TFE4141 – Design of Digital Systems 1

# Introductory Presentation of the Term Project

## 12. September 2017

**Resources:**

| | |
|---|---|
| **Ali Asghar Vatanjou** | **(**ali.vatanjou@ntnu.no**)** |
| **Johan Austlid Fjeldtvedt** | **(**johanaus@stud.ntnu.no**)** |
| **Tore Våland Bø** | **(**torevb@stud.ntnu.no) |
| **Øystein Gjermundnes** | **(**oystein.gjermundnes@ntnu.no**)** |

Department of Electronics and Telecommunications

# Outline

- Term project - goals
- RSA
- Plan – Specification

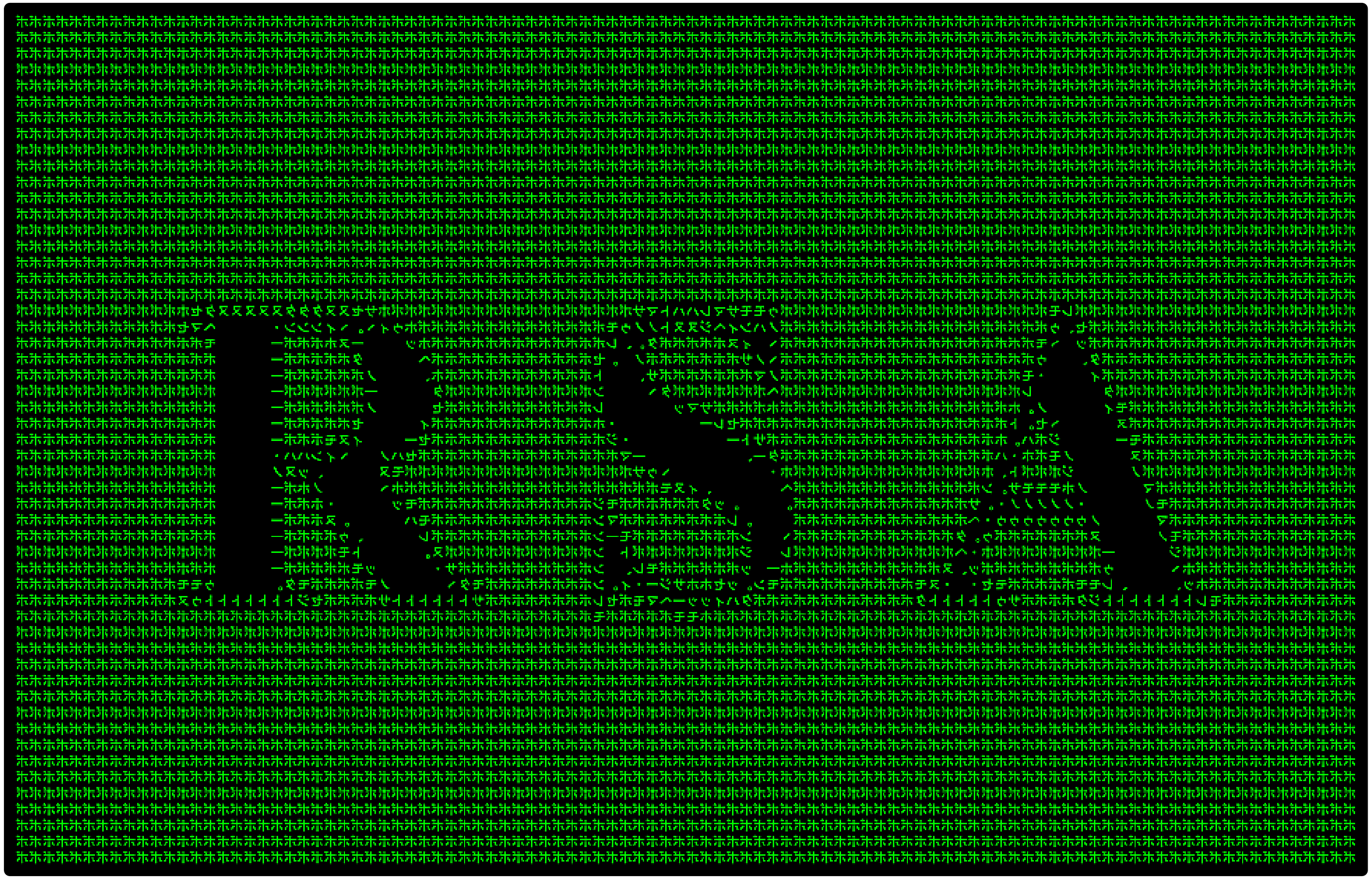
# Term Project

Learning goals

- **Important part of learning in TFE4141**
- **Learning – design and practical skills in using EDA tools**

- The work on this term project will give you experience with :
  - Implementation of algorithms in hardware
  - Verification by simulation
  - Use of tools for synthesis
  - To understand and evaluate other people's work by a peer-review system. This will also help you to receive critique (and praise..)

