

# Operator Theory and Approximation

Vienna University of Technology, July 8-12, 2024

## Program & Abstracts

(updated July 5, 2024)

# Monday 8.7.2024

09'00 - 9'40 Registration (Sem 03 A, 3rd floor/green)

- HS 3, 2nd floor/yellow -

9'45 - 10'00	Opening
10'00 - 11'00	<i>Misha Sodin</i> : The Szegő minimum problem

11'00 - 11'45 Coffee Break / Late Registration (Sem 03 A, 3rd floor/green)

- HS 3, 2nd floor/yellow -

11'45 - 12'45	<i>Roman Bessonov</i> : Canonical Hamiltonian systems with periodic Weyl functions
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12'45 - 14'30 Lunch Break

- HS 3, 2nd floor/yellow -

- HS 2, 2nd floor/yellow -

14'30 - 15'00	<i>Aleksey Kostenko</i> : Trace formulas and inverse spectral theory for strings	<i>Marlena Nowaczyk</i> : Spectrum of the Laplace operator on quantum graphs and defects in a graphene lattice
15'00 - 15'30	<i>Andrea Mantile</i> : Scattering theory with both regular and singular perturbations and applications	<i>Anna Muranova</i> : Capacity of graphs over non-Archimedean ordered fields
15'30 - 16'00	<i>Mateusz Piorkowski</i> : A finer limit circle/limit point classification for trace class Sturm-Liouville operators	<i>Chung-Chuan Chen</i> : Recurrent operators on Orlicz spaces

16'00 - 16'45 Coffee Break (Sem 03 A, 3rd floor/green)

- HS 3, 2nd floor/yellow -

- HS 2, 2nd floor/yellow -

16'45 - 17'15	<i>Jakob Reiffenstein</i> : Eigenvalue density of limit circle Jacobi operators and related canonical systems	<i>Yosra Barkaoui</i> : Unbounded product of two non negative selfadjoint operators
17'15 - 17'45	<i>Grzegorz Świdorski</i> : Asymptotic zeros' distribution of orthogonal polynomials with unbounded recurrence coefficients	<i>Nigar Aslanova</i> : On one class eigenvalue problems for fourth order differential operator equation

## Tuesday 9.7.2024

- HS 3, 2nd floor/yellow -

9'00 - 10'00	<i>Christiane Tretter</i> : Challenges for non-selfadjoint spectral problems in analysis and computation
10'15 - 11'15	<i>Pavel Exner</i> : On time-reversal invariance violation in quantum graphs

11'15 - 12'00 Coffee Break (Sem 03 A, 3rd floor/green)

- HS 3, 2nd floor/yellow -

- HS 2, 2nd floor/yellow -

	<b>Thematic session: Approximation Theory</b>	12'00 - 12'30 <i>Dong Liang</i> : Legendre-tau Chebyshev Spectral Method for Maxwell's Equations with Interfaces and its Theoretical Analysis
12'00 - 13'00	<i>Maxim Zinchenko</i> : Chebyshev and Extremal Polynomials	12'30 - 13'00 <i>Hong Hai Ly</i> : Stabilization by multiplicative Ito noise for Chafee-Infante equation in perforated domains

13'00 - 14'30 Lunch Break

- HS 3, 2nd floor/yellow -

- HS 2, 2nd floor/yellow -

14'30 - 15'00	<i>Jacob Christiansen</i> : Chebyshev polynomials and Widom factors	<i>Andreas Buchinger</i> : Strong Operator Convergence in Homogenization of PDEs with Nonlocal Coefficients
15'00 - 15'30	<i>Olof Rubin</i> : Minimal polynomials with prescribed zeros	<i>Michel Alexis</i> : Quantum Signal Processing and the nonlinear Fourier Transform

15'30 - 16'15 Coffee Break (Sem 03 A, 3rd floor/green)

- HS 3, 2nd floor/yellow -

- HS 2, 2nd floor/yellow -

16'15 - 16'45	<i>Nikos Stylianopoulos</i> : Grunsky Operator Connecting Christoffel Functions to Faber Polynomials	<i>Phuoc-Tai Nguyen</i> : Semiclassical Moser-Trudinger inequalities
16'45 - 17'15	<i>Benedikt Buchecker</i> : Extremal polynomials on a Jordan curve or arc	<i>Thi Minh Thao Le</i> : Cwikel-Lieb-Rozenblum type inequalities for Hardy-Schrödinger operator

19'00 Conference Dinner (Wieden Bräu, Waaggasse 5)

## Wednesday 10.7.2024

- HS 3, 2nd floor/yellow -

10'00 - 11'00	<i>Milivoje Lukić</i> : Universality limits via canonical systems
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11'00 - 11'45 Coffee Break (Sem 03 A, 3rd floor/green)

- HS 3, 2nd floor/yellow -

11'45 - 12'45	<i>Peter Yuditskii</i> : Periodically modulated rescaling limits and canonical systems with cantor spectrum
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Free Afternoon

18'00 - 20'00 Guided tour (meeting point: Vienna state opera, Herbert-von-Karajan Platz)

## Thursday 11.7.2024

- HS 3, 2nd floor/yellow -

9'30 - 10'30	<i>Arno Kuijlaars</i> : Multiple orthogonal polynomials in random matrix theory
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10'30 - 11'15    **Coffee Break** (Sem 03 A, 3rd floor/green)

- HS 3, 2nd floor/yellow -

11'15 - 12'15	<i>Sergey Denisov</i> : Strong asymptotics of MOP
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12'15 - 14'00    **Lunch Break**

- HS 3, 2nd floor/yellow -

- HS 2, 2nd floor/yellow -

14'00 - 14'30	<i>Gordon Blower</i> : Linear systems, differentials and determinants	<i>Janko Bračič</i> : Ultrainvariant subspaces
14'30 - 15'00	<i>Felix Schwenninger</i> : Reproducing kernel theses and Pisier's example	<i>Mark Malamud</i> : To the Birman-Krein-Visic Theory. Solution to the Birman problem.
15'00 - 15'30	<i>Giorgio Young</i> : Rational solutions to the mKdV equation	<i>Elhadj Dahia</i> : p-regularity of bilinear Operators between Banach lattices

15'30 - 16'15    **Coffee Break** (Sem 03 A, 3rd floor/green)

- HS 3, 2nd floor/yellow -

- HS 2, 2nd floor/yellow -

16'15 - 16'45	<i>Alexander Kheifets</i> : Automorphic Caratheodory - Julia Theorem and Related Boundary Interpolation	<i>Yuri Karlovich</i> : Mellin pseudodifferential operators with non-regular symbols and their algebras
16'45 - 17'15	<i>Antti Haimi</i> : Zeros of Gaussian Weyl-Heisenberg functions	<i>Karen Avetisyan</i> : Some Bergman type operators and projections on mixed norm and Besov spaces
17'15 - 17'45	<i>Ahmed Abdeljawad</i> : Approximation with Neural Networks: Impact of Activation Functions	<i>Wentao Teng</i> : Imaginary Powers of (k,a)-Generalized Harmonic Oscillator

## Friday 12.7.2024

- HS 3, 2nd floor/yellow -

10'00 - 11'00	<i>Marius Lemm</i> : Spectra of mildly random matrices
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11'00 - 11'45 Coffee Break (! Sem 03 C!, 3rd floor/green)

- HS 3, 2nd floor/yellow -

11'45 - 12'45	<i>Matthias Langer</i> : Canonical systems, Weyl coefficients and eigenvalues
12'45 - 13'00	Closing

13'00 - Coffee (! Sem 03 C!, 3rd floor/green)

# Abstracts

## Approximation with Neural Networks: Impact of Activation Functions

Abdeljawad Ahmed

Thursday 17'15 (HS3)

In this talk, we explore the approximation capabilities of deep neural networks in two distinct contexts. First, we examine the challenges and solutions associated with different activation functions, focusing on the impact of using the Rectified Power Unit (RePU) activation for approximating Sobolev-regular functions.

