Operator Theory and Indefinite Inner Products

TU Wien, February 7-9, 2025

Program & Abstracts

(updated January 29, 2025)

Friday 7.2.2025

8'30 `Registration (Sem 03 A, 3rd floor/green)`

`- HS 7, 2nd floor/yellow -`

9'00 - 9'30	Opening
9'30 - 10'10	*Christiane Tretter:*
10'10 - 10'40	*Vadym Adamyan:* Asymptotic properties of systems of orthogonal entire functions generated by Hermite positive and Hermite indefinite functions

10'40 `Coffee Break & Late Registration (Sem 03 A, 3rd floor/green)`

`- HS 7, 2nd floor/yellow -`

11'20 - 12'00	*Aad Dijksma:* Self-adjoint coupling, Straus subspaces, and the equation $\mathcal{W}(z)\mathcal{P}(z) \equiv \mathcal{P}(z)V$
12'00 - 12'40	*Aurelian Gheondea:* On Singularity of Critical Points of Definitisable Operators

12'40 `Lunch`

`- HS 7, 2nd floor/yellow -`

14'50 - 15'30	*Michael Kaltenbäck:* Heinz's theorem on definitizable selfadjoint operators on Krein spaces and beyond
15'30 - 16'10	*Matthias Langer:*

16'10 `Coffee Break (Sem 03 A, 3rd floor/green)`

`- HS 7, 2nd floor/yellow -`

16'50 - 17'30	*Kresimir Veselic:* Optimising damped systems
17'30 - 18'10	*Ilia Spitkovsky:*

Saturday 8.2.2025

- HS 7, 2nd floor/yellow -

9'00 - 9'40	*Jussi Behrndt:* Spectral and perturbation theory for definitizable operators
9'40 - 10'20	*Mark Malamud:* Trace formulas for pairs of nonselfadjoint operators. Langer's contribution to the theory and its development.

10'20 Coffee Break (Sem 03 A, 3rd floor/green)

- HS 7, 2nd floor/yellow -

11'00 - 11'40	*Henk de Snoo:* The Langer-Textorius paper, almost fifty years ago
11'40 - 12'20	*Seppo Hassi:* Unitary boundary pairs for isometric operators in Pontryagin spaces, generalized coresolvents, and Krein's formula

12'20 Lunch

- HS 7, 2nd floor/yellow -

14'30 - 15'10	*Daniel Alpay:* Hyper-positive functions and dissipativity
15'10 - 15'50	*Annemarie Luger:* From Nevanlinna functions to quasi-Herglotz and beyond

15'50 Coffee Break (Sem 03 A, 3rd floor/green)

- HS 7, 2nd floor/yellow -

16'30 - 17'00	*Manfred Möller:* On continuous parameter dependence of roots of analytic functions
17'00 - 17'30	*:*

19'30 Conference Dinner (Cafe Wortner, Wiedner Hauptstrasse 55)

Sunday 9.2.2025

- HS 7, 2nd floor/yellow -

9’30 - 10’10	*Zoltan Sasvari:* On measurability and decomposition of functions with a finite number of negative squares. Two questions of Heinz Langer
10’10 - 10’50	*Marco Marletta:* Essential spectra of Maxwell systems and some remarks on analytic pencils

10’50 Coffee Break (Sem 03 A, 3rd floor/green)

- HS 7, 2nd floor/yellow -

11’30 - 12’10	*Harald Woracek:* Indefinite canonical systems and applications
12’10 - 12’50	*Henrik Winkler:* Canonical systems of differential equations on star-like graphs

12’50 Closing

Abstracts

Asymptotic properties of systems of orthogonal entire functions generated by Hermite positive and Hermite indefinite functions

