No :

Tanggal :

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1 $S = \{x \mid x$ bilangan bulat non negatif yang tidak habis dibagi 3 dan merupakan bilangan kuadrat sempurna$\}$

$= \{x \mid x \in N, x \% 3 \; != 0 \cap x = k^2, k \in Z^+\}$

2 $M = \{x \in R : -10 < x < 10\} = \{-9, -8, \ldots, 8, 9\}$

$N = \{x \in R : 1 < x < 20\} = \{2, 3, 4, \ldots, 17, 18, 19\}$

a. $M \cap N = \{x \in R : 1 < x < 10\}$

b. $N - M = \{x \in R : 10 < x < 20\}$

c. $M - \{1,2,3,4,5\} = \{x \in R : -10 < x < 1 \lor 1 < x < 2 \lor 2 < x < 3 \lor 3 < x < 4 \lor 4 < x < 5 \lor 5 < x < 10\}$

d. $N^c = \{x \in R : x < 1 \lor x > 20\}$

3 $U = \{x \in Z\}$

$A = \{1, 2, 5, 10\}$

a. $B - C = \{7, 9\}$

$C - B = \{2, 8\}$

$B \cap C = \{1, 3, 5\}$

$B = \{1, 3, 5, 7, 9\}$

$C = \{1, 2, 3, 5, 8\}$

$B - \{1,3,5\} = \{7, 9\}$

$C - \{1,3,5\} = \{2, 8\}$

b. $(A \cup B) \cap (A \cup C) \rightarrow A \cup (B \cap C)$

$\{1,2,3,5,7,9,10\} \cap \{1,2,3,5,8,10\}$

$= \{1, 2, 3, 5, 10\}$

c. $(A^c \cap B) \oplus (B - C^c)$

$\{3, 7, 9\} \oplus \{1, 3, 5\}$

$\{1, 5, 7, 9\}$

$A^c = \{x \in Z \mid x \notin A\}$

$C^c = \{x \in Z \mid x \notin C\}$

No :

Tanggal :

4 $X = \{\{\emptyset, \{\emptyset\}\}, \{\emptyset\}, \emptyset\}$
$= \{\{\{\}, \{\{\}\}\}, \{\{\}\}, \{\}\}$ → $\emptyset = \{\}$

a $|X| = 4$ → False

Ada 3 elemen di dalam X , $\{\{\},\{\{\}\}\}$, $\{\{\}\}$, $\{\}$
1 2 3

b $Y = \{\{\}\}$ → $Y \in X \cap Y \subseteq X$ True, $\{\{\}\}$ atau Y ada di dalam X

c $\{\{\}, \{\{\{\}\}\}\} \subseteq P(X)$ False , X tidak memiliki elemen $\{\{\emptyset\}\}$ sehingga P(X) juga tidak ada elemen $\{\{\emptyset\}\}$

5 $f: \mathbb{Z} \to \mathbb{Z}$, $f(x) = \lfloor x^3/13 \rfloor$

a $S = \{-13, -7, 0, 7, 13\}$

$f(-13) = -169$ $f(13) = 169$ $f(0) = 0$

$f(-7) = -27$ $f(7) = 26$

$f(S) = \{-169, -27, 0, 26, 169\}$

b f(x) bukan injektif

$f(1) = 0$, $f(0) = 0$.

$f(1) = f(0) = 0$

$1 \neq 0$ → ada elemen domain yang punya nilai sama

c f(x) bukan surjektif

untuk kodomain bilangan bulat , 3 tidak memiliki pasangan di domain

$2 = f(3)$, $4 = f(4)$

6 $f: R \to R$

a $f(x) = \lfloor(\lceil x \rceil)\rfloor$ → fungsi tidak bijektif , tidak ada invers

$f(a) = f(b)$

$f^{-1}(b) = f(a)$

$f(0,1) = 1$

$f^{-1}(1) \neq 0,1$ → floor function / roof tidak mengubah angka bulat sehingga kodomain tidak bisa di inverse kembali ke domain / tidak ada fungsi inverse

6 b $f(x) = 3x/2$ → fungsi bijektif

$y = 3x/2$ → $x = \frac{2y}{3}$

$2y = 3x$ → $f^{-1}(x) = \frac{2x}{3}$

cth: $f(1) = 3/2$

$f^{-1}(3/2) = 1$ ✓

fungsi memiliki invers

(aturan invers terpenuhi)

c $f(x) = \pi(x+4)$ → fungsi bijektif

$y = \pi(x+4)$

$x = y/\pi - 4$

$f^{-1}(x) = x/\pi - 4$

cth: $f(-3) = \pi$

$f^{-1}(\pi) = -3$

fungsi memiliki invers

(aturan invers terpenuhi)

