EE 7510 – Advanced Linear Systems

Project Report

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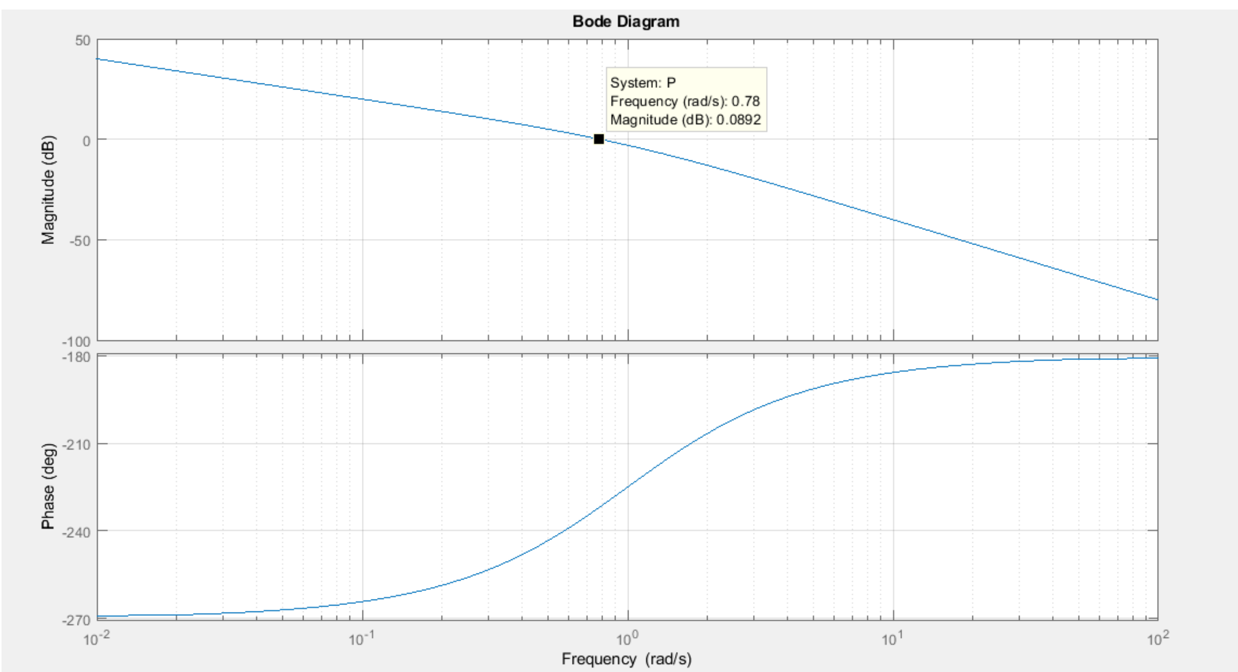
Project Summary

We are given a plant model P and we need to obtain an Observer based controller using LQR/LTR Control technique and then apply this model to a True system P_{true}, which has perturbations. Our main aim is to make this a robust controller. In the way of achieving our goal, we may lose the system rise time. For this, we incorporate a non-linear input r(t). At the end of the project, we can conclude that robustness is achieved in the design, along with an improvement of about 30% rise time.

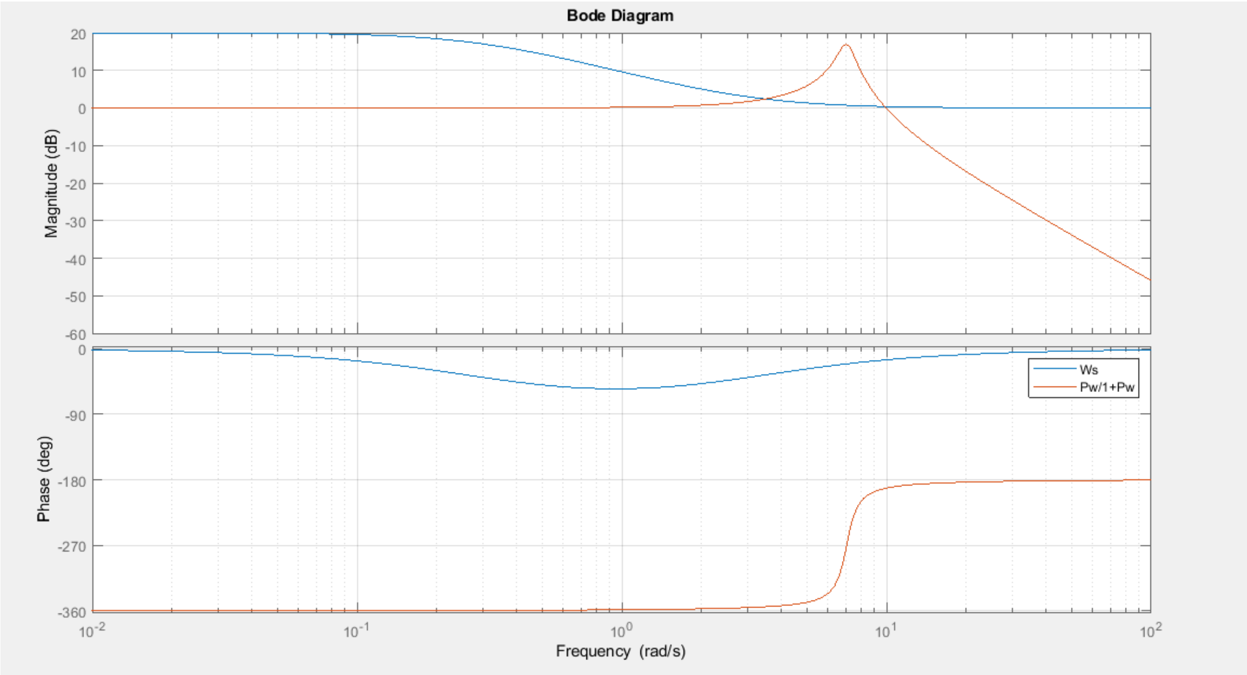
Introduction

We are given the Plant model P(s) = 1/s*(s-1); which has two poles and no zeros. Initially, let us not consider the uncertainty (ε) here for an instance. Uncertainty is included in the true model of the system, which will come later. We have the cut off frequency $w_c = 0.77$, and we can clearly see that the gain margin is infinite and the phase margin is about 52^0.



Let us take the stabilizing function W_s given as (s+3a)/(s+0.3a) and multiply it with P to get Pw. Now, we have to choose a magnitude for the transfer function Pw, so that it satisfies the condition $|W_s| > L/(1+L)$, where L is the closed loop gain. As we only have the plant model here, let's use Pw as L and set the magnitude level. When W(weighting function) is taken very large, then the above condition is not satisfied.

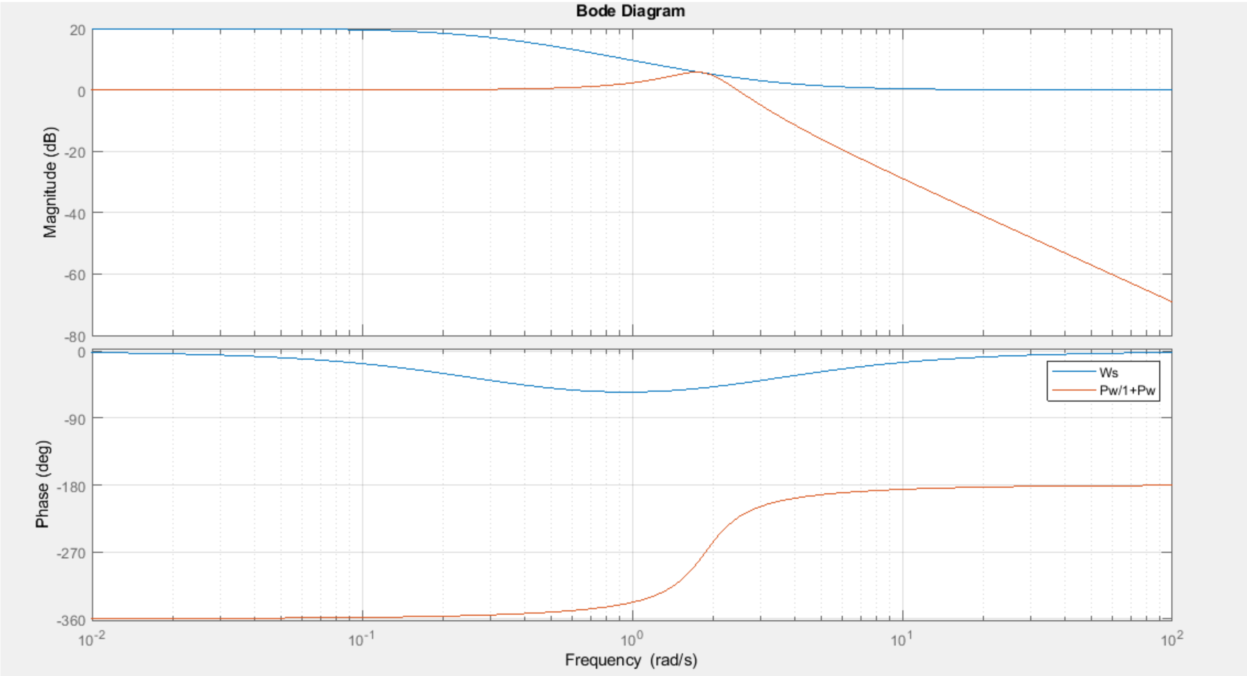
For W = 20, the magnitude response is given as



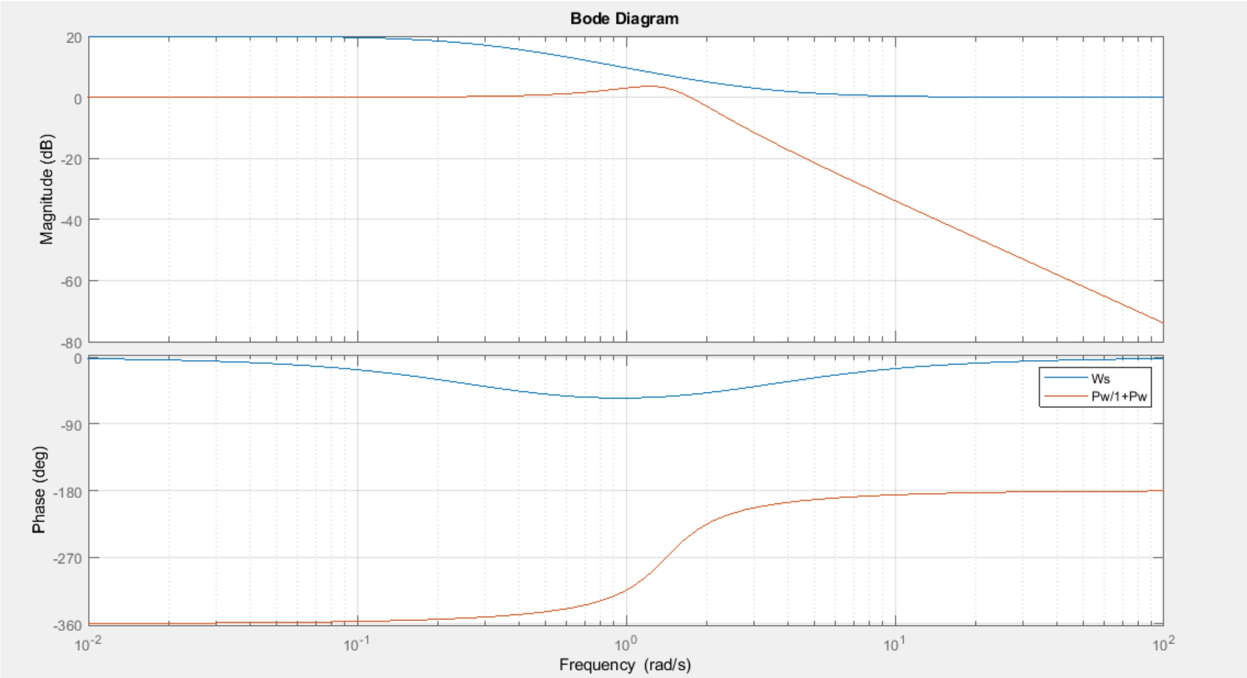
W = 25

where we can clearly observe the condition is not satisfying at higher frequencies.

As we go on reducing the value of W, to a value of about 3.5,and then to 2, we can observe the shift in gain and now the condition is satisfied.