Next, we delve into the practical challenges of computing these approximations from point samples, addressing the “theory-to-practice gap” in deep learning. Our study reveals that functions approximable by neural networks with RePU activation require an exponentially growing number of samples as the input dimension increases. This finding provides a comprehensive theoretical understanding of the sampling complexity bounds for neural network approximation spaces in connection with the chosen activation.

This presentation is based on a series of works including:

- “Sampling Complexity of Deep Approximation Spaces” by A. A. and Philipp Grohs, *Analysis and Applications*, (2023).
- “Approximations with Deep Neural Networks in Sobolev Time-Space” by A. A. and Philipp Grohs, *Analysis and Applications*, (2022).
- “Deep Neural Network Approximation for Hölder Functions” by A. A. , *arXiv:2201.03747*, (2022).

## Quantum Signal Processing and the nonlinear Fourier Transform

Alexis Michel

Tuesday 15'00 (HS2)

Quantum Signal Processing (QSP) is an algorithmic process by which one represents a signal  $f : [0, 1] \rightarrow (-1, 1)$  as the upper left entry of a product of  $SU(2)$  matrices parametrized by the input variable  $x \in [0, 1]$  and some “phase factors”  $\{\psi_k\}_{k \geq 0}$  depending on  $f$ . QSP was well-defined for polynomial signals  $f$ , but how to represent arbitrary signals  $f : [0, 1] \rightarrow (-1, 1)$  was not fully understood till our recent work relating QSP to the nonlinear Fourier transform. We will

see that, after a change of variables, QSP is actually the  $SU(2)$  model of the nonlinear Fourier transform, and the phase factors  $\{\psi_k\}_k$  correspond to the nonlinear Fourier coefficients. Then, by exploiting a nonlinear Plancherel identity and using some basic operator theory, we will show that QSP can be extended to all signals  $f$  satisfying the log integrability condition

$$\int_0^1 \log(1 - f(x)^2) \frac{dx}{\sqrt{1 - x^2}} > -\infty.$$

## On one class eigenvalue problems for fourth order differential operator equation

Aslanova Nigar

Monday 17'15 (HS2)

We consider the next eigenvalue problem in space  $L_2(H, (0, 1))$

$$y^{IV}(t) + Ay(t) + q(t)y(t) = \lambda y(t) \quad (1)$$

$$y(0) = y''(0) = 0 \quad (2)$$

$$-y'''(1) = \lambda Q_1 y(1) \quad (3)$$

$$y'(1) = \lambda Q_2 y'(1) \quad (4)$$

where  $H$  is abstract separable Hilbert space. Coefficients of equation are operators, namely  $A$  is unbounded self-adjoint operator with compact inverse,  $A^{-1} \in \sigma_\infty$ , and  $q(t)$  is bounded in  $H$  for each  $t \in [0, 1]$  from which follows also boundedness of it in  $L^2(0, 1; H)$ . Coefficients of boundary conditions  $Q_1, Q_2$  are also unbounded selfadjoint, positive-definite operators in  $H$ . Obviously, one can't give operator formulation of that problem with some self-adjoint operator without leaving the space  $L^2(0, 1; H)$ . Thus, our aim is definition of domain of minimal operator in exit space, give description of domains of maximal operator and self-adjoint extensions in terms of boundary conditions. And finally, give characterization of spectrum, find asymptotics of spectrum and derive formula for regularized trace of operator in operator setting of boundary value problem (1)-(4).

Introduce the next direct sum of Hilbert spaces

$$H = L_2(H, (0, 1)) \bigoplus H_Q^2$$

Scalar product of its elements  $Y = (y(t), y_1, y_2)$ ,  $Z = (z(t), z_1, z_2)$  is defined by

$$(Y, Z)_H = (y(t), z(t))_{L_2(H, (0, 1))} + (Q_1^{-1} y_1, z_1) + (Q_2^{-1} y_2, z_2).$$

Let the operator  $L'_0$  has the domain

$$D(L'_0) = \{Y \in H / Y = (y(t), Q_1 y(1), Q_2 y'(1))\},$$

$$y(t) \in C_0^\infty(H_\infty, (0, 1]), \quad y(1) \in D(Q_1), \quad y'(1) \in D(Q_2), \\ y(1), y''(1), y'''(1) \in H\},$$

where  $H_\infty = \bigcap_{j=1}^\infty (D(A^j))$ , and

$$L'_0(Y) = (ly, -y'''(1), y''(1)).$$

Closure of  $L'_0$  in  $H$  call the minimal operator. Its adjoint call maximal operator. We investigate spectral questions related to that operators and their self-adjoint extensions.

Given one relation between characteristic determinant and norming constants (reciprocal of norms of eigen-vectors) which is later applied to deriving the regularized trace formula.

## Some Bergman type operators and projections on mixed norm and Besov spaces

*Avetisyan Karen*

Thursday 16'45 (HS2)

As is well known, Bergman (type) projection operators  $T$  continuously map weighted Lebesgue and some more general mixed norm spaces  $L(p, q, \alpha)$  onto their holomorphic or harmonic subspaces  $h(p, q, \alpha)$  for suitable parameters. First, we find a necessary and sufficient condition for the operators  $T$  to be bounded on mixed norm spaces  $L(p, q, \alpha)$  over the unit ball in  $\mathbb{R}^n$ . To this end, we define harmonic reproducing kernels  $P_\alpha$  of Poisson–Bergman type given by a version of fractional derivative, and next prove sharp lower estimates for the kernels  $P_\alpha$  and their mixed norms. Second, for non-positive  $\alpha$ , Bergman projection  $T$  continuously maps mixed norm space  $L(p, q, \alpha)$  onto a harmonic Besov space. Then we turn to Besov spaces and define three-parameter Besov spaces  $\Lambda_\alpha^{p,q}$  of smooth functions over the unit ball in  $\mathbb{R}^n$ . A new family of Bergman type operators is constructed whose members are true projections from the Besov space  $\Lambda_\alpha^{p,q}$  onto its harmonic subspace  $h\Lambda_\alpha^{p,q}$ , see [1], [2].

