Acta Futura 8 (2014) 37-44
DOI: 10.2420/AF08.2014.37

Acta Futura

GTOC5: Results from the Tsinghua University

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Abstract. The Lab of Aerospace Dynamics at Tsinghua University ranked three in the 5th global trajectory optimization competition. The results found are presented and the methods used are described briefly. The search for the greatest number of asteroids that must be visited first by rendezvous and possibly revisited thereafter by flyby is performed in two steps. The first step is a rough global search on the reduced candidate space that represents 1000 asteroids with orbit energies differing smallest from the Earth. The launch window and flight time of each leg are discretized with admissible time step, and then series of Lambert problems are solved. Branch, bound and prune algorithms are used to advance the achievable impulsive trajectories. The second step involves local optimization of the impulsive trajectories by indirect method combined with homotopic approach. The techniques of normalization of the unknown multipliers and a fixed step Runge-Kutta integrator with switching function detection are significantly exploited to solve series of multi-point boundary value problems The best trajectory of 17 asteroid rendezvous and successive flybys, and especially of fully bang-bang control, is obtained.

1 Introduction

The Lab of Aerospace Dynamics at Tsinghua University (THU LAD) has taken part in all five trajectory optimization competitions but did not obtain top-level results until this competition. It should be realized that to obtain somewhat wonderful results for this kind of complex optimization problem, the strategy of first rough global search and then local optimization, which was not well performed in previous competition resolution of this group, is crucial. The global search requires the low-thrust trajectories to be approximated simply enough to carry out large amount of time saving exhaustive search methods. Ballistic approximation to each leg, which results in bi-impulse transfer, is much simpler than shape-based methods [1]. Fortunately, for this competition problem, within admissible 15 years of mission time, the number of candidate asteroids are 7075. This implies that the flight time of each leg connecting two targets may be short enough to not exceed one revolution, so that ballistic approximation is somewhat competent. After rough search in the large solution space, lots of impulsive sequences are determined. Though bounded and pruned carefully, some sequences cannot be transferred into low-thrust trajectories. Recently, this group devotes more time to indirect method combined with homotopic approach [2] for low-thrust trajectory optimization. Some practical techniques [3] are used to transfer the local optimization problem from impulsive propulsion to low-thrust one, so that eventually the competition result is obtained, more techniques details cloud be found in reference [3].

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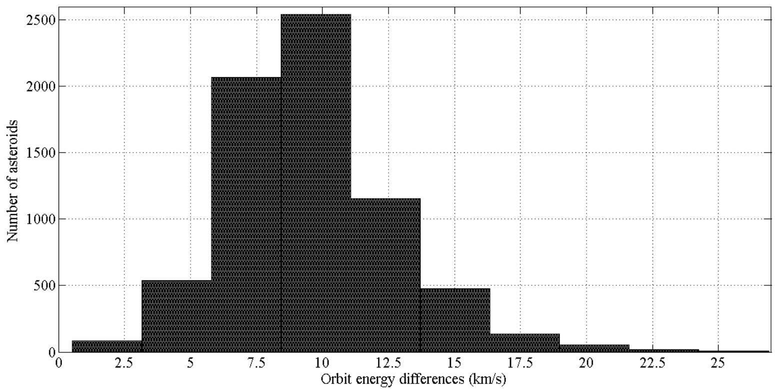

FIGURE 1. *Distribution of orbit energy differences between asteroids and the Earth.*

2 General description of the method

2.1 Preliminary consideration

Before solving this problem, the first conceptual problem one should consider is the upper bound the performance index may reach. Though reasonable estimation mainly depends on experience and intuition, the upper bound may be 20, which means at least 20 asteroid first rendezvous and successive flybys. Because the initial mass is 4000 kg, the dry mass is at least 500 kg, one rendezvous loses 40 kg equipment, one flyby loses 1 kg penetrator, and the thruster parameters give that one year of full thrust consumes about 322 kg fuel, it can be evaluated that the accumulative time of full thrust is about $(4000-500-20\times40-20\times1)/322$=8.3 years, a little larger than half of the maximum mission time. The index of 20 requires at least 40 legs of trajectories, which means the average flight time of each leg is 4.5 months, somewhat urgent for a rendezvous mission. So the index 20 is regarded as the upper bound.