- **30 %** of the final grade is based upon the project report



RSA

# RSA

Ron **R**ivest, Adi **S**hamir, Len **A**dleman

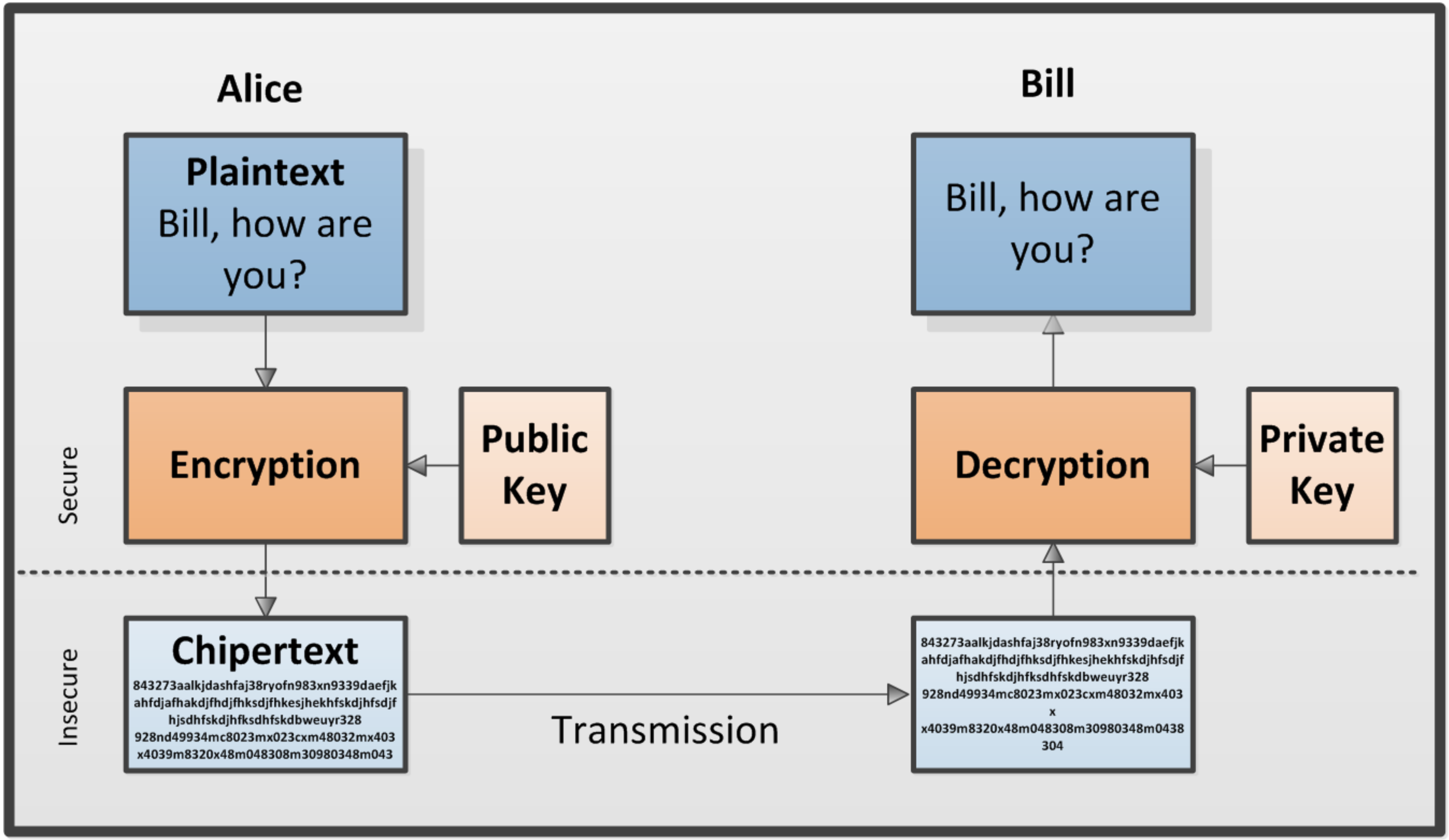
- Block cipher: both plaintext and cipher text are integers between 0 and n-1.