## References

- [1] K. Avetisyan, *Estimates for harmonic reproducing kernel and Bergman type operators on mixed norm and Besov spaces in the real ball*, Annals Funct. Anal. **14** (2023), no. 2, Article 40, 29 pp.
- [2] K. Avetisyan, *Harmonic Poisson-Bergman kernel and Besov spaces in the real ball*, Indian J. Pure Appl. Math., 2024, (to appear).

## Unbounded product of two non negative selfadjoint operators

*Barkaoui Yosra*

Monday 16'45 (HS2)

The class of closed operators which can be written as the product  $AB$  of two non-negative selfadjoint operators  $A$  and  $B$ , where  $A$  is bounded and  $B$  is unbounded, is studied involving spectral and local spectral theory.

## Canonical Hamiltonian systems with periodic Weyl functions

*Bessonov Roman*

Monday 11'45 (HS3)

We characterize canonical Hamiltonian systems whose Weyl functions satisfy relation  $m(z+a) = m(z)$  for all  $z \in \mathbb{C}_+$  and some  $a > 0$ . We also discuss the problem of stable reconstruction of Hamiltonians corresponding to periodic Weyl functions.

## Linear systems, differentials and determinants

*Blower Gordon*

Thursday 14'00 (HS3)

Let  $(-A, B, C)$  be a continuous time linear system with state space a separable complex Hilbert space  $H$ , where  $-A$  generates a strongly continuous contraction semigroup  $(e^{-tA})_{t \geq 0}$  on  $H$ , and  $\phi(x) = Ce^{-x A} B$  is the impulse response function. Let  $\Gamma_\phi$  be the corresponding Hankel integral operator on  $L^2(0, \infty)$ . The paper introduces an algebra  $\mathcal{E}$  of operators on  $H$  in which one solves the Lyapunov equation  $dR_x/dx = -AR_x - R_x A$ , so that  $\det(I + R_0) = \det(I + \Gamma_\phi)$ . The paper gives several determinant formulas related to the Carey–Pincus formulas for multiplicative commutators. Special results hold when the quotient  $\mathcal{A}$  of  $\mathcal{E}$  by the algebra of compact operators is quasi-free in the sense of Cuntz and Quillen [J. Amer. Math. Soc. **8** (1995), 251-289]. Under suitable conditions on  $(-A, B, C)$ , this  $\mathcal{A}$  gives a commutative and finitely generated algebra of differential operators such that the maximal ideal space of determines a hyperelliptic spectral curve. Work of Gordon Blower (School of Mathematical Sciences, Lancaster University, UK) and Ian Doust (UNSW Sydney, Australia)

Keywords: spectral measures; Hankel operators; cyclic theory



## Ultrainvariant subspaces

*Bračić Janko*

Thursday 14'00 (HS2)

For an operator  $A$  on a complex Banach space  $X$  and a closed subspace  $M \subseteq X$ , the *local commutant* of  $A$  at  $M$  is the set  $C(A; M)$  of all operators  $T$  such that  $TAx = ATx$ , for all vectors  $x \in M$ . It is clear that  $C(A; M)$  is a closed space of operators, however, it is not an algebra, in general. One can show that  $C(A; M)$  is an algebra if and only if the subspace  $\text{gir}_A(M) = \{x \in X; TAx = ATx, \forall T \in C(A; M)\}$  is *ultrainvariant*, that is, invariant for every operator in  $C(A; M)$ . Every ultrainvariant subspace is hyperinvariant, but the opposite does not hold, in general. It is an open question of whether every operator with a non-trivial hyperinvariant subspace has a non-trivial ultrainvariant subspace.

## Extremal polynomials on a Jordan curve or arc

*Buchecker Benedikt*

Tuesday 16'45 (HS3)

For a given Jordan curve or arc  $\Gamma$  in the complex plane a classic problem is to consider a measure  $\mu$  supported on  $\Gamma$ ,  $z_0 \in \mathbb{C}$  and

$$\lambda_n(\mu, z_0) = \inf \left\{ \int |P|^2 d\mu \mid P \text{ is a polynomial of degree } \leq n \text{ and } P(z_0) = 1 \right\}.$$

This can be extended to  $z_0 = \infty$  by considering the norm of the  $n$ -th monic orthogonal polynomial. Instead of  $L^2$  it is also possible to consider the sup-norm on  $\Gamma$ . In both cases the right way to rescale is with the conformal mapping from the outside of  $\Gamma$  to the outside of the unit circle which is the notion of Widom factors. We will give a framework to characterize existing and new results which is related to the Szegő-function and Hardy space associated with a measure. We will also talk about the Ahlfors problem, a related polynomial minimization problem.

## Strong Operator Convergence in Homogenization of PDEs with Nonlocal Coefficients

*Buchinger Andreas*

Tuesday 14'30 (HS2)

In this talk, we will revisit the classical notion of homogenization of div-grad-systems (H-convergence) and its operator-theoretic description that allows for more general systems with possibly nonlocal coefficients (nonlocal H-convergence provided by M. Waurick). We will introduce a convergence theorem for the corresponding solution operators, and we will discuss its sharpness in the sense

of weak vs. strong operator convergence. This is joint work with S. Franz, N. Skrepek and M. Waurick.

## Recurrent operators on Orlicz spaces

*Chen Chung-Chuan*

Monday 15'30 (HS2)

In this talk, we will characterize topologically multiple recurrence for a single operator and sequences of operators on the Orlicz spaces of locally compact groups. These sequences of operators can be regarded as cosine operator functions. The sufficient conditions for sequences of operators to be disjoint topologically transitive and disjoint topologically mixing will be given as well.

## Chebyshev polynomials and Widom factors

*Christiansen Jacob S.*

Tuesday 14'30 (HS3)

Let  $E \subset \mathbb{C}$  be an infinite compact set, and denote by  $T_n$  the minimax (or Chebyshev) polynomials of  $E$ , i.e., the monic degree  $n$  polynomials minimizing the sup-norm on  $E$ . A well-known result by Szegő asserts that  $\|T_n\|_E \geq \text{Cap}(E)^n$  for all  $n$ , a lower bound that doubles when  $E \subset \mathbb{R}$ , as proven by Schiefermayr. More recently, Totik proved that for real subsets,  $\|T_n\|_E / \text{Cap}(E)^n \rightarrow 2$  if and only if  $E$  is an interval.

We will introduce the Widom factors, denoted by

$$W_n(E) := \frac{\|T_n\|_E}{\text{Cap}(E)^n}$$

and investigate whether there exist additional subsets of  $\mathbb{C}$  for which  $W_n(E) \rightarrow 2$ . It appears that the answer is affirmative for certain polynomial preimages. Interestingly, our proof relies on properties of the Jacobi orthogonal polynomials established by Bernstein. We will also discuss the symmetry properties underlying this phenomenon and explore related open problems.

The talk is based on joint work with B. Eichinger (TU Wien) and O. Rubin (Lund).