The average leg flight time not more than half year implies the trajectories from one asteroids to the successive are generally not longer than one revolution, thus ballistic approximation for each leg is somewhat competent. A series of Lambert problems will be rapidly solved, whereas, the number of 7075 asteroids given by the organizer is still over-burdensome for performing exhaustive search methods. Since the spacecraft must be launched from the Earth and rendezvous is necessary to score one point, reserving a certain number of asteroids with smallest orbit energy differences from the Earth may be a good trade-off. Here the expressions presented by Izzo et al. [4] are used to evaluate orbit energy differences. The distribution of orbit energy differences is plot by Fig. 1, which shows that most asteroids are of the values [7.5, 10] km/s. The Beletskij asteroid with bonus point is not included because its orbit energy difference is 10.246 km/s, too high order.

2.2 Global search in the reduced search space

The global search in the reduced search space of 1000 asteroids applies the branch, bound and prunes algorithms. Brand, bound and prune algorithm is a general algorithm for finding optimal solutions of optimization problems, especially in discrete and combinatorial optimization. The branch, bound and prune method is adopted as global optimization method by most of the participating teams in previous competitions.

The launch window and flight time of each leg are discretized. The steps for grid search of launch date and flight time of each leg are varied from 0.05 years to 0.01 years. The flight time of each leg of trajectory is bound by the upper and lower limits, which are set to be 0.8 years and 0.2 years, respectively. The state at each target asteroid is regarded as the start point of the next leg of trajectory. And the end point of each leg is determined by selecting the candidate target. Then, series of Lambert problems are solved to determine the rendezvous or flyby objects. Four types of transfer legs are

 DOI: 10.2420/AF08.2014.37

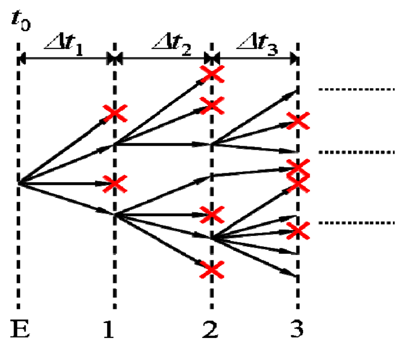

Figure 2. *Searching procedure of the branch, bound and prune algorithm.*

distinguished: rendezvous to rendezvous, rendezvous to flyby, flyby to rendezvous and flyby to flyby. ΔV of each trajectory leg can be obtained from the computation results of Lambert problem. The prune criterion is set to be that the total ΔV of each leg should be not more than $w \times T \times \Delta t / m$, where, T is the maximal thrust, Δt is the flight time, m is the mass of the spacecraft, and importantly w is a factor that usually we set it with 0.5. The possible sequences are propagated with up to millions each time. When the number of reserved sequences is too less, the factor or the upper bound of flight time is either enlarged. When the cumulate ΔV is up to about 40 km/s or the total flight time is up to 15 years, the propagation is ended. Fig. 2 shows the searching procedure.

2.3 Local optimization by indirect method

Indirect method combined with homotopic approach

The dynamic equations of the spacecraft subject only to the central force of Sun's gravity and the thrust of its own electric propulsion system are

$$\dot{\boldsymbol{r}} = \boldsymbol{v}, \dot{\boldsymbol{v}} = -\frac{\mu}{r^3}\boldsymbol{r} + \frac{T_{max}u}{m}\boldsymbol{\alpha}, \dot{m} = -\frac{T_{max}u}{I_{sp}g_0} \tag{1}$$

where $\boldsymbol{r}$ and $\boldsymbol{v}$ denote the state of spacecraft in the heliocentric ecliptic reference frame (HERF), m is the instantaneous mass, T_{max} is the maximal thrust magnitude, $\boldsymbol{\alpha}$ is the unit thrust direction vector, $u \in [0, 1]$ is the engine thrust ratio, I_{sp} is the thruster specific impulse, g_0 is the standard acceleration of gravity at sea level, and μ is the Sun's gravitational constant. For computing convenience, the quantities about length, time, and mass are made non-dimensional by astronomical unit (AU, 149597870.66 km), year (yr, 365.25×86400 s), and spacecraft initial mass, respectively. Under model (1), it can be deduced through using minimum principle to optimal control theory that for fuel-optimal problem, the optimal control is bang-bang control, and for time-optimal problem, the optimal thrust magnitude is always maximal.

