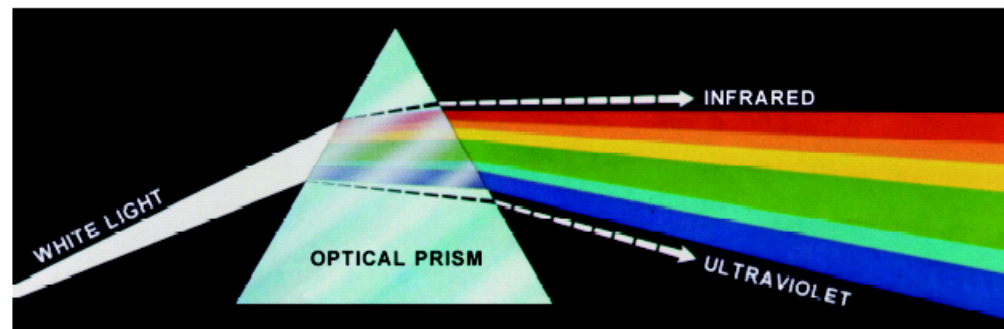
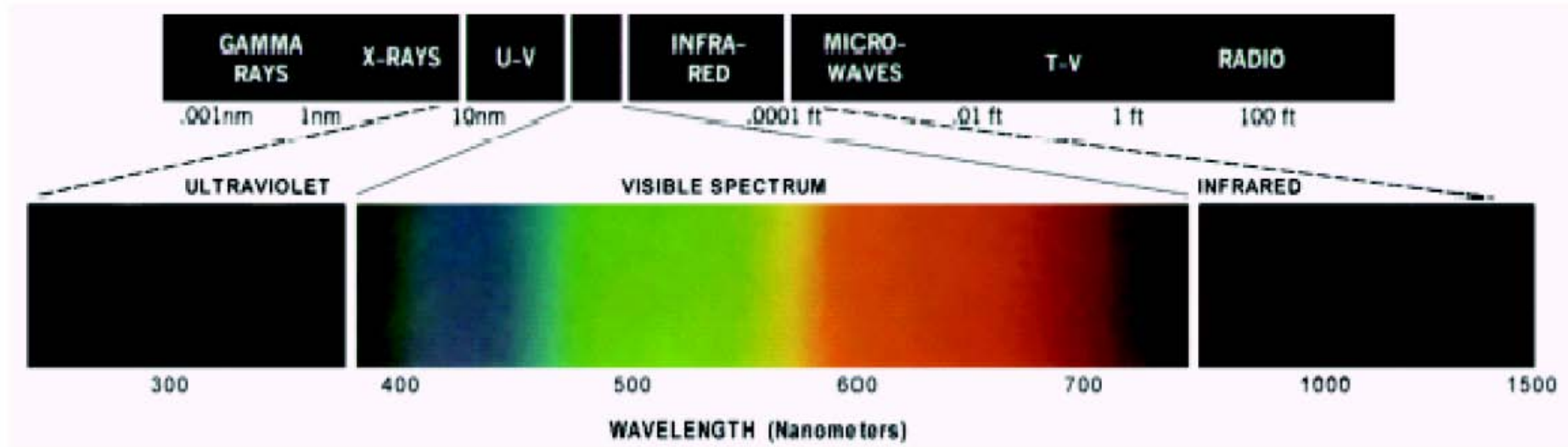


# Color

- Trichromacy
- Spectral matching curves
- CIE XYZ color system
- xy-chromaticity diagram
- Color gamut
- Color temperature
- Color balancing algorithms

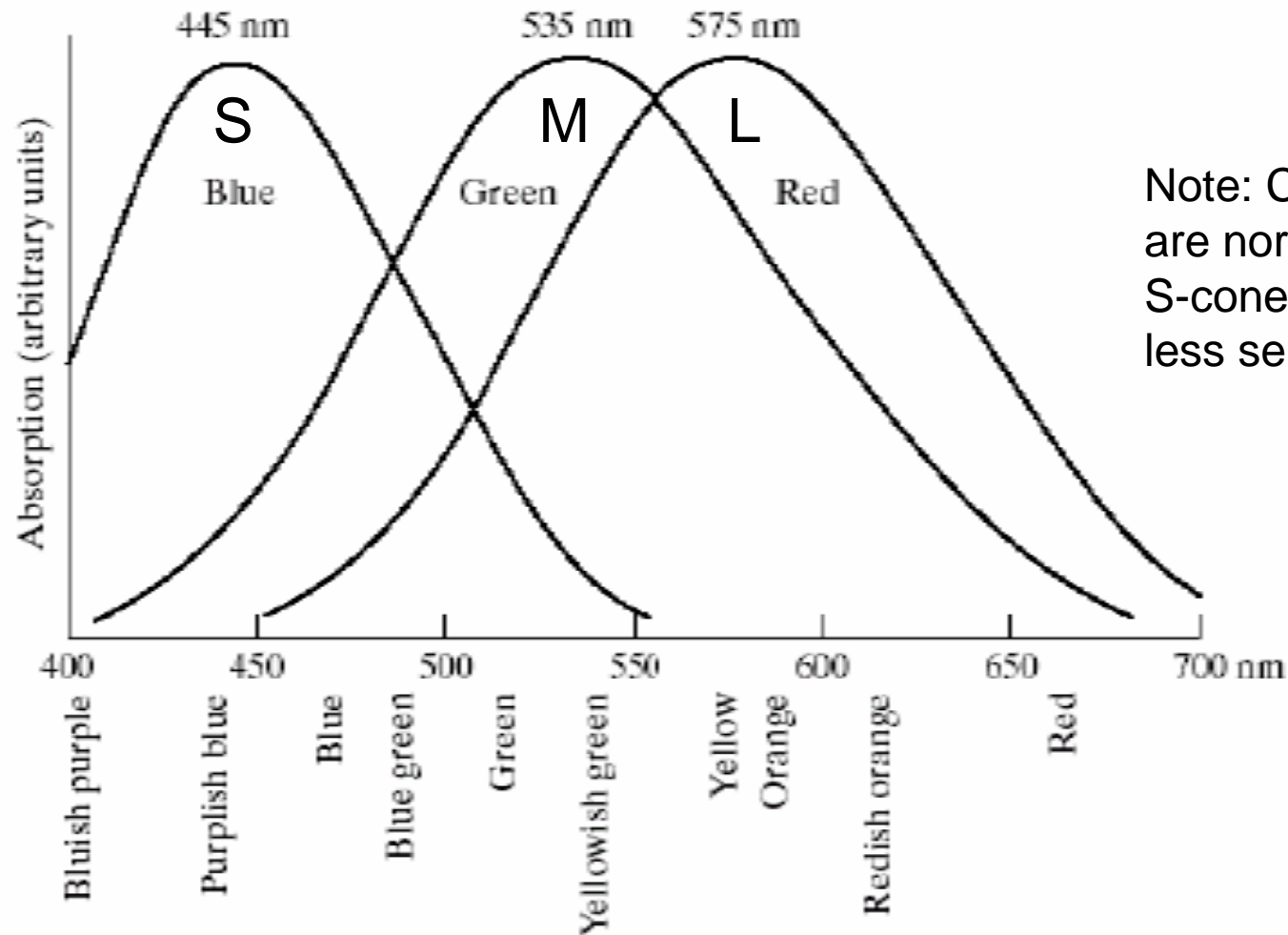
# Visible Range of the Electromagnetic Spectrum