## **$p$ -regularity of bilinear Operators between Banach lattices**

*Dahia Elhadj*

Thursday 15'00 (HS2)

We introduce the new class of the  $(p; r, q)$ -regular bilinear operators between Banach lattices, that is defined using a summability property that provides the bilinear version of the  $(p, q)$ -regular operators. We show that every continuous bilinear operators are  $(p; r, q)$ -regular under some requirements. We find the trace duality representation of this class of bilinear operators by presenting a reasonable crossnorm that satisfies that the topological dual space of an 3-fold tensor product is isometric to the space of  $(p; r, q)$ -regular bilinear operators.

## **Strong asymptotics of MOP**

*Denisov Sergey*

Thursday 11'15 (HS3)

I will explain how the fixed point argument can be used to prove the strong asymptotics of multiple orthogonal polynomials. We will consider the Angelesco system and discuss the sharp analog of the celebrated Szegő theorem on asymptotics for polynomials orthogonal on a segment. The recently discovered connection between MOP and Jacobi matrices on trees provides applications of our results in spectral theory. Based on the joint work with A. Aptekarev and M. Yattselev.

## **On time-reversal invariance violation in quantum graphs**

*Exner Pavel*

Tuesday 10'15 (HS3)

The talk is concerned with quantum graphs the vertex coupling of which does not preserve the time-reversal invariance; the motivation for such a problem comes from recent attempts to use quantum graphs to model the anomalous Hall effect. As a case study we analyze the simplest example with the asymmetry being maximal at a fixed energy. In this situation the high-energy scattering depends crucially on the vertex parity; we will demonstrate implications of this fact for spectral and transport properties in several classes of graphs, both finite and infinite periodic ones. In particular, we prove the Band-Berkolaiko universality for Kagome lattices with this coupling. Furthermore, we discuss other time-asymmetric graphs and identify a class of such couplings which exhibits a nontrivial  $\mathcal{PT}$ -symmetry despite being self-adjoint; we also illustrate the role of

the Dirichlet component in the vertex coupling and discuss spectrum of the Cairo lattice. Finally, we show how a square lattice with such a coupling behaves in the presence of a homogeneous magnetic field when the two time-asymmetry mechanisms compete, the field effect being dominant at high energies. The results come from a common work with Marzieh Baradaran, Jiří Lipovský, and Miloš Tater.

## **Zeros of Gaussian Weyl-Heisenberg functions**

*Haimi Antti*

Thursday 16'45 (HS3)

We study zero sets of Gaussian random functions on the complex plane enjoying twisted stationarity, that is, stochastic invariance under the action of the Weyl-Heisenberg group. This family of models includes translation invariant the Gaussian entire function (GEFs), and also many other non-analytic examples originating from signal analysis and quantum mechanics. In general, winding numbers around zeros can be either positive or negative, while for GEF they are always positive due to analyticity. We investigate zero statistics both when zeros are weighted with their winding numbers (charged zero set) and when they are not (uncharged zero set). Firstly, we show that the charged zero sets are hyperuniform, which means informally that charge fluctuations are suppressed at large scales. Secondly, we show that in some central model cases hyperuniformity does not hold for uncharged zero sets.

This is joint work with Naomi Feldheim, Günther Koliander and José Luis Romero.

## **Mellin pseudodifferential operators with non-regular symbols and their algebras**

*Karlovich Yuri*

Thursday 16'15 (HS2)

Mellin pseudodifferential operators with bounded measurable symbols whose values belong to some subclasses of Fourier multipliers are studied on Lebesgue spaces. Criteria of the boundedness and compactness of these operators are established. The compactness of commutators and semicommutators of Mellin pseudodifferential operators is obtained for a subclass of non-regular symbols of limiting smoothness. Banach algebras of Mellin pseudodifferential operators with mentioned classes of non-regular symbols are investigated, and the corresponding symbol calculus is constructed. Applications of such Mellin pseudodifferential operators and their algebras are considered.

## Automorphic Carathéodory - Julia Theorem and Related Boundary Interpolation

*Kheifets Alexander*

Thursday 16'15 (HS3)

Let  $w$  be an analytic function on the unit disk,  $|w(\zeta)| \leq 1$ . Let  $t_0$  be a point on the unit circle,  $|t_0| = 1$ . The classical Carathéodory - Julia Theorem states in particular that if  $w$  and  $w'$  have nontangential boundary values  $w_0$ ,  $|w_0| = 1$  and  $w'_0$ , respectively, at this point  $t_0$ , then

$$t_0 \frac{w'_0}{w_0} \geq 0. \quad (5)$$

Moreover, the theorem states that  $\frac{w(\zeta) - w_0}{\zeta - t_0}$  belongs to the Hardy class  $H^2$ .

Conversely, for every numbers  $w_0$ ,  $|w_0| = 1$  and  $w'_0$  such that (5) holds there exists an analytic function  $w$  on the unit disk,  $|w(\zeta)| \leq 1$  with nontangential boundary values of  $w$  and  $w'$  at  $t_0$  equal  $w_0$  and  $w'_0$ , respectively.

Let  $\Gamma$  be a Fuchsian group acting on the unit disk. Let  $\beta$  be a unitary character of this group. Let  $w$  be an analytic function on the unit disk,  $|w(\zeta)| \leq 1$  which is  $\beta$ -automorphic, that is

$$w(\gamma(\zeta)) = \beta(\gamma)w(\zeta)$$

for every  $\gamma \in \Gamma$ . The goal of the work is to establish an analogue of (5) in this case: 0 in the righthand side of (5) will be replaced with a positive quantity that depends on  $\beta$ .

To have a meaningful construction one needs a condition on group  $\Gamma$  that guarantees existence for every character  $\alpha$  of an  $\alpha$ -automorphic function  $h$  such that  $\frac{h(\zeta) - 1}{\zeta - t_0} \in H^2$ . Recall that existence for every character  $\alpha$  of an  $\alpha$ -automorphic function  $h \in H^2$  is equivalent to the famous Widom condition (given in terms of the Green function of group  $\Gamma$ ) and necessary and sufficient condition of existence for every character  $\alpha$  of an  $\alpha$ -automorphic function  $h$  such that  $\frac{h(\zeta)}{\zeta - t_0} \in H^2$  was established in a recent joint work with Peter Yuditskii (given in terms of the Martin function of  $\Gamma$  with singularity at  $t_0$ ).

## Trace formulas and inverse spectral theory for strings

*Kostenko Aleksey*

Monday 14'30 (HS3)

Generalized indefinite strings provide a canonical model for self-adjoint operators with simple spectrum (other classical models are Jacobi matrices, Krein strings and  $2 \times 2$  canonical systems). We'll present several Szego-type theorems for generalized indefinite strings and related spectral problems (including Krein strings and Dirac operators). More specifically, for several classes of coefficients (that can be regarded as Hilbert-Schmidt perturbations of model problems), we provide a complete characterization of the corresponding set of spectral measures. The talk is based on joint work with J.Eckhardt.

## Multiple orthogonal polynomials in random matrix theory

*Kuijlaars Arno*

Thursday 9'30 (HS3)

I will give an overview of some of the uses of multiple orthogonal polynomials (MOPs) in the theory of random matrices. Multiple orthogonal polynomials have orthogonality properties with respect to several orthogonality measures. They arise as averages of characteristic polynomials in a number of random matrix ensembles, including random matrices with external source, two matrix models, Muttalib-Borodin ensembles, and normal random matrices.

