Autonomous Determination of an Orbit around an irregular-shaped Celestial Body

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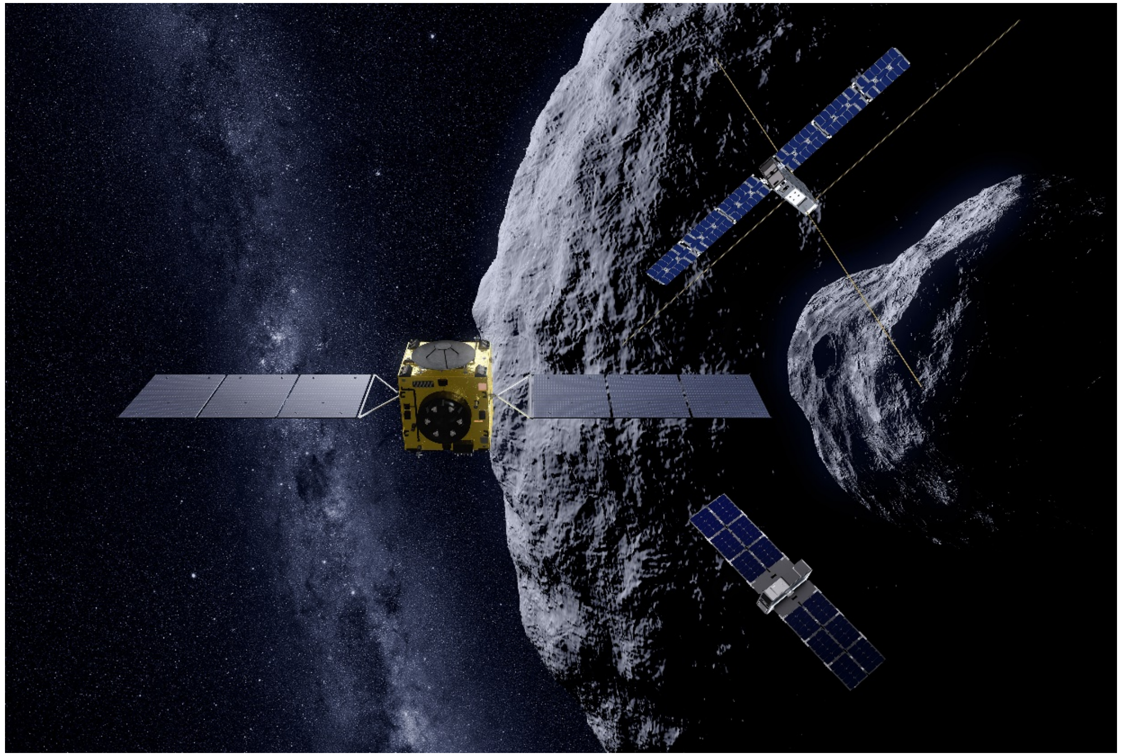
Introduction

- There is a growing interest in deep space missions for the exploration of asteroids and comets.
- NEAR-Shoemaker [Eros, 2001],
  Rosetta [Churyumov-Gerasimenko, 2014],
  Hayabusa [Itokawa, 2005],
  Hayabusa-2 [Ryugu, 2018],
  OSIRIS-Rex [Bennu, 2018-21],
  demonstrated close proximity operations near asteroids.
- Trend toward more autonomous functionalities for guidance, navigation and control (GNC)



ESA's Hera mission seen with its CubeSats in orbit around its target asteroid. Launch planned for October 2024.
Credit: ESA/Science Office

Small Celestial Bodies

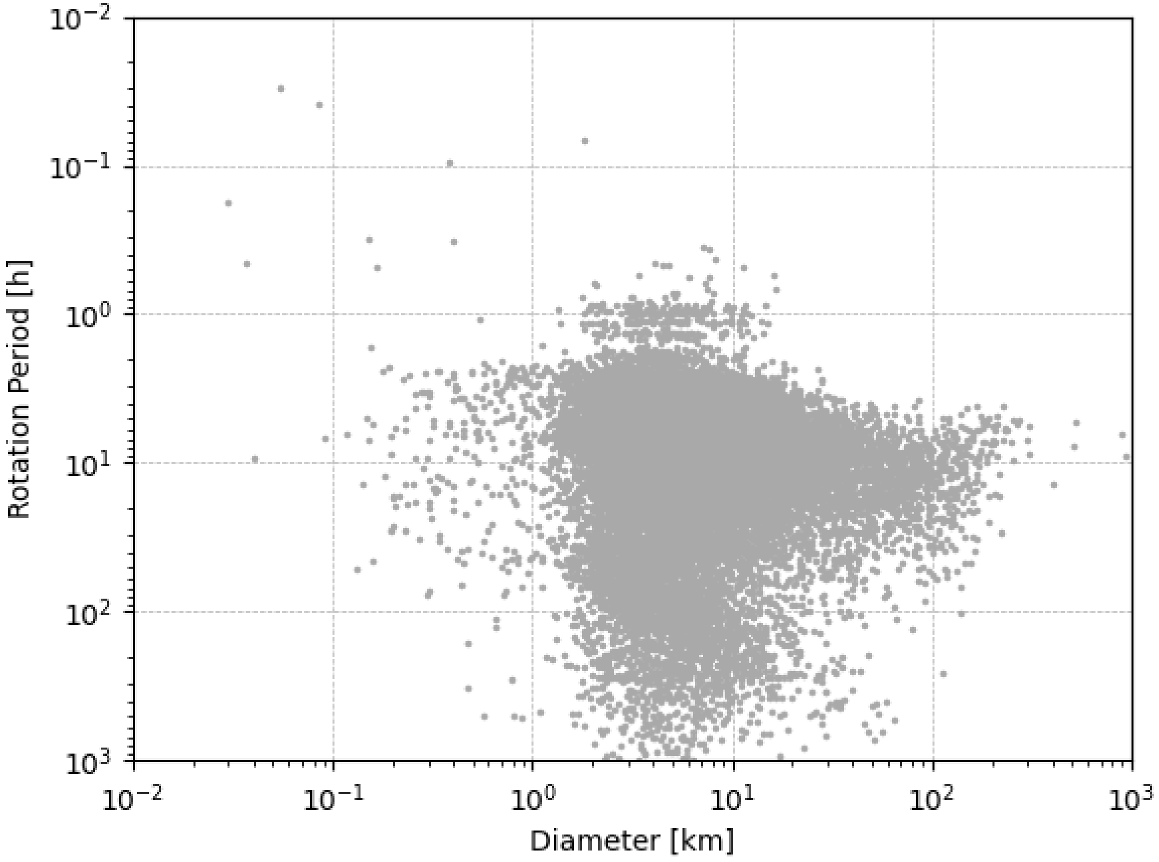


Fig 2: Distribution of small bodies (rotation period vs. diameter)