- RSA uses two different keys. One is for encryption (public key); the other is for decryption (private key). This is an **asymmetric** algorithm.

- The method relies on a one-way function **$f$** such that it is easy to compute **$Y = f(X)$**, but impossible to compute **$X = f^{-1}(Y)$**, if you don’t have the correct key **$k$**.

- RSA is widely used in electronic commerce protocols, and is believed to be secure, given sufficiently long keys and the use of up-to-date implementations.

# RSA Encryption/Decryption Mechanism



Alice

Bill

Plaintext

Bill, how are you?

Bill, how are you?

Secure

Encryption

Public Key

Decryption

Private Key

Insecure

Chipertext

843273aalkjdashfaj38ryofn983xn9339daefjk
ahfdjafhakdjfhdjfhksdjfhkesjhekhfskdjhfsdjf
hjsdhfskdjhfksdhfskdbweuyr328
928nd49934mc8023mx023cxm48032mx403
x4039m8320x48m048308m30980348m043

Transmission

843273aalkjdashfaj38ryofn983xn9339daefjk
ahfdjafhakdjfhdjfhksdjfhkesjhekhfskdjhfsdjf
hjsdhfskdjhfksdhfskdbweuyr328
928nd49934mc8023mx023cxm48032mx403
x
x4039m8320x48m048308m30980348m0438
304

# RSA

- The **same computation** is performed for encryption and decryption, but with different parameters. The encryption and decryption keys (***d, e***) are related through the following equation:

- **Encryption**: $\mathbf{C = M^e \bmod n, M < n}$
  public key: {e, n}

- **Decryption**: $\mathbf{M = C^d \bmod n = (M^e)^d \bmod n}$
  private key: {d, n}

- $\mathbf{M = M^{ed} \bmod n}$ restricts the values for **e, d** and **n.**
- to be found by **Euler's theorem**.

**Wikipedia** http://en.wikipedia.org/wiki/RSA

# Generating the keys

1) Select two prime numbers **p** and **q**

2) Compute **$n = pq$**

3) Compute **$\Phi(n) = \Phi(p)* \Phi(q)=(p-1)(q-1)$**

4) Select **e** such that **$gcd(\Phi(n),e) = 1, 1<e< \Phi(n)$**

5) Compute **$d=e^{-1} mod\ \Phi(n)$**

**gcd: greatest common divisor**

**Φ(k): Euler’s totient function.**

# A simple example

**Key generation:**

- Find two primes p = 7, q=17
- n = pq = 7*17=119
- Φ(n) = 6*16=96
- Select *e* such that *e* and *Φ(n)* are relative primes. e=5 can be used.
- Determine *d* such that de=1 mod 96 and d < 96.
  77*5=385=4*96+1 => d=77.
- Public key : {e,n} = {5,119}
- Private key: {d,n} = {77,119}

**Encrypt/Decrypt:**

- Message: M=**19**
- Encryption: $19^5$ mod 119 = 2476099 mod 119 = 20807*119 + 66 mod 119 = 66
- Decryption: $66^{77}$ mod 119 = …”Large numbers”… = **19**

# $M^e$ mod n

“Brute force”

```
# 1) Very naive method for computing M^E:

#    Say we were to compute M^55. One possible solution would be:
#    C = M*M*M*M*M*M*M*M*M*M*
#        M*M*M*M*M*M*M*M*M*M*
#        M*M*M*M*M*M*M*M*M*M*
#        M*M*M*M*M*M*M*M*M*M*
#        M*M*M*M*M*M*M*M*M*M*
#        M*M*M*M*M
#    which requires 54 multiplication operations. It cannot be a very good
#    method since it would take us quite some time with very large numbers.
#
#    This is an algorithm with complexity O(E)
```

- Modular exponentiation is achieved by
- replacing all multiplications in the
- algorithm by modular multiplications.

