No :

Tanggal :

Clement Samuel Marly 220608244 Kalkulus - B PR - 5

1

$\int_0^{\pi/2} \sin^2 x \, (1-\sin^2 x)^2 \, dx$

$\int_0^{\pi/2} \sin^2 x \, . (\cos^2 x)^2 \, dx$

$\int_0^{\pi/2} 1-\cos^2 x \, . \cos^4 x \, dx$

$\int_0^{\pi/2} \cos^4 x - \cos^6 x \, dx \to \int_0^{\pi/2} \cos^4 x \, dx - \int_0^{\pi/2} \cos^6 x \, dx$

$\int_0^{\pi/2} \cos^4 x \, dx$

$\int_0^{\pi/2} \left(\frac{1+\cos 2x}{2}\right)^2 dx$

$\int_0^{\pi/2} \frac{1}{4} . \cos^2 2x + 2\cos 2x + 1 \, dx$

$\frac{1}{4} \cdot \left(\int_0^{\pi/2} \cos^2 2x \, dx + \int_0^{\pi/2} 2\cos 2x \, dx + \int_0^{\pi/2} 1 \, dx \right)$

$\frac{1}{4} \cdot \left(\int_0^{\pi/2} \frac{1}{2} (1+\cos 4x) \, dx + \frac{2}{2} [\sin 2x]_0^{\pi/2} + [x]_0^{\pi/2} \right)$

$\frac{1}{4} \cdot \left(\frac{1}{2} \int_0^{\pi/2} x + \frac{1}{2} \int_0^{\pi/2} \cos 4x + 0 + \frac{\pi}{2} \right)$

$\frac{1}{8} [x]_0^{\pi/2} + \frac{1}{32} [\sin 4x]_0^{\pi/2} + \frac{\pi}{8}$

$\frac{\pi}{16} + 0 + \frac{\pi}{8}$

$\frac{3\pi}{16}$

$\int_0^{\pi/2} \cos^6 x \, dx$

$\int_0^{\pi/2} \left(\frac{1}{2} \cdot (1+\cos 2x) \right)^3 dx$

$\frac{1}{8} \int_0^{\pi/2} \cos^3 2x + 3\cos^2 2x + 3\cos 2x + 1 \quad dx$

$\frac{1}{8} \left(\int_0^{\pi/2} \cos^2 2x . \cos 2x \, dx + \int_0^{\pi/2} 3 \cdot \frac{1}{2} (1+\cos 4x) \, dx + \int_0^{\pi/2} 3 \cos 2x \, dx + \int_0^{\pi/2} 1 \, dx \right)$

$\frac{1}{8} \left(\int_0^{\pi/2} (1-\sin^2 2x) \cos 2x \, dx + \frac{3}{2} \int_0^{\pi/2} 1 \, dx + \frac{3}{2} \int_0^{\pi/2} \cos 4x \, dx + \frac{3}{2} [\sin 2x]_0^{\pi/2} + [x]_0^{\pi/2} \right)$

$\frac{1}{8} \left(\int_0^{\pi/2} \cos 2x - \cos 2x \sin^2 2x \, dx \right) + \frac{3}{16} [x]_0^{\pi/2} + \frac{3}{16} \cdot \frac{1}{4} [\sin 4x]_0^{\pi/2} + 0 + \frac{1}{8} \cdot \frac{\pi}{2})$

$\frac{1}{8} \int_0^{\pi/2} \cos 2x \, dx - \frac{1}{8} \int_0^{\pi/2} \cos 2x \sin^2 2x + \frac{3\pi}{32} + 0 + \frac{\pi}{16})$

$\frac{1}{8} \cdot \frac{1}{2} [\sin 2x]_0^{\pi/2} - \frac{1}{8} \cdot \frac{1}{6} [\sin^3 2x]_0^{\pi/2} + \frac{3\pi}{32} + \frac{2\pi}{36})$

$0 - 0 + \frac{5\pi}{32}$

$\int_0^{\pi/2} \sin^2 x \, (1-\sin^2 2x)^2 \, dx = \frac{3\pi}{16} - \frac{5\pi}{32}$

$= \frac{\pi}{32}$ //

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2 $\int_0^{\pi/2} f(t)\,dt = \sin x + \int_0^x f(t)\cos^2 t\,dt$

$\frac{d}{dx}\int_0^{\pi/2} f(t)\,dt = \frac{d}{dx}\left(\sin x + \int_0^x f(t)\cos^2 t\,dt\right)$