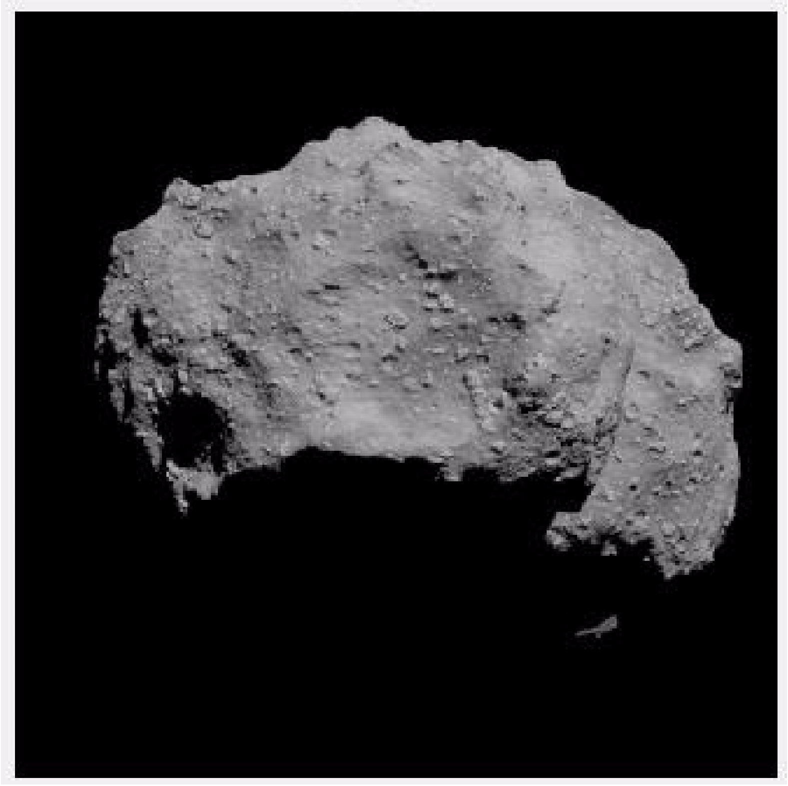
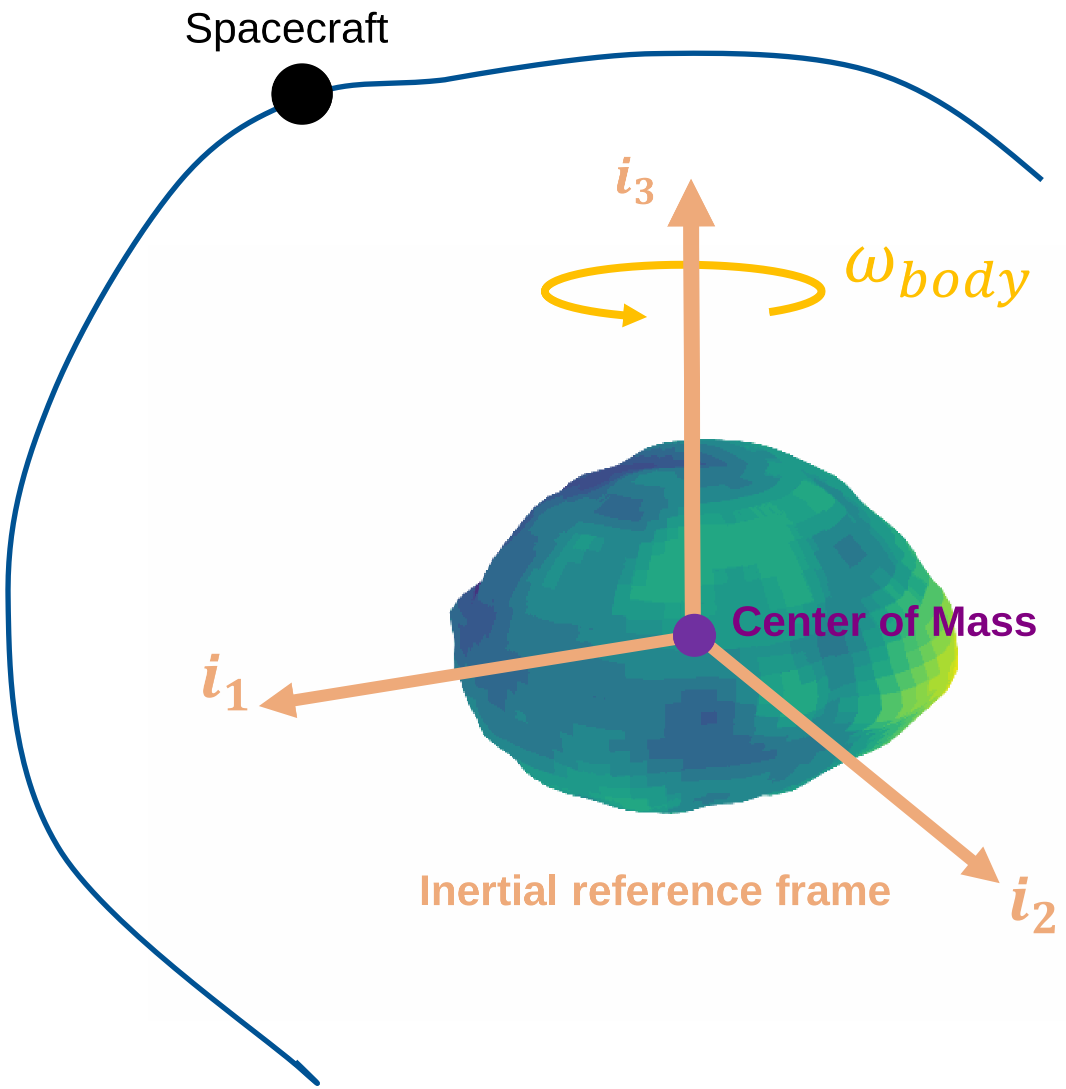


Fig 3: Animation of 67P/Churyumov-Gerasimenko in Blender

Problem Setting

Given a spacecraft in a bounded orbit around a rotating small celestial body.

1. What is the detailed shape of the non-Keplerian orbit?
2. What is the shape of the celestial body?
3. Where is its center of mass?
4. What is its rotation axis?
   $\rightarrow$ reference frame



Traditional Deep-Space Missions

- They require communication with an Earth ground-segment for navigation
- Appart from some autonomous functionality, all exisiting spacecraft operate by uploaded command sequences
- During cruise and approach phase, navigation & control heavily relies on radiometric tracking from Earth

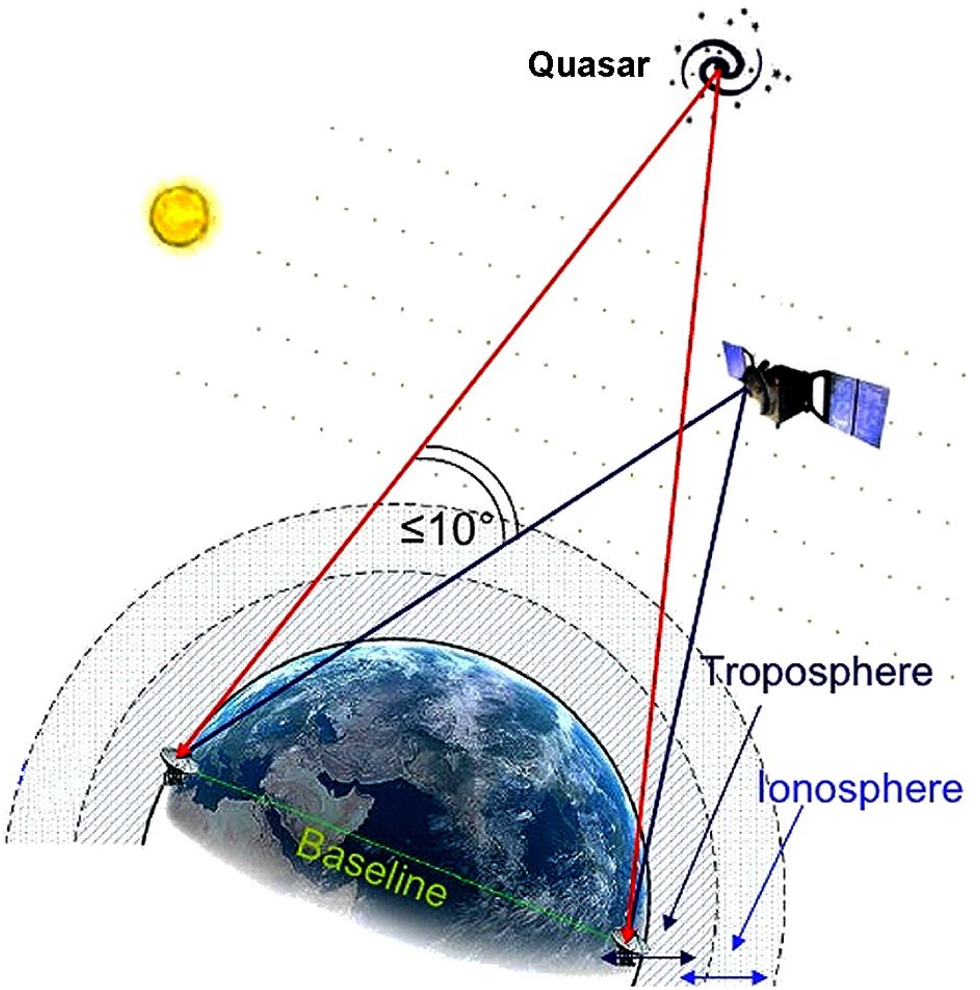


Fig 4: Concept of ground-based radiometric tracking (Delta-DOR)

Optical Navigation

- Extract observables from images using image-processing techniques:
  - ✓ Line-of-Sight vector
  - ✓ Apparent diameter
- Observables are usually fed into a state estimation filter using a dynamical model
- For closer distance, the body becomes resolved: Feature-based methods