# $M^e$ mod n

“Repeated Squaring”

```
•   # 2) A much better solution for computing M^E
•   #
•   #    The exponent written with different base numbers:
•   #    E = "55"dec = "37"hex = "00110111"bin
•   #
•   #    The exponent written in order to show the bit indexes
•   #    E : 0  0  1  1  0  1  1  1
•   #    i : 7  6  5  4  3  2  1  0
•   #
•   #    The exponent written as a sum of powers of two:
•   #    E = 0*2^7 + 0*2^6 + 1*2^5 + 1*2^4 + 1*2^2 + 1*2^1 + 1*2^0= 55
•   #
•   #    We can now easily observe that M**55 can be rewritten like this:
•   #    M^55 = M^[0*2^7 + 0*2^6 + 1*2^5 + 1*2^4 + 1*2^2 + 1*2^1 + 1*2^0]
•   #         = M^[2^5] * M^[2^4] * M^[2^2] * M^[2^1] * M^[2^0]
•   #
```

# $M^e$ mod n

“Repeated Squaring”

```
•   #   From this we can easily device a very efficient algorithm:
•   #
•   #   E0 = 1:  C = M    = M^[2^0]
•   #            P = M*M  = M^[2^1]
•   #
•   #   E1 = 1:  C = C*P  = M^[2^0 + 2^1]
•   #            P = P*P  = M^[2^2]
•   #
•   #   E2 = 1:  C = C*P  = M^[2^0 + 2^1 + 2^2]
•   #           P = P*P  = M^[2^3]
•   #
•   #   E3 = 0:  C = C*P  = M^[2^0 + 2^1 + 2^2]
•   #           P = P*P  = M^[2^4]
•   #
•   #   E4 = 1:  C = C*P  = M^[2^0 + 2^1 + 2^2 + 2^4]
•   #           P = P*P  = M^[2^5]
•   #
•   #   E5 = 1:  C = C*P  = M^[2^0 + 2^1 + 2^2 + 2^4 + 2^5]
•   #           P = P*P  = M^[2^6]
•   #
•   #   E6 = 0:  C = C*P  = M^[2^0 + 2^1 + 2^2 + 2^4 + 2^5]
•   #           P = P*P  = M^[2^7]
•   #
•   #   E7 = 0:  C = C*P  = M^[2^0 + 2^1 + 2^2 + 2^4 + 2^5]  <-- CORRECT
    ANSWER
•   #           P = P*P  = M^[2^8]
•   #
•   #   Only 15 multiplications were needed here....
```

```
#    Pseduocode for the algorithm:
#
#    C := 1
#    P := M
#    for i=0 to k-1
#      if Ei = 1
#        C := C*P
#      P := P*P
#    return C
#
#    This is an algorithm with
#    complexity O(log E) which is a lot
#    better than O(E).
```

# A*B

```
•    # 1) Paper and pencil method for multiplying two nonnegative integers:
•    #
•    #    A = 1001 = 9
•    #    B = 1011 = 11
•    #    P = 110011 = 2**6 + 2**5 + 2**1 + 2**0 = 64 + 32 + 2 + 1 = 99
•    #    P = A*B = 9*11 = 99
•    #
•    #
•    #           1001 * 1011
•    #       ------------------
•    #          1
•    #         1-1001
•    #         -1001
•    #         0000
•    #        1001
•    #       ------------------
•    #     =  1100011
•    #       ------------------
•    #       ------------------
•    #
•    #    An algorithm for multiplying two nonnegative integers:
•    #
•    #    P := 0                  | Initial value for partial sum
•    #    for i=0 to k-1          | k is the number of bits in B
•    #      P = 2P + $A*B_{k-1-i}$          | Bi is the i'th bit in B
•    #    return P
```

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# A*B mod n

```
#  2) Paper and pencil method for modular multiplication of two nonnegative
#     integers:
#
#     This can be done very similar to the algorithm explained in 1)
#
#
#     P := 0                       | Initial value for partial sum
#     for i=0 to k-1               | k is the number of bits in B
#       P = 2P + A*$B_{k-1-i}$        | Bi is the i'th bit in B
#       P = P mod N                | Interleaved reduction of P
#     return P
#
#     The largest possible number produced by P mod N is: N-1
#     The largest possible number produced by A*Bi is: N-1
#     Due to the interleaved reduction we will always have P in the range
#     0 <= P <= 2*(N-1) + (N-1) = 3N - 3
#     In order to reduce P to the range 0 <= P <= N-1 it is necessary with
#     at most two subtractions. The new pseudocode will then be:
#
#     P := 0                       | Initial value for partial sum
#     for i=0 to k-1               | k is the number of bits in B
#       P = 2P + A*$B_{k-1-i}$         | Bi is the i'th bit in B
#       if P >= N                  |
#         P = P - N                |
#       if P >= N                  | Reduction ...
#         P = P - N                |
#     return P                     |
```