$\frac{d}{dx}\int_0^x f(t)\,dt - \int_{\pi/2}^x f(t)\,dt = \cos x + f(x)\cos^2 x$

$f(x) - f(x) = \cos x + f(x)\cos^2 x$

$f(x) = -\frac{\cos x}{\cos^2 x}$

$f(x) = -\sec x$

$\int_0^{\pi/4} f(x)\,dx = \int_0^{\pi/4} -\sec x\,dx$

$= -\ln\left[|\tan x + \sec x|\right]_0^{\pi/4}$

$= -(\ln|\tan \pi/4 + \sec \pi/4| - \ln|\tan 0 + \sec 0|)$

$= -(\ln|\sqrt{2}+1| - \ln|1+0|)$

$= -\ln(1+\sqrt{2})$ //

3 a $\int_{-5}^{5}(f(x)+f(-x))\,dx$

$\int_{-5}^{5}(f(x)+f(x))\,dx \rightarrow$ fungsi genap

$2\int_{-5}^{5}(f(x))\,dx$

$2\left(\int_{-5}^{0} f(x)\,dx + \int_0^5 f(x)\,dx\right)$ | $\int_0^5 f(x) = -25$

$2\left(\int_0^5 f(x)\,dx + -25\right)$

$2(-50)$

-100 //

b $\int_{-5}^{5}|f(x)|\,dx$

$\int_0^5 f(x)\,dx + \int_0^5 -f(x)\,dx \longrightarrow f(x) \le 0$

$-\left(\int_0^5 f(x)\,dx + \int_0^5 f(x)\,dx\right)$

$-(-50)$

50 //

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$f(x) = \frac{x}{\sqrt{x^2+9}}$ pada $(0,2)$

$= \frac{1}{2-0} \int_0^2 \frac{x}{\sqrt{x^2+9}}\, dx$

$= \frac{1}{2} \int_0^2 \frac{3\tan\theta}{\sqrt{9\tan^2\theta+9}} \cdot 3\sec^2\theta\, d\theta$

$= \frac{1}{2} \int_0^2 \frac{3\tan\theta}{3\sqrt{\sec^2\theta}} \cdot 3\sec^2\theta\, d\theta$

$= \frac{1}{2} \int_0^2 3\tan\theta \cdot \sec\theta\, d\theta$

$= \frac{1}{2} \cdot 3 \left[\sec\theta\right]_0^2$

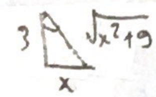
$= \frac{3}{2} \left[\frac{\sqrt{x^2+9}}{3}\right]_0^2$

$= \frac{3}{2} \left(\frac{\sqrt{13}}{3} - \frac{3}{3}\right)$

$= \frac{\sqrt{13}-3}{2}$ //

$\frac{x}{\sqrt{x^2+9}}$ | $x = 3\tan\theta \rightarrow \tan\theta = \frac{x}{3}$

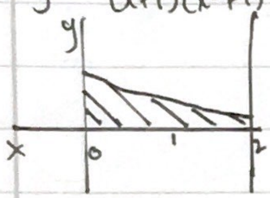
$dx = 3\sec^2\theta\, d\theta$



$\sec\theta = \frac{\sqrt{x^2+9}}{3}$

5

$y = \frac{5x^2+2x+3}{(x+1)(x^2+1)}$, $y=0 \rightarrow x = 2 \wedge 0$



tool = photomath

$y=0$ untuk $x = 2 \wedge 0$

$\int_0^2 y\, dx$

$\int_0^2 \frac{5x^2+2x+3}{(x+1)(x^2+1)}\, dx$

$\downarrow$

$\frac{A}{(x+1)} + \frac{Bx+C}{(x^2+1)} = \frac{5x^2+2x+3}{(x+1)(x^2+1)}$

$Ax^2 + A + Bx^2 + Cx + Bx + C$

$5x^2 = Ax^2 + Bx^2 \rightarrow 5 = A+B$

$2x = Cx + Bx \rightarrow 2 = C+B$

$3 = A + C \rightarrow 3 = 5 - B + 2 - B$

$A = 3 \quad B = 2 \quad C = 0$

$\int_0^2 \frac{3}{x+1} + \frac{2x}{x^2+1}\, dx$

$3\int_0^2 \frac{1}{x+1}\, dx + \int_1^5 \frac{2x}{x^2+1} \frac{dx^2+1}{2x}$ | $(x^2+1 = a)$

