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1a. $f(x) = x^5 + 3x^4 + \frac{2}{3}x^3 + 7x^2 + 6x + 6$

$\int f(x) = \int x^5 + 3x^4 + \frac{2}{3}x^3 + 7x^2 + 6x + 6$

$= \int d(\frac{1}{6}x^6 + \frac{3}{5}x^5 + \frac{2}{12}x^4 + \frac{7}{3}x^3 + 3x^2 + 6x)$

$= \frac{1}{6}x^6 + \frac{3}{5}x^5 + \frac{1}{6}x^4 + \frac{7}{3}x^3 + 3x^2 + 6x + c$,

b. $f''(x) = 24x^2 - 48x + 2$

$\int f''(x) = \int 24x^2 - 48x + 2$

$f'(x) = \int d(8x^3 - 24x^2 + 2x)$

$f'(x) = 8x^3 - 24x^2 + 2x + c$

$\int f'(x) = \int 8x^3 - 24x^2 + 2x + c$

$= \int d(2x^4 - 8x^3 + x^2 + cx)$

$f(x) = 2x^4 - 8x^3 + x^2 + cx + a$

$f(1) = -9$

$-9 = 2.1 - 8.1 + 1 + c + a$

$-9 = 2 - 8 + 1 + c + a$

$c + a = -4$

$a = -4 - c$

$f(-2) = -4$

$-4 = 2.16 - 8.-8 + 4 - 2c + a.$

$-4 = 100 - 2c + a$

$-2c + a = -104$

$-2c - 4 - c = -104$

$-3c = -100$

$c = 100/3$ $\quad a = -4 - 100/3$

$= -112/3$

$f(x) = 2x^4 - 8x^3 + x^2 + \frac{100}{3}x - \frac{112}{3}$ //

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2a. $\int \frac{x^2-1}{x-1}\, dx$

$\int \frac{(x+1)(x-1)}{x-1}\, dx$

$\int x+1\, dx$

$= \frac{x^2}{2} + x + C_{11}$

2b. $\int x \ln(x)\, dx = x^2/_2 \cdot \ln(x) - \int \frac{1}{x} \cdot \frac{x^2}{2}\, dx \rightarrow \left(\int f dg = fg - \int g df\right)$

$= x^2/_2 \cdot \ln(x) - \frac{1}{2} \int x\, dx$

$= x^2/_2 \cdot \ln(x) - \frac{1}{2} \cdot \frac{x^2}{2}$

$= x^2/_2 \ln(x) - x^2/_4 + C$

2c. $\int \frac{1}{(x^2+1)(x^2-1)}\, dx = \int \frac{ax+b}{(x^2+1)} + \frac{c}{x+1} + \frac{d}{x-1}\, dx$

$= \int \frac{(ax+b)(x+1)(x-1) + c(x^2+1)(x-1) + d(x+1)(x^2+1)}{(x^2+1)(x^2-1)}$

$x=1,\ 1 = (ax+b)(2)(0) + c(2)(0) + d(2)(2)$

$1 = 4d \rightarrow d = \frac{1}{4}$

$x=-1,\ 1 = (ax+b)(0)(-2) + c(2)(-2) + d(0)(2)$

$1 = -4c \rightarrow c = -\frac{1}{4}$

$x=0,\ 1 = (ax+b)(-1) + c(-1) + d(1)$

$1 = -b + \frac{1}{4} + \frac{1}{4}$

$b = -\frac{1}{2}$

$ax^3 + bx^2 \ldots + cx^3 \ldots + dx^3 \ldots$

$0 = a + c + d$ (Choefisien3 yang diambil)

$a = -\frac{1}{4} + \frac{1}{4}$

$a = 0$

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$a = 0 \quad , b = -\frac{1}{2} \quad , c = -\frac{1}{4} \quad , d = \frac{1}{4}$

$$\int \frac{-\frac{1}{2}}{(x^2+1)} dx + \int \frac{-\frac{1}{4}}{x+1} dx + \int \frac{\frac{1}{4}}{x-1} dx$$

$$= -\frac{1}{2} \int \frac{1}{(x^2+1)} dx - \frac{1}{4} \int \frac{1}{x+1} dx + \frac{1}{4} \int \frac{1}{x-1} dx$$