→
- 3D-shape of the body (polyhedron)
- rotation axis

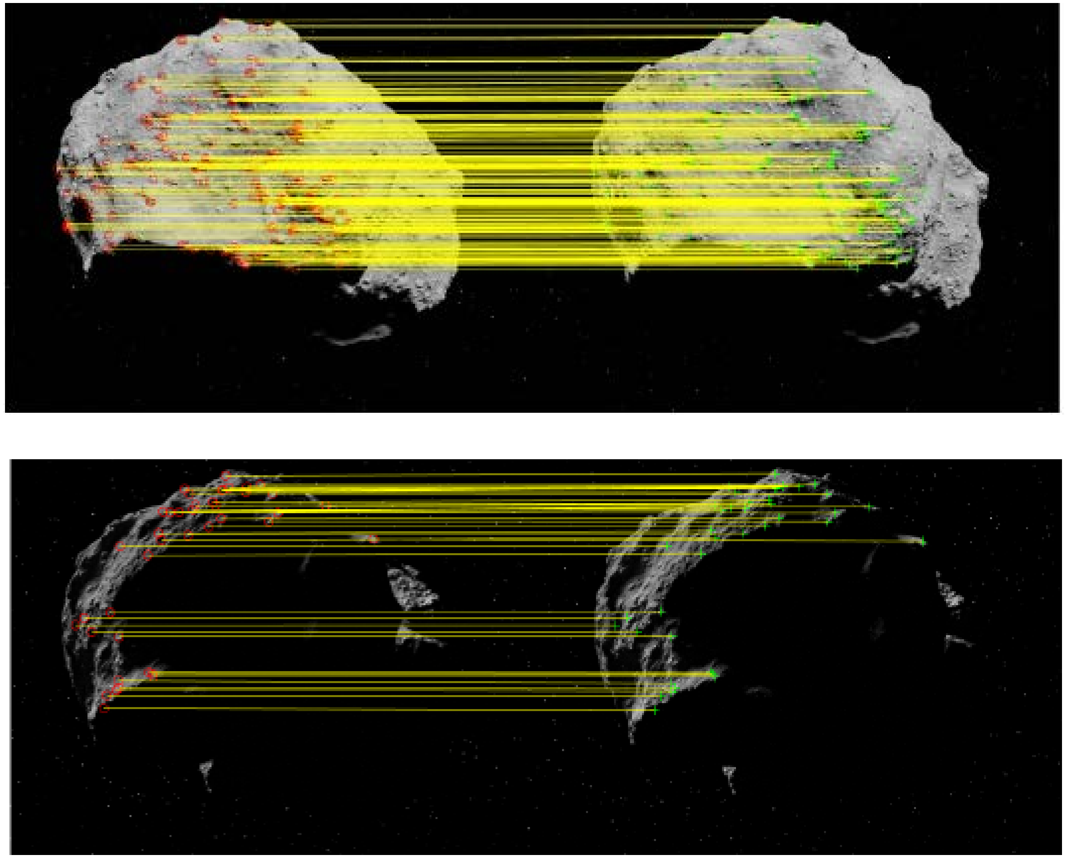
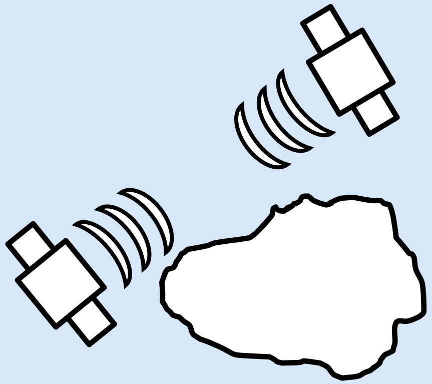


Fig 5: Examples of feature-matching with different lighting conditions

Further Studies on Automous Navigation

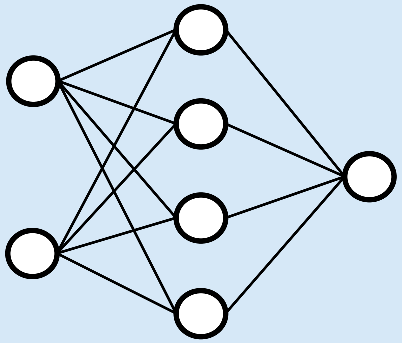
LiAISON, Satellite Swarming



- Orbit determination by inter-satellite measurements
- Multiple satellites are utilized

LiAISON – Linked Autonomous Inter Satellite Orbit Navigation

Artificial Intelligence



- Navigation techniques benefit from artificial intelligence
- Can be used in an end-to-end manner or for a specific low-level task

Limitations and Opportunities

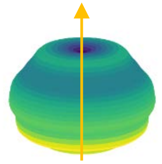
- Ground Segment Interaction → Time Delays, „Human-in-the-Loop“
- Dependency on Optical Navigation → Irregular Shapes, Lighting Conditions
- Multiple satellites → Complex operations
- Data-hungry and complex algorithms → Challenges in deep-space

Proposed new Technology

Use Gradiometer to support autonomous navigation near irregular-shaped bodies

Main Sensors

Gradiometer | *Star Tracker* | *Gyroscope*



Light-of-sight direction

Averaging of Gradiometer Measurements

Reduce the dependency/necessity

- Ground Segment Interaction
- Dependency on Optical Navigation
- Multiple satellites
- Data-hungry and complex algorithms

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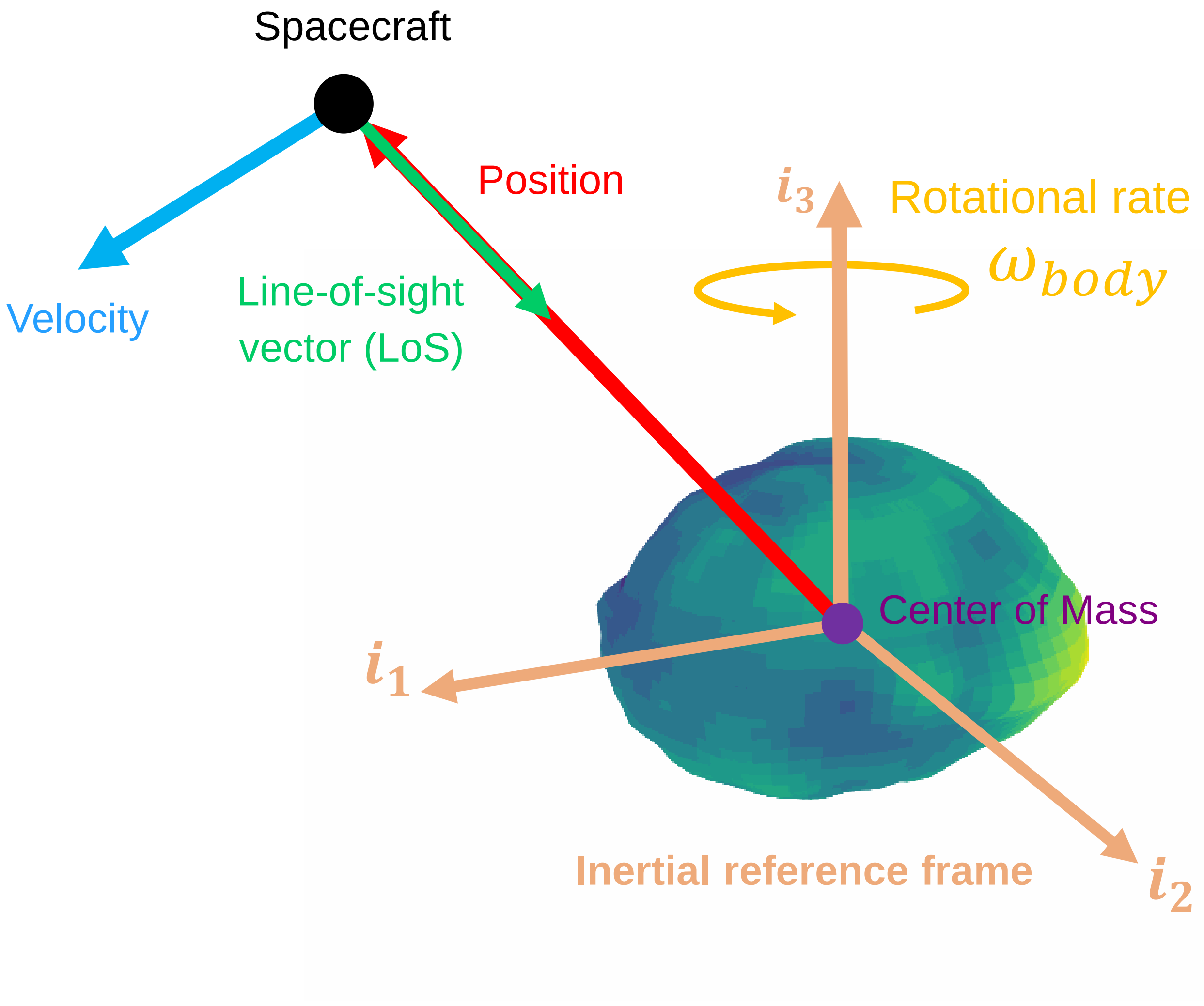
Solution Approach

The spacecraft state (state vector) is defined by 6 degrees of freedom: position and velocity in a given inertial reference frame

- We focus on determining the line-of-sight vector to the center of mass.



Approach and Assumptions

Goal: Extract Line-of-Sight vector from measurements

$\langle \mathbf{1} \rangle$: Average using body's rotational rate

$$\boxed{P_{body} \ll P_{orbit}}$$

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$$\epsilon = \frac{r}{R}$$

r Orbital radius
R Mean radius of body

Trade-off: Best distance to body?