Due to the discontinuous bang-bang control, the fuel-optimal problem is very difficult to solve. To overcome the difficulty, homotopic approach is introduced to solve the fuel-optimal problem by modifying the performance index as

$$\begin{aligned} J &= \lambda_0 \int_{t_0}^{t_f} L(\boldsymbol{x}, \boldsymbol{u}, t)\, dt \\ &= \lambda_0 \frac{T_{max}}{I_{sp}g_0} \int_{t_0}^{t_f} [u - \varepsilon u (1 - u)]\, dt \end{aligned} \tag{2}$$

where λ_0 is a positive numerical factor introduced to make multiplier normalization, and the homotopy parameter $\varepsilon \in [0, 1]$ connects the fuel-optimal problem ($\varepsilon = 0$) with the energy-optimal problem ($\varepsilon = 1$). For the modified performance index, the Hamiltonian is

$$\begin{aligned} H = {} & \boldsymbol{\lambda_r} \cdot \boldsymbol{v} + \boldsymbol{\lambda_v} \cdot \left(-\frac{\mu}{r^3}\boldsymbol{r} + \frac{T_{max}u}{m}\boldsymbol{\alpha} \right) \\ & - \lambda_m \frac{T_{max}}{I_{sp}g_0} u + \lambda_0 \frac{T_{max}}{I_{sp}g_0} [u - \varepsilon u (1 - u)] \end{aligned} \tag{3}$$

where $\boldsymbol{\lambda_r}$, $\boldsymbol{\lambda_v}$, and λ_m are the co-state variables, also known as functional Lagrange multipliers.

According to the minimum principle, the optimal control that minimizes the Hamiltonian holds the forms

$$\alpha = -\frac{\boldsymbol{\lambda_v}}{\|\boldsymbol{\lambda_v}\|} \tag{4}$$

$$\begin{cases} u = 0 \text{ if } \rho > \varepsilon \\ u = 1 \text{ if } \rho < -\varepsilon \\ u = \frac{1}{2} - \frac{\rho}{2\varepsilon} \text{ if } |\rho| \leqslant \varepsilon \end{cases} \tag{5}$$

where the switching function (SF) is of the form

$$\rho = 1 - \frac{I_{sp}g_0 \|\boldsymbol{\lambda_v}\|}{\lambda_0 m} - \frac{\lambda_m}{\lambda_0} \tag{6}$$

The Euler-Lagrange equations are given as

$$\begin{aligned}\dot{\boldsymbol{\lambda}}_r &= \frac{\mu}{r^3}\boldsymbol{\lambda}_v - \frac{3\mu \boldsymbol{r}\cdot\boldsymbol{\lambda}_v}{r^5}\mathrm{r},\\ \dot{\boldsymbol{\lambda}}_v &= -\boldsymbol{\lambda}_r,\ \dot{\lambda}_m = -\frac{T_{\max}u}{m^2}\left\|\boldsymbol{\lambda}_v\right\|\end{aligned} \tag{7}$$

Expression (5) shows that the optimal thrust ratio is always continuous and sometimes even differentiable unless SF equals to zeros. This attribute causes the optimal control problem, especially the energy-optimal problem, greatly easy to be solved.

Denote the spacecraft state $[\boldsymbol{r}, \boldsymbol{v}, m]$ by $\boldsymbol{x}$, and corresponding co-state by $\boldsymbol{\lambda}$. Consider general boundary and inner (termed an event) constraints combined by the p-D equality constraint

$$\boldsymbol{\psi}\left(\boldsymbol{x}(t_0), \boldsymbol{x}(t_f), \boldsymbol{x}\left(t_m^-\right), \boldsymbol{x}\left(t_m^+\right), t_0, t_f, t_m^-, t_m^+\right) = 0 \tag{8}$$

and the q-D inequality constraint

$$\boldsymbol{\sigma}\left(\boldsymbol{x}(t_0), \boldsymbol{x}(t_f), \boldsymbol{x}\left(t_m^-\right), \boldsymbol{x}\left(t_m^+\right), t_0, t_f, t_m^-, t_m^+\right) \leqslant 0 \tag{9}$$

where - and + denote the date just before and after the event m, respectively. Then, under the index (2), by introducing p- and q-D numerical multiplier vectors $\boldsymbol{\chi}$ and $\boldsymbol{\kappa}$, the first-order necessary conditions are derived to include the transversality conditions