Adamyan Vadym `Friday 10'10 (online)`

We discuss asymptotic relations in systems of orthogonal entire functions $\{e_t(\lambda)\}_{t=0}^{\infty}$ on the axis $-\infty < \lambda < \infty$ derived from the Gram-Schmidt orthogonalization of the exponents $\{e^{it\lambda}\}_{t=0}^{\infty}$ with the Gram-Schmidt matrices for $0 < t \le r$ replaced by the Fredholm integral operators in $L^2(0,r)$ with generalized kernels

$$\delta(t-t') + H(t-t'), \quad 0 \le t, t' \le r,$$

where $\delta(t)$ is the Dirac delta function, and $H(t) = \overline{H(-t)}$ are continuous functions that admit the representation

$$H(t) = \int_{-\infty}^{\infty} e^{i\lambda t}\, d\Omega(\lambda)$$

with some bounded variation function $\Omega(\lambda)$.

The results can be used to create algorithms for addressing problems related to the continuation of Hermite-positive functions.

Hyper-positive functions and dissipativity

Alpay Daniel `Saturday 14'30`

Positive functions map the open right half-plane $\mathbb{C}_r$ into its closure and serve as a model for passive continuous-time, linear, time-invariant systems. Within this set, functions mapping $\mathbb{C}_r$ into a sub-region of $\mathbb{C}_r$, which can be contained in a finite disk, can be identified with quantitative dissipativity. Such functions are called hyper-positive and, in the scalar case, are characterized by an inequality

$$F(z) + F(z)^* \ge \beta(1 + |F(z)|^2), \quad z \in \mathbb{C}_r$$

for some $\beta \in (0,1)$. In the talk we consider such functions in the matrix-valued setting, replacing the above condition by the positive-definiteness of the kernel

$$\frac{F(z) + F(w)^* - T - F(z)TF(w)^*}{z + \overline{w}}.$$

where $T \in \mathbb{C}^{n\times n}$ satisfies $0 < T < I_n$. We present their main properties such as minimal realizations in the rational case, and study the associated reproducing kernels spaces. Connections to stability in the spirit of the Kalman-Yakubovich-Popov lemma are also presented.

This is joint work with Izchak Lewkowicz (Ben-Gurion University of the Negev, Beer-Sheva, Israel).

References:

D.A and Izchak Lewkowicz, Quantitatively hyper-positive real functions. LAA 623 (2021) 316-334
D.A and Izchak Lewkowicz, Quantitatively hyper-positive real functions II. LAA 697 (2024) 332-364
D.A and Izchak Lewkowicz, Quantitatively hyper-positive real functions III, submitted.

Spectral and perturbation theory for definitizable operators

Behrndt Jussi `Saturday 9'00`

One of Heinz's most important contributions to operator theory in Krein spaces is the notion of definitizable self-adjoint operators introduced in his Habilitation thesis from 1965. The aim of this talk is to provide a brief introduction to spectral and perturbation theory of this class of operators and to highlight some of Heinz's main results in this area. A few more recent developments and typical applications to indefinite Sturm-Liouville operators will be discussed as well.

Self-adjoint coupling, Straus subspaces, and the equation $\mathcal{W}(z)\mathcal{P}(z) \equiv \mathcal{P}(z)\mathsf{V}$

Dijksma Aad `Friday 11'20`

The lecture concerns recent joint work with Branko Ćurgus (Bellingham).

In a coupling theorem from 2001 we described a special class of canonical self-adjoint extensions of the direct sum of symmetric linear relations S_1 and S_2 in Krein spaces $\mathfrak{H}_1$ and $\mathfrak{H}_2$ and assigned a unique parameter to each of these extensions. Assuming that $\mathfrak{H}_2$ is finite dimensional and that S_2 is an operator without eigenvalues, we construct a model for $(\mathfrak{H}_2, S_2)$ based on an essentially unique polynomial matrix $\mathcal{P}(z)$. The families of Straus subspaces associated with the self-adjoint extensions are characterized as restrictions of S_1^* by polynomial boundary conditions involving $\mathcal{P}(z)$ and the parameters. We establish necessary and sufficient conditions on the parameters under which the extensions are similar and the corresponding families of Straus subspaces coincide. Related to our results is the equation $\mathcal{W}(z)\mathcal{P}(z) = \mathcal{P}(z)\mathsf{V}$ in which the unimodular matrix polynomial $\mathcal{W}(z)$ and the invertible matrix V are the unknowns. Explicit examples are given.

