$

- Special case: autocorrelation function

$$\phi_{ff}[m,n] = E\left\{f[x+m, y+n]f^*[x,y]\right\}$$

- Cross spectral density

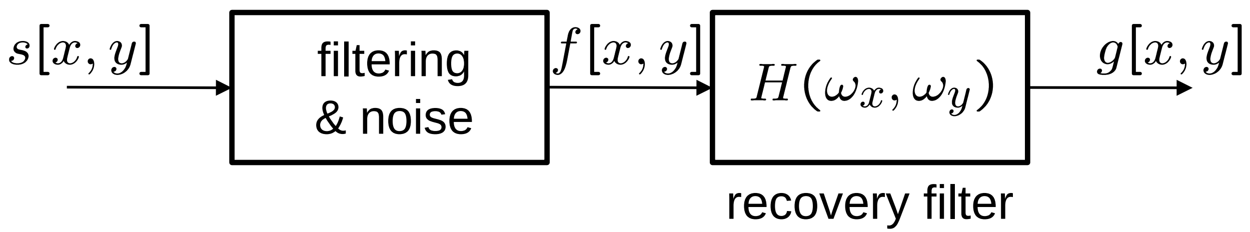
$$\Phi_{fg}(\omega_x,\omega_y) = \sum_{n=-\infty}^{\infty}\sum_{m=-\infty}^{\infty}\phi_{fg}[m,n]e^{-j\omega_x m - j\omega_y n}$$

- Power spectral density

$$\Phi_{ff}(\omega_x,\omega_y) = \sum_{n=-\infty}^{\infty}\sum_{m=-\infty}^{\infty}\phi_{ff}[m,n]e^{-j\omega_x m - j\omega_y n}$$

Wiener Filtering

- Model



- Power spectral density of estimation error e = g-s

$$\begin{aligned}
\Phi_{ee}(\omega_x,\omega_y) &= \Phi_{gg}(\omega_x,\omega_y) - \Phi_{gs}(\omega_x,\omega_y) - \Phi_{sg}(\omega_x,\omega_y) + \Phi_{ss}(\omega_x,\omega_y) \\
&= \Phi_{ff}(\omega_x,\omega_y)|H(\omega_x,\omega_y)|^2 \\
&\quad -\Phi_{fs}(\omega_x,\omega_y)H(\omega_x,\omega_y) - \Phi_{sf}(\omega_x,\omega_y)H^*(\omega_x,\omega_y) \\
&\quad +\Phi_{ss}(\omega_x,\omega_y)
\end{aligned}$$

Wiener Filtering

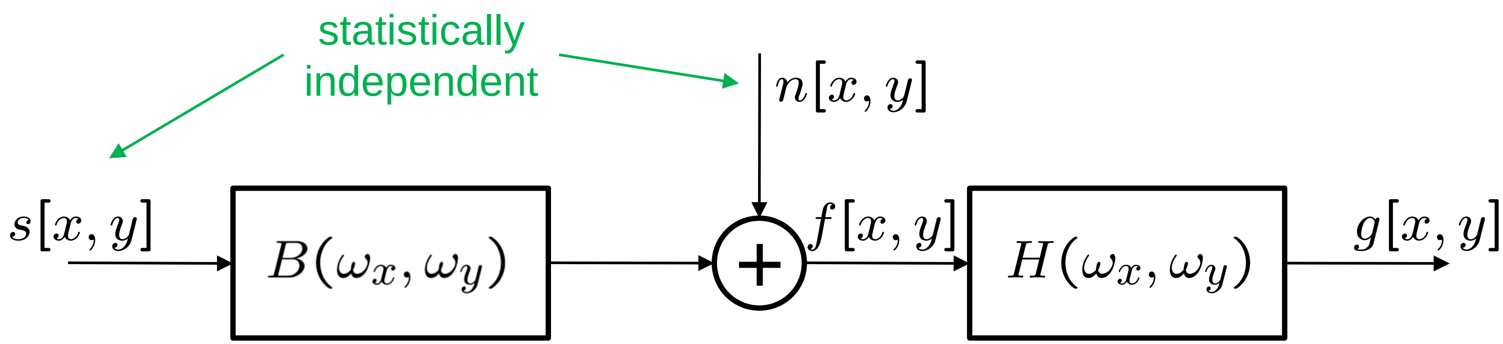
- Power spectrum $\Phi_{ee}(\omega_x, \omega_y)$ is minimized separately at each frequency ω_x, ω_y if

$$H(\omega_x, \omega_y) = \frac{\Phi^*_{fs}(\omega_x, \omega_y)}{\Phi_{ff}(\omega_x, \omega_y)} = \frac{\Phi_{sf}(\omega_x, \omega_y)}{\Phi_{ff}(\omega_x, \omega_y)}$$