[Newton, 1666]

Source: Gonzalez, Woods, Figs. 6.1, 6.2

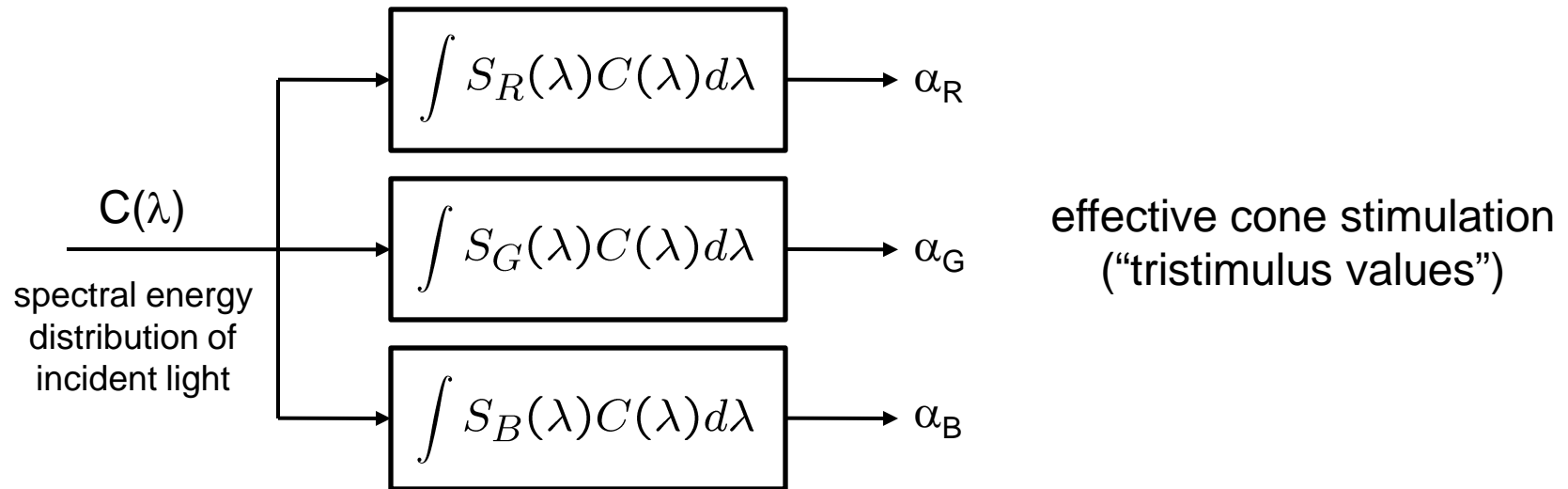
# Absorption of Light in the Cones of the Human Retina



Note: Curves are normalized. S-cones are much less sensitive.

Source: Gonzalez, Woods, Fig. 6.3

# Three-Receptor Model of Color Perception

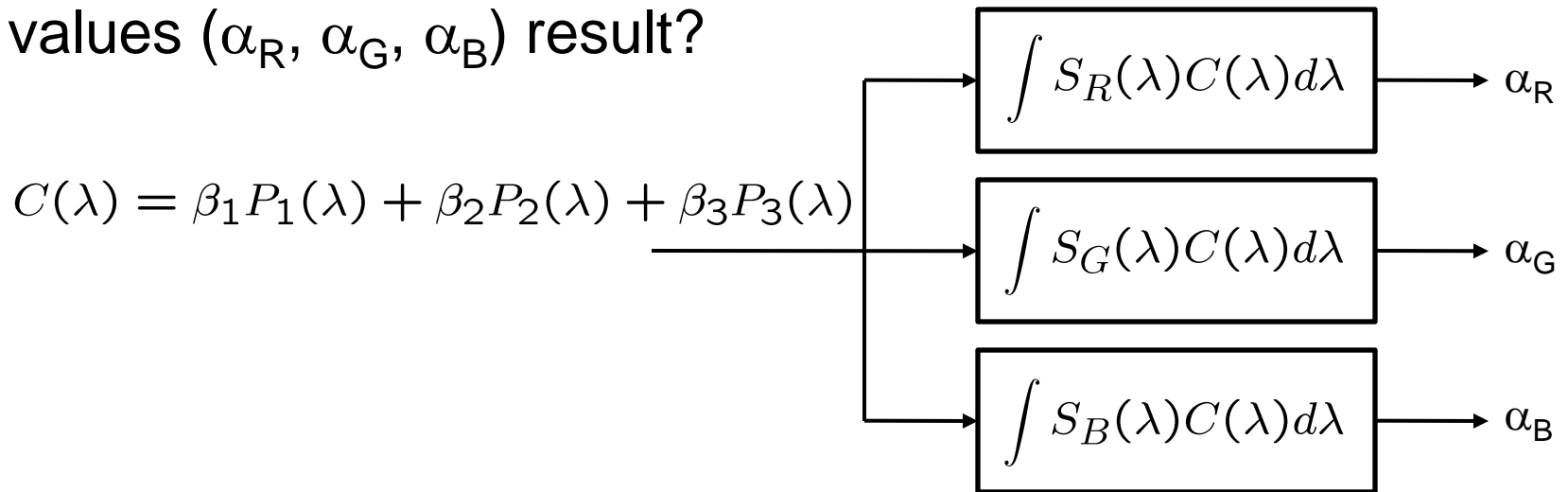


[T. Young, 1802], [J.C. Maxwell, 1890]

- Different spectra can map into the same tristimulus values and hence look identical ("metamers")
- Three numbers suffice to represent any color