On Singularity of Critical Points of Definitisable Operators

Gheondea Aurelian `Friday 12'00`

One of the most important legacy of Heinz Langer in mathematics is the spectral theory of selfadjoint definitisable operators in Krein spaces which started with his Habiliationsschrift in 1965 and continues to our days to be one of the most important achievements and tools in operator theory on indefinite inner product spaces. The behaviour of the spectral function in the neighbourhood of critical points is one of the technical thing that is mostly challenging and there are many attempts to characterise it by different analytical methods. We focus on discussing how intuition can be mislead in understanding the singularity and to what kind of mathematical problems it is related to. Along the way, we point out certain mathematical facts that point out the extraordinary talent of Heinz Langer to trigger important mathematical problems and new perspectives.

Unitary boundary pairs for isometric operators in Pontryagin spaces, generalized coresolvents, and Kreĭn's formula

Hassi Seppo `Saturday 11'40`

Extension theory for symmetric and isometric operators in Pontryagin spaces was initiated by I.S. Iokhvidov and M.G. Kreĭn in the 1950s. Generalized resolvents of such operators were described by M.G. Kreĭn and H. Langer in early 1970s and later further developed by A. Dijksma, H. Langer, and H.S.V. de Snoo for (standard) isometric and symmetric relations even in the Kreĭn space setting. For a nonstandard isometric operator in a Pontryagin space, a description of its regular generalized resolvents in a Pontryagin space, without the growth of the negative index, was given by P. Sorjonen (1985), while the non-regular case allowing also a growth of the negative index was studied by O. Nitz (2000).

In the present talk we describe a coupling approach to study generalized coresolvents of isometric operators in the Pontryagin space setting. The methods used rely on a new general notion of boundary pairs for isometric operators in the Pontryagin space setting. Even in the Hilbert space case this notion generalizes the concept of boundary triples and associated Weyl functions for isometric operators introduced (more generally for dual pairs of operators) by M.M. Malamud and V.I. Mogilevskii (2003, 2004). The notion of boundary pairs for isometric operators offers an alternative approach to study operator valued generalized Schur functions without any additional invertibility requirements (at the origin). Combining this realization method for generalized Schur functions with the coupling method of boundary pairs, the generalized coresolvents of isometric operators via an analog of Krein'n formula can be obtained in the general case.

The coupling method was in fact initially appearing in order to give a simple solution to a problem introduced to me by Heinz Langer in Vienna 1992. That problem was concerning an invariance result of exit spaces when the exit spaces are constructed with the so-called ε-method simultaneously for a family of Nevanlinna functions, which were used as spectral parameters in the boundary conditions.

The talk is based on joint work with Vladimir Derkach and Dmytro Baidiuk.

Heinz' s theorem on definitizable selfadjoint operators on Krein spaces and beyond

Kaltenbäck Michael `Friday 14'50`

The Spectral Theorem for selfadjoint operators on Hilbert spaces nowadays is one of the cornerstones of operator theory. In the situation of a selfadjoint operator A on a Krein space $(K, [.,.])$ there is no analogue of this resut in general. Nevertheless assuming the existence of a definitizing polynomial p, which means $[p(A)x, x] \geq 0$ for all $x \in K$, Heinz found a way to construct spectral projections related to A by using the Riesz-Dunford functional calculus.

In the talk I am going to deliver I will recall some of Heinz's methods in proofing his 'Spectral Theorem for definitizable selfadjoint operators on Krein spaces'. Moreover, I will give an overview of the results that we achieved for definitizable operators by putting focus more on the functional calculus coming along with the spectral theorem and less on the spectral projections. With this approach also a spectral theorem for normal definitizable operators could be found.