$$\begin{aligned}\boldsymbol{\lambda}(t_0) &= -\frac{\boldsymbol{\chi}\cdot\partial\boldsymbol{\psi}}{\partial\boldsymbol{x}(t_0)} - \frac{\boldsymbol{\kappa}\cdot\partial\boldsymbol{\sigma}}{\partial\boldsymbol{x}(t_0)},\\ \boldsymbol{\lambda}(t_f) &= \frac{\boldsymbol{\chi}\cdot\partial\boldsymbol{\psi}}{\partial\boldsymbol{x}(t_f)} + \frac{\boldsymbol{\kappa}\cdot\partial\boldsymbol{\sigma}}{\partial\boldsymbol{x}(t_f)}\end{aligned} \tag{10}$$

$$\begin{aligned}\boldsymbol{\lambda}\left(t_m^-\right) &= \frac{\boldsymbol{\chi}\cdot\partial\boldsymbol{\psi}}{\partial\boldsymbol{x}\left(t_m^-\right)} + \frac{\boldsymbol{\kappa}\cdot\partial\boldsymbol{\sigma}}{\partial\boldsymbol{x}\left(t_m^-\right)},\\ \boldsymbol{\lambda}\left(t_m^+\right) &= -\frac{\boldsymbol{\chi}\cdot\partial\boldsymbol{\psi}}{\partial\boldsymbol{x}\left(t_m^+\right)} - \frac{\boldsymbol{\kappa}\cdot\partial\boldsymbol{\sigma}}{\partial\boldsymbol{x}\left(t_m^+\right)}\end{aligned} \tag{11}$$

and the stationary conditions

$$\begin{aligned}H(t_0) &= \frac{\boldsymbol{\chi}\cdot\partial\boldsymbol{\psi}}{\partial t_0} + \frac{\boldsymbol{\kappa}\cdot\partial\boldsymbol{\sigma}}{\partial t_0},\\ H(t_f) &= -\frac{\boldsymbol{\chi}\cdot\partial\boldsymbol{\psi}}{\partial t_f} - -\frac{\boldsymbol{\kappa}\cdot\partial\boldsymbol{\sigma}}{\partial t_f}\end{aligned} \tag{12}$$

$$\begin{aligned}H\left(t_m^-\right) &= -\frac{\boldsymbol{\chi}\cdot\partial\boldsymbol{\psi}}{\partial t_m^-} - \frac{\boldsymbol{\kappa}\cdot\partial\boldsymbol{\sigma}}{\partial t_m^-},\\ H\left(t_m^+\right) &= \frac{\boldsymbol{\chi}\cdot\partial\boldsymbol{\psi}}{\partial t_m^+} + \frac{\boldsymbol{\kappa}\cdot\partial\boldsymbol{\sigma}}{\partial t_m^+}\end{aligned} \tag{13}$$

For the case$t_m^- = t_m^+ = t_m$, the stationary conditions can be combined to be

$$H\left(t_m^+\right) - H\left(t_m^-\right) = \frac{\boldsymbol{\chi}\cdot\partial\boldsymbol{\psi}}{\partial t_m} + \frac{\boldsymbol{\kappa}\cdot\partial\boldsymbol{\sigma}}{\partial t_m} \tag{14}$$

where the Hamiltonian at t_m^- differs from its value at t_m^+ because the costate and state may both be different at the date just before and after the event. Besides, the complementary slackness conditions