- Can be shown to be global minimum by considering filter

$$H(\omega_x, \omega_y) = \frac{\Phi^*_{fs}(\omega_x, \omega_y)}{\Phi_{ff}(\omega_x, \omega_y)} + \Delta H(\omega_x, \omega_y)$$

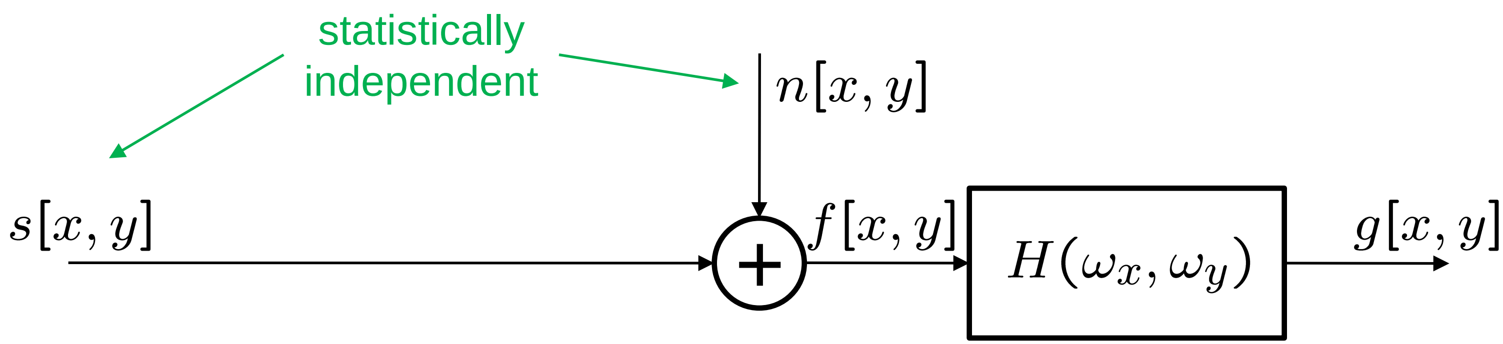
Wiener Filter for Linear Distortion and Additive Noise



$$
\begin{aligned}
\Phi_{sf}(\omega_x,\omega_y) &= \Phi_{ss}(\omega_x,\omega_y)B^*(\omega_x,\omega_y) \\
\Phi_{ff}(\omega_x,\omega_y) &= \Phi_{ss}(\omega_x,\omega_y)|B(\omega_x,\omega_y)|^2 + \Phi_{nn}(\omega_x,\omega_y)
\end{aligned}
$$

$$
H(\omega_x,\omega_y) = \frac{\Phi_{ss}(\omega_x,\omega_y)B^*(\omega_x,\omega_y)}{\Phi_{ss}(\omega_x,\omega_y)|B(\omega_x,\omega_y)|^2 + \Phi_{nn}(\omega_x,\omega_y)}
$$

Wiener Filter for Additive Noise



$$H(\omega_x, \omega_y) = \frac{\Phi_{ss}(\omega_x, \omega_y)}{\Phi_{ss}(\omega_x, \omega_y) + \Phi_{nn}(\omega_x, \omega_y)}$$