In such models, the limiting behavior of MOPs as their degrees tend to infinity is of interest for the eigenvalue behavior as the size of the random matrix increases. In typical examples, the limiting behavior of the zeros of the MOPs is given in terms of a vector equilibrium problem from logarithmic potential theory. New types of critical behavior and phase transitions appear beyond those that arise in models that are associated with ordinary orthogonal polynomials.

## Canonical systems, Weyl coefficients and eigenvalues

Langer Matthias

Friday 11'45 (HS3)

Two-dimensional canonical systems appear in many situations as they cover one-dimensional Schrödinger equations, Sturm–Liouville equations, Jacobi operators, Dirac systems and (generalised) Krein strings as special cases. In the first part of the talk I shall discuss uniform estimates for Weyl coefficients and related properties of the spectral measure. The second part of the talk deals with asymptotic properties of eigenvalues in the case when the spectrum is discrete. My focus will be on the case when the spectrum is sparse. It turns out that in this situation the results are quite different from the case of dense spectrum. In the proofs the uniform estimates of the Weyl coefficients play a crucial role.

This talk is based on joint work with Raphael Pruckner, Jakob Reiffenstein and Harald Woracek.

## Cwikel-Lieb-Rozenblum type inequalities for Hardy-Schrödinger operator

Le Thi Minh Thao

Tuesday 16'45 (HS2)

We prove a Cwikel–Lieb–Rozenblum type inequality for the number of negative eigenvalues of the Hardy–Schrödinger operator  $-\Delta - (d-2)^2/(4|x|^2) - W(x)$  on  $L^2(\mathbb{R}^d)$ . The bound is given in terms of a weighted  $L^{d/2}$ -norm of  $W$  which is sharp in both large and small coupling regimes. We also obtain a similar bound for the fractional Laplacian.

## Spectra of mildly random matrices

Lemm Marius

Friday 10'00 (HS3)

Abstract: Random-matrix eigenvalue statistics are believed to be universal in the sense that they are conjectured to arise for many strongly correlated point processes, even some deterministic ones. It is therefore of interest to study matrix ensembles containing relatively few random variables. We introduce ensembles of  $N \times N$  Hermitian matrices, which depend on only  $\mathcal{O}(N)$  random variables. The construction is motivated by ergodic theory and involves evaluating the complex exponential function at integer arguments of random polynomials. We prove that the global and local eigenvalue statistics of these ensembles are described by the Wigner semicircle law, a hallmark of random matrix statistics. The result relies on showing that oscillatory deterministic cancellations combine well with

the better understood probabilistic cancellations. Based on joint work with Arka Adhikari and Horng-Tzer Yau.

## Legendre-tau Chebyshev Spectral Method for Maxwell's Equations with Interfaces and its Theoretical Analysis

Liang Dong

Tuesday 12'00 (HS2)

Computational electromagnetics plays an important role in many applications in modern society, such as internet and satellite communication systems, radar systems, medical imaging systems, telecommunication chips, and so on. The propagation and scattering of electromagnetic waves are governed by Maxwell's equations. The problems of inhomogeneous media are universal in practical engineering.

In this study, we develop and analyze multidomain Legendre-tau Chebyshev spectral method for solving Maxwell's equations with material interfaces. The computational domain is decomposed into some non-overlapping sub-domains naturally along material interfaces, while the interface conditions are treated in a way like the natural boundary conditions based on the constructed weak formulation. The electric and magnetic fields are approximated by using Legendre-tau Chebyshev polynomial spaces of different degrees, which can be solved separately in computation. The important feature is that the scheme makes the numerical solution retain the original physical properties. We further analyze theoretically the multidomain Legendre-tau Chebyshev spectral scheme for the Maxwell's equations with interfaces. We prove its energy conservation in the discrete form and its optimal error estimates. Numerical experiments confirm that the spectral accuracy is achieved being not affected by the discontinuity of solutions. Compared with some related methods, the computational cost times of the schemes are shorter. This is a joint work with C. Niu and H. Ma.

## Universality limits via canonical systems

Lukić Milivoje

Wednesday 10'00 (HS3)

It is often expected that the local statistical behavior of eigenvalues of some system depends only on its local properties; for instance, the local distribution of zeros of orthogonal polynomials should depend only on the local properties of the measure of orthogonality. The most commonly studied case is known as bulk universality, where Christoffel-Darboux kernels have a double scaling limit given by the sine kernel. In this talk, I will discuss the first completely local sufficient

condition for bulk universality and, much more generally, necessary and sufficient conditions for regularly varying universality limits. The proofs of these results rely on the de Branges theory of canonical systems, and the results also apply to other self-adjoint systems with 2x2 transfer matrices such as Schrodinger operators.

The talk is based on joint work with Benjamin Eichinger (TU Wien), Brian Simanek (Baylor University), and Harald Woracek (TU Wien).

## Stabilization by multiplicative Itô noise for Chafee-Infante equation in perforated domains

Ly Hong Hai

Tuesday 12'30 (HS2)

The stabilization by noise for parabolic equations in perforated domains, i.e. domains with small holes, is investigated. We show that when the holes are small enough, one can stabilize the unstable equations using suitable multiplicative Itô noise. The results are quantitative, in the sense that we can explicitly estimate the size of the holes and diffusion coefficients for which stabilization by noise takes place. This is done by using the asymptotic behaviour of the first eigenvalue of the Laplacian as the hole shrinks to a point.

## To the Birman-Krein-Visic Theory. Solution to the Birman problem.

Malamud Mark

Thursday 14'30 (HS2)

Let  $A$  be a closed non-negative symmetric densely defined operator in a Hilbert space  $\mathfrak{H}$  and let  $\mathfrak{H}_1 := \text{ran}(A + I)$ . Stone and Friedrichs proved that the set  $\text{Ext}_A(0, \infty)$  of all nonnegative selfadjoint extensions  $\tilde{A} = \tilde{A}^*$  of  $A$  is nonempty. Complete theory of extensions of  $A \geq 0$  was built by M. Krein. In particular, he proved that  $\text{Ext}_A(0, \infty)$  contains the maximal (the Friedrichs) and the minimal (the Krein) extensions  $\hat{A}_F$  and  $\hat{A}_K$ . They are uniquely characterized by means of the following inequalities:  $\hat{A}_K \leq \tilde{A} \leq \hat{A}_F$  for each  $\tilde{A} \in \text{Ext}_A(0, \infty)$  which are understood in the sense of either quadratic forms or the resolvents.

Krein's theory has substantially been completed by M. Vicik and M. Birman.

If  $A$  is positive definite, then  $\hat{A}_K$  admits a representation  $\hat{A}_K = \hat{A}'_K \oplus (\mathbb{O} \upharpoonright \mathfrak{N}_0)$  where  $\mathfrak{N}_0 := \ker A^*$ . The operator  $\hat{A}'_K$  is called the reduced Krein extension.

