Clement Samuel Marly 220608 2114 Kalkulus - B PR2

1a. $f(x) = 4x^4 + 4x^2 + 1$

$$\lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h \to 0} \frac{4(x+h)^4 + 4(x+h)^2 + 1 - (4x^4 + 4x^2 + 1)}{h}$$

$$= \lim_{h \to 0} \frac{4(x^4 + 4x^3h + 6x^2h^2 + 4xh^3 + h^4) + 4(x^2 + 2xh + h^2) + 1 - 4x^4 - 4x^2 - 1}{h}$$

$$= \lim_{h \to 0} \frac{4x^4 + 16x^3h + 24x^2h^2 + 16xh^3 + 4h^4 + 4x^2 + 8xh + 4h^2 + 1 - 4x^4 - 4x^2 - 1}{h}$$

$$= \lim_{h \to 0} \frac{16x^3h + 24x^2h^2 + 16xh^3 + 4h^4 + 8xh + 4h^2}{h}$$

$$= \lim_{h \to 0} 16x^3 + 24x^2h + 16xh^2 + 4h^3 + 8x + 4h$$

$$\therefore = 16x^3 + 8x$$

b. $f(x) = \frac{4}{x^2}$

$$= \lim_{h \to 0} \frac{\frac{4}{(x+h)^2} - \frac{4}{x^2}}{h}$$

$$= \lim_{h \to 0} \frac{4}{x^2 + 2xh + h^2} - \frac{4}{x^2} \cdot \frac{1}{h}$$

$$= \lim_{h \to 0} \frac{4x^2 - 4(x^2 + 2xh + h^2)}{(x^2 + 2xh + h^2) \cdot x^2 \cdot h}$$

$$\lim_{h \to 0} \frac{4x^2 - 4x^2 - 8xh - 4h^2}{h(x^4 + 2x^3h + x^2h^2)}$$

$$= \lim_{h \to 0} \frac{h(-8x - 4h)}{h(x^4 + 2x^3h + x^2h^2)}$$

$$= \lim_{h \to 0} \frac{-8x - 4h}{x^4 + 2x^3h + x^2h^2}$$

$$= \frac{-8x}{x^4}$$

$$\therefore = -\frac{8}{x^3}$$

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2a. $f(x) = \sin x \cos x \tan x$

$df(x) = d \sin x \cos x \tan x$

$df(x) = d \sin x \cos x \cdot \frac{\sin x}{\cos x}$

$df(x) = d \sin^2 x$

$df(x) = 2 \sin x \, d \sin x$

$df(x) = 2\sin x \cos x \, dx \rightarrow \frac{d}{dx} = 2\sin x \cos x$

$\therefore f'(x) = 2\sin x \cos x$

b $f(x) = \frac{x^2 \sin x}{x^2+1}$

$df(x) = \frac{(x^2+1)\, d(x^2 \sin x) - x^2 \sin x \, d(x^2+1)}{(x^2+1)^2}$

$df(x) = \frac{(x^2+1)(\sin x \, dx^2 + x^2 d \sin x) - x^2 \sin x \, dx^2 + d1}{(x^2+1)^2}$

$df(x) = \frac{(x^2+1)(\sin x \, 2x dx + x^2 \cos x dx) - x^2 \sin x . \, 2x \, dx}{(x^2+1)^2}$

$df(x) = \frac{2x^3 \sin x \, dx + x^4 \cos x dx + \sin x \, 2x \, dx + x^2 \cos x dx - 2x^3 \sin x \, dx}{(x^2+1)^2}$

$df(x) = \frac{x^4 \cos x \, dx + \sin x \, 2x \, dx + x^2 \cos x \, dx}{(x^2+1)^2}$

$f'(x) = \frac{x^4 \cos x + x^2 \cos x + 2x \sin x}{(x^2+1)^2}$

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2c $f(x) = \cos nx$

$df(x) = d\cos nx$

$df(x) = -\sin nx \ dnx$

$df(x) = -\sin nx \, . \, n \ dx$

$f'(x) = n . -\sin nx$

3a $xy + 3y = 3x^2 - 7y^2$

$x\,dy + y\,dx + d3y = d3x^2 - d7y^2$

$x\,dy + y\,dx + 3\,dy = 6x\,dx - 14y\,dy$

$x\,dy + 3dy + 14y\,dy = 6x\,dx - y\,dx$

$dy\,(x+3+14y) = dx\,(6x-y)$

$$\frac{dy}{dx} = \frac{6x-y}{x+3+14y}$$

b $x^2 + y^2 = \sin xy$

$dx^2 + dy^2 = d\sin xy$

$2x\,dx + 2y\,dy = y\cos xy\,dx + x\cos xy\,dy$

$2x\,dx - y\cos y xy\,dx = x\cos xy\,dy - 2y\,dy$

$dx\,(2x - y\cos xy) = dy\,(x\cos xy - 2y)$

$$\frac{dy}{dx} = \frac{2x - y . \cos xy}{x \cdot \cos xy - 2y}$$

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4.a $y = 2^{3x+2} + e^{-3x} + \ln(x)$