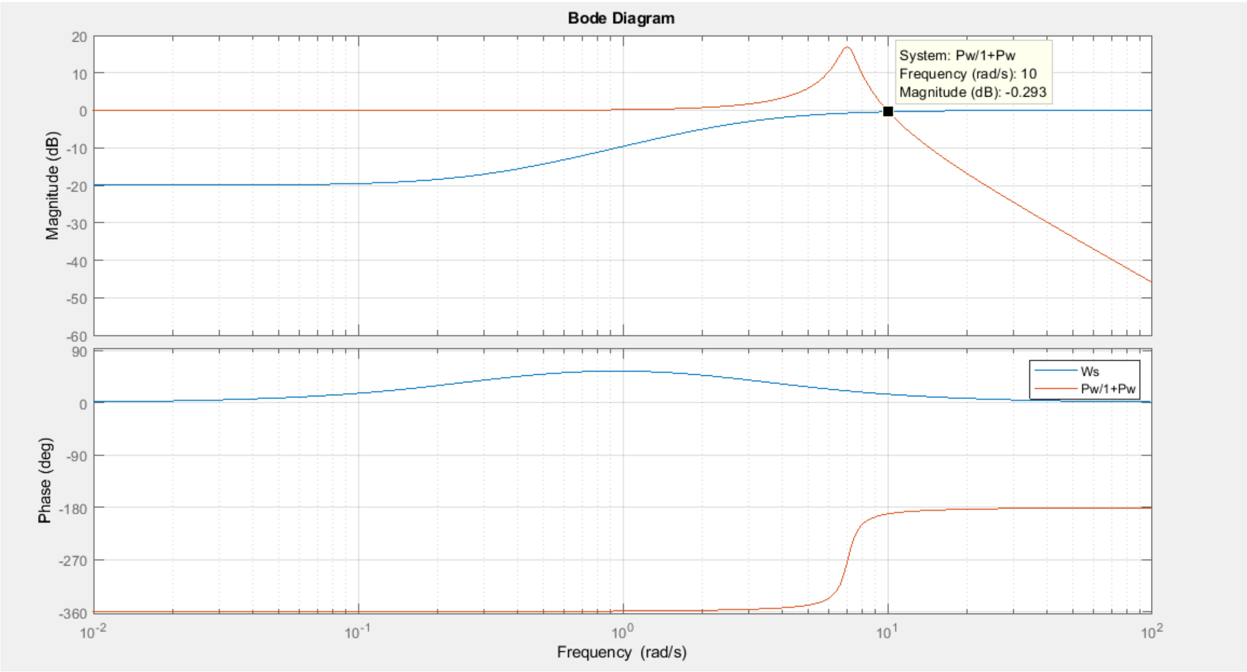


W = 3.5



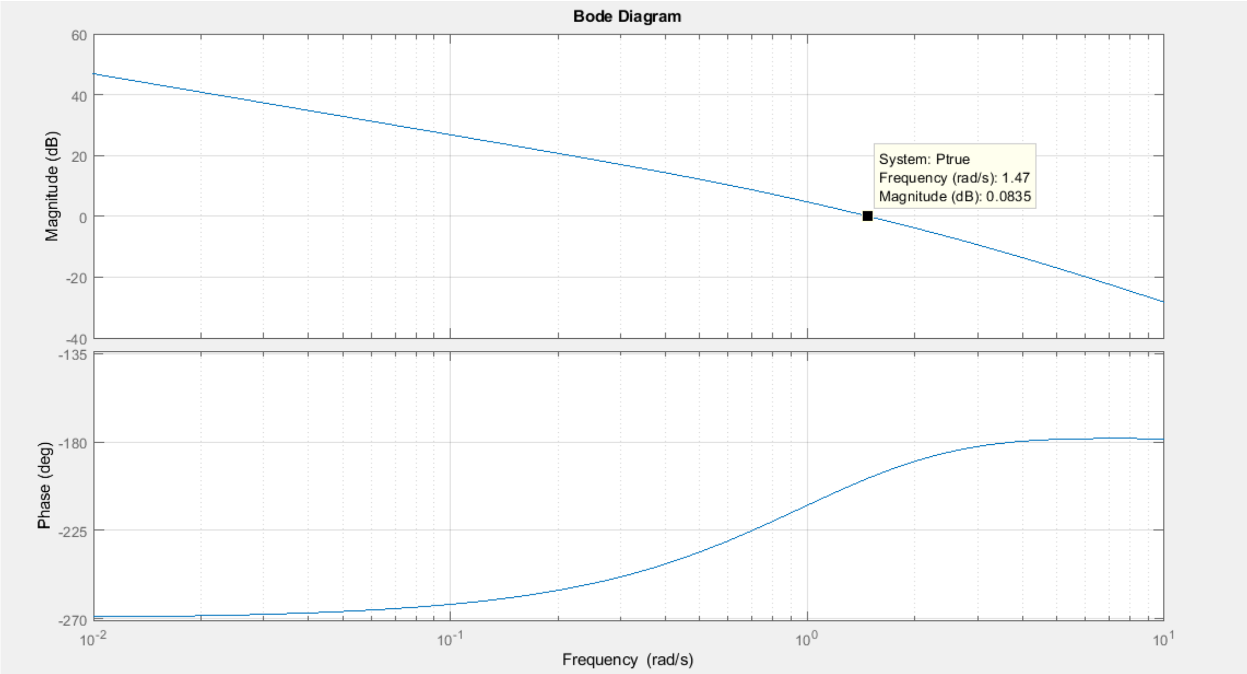
W = 2

In the initially given problem, there is a confusion in W_s and W_s^{-1}. And because of this, whatever might be the value of W, W_s is always less than $P_w/1+P_w$ (as shown in the figure below)

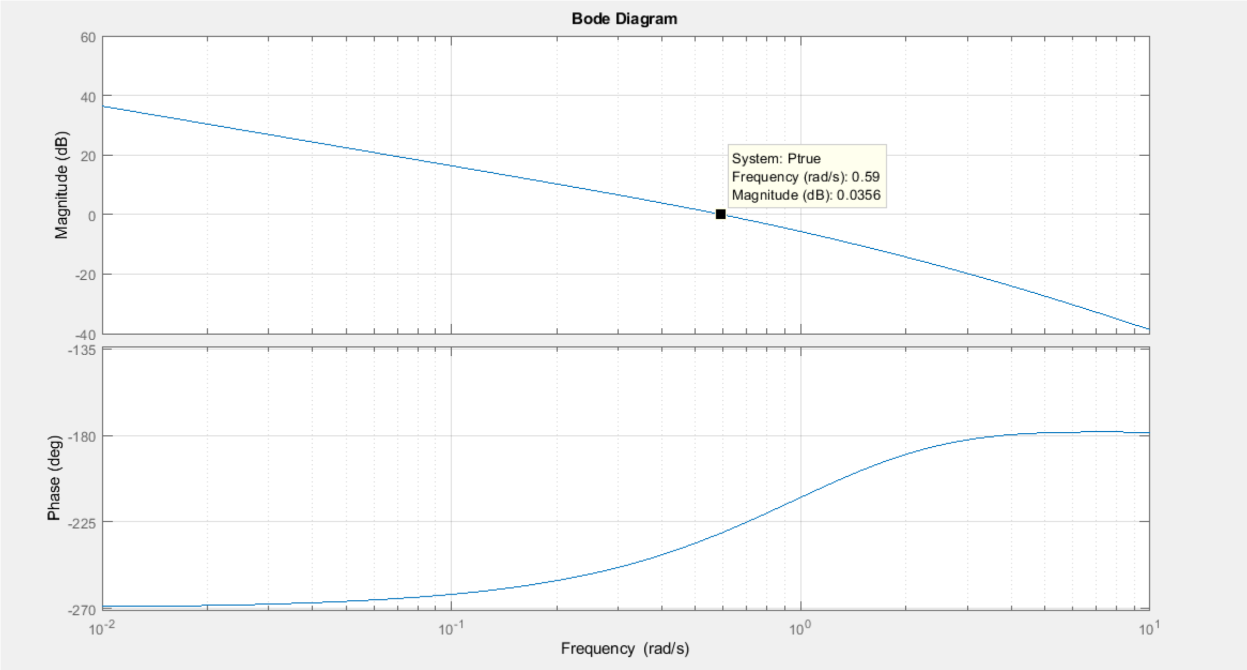


These is a greater possibility to change the value of W after the true system is implemented. But for now, let's move on with this value of W.

Now, let us proceed to the design of state feedback and estimate the gains using LQR/LTR technique. Here, there is a degree difference of 2 for the function P with ε = 0. Because of this degree difference, Pw does not satisfy $C_W B_W = 0$. This may cause a problem in the LTR procedure.



W = 2 for Ptrue



W = 0.6 for Ptrue.

Report:

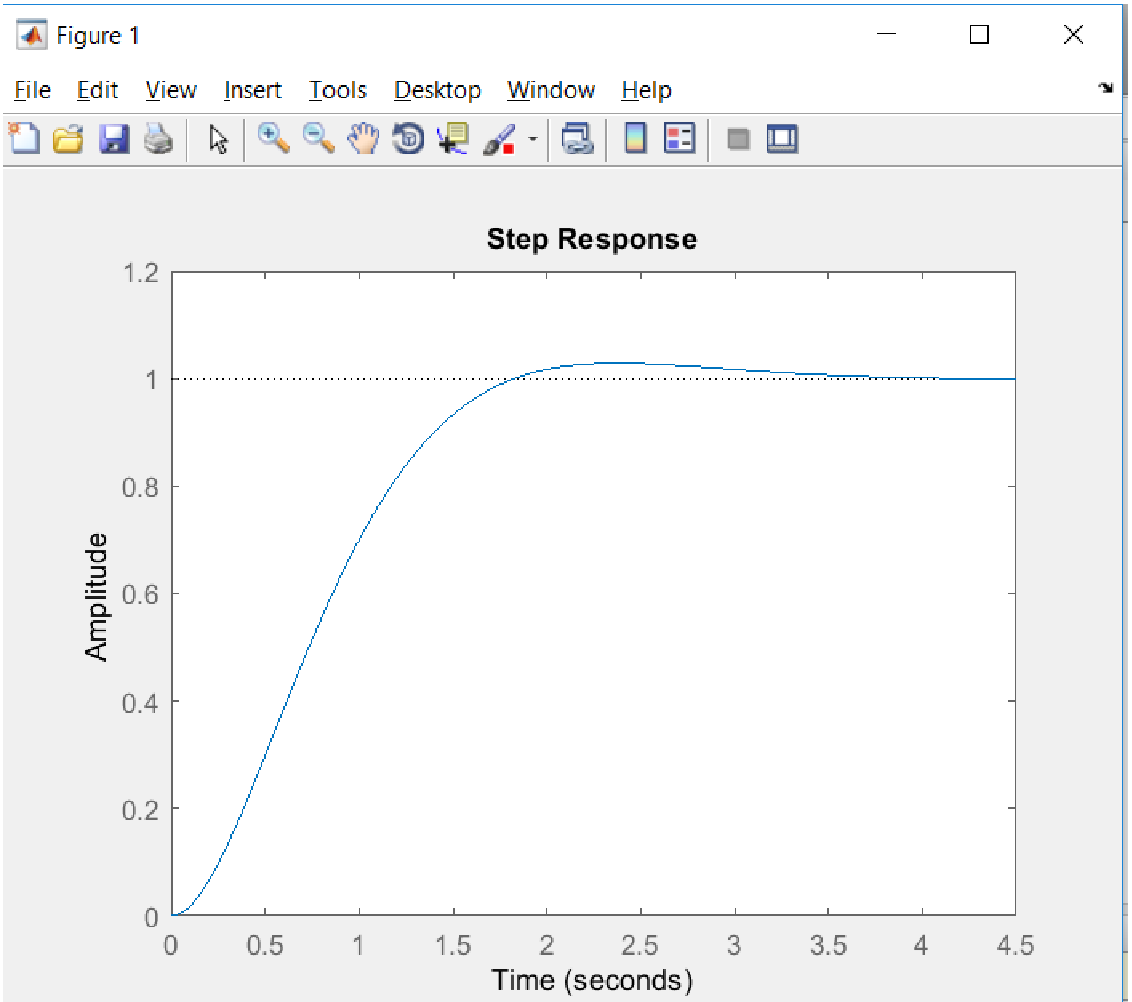
For now, as we have calculated what is P_{true}, by not considering the value of the uncertainty as 0, Let's move on to evaluate the ARE formed by the state space variables of P_W to find out the value of gain matrix F_W along with the stabilizing solution X_W.