$3\left[\ln(x+1)\right]_0^2 + \left[\ln(a)\right]_1^5$

$3(\ln(3) - \ln(1)) + \ln(5) - \ln(1) \rightarrow \ln 1 = 0$

$3\ln(3) + \ln(5)$ //

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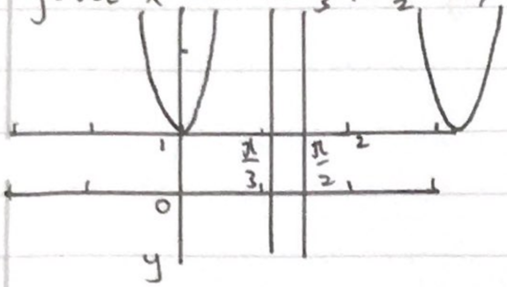
Clemont Samuel Marly 2206082114 Kalkulus -B PR-5

6 $y = \sec^6 x \to x = \frac{\pi}{3} \wedge \frac{\pi}{2}$, $y = 1$



tool = photomath

$$V = \pi \int_{\pi/3}^{\pi/2} (\sec^6 x - 1)^2 \, dx$$

$$= \pi \left(\int_{\pi/3}^{\pi/2} \sec^{12} x \, dx + \int_{\pi/3}^{\pi/2} 2\sec^6 x \, dx + \int_{\pi/3}^{\pi/2} 1 \, dx \right)$$

$$\int_{\pi/3}^{\pi/2} \sec^{12} x \, dx$$

$$= \int_{\pi/3}^{\pi/2} (\sec^2 x)^5 \, dx$$

$$= \int_{\pi/3}^{\pi/2} (1 - \tan^2 x)^5 \, dx$$

$$= \int_{\pi/3}^{\pi/2} -\tan^{10} x + 5\tan^8 x - 10\tan^6 x + 10\tan^4 x - 5\tan^2 x + 1$$

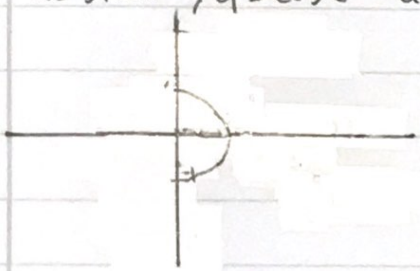
$$= \left[-\frac{1}{11}\tan^{11} x + \frac{5}{9}\tan^9 x - \frac{10}{7}\tan^7 x + \frac{10}{5}\tan^5 x - \frac{5}{3}\tan^3 x + \tan x \right]_{\pi/3}^{\pi/2}$$

$$= \left(-\frac{1}{11}\tan^{11}\frac{\pi}{2} + \frac{5}{9}\tan^9\frac{\pi}{2} - \frac{10}{7}\tan^7\frac{\pi}{2} + \frac{10}{5}\tan^5\frac{\pi}{2} - \frac{5}{3}\tan^3\frac{\pi}{2} + \tan\frac{\pi}{2} \right) - \ldots\ldots$$

$$\tan\frac{\pi}{2} = \infty$$

Volume $= \infty$ atau tidak terdefinisi,,

7 $x = \sin t$, $y = \cos t$ untuk $0 \le t \le \pi \to \int_0^\pi$



tool = geogebra $\to x = \sin t$, $t = \arcsin x$, $y = \cos(\arcsin x)$

$$= \int_0^\pi \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2} \, dt$$

$dx = \cos t$

$dy = \sin t$

$$= \int_0^\pi \sqrt{\cos^2 t + \sin^2 t} \, dt$$

$$= \int_0^\pi 1 \, dt$$

$$= [t]_0^\pi$$

$$= \pi - 0$$

Panjang busur kurva $= \pi$,,

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8 $y = \sqrt{4-x^2} \rightarrow -1 \leq x \leq 1$, mengelilingi x

$ds = \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\, dx \quad \rightarrow \int_{-1}^{1}$

$dy = d\sqrt{4-x^2}$

$dy = \frac{-x}{\sqrt{4-x^2}}$

$ds = \sqrt{1 + \left(\frac{-x}{4-x^2}\right)^2}\, dx$

$ds = \frac{2}{\sqrt{4-x^2}}\, dx$

$\text{Luas} = 2\pi \int_{-1}^{1} \sqrt{4-x^2} \quad \frac{2}{\sqrt{4-x^2}} \quad dx$

$= 2\pi \int_{-1}^{1} 2 \; dx$

$= 2\pi \; [2x]_{-1}^{1}$

$= 2\pi (2 + 2)$

$= 2 \cdot 4\pi$

$= 8\pi$,,