From Nevanlinna functions to quasi-Herglotz and beyond

Luger Annemarie `Saturday 15'10`

Starting from Nevanlinna functions, which basically are resolvents of self-adjoint relations in Hilbert spaces, we will give an overview on different generalizations as well as applications.

Trace formulas for pairs of nonselfadjoint operators. Langer's contribution to the theory and its development.

Malamud Mark Saturday 9'40

The talk is devoted to perturbation determinants and trace formulas for a pair of operators with the trace class resolvent difference.

For a pair of selfadjoint operators $\{A_0, A_1\}$ with the trace class resolvent difference this formula as well as a concept of the spectral shift function were introduced by I.M. Lifshitz (the case of finite dimensional perturbations) and M. G. Krein (general case). The first work treated a pair of non-selfadjoint operators is due to H. Langer [1]. More precisely, he generalized Krein's formula for a pair $\{T_1, T_2\}$ of bounded operators with the spectra lying within the unit disc and computed the spectral shift function via the perturbation determinant of the pair $\{T_1, T_2\}$.

We will discuss the present state of this topic including certain recent results as well as Langer's influence on the theory.

Following [2]-[5] we will discuss the existence of complex valued spectral shift function for a pair of maximal dissipative operators $\{L_1, L_2\}$ with trace class resolvent difference. For such pairs of operators the Krein-Langer type trace formulas are established for a class of operator Lipschitz functions. The proof is substantially relied on boundary triplet technique and the method of double operator integrals.

For instance, we plan to discuss a formula for the spectral shift function of a pair of m-dissipative extensions $\{L_1, L_2\}$ of a symmetric operator as well as the formula for the corresponding perturbation determinant of the pair $\{L_1, L_2\}$. Both objects are expressed via the Weyl function and boundary operators.

The problem of existence **a real valued spectral shift function** for such pairs of operators will also be discussed. Applications to boundary value problems for differential operators will be discussed too.

The talk is based on our joint works with H. Neidhardt and V. Peller [2]–[5]. Some recent results in this direction will be discussed too.

References

[1] Heinz Langer, *Eine Erweiterung der Spurformel der Storungstheorie.*
Math. Nachr., **30**, (1965), 123–135.
[2] M.M. Malamud, H. Neidhardt, *Perturbation determinants for singular perturbations.*
Russ. J. Math. Phys. **21**, (2014), 55–98.
[3] M. Malamud, H. Neidhardt, *Trace formulas for additive and non-additive perturbations.*
Adv. Math. **274**, (2015), 736–832.
[4] M.M. Malamud, H. Neidhardt, V.V. Peller, *Absolute continuity of spectral shift.*
J. Funct. Anal. **276**, (2019), 1575–1621.
[5] M. M. Malamud, H. Neidhardt, and V. V. Peller,
Real-Valued Spectral Shift Functions for Contractions and Dissipative Operators,
Doklady Mathematics, **110, No. 2**, (2024), 399–403.

Essential spectra of Maxwell systems and some remarks on analytic pencils

Marletta Marco Sunday 10'10

We consider how to calculate the essential spectra of time-harmonic dissipative Maxwell systems and Drude-Lorentz pencils. We show how additional unexpected essential spectrum arises at discontinuity interfaces between conductive and non-conductive regions, somewhat related to 'black hole modes? and plasmons observed for second order elliptic equations with sign-changing leading coefficients by several authors. I shall make a case for a particular definition of the $\sigma_{e,5}$ essential spectrum when the problems depend on the spectral parameter in a rather general way. (Hint: it is not the pointwise definition that is natural for the other σ_{ek}.) If time permits I shall also discuss a model of a Faraday layer, for which these ideas turn out to be indispensable.