Distance should be large enough

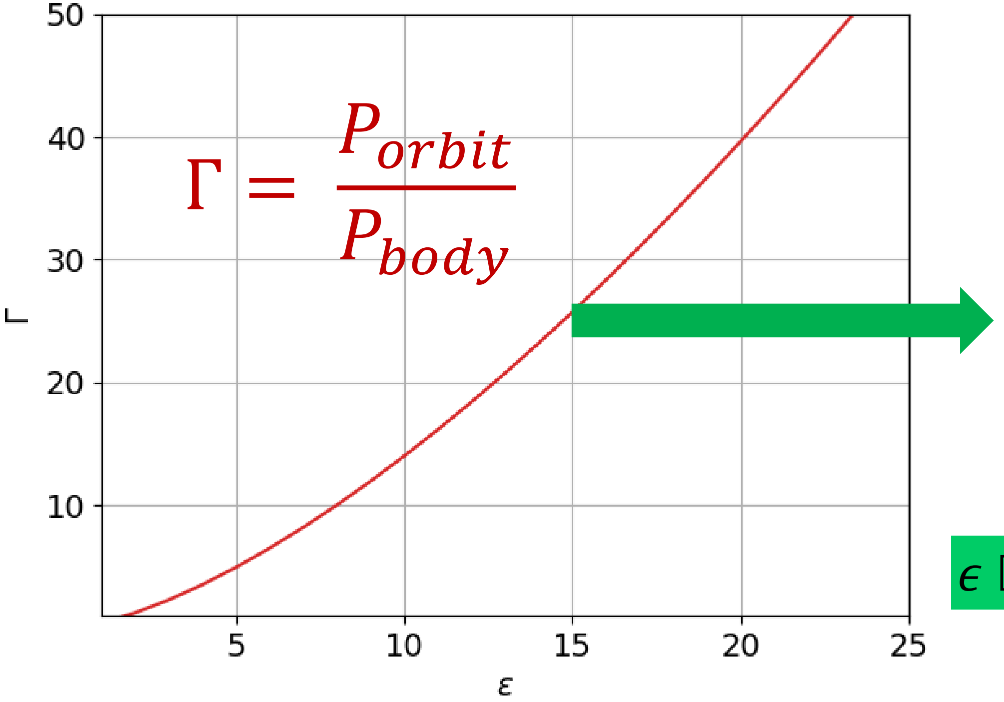


Fig 6: Example how the period ratio depends on the distance

$\epsilon \;\square\; 15 - 40$

Distance should be small enough

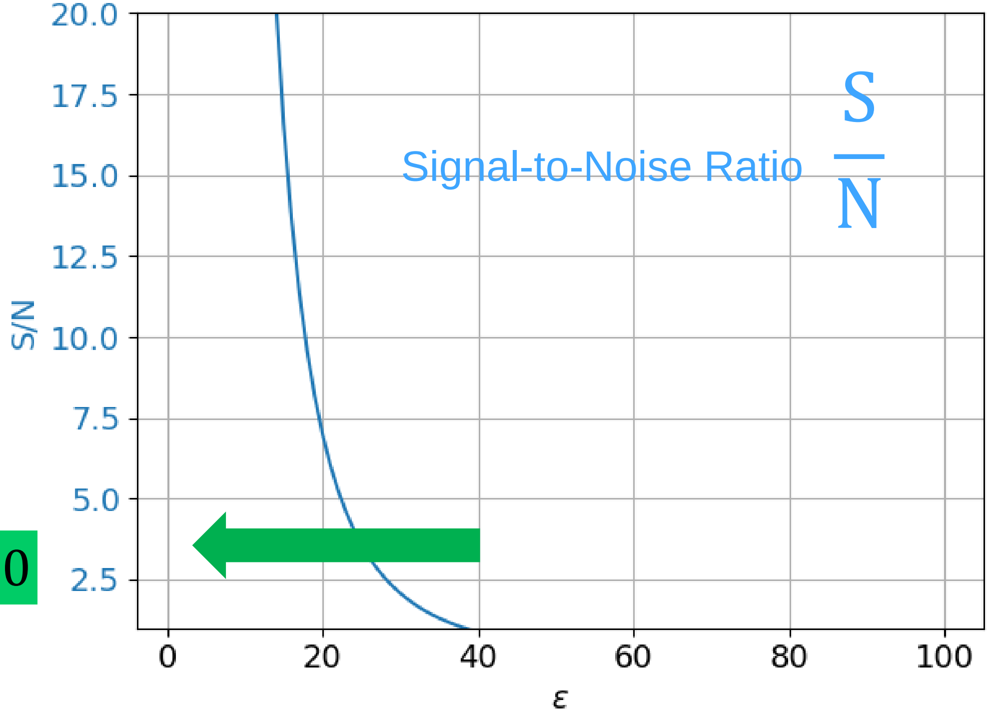
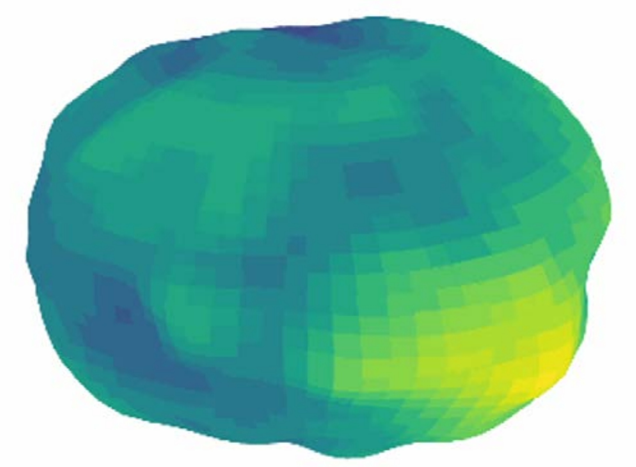


Fig 7: Example how the signal-to-noise ratio depends on the distance

Gravitational Field Representation



Different representations:

- Polyedron Model
- Mascon Model
- Spherical Harmonics

Gravitational Potential

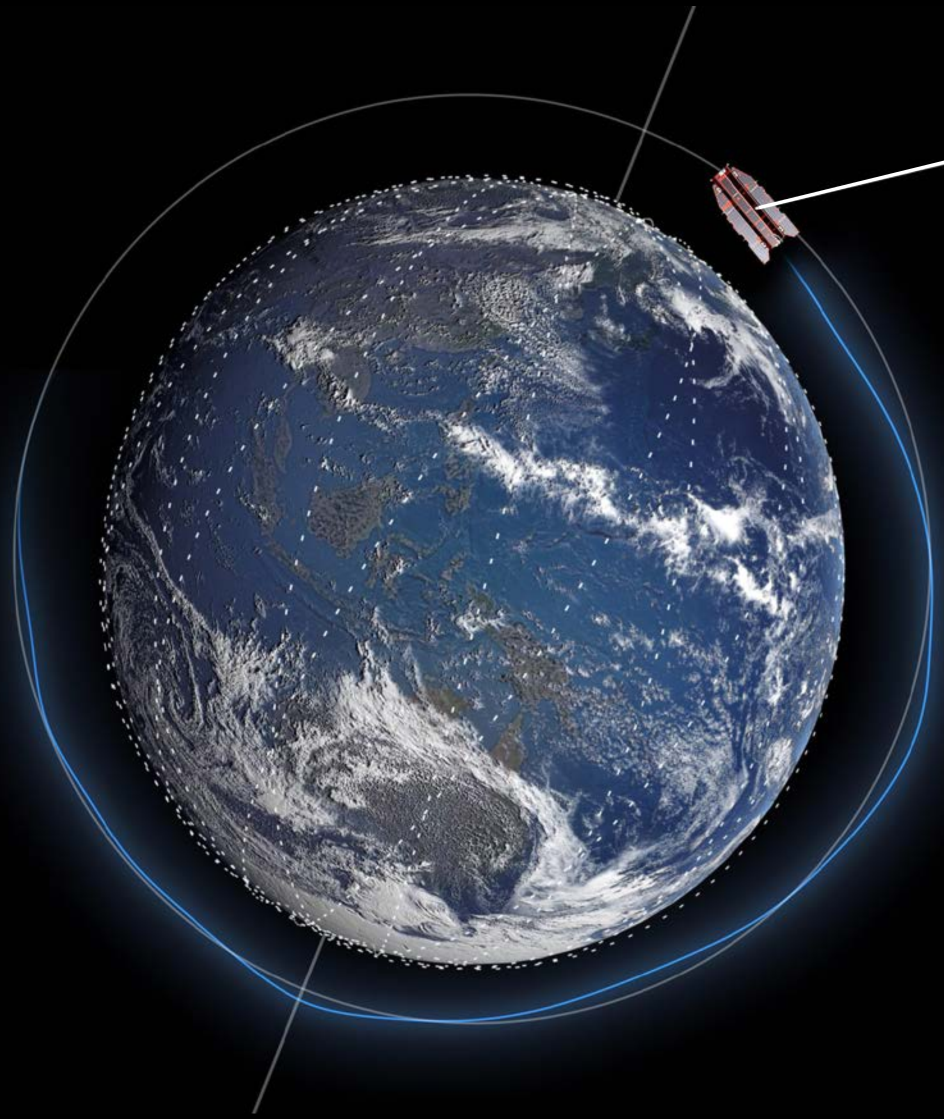
$$U(r,\lambda,\phi)=\frac{\mu}{r}\sum_{n=0}^{\infty}\sum_{m=0}^{n}\frac{R^n}{r^n}(C_{nm}\cos m\lambda+S_{nm}\sin m\lambda)P_{nm}(\sin\phi)$$