For different values of W, the step response under state feedback – T_W is obtained as (A stable step response is obtained at about W = 0.7)

W =4;

Xw =	4.0000	4.0000
	4.0000	12.0000

Fw =	4.0000	4.0000



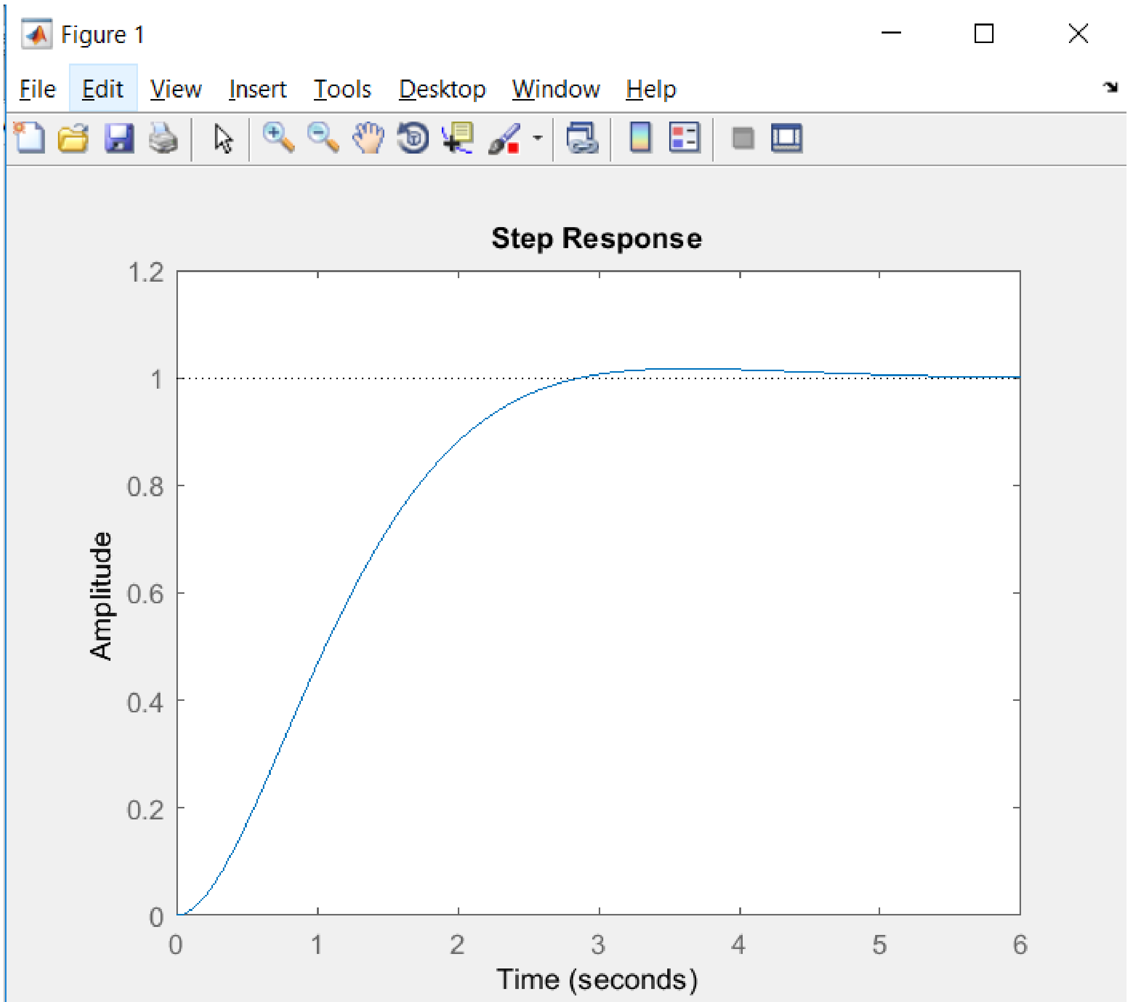

```
     RiseTime: 1.1442
 SettlingTime: 2.8713
  SettlingMin: 0.9049
  SettlingMax: 1.0284
    Overshoot: 2.8369
   Undershoot: 0
         Peak: 1.0284
     PeakTime: 2.3640
```

W = 2.5

Xw =   3.2361   2.0000

2.0000   4.4721

Fw =   3.2361   2.0000



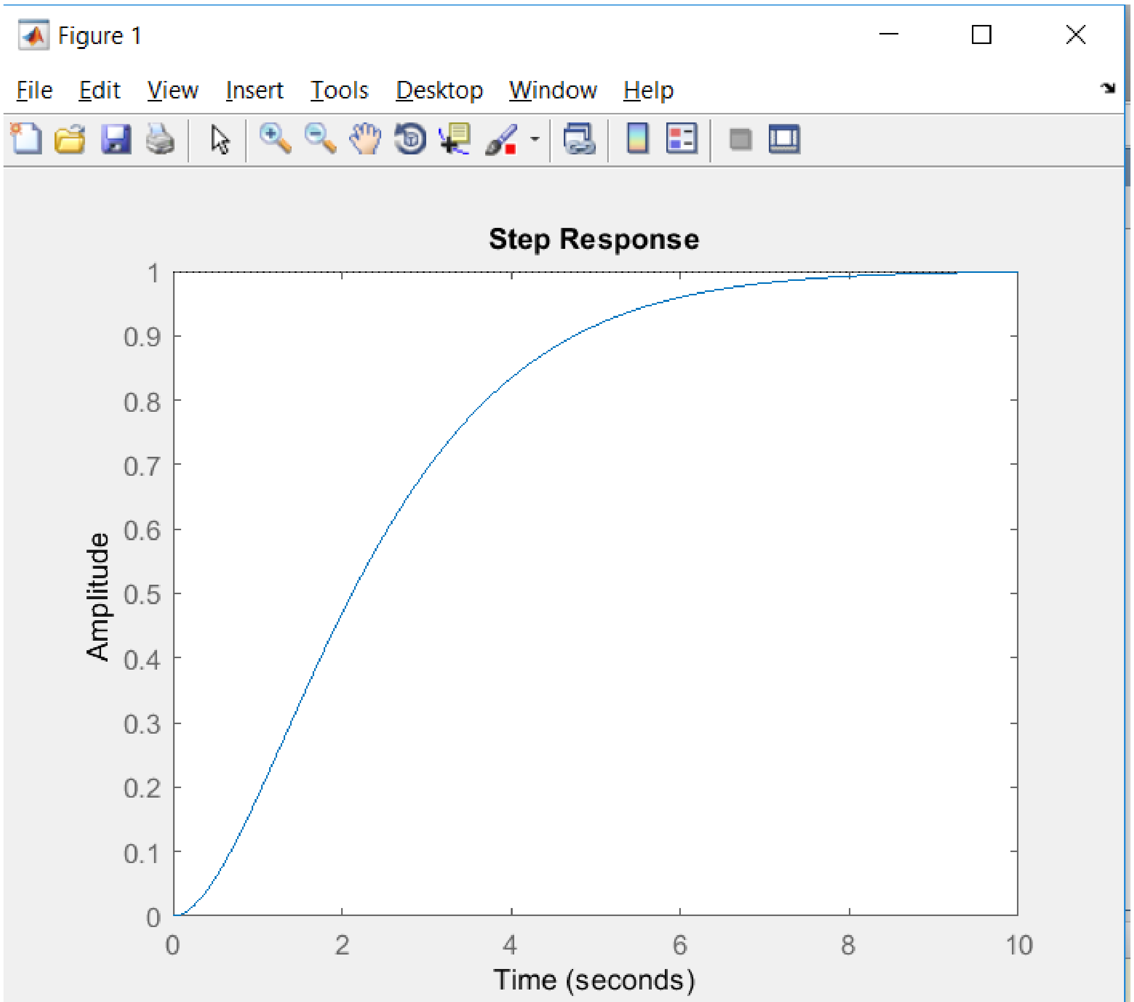

```
       RiseTime: 1.7202
   SettlingTime: 2.6046
    SettlingMin: 0.9040
    SettlingMax: 1.0173
      Overshoot: 1.7322
     Undershoot: 0
           Peak: 1.0173
       PeakTime: 3.6247
```

W = 0.6

Xw = 2.4832 0.6000

0.6000 0.8899

Fw = 2.4832 0.6000



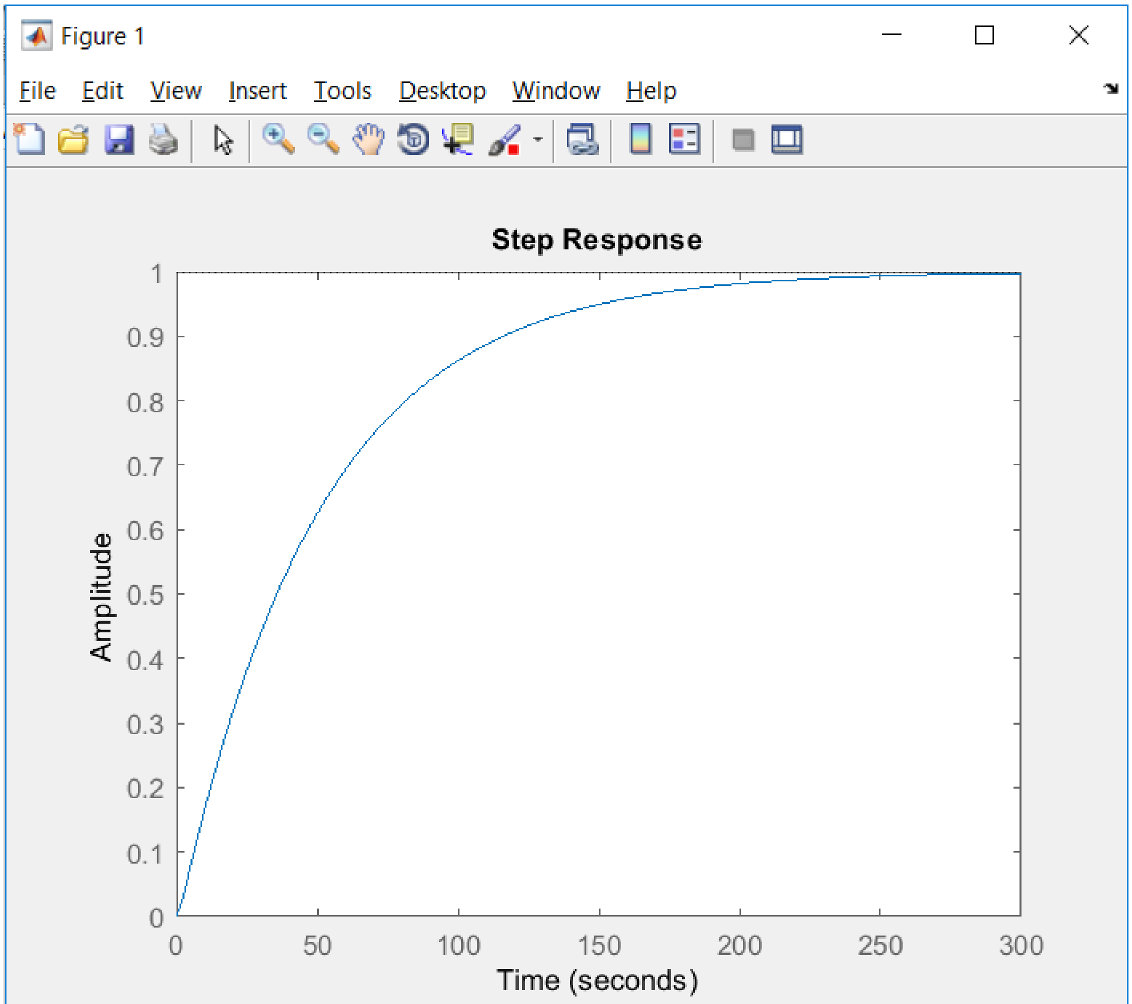

```
        RiseTime: 4.0673
    SettlingTime: 6.9024
     SettlingMin: 0.9023
     SettlingMax: 0.9997
       Overshoot: 0
      Undershoot: 0
            Peak: 0.9997
        PeakTime: 11.1152
```

W = 0.02

Xw = 2.0198 0.0200

0.0200 0.0204

Fw = 2.0198 0.0200




```
     RiseTime: 109.8457
 SettlingTime: 196.5850
  SettlingMin: 0.9026
  SettlingMax: 0.9993
    Overshoot: 0
   Undershoot: 0
         Peak: 0.9993
     PeakTime: 366.0378
```

We have a decent transient response at W = 0.6, with no overshoot but with an increased rise time.

Loop Transfer Recovery:

Now that we have calculated the first ARE, lets compute the stabilizing solution (Y_W) and the gain matrix(L_q) to the Equation

$$A_W Y_q + Y_q A_W' - Y_q C_W' C_W Y_q + q^2 B_W B_W' = 0.$$

with q as very high value, say 100000, for W = 0.6, we get

Yq = 1.0e+04 * 1.1391 0.0584

0.0584 0.0057

Lq = 350.4040 34.1762.

