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1 Kohi training 1 tahun → Kohi tetap 1 tahun → kohi senior

↓ 1 kohi training ↓ 2 kohi training

Kohi awal = 2 kohi training

$A_0 = 2$ kohi

n	Kohi training	Kohi tetap	Kohi senior	Jumlah
0	2	0	0	2
1	2	2	0	4
2	6	2	2	10
3	14	6	4	24
4	34	14	10	58

$A_n =$ jumlah kohi

$A_n = 2A_{n-1} + A_{n-2}, A_0 = 2, A_1 = 4, n \geq 2$

$A_7 = 2 . A_6 + A_5$

$A_7 = 2.(2.A_5 + A_4) + (2 . A_4 + A_3)$

$A_7 = 2.(2.A_5 + 58) + (2.58 + 24)$

$A_7 = 2.(2.A_5 + 58) + (140)$

$A_7 = 2.(2.140 + 58) + 140$

$A_7 = 2.\ 338 + 140$

$A_7 = 816$

2 a. $A_n = 2A_{n-1} - A_{n-2}, n \geq 2, A_0 = 4, A_1 = 1 \nearrow \sum_{n=2}^{\infty} A_n z^n = \sum_{n=2}^{\infty} 2A_{n-1} z^n - \sum_{n=2}^{\infty} A_{n-n} z^n$

$A_2 z^2 + A_3 z^3 + \ldots = 2(A_1 z^2 + A_2 z^3 + \ldots) - (A_0 z^2 + A_1 z^3 + \ldots)$

$G(z) = A_0 z^0 + A_1 z^1 + \ldots \quad \downarrow (A_1 z^1 + A_2 z^2 + \ldots).z \quad \downarrow z^2(A_0 z^0 + A_1 z^1 +)$

$G(z) - A_0 - A_1 z^1 = 2(G(z) - A_0).z - z^2(G(z))$

$G(z) - 4 - 1.z = 2(G(z) - 4).z - z^2(G(z))$

$G(z) - 2z\,G(z) + z^2 G(z) = 4 + z - 8z$

$G(z)(1 - 2z + z^2) = 4 - 7z$

No :
Tanggal :

$$G(z) = \frac{4-7z}{1-2z+z^2} \rightarrow G(z) = \frac{4-7z}{(z-1)(z-1)}$$

$$G(z) = \frac{A}{(z-1)} + \frac{B}{(z-1)^2}$$

$$G(z) = \frac{A(z-1)^2 + B(z-1)}{(z-1)^3}$$

$4-7z = Az - A + B$

$4 = (-A + B)$

$-7z = Az$

$A = -7$

$-A + B = 4$

$7 + B = 4$

$B = -3$

$$G(z) = \frac{-7}{(z-1)} + \frac{-3}{(z-1)^2}$$

$G(z) = -7\left(\frac{1}{z-1}\right) + -3\left(\frac{1}{(z-1)^2}\right)$

$G(z) = 7\left(\frac{1}{z-1}\right) - 3\left(\frac{1}{(z-1)^2}\right)$

$A_n = 7 - 3(n+1)$

$A_n = 4 - 3n$ //

6 $A_n = 7A_{n-1} - 5^n$, $n \geq 1$, $A_0 = 8$, $A_1 = 51$

$\sum_{n=1}^{\infty} A_n z^n = 7 \sum_{n=1}^{\infty} A_{n-1} z^n - \sum_{n=1}^{\infty} 5^n z^n$

$G(z) = A_0 z^0 + A_1 z^1 + \ldots$

$A_1 z^1 + A_2 z^2 + \ldots = 7(A_0 z^1 + A_1 z^2 + \ldots) - (5^1 z^1 + 5^2 z^2 + \ldots)$

$G(z) - A_0 z^0 = 7 . z . G(z) - \left(\frac{1}{1-5z} - 1\right)$

$G(z)(1-7z) = 8 - \left(\frac{1}{1-5z}\right.$

$G(z) = 8 - \left(\frac{1}{1-5z} - 1\right) / 1-7z$

$G(z) = 9 - \frac{1}{1-5z} / 1-7z$

$$G(z) = \frac{9(1-5z) - 1}{(1-7z)(1-5z)}$$

$$G(z) = \frac{8 - 45z}{(1-7z)(1-5z)}$$

$$G(z) = \frac{A}{(1-5z)} + \frac{B}{(1-7z)}$$

$$G(z) = \frac{A(1-7z) + B(1-5z)}{(1-7z)(1-5z)}$$

$$G(z) = \frac{A - 7Az + B - 5Bz}{(1-7z)(1-5z)} \rightarrow \frac{(A+B) - (7A+5B)z}{(1-7z)(1-5z)}$$