Krein proved the implication  $(I_{\mathfrak{H}} + \hat{A}_F)^{-1} \in \mathfrak{S}_{\infty} \implies (I_{\mathfrak{M}_0} + \hat{A}'_K)^{-1} \in \mathfrak{S}_{\infty}$ . We improve and complete Krein's result by showing that replacing  $\hat{A}_F$  by  $A$  turns this implication into the equivalence:

$$P_1(I_{\mathfrak{H}} + A)^{-1} \in \mathfrak{S}_{\infty}(\mathfrak{H}_1) \iff (I_{\mathfrak{M}_0} + \hat{A}'_K)^{-1} \in \mathfrak{S}_{\infty}(\mathfrak{M}_0), \quad (6)$$

where  $P_1$  is the orthoprojection in  $\mathfrak{H}$  onto  $\mathfrak{H}_0$  and  $\mathfrak{M}_0 = \mathfrak{N}_0^{\perp}$ . It happens that this equivalence remains valid with the ideal  $\mathfrak{S}_{\infty}$  of compact operators replaced by any symmetrically normed ideal  $\mathfrak{S}$  (including ideals  $\mathfrak{S}_p$ ,  $\Sigma_p$ ,  $\Sigma_p^0$ , etc.)

Moreover, it turns out that under certain additional assumption on  $A$  the power asymptotic behaviour of the eigenvalues of these operators coincide, i.e. the following equivalence holds as  $n \rightarrow \infty$ :

$$\lambda_n(\hat{A}_F) = a^{-1}n^{1/p}(1 + o(1)) \iff \lambda_n(\hat{A}'_K) = a^{-1}n^{1/p}(1 + o(1)).$$

In accordance with the Grubb result an extension  $A_B = A_B^*$  is semibounded below only simultaneously with its boundary operator  $B$  (LSB-property of  $A$ ) whenever  $(I_{\mathfrak{H}} + A)^{-1} \in \mathfrak{S}_{\infty}$ . An improvement of this result will also be discussed.

In early 2000s M.S. Birman posed the following problem.

**Problem.** Assume that the operator  $(I + A)^{-1} : \mathfrak{H}_1 \rightarrow \mathfrak{H}$  is compact. Is it true that the resolvent of the Friedrichs' extension  $\hat{A}_F$  of  $A$  is also compact?

An answer to this question is negative and abstract counterexamples easy to build. In particular, they show that the Krein implication is not reversible.

Birman asked also to present examples of a non-negative symmetric differential operator  $A$  with compact inverse  $(I + A)^{-1}$  and such that  $(I + \hat{A}_F)^{-1}$  is not. We will discuss a solution to this Birman problem for certain restrictions of Schrödinger operators  $H(q) = -\Delta + q \geq 0$  in  $\mathbb{R}^n$  with  $\text{dom}(H(q)) = W^{2,2}(\mathbb{R}^n)$ .

Moreover, it will be shown that for certain  $q \geq 0$  the spectrum of  $\hat{A}_F$  is purely absolutely continuous while  $(I + A)^{-1}$  is compact.

The main results of the talk were announced in [1] and [2].

1. M. M. Malamud, To Birman–Krein–Vishik Theory, Doklady Mathematics, Vol. 107, No. 1 (2023), pp. 44–48.

2. M.M. Malamud, On the Birman problem on positive symmetric operators with compact inverse, Func. Anal. Appl., V.57, No 2 (2023), p. 111–116.

## Scattering theory with both regular and singular perturbations and applications

Mantile Andrea

Monday 15'00 (HS3)

The mathematical scattering theory for short-range potential is a well studied subject which was developed by two essentially different approaches: the trace-class method and the smooth method. The scattering problem for singular perturbations of self-adjoint operators, which is outside the original scope of these methods, is connected with scattering from obstacles with impenetrable or semi-transparent boundary conditions. On this side, a general scheme has been developed by combining the construction of singular perturbations, following Posilicano's approach, with an abstract version of the Limiting Absorption Principle (LAP in the following) due to W. Renger and a variant of the smooth

method due to M. Schechter. Let us recall that boundary triple theory and properties of the associated operator-valued Weyl functions were also used to obtain similar representation of the scattering matrix for singularly coupled self-adjoint extensions.

The target of this talk is to present a general framework for the multiple scattering with both potential type and singular perturbations. Our concern is the scattering theory with respect to the free Laplacian and the regular and the singular parts of the perturbation are dealt as a single object: this constitutes the main novelty of our approach. At first we provide an abstract resolvent formula for a perturbations  $A_B$  of the self-adjoint  $A$  by a linear combination of the adjoint of two bounded trace-like maps. The LAP for  $A_B$  and then an asymptotic completeness criterion for the scattering couple  $(A_B, A)$  are provided, under suitable hypothesis, as a generalisation of the scheme adopted for purely singular perturbations. Then, by a combination of LAP with stationary scattering theory in the Birman-Yafaev scheme and the invariance principle, we obtain a representation formula for the scattering matrix of the couple  $(A_B, A)$ . Whenever  $A$  is the free Laplacian in  $L^2(\mathbb{R}^3)$ , such a formula contains, as subcases, both the usual formula for the perturbation given by a short-range potential and the formula for the case of a singular perturbation describing self-adjoint boundary conditions on a hypersurface.

After introducing the main features of this theory, we present applications where our construction is used to describe the stationary-scattering in composite acoustic or electromagnetic dispersive media. In particular, we show how the stationary resolvent and the scattering solutions, for scalar-wave equations with the divergence-form Laplacian having discontinuous density, can be represented in terms of a class of perturbations involving both potential terms and specific frequency-dependent interface conditions.

## Capacity of graphs over non-Archimedean ordered fields

*Muranova Anna*

Monday 15'00 (HS2)

We introduce and study the notion of capacity of a vertex for infinite graphs over non-Archimedean fields. In this talk we present its connection to minimization of the energy, solutions of the Dirichlet problem and existence of a Green's function. In contrast to graphs over the real field monotone limits do not need to exist. Thus, in our situation next to positive and null capacity there is a third case of divergent capacity and we show that the type of capacity does not depend on the choice of a vertex. Moreover, we discuss the existence of positive superharmonic functions for the Laplace operator. The talk is based on joint works with Florian Fischer, Matthias Keller and Noema Nicolussi.

## Semiclassical Moser-Trudinger inequalities

*Nguyen Phuoc-Tai*

Tuesday 16'15 (HS2)

In this talk, I will discuss the extension of the Moser-Trudinger inequality of one function to systems of orthogonal functions. The obtained results are asymptotically sharp when applied to the collective behavior of eigenfunctions of Schrödinger operators on bounded domains.

The talk is based on a joint work with Rakesh Arora and Phan Thành Nam.

## Spectrum of the Laplace operator on quantum graphs and defects in a graphene lattice

*Nowaczyk Marlena*

Monday 14'30 (HS2)

We consider the two-dimensional honeycomb structure of graphene that is a one-atom-thick layer of covalently bonded carbon atoms. The Laplace equation together with Kirchhoff boundary conditions at the nodes model the movement of low-energy free electrons in such structure. Our research is based on the trace formula that combines the spectrum properties of the Laplace operator with the geometric properties of the underlying quantum graph. To be more specific, we use a one-to-one correspondence between the eigenvalues and the lengths of the closed paths. We investigate the four common types of defects in graphene, and based on the closed paths of odd lengths, we show the method for determining the type and the position of a defect.