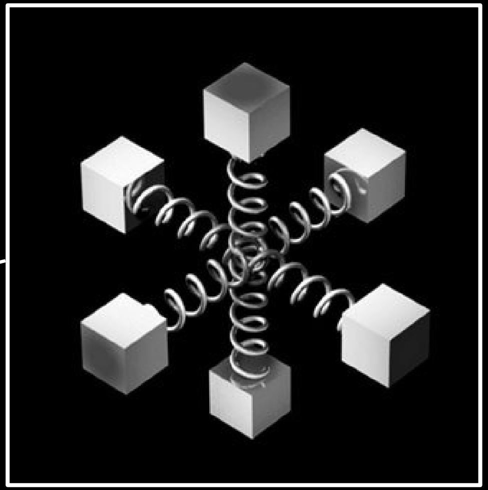
r,λ,ϕ	Radius, geocentric longitude, latitude
μ	Gravity parameter
R	Reference radius
$P_{nm}(\sin\phi)$	Associated Legendre polynomials
C_{nm}, S_{nm}	Gravitational coefficients
n,m	Degree, order

$$T=\nabla\nabla U=\begin{bmatrix}\frac{\partial^2U}{\partial^2x} & \frac{\partial^2U}{\partial x\partial y} & \frac{\partial^2U}{\partial x\partial z}\\ \frac{\partial^2U}{\partial y\partial x} & \frac{\partial^2U}{\partial^2x} & \frac{\partial^2U}{\partial y\partial z}\\ \frac{\partial^2U}{\partial z\partial y} & \frac{\partial^2U}{\partial z\partial y} & \frac{\partial^2U}{\partial^2z}\end{bmatrix}=\begin{bmatrix}U_{xx} & U_{xy} & U_{xz}\\ U_{yx} & U_{yy} & U_{yz}\\ U_{zx} & U_{zy} & U_{zz}\end{bmatrix}$$

Gravitational Gradient Tensor

Satellite Gradiometry – GOCE





Satellite gradiometry

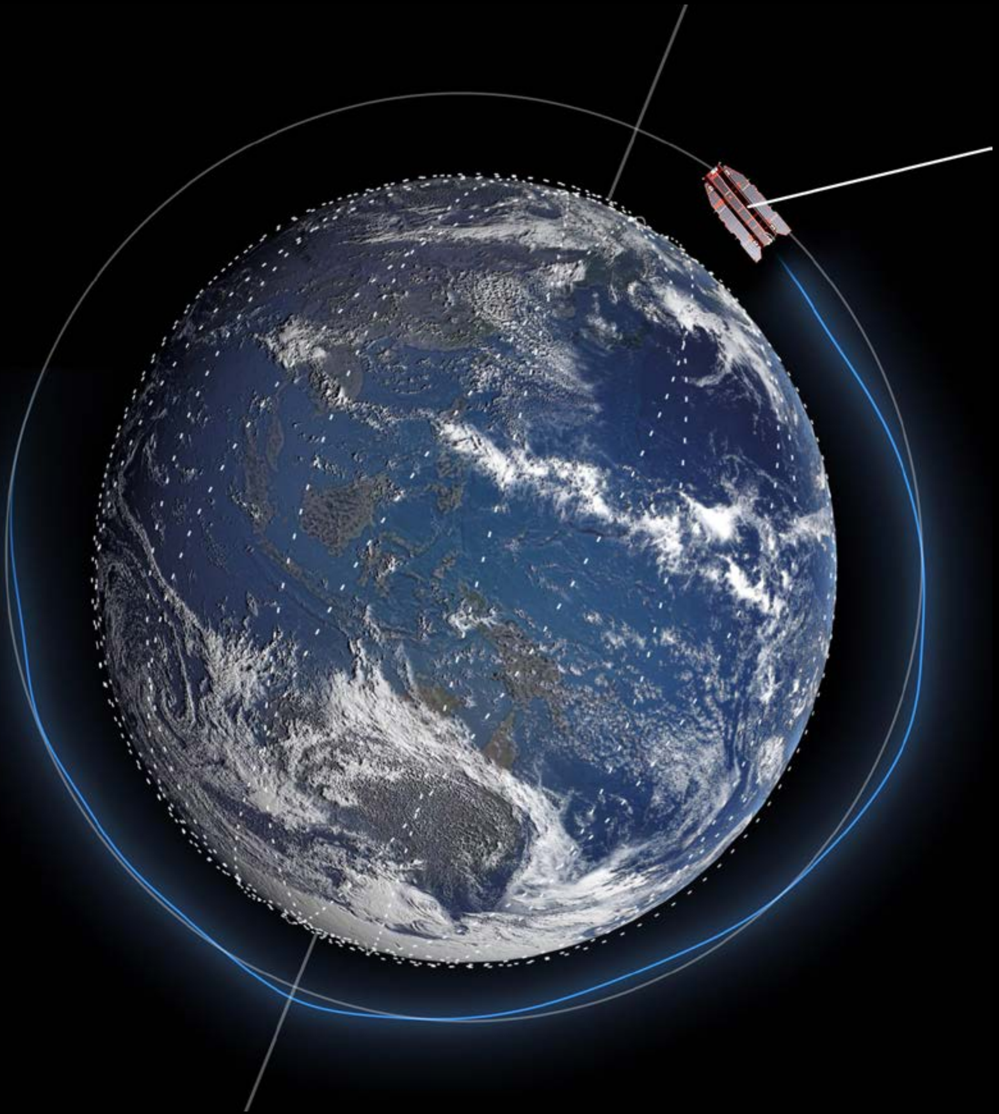
3 pairs of proof masses. A capacitive measuring principle measures the differential acceleration in one spatial direction = 3D tidal force at one orbital point.

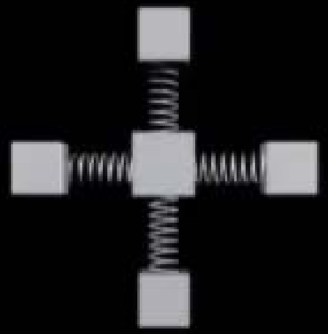
→ Medium and short wavelength part of the gravitational potential

GPS-Measurement

→ Long wavelength portion

Satellite Gradiometry – GOCE





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3 pairs of proof masses. A capacitive measuring principle measures the
differential acceleration in one spatial direction
= 3D tidal force at one orbital point.

→ Medium and short wavelength part of the gravitational potential

GPS-Measurement

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Measurement Principle

Measurements from six accelerometers:

$$\boldsymbol{a}_i = -\boldsymbol{T}\boldsymbol{r} + \underbrace{\dot{\boldsymbol{\omega}} \times \boldsymbol{r}}_{\text{Euler acceleration}} + \underbrace{\boldsymbol{\omega} \times (\boldsymbol{\omega} \times \boldsymbol{r})}_{\text{Centrifugal acceleration}}$$

$\boldsymbol{a}_i$ Measurered acceleration vector
$\boldsymbol{T}$ Gravity gradient tensor
$\boldsymbol{r}$ Distance to center of mass
$\boldsymbol{\omega}$ Angular velocity
$\dot{\boldsymbol{\omega}}$ Angular acceleration

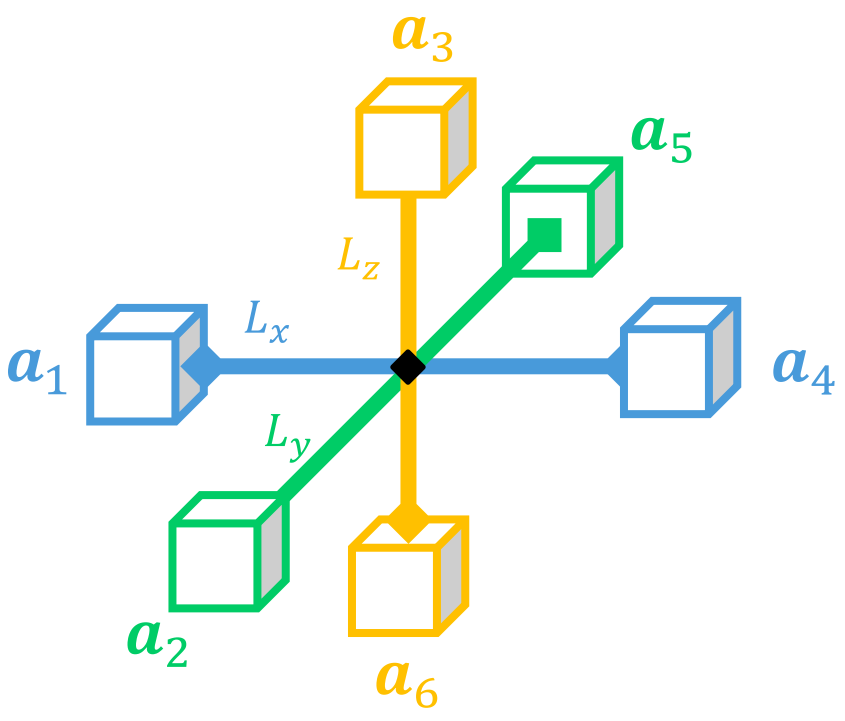


Fig 8: Concept of a Gradiometer

Common-Mode and Differential-Mode Accelerations

Can be extracted from gradiometer data

Can be extracted from gyroscope/startracker data

$$\boldsymbol{a}_i = -\boldsymbol{T}\boldsymbol{r} + \dot{\boldsymbol{\omega}} \times \boldsymbol{r} + \boldsymbol{\omega} \times (\boldsymbol{\omega} \times \boldsymbol{r}) \longrightarrow \boldsymbol{T}$$