$8 - 45z = A+B - (7A+5B)z$

$8 = A+B$ → $40 = 5A+5B$ → $8 = \frac{5}{2} + B$

$45 = 7A+5B$ → $45 = 7A+5B$ -

$-5 = -2A$ → $B = \frac{11}{2}$

$A = \frac{5}{2}$

$$G(z) = \frac{\frac{5}{2}}{(1-5z)} + \frac{\frac{11}{2}}{(1-7z)}$$

$$A_n = \frac{5}{2} \cdot 5^n + \frac{11}{2} \cdot 7^n$$

$$A_n = \frac{(5.5^n + 11.7^n)}{2} \rightarrow A_n = \frac{(5^{n+1} + 11.7^n)}{2} \text{ //}$$

3 $A_n = 10A_{n-1} - 25A_{n-2}$

a $A_n = 0 \rightarrow A_0 = 0, A_1 = 0, A_2 = 1$

$A_2 = 10.A_1 - 25A_0$

$A_2 = 10.0 - 25.0$

$A_2 = 0$ // solusi

b $A_n = 1 \rightarrow A_0 = 1, A_1 = 1, A_2 = 1$

$A_2 = 10.1 - 25.1$

$A_2 = -15$

$1 \neq -15$ bukan solusi

3 c $A_n = n5^n$

$A_{n-1} = (n-1)5^{n-1}$ | $A_{n-2} = (n-1)5^{n-2}$

$A_{n-1} = \frac{(n-1)5^n}{5}$ | $A_{n-2} = \frac{(n-2)5^n}{25}$

$A_n = 10\left(\frac{(n-1)5^n}{5}\right) - 25\left(\frac{(n-2)5^n}{25}\right)$

$n5^n = 2(n-1)5^n - ((n-2)5^n)$

$n5^n = 2n5^n - \cancel{2.5^n} - n5^n + \cancel{2.5^n}$

$n5^n = n5^n$ // solusi

d $A_n = n^2 5^n$

$A_{n-1} = (n-1)^2 5^{n-1}$ | $A_{n-2} = (n-2)^2 5^{n-2}$

$A_{n-1} = \frac{(n^2-2n+1)5^n}{5}$ | $A_{n-2} = \frac{(n^2-4n+4)5^n}{25}$

$A_n = 10\left(\frac{(n^2-2n+1)5^n}{5}\right) - 25\left(\frac{(n^2-4n+4)5^n}{25}\right)$

$n^2 5^n = 2n^2 5^n - 4n5^n + 2.5^n - n^2 5^n + 4n5^n - 1.5^n$

$n^2 5^n \neq n^2 5^n - 2.5^n$ // bukan solusi

4 a $G(z) = 3 + 3z + 3z^2 + 3z^3 + \frac{z}{1-2z^2}$

$\frac{z}{1-2z^2} = z + 2z^3 + 4z^5 + \ldots$

$G(z) = 3 + 3z + 3z^2 + 3z^3 + z + 2z^3 + 4z^5 + \ldots$

$G(z) = 3 + 4z + 3z^2 + 5z^3 + 4z^5 + \ldots$

$X_n = \langle 3, 4, 3, 5, 0, 4, \ldots \rangle$ //

b $G(z) = \frac{4z^2 + z - 1}{(1-z)^2}$

$= \frac{4z^2}{(1-z)^2} + \frac{z}{(1-z)^2} - \frac{1}{(1-z)^2}$

No :

Tanggal :