Once the value of L_q and F_W are handy, we can easily calculate the gain of the observer based Controller K.

```
K =

    -890.6 s - 189.7
  ---------------------
  s^2 + 21.99 s + 241.3
```

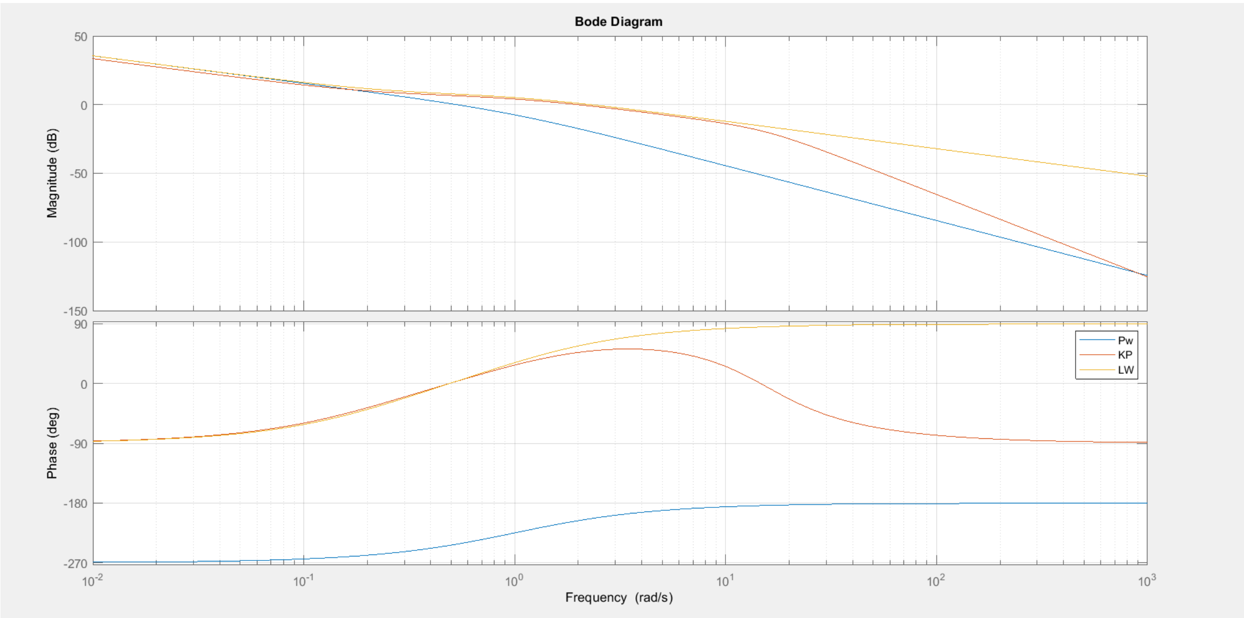
Multiplying this with the actual plant model P_W, we obtain Loop gain at output feedback (KP).

```
KP =

              -534.4 s - 113.8
  -------------------------------------
  s^4 + 20.99 s^3 + 219.3 s^2 - 241.3 s
```

To obtain the Loop gain at state feedback, we make use of the equation $L_W = F_W(sI\text{-}A_W)^{-1}B_W$

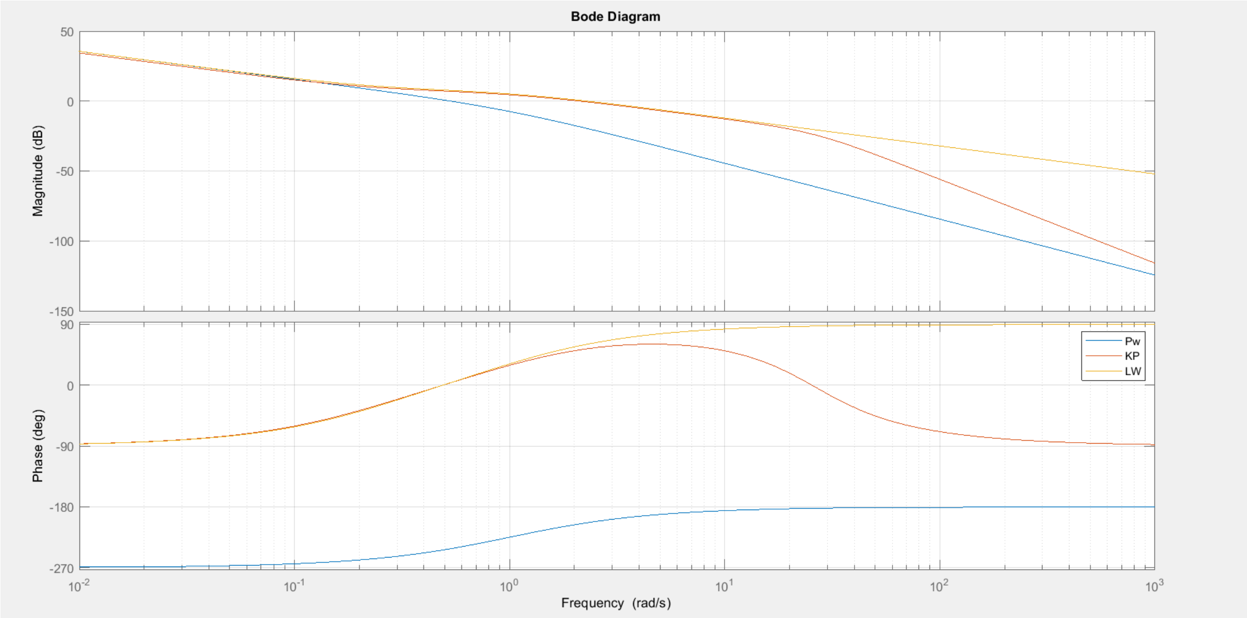
```
LW =

  -2.483 s - 0.6
  --------------
     s^2 - s
```

Taking these three magnitude responses, we obtain



The above gives the magnitude response at W = 0.6 and q^2 = 100000.

We can see a small distortion coming in the low frequencies. That is L_W is not tracking the KP value at low frequencies. This can be overcome when q^2 is increased to a higher value 1000000 (say) with same W value,where there won't be much difference in the loop gains at low frequencies.



As frequencies grows higher, there will be much difference in between them and as the plant model has two degree difference, even q = ∞ can't bridge the gap between these output gains. A one degree difference plant model is a different issue though.

The step responses under state feedback and output feedback will be the same, as the closed loop system has same transfer function $T_W(s)$.

Perturbation Analysis:

As we have designed a observer based control model using LQR/LTR method, let us include perturbation to the existing plant model and test it for different values of this disturbance constant. We will introduce perturbation as $\Delta = (1- \varepsilon s)/(1+ \varepsilon s)$.

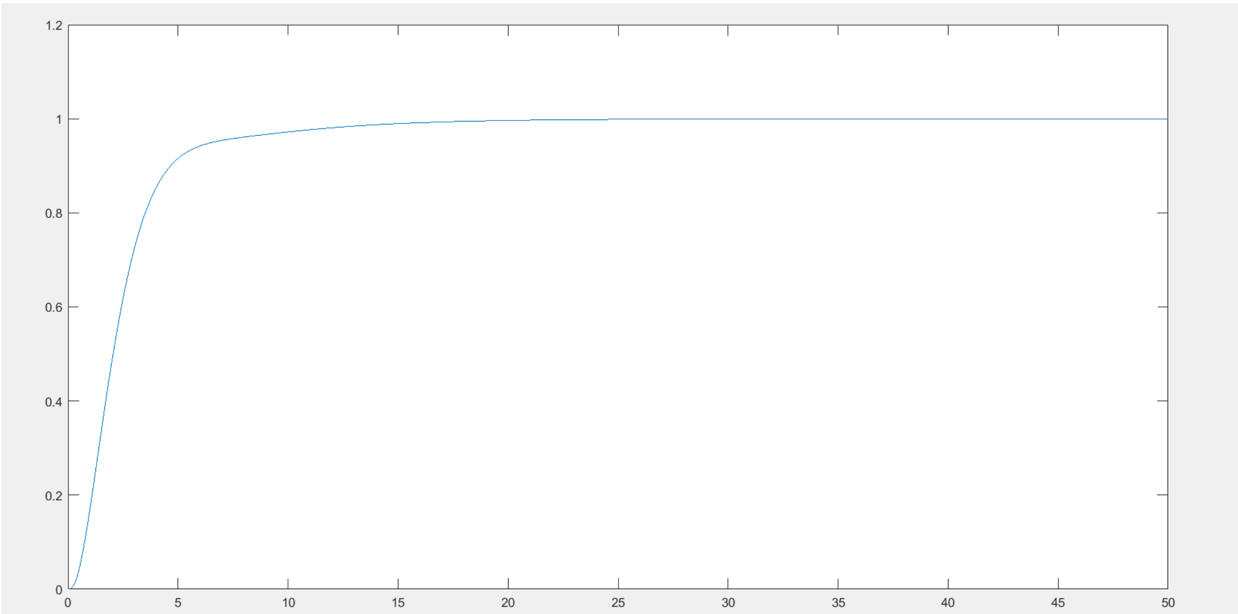
When ε is taken a minimal value, (say 0.1), the plant model P_{true} is given as (this value changes as ε changes)

```
Ptrue =

              2.01 s + 9.9
  -----------------------------------
  0.1 s^4 + 2.4 s^3 + 12.5 s^2 - 15 s
```

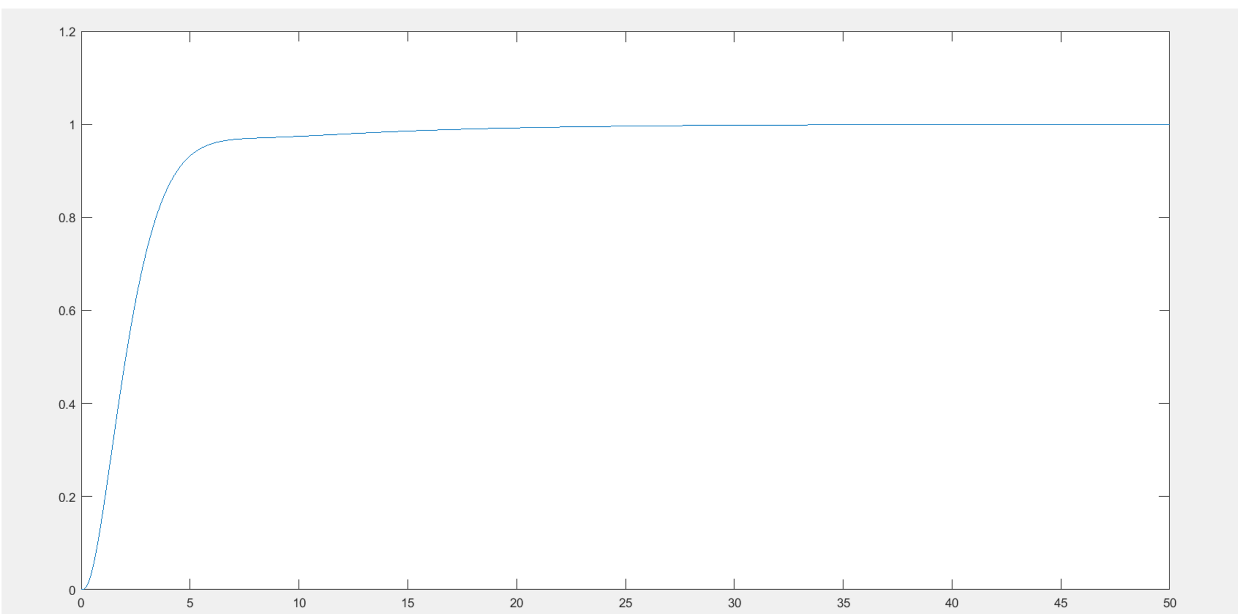
Using the values A_k , and using the state space values from the true system, P_{true}, we can find out the state space description of the closed loop system $T_{ry}(s)$(when true system is taken into consideration)

Finding the state space realization, we'll get the step response as follows

When ε = 5 and a =5,



When ε = 9 and a =5



We can observe that the rise time is more in these two graphs.