$x = \tan\alpha \quad , dx = \sec^2\alpha \, d\alpha$

$$= -\frac{1}{2} \int \frac{1}{(\tan^2\alpha+1)} dx - \frac{1}{4} \ln|x+1| + \frac{1}{4} \ln|x-1|$$

$$= -\frac{1}{2} \int \frac{\sec^2\alpha}{\sec^2\alpha} d\alpha - \frac{1}{4} \ln|x+1| + \frac{1}{4} \ln|x-1|$$

$$= -\frac{1}{2} \int d\alpha - \frac{1}{4} \ln|x+1| + \frac{1}{4} \ln|x-1|$$

$$= -\frac{1}{2} \alpha - \frac{1}{4} \ln|x+1| + \frac{1}{4} \ln|x-1| \quad \rightarrow \alpha = \arctan x$$

$$= -\frac{1}{2} \arctan x - \frac{1}{4} \ln|x+1| + \frac{1}{4} \ln|x-1| + C_{//}$$

3a. $\int \sin x \operatorname{cosec} x \, dx$

$\int \frac{\sin x}{\sin x} dx$

$\int 1 dx$

$x + C_{//}$

3b. $\int \frac{\sec^2 x}{\tan x} dx$

$\int \frac{\sec^2 x}{\tan x} \frac{d\tan x}{\sec^2 x}$

$\int \tan^{-1} x \, d\tan x$

$\ln|\tan x| + C_{//}$

3c. $\int \frac{\sin \frac{1}{x}}{x^2} dx$

$u = \frac{1}{x} = x^{-1}$

$du = -x^{-2} dx \rightarrow dx = -x^2 du$

$\int \frac{\sin u}{x^2} \cdot -x^2 du$

$-\int \sin u \, du$

$= \cos u + C$

$= \cos \frac{1}{x} + C_{//}$

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3d. $\int e^{2x} \sin 2x \, dx$

$\int e^{2x} \sin 2x \frac{d\cos 2x}{-2\sin 2x}$

$\int -\frac{1}{2} e^{2x} \, d\cos 2x \qquad \rightarrow \int f\,dg = fg - \int g\,df$

$-\frac{1}{2} e^{2x} . \cos 2x + \frac{1}{2} \int \cos 2x . 2e^{2x} dx$

$-\frac{1}{2} e^{2x} \cos 2x + \int \cos 2x . e^{2x} dx$

$-\frac{1}{2} e^{2x} \cos 2x + \int \cos 2x . e^{2x} \frac{d\sin 2x}{2\cos 2x}$

$-\frac{1}{2} e^{2x} \cos 2x + \frac{1}{2} \int e^{2x} d\sin 2x$

$-\frac{1}{2} e^{2x} \cos 2x + \frac{1}{2} e^{2x} \sin 2x - \frac{1}{2} \int \sin 2x \, 2e^{2x} dx$

$-\frac{1}{2} e^{2x} \cos 2x + \frac{1}{2} e^{2x} \sin 2x - \int \sin 2x \, e^{2x} dx$

$\int e^{2x} \sin 2x \, dx = -\frac{1}{2} e^{2x} \cos 2x + \frac{1}{2} e^{2x} \sin 2x - \int \sin 2x \, e^{2x} dx$

$2 \int e^{2x} \sin 2x \, dx = -\frac{1}{2} e^{2x} . \cos 2x + \frac{1}{2} e^{2x} \sin 2x$

$\int e^{2x} \sin 2x \, dx = -\frac{1}{4} e^{2x} \cos 2x + \frac{1}{4} e^{2x} \sin 2x + C$ //

3e. $\int \frac{1}{1+\cos 2x} dx$

$\int \frac{1}{1+\cos 2x} \cdot \frac{1-\cos 2x}{1-\cos 2x} dx$

$\int \frac{1-\cos 2x}{1-\cos^2 2x} dx$

$\int \frac{1}{1-\cos^2 2x} dx - \frac{\cos 2x}{1-\cos^2 2x} dx$

$\int \frac{1}{\sin^2 2x} dx - \int \frac{\cos 2x}{\sin^2 2x}$

$\int \csc^2 2x \frac{d\,2x}{2} - \int \frac{\cos 2x}{\sin^2 2x} \frac{d\sin 2x}{2\cos 2x}$

$\frac{1}{2} \int \csc^2 2x \, d2x - \frac{1}{2} \int \frac{d\sin 2x}{\sin^2 2x}$