$\frac{4z^2}{(1-z)^2} = 4z^2 + 8z^3 + 12z^4 + 16z^5$

$\frac{z}{(1-z)^2} = 0 + z + 2z^2 + 3z^3 + 4z^4 + 5z^5$ +

$= 0 + z + 6z^2 + 11z^3 + 16z^4 + 21z^5$

$\frac{1}{(1-z)^2} = 1 + 2z + 3z^2 + 4z^3 + 5z^4 + 6z^5$ −

$-1 - z + 3z^2 + 7z^3 + 11z^4 + 15z^5$

$X_n = \langle -1, -1, 3, 7, 11, 15, \ldots \rangle$ //

4 c $G(z) = \frac{3z^2 + 7z - 2}{1 - z - 2z^2} \rightarrow (1+z)(1-2z)$

$= -\frac{3}{2} + \frac{11z - 1}{2(1-z-2z^2)}$

$\frac{11z-1}{2(1-z-2z^2)} = \frac{A}{(1+z)} + \frac{B}{(1-2z)}$

$\frac{\frac{11z-1}{2}}{(1-z-2z^2)} = \frac{A(1-2z) + B(1+z)}{(1-z-2z^2)}$

$\frac{11z-1}{2} = A - 2zA + B + Bz$

$11z - 1 = 2A - 4zA + 2B + 2Bz$

$11z - 1 = (2A + 2B) + (2Bz - 4zA)$

$11 = 2B - 4A$

$-1 = 2A + 2B$ −

$12 = -6A$

$A = -2$

$\rightarrow 11 = 2B + 8$

$3 = 2B$

$B = \frac{3}{2}$

$G(z) = -\frac{3}{2} - 2\left(\frac{1}{1+z}\right) + \frac{3}{2}\left(\frac{1}{1-2z}\right)$

$G(z) = -\frac{3}{2} - 2(-1)^n + \frac{3}{2}(2)^n$

$-\frac{3}{2} = -\frac{3}{2}$

$-2(-1)^n - \frac{3}{2} = -3\frac{1}{2} + 2z + -2z^2 + 2z^3 - 2z^4 + 2z^5 + \ldots$

$\frac{3}{2}(2)^n = \frac{3}{2} + 3z + 6z^2 + 12z^3 + 24z^4 + 48z^5$ +

$X_n = \langle -2, 5, 4, 14, 22, 50, \ldots \rangle$ //

5 $K_n = \langle 0, 3, 13, 44, \dots \rangle = L(z) \times M(z)$

$L_n = \langle 3, 4, 5, 6, \dots \rangle$

$M_n = \langle 0, 1, 3, 9, \dots \rangle$

$L_n = \langle 2, 2, 2, 2, 2 \dots \rangle + \langle 1, 2, 3, 4, \dots \rangle$

$L(z) = 2 \cdot \frac{1}{1-z} + \frac{1}{(1-z)^2}$

$= \frac{2(1-z)^2 + (1-z)}{(1-z)^3}$

$= \frac{2(1-z) + 1}{(1-z)^2}$

$L(z) = \frac{3-2z}{(1-z)^2}$

$M_n = \langle 0, 1, 3, 9, \dots \rangle$

$M(z) = z + 3z^2 + 9z^3 + \dots$

$= z\left(\frac{1}{1-3z}\right)$

$= \frac{z}{1-3z}$

$K(z) = M(z) \times L(z)$

$= \frac{3-2z}{(1-z)^2} \cdot \frac{z}{1-3z}$

$K(z) = \frac{3-2z \cdot z}{(1-2z+z)(1-3z)}$

$= \frac{3z - 2z^2}{-3z^3 + 7z^2 - 5z + 1}$ //

6 a $X_n = \langle 2, -2, 2, -2, 2, \dots \rangle$

$G(z) = 2 - 2z + 2z^2 - 2z^3 + 2z^4 + \dots$

$G(z) = 2(1 - z + z^2 - z^3 + z^4 \dots)$

$G(z) = 2\left(\frac{1}{1+z}\right)$

$= \frac{2}{1+z}$ //

No :

Tanggal :

6 b $X_n = \langle 0, 2, 5, 9, 14, 20, 27, \dots \rangle$

$G(z) = 2z + 5z^2 + 9z^3 + 14z^4 + 20z^5 + 27z^6 + \dots$ $\rightarrow z.G(z) = \langle 2, 5, 9, 14, 20, 27, \dots \rangle$

$z.G(z) = 2z^2 + 5z^3 + 9z^4 + 14z^5 + 20z^6 + \dots$ $-$

$G(z)(1-z) = 2z + 3z^2 + 4z^3 + 5z^4 + 6z^5 + 7z^6 + \dots$ $\rightarrow \langle 0, 2, 3, 4, 5, 6, 7, \dots \rangle$