Common-mode accelerations:

$$a_{c,1,4,i} = \frac{1}{2}(a_{1,i} + a_{4,i}) = 0$$

$$a_{c,2,5,i} = \frac{1}{2}(a_{2,i} + a_{5,i}) = 0$$

$$a_{c,3,6,i} = \frac{1}{2}(a_{3,i} + a_{6,i}) = 0$$

Vanish for non-gravitational accelerations
(e.g. drag and solar radiation pressure)

Differential-mode accelerations:

$$a_{c,1,4,i} = \frac{1}{2}(a_{1,i} - a_{4,i}) = f(\mathbf{T}, \boldsymbol{\omega}, \dot{\boldsymbol{\omega}})$$

$$a_{c,2,5,i} = \frac{1}{2}(a_{2,i} - a_{5,i}) = f(\mathbf{T}, \boldsymbol{\omega}, \dot{\boldsymbol{\omega}})$$

$$a_{c,3,6,i} = \frac{1}{2}(a_{3,i} - a_{6,i}) = f(\mathbf{T}, \boldsymbol{\omega}, \dot{\boldsymbol{\omega}})$$

$\mathbf{T}$ can be exstracted using angular rates/accelerations

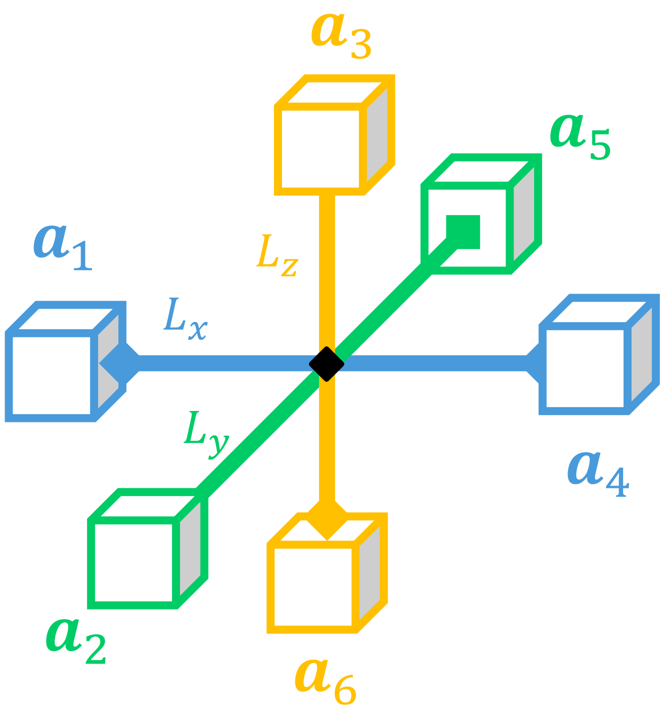


Fig 8: Concept of a Gradiometer

Eigendecomposition

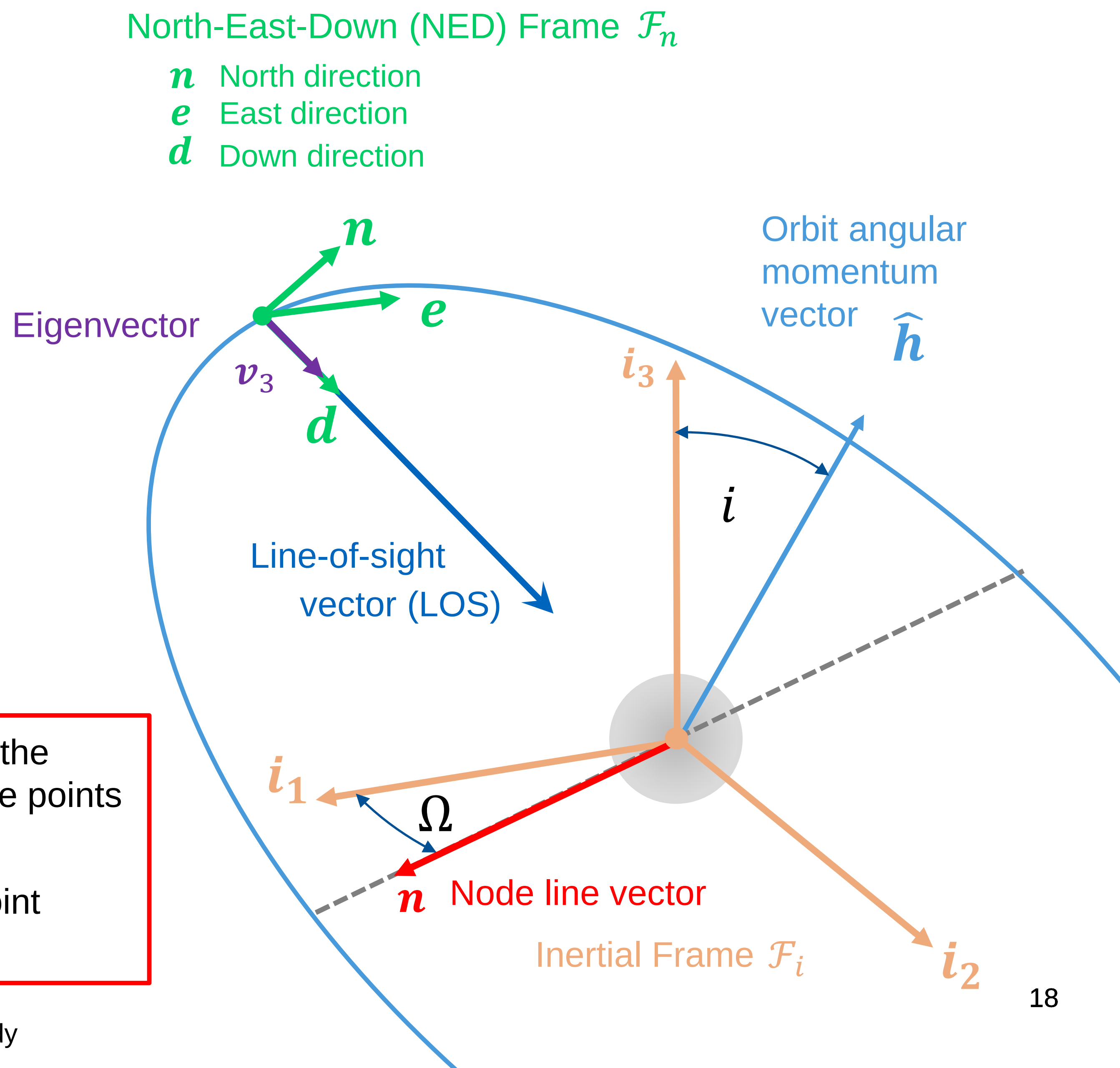
$$\boldsymbol{T}^i = \begin{bmatrix} U_{xx} & U_{xy} & U_{xz} \\ U_{yx} & U_{yy} & U_{yz} \\ U_{zx} & U_{zy} & U_{zz} \end{bmatrix} = \mathbf{V \Lambda V}^T =$$

$$[\boldsymbol{v}_1 \quad \boldsymbol{v}_2 \quad \boldsymbol{v}_3] \begin{bmatrix} \lambda_1 & 0 & 0 \\ 0 & \lambda_2 & 0 \\ 0 & 0 & \lambda_3 \end{bmatrix} [\boldsymbol{v}_1 \quad \boldsymbol{v}_2 \quad \boldsymbol{v}_3]^T$$

$\boldsymbol{T}^i$	Gravity Gradient Tensor resolved in $\mathcal{F}_i$
$\lambda_1, \lambda_2, \lambda_3$	Eigenvalues
$\boldsymbol{v}_1 \quad \boldsymbol{v}_2 \quad \boldsymbol{v}_3$	Eigenvectors resolved in $\mathcal{F}_i$

For point mass: $\boldsymbol{T}^n = \frac{\mu}{r^3}\begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 2 \end{bmatrix}$ Eigenvector with the largest eigenvalue points to body‘s CM