$\rightarrow \frac{1}{2} \cdot -\cot 2x - \frac{1}{2} \cdot \int \sin^{-2} 2x \, d(\sin 2x)$

$= -\frac{1}{2} \cot 2x - \frac{1}{2} . -1 \sin^{-1} 2x$

$= -\frac{1}{2} \cot 2x + \frac{1}{2} \csc 2x + C$

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3f. $\int \sec^3 x \, dx$

$\int \sec^n x \, dx = \frac{1}{n-1} \sec^{n-2} x \tan x + \int \frac{n-2}{n-1} \sec^{n-2} x \, dx$

$\int \sec^3 x \, dx = \frac{1}{2} \sec^1 x \tan x + \frac{1}{2} \int \sec^1 x dx$

$= \frac{1}{2} \sec x \tan x + \frac{1}{2} \ln |\sec x + \tan x| + C$

3g. $\int \cos^2 3x \sin 2x \, dx$

$\int \cos 3x . \cos 3x \sin 2x \, dx$

$\cos 3x \sin 2x = \frac{1}{2} (\sin 5x - \sin x) dx$

$\int \cos 3x \cdot \frac{1}{2} \sin 5x - \frac{1}{2} \sin x \, dx$

$\frac{1}{2} \int \cos 3x \sin 5x - \cos 3x \sin x \, dx$

$\frac{1}{2} \int \sin 5x \cos 3x - \frac{1}{2} (\sin 4x - \sin 2x) \, dx$

$\frac{1}{4} \int \frac{1}{2} (\sin 8x + \sin 2x) - \frac{1}{2} \sin 4x + \sin 2x \, dx$

$\frac{1}{4} \left(\int \sin 8x dx + \int \sin 2x dx - \int \sin 4x dx + \int \sin 2x dx \right)$

$\frac{1}{4} \left(-\frac{1}{8} \cos 8x - \frac{1}{2} \cos 2x + \frac{1}{4} \cos 4x - \frac{1}{2} \cos 2x \right)$

$-\frac{1}{32} \cos 8x - \frac{1}{8} \cos 2x + \frac{1}{16} \cos 4x - \frac{1}{8} \cos 2x + C$

$-\frac{1}{32} \cos 8x + \frac{1}{16} \cos 4x - \frac{1}{4} \cos 2x + C$

4a. $\int \frac{1}{x^2+1} dx$ $\rightarrow$ $x = 1 \tan \theta$ $\rightarrow$ $\theta = \arctan x$

$dx = \sec^2 \theta \, d\theta$

$\int \frac{1}{\tan^2 \theta + 1} . \sec^2 \theta \, d\theta$

$= \arctan x + C$,,

$\int \frac{\sec^2 \theta}{\sec^2 \theta} d\theta$

$\int d\theta$

$= \theta + C$

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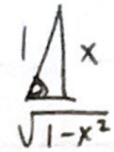
4b $\int x\sqrt{1-x^2}\,dx \rightarrow x = 1\sin\theta \rightarrow$



$dx = \cos\theta\,d\theta$

$\cos\theta = \frac{\sqrt{1-x^2}}{1}$

$\int \sin\theta\sqrt{1-\sin^2\theta}\,.\cos\theta\,d\theta$

$\int \sin\theta\sqrt{\cos^2\theta}\,.\cos\theta\,d\theta$

$\int \sin\theta\cos^2\theta\,d\theta$

$\int \sin\theta\cos^2\theta\,\frac{d\cos\theta}{-\sin\theta}$

$\int -\cos^2\theta\,d\cos\theta$

$-\frac{1}{3}\cos^3\theta + c$

$-\frac{1}{3}\cos^3\sqrt{1-x^2} + c$ //

5a. $\int \sec^5 x\,dx \rightarrow \int \sec^n x\,dx = \frac{1}{n-1}\sec^{n-2}x\tan x + \int \frac{n-2}{n-1}\sec^{n-2}x\,dx$

$= \frac{1}{4}\sec^3 x\tan x + \int \frac{3}{4}\sec^3 x\,dx$

$= \frac{1}{4}\sec^3 x\tan x + \frac{3}{4}\left(\frac{1}{2}\sec x\tan x + \frac{1}{2}\int \sec x\,dx\right)$

$= \frac{1}{4}\sec^3 x\tan x + \frac{3}{8}\sec x\tan x + \frac{3}{8}\ln|\sec x + \tan x| + c$ //