This is a joint work with Margaret Archibald and Sonja Currie.

## A finer limit circle/limit point classification for trace class Sturm-Liouville operators

*Piorkowski Mateusz*

Monday 15'30 (HS3)

In this talk we study an index  $\ell \in \mathbb{N}_0 \cup \{+\infty\}$  associated to the endpoints of nonoscillatory Sturm-Liouville differential expressions with trace class resolvents. This notion extends the limit circle/limit point dichotomy in the sense that  $\ell = 0$  at some endpoint if and only if the expression is in the limit circle case. In the limit point case  $\ell > 0$ , we present a natural interpretation of  $\ell$  in terms of iterated Darboux transforms. We also show stability of the index  $\ell$  for a suitable class of perturbations, extending earlier work on perturbations of spherical Schrödinger operators to the case of general three terms Sturm-Liouville

operators.

This is joint work with Jonathan Stanfill from Ohio State University.

## Eigenvalue density of limit circle Jacobi operators and related canonical systems

Reiffenstein Jakob

Monday 16'45 (HS3)

The eigenvalues of a limit circle Jacobi operator can be described on a quantitative level in terms of the growth of the Nevanlinna matrix. We translate this problem to the setting of canonical systems. In this setting, rather mild assumptions on the data are sufficient to determine the growth of the Nevanlinna matrix: Certain sequences of coefficients need to be close enough to some regularly varying sequences. Our method gives particular insight into the different behavior for orders less than  $1/2$  and orders larger than  $1/2$ .

## Minimal polynomials with prescribed zeros

Rubin Olof

Tuesday 15'00 (HS3)

Let  $E$  denote a compact subset of the complex plane  $\mathbb{C}$  containing an infinite number of points. Then, there exists a unique monic polynomial of degree  $n$  that minimizes the infinity norm on  $E$ . This polynomial is known as the Chebyshev polynomial associated with  $E$ . In this presentation, we will explore a related problem by considering monic minimizers with respect to the infinity norm on the unit circle having prescribed zeros on the boundary. Building upon work in [3], we will see that prescribing a zero on the boundary dramatically changes the behavior of the corresponding minimizers in terms of the asymptotic zero distribution as well as the corresponding norms. Alternatively, this can be viewed as a weighted Chebyshev problem and we will extend the analysis to allow for fractional powers of the boundary zero in order to draw conclusions regarding Chebyshev polynomials corresponding to the lemniscatic family  $\{z : |z^m - 1| = 1\}$ . Finally, we will supplement our theoretical discussion with numerical experiments conducted using the complex Remez algorithm [5]. These experiments will serve to suggest directions for further study. This is based on joint work with A. Bergman, J. S. Christiansen and B. Eichinger.

*Acknowledgements: The Royal Physiographic Society in Lund is gratefully acknowledged for their generous support*

## References

- [1] A. Bergman & O. Rubin, *Chebyshev polynomials corresponding to a vanishing weight*, J. Approx. Theory **301** (2024) 106048.
- [2] J. S. Christiansen, B. Eichinger & O. Rubin, *Chebyshev polynomials and sets of minimal capacity* Constr. Approx. *to appear*.
- [3] M. Lachance, E. B. Saff & R. S. Varga, *Inequalities for polynomials with a prescribed zero* Math. Z. **168** (1979)
- [4] O. Rubin, *Computing Chebyshev polynomials using the complex Remez algorithm*, (in progress)
- [5] P. T. P. Tang *A fast algorithm for linear complex Chebyshev approximations* Math. Comput. **51** (1988)

## Reproducing kernel theses and Pisier's example

Schwenninger Felix

Thursday 14'30 (HS3)

We study an abstract form of reproducing kernel theses (RKT), covering several well-known examples of RKTs. More precisely, we study the boundedness of mappings of the form

$$H^2(\mathbb{D}) \rightarrow X, f \mapsto f(T)c,$$

where  $T$  is a fixed, power-bounded operator on a Hilbert space  $X$  and  $c \in X$ . Particularly, we study the case of Foguel–Hankel operators  $T$  as arising in Pisier's example for the Halmos problem. This relates to previous results by Jacob-Partington and Harper for Hilbert space contractions  $T$  and Le Merdy for Ritt operators  $T$ , underlining their common feature—the bounded polynomial calculus—, despite their seemingly unrelated proof techniques. This is joint work with Eskil Rydhe (Lund).

## The Szegő minimum problem.

Sodin Mikhail

Monday 10'00 (HS3)

Given a non-negative measure  $\rho$  on the unit circle  $\mathbb{T}$ , the Szegő minimum problem is to find the quantity

$$e_n(\rho)^2 = \min_{q_1, \dots, q_n} \int_{\mathbb{T}} |1 + q_1 t + \dots + q_n t^n|^2 d\rho(t).$$

A celebrated Szegő's theorem states that

$$\lim_{n \rightarrow \infty} e_n(\rho) = \exp\left(\frac{1}{2} \int_{\mathbb{T}} \log \rho' dm\right),$$

where  $m$  is the Lebesgue measure, and  $\rho' = d\rho/dm$  is the Radon-Nikodym derivative. Thus,  $\lim_{n \rightarrow \infty} e_n(\rho) = 0$  if and only if the measure  $\rho$  has a divergent logarithmic integral. In spite of the classical nature and omnipresence of this result, little is known how properties of a measure  $\rho$  with divergent logarithmic integral affect the rate of decay of the sequence  $e_n(\rho)$ .

Several quantitative results in that direction were proven recently in joint works with Alexander Borichev, Anna Kononova, and Fedor Nazarov. Among them are a necessary and sufficient condition on  $\rho$  that guarantees a subexponential decay of  $e_n(\rho)$ , i.e., a solution to the Erdős-Turán problem, upper and lower bounds for  $\log_- e_n(\rho)$  that match each other up to positive numerical factors, and the refutation of the possibility of a relative Szegő asymptotics.

## Grunsky Operator Connecting Christoffel Functions to Faber Polynomials

*Stylianopoulos Nikos*

Tuesday 16'15 (HS3)

We review the Grunsky operator defined by quasiconformal domains in the complex plane and show how it can be used in order obtain new results connecting the associated Christoffel functions to Faber polynomials. In addition, we provide new estimates for the Grunsky coefficients, for domains bounded by piecewise analytic curves.

## Asymptotic zeros' distribution of orthogonal polynomials with unbounded recurrence coefficients

*Świderski Grzegorz*

Monday 17'15 (HS3)

We study spectrum of finite truncations of unbounded Jacobi matrices with periodically modulated entries. In particular, we show that under some hypotheses a sequence of properly normalized eigenvalue counting measures converge vaguely to an explicit infinite Radon measure. Finally, we derive strong asymptotics of the associated orthogonal polynomials in the complex plane, which allows us to prove that Cauchy transforms of the normalized eigenvalue counting measures converge pointwise and which leads to a stronger notion of convergence. This is a joint work with Bartosz Trojan (Wrocław University of Science and Technology).