For any irregular-shaped body: Eigenvector does not necessarily point to body‘s center of mass



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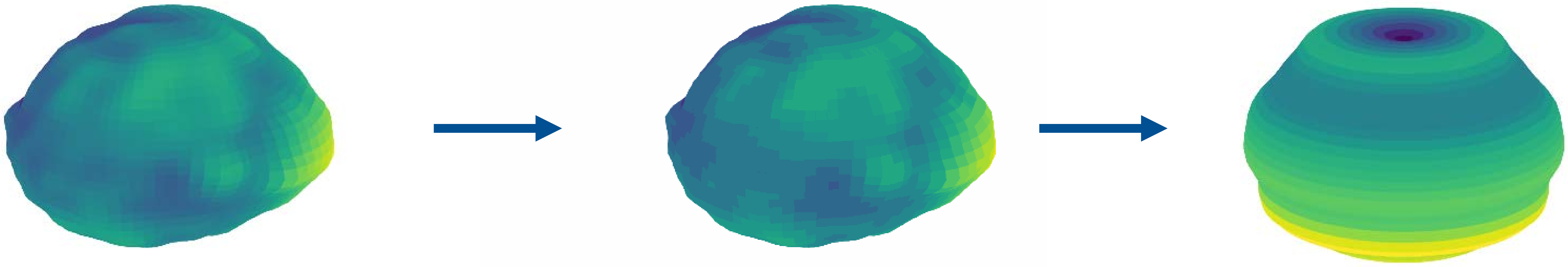
$\langle\mathbf{1}\rangle$: Average using Body's Rotational Rate

- Assume: $P_{body} \ll P_{orbit}$
- Collect gradiometer measurements: $\boldsymbol{T_j}, \boldsymbol{T_{j+1}}, \ldots$
- Take the average over period P_{body}: $\langle \boldsymbol{T} \rangle$

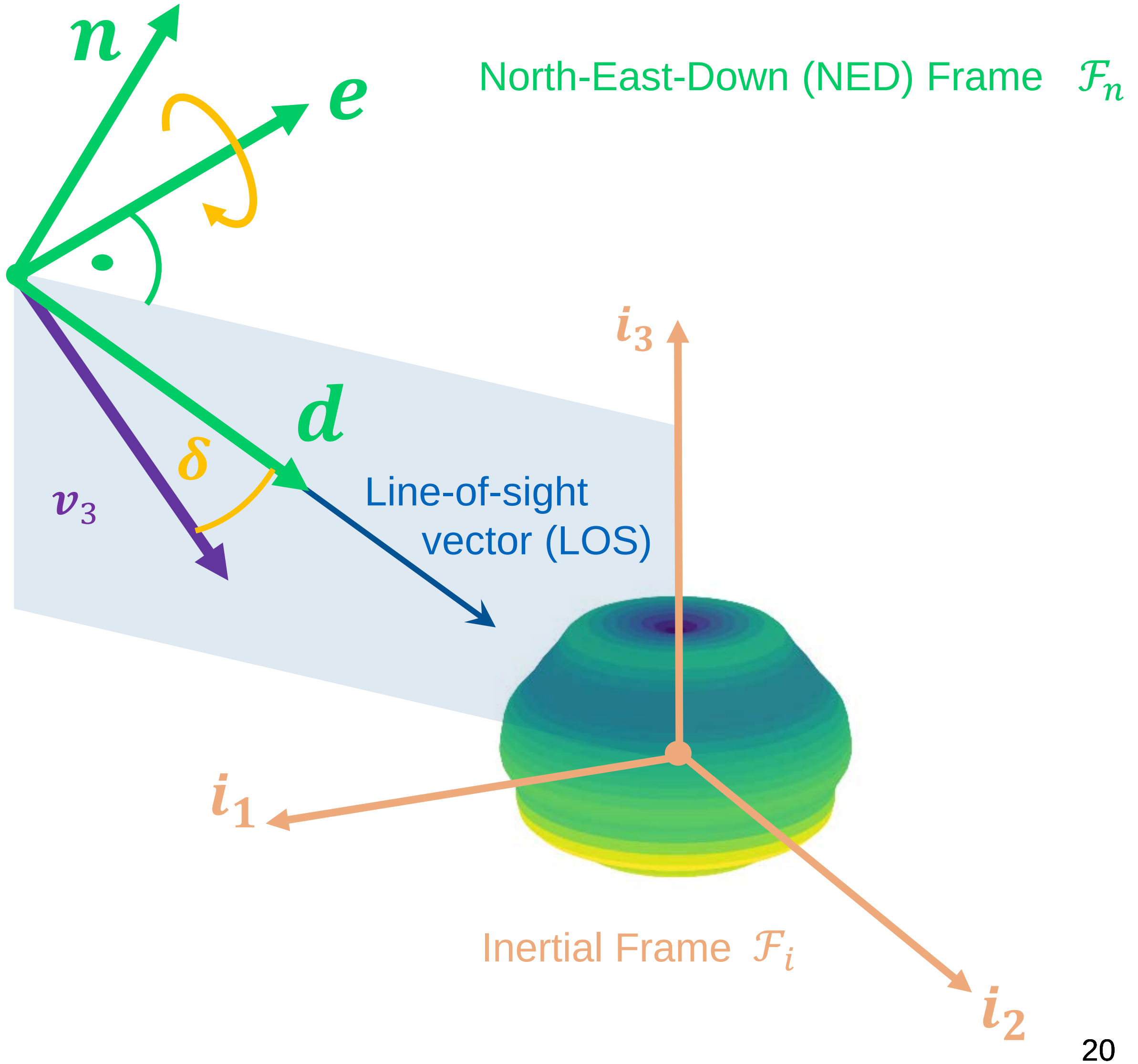


Eigendecomposition after Averaging Process

$$
\boldsymbol{T}^n = \begin{bmatrix} U_{xx} & 0 & U_{xz} \\ 0 & U_{yy} & 0 \\ U_{zx} & 0 & U_{zz} \end{bmatrix}
= \begin{bmatrix} \cos\delta & 0 & -\sin\delta \\ 0 & 1 & 0 \\ \sin\delta & 0 & \cos\delta \end{bmatrix}
\begin{bmatrix} \lambda_1 & 0 & 0 \\ 0 & \lambda_2 & 0 \\ 0 & 0 & \lambda_3 \end{bmatrix}
\begin{bmatrix} \cos\delta & 0 & \sin\delta \\ 0 & 1 & 0 \\ -\sin\delta & 0 & \cos\delta \end{bmatrix}
$$

$\boldsymbol{T}^i$	Gravity Gradient Tensor resolved in $\mathcal{F}_n$
$\lambda_1, \lambda_2, \lambda_3$	Eigenvalues
δ	Eigenvectors resolved in $\mathcal{F}_n$

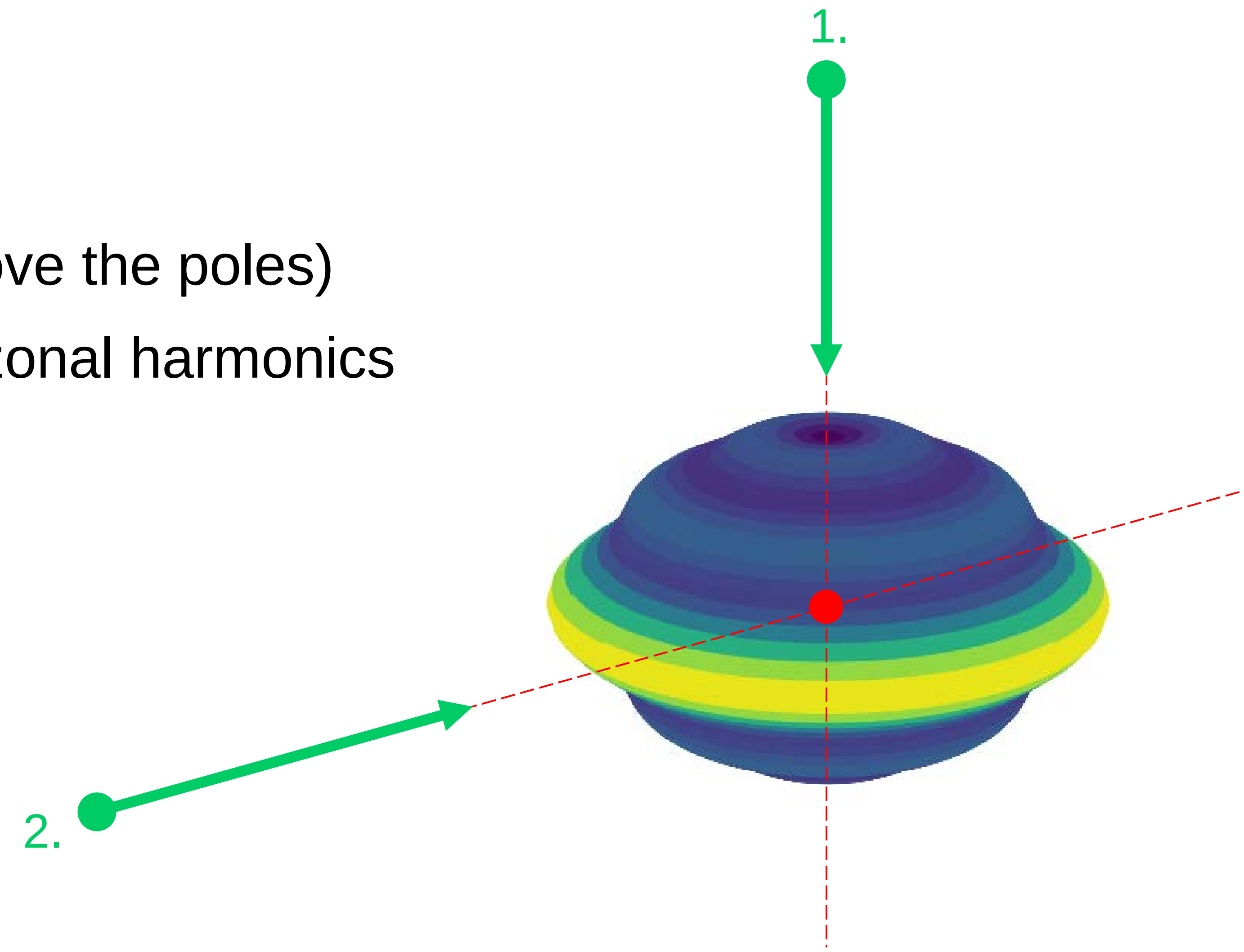
The Eigenvector $\boldsymbol{v}_3$ lies in the $\boldsymbol{n}, \boldsymbol{d}$ -plane and is rotated by the angle $\boldsymbol{\delta}$