# Color Matching

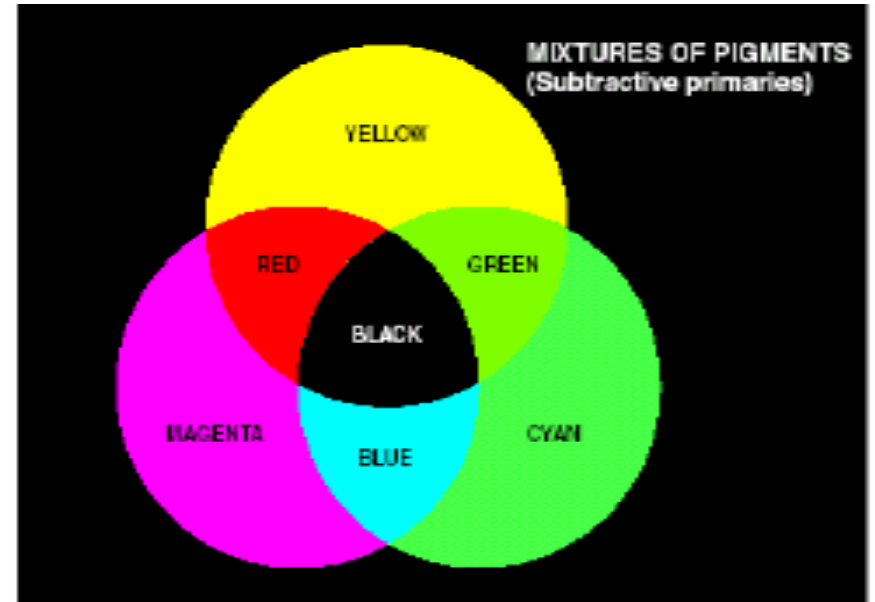
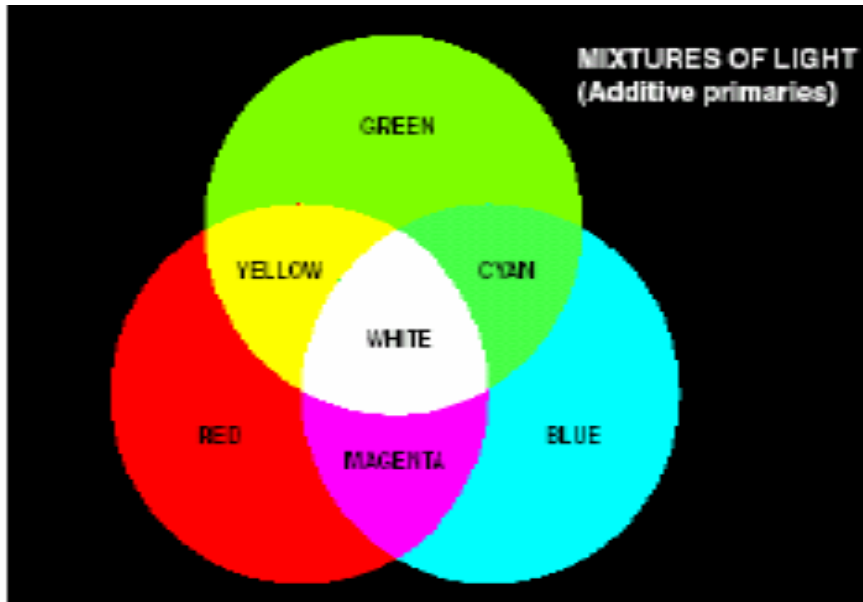
- Suppose 3 primary light sources with spectra  $P_k(\lambda)$ ,  $k=1,2,3$
- How to choose  $\beta_k$ ,  $k=1,2,3$ , such that desired tristimulus values  $(\alpha_R, \alpha_G, \alpha_B)$  result?



$$\begin{aligned}\alpha_i &= \int S_i(\lambda) [\beta_1 P_1(\lambda) + \beta_2 P_2(\lambda) + \beta_3 P_3(\lambda)] d\lambda \\ &= \beta_1 K_{i,1} + \beta_2 K_{i,2} + \beta_3 K_{i,3} \quad \text{with} \quad K_{i,j} = \int S_i(\lambda) P_j(\lambda) d\lambda\end{aligned}$$

- Color matching is **linear** (“Grassman’s Laws”)

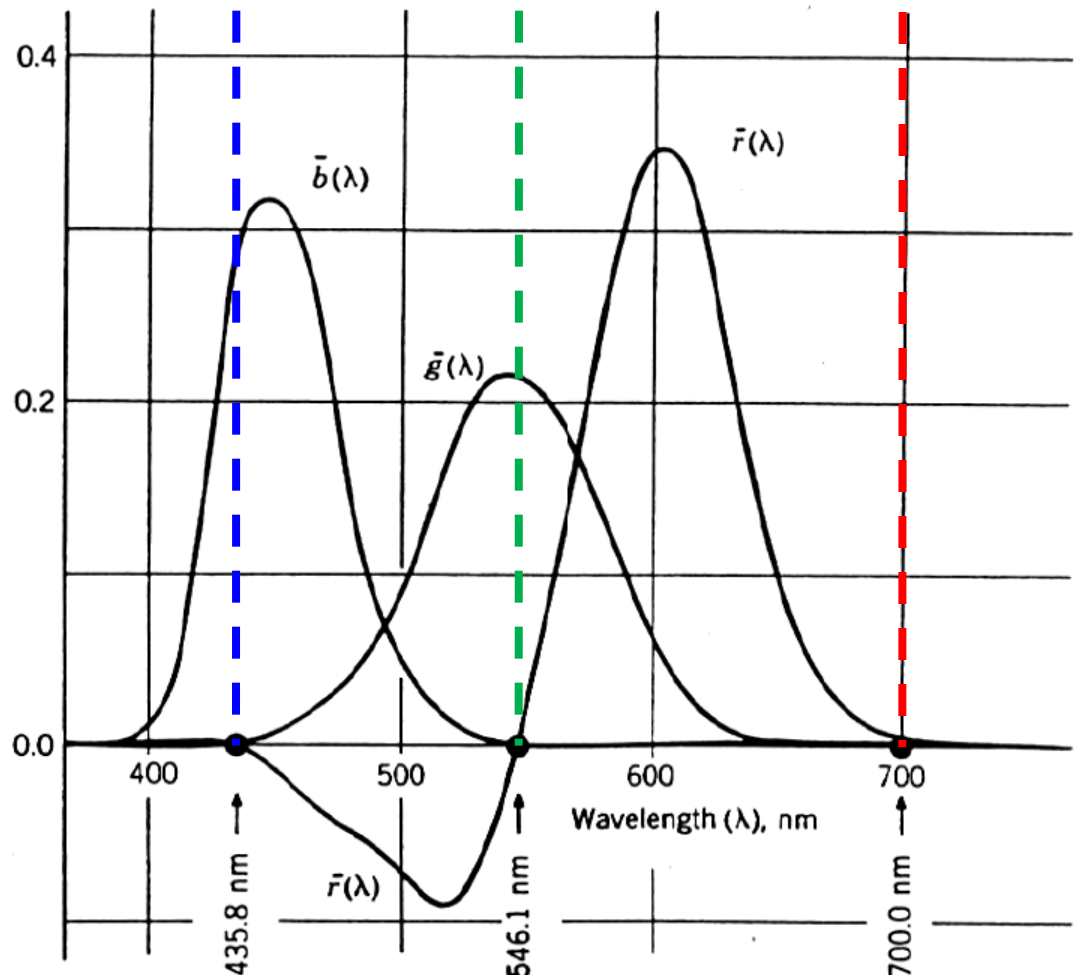
# Additive vs. Subtractive Color Mixing



Source: Gonzalez, Woods, Fig. 6.4

# Spectral Matching Curves

- **Experiment:**  
Match monochromatic light with 3 monochromatic primaries
- “Negative intensity”:  
Color is added to test color
- **CIE** (*Commission Internationale de L’Eclairage*), **1931**:  
Spectral RGB primaries (scaled such that  $R_\lambda = G_\lambda = B_\lambda$  matches spectrally flat white)