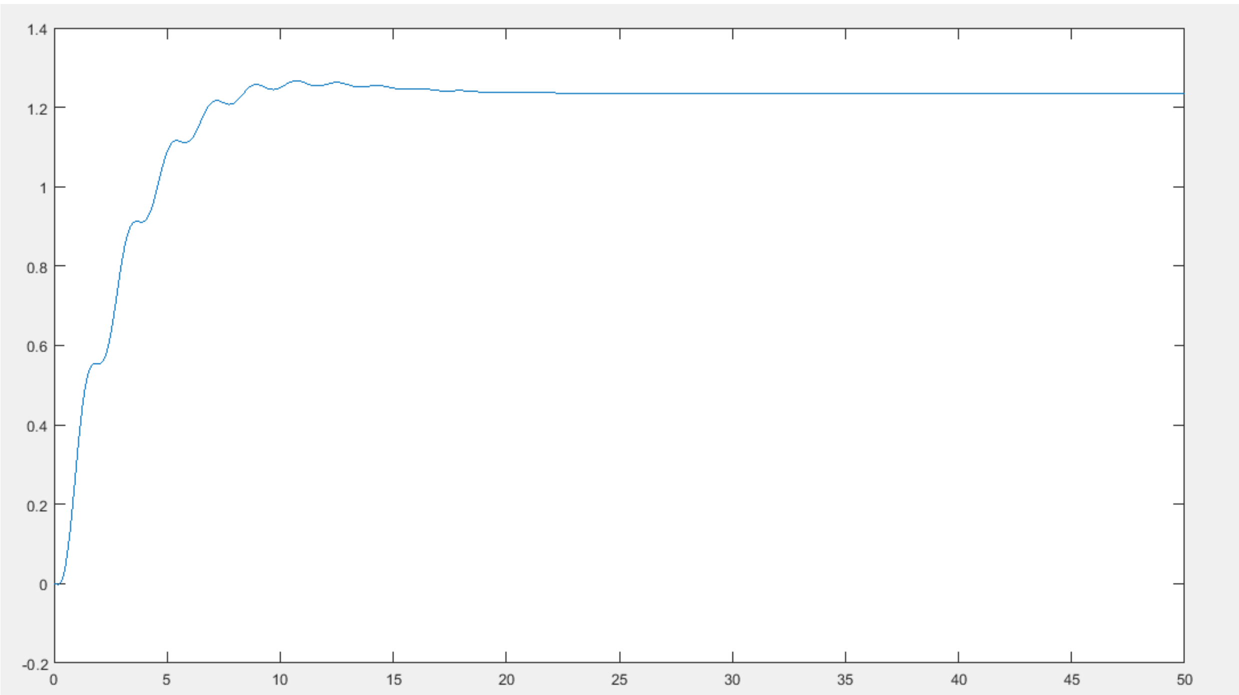
Problems with change in w_c value:

Now that the Weighting function is given as (s+3a)/(s+0.3a), for W = 0.6, and for the given Plant model, we are able to find the observer controller K using LQR/LTR technique, so that the system is robust to uncertainties (over the given range). There are two things to be considered here.

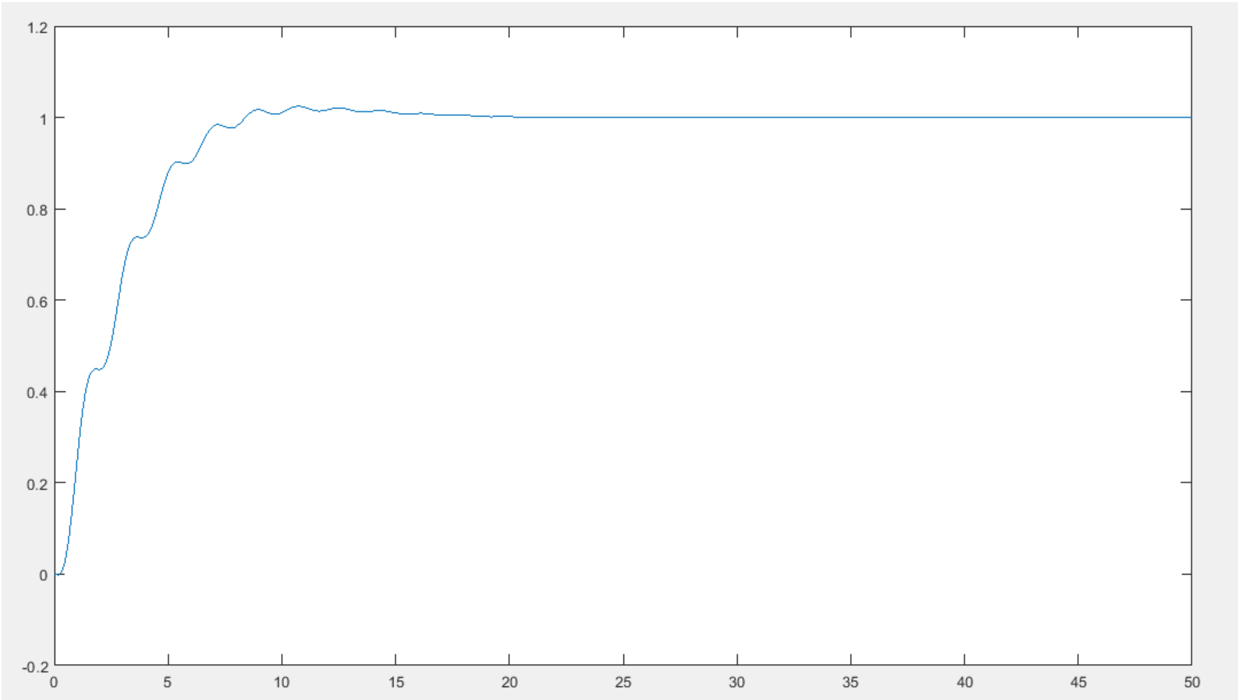
1. If the Weighting function is the old one, i.e., as 2*(s+1) (s+10a) / (s+0.1a) (s+5a), then, the observer controller can't produce robustness as in this case. In other words, it is very sensitive to the perturbations.
2. If the cutoff frequency is driven to a very small value, the system will go out of bound in both the cases

Case 1:

When $\varepsilon = 0.5$, W =0.65 and a = 1;

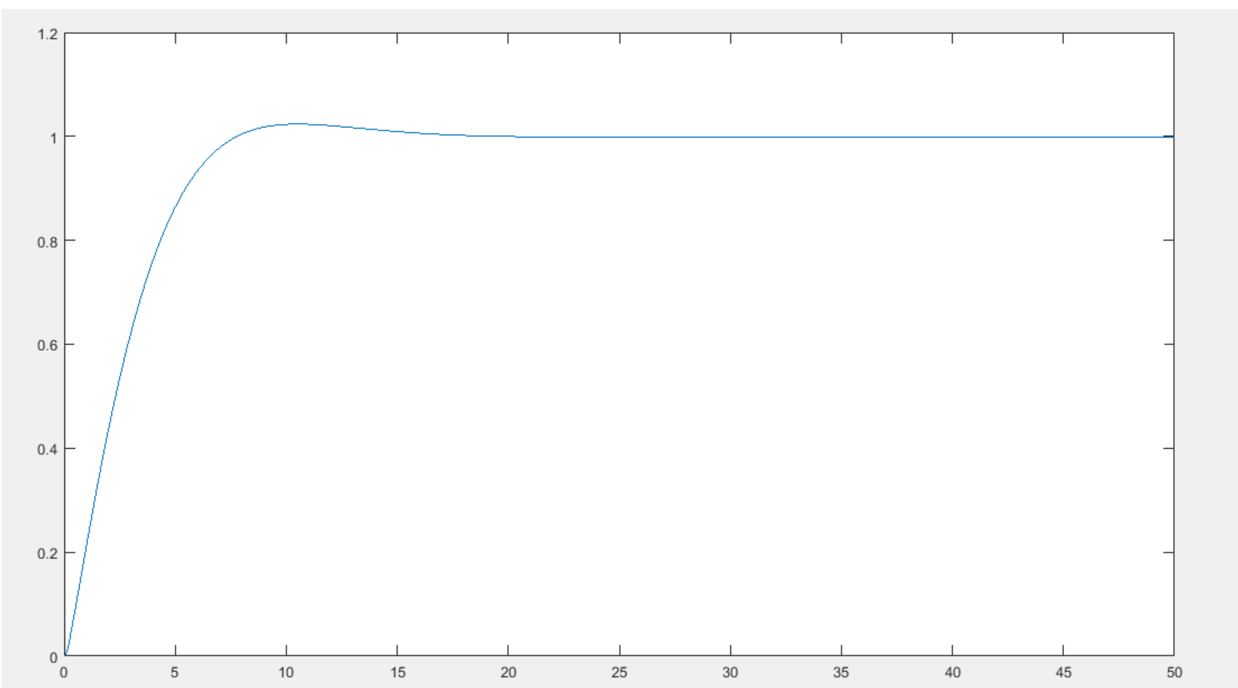


We can observe the amplitude of the transfer function Try(s) = $C_t*(sI-A_t)^{-1}*B_t$ as a bit higher than the unity in the step response. To compensate this, we'll normalize the transfer function to limit its magnitude to 1.

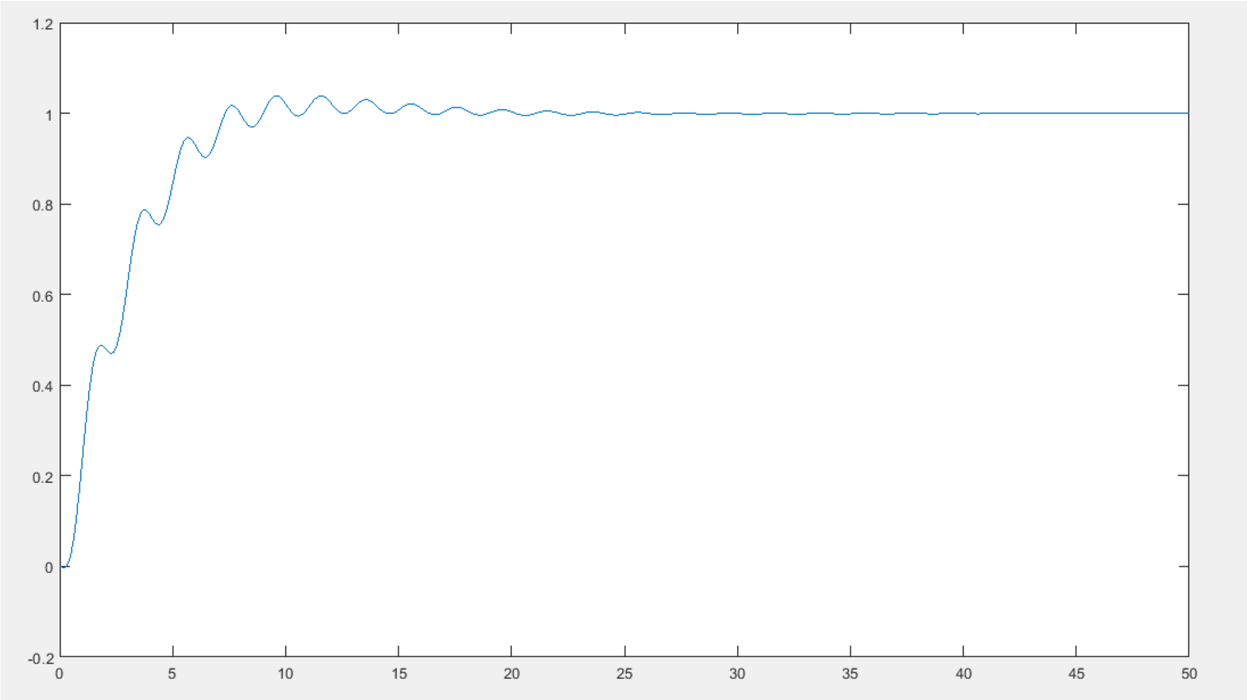


This is the same case as above, but with the normalized magnitude.

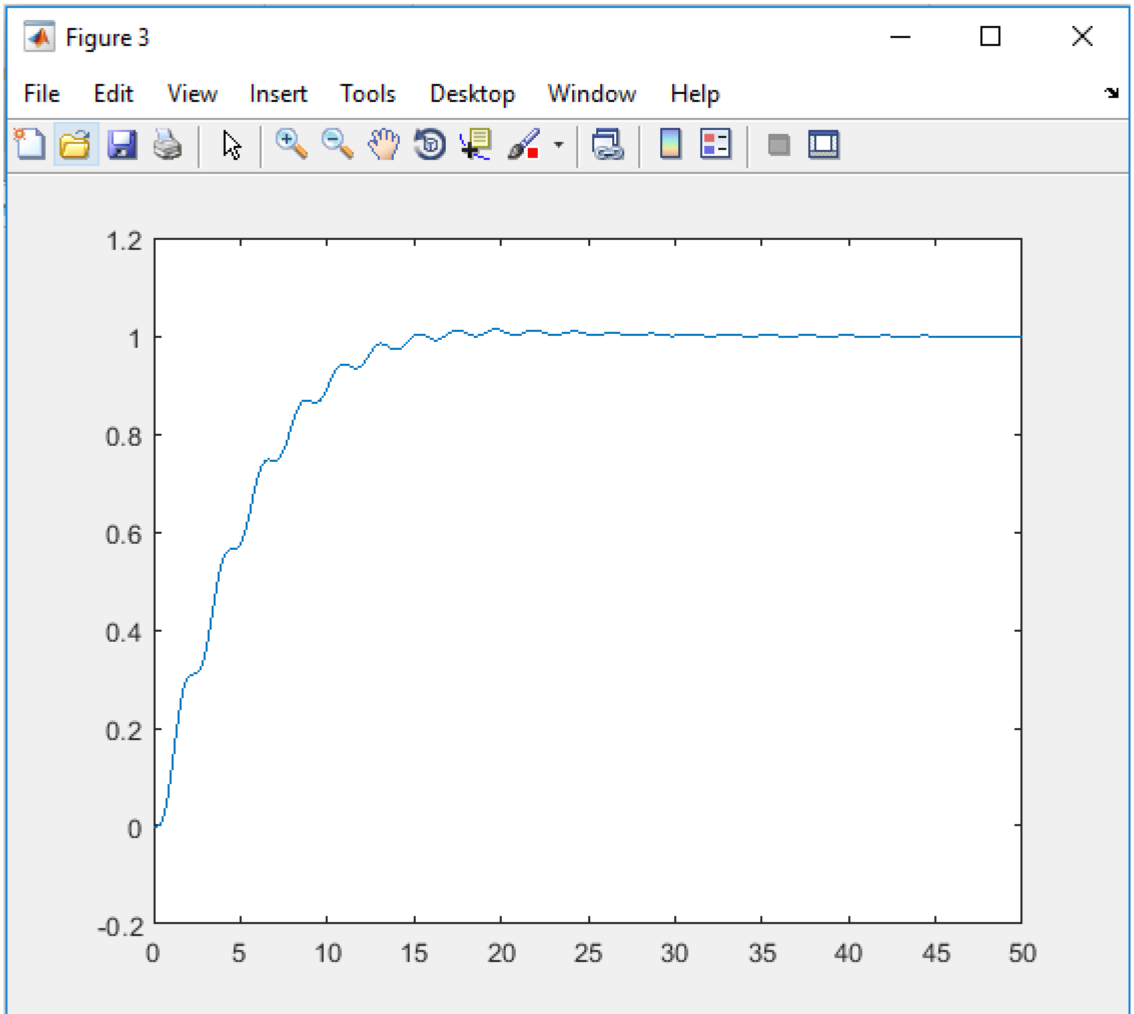
Lets take the very initial case ε = 0, W =0.65 and a = 1. Then the system acts perfectly (ofcourse with an increased rise time)