$dy = d\,2^{3x+2} + d\,e^{-3x} + d\ln(x)$

$dy = 3 . 2^{3x+2} \ln(2) dx + -3 . e^{-3x} dx + \frac{dx}{x}$

$\frac{dy}{dx} = 3 . 2^{3x+2} . \ln(2) - \frac{3}{e^{3x}} + \frac{1}{x}$

b. $y = \ln\left(\frac{1}{x^3}\right) + \ln(x^4)$

$dy = d\ln\left(\frac{1}{x^3}\right) + d\ln(x^4)$ $\qquad d\ln x = \frac{dx}{x}$

$dy = -\frac{3x^2}{(x^3)^2} \cdot \frac{1}{\frac{1}{x^3}} dx + \frac{4x^3}{x^4} dx$

$dy = -\frac{3}{x^4} . x^3 dx + \frac{4}{x} dx$

$dy = -\frac{3}{x} dx + \frac{4}{x} dx$

$dy = \frac{1}{x} dx$

$\frac{dy}{dx} = \frac{1}{x}$

5. $y = Ae^{px}$

$dy = d\,Ae^{px}$

$dy = Ape^{px} dx$

$\frac{dy}{dx} = Ape^{px}$

$\frac{d^2y}{dx} = d(Ap\,e^{px})$

$\frac{d^2y}{dx} = A . p . d\,e^{px}$

$\frac{d^2y}{dx} = A . p . e^{px} d\,px$

$\frac{d^2y}{dx^2} = A . p . e^{px} . p$

$\frac{d^2y}{dx^2} - 2p\frac{dy}{dx} + p^2 y = 0$

$A . p^2 . e^{px} - 2p . Ap . e^{px} + p^2 . A . e^{px} = 0$

$A . p^2 . e^{px} - 2A . p^2 . e^{px} + A . p^2 . e^{px} = 0$

$0 = 0$

$\therefore$ Terbukti, $\frac{d^2y}{dx^2} - 2p\frac{dy}{dx} + p^2 y = 0$

Clement Samuel Marly 2206082444 Kalkulus B PR 2

6. $f(x) = \arcsin\left(\frac{2^{x+1}}{1+4^x}\right)$

$df(x) = d \arcsin\left(\frac{2^{x+1}}{1+4^x}\right)$

$df(x) = \frac{1}{\sqrt{1-\left(\frac{2^{x+1}}{1+4^x}\right)^2}} \; d\left(\frac{2^{x+1}}{1+4^x}\right)$

$df(x) = \frac{1}{\sqrt{1-\left(\frac{2^{x+1}}{1+4^x}\right)^2}} \cdot \frac{1+4^x \, d\, 2^{x+1} - 2^{x+1} \, d\, 1+4^x}{(1+4^x)^2}$

$df(x) = \frac{1}{\sqrt{1-\left(\frac{2^{x+1}}{1+4^x}\right)^2}} \cdot \frac{(1+4^x) \ln(2) . 2^{x+1} \, dx - 2^{x+1} . \ln(4) . 4^x \, dx}{(1+4^x)^2}$

$df(x) = \frac{(1+4^x) . \ln(2) . 2^{x+1} dx - 2^{x+1} . \ln(4) . 4^x dx}{\sqrt{1-\left(\frac{2^{x+1}}{1+4^x}\right)^2} \cdot (1+4^x)^2}$

$f'(0) = \frac{(1+1) . \ln(2) . 2^1 - 2^1 . 2\ln(2) . 1}{\sqrt{1-\left(\frac{2^1}{1+1}\right)^2} \cdot (1+1)^2}$

$f'(0) = \frac{2 . 2 . \ln(2) - 2 . 2 \ln(2)}{\sqrt{1-1^2} \cdot 4}$

$f'(0) = \frac{0}{0}$ $\therefore$ tidak terdefinisi