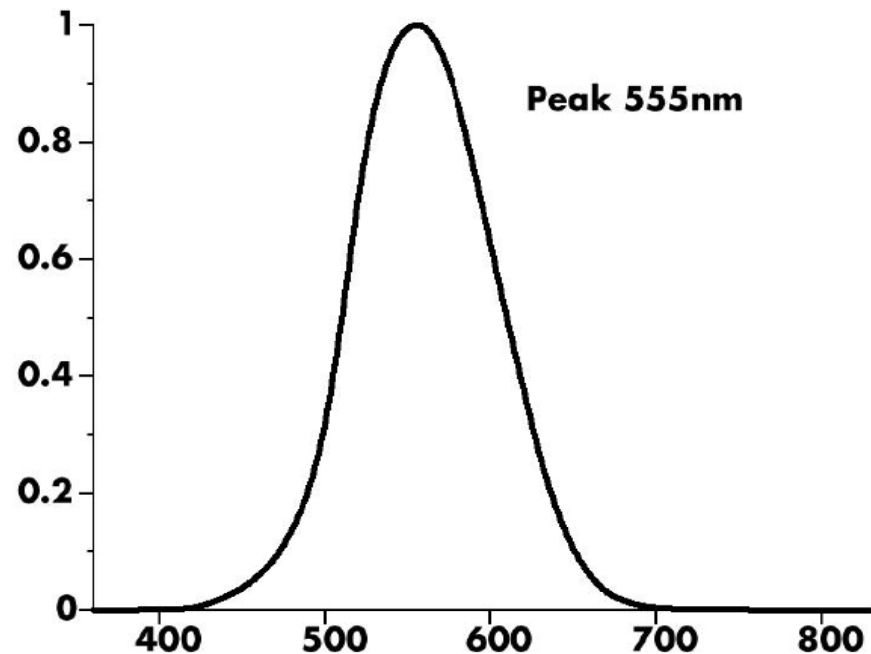


# Luminous Efficiency Curve

- **Experiment:**

Match the brightness of a monochromatic light with a white reference light

- Links photometric and radiometric quantities



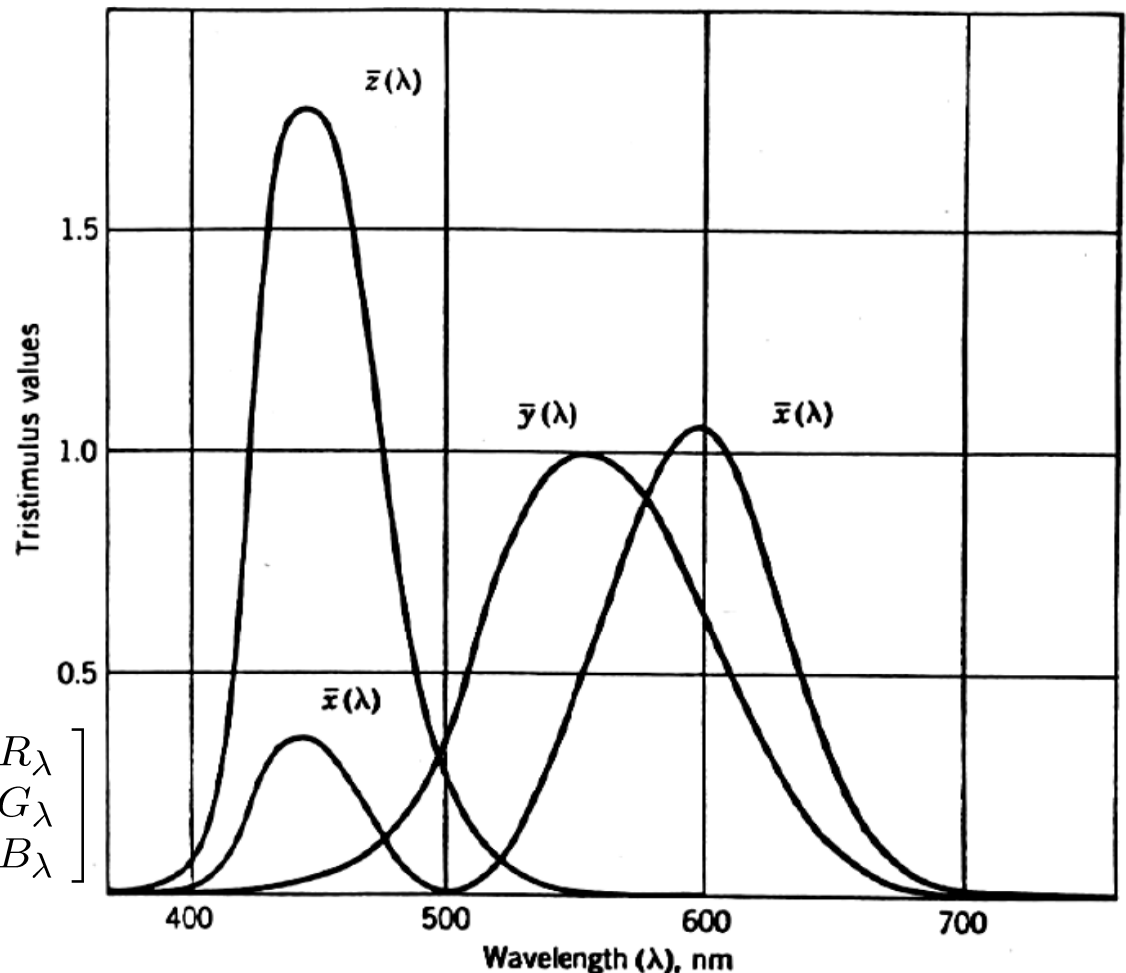


# CIE 1931 XYZ Color System

## Properties:

- All positive spectral matching curves
- Y corresponds to luminance
- Equal energy white:  $X=Y=Z$
- Virtual primaries

$$\begin{bmatrix} X \\ Y \\ Z \end{bmatrix} = \begin{bmatrix} .490 & .310 & .200 \\ .177 & .812 & .011 \\ .000 & .010 & .990 \end{bmatrix} \begin{bmatrix} R_\lambda \\ G_\lambda \\ B_\lambda \end{bmatrix}$$

