EN2401 – Image and Video Processing

Assignment #1

due Friday, November 6, 2015, 13:00

The following preparatory assignment is to be solved before the next exercise session indicated by the due date of the assignment. You bring your solution to the exercise session and one of your peers will correct it during that session. After that you will discuss the correction with your peers and resolve any open questions. If necessary the teaching assistant can help you. It is required to solve all the assignments and correct at least one peer solution of each assignment in order to pass the course. More information is available at the course webpage: `https://www.kth.se/social/course/EN2401/`.

Problem

We consider the gray-level continuous image *desert*, shown in Figure 1.

Figure 1: The Namib desert.

Assume that the probability density function for the gray-levels is given by

$$f_X(x) = \frac{1}{|b|} \sin(a\pi x), \quad x \in [0,1] \tag{1}$$

where a and b are constants.

1. Give the general conditions (independently of the image *desert*) on a and b that ensure that equation (1) represents a true probability density function.

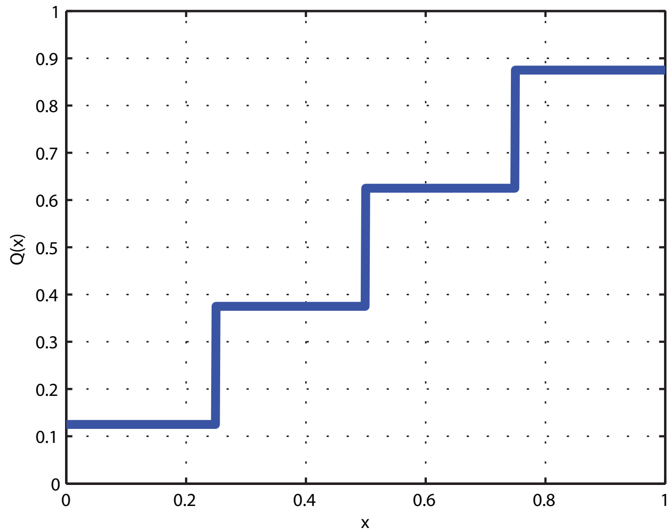

Figure 2: Uniform quantizer $Q(x)$.

2. Estimate (approximately) the values of a and b that apply to the image *desert*. Sketch the approximate histogram for these values. Motivate!

3. The image *desert* is transformed using

$$y = T(x) = \int_0^x f_X(\tau)\ d\tau. \tag{2}$$

Give the explicit expressions for y and the probability density function $f_Y(y)$.

4. A uniform two-bit quantizer (as depicted in Fig. 2) is applied to *desert*,

$$\bar{x} = Q(x). \tag{3}$$

The quantization function $Q(x)$ is defined as

$$Q(x) = \begin{cases} \frac{1}{8} \text{ for } 0 \le x < \frac{1}{4} \\ \frac{3}{8} \text{ for } \frac{1}{4} \le x < \frac{1}{2} \\ \frac{5}{8} \text{ for } \frac{1}{2} \le x < \frac{3}{4} \\ \frac{7}{8} \text{ for } \frac{3}{4} \le x \le 1 \end{cases}. \tag{4}$$

Find the probability mass function $p_{\bar{X}}(\bar{x})$ for a and b you have selected before.

5. Histogram equalization is applied to the quantized image, using the transform

$$z = H(\bar{x}) = \sum_{\tau \in \{0,\ldots,\bar{x}\}} p_{\bar{X}}(\tau). \tag{5}$$

Give the explicit expressions for the values that z can attain and the probability mass function $p_Z(z)$. Comment on the effect of histogram equalization. *Hint:* Compare the discrete and the continuous case.