$$\kappa_j\boldsymbol{\sigma}_j = 0,\ j = 1,2,\cdots,q \tag{15}$$

and the nonnegativity conditions

$$\lambda_0 > 0, \boldsymbol{\kappa}_j \geqslant 0,\ j = 1,2,\cdots,q \tag{16}$$

should be satisfied at the same time. For a general optimal control problem with both boundary and inner constraints, the calculus of variations and minimum principle can transfer it to a boundary value problem. The unknowns are chosen to be $\{\boldsymbol{x}(t_0), \boldsymbol{x}\left(t_m^+\right), t_0, t_m^-, t_m^+, t_f, \boldsymbol{\chi}, \boldsymbol{\kappa}, \boldsymbol{\lambda_0}\}$. The states $\boldsymbol{x}\left(t_m^-\right)$ and $\boldsymbol{x}(t_f)$ are obtained through integrating the dynamic equations (1). $\boldsymbol{\lambda}(t_0)$ and $\boldsymbol{\lambda}\left(t_m^+\right)$ are determined by the first parts of Eq. (10) and (11), and then they are used as the initial value to integrate co-state equations (7) to obtain $\boldsymbol{\lambda}\left(t_m^-\right)$ and $\boldsymbol{\lambda}(t_f)$, which should satisfy the second parts of Eq. (10) and (11). The optimal datas $[t_0, t_m^-, t_m^+, t_f]$ are determined by the stationary conditions (12)-(13), and numerical multipliers $[\boldsymbol{\chi}\ ,\ \boldsymbol{\kappa}]$ by the constraints (8) and the complementary slackness conditions (15). Note that the non negativity conditions (16) are just used to check the validity of the obtained multipliers λ_0 and $\boldsymbol{\kappa}$. Except the positive factor λ_0, the number of unknowns, however they are chosen, is always equal to that of equations.

An interesting attribute is that when regarding the functional co-states $\boldsymbol{\lambda}(t)$, numerical multipliers $[\boldsymbol{\chi}\ ,\ \boldsymbol{\kappa}]$, and numerical factor λ_0 together as combined multipliers, then all the equations needed to solve the boundary value problem are homogeneous to the combined multipliers. Therefore, they can be scaled arbitrarily. One way, for example, is dividing them by the Euclidian norm of the combined vector$[\lambda_0, \boldsymbol{\chi}, \boldsymbol{\kappa}]$ so that these unknown multipliers satisfy

$$\sqrt{\lambda_0^2 + \boldsymbol{\chi}\cdot\boldsymbol{\chi} + \boldsymbol{\kappa}\cdot\boldsymbol{\kappa}} = 1 \tag{17}$$

Note that when the chosen unknowns includes the boundary co-state values, the normalization condition should also take them into account. The normalization will constraint the solution space of unknown multipliers and greatly increase the possibility to find the solutions.

To solve the nonlinear equations including integral, MinPack-1 [5], a program package implements a modification of Powell's hybrid algorithm [6] that is a combination of Newton's method and the method of gradient, is used. With the application of homotopic approach, the fuel-optimal problem is solved by starting from the energy-optimal problem ($\varepsilon = 1$). The parameter ε is decreased slowly and successive related problem is solved by using the preceding solution as the initial guess of the following resolution. When the parameter ε is not close to zero, integrators such as Runge-Kutta methods with adaptive step size are competent to ensure the integration accuracy because the optimal thrust varies not too rapidly. when the parameter ε of homotopic problem is close to zero, Runge-Kutta method with adaptive step size is not able to insure the integration accuracy because the right-hand sides of ODEs vary rapidly around switching points. Here the fourth-order Runge-Kutta algorithm with fixed step size (RK4) combined with switching detection will be considered to overcome this problem.

If the fixed step size h is small enough, the value of SF at the (k+1)-th iteration can be approximately examined by the value at the k-th iteration through

$$\rho_{k+1} = \rho_k + \dot{\rho}_k h + \frac{1}{2}\ddot{\rho}_k h^2 \tag{18}$$

where the first and second derivatives of SF with respect to time both exist

$$\dot{\rho} = I_{sp} g_0 \frac{1}{\lambda_0} \frac{\boldsymbol{\lambda_v} \cdot \boldsymbol{\lambda_r}}{m \left\| \boldsymbol{\lambda_v} \right\|} \tag{19}$$

$$\ddot{\rho} = I_{sp} g_0 \frac{1}{\lambda_0} \left[-\frac{\boldsymbol{\lambda_r} \cdot \boldsymbol{\lambda_r}}{m \left\| \boldsymbol{\lambda_v} \right\|} + \frac{\mu}{r^3} \frac{\left\| \boldsymbol{\lambda_v} \right\|}{m} - \frac{3\mu}{r^5} \frac{\left\| \boldsymbol{\lambda_v} \cdot \boldsymbol{r} \right\|^2}{m \left\| \boldsymbol{\lambda_v} \right\|} + \right.$$
$$\left. + \frac{T_{max} u}{I_{sp} g_0 m^2} \frac{\boldsymbol{\lambda_v} \cdot \boldsymbol{\lambda_r}}{\left\| \boldsymbol{\lambda_v} \right\|} + \frac{\left(\boldsymbol{\lambda_v} \cdot \boldsymbol{\lambda_r} \right)^2}{m \left\| \boldsymbol{\lambda_v} \right\|^3} \right] \tag{20}$$

Dividing the range of SF into three intervals: $(-\infty, -\varepsilon)$, $(-\infty, -\varepsilon)$, and $[-\varepsilon, \varepsilon]$, the iteration from xk to xk+1 is carried out by first examining which intervals the values of ρ_k and ρ_{k+1} respectively fall into. There are nine cases with respect to the values of ρ_k and ρ_{k+1}. The details about the switching detection technique and integration strategy can be found in published work [3].