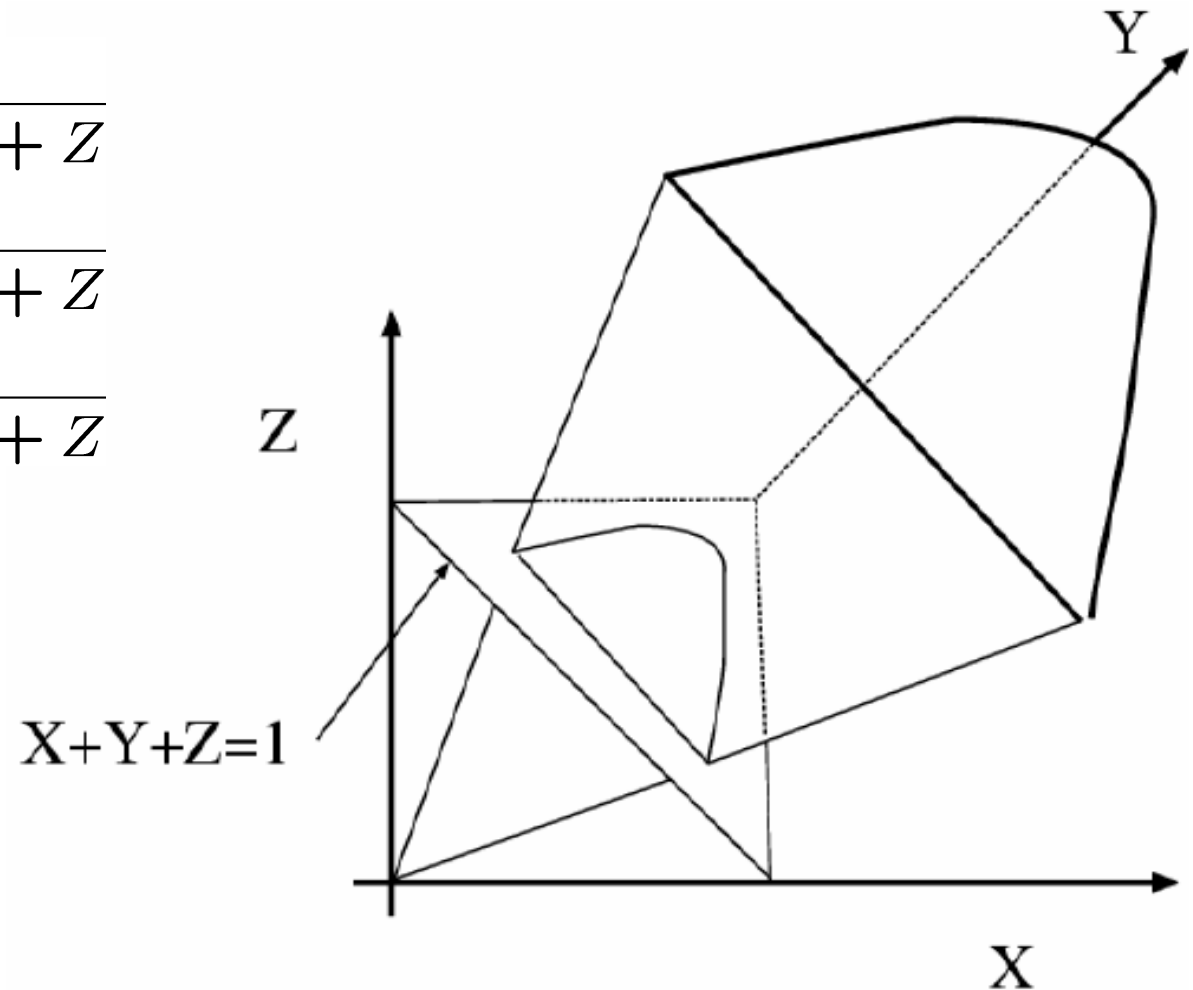


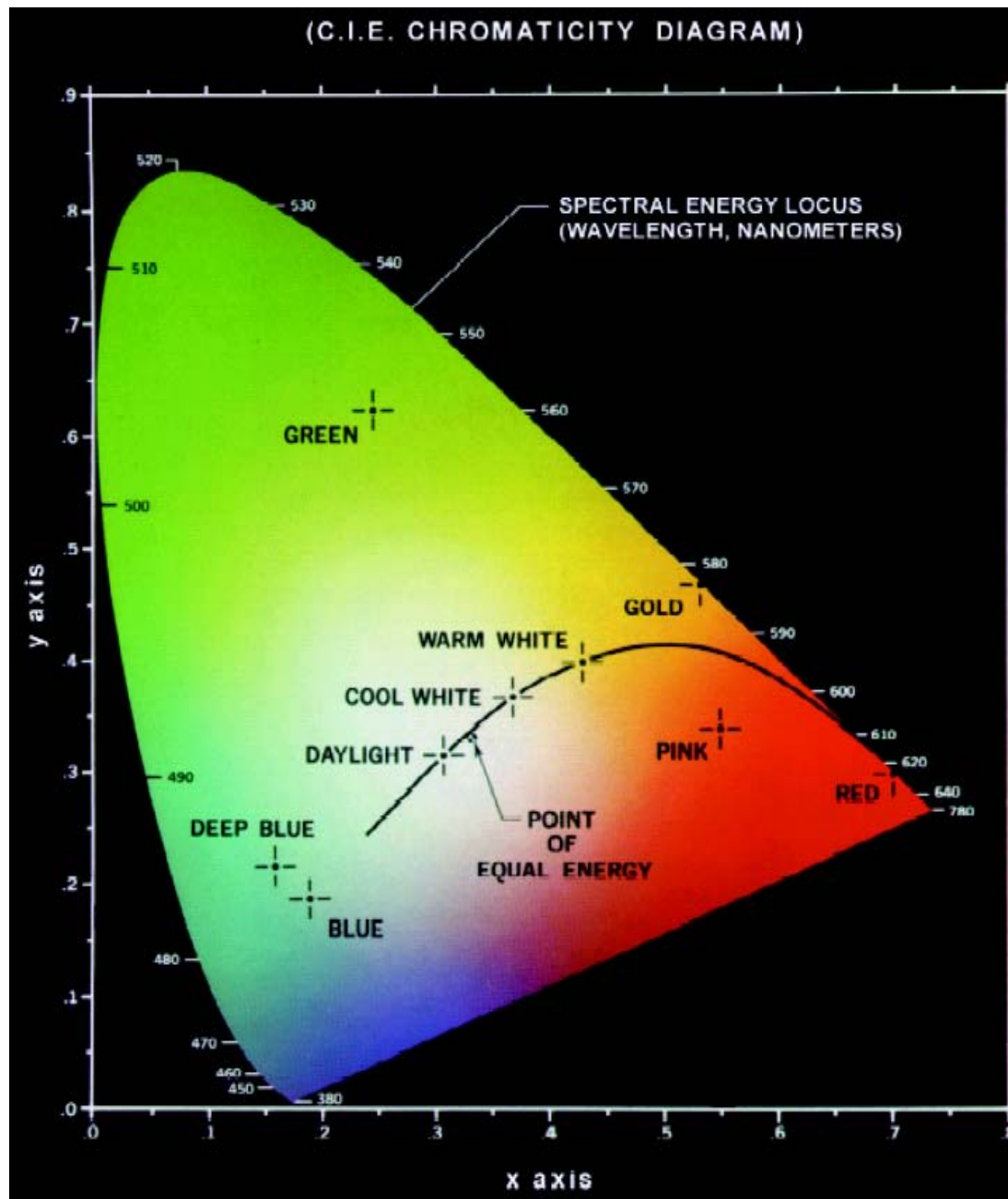
# Chromaticity Diagram

$$x = \frac{X}{X + Y + Z}$$

$$y = \frac{Y}{X + Y + Z}$$

$$z = \frac{Z}{X + Y + Z}$$