# Design specification

Design an RSA encryption/decryption circuit that meets the following requirements:

**REQ1: The design must implement the RSA encryption algorithm.**

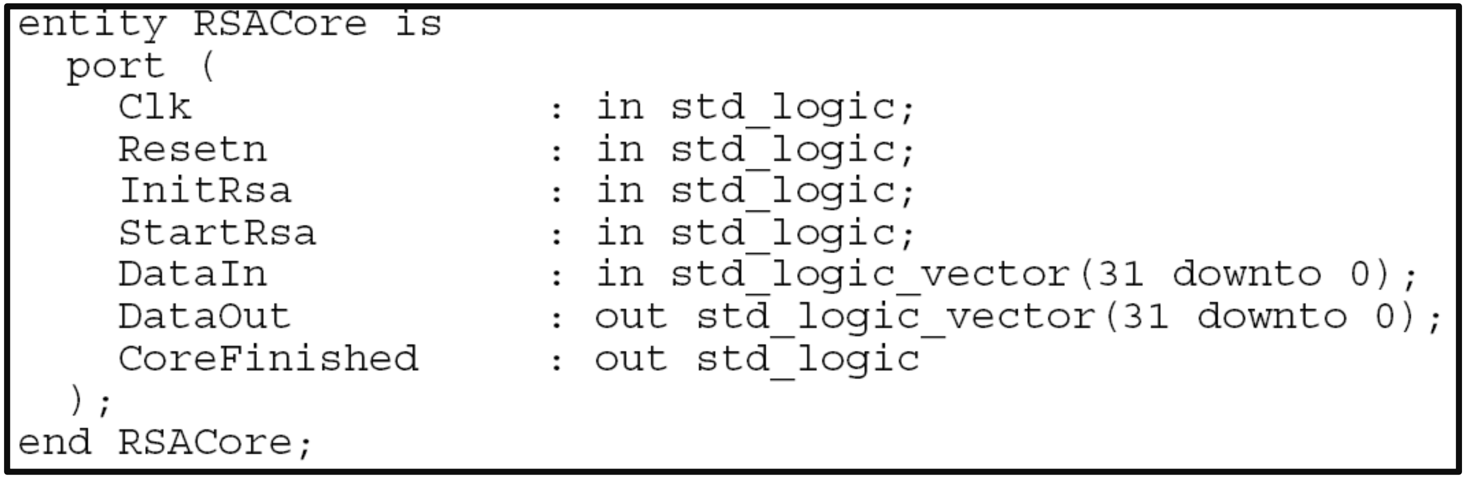
**REQ2: Encrypt/decrypt a message of length 128 bits as fast as possible.**

**REQ3: The target frequency is 50MHz.**

**REQ4: The design must use less that 50% of the resources in a Xilinx Zynq®-7000 device.**

**REQ5: The design entity declaration must match Fig 1.**

**REQ6: The design must implement the interface in Fig 2.**



```
entity RSACore is
  port (
    Clk                 : in std_logic;
    Resetn              : in std_logic;
    InitRsa             : in std_logic;
    StartRsa            : in std_logic;
    DataIn              : in std_logic_vector(31 downto 0);
    DataOut             : out std_logic_vector(31 downto 0);
    CoreFinished        : out std_logic
  );
end RSACore;
```

Fig 1. RSACore entity declaration

```
entity RSACore is
  port (
    Clk
    Resetn
    InitRsa
    StartRsa
    DataIn
    DataOut
    CoreFinished
  );
end RSACore;
```