The first-order necessary conditions for competition problem

The events of launch from the Earth with limited hyperbolic excess velocity, rendezvous with asteroids, and flyby with asteroids with lower bound relative velocity are thoroughly considered as constraints and the corresponding quantities are determined through solving the consequent first-order necessary conditions (FONCs). Set the lower bound of launch window 57023 MJD as the reference date, and all quantities are non-dimensional. For the launch from the Earth, the equality and inequality constraints are

$$\boldsymbol{\psi} = \begin{Bmatrix} \boldsymbol{r}(t_0) - \mathbf{r}_E(t_0) \\ m(t_0) - 1 \end{Bmatrix} = \mathbf{0} \tag{21}$$

$$\sigma = v_{rel} - 1.0547 \leqslant 0 \tag{22}$$

where v_{rel} denotes the magnitude of relative velocity $\boldsymbol{v}_{rel} = \boldsymbol{v}(t_0) - \boldsymbol{v}_E(t_0)$, and $\mathbf{r}_E$ and $\boldsymbol{v}_E$ denote the position and velocity of the Earth, and 1.0547 AU/yr is equal to 5 km/s. The FONCs are derived as

$$\begin{aligned} &\boldsymbol{\lambda_r}(t_0) = -\boldsymbol{\chi}_{1:3}, \boldsymbol{\lambda_v}(t_0) = -\kappa \frac{\boldsymbol{v}_{rel}}{v_{rel}}, \lambda_m(t_0) = -\chi_4 \\ &H(t_0) + \boldsymbol{\chi}_{1:3} \cdot \boldsymbol{v}_E + \kappa \boldsymbol{v}_{rel} \cdot \boldsymbol{a_E}(t_0) = 0 \end{aligned} \tag{23}$$

where $\chi_{i:j}$ denotes the i-th to j-th components of the vector $\boldsymbol{\chi}$, and $\boldsymbol{a}_E$ is the acceleration vector of the Earth.

For an intermediate rendezvous at date tm with an asteroid, the constraints are

$$\boldsymbol{\psi} = \begin{Bmatrix} \boldsymbol{r}(t_m) - \boldsymbol{r}_a(t_m) \\ \boldsymbol{v}(t_m) - \boldsymbol{v}_a(t_m) \\ m(t_m^-) - m(t_m^+) - 0.01 \end{Bmatrix} = \mathbf{0} \tag{24}$$

where non-dimensional mass 0.01 equals to 40 kg, and subscript a denotes the asteroid. The FONCs are derived as

DOI: 10.2420/AF08.2014.37

Table 1. *Competition design results*

Asteroid name	Date (MJD)	Event	Mass (kg)
Earth	58684.8875	Launch	4000.000
2009BD	58995.6407	Rendezvous	3902.2753
2009BD	59132.1819	Flyby	3786.7079
2008EA9	59468.8073	Rendezvous	3623.2240
2008EA9	59589.7345	Flyby	3503.0538
1991VG	59674.6080	Rendezvous	3427.2773
1991VG	59803.9887	Flyby	3320.7407
2000SG344	60033.4813	Rendezvous	3212.4910
2000SG344	60155.9597	Flyby	3112.2227
2007UN12	60513.4954	Rendezvous	2903.5742
2007UN12	60618.0116	Flyby	2800.4630
2008KT	60902.0098	Rendezvous	2664.3218
2008KT	60993.2060	Flyby	2559.6191
2001GP2	61322.5804	Rendezvous	2383.0377
2001GP2	61401.5696	Flyby	2279.2183
2007VU6	61720.5920	Rendezvous	2121.0774
2007VU6	61799.6402	Flyby	2034.8921
2009YR	62005.2438	Rendezvous	1903.6709
2009YR	62072.3511	Flyby	1817.6482
2004QA22	62360.9638	Rendezvous	1714.3367
2004QA22	62427.6511	Flyby	1625.4205
2006RH120	62664.1824	Rendezvous	1441.7554
2006RH120	62715.2479	Flyby	1362.9280
1993HD	62937.4473	Rendezvous	1260.4956
1993HD	62993.2455	Flyby	1195.4798
2006JY26	63204.8926	Rendezvous	1118.9827
2006JY26	63255.4615	Flyby	1056.9635
2007YF	63376.8418	Rendezvous	992.4407
2007YF	63432.1415	Flyby	935.3241
2008UA202	63609.9229	Rendezvous	849.7547
2008UA202	63642.3136	Flyby	790.3012
2009WR52	63821.1021	Rendezvous	679.2205
2009WR52	63846.0501	Flyby	623.1293
2003WE	63943.4974	Rendezvous	563.0469
2003WE	63962.7500	Flyby	508.0949