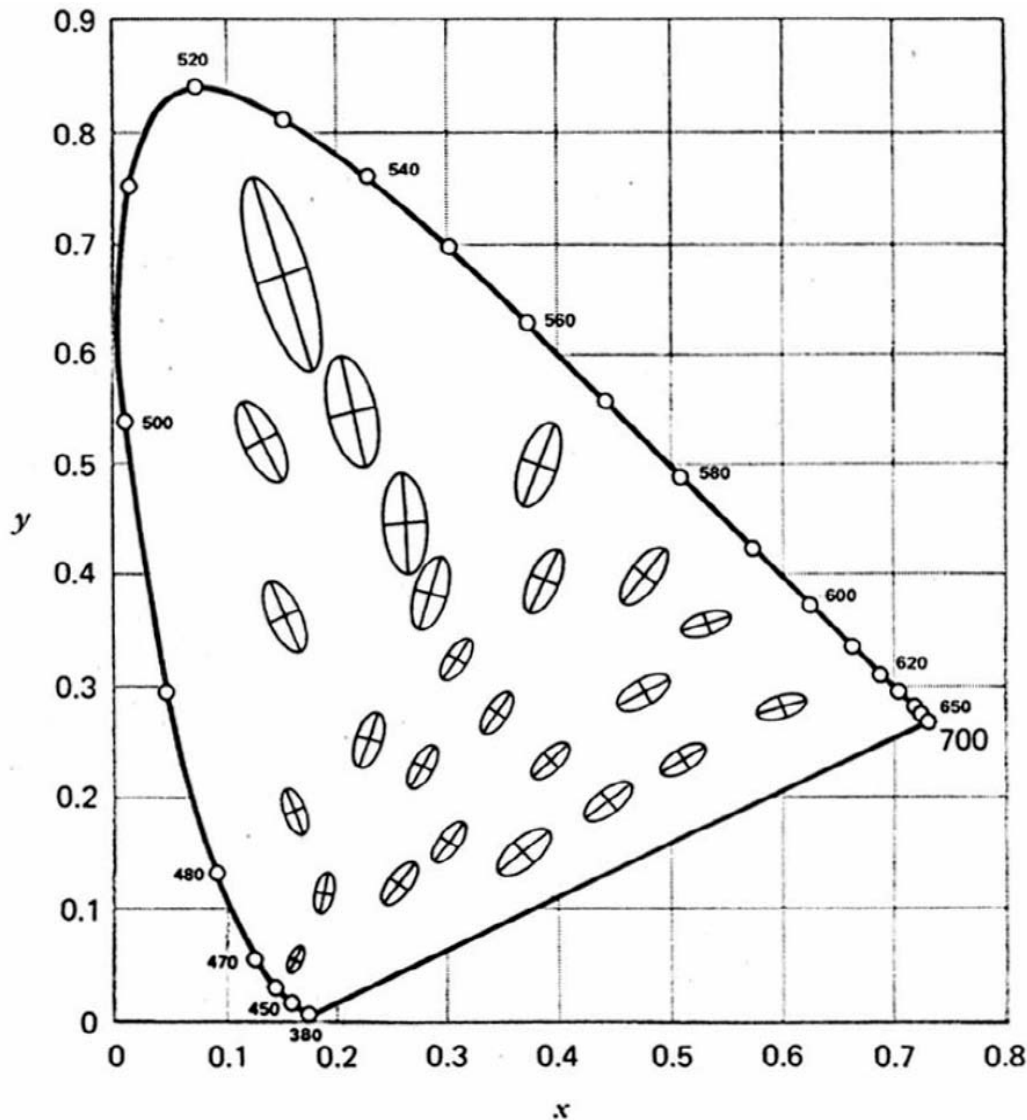




Source:  
Gonzalez, Woods, Fig. 6.5

**Color no. 11**

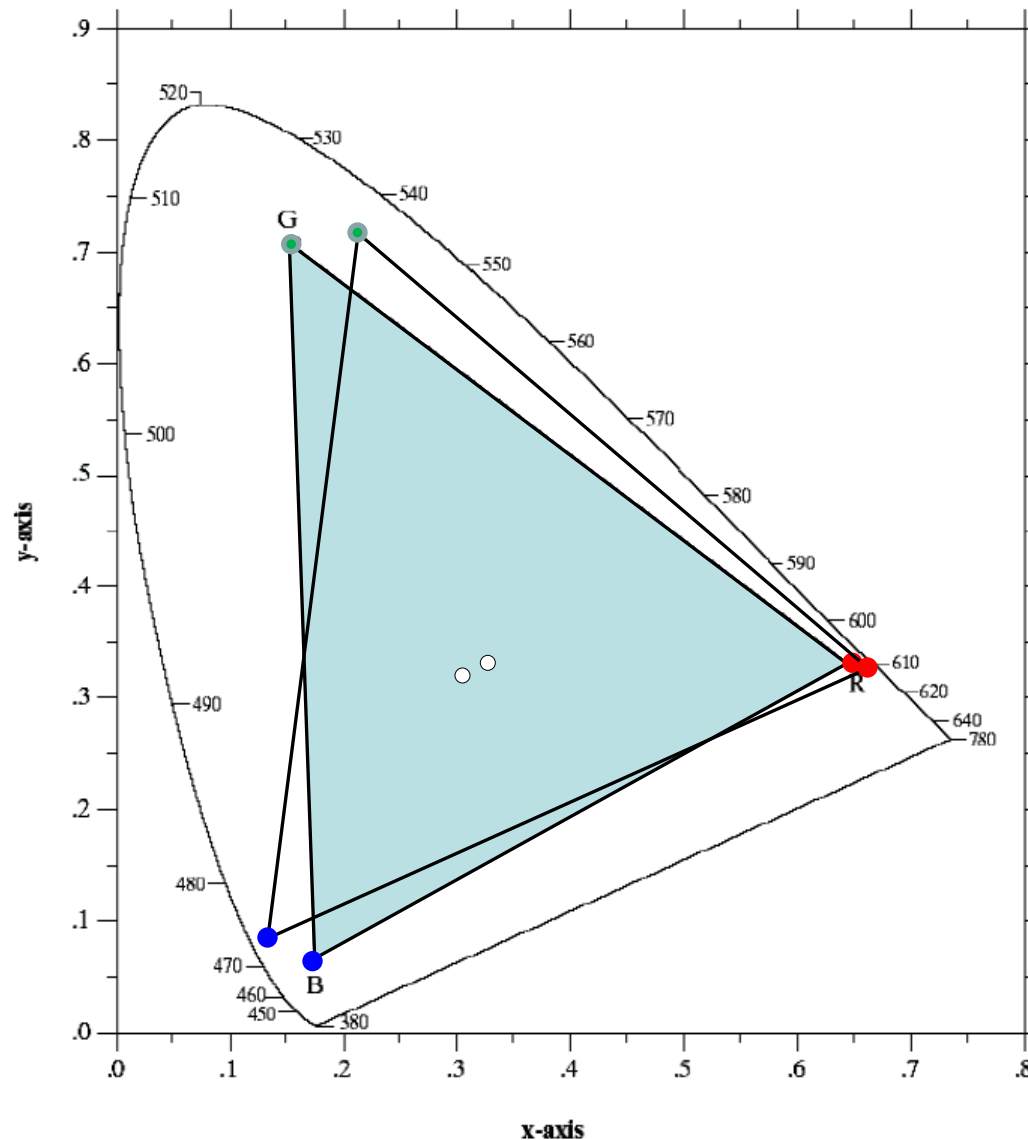
# Inaccuracy for Color Matches



Just noticeable  
chromaticity  
differences  