7 $g : A \to B$ | $f = B \to C$

a $f(g) : A \to C$ → surjektif

Setiap elemen C ada pasangan di A

$f : B \to C$

Setiap elemen C ada pasangan di B

∴ Karena setiap elemen C ada pasangan, f adalah fungsi surjektif

b f dan g injektif

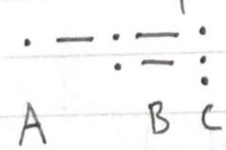
f injektif : $A \to B$

setiap elemen di a ada 1 pasangan di elemen b

g injektif : $B \to C$

setiap elemen di b ada 1 pasangan di elemen c

cth:



$f \circ g = A \to C$

→ setiap elemen di A ada 1 pasangan di elemen C

∴ $f \circ g$ injektif //

8 $f(x) = x^2+4$ $\quad f: R \to R$

$g(x) = 2x+1$

a $f(g) \to f(2x+1)$

$= (2x+1)^2 + 4$

$= 4x^2 + 4x + 1 + 4$

$= 4x^2 + 4x + 5$,, $x \in R$

b $f+g = f(x) + g(x)$

$= x^2 + 4 + 2x + 1$

$= x^2 + 2x + 5$,, $x \in R$

c $fg = f(x) \cdot g(x)$

$= (x^2+4)(2x+1)$

$= 2x^3 + x^2 + 8x + 4$,, $x \in R$

9 a $\{a_n\} = \{6, 10, 15, 21, 28, \ldots\} \to a_1 = 6 \mid a_n = a_{n-1} + n + 3$ untuk $n \geq 1$

$\{b_n\} = \{8, 12, 24, 44, \ldots\} \to a_1 = 8 \mid a_n = a_{n-1} + 8n - 4$ untuk $n \geq 1$

$\{c_n\} = \{12, 21, 30, 39, \ldots\} \to a_1 = 12 \mid a_n = a_{n-1} + 9n$ untuk $n \geq 1$

formula tertutup $\{a_n\} = \frac{1}{2}(n^2 + 5n + 6)$

formula tertutup $\{b_n\} = 4(n^2 - 2n + 3)$

formula tertutup $\{c_n\} = 3(3n+1)$

b $\sum_{n=1}^{7} b_n = \sum_{1}^{7} (4(n^2 - 2n + 3)) = \sum_{1}^{7} (4n^2 - 8n + 12)$

$= \sum_{1}^{7} 4n^2 - \sum_{1}^{7} 8n + 7.12$

$= 560 - 224 + 84$

$= 420$,,

No :

Tanggal :

9 c $\sum_{n=5}^{9} \frac{1}{2}n^2 + \frac{5}{2}n + 3 = \sum_{n=5}^{9} \frac{1}{2}n^2 + \sum_{n=5}^{9} \frac{5}{2}n + 3.5$

$\boxed{\sum_{n=5}^{9} a_n}$

$= \frac{255}{2} + \frac{175}{2} + 15$

$= 15 + 215$

$= 230$

$\sum_{n=7}^{11} c_n = \sum_{n=7}^{11} 9n+3 = \sum_{n=7}^{11} 9n + 5.3$

$= 405 + 15$

$= 420$

$420 . 230 = 96600$,,

10 a $A = \{x(x+2) \mid x \text{ bilangan bulat positif}\}$

$= \{x(x+2) \mid x \in Z^+\}$

$f : Z^+ \to A$ dengan $f(x) = x(x+2)$

f ditunjukkan bijektif

f injektif : $f(x) = f(y)$, maka $x(x+2) = y(y+2)$, $x \in Z^+, y \in Z^+$

Didapat $x^2 + 2x = y^2 + 2y$

$x^2 = y^2$

$\underset{\substack{y \in Z^+ \\ TM}}{x = -y} \vee x = y$

$x = y$, terbukti fungsi injektif

f surjektif : $f(x) = y$

$y = x^2 + 2x \to x = \frac{y}{(x+2)}$

ada x untuk setiap y , terbukti fungsi surjektif

$\therefore$ f terbukti bijektif, maka A countable , countably infinite dan $|A| = \aleph_0$

Tanggal :

10 b $B = \left\{ \frac{1}{x+1} - 1 \mid x \in Z^+ \right\} \cup \left\{ \frac{x-1}{x+1} \mid x \in Z^+ \right\}$

$f : Z^+ \to B$ dengan $f(x) = \frac{1}{x+1} - 1 \vee \frac{x-1}{x+1}$

f ditunjukkan bijektif

f injektif : $f(x) = f(y)$

$\frac{1}{x+1} - 1 = \frac{1}{y+1} - 1$

$y + 1 = x + 1$

$y = x$

$f(x) = f(y)$

$\frac{x-1}{x+1} = \frac{y-1}{y+1}$

$(x-1)(y+1) = (x+1)(y-1)$

$xy - 1 = xy - 1$

$xy = xy$, $x = x \vee y = y$

fungsi terbukti injektif

f surjektif : $f(x) = y$

$y = \frac{1}{x+1} - 1$

$xy + y + x + 1 = 1$

$x = \frac{-y}{y+1}$

$f(x) = y$

$y = \frac{x-1}{x+1}$

$xy + y - x + 1 = 0$

$x(y-1) = -y-1$

$x = \frac{-y-1}{y-1}$

fungsi terbukti surjektif

$\therefore$ f terbukti bijektif, maka B countable, countably infinite dan $|B| = \aleph_0$