# Communication

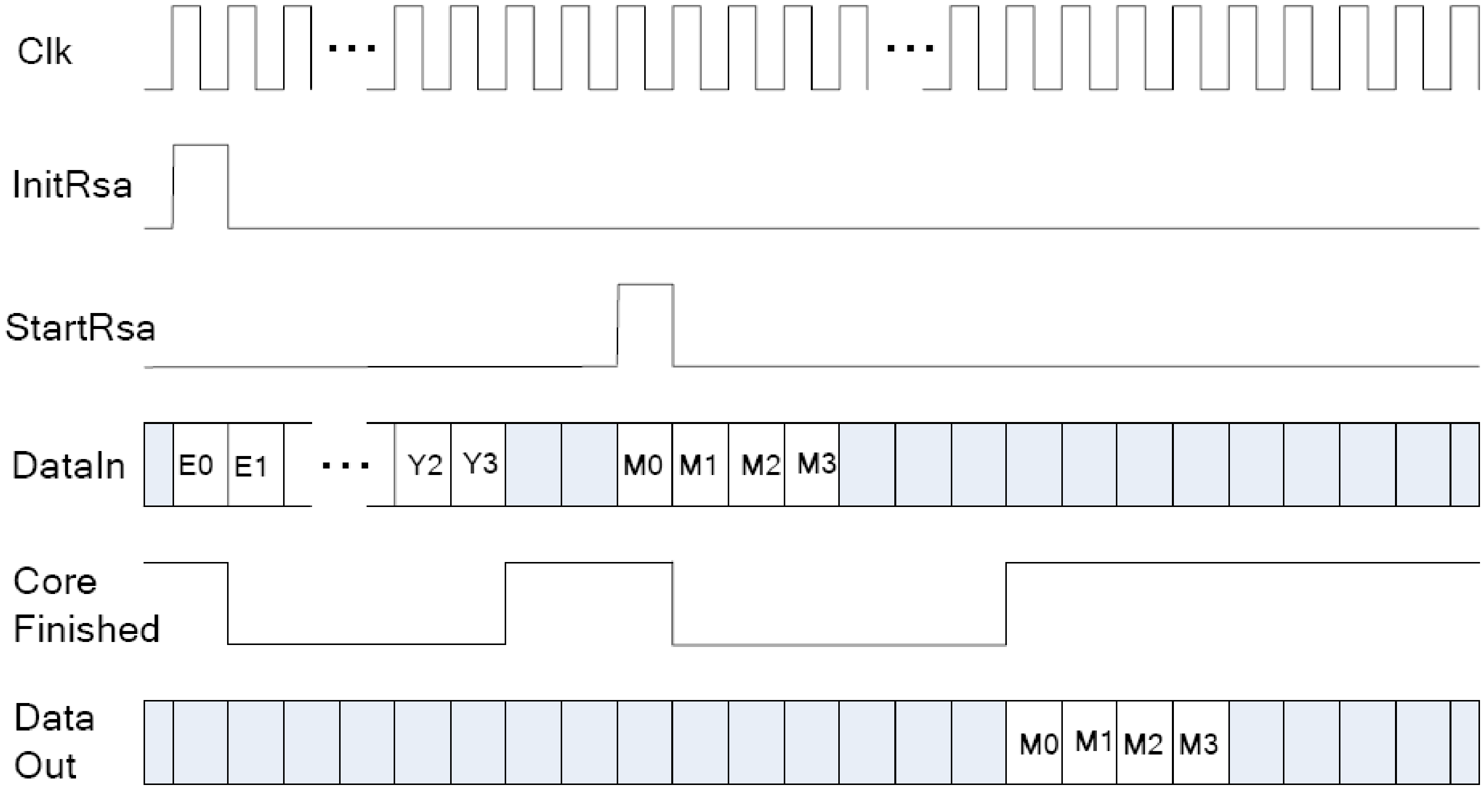
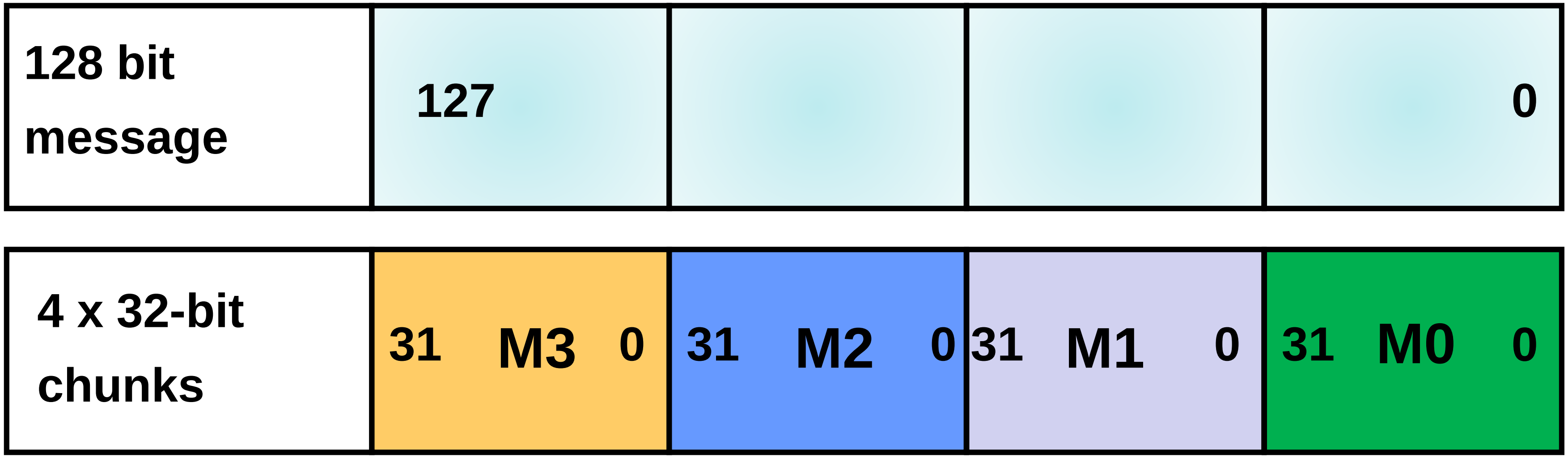