## Imaginary Powers of $(k, a)$ -Generalized Harmonic Oscillator

*Teng Wentao*

Thursday 17'15 (HS2)

We will define and investigate the imaginary powers  $(-\Delta_{k,a})^{-i\sigma}$ ,  $\sigma \in \mathbb{R}$  of the  $(k, a)$ -generalized harmonic oscillator  $-\Delta_{k,a} = -\|x\|^{2-a} \Delta_k + \|x\|^a$  for  $a = 2$  and 1 respectively, and prove the  $L^p$ -boundedness ( $1 < p < \infty$ ) and weak  $L^1$ -boundedness of such operators. To prove this result, we develop the Calderón-Zygmund theory adapted to the  $(k, a)$ -generalized setting for  $a = 2$  and 1, and show that  $(-\Delta_{k,a})^{-i\sigma}$  are singular integral operators satisfying the corresponding Hörmander type condition.

## Challenges for non-selfadjoint spectral problems in analysis and computation

*Tretter Christiane*

Tuesday 9'00 (HS3)

Non-selfadjoint spectral problems appear frequently in a wide range of applications. Reliable information about their spectra is therefore crucial, yet extremely difficult to obtain by approximations. This talk focuses on tools to master these challenges such as spectral pollution or spectral invisibility. In particular, the concept of essential numerical range for unbounded linear operators is introduced and studied, including possible equivalent characterizations and perturbation results. Compared to the bounded case, new interesting phenomena arise which are illustrated by some striking examples. A key feature of the essential numerical range is that it captures, in a unified and minimal way, spectral pollution which may affect e.g. spectral approximations of PDEs by projection methods or domain truncation methods. As an application, Maxwell's equations with conductivity will be considered.

(joint work with S. Boegli, M. Marletta, and F. Ferrarezzo)

## Rational solutions to the mKdV equation

*Young Giorgio*

Thursday 15'00 (HS3)

This talk will focus on ongoing work that uses a modified inverse scattering transform to produce solutions to the modified Korteweg-de Vries (mKdV) equation that may be of particular interest. After a brief discussion of this equation and the methodology, I will present some rational solutions to the mKdV equation of arbitrary order that we obtain through this process, as well as a "rational solution of infinite order" that we are able to realize as a limit of these finite order



solutions. Finally, I will describe some preliminary results on the asymptotic properties of this infinite order solution. This is joint work with Deniz Bilman, Elliot Blackstone, and Peter Miller.

## Periodically modulated rescaling limits and canonical systems with cantor spectrum

*Yuditskii Peter*

Wednesday 11'45 (HS3)

Recent advances in the study of universality limits for fixed measures show that regularly varying rescalings of Christoffel–Darboux kernels have a limit if and only if rescalings of the associated Weyl  $m$ -functions have a limit. In this talk, we discuss the case when there is not a single limit kernel but a full limit cycle. We show that balanced measures on a real Julia set of an arbitrary expanding polynomial provide natural examples for this kind of scaling behavior.

Joint work with B. Eichinger and M. Lukić.

P. Y. was supported by the Austrian FWF project P34414.

## Chebyshev and Extremal Polynomials

*Zinchenko Maxim*

Tuesday 12'00 (HS3)

In this talk I will give an overview of some of the classical and more recent results for Chebyshev and  $L_p$  extremal polynomials. In particular, I will discuss norm estimates and asymptotics for such polynomials on subsets of the real line and the unit circle.

# Participants

*Abdeljawad Ahmed* (RICAM, Austria)  
*Alexis Michel* (University of Bonn, Germany)  
*Aslanova Nigar* (Azerbaijan University of Architecture and Construction)  
*Avetisyan Karen* (Yerevan State University, Armenia)  
*Baba Nossai* (Hassan II University of Casablanca, Morocco)  
*Barkaoui Yosra* (University of Vaasa, Finland)  
*Bessonov Roman* (St. Petersburg State University, Russia)  
*Blower Gordon* (Lancaster University, UK)  
*Bračič Janko* (University of Ljubljana, Slovenia)  
*Buchecker Benedikt* (TU Wien, Austria)  
*Buchinger Andreas* (TU Bergakademie Freiberg, Germany)  
*Chafai Ezzeddine* (University of Sfax, Tunisia)  
*Chen Chung-Chuan* (National Taichung University of Education, Taiwan)  
*Christiansen Jacob* (Lund University, Sweden)  
*Ciepliński Krzysztof* (AGH University of Krakow, Poland)  
*Currie Sonja* (University of the Witwatersrand, South Africa)  
*Dahia Elhadj* (ENS Bousaada, Algeria)  
*Denisov Sergey* (University of Wisconsin-Madison, USA)  
*Eichinger Benjamin* (TU Wien / Lancaster University, Austria / UK)  
*Exner Pavel* (Czech Technical University, Czech Republic)  
*Gatt Juanita* (University of Malta, Malta)  
*Haimi Antti* (University of Vienna, Austria)  
*Jeribi Aref* (University of Sfax, Tunisia)  
*Kaltenböck Michael* (TU Wien, Austria)  
*Karlovich Yuri* (Universidad Autónoma del Estado de Morelos, Mexico)  
*Kheifets Alexander* (University of Massachusetts Lowell, USA)  
*Kuijlaars Arno* (Katholieke Universiteit Leuven, Belgium)  
*Langer Matthias* (University of Strathclyde, UK)  
*Le Thi Minh Thao* (Masaryk University, Czech Republic)  
*Lemm Marius* (University of Tübingen, Germany)  
*Liang Dong* (York University, Canada)  
*Lukić Milivoje* (Rice University, USA)

*Ly Hong Hai* (University of Ostrava, Czech Republic)  
*Malamud Mark* (.)  
*Mantile Andrea* (Université de Reims, France)  
*Meradjah Ibrahim* (Djillali Liabes University, Algeria)  
*Moalla Nedra* (University of Sfax, Tunisia)  
*Muranova Anna* (University of Warmia and Mazury in Olsztyn, Poland)  
*Nguyen Phuoc-Tai* (Masaryk University Brno, Czech Republic)  
*Nowaczyk Marlena* (Halmstad University, Sweden)  
*Piorkowski Mateusz* (KU Leuven, Belgium)  
*Reiffenstein Jakob* (Stockholm University, Sweden)  
*Romero Jose Luis* (University of Vienna, Austria)  
*Rubin Olof* (Lund University, Sweden)  
*Schneider Baruch* (University of Ostrava, Czech Republic)  
*Schneiderová Diana* (University of Ostrava, Czech Republic)  
*Schwenninger Felix* (University of Twente, The Netherlands)  
*Snoun Safa* (University of Gabes, Tunisia)  
*Sodin Mikhail* (Tel Aviv University, Israel)  
*Stylianopoulos Nikos* (University of Cyprus, Cyprus)  
*Świdorski Grzegorz* (Polish Academy of Sciences, Poland)  
*Teng Wentao* (University of Tokyo, Japan)  
*Tretter Christiane* (Universität Bern, Switzerland)  
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