$$\begin{aligned}
&\boldsymbol{\lambda}_{\boldsymbol{r}}\left(t_m^+\right) - \boldsymbol{\lambda}_{\boldsymbol{r}}\left(t_m^-\right) + \boldsymbol{\chi}_{1:3} = \mathbf{0}, \\
&\boldsymbol{\lambda}_{\boldsymbol{v}}\left(t_m^+\right) - \boldsymbol{\lambda}_{\boldsymbol{v}}\left(t_m^-\right) + \boldsymbol{\chi}_{4:6} = \mathbf{0} \\
&\lambda_m\left(t_m^-\right) = \lambda_m\left(t_m^+\right) = \chi_7, \\
&H\left(t_m^-\right) - H\left(t_m^+\right) - \boldsymbol{\chi}_{1:3} \cdot \boldsymbol{v}_a\left(t_m\right) \\
&\quad - \boldsymbol{\chi}_{4:6} \cdot \boldsymbol{a}_a\left(t_m\right) = 0
\end{aligned} \tag{25}$$

where $\boldsymbol{a}_a$ is the acceleration vector of the asteroid.

For an intermediate flyby at date t_m with an asteroid, the constraints are

$$\boldsymbol{\psi} = \begin{Bmatrix} \boldsymbol{r}\left(t_m\right) - \boldsymbol{r}_a\left(t_m\right) \\ m\left(t_m^-\right) - m\left(t_m^+\right) - 0.00025 \end{Bmatrix} = \mathbf{0} \tag{26}$$

$$\sigma = 0.0844 - v_{rel} \leqslant 0 \tag{27}$$

where v_{rel} denotes the magnitude of relative velocity $\boldsymbol{v}_{rel} = \boldsymbol{v}\left(t_m\right) - \boldsymbol{v}_a\left(t_m\right)$, and nondimensional mass 0.00025 and velocity 0.0844 equal to 1 kg and 400 m/s, respectively. The FONCs are derived as

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$$\begin{aligned}
&\boldsymbol{\lambda}_{\boldsymbol{r}}\left(t_m^+\right)-\boldsymbol{\lambda}_{\boldsymbol{r}}\left(t_m^-\right)+\boldsymbol{\chi}_{1:3}=\mathbf{0},\\
&\boldsymbol{\lambda}_{\boldsymbol{v}}\left(t_m^+\right)-\boldsymbol{\lambda}_{\boldsymbol{v}}\left(t_m^-\right)-\kappa\frac{\boldsymbol{v}_{\text{rel}}}{v_{\text{rel}}}=\mathbf{0}\\
&\lambda_m\left(t_m^-\right)=\lambda_m\left(t_m^+\right)=\chi_7,\\
&H\left(t_m^-\right)-H\left(t_m^+\right)-\boldsymbol{\chi}_{1:3}\cdot\boldsymbol{v}_a\left(t_m\right)\\
&+\kappa\frac{\boldsymbol{v}_{\text{rel}}\cdot\boldsymbol{a}_a\left(t_m\right)}{v_{\text{rel}}}=0
\end{aligned} \tag{28}$$

Local optimization of the impulsive results

By global searching, a sequence with 17 asteroid rendezvous and successive flybys is found. There are altogether 34 legs of trajectory. If these legs of trajectory are optimized separately and then patched as the whole trajectory, it is possible to miss the global optimal. However, it is too difficult for our algorithms to optimize the entire trajectory. That because, one rendezvous introduces 7 design variables (including 3-D position vector, 3-D velocity vector and 1-D rendezvous time) and one flyby introduces 5 variables (including 3-D position vector, 1-D velocity inequality constraint multiplier and 1-D flyby time), respectively. 17 rendezvous and flybys correspond to about 200 variables and the total integration duration is about 15 years. As a result, we solved 3 or 4 legs of trajectory for one time. The initial state and final state are based on the results of global optimization.