Fig 2. Interface communication protocol

# Data packets



| | | | | |
|---|---|---|---|---|
| 128 bit message | 127 | | | 0 |
| 4 x 32-bit chunks | 31 M3 0 | 31 M2 0 | 31 M1 0 | 31 M0 0 |

- The message blocks to be encrypted will be 128-bit wide.
- One 128-bit message will be divided into four 32-bit packets

# Configuration packets - Encryption

<table>
<tr><td colspan="4">ParY</td><td colspan="4">ParX</td><td colspan="4">KeyN</td><td colspan="4">KeyE</td></tr>
<tr><td>Y3</td><td>Y2</td><td>Y1</td><td>Y0</td><td>X3</td><td>X2</td><td>X1</td><td>X0</td><td>N3</td><td>N2</td><td>N1</td><td>N0</td><td>E3</td><td>E2</td><td>E1</td><td>E0</td></tr>
</table>

- The RSA-module has to be configured with the correct keys
- Each key consists of two 128-bit integers
- The extra parameters ParX and ParY are user defined.

# Summary of your task

- Major Goal - Design and Implement the RSA Circuit:

The circuit should be able to encrypt and decrypt a message of 128 bits as quick as possible, within certain area requirements.

- Understand Algorithm and create model

Literature study of RSA and HW realization of RSA.

Create a high level model of the RSA algorithm.

- Create microarchitecture for HW design

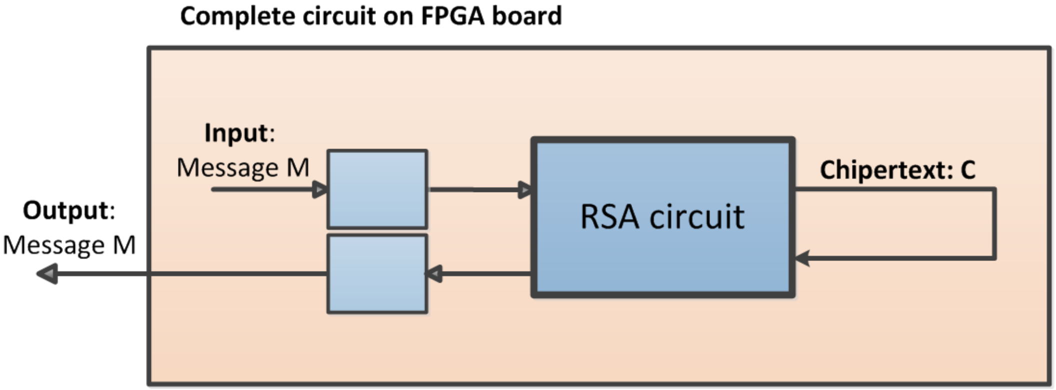
Suggest a design that implements RSA according to the specification. Create block diagram for your solution. It must be possible to identify mux-es, registers, adders etc.

▪ Write RTL code and test

Write RTL code for your design based on the microarchitecture.

# Summary of your task

- Subsequent works of this project:
- ➢ Simulate and Verify your circuit function
- ➢ Construct the system in hardware and perform physical implementation on a FPGA
- ➢ (Circuit shall be tested and verified in the lab )
- Expected result
- ➢ After the circuit has been verified and implemented on an FPGA, then it should be able to encrypt and decrypt the given message.