11 $P(n) = 3^{2n} + 2^{2n+2}$, $n \in Z^+$

$P(n) = 5x$, $x \in Z^+$

Basis step:

n terkecil adalah 1

$P(1)$ dibuktikan :

$P(1) = 3^2 + 2^4 = 5x$

$= 9 + 16 = 5x$

$= 25 = 5x$

$P(1)$ terbukti benar (kelipatan 5)

11 Inductive step:

$P(k) \to P(k+1)$ dibuktikan , dimana $k \in Z^+$

Asumsi $P(k)$ benar :

$P(k) = 3^{2k} + 2^{2k+2} = 5x$

$P(k+1)$ dibuktikan :

$P(k+1) = 3^{2(k+1)} + 2^{2(k+1)+2} = 5x$

$= 3^{2k+2} + 2^{2k+4} = 5x$

$= 9.3^{2k} + 4(2^{k+2}) = 5x \to 5.3^{2k} + 4.3^{2k} + 4(2^{k+2})$

$= \underbrace{5.3^{2k}}_{\text{kelipatan 5}} + 4\underbrace{(3^{2k} + 2^{k+2})}_{P(k) = \text{kelipatan 5}} = 5x$

$P(k+1)$ terbukti

$P(k) \to P(k+1)$ terbukti

$\therefore$ Setiap bilangan bulat positif n , $3^{2n} + 2^{2n+2}$ adalah kelipatan 5 berlaku

12 $P(n) = 1^3 + 3^3 + 5^3 + \ldots (2n+1)^3 = (n+1)^2 (2n^2 + 4n + 1)$, $n \in N$

Basis step:

n terkecil adalah 0

$P(0)$ dibuktikan :

$P(0) = 1^3 = 1^2 (0+0+1)$

$1 = 1$

$P(0)$ terbukti benar

Induction step:

$P(k) \to P(k+1)$ dibuktikan , $k \in N$

Asumsi $P(k)$ benar :

$P(k) = 1^3 + 3^3 + 5^3 + \ldots + (2k+1)^3 = (k+1)^2 (2k^2 + 4k + 1)$, $k \in N$

$P(k+1)$ dibuktikan :

$P(k+1) = 1^3 + 3^3 + 5^3 + \ldots + (2k+1)^3 + (2k+1+2)^3 = (k+1+1)^2 (2(k+1)^2 + 4(k+1) + 1)$

$= 1^3 + 3^3 + 5^3 + \ldots + (2k+1)^3 + (2k+3)^3 = (k^2+4k+4)(2k^2+8k+7)$

12 $P(k+1) = \overbrace{1^3+3^3+5^3+\ldots+(2k+1)^3}^{P(k)} + (2k+3)^3 = 2k^4+16k^3+47k^2+60k+28$

$= (k^2+2k+1)(2k^2+4k+1)+(2k+3)^3 = 2k^4+16k^3+47k^2+60k+28$

$= (2k^4+8k^3+11k^2+6k+1)+(8k^3+36k^2+54k+27) = \text{—}\ \text{''}\ \text{—}$

$= 2k^4+16k^3+47k^2+60k+28 = 2k^4+16k^3+47k^2+60k+28$ ✓

P(k+1) terbukti

$P(k) \rightarrow P(k+1)$ terbukti benar

∴ Setiap bilangan cacah, $1^3+3^3+5^3+\ldots+(2n+1)^3 = (n+1)^2(2n^2+4n+1)$ berlaku

13 $P(n) = 2^n > n^2+n$, $n>4$, $n \in Z$

Basis step:

n terkecil adalah 5

P(5) dibuktikan:

$P(5) = 2^5 > 5^2+5$

$= 32 > 25+5$

$= 32 > 30$

P(5) terbukti

Induction step:

Asumsikan P(j) benar untuk semua k, $4<j<k$, Asumsikan P(k) benar,

$P(k) = 2^k > k^2+k$, $k>4$, $k \in Z$

P(k+1) dibuktikan

$P(k+1) = 2.2^k > 2(k^2+k) > (k+1)^2+k+1$

$\underbrace{2.2^k > 2k^2+2k}_{P(k),\text{ benar}} > k^2+3k+2$ $k>4$

$2k^2+2k > k^2+3k+2$

$k>4$, $k^2-k>2$ terbukti

P(k+1) terbukti, $P(k) \rightarrow P(k+1)$ terbukti

∴ Setiap bilangan bulat $n>4$, $2^n > n^2+n$ terbukti benar.