Wiener Filtering Example

image with motion blur

restored by Wiener filter

Source: http://www.cs.kun.nl/~ths/rt2/col/h5/5restoratieENG.html

Wiener Filtering Example

original

additive
white noise

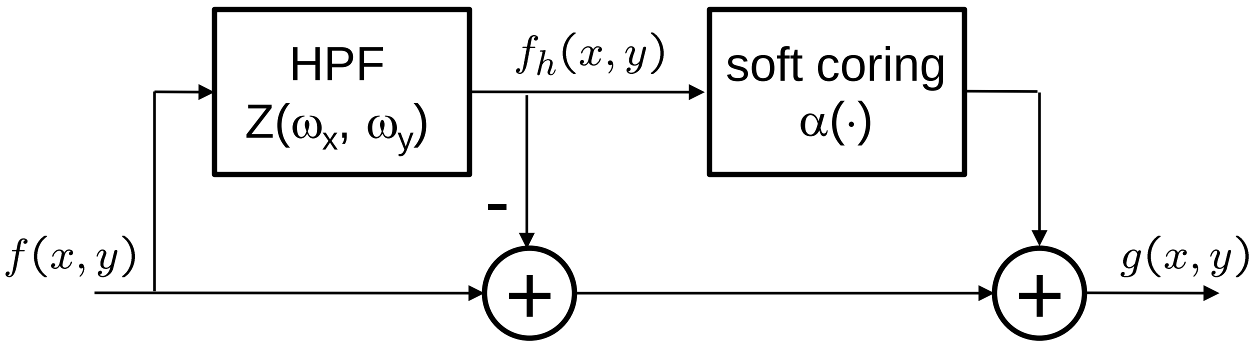
rmse = 18.9

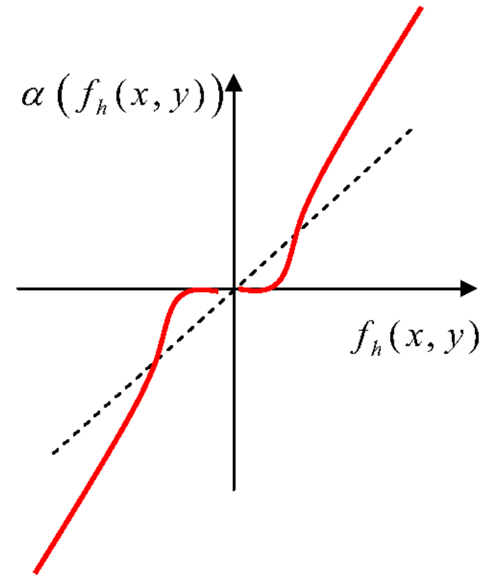
noise reduction
by Wiener filtering

rmse = 10.5

Nonlinear Noise Reduction/Sharpening

- Noise reduction: smooth the image, low-pass filtering
- Deblurring: sharpen edges, high-pass filtering
- How can both be achieved simultaneously?
- Key insight: large amplitude of high-pass filtered image indicates presence of edge