This talk is based on a series of five papers I have written over the last 6 years with Giovanni Alberti (Genova), Sabine Boegli (Durham), Malcolm Brown (deceased), Francesco Ferraresso (Sassari) and Christiane Tretter.

On continuous parameter dependence of roots of analytic functions

Möller Manfred Saturday 16'30 (online)

Considering pencils of ordinary differential operators with eigenvalue parameter dependent boundary conditions whose spectrum only consists of eigenvalues with finite multiplicity, the spectrum is given by the zeros of the characteristic function. Often, such a problem can be considered as a perturbation of a simpler problem, with corresponding perturbation of the characteristic functions. To the best of my knowledge, the global behaviour of the zeros have so far always been considered in an ad hoc way. I will present a general result on continuous dependence of the zeros of continuous families of analytic functions [2]. This is inspired by corresponding isomorphism results between the sets of polynomials and the (multi-)sets of their zeros; in particular, the topological approach used in [1].

References

[1] Ćurgus, B.; Mascioni, V., Roots and polynomials as homeomorphic spaces, Expo. Math. 24, No. 1, 81-95 (2006).

[2] Möller, M., On continuous parameter dependence of roots of analytic functions, Result. Math. 79, issue 2, 9 pages, (2024), Url 10.1007/s00025-023-02079-y.

On measurability and decomposition of functions with a finite number of negative squares. Two questions of Heinz Langer

Sasvári Zoltán `Sunday 9'30`

We consider some problems concerning measurability and decomposition of positive definite functions and of functions with a finite number of negative squares. The decompositions are of the form $f_c + f_0$ where f_c has a finite number of negative squares and is continuous while f_0 is positive definite, not continuous and in some sense near to zero. Our attention was drawn to these problems by Heinz Langer.

The Langer-Textorius paper, almost fifty years ago

de Snoo Henk `Saturday 11'00`

In 1977 appeared "On generalized resolvents and Q-functions of symmetric linear relations (subspaces) in Hilbert space" by Heinz Langer and Björn Textorius. I will give a little survey of some of the consequences of this paper.

Optimising damped systems

Veselic Kresimir `Friday 16'50`

We give a short overview on controlling damped linear systems by use of the Lyapunov equation. We then show that different approaches, some of them known, some new - can be covered by the said equation.

Canonical systems of differential equations on star-like graphs

Winkler Henrik `Sunday 12'10`

For n canonical systems of differential equations the corresponding n copies of their domain $[0,\infty)$ are thought of as a graph with vertex 0. An interface condition at 0 is given by a so-called Nevanlinna pair. Explicit formulas are deduced for the spectral representation of the corresponding underlying selfadjoint relation and the generalized Fourier transformation. Further, results on compressions of the Fourier transformation to closed linear subspaces and the multiplicity of the eigenvalues if the spectrum is discrete are presented.

Joint work with Henk de Snoo

Indefinite canonical systems and applications

Woracek Harald `Sunday 11'30`

By de Branges inverse spectral theorem the set of all Nevanlinna functions (analytic functions q in the upper half plane with positive semidefinite kernel $N_q(w,z) := \frac{q(z)-\overline{q(w)}}{z-\overline{w}}$) corresponds via Weyl's limit point construction bijectively to the set of all positive semidefinite and trace-normalised Hamiltonians on the half line $(0,\infty)$.

M.G.Krein and H.Langer introduced and studied the class of generalised Nevanlinna functions (meromorphic function in the upper half plane whose kernel N_q has a finite number of negative squares). We discuss a generalisation of the notion of a canonical systems adapted to the setting of "finitely many negative squares", these are systems with a finite number of inner singularities behaving not too badly. Further we present some applications.

The task to find a notion of "indefinite canonical system" and prove a generalisation of de Branges' theorem was given to me by Heinz as the first thing when I started my PhD (I didn't understand a word then). It took about 15 years of hard work together with my friend and colleague M.Kaltenbäck to develop that theory.

Participants

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