When ε is taken as 0.6, with same values of W and a, then the output is



There is a lot of distortion in the step response. So, we thought to decrease the value of W, which indeed decreases the cut off frequency. So the new values for ε = 0.6, W =0.3 and a = 1

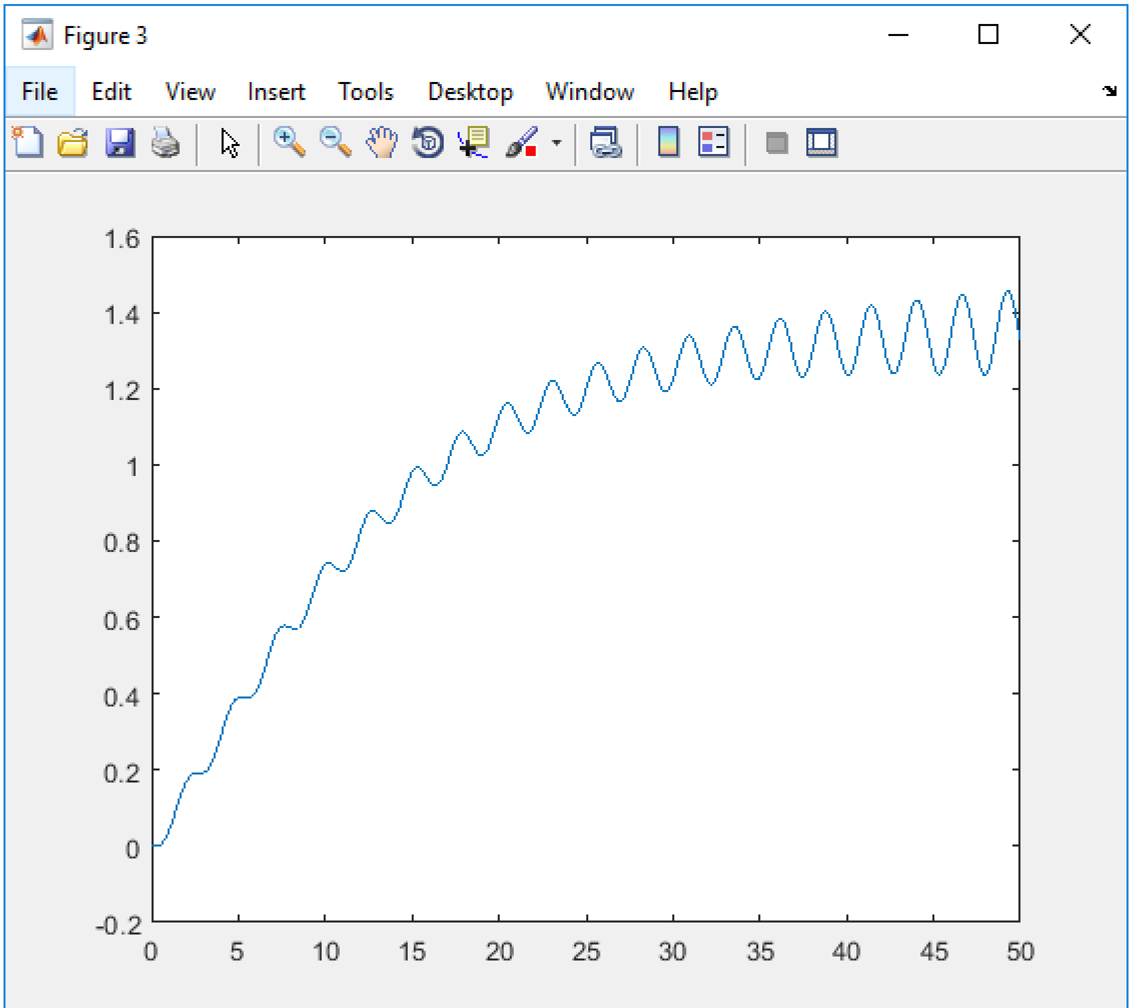



```
-10.6011 + 8.3056i
-10.6011 - 8.3056i
 -5.8813 + 0.0000i
 -0.1155 + 2.8016i
 -0.1155 - 2.8016i
 -0.2139 + 0.1284i
 -0.2139 - 0.1284i
```

Here, the poles of Try are gradually moving towards the positive half of s-plane.

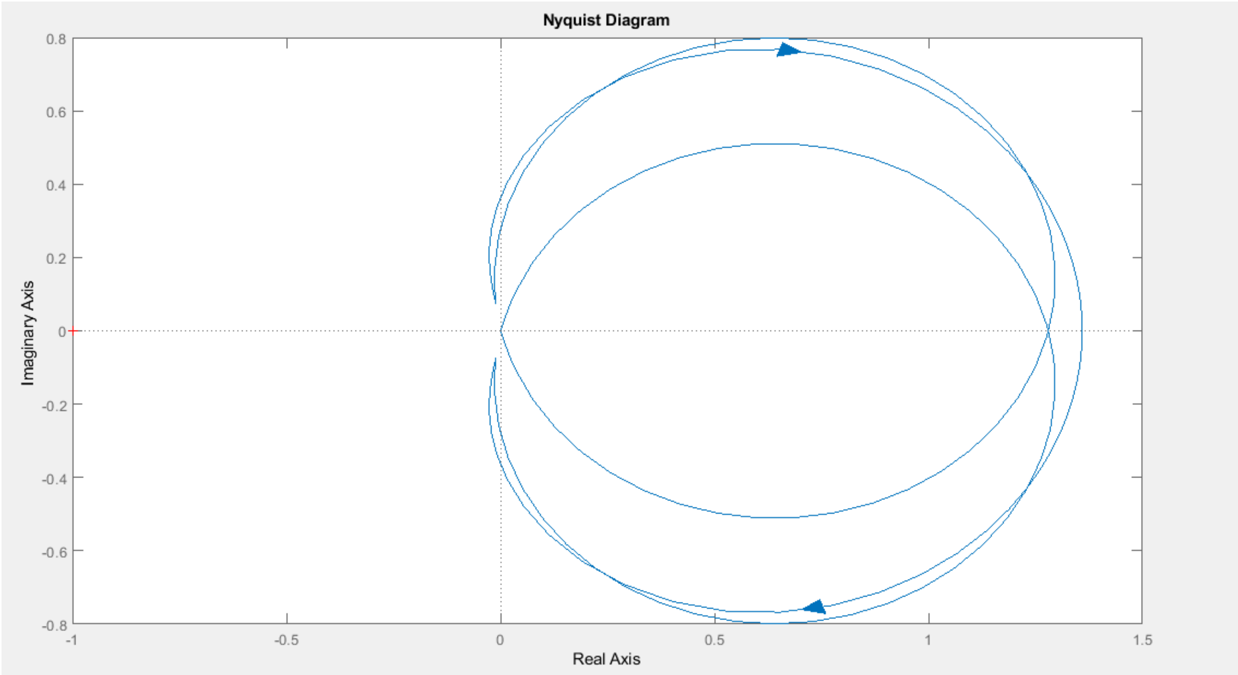
Case 2:

We decreased the value of W further to 0.1, To our utter surprise, the system went into unstable mode. We can clearly observe two poles of the transfer function on the Right hand side of the s-plane.

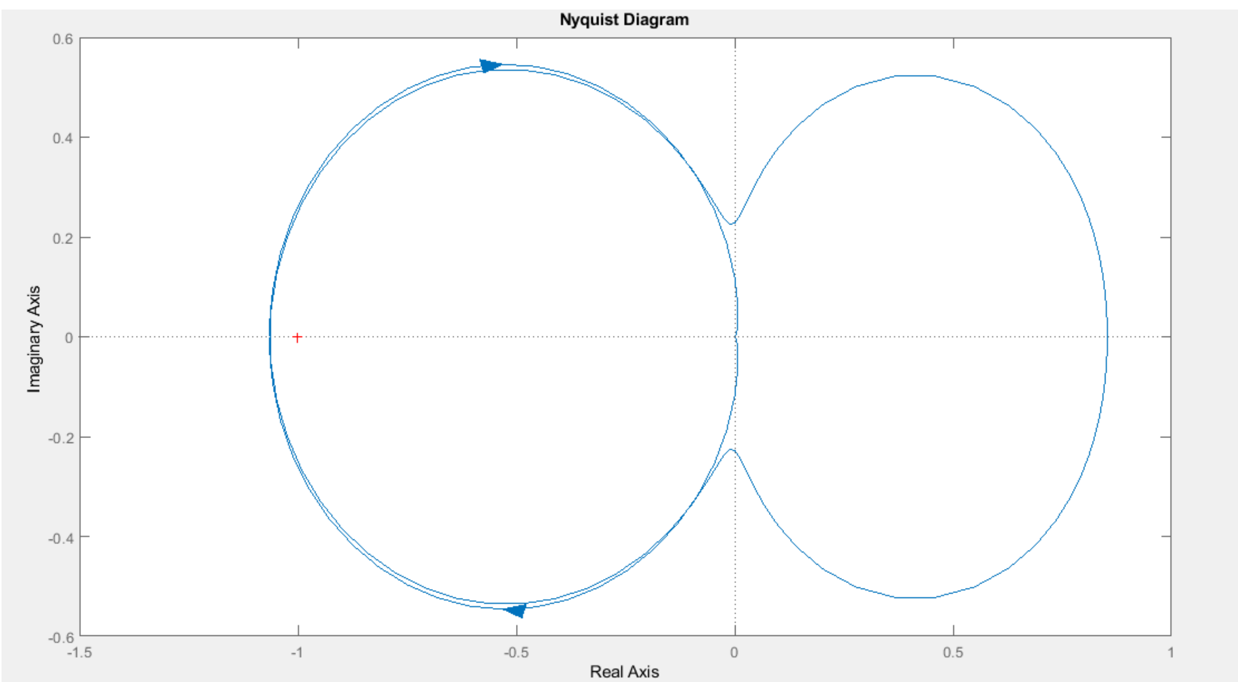



```
-7.7059 + 4.4106i
-7.7059 - 4.4106i
-6.1106 + 0.0000i
 0.0270 + 2.3949i
 0.0270 - 2.3949i
-0.1007 + 0.0000i
-0.2084 + 0.0000i
```

Also, the nyquist plot shows zero encirclements about -1+j0 as shown below.



When we look at the Nyquist plot at W=0.7we can observe two encirclements.



