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1 Analisis Pendahuluan

Untuk $\varepsilon > 0$, akan dicari $\delta > 0$ yang memenuhi $|x-k| < \delta \rightarrow |(b+mx)-(b+mk)| < \varepsilon$

Perhatikan :

$$|x-c| < \delta \rightarrow |(b+mx) - (b+mk| < \varepsilon$$
$$|(b+mx) - b+mk| < \varepsilon$$
$$|mx - mk| < \varepsilon$$
$$m\,|x-k| < \varepsilon$$
$$|x-k| < \frac{\varepsilon}{m}$$

Bukti formal

Untuk sembarang $\varepsilon > 0$, pilih $\delta = \frac{\varepsilon}{m}$ sehingga memenuhi

$|x-k| < \delta \rightarrow |(b+mx) - (b+mk| = |mx-mk| = m|x-k| < m\delta = \varepsilon$

$\therefore$ terbukti apabila $\lim_{x \to k} (b+mx) = b+mk$ diambil sembarang $\varepsilon > 0$, akan menghasilkan $\delta = \frac{\varepsilon}{m}$ yang memenuhi $|x-k| < \delta$ maka $|(b+mx) - (b+mk)| < \varepsilon$

2.a $\lim_{x \to -1} (x^{2022} + 3)^5 = ((-1)^{2022} + 3)^5$

$$= (1+3)^5$$
$$= 4^5$$
$$\therefore = 1024$$

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2.b. $\lim_{x \to \infty} (\sqrt{x+3} - \sqrt{x-1}) = \lim_{x \to \infty} (\sqrt{x+3} - \sqrt{x-1}) \cdot \frac{(\sqrt{x+3} + \sqrt{x-1})}{(\sqrt{x+3} + \sqrt{x-1})}$

$= \lim_{x \to \infty} \frac{x+3-(x-1)}{\sqrt{x+3} + \sqrt{x-1}}$

$= \lim_{x \to \infty} \frac{4}{\sqrt{x+3} + \sqrt{x-1}}$

$= \lim_{x \to \infty} \frac{\cancel{\sqrt{x}} \cdot \frac{4}{\sqrt{x}}}{\cancel{\sqrt{x}} \left(\sqrt{1+\frac{3}{x}} + \sqrt{1-\frac{1}{x}}\right)}$

$= \frac{0}{\sqrt{1} + \sqrt{1}}$

$= \frac{0}{2}$

$\therefore = 0$

2.c $\lim_{x \to 0} \frac{\sqrt{x+2} - \sqrt{2}}{\sqrt{x}} = \lim_{x \to 0} \frac{\sqrt{x+2} - \sqrt{2}}{\sqrt{x}} \cdot \frac{\sqrt{x+2} + \sqrt{2}}{\sqrt{x+2} + \sqrt{2}}$

$= \lim_{x \to 0} \frac{x+2-2}{\sqrt{x} \cdot (\sqrt{x+2} + \sqrt{2})}$

$= \lim_{x \to 0} \frac{x}{\sqrt{x}} \cdot \frac{1}{\sqrt{x+2} + \sqrt{2}}$

$= \lim_{x \to 0} \sqrt{x} \cdot \frac{1}{\sqrt{x+2} + \sqrt{2}}$

$= 0 \cdot \frac{1}{\sqrt{2} + \sqrt{2}}$

$\therefore = 0$

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3a. $\lim_{x \to 0} \frac{x^3 (\cos 5x)^{-1}}{2x+1} = \lim_{x \to 0} \frac{x^3}{2x+1} \cdot \frac{1}{\cos 5x}$

$= \frac{0^3}{2.0+1} \cdot \frac{1}{1}$

$\therefore = 0$

3b. $\lim_{x \to 0} \frac{(\sin(3x))^{-2}}{x^{-2}} = \lim_{x \to 0} \frac{x^2}{(\sin 3x)^2}$

$= \lim_{x \to 0} \frac{x}{\sin 3x} \cdot \frac{3}{3} \quad \lim_{x \to 0} \frac{x}{\sin 3x} \cdot \frac{3}{3}$

$= \lim_{x \to 0} \frac{3x}{3 \sin 3x} \quad \lim_{x \to 0} \frac{3x}{3 \sin 3x}$