# Verification

- Each group will get their own message to decrypt.
- You have to decrypt it by an FPGA implementation of the RSA algorithm.
- In the message, you will read how to get a prize for decrypting the message
- What is the prize? -Wait and see.. ☺

# Important points to consider…

If you choose to implement Montgomery modular multiplication as a part of your design, then you may want to use the extra user defined parameters for sending in:

1. r mod n
2. r^2 mod n

# Practical Questions and Ans…

**Actually the encryption and decryption basic formulas are,**

**Encryption:** $C = M^e \bmod n, M < n$,
**public key: {e, n}**

**Decryption:** $M = C^d \bmod n$
**private key: {d, n}**

**Note: Encryption and Decryption are essentially the same operations**

**Question:**
**When we implement them using either Normal or Montgomery algorithm in VHDL, what should be the first, second and so on?**

**Answer:**
**Normal: e (exponent) sent first, n (modulator) sent second**
**Montgomery: e (exponent) sent first, n (modulator) sent second, third R mod n and fourth $R^2$ mod n.**

**These are configuration packages during the setup of the chip.**

# Practical Question and Ans…

**Question:**
**Do we just repeat the same modules/operations for decryption?**

**Answer: Yes**
**RSA is just reversible. You can just repeat the same procedure for decryption.**

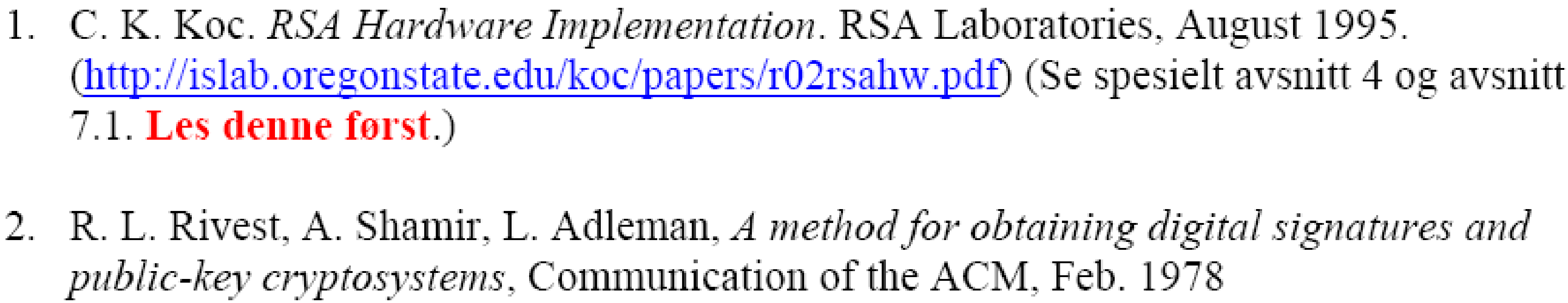
**The configuration packages for decryption is:**

**Normal: d first, then n**
**Montgomery: d first, n second, third R mod n and fourth $R^2$ mod n**

**Note:**
**1. The same configuration package as with encryption, just the symbol e exchanged with the symbol d**

**2. When the chip is set up (has received a configuration package), it is ready to receive messages. It will then receive the message (M or C, depending on whether you are decrypting or encrypting) for each message-block you want encrypted/decrypted. (usually we send 0-31 first).**

# Recommended literature



1. C. K. Koc. *RSA Hardware Implementation*. RSA Laboratories, August 1995. (http://islab.oregonstate.edu/koc/papers/r02rsahw.pdf) (Se spesielt avsnitt 4 og avsnitt 7.1. **Les denne først**.)

2. R. L. Rivest, A. Shamir, L. Adleman, *A method for obtaining digital signatures and public-key cryptosystems*, Communication of the ACM, Feb. 1978

# Time schedule term project - tentative

| **Date** | **Milestone** |
|---|---|
| Tuesday 12. Sept | Term project introduction |
| Tuesday 03. Oct | Microarchitecture review day. Whiteboard presentations for all groups .<br>Each group prepare 2-3 powerpoint slides that describe the microarchitecture of their design. Each group will present for two one other groups as well as one representative from the staff (Ali, Johan, Tore and Øystein) |
| Tuesday 21. Nov | Each group presents their solution. Focus on presenting the architecture, performance and area. |
| Thursday 24. Nov | Hand in the term project report |