(10x enlarged)

[MacAdam, 1942]

# Color Gamut



**NTSC phosphors:**

R:  $x=0.67$ ,  $y=0.33$

G:  $x=0.21$ ,  $y=0.71$

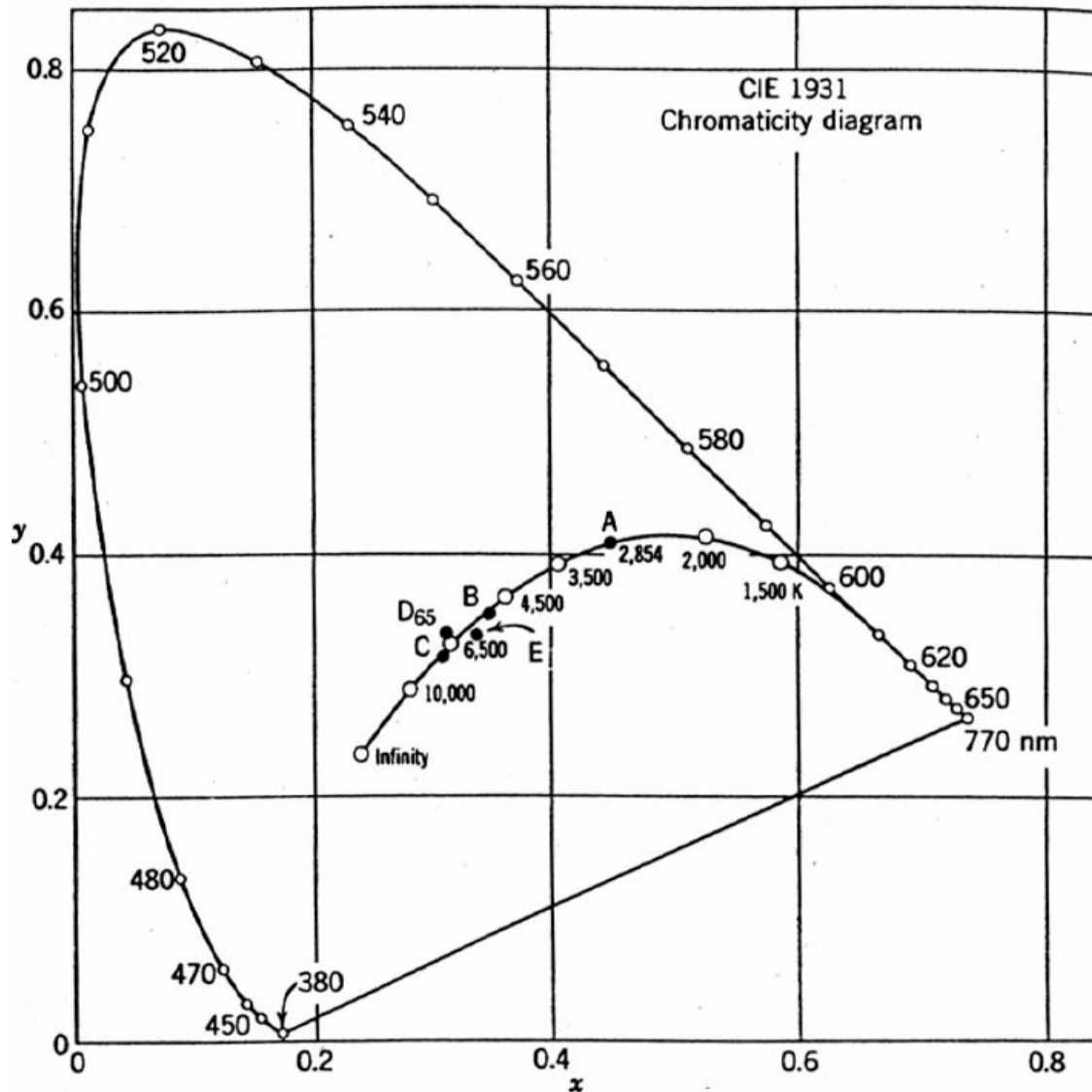
B:  $x=0.14$ ,  $y=0.08$

Reference white:

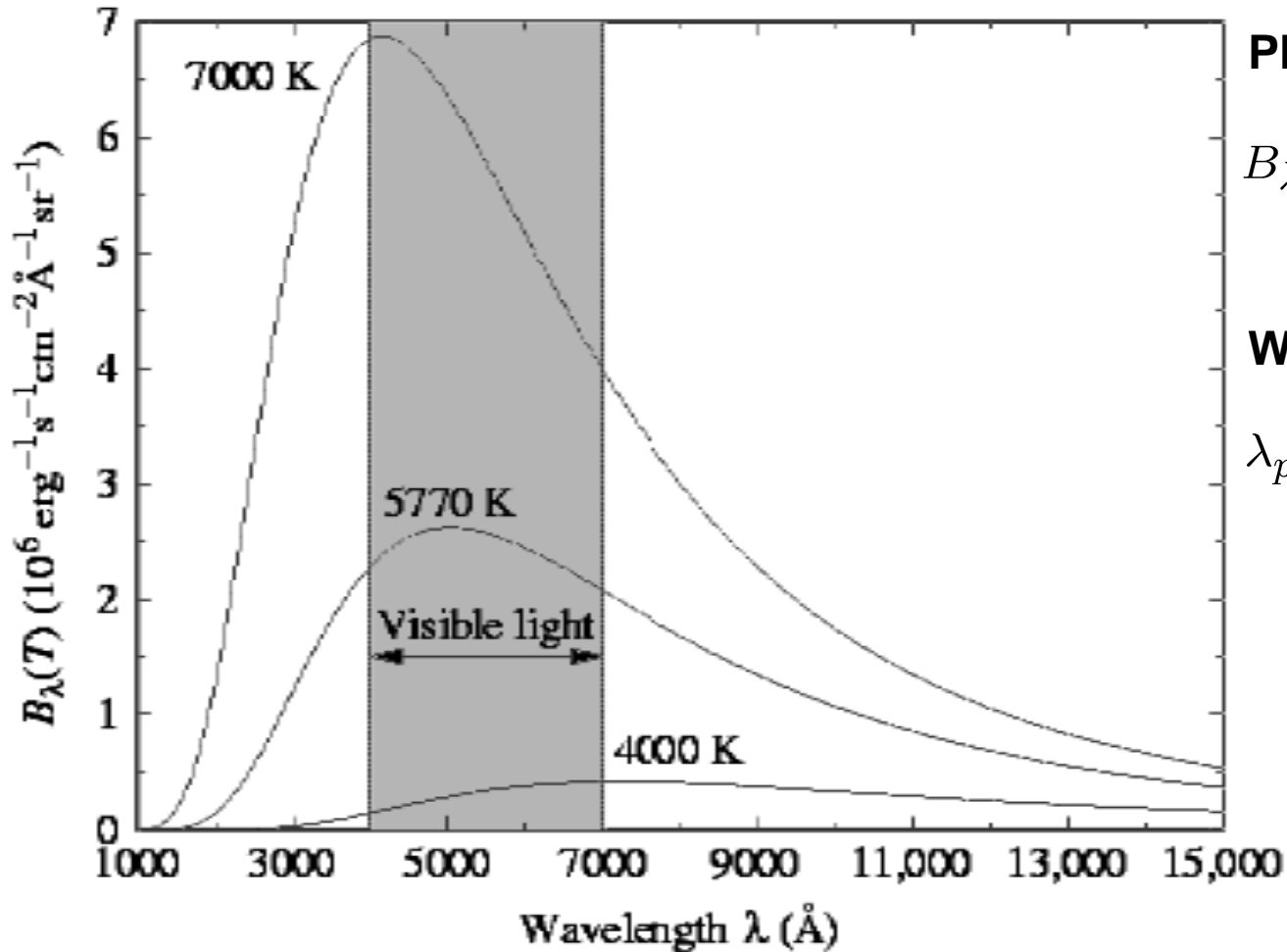
$x=0.31$ ,  $y=0.32$

Illuminant C

# White at Different Color Temperatures



# Blackbody Radiation



## Planck's Law

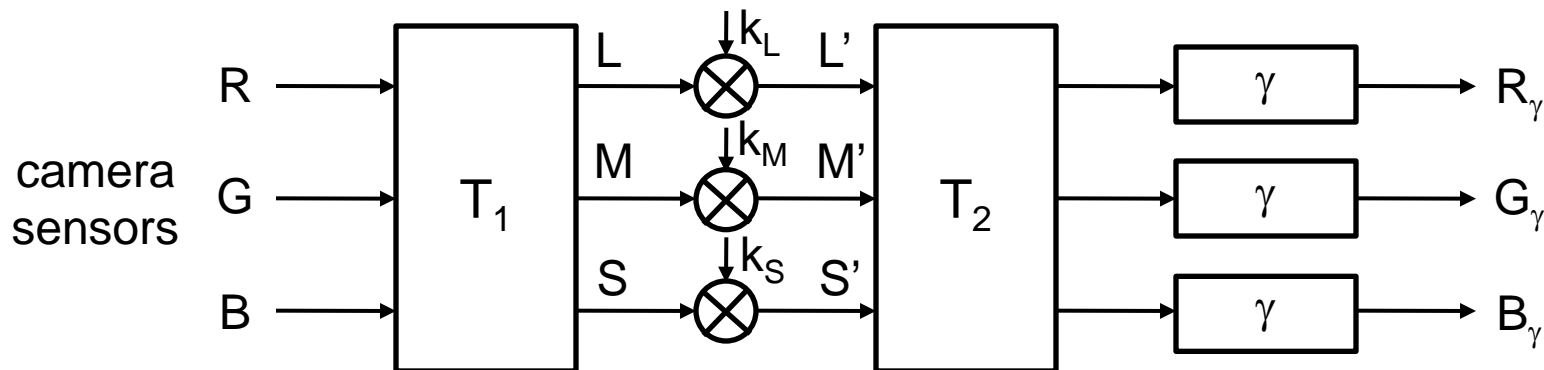
$$B_\lambda(T) = \frac{2hc^2}{\lambda^5} \frac{1}{e^{\left(\frac{hc}{\lambda kT}\right)} - 1}$$

## Wien's Law

$$\lambda_{peak} = \frac{2900 \mu m K}{T}$$

# Color Balancing

- Effect of different illuminants can be cancelled only in the spectral domain (impractical)
- Color balancing in 3-d color space is a practical approximation
- Color constancy in human visual system: Gain control in LMS cone space [\[von Kries, 1902\]](#)
- Von Kries hypothesis applied to image acquisition devices (cameras, scanners)



- Which color space is best?
- How to determine  $k_L$ ,  $k_M$ ,  $k_S$  automatically?



# Color Balancing

- Von Kries hypothesis

$$\begin{bmatrix} L' \\ M' \\ S' \end{bmatrix} = \begin{bmatrix} k_L & 0 & 0 \\ 0 & k_M & 0 \\ 0 & 0 & k_S \end{bmatrix} \begin{bmatrix} L \\ M \\ S \end{bmatrix}$$

- If illumination (or a patch of white in the scene) is known, calculate

$$k_L = \frac{L_{desired}}{L_{actual}} \quad k_M = \frac{M_{desired}}{M_{actual}} \quad k_S = \frac{S_{desired}}{S_{actual}}$$

# Color Balancing with Unknown Illumination

- Gray-world

$$\sum_{image} k_L L = \sum_{image} k_M M = \sum_{image} k_S S$$

- Apply gray-world algorithm to a subset of pixels
  - Exclude saturated colors
  - Bright pixels only
- Scale-by-max algorithm
  - Determine  $\max(L)$ ,  $\max(M)$ ,  $\max(S)$  separately in each channel
  - Scale each channel by its max
  - Sensitive to saturation

# Color Balancing Example



original



scale-by-max  
color balancing

# Color Balancing Example



original



scale-by-max  
color balancing