$G(z)(1-z)+1 = 1 + 2z + 3z^2 + 4z^3 + 5z^4 + 6z^5 + 7z^6 + \dots$ $\rightarrow \langle 1, 2, 3, 4, 5, 6, 7, \dots \rangle$

$(G(z)(1-z)+1)z = z + 2z^3 + 3z^3 + 4z^4 + 5z^5 + 6z^6 + 7z^7$ $-$

$(G(z)(1-z)+1)(1-z) = 1 + z + z^2 + z^3 + z^4 + z^5 + z^6 + z^7$ $\rightarrow \langle 1, 1, 1, 1, 1, 1, \dots \rangle$

$(G(z)(1-z)+1)(1-z)z = z + z^2 + z^3 + z^4 + z^5 + z^6 + z^7$ $-$

$(G(z)(1-z)+1)(1-z)(1-z) = 1$

$(G(z)(1-z)+1)(1-z)^2 = 1$

$G(z)(1-z)^3 + (1-z)^2 = 1$

$$G(z) = \frac{1-(1-z)^2}{(1-z)^3}$$

$$G(z) = \frac{-z^2+2z}{(1-z)^3} \;//$$

c $X_n \langle 4, 5, 9, 27, 123, \dots \rangle$

$$L(z) = \sum_{n=0}^{\infty} n!\, z^n = \frac{e^{-1/z} E_1(-1/z)}{z}$$

$L(z) = 0!\,x^0 + 1!\,x^1 + 2!\,x^2 + 3!\,x^3 + 4!\,x^4 + \dots$

$L_n : \langle 1, 1, 2, 6, 24, \dots \rangle$

$M_n = \langle 3, 3, 3, 3, \dots \rangle$

$K_n = \langle 1, 2, 6, 24, \dots \rangle$

$K(z) = \frac{1}{z}(L(z) - 1)$

$= \frac{1}{z}\left(\frac{e^{-1/z} E_1(-1/z)}{z} - 1\right)$

$M(z) = \frac{3}{1-z}$

$G(z) = K(z) + M(z)$

$= \left(\frac{L(z)-1}{z}\right) + \frac{3}{1-z}$

$= \frac{(L(z)-1)(1-z) + 3z}{z - z^2}$

6 c

$$= \frac{L(z) - L(z).z - 1 + z + 3z}{z - z^2}$$

$$= \frac{L(z)(1-z) - 1 + 4z}{z(1-z)}$$

$$= \frac{L(z)}{z} + \frac{-1+4z}{z-z^2}$$

$$= \frac{e^{-1/z} E_1(-1/z)}{z^2} + \frac{-1+4z}{z-z^2} \,//$$

7

$n = 0, 1, 2, \dots$

a $A_n = 2^n + 1$

$2^n = \frac{1}{1-2z}$

$1 = \frac{1}{1-z}$

$$A_n = \frac{1}{1-2z} + \frac{1}{1-z}$$

$$= \frac{1-z+1-2z}{(1-2z)(1-z)}$$

$$= \frac{2-3z}{1-3z+2z^2} \,//$$

b $A_n = \frac{n-1}{n}$, $n != 0$

$$G(z) = \sum_{n=1}^{\infty} \frac{n-1}{n} \cdot z^n$$

$$= 0 + \frac{1}{2}z^2 + \frac{2}{3}z^3 + \frac{3}{4}z^4 + \dots$$

$G_n = \langle 0, 0, \frac{1}{2}, \frac{2}{3}, \frac{3}{4}, \dots \rangle$

$H_n = \langle 0, -1, -\frac{1}{2}, -\frac{1}{3}, -\frac{1}{4}, \dots \rangle$

$H(z) = \ln(1-z)$

$G_n - H_n = \langle 0, 1, 1, 1, 1, 1 \dots \rangle$

$G_n = \langle 0, 1, 1, 1, \dots \rangle + H_n$

$$G(z) = \frac{z}{1-z} + \ln(1-z)$$

$$= \frac{z + (1-z)\ln(1-z)}{1-z}$$

No :

Tanggal :