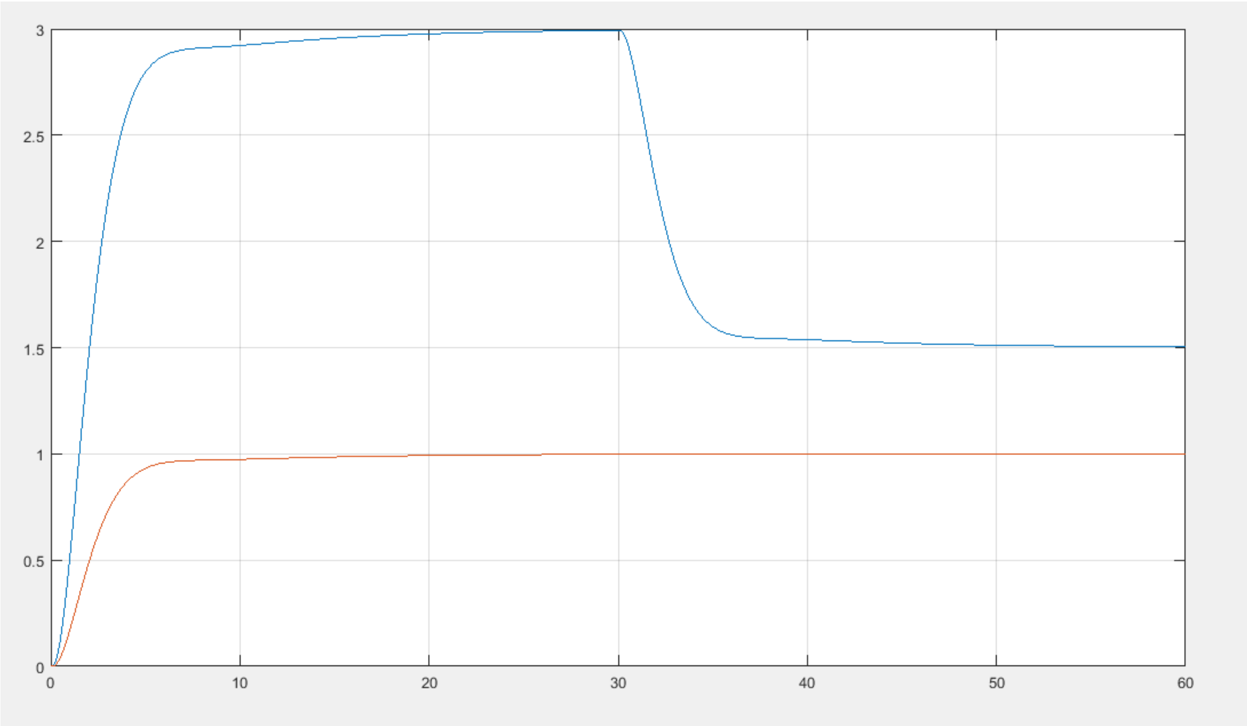
So, there's a limit for minimum value of w_c and even reaching that minimum value is no good when ε is taken more than 0.6

This is the reason why, the previous weighting function is not a part of the good design, when the given plant model is taken into consideration. And this is the reason why a new plant model is chosen.

Fourth Problem

As we have seen in the previous part, the system is robust irrespective of the value of ε (over the given range). But this stability comes at the cost of the system response. We can observe the increase in rise time due to the decrease in the cutoff frequency (which is a result of change in magnitude).

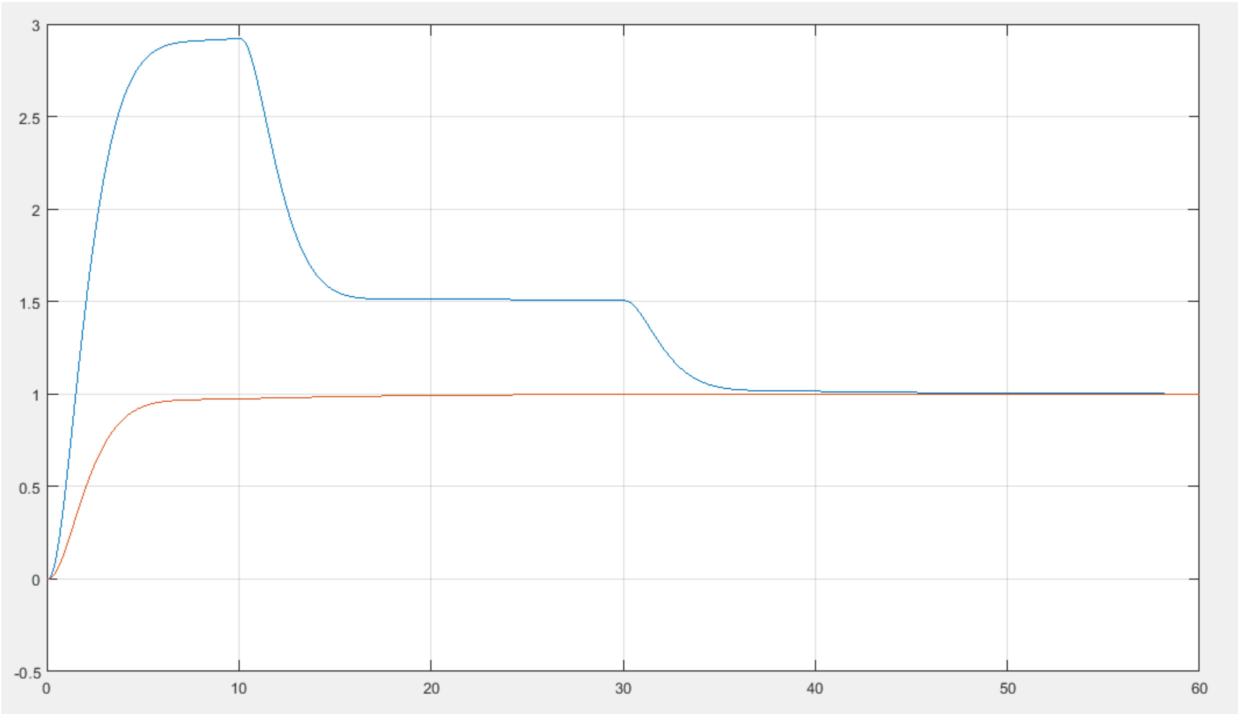
Let's try to take the input to the system r(t) as nonlinear rather than step (using lsim), We can refer to a nonlinear piecewise response in the figure below.



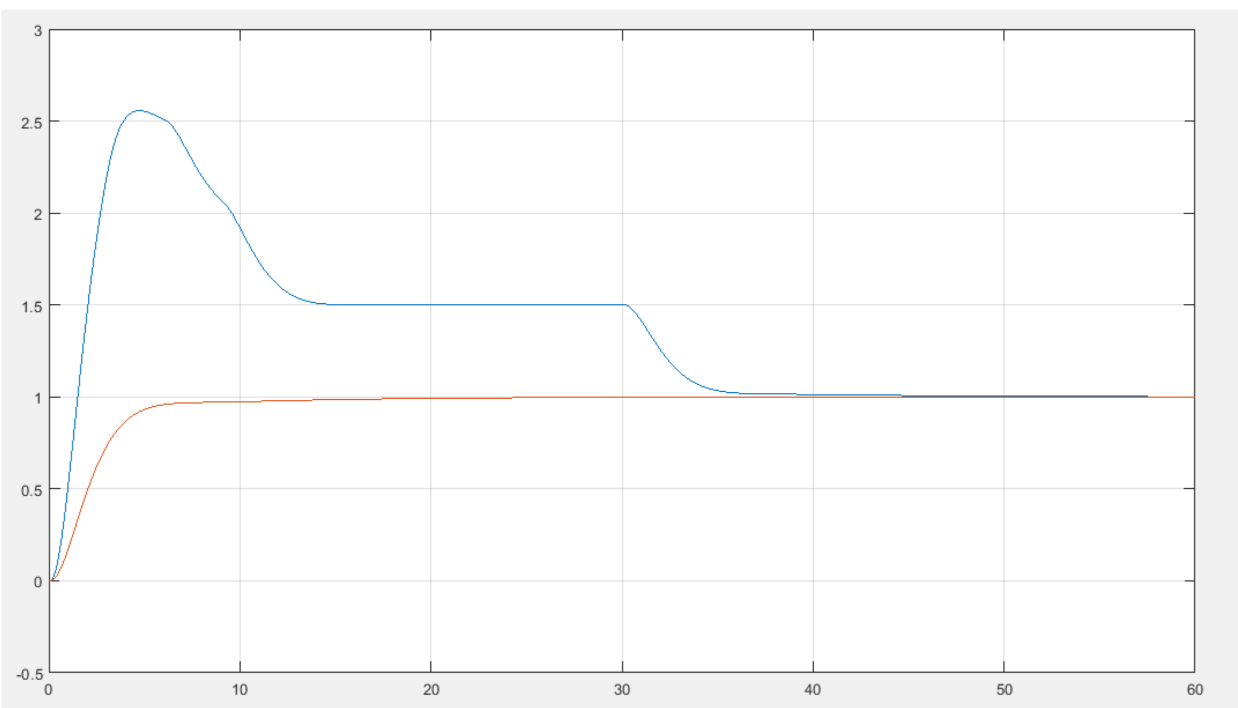
These are all taken for ε = 9 and a =5(unless specified).

We can clearly observe that the rise time is improved in the piecewise r(t) compared with the step r(t).

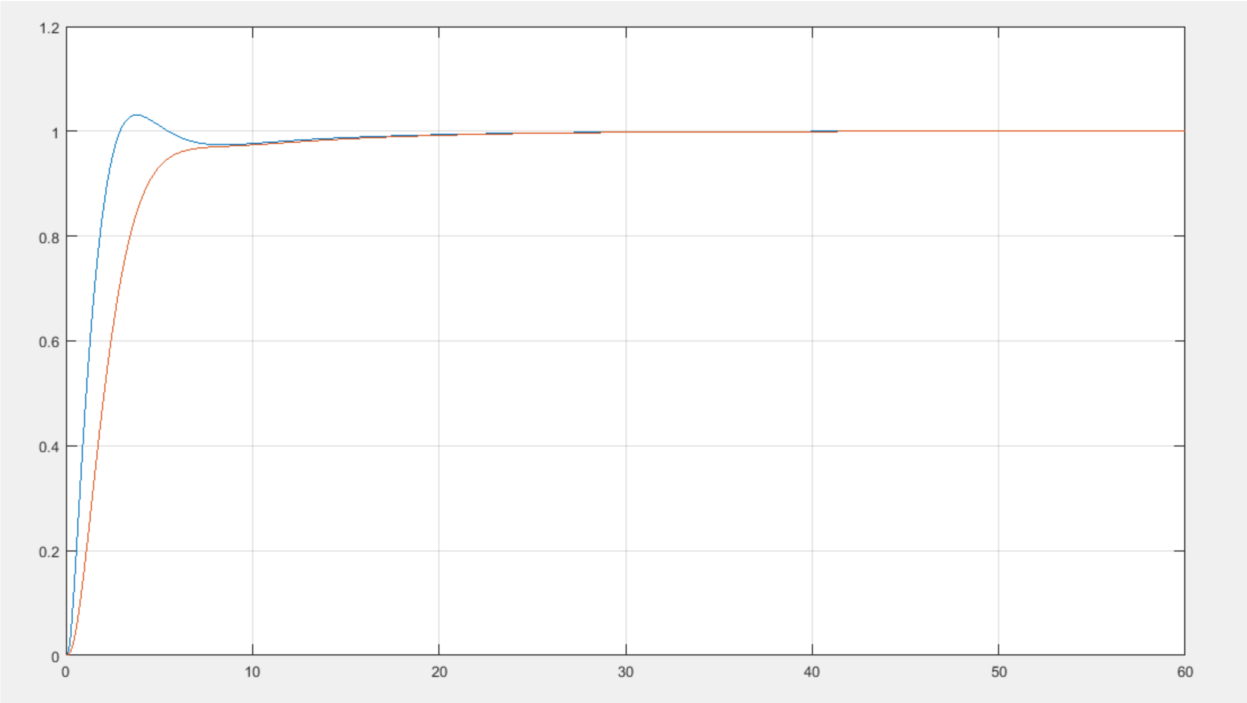
Adding some more non-linearity to the system, we can still see the increase in rise time, and now we have to eliminate the instability.