- Can be extended to multiple HPFs

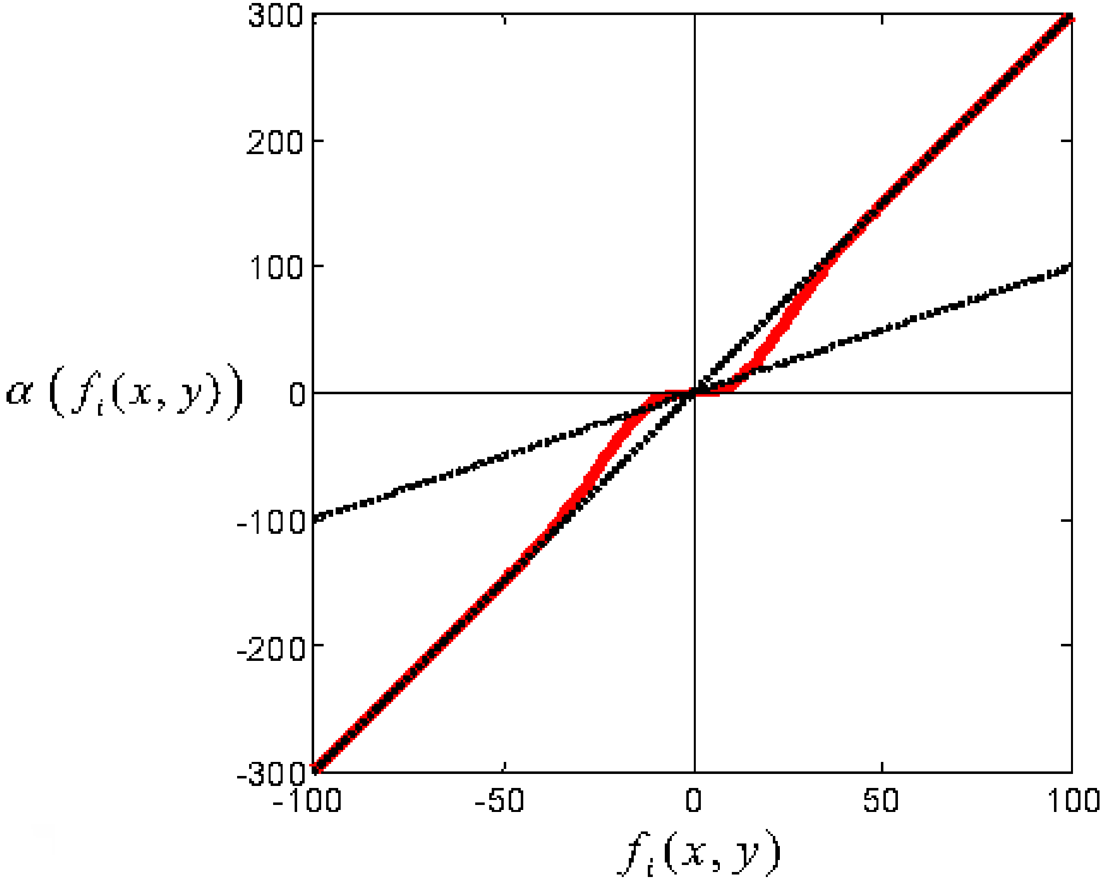
Example

blurred, noisy image

noise-reduced and sharpened

Soft Coring Function

$$\alpha(f_i(x,y)) = m \cdot f_i(x,y) \cdot \left[1 - e^{-\left(\frac{f_i(x,y)}{\tau}\right)^{\gamma}}\right]$$

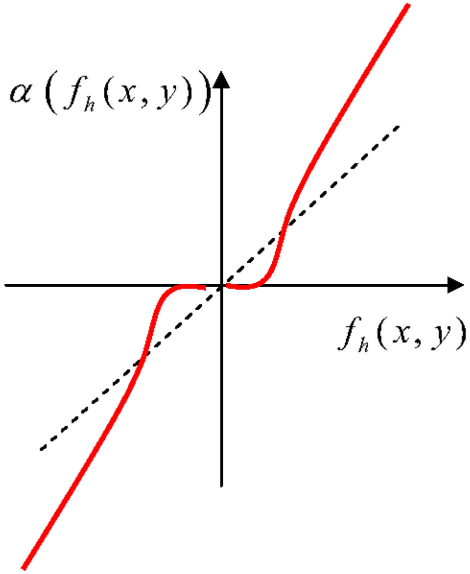


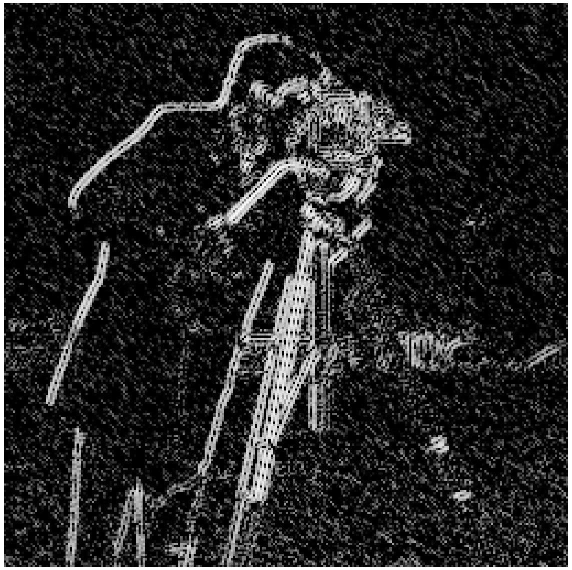
Example:

m = 3
γ = 2
τ = 20

Soft Coring of High-Pass Filtered Images

high-pass filtered





soft coring output

Linear vs. Nonlinear Noise Reduction/Sharpening

noise reduction
by low-pass filter
(linear)

sharpening
by high-pass filter
(linear)

combined
noise reduction
and sharpening
(nonlinear)