8 $G(z) = \frac{1+3z-z^2}{(1-z)(1-2z)(z+1)}$

$$\frac{1+3z-z^2}{(1-z)(1-2z)(z+1)} = \frac{A}{1-z} + \frac{B}{1-2z} + \frac{C}{z+1}$$

$$1+3z-z^2 = A(1-2z)(z+1) + B(1-z)(z+1) + C(1-2z)(1-z)$$

$$1+3z-z^2 = A(1-z-2z^2) + B(1-z^2) + C(1-3z+2z^2)$$

$$1+3z-z^2 = A - Az - A2z^2 + B - Bz^2 + C - C3z + C2z^2$$

$$1+3z-z^2 = (A+B+C) + (-A - C3)z + (-A2 - B + C2)z^2$$

$1 = A+B+C$

$3 = -A-3C$ +

$4 = B-2C$

$-1 = -2A - B + 2C$ +

$3 = -2A$

$A = -\frac{3}{2}$

$\rightarrow 3 = \frac{3}{2} - 3C$

$1 = \frac{1}{2} - C$

$-\frac{1}{2} = C$

$4 = B+1$

$3 = B$

$$\frac{1+3z-z^2}{(1-z)(1-2z)(z+1)} = -\frac{3}{2}\left(\frac{1}{1-z}\right) + 3\left(\frac{1}{1-2z}\right) + -\frac{1}{2}\left(\frac{1}{1+z}\right)$$

$$= -\frac{3}{2} + 3(2^n) - \frac{1}{2}(-1)^n \quad \rightarrow X_0 = 1, X_1 = 5, X_2 = 10, X_3 = 23$$

$$X_n = \langle 1, 5, 10, 23, 46, \ldots \rangle$$

9 Eskrim awal = 200

Deh Depe min = 20 , Pak Esde min = 25 , Sofita min = 35

Sisa = 200 − (20 + 25 + 35)

= 120

a Maksimal :

Deh Depe = 20 + 120

= 140

Pak Esde = 25 + 120

= 145

Sofita = 35 + 120

= 155

9 b $A_n = \langle 20, \dots, 140 \rangle \rightarrow$ Deh Depe

$B_n = \langle 25, \dots, 145 \rangle \rightarrow$ Pak Esde

$C_n = \langle 35, \dots, 155 \rangle \rightarrow$ Sofia

$A + B + C \leq 200$

$A(z) = z^{20} + z^{21} + \dots + z^{140}$

$B(z) = z^{25} + z^{26} + \dots + z^{145}$

$C(z) = z^{35} + z^{36} + \dots + z^{155}$

$z^{200} = A(z)\ B(z)\ C(z)$

$z^{200} = (z^{20} + z^{21} + \dots + z^{140})(z^{25} + z^{26} + \dots + z^{145})(z^{35} + z^{36} + \dots + z^{155})$ "

z^{200} dicari untuk dapat jumlah cara membagi eskrim

c Vanilla : $\langle 0, 1, \dots, 7 \rangle$

Strawberry : $\langle 0, 1, \dots, 10 \rangle$

Cohlot : $\langle 0, 1, \dots, 15 \rangle$ $\rightarrow \leq 50$

Matcha : $\langle 0, 1, \dots, 18 \rangle$

$V(z) = z^0 + z^1 + \dots + z^7$

$S(z) = z^0 + z^1 + \dots + z^{10}$

$C(z) = z^0 + z^1 + \dots + z^{15}$

$M(z) = z^0 + z^1 + \dots + z^{18}$

Banyak cara memberikan label :

$V = \frac{z^0}{0!} + \frac{z^1}{1!} + \frac{z^2}{2!} + \frac{z^3}{3!} + \dots + \frac{z^7}{7!}$

$S = \frac{z^0}{0!} + \frac{z^1}{1!} + \dots + \frac{z^{10}}{10!}$

$C = \frac{z^0}{0!} + \frac{z^1}{1!} + \dots + \frac{z^{15}}{15!}$

$M = \frac{z^0}{0!} + \frac{z^1}{1!} + \dots + \frac{z^{18}}{18!}$

$FP = 50!\left(\left(\frac{z^0}{0!} + \frac{z^1}{1!} + \dots + \frac{z^7}{7!}\right)\left(\frac{z^0}{0!} + \frac{z^1}{1!} + \dots + \frac{z^{10}}{10!}\right)\left(\frac{z^0}{0!} + \frac{z^1}{1!} + \dots + \frac{z^{15}}{15!}\right)\left(\frac{z^0}{0!} + \frac{z^1}{1!} + \dots + \frac{z^{18}}{18!}\right)\right)$

Karena kode unik dan jumlah kode = 50, koefisien z^{50} dicari

$= \frac{50!}{7!\,10!\,15!\,18!}$