When is $\delta = 0$?

Condition $\delta = 0$ holds for:

1. Position on rotation axis (above the poles)
2. In equatorial plane for even zonal harmonics
3. Far away from body



Approach and Assumptions

Goal: Extract Line-of-Sight vector from measurements

$\langle \mathbf{1} \rangle$: Average using body‘s rotational rate

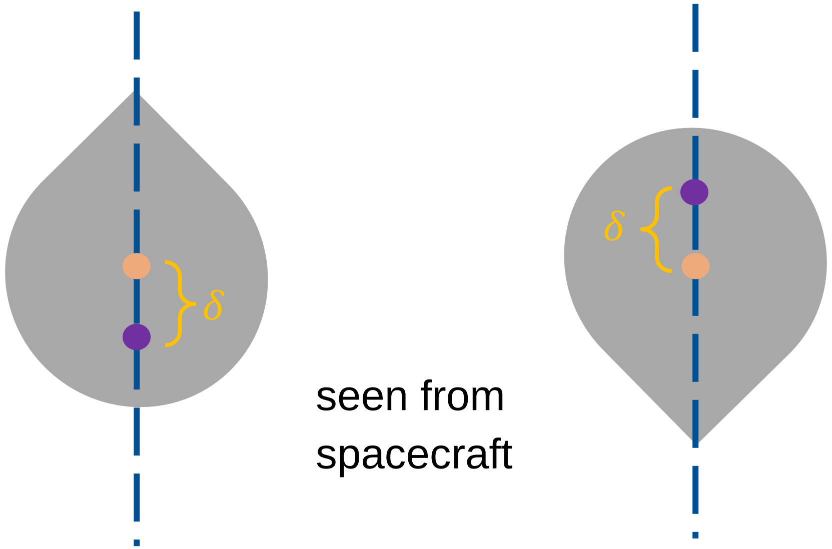
$$P_{body} \ll P_{orbit}$$

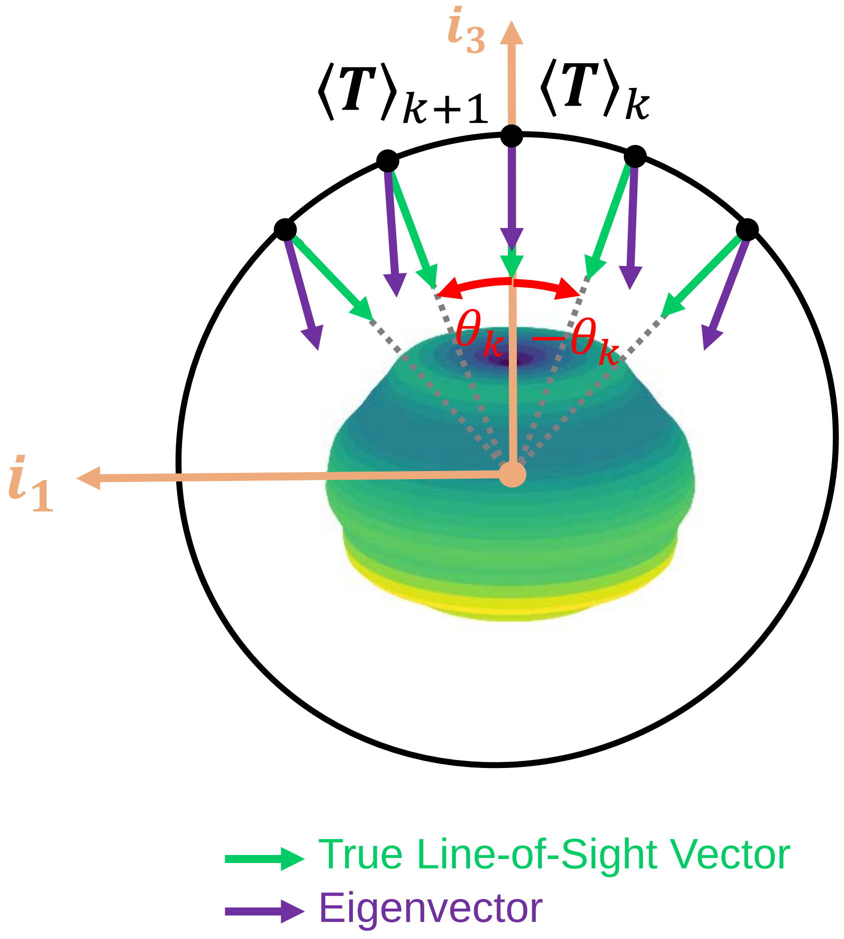
$\langle \mathbf{2} \rangle$: Average using orbital rate

Forced circular polar motion of spacecraft

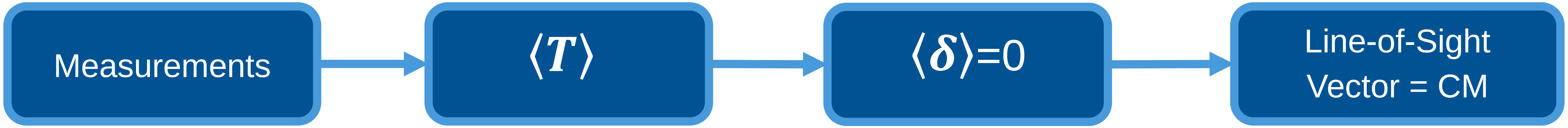
$\langle \mathbf{2} \rangle$: Average by spacecraft polar orbit

- Assume polar orbital motion: $-\delta(\theta) = \delta(-\theta)$
- Collect measurements: $\langle \boldsymbol{T} \rangle_k, \langle \boldsymbol{T} \rangle_{k+1}, \ldots$
- Take the average over period: $P_{orbit} \rightarrow \langle \delta \rangle = 0$

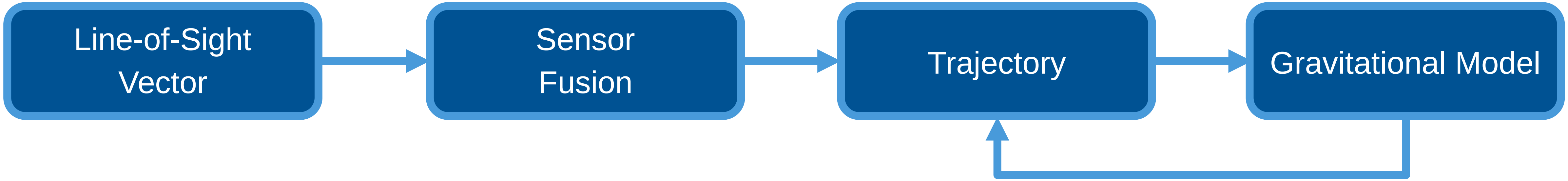




Summary of Approach



Future potentials to the complete full trajectory and gravitational model:



Conclusion

Major Findings

- Gradiometry allows to determine the line-of-sight vector $\rightarrow$ CM for an irregular-shaped body without knowing the body‘s shape
- Determination is accomplished by averaging the measurements
- Approach addresses shortcomings of optical navigation

Future Challenges

Realistic Sensor Model

Full Determination of Trajectory

Orbit Control

Non-polar Orbits

Low-spinning Bodies

Thank you for your attention!