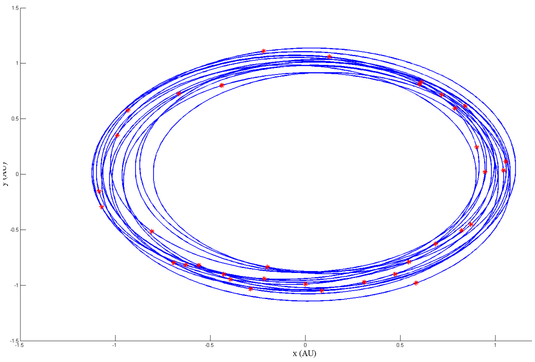

Figure 3. *Trajectory projected onto the ecliptic plane*

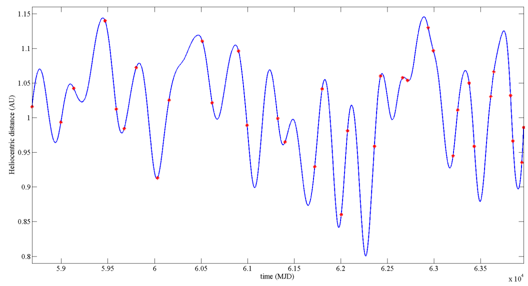

Figure 4. *Distribution of orbit energy differences between asteroids and the Earth.*

3 Competition results

The performance index we obtained is 17.0, i.e. 17 asteroids are rendezvoused and then immediately revisited. The total flight time is 5277.862 days, i.e. 14.45 years. The launch v_∞ is [-0.935, 1.906, 0.034] km/s, and so the magnitude is 2.123 km/s. The total thrust durations is 3173.433 days, i.e. 8.689 years. The final mass is 507.095 kg. The events are listed in detail as follows, noting that the listed mass at rendezvous date includes the 40 kg scientific mass that will be delivered immediately, and that at flyby date also includes the 1 kg penetrator. The trajectory is shown in Fig. 3 and the Sun-centered distance is shown in Fig. 4.

4 Conclusions

The Lab of Aerospace Dynamics at Tsinghua University has taken part in the five GTOCs. The aims of us are experience accumulating and learning from the research teams all over the world. Although we have made progress in GTOC5, there exists a great gap in mission analysis and trajectory design between us and the top-level teams, such JPL, ESA Advanced Concepts Team, Moscow state university and so on. In recent years, we have investigated the literatures of winners of previous GTOCs and been improving our theories and tools on deep space exploration trajectory optimization. It is hoped that the trajectory optimization techniques could be used in Chinese future deep space exploration missions.

There are still a lot of works for us to do in trajectory design and optimization. First of all, compared with the winner JPL, our searching program is just run in personal computer, rather than the parallel computer. To shorten the computational time, some potential better asteroid-visiting sequences may be missed. Parallel computing is our next work. Secondly, the optimal control problems with multiple intermediate constraints are not able to solve for our local optimization algorithm, which also needs to be improved. In addition, the approximation of heliocentric two body problem, namely the spacecraft moves only under the solar gravity force

DOI: 10.2420/AF08.2014.37

and the thrust of on-board propulsion system, is employed for all the five GTOCs. And the simplified constraints are considered. For example, the details of how the rendezvous or flyby would occur are not addressed. The gravity assists are approximated to the impulsive model. That is, only the preliminary mission design problems are concerned about in GTOCs. However, in practical engineering, an accurate dynamical model, which includes the perturbation forces of third bodies and solar radiation pressure, should be used. And the gravity assist should be studied in a full ephemeris model. The trajectory optimization problem in an accurate dynamical model is expected to be investigated.

5 Acknowledgments

This work was supported by the National Natural Science Foundation of China (Grant No. 11302112) and the Tsinghua University Initiative Scientific Research Program (Grant No. 20131089268)

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