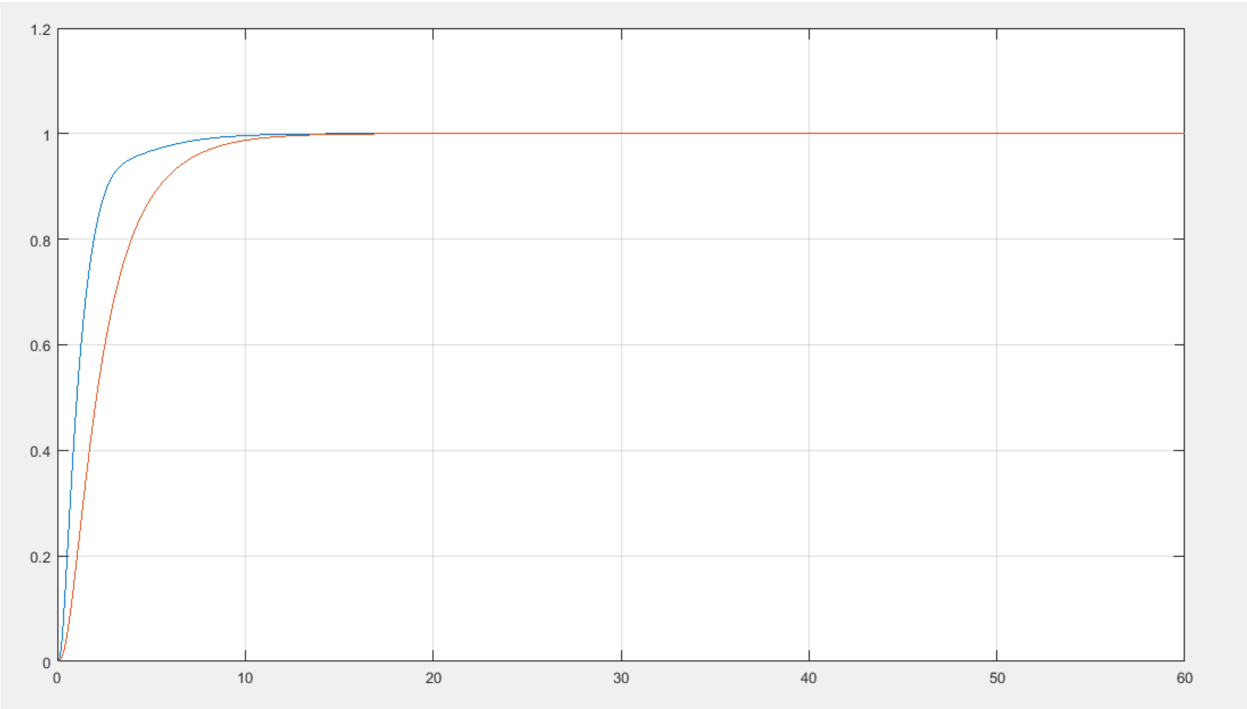
Making the function much more non linear, we will get the the figure as



Now, let's start to minimize the overshoot using some variations in r(t)

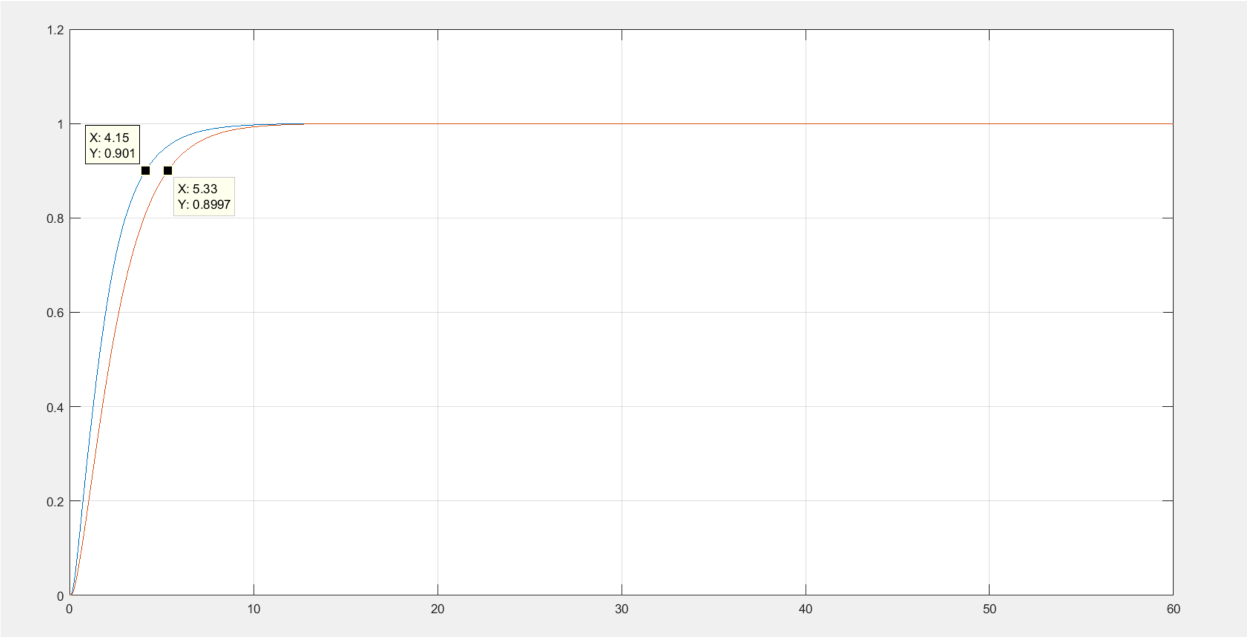


When ε = 1, this over shoot won't be there.

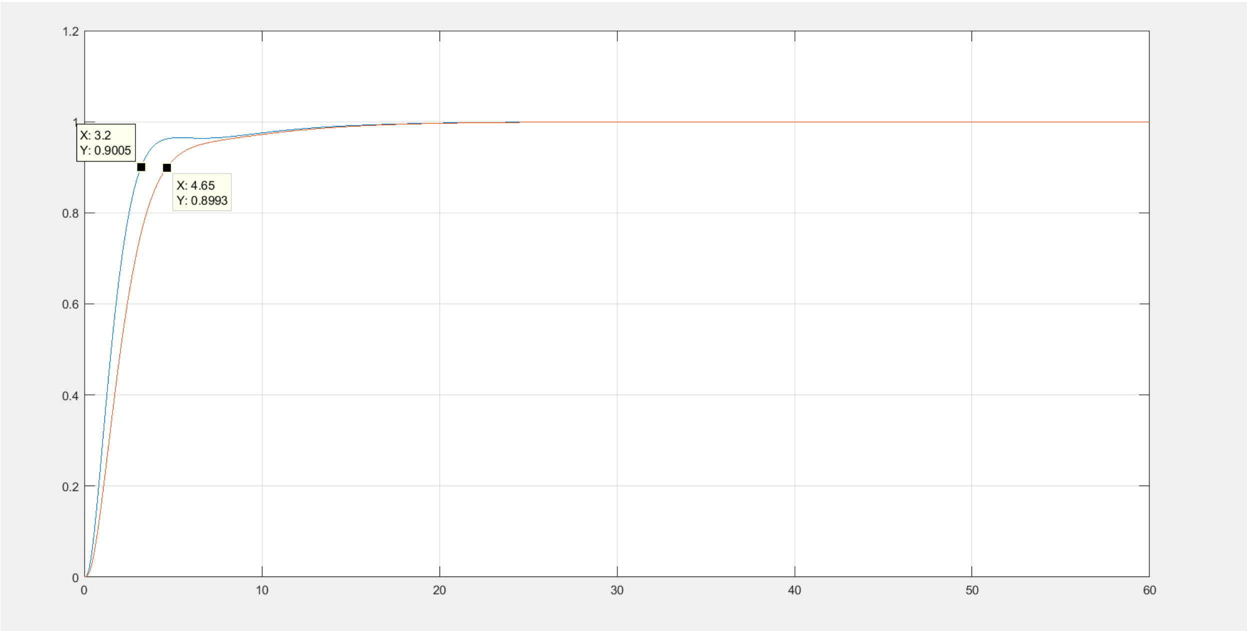


Eliminating this overshoot and getting the final results, we can have upto a 30 percent decrease in the rise time, by taking a non linear value of r(t).

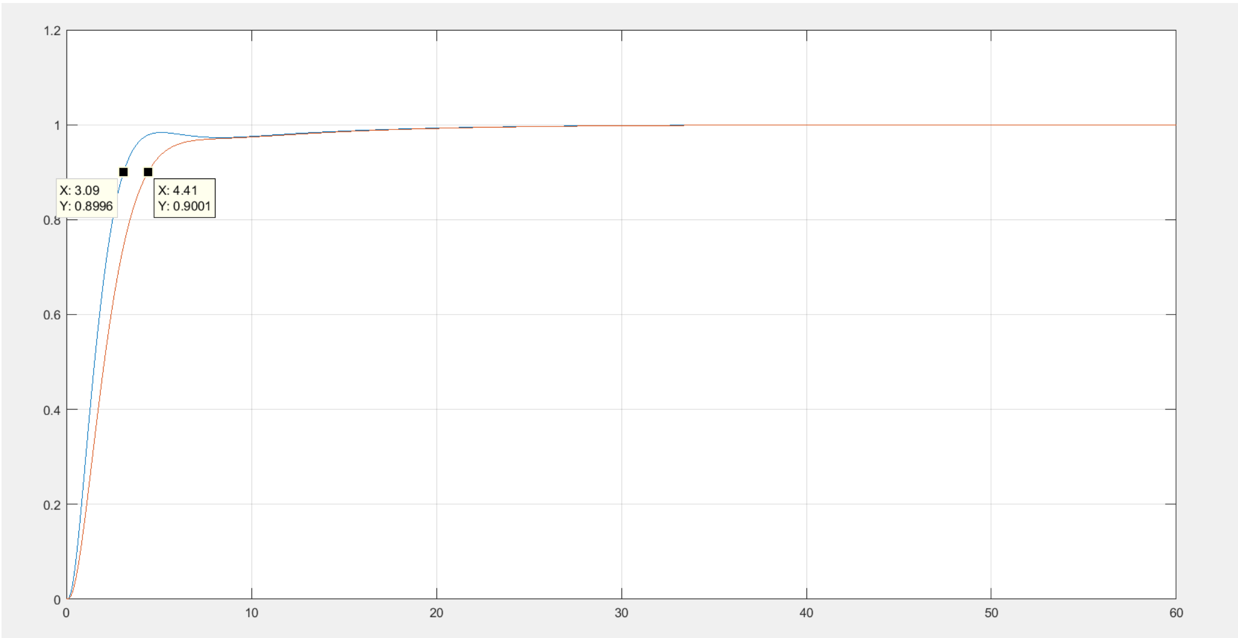
When ε = 0.1 and a = 5



When ε = 5 and a = 5



When ε = 9 and a = 5



Code

```
% Design of Unity Feedback control System
% for a given plant model, using LQR/LTR Method.
% in completion of a project for the course
% EE7501 - Advanced Control Systems
% for Dr. Guoxiang Gu
% by Haranath Manikonda
% to be submitted on Dec 4th 2018

% Given Plant model
 s = tf('s');
P = 1 / (s*(s-1))
W = 0.6;
Pw = P * W

% Weighting Function
a = 5;
epi = 9;
Ws =(((s+.3*a)/(s+3*a)));
ds = ((1-epi*s)/(1+epi*s))

%True System
Ptrue = Pw*(1+(Ws*ds))

%Finding ARE for given system
[b,a] = tfdata(Pw,'v');
[Aw,Bw,Cw,Dw] = tf2ss(b,a);
Qw = Cw'*Cw;
[Xw,L,Fw] = care(Aw,Bw,Qw)

% ARE for true system
[bd,ad] = tfdata(Ptrue,'v');
[Adw,Bdw,Cdw,Ddw] = tf2ss(bd,ad)
Qdw = Cdw'*Cdw;
[Xdw,Ld,Fdw] = care(Adw,Bdw,Qdw);

% Plotting Step response for Tw(s)
Aw1 = Aw - (Bw*Fw);
Cw1 = Cw - (Dw*Fw);
Tw = ss(Aw1,Bw,Cw1,0);
figure(1);
step(Tw);
title('Step response for Tw(s)');
grid on

% Loop Transfer Recovery
Aw2 = Aw';
Bw2 = Cw';
q2  = 100000;
Qw2 = q2*Bw*Bw'
[Yq,L2,Lq] = care(Aw2,Bw2,Qw2)

% Finding K (Observer based Controller) value
Ak = Aw-(Bw*Fw)-(Lq'*Cw)+(Lq'*Dw*Fw) ;
```

```
Bk = -Lq';
Ck = Fw;
Dk = Dw
[b,a]= ss2tf(Ak,Bk,Ck,Dk)
K = tf(b,a)

% Loop Gain at Output Feedback
KP = K*Pw

% Finding LW(s)- Loop Gain at State Feedback
Alw = Aw;
Blw = Bw;
Clw = -Fw;
Dlw = 0;
[bL,aL]= ss2tf(Alw,Blw,Clw,Dlw)
LW = tf(bL,aL)

% Plotting the magnitude responses
% of Pw, KP, LW.
figure(2)
bode(Pw);
hold on
bode(KP);
hold on
bode(LW)
grid on
legend('Pw','KP','LW');

% Realization of Closed Loop System
At = [Adw,Bdw*Fw;-(Lq'*Cdw),Ak-(Lq'*Ddw*Fw)]
Bt = [Bdw/1.745;Bw]
Ct = [Cdw Ddw*Fw]
Dt = Ddw

% Plotting the step responses for Try
% using lsim with linear and non linear
% r(t)inputs.
tmax = 60;
sz = size(At);
n = sz(1);
t= 0:0.01:tmax;
N = length(t);
Try = ss(At,Bt,Ct,Dt);
u =
[2*ones(20,1);1.5*ones(50,1);1.20*ones(30,1);1.15*ones(50,1);1.1*ones(50,1);1
.05*ones(50,1);ones(3750,1);ones(2001,1)];
y = lsim(Try,u,t,zeros(n,1));
y1 = lsim(Try, ones(1,N),t,zeros(n,1));
figure(3);
plot(t,y,t,y1)
legend('Try improved','Try actual');
title('Step response for Try(s)with linear and non linear r(t)')
grid on
```