$= \frac{3}{9} \cdot \frac{3}{9}$

$\therefore = \frac{1}{9}$

3c. $\lim_{x \to 0} \frac{1-\cos x}{2x \sin x} = \lim_{x \to 0} \frac{1-\cos x}{2x \sin x}$

$= \lim_{x \to 0} \frac{2 \sin^2 \frac{1}{2} x}{2x \sin x}$

$= \lim_{x \to 0} \frac{2}{2} \cdot \frac{\sin \frac{1}{2} x}{\sin x} \cdot \frac{\sin \frac{1}{2} x}{x}$

$= 1 \cdot \frac{1}{2} \cdot \frac{1}{2}$

$\therefore = \frac{1}{4}$

4a. $f(x) = \frac{1}{x^2+3x-10} \longrightarrow \frac{1}{(x+5)(x-2)}$

$x_1 = -5 \quad x_2 = 2$

$\therefore$ $f(x)$ tidak kontinu di -5 dan 2 karena apabila $x = -5 \vee 2$, nilai $f(x) = \frac{1}{0}$ atau tidak terdefinisi

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4b. $f(x) = \frac{4x^2-64}{x-4}, x \neq 4$

agar kontinu di $x = 4$, $f(x) = \lim_{x\to 4} \frac{4x^2-64}{x-4}$

$= \lim_{x\to 4} \frac{(2x-8)(2x+8)}{x-4}$

$= \lim_{x\to 4} 2(2x+8)$

$\therefore = \lim_{x\to 4} 4x+16$

5. $\lim_{x\to -2} \frac{3x^2+ax+a+3}{x^2+x-2} = \lim_{x\to -2} \frac{3x^2+ax+a+3}{(x+2)(x-1)}$

$(x+2)$ = harus hilang

$3x^2+ax+a+3 = (x+2). \ldots$

$x = -2$

$3.4 + -2a + a+3$

$12 - a + 3$

$a = 15$

$= \lim_{x\to -2} \frac{3x^2+15x+15+3}{(x+2)(x-1)}$

$= \lim_{x\to -2} \frac{3x^2+15x+18}{(x+2)(x-1)}$

$= \lim_{x\to -2} \frac{3(x^2+5x+6)}{(x+2)(x-1)}$

$= \lim_{x\to -2} 3\frac{(x+2)(x+3)}{(x+2)(x-1)}$

$= \lim_{x\to -2} 3 \cdot \frac{(x+3)}{(x-1)}$

$= 3 \cdot \frac{(-2+3)}{(-2-1)}$

$= 3 \cdot \frac{1}{-3}$

$\therefore = -1$

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6. f kontinu di 0,1 $\rightarrow$ $0 < f(x) < 1$, $f(c) = c$

Intermediate value theorem

$$f(0) < f(c) < f(1)$$
$$f(0) < c < f(1)$$

$\rightarrow$ $g(x) = x - f(x)$

$g(c) = c - f(c)$

$= c - c$

$g(c) = 0$

$\rightarrow$ $g(0) = 0 - f(0)$

$-g(0) > 0$

$g(0) < 0$

$\rightarrow$ $g(1) = 1 - f(1)$

$g(1) > 0$

$\therefore$ $g(0) < g(c) < g(1)$

$g(0) < 0 < g(1)$

terbukti f(x) fungsi kontinu di (0,1)

7. $$\lim_{x \to 0} \frac{2^{\frac{1}{x^2}} + 2^{\frac{1}{x}} + 3}{2^{\frac{1}{x^2}+1} + 2^{\frac{1}{x}+1} + 5} = \lim_{a \to \infty} \frac{2^{a^2} + 2^{a} + 3}{2^{a^2+1} + 2^{a+1} + 5} \qquad \left(\frac{1}{x} = a\right) \quad a = \infty$$

$$= \lim_{a \to \infty} \frac{2^{a^2} + 2^{a} + 3}{2^{a^2}.2 + 2^{a}.2 + 5}$$

$$= \lim_{a \to \infty} \frac{2^{a^2}/2^{a^2} + 2^{a}/2^{a^2} + 3/2^{a^2}}{2^{a^2}/2^{a^2}.2 + 2^{a}/2^{a^2}.2 + 5/2^{a^2}}$$

$$= \frac{1 + 0 + 0}{1.2 + 0 + 0}$$

$$\therefore = \frac{1}{2}$$