GTOC12: Methods and Results from the Jet Propulsion Laboratory Team

GTOC12 Workshop
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Objective: Maximize total weighted collected mass

- From a set of 60,000 asteroids, between 2035 and 2050,
- Maximize total weighted collected mass:

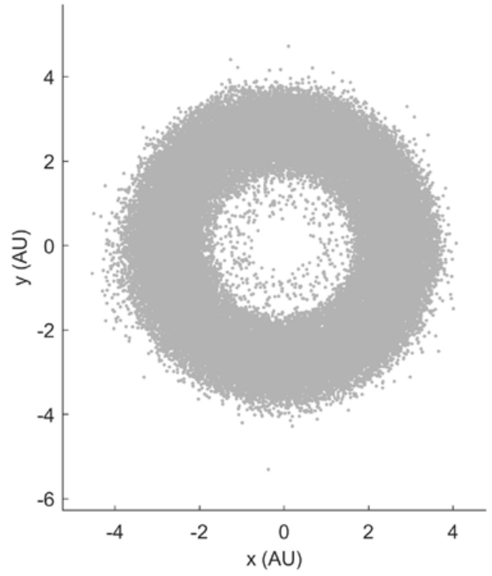
$$\boldsymbol{J = \sum_{i=1}^{60{,}000} B_i M_i}$$

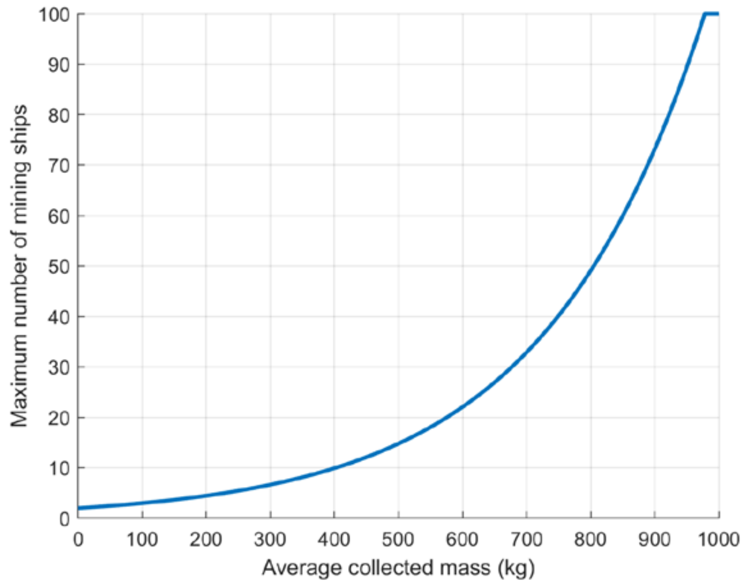
where $\boldsymbol{B_i}$ is the bonus coefficient of the *i*-th asteroid, and $\boldsymbol{M_i}$ is the collected mass of the *i*-th asteroid

- The number of Mining Ships used in each solution, N, is constrained by

$$\boldsymbol{N \leq min(100, 2exp(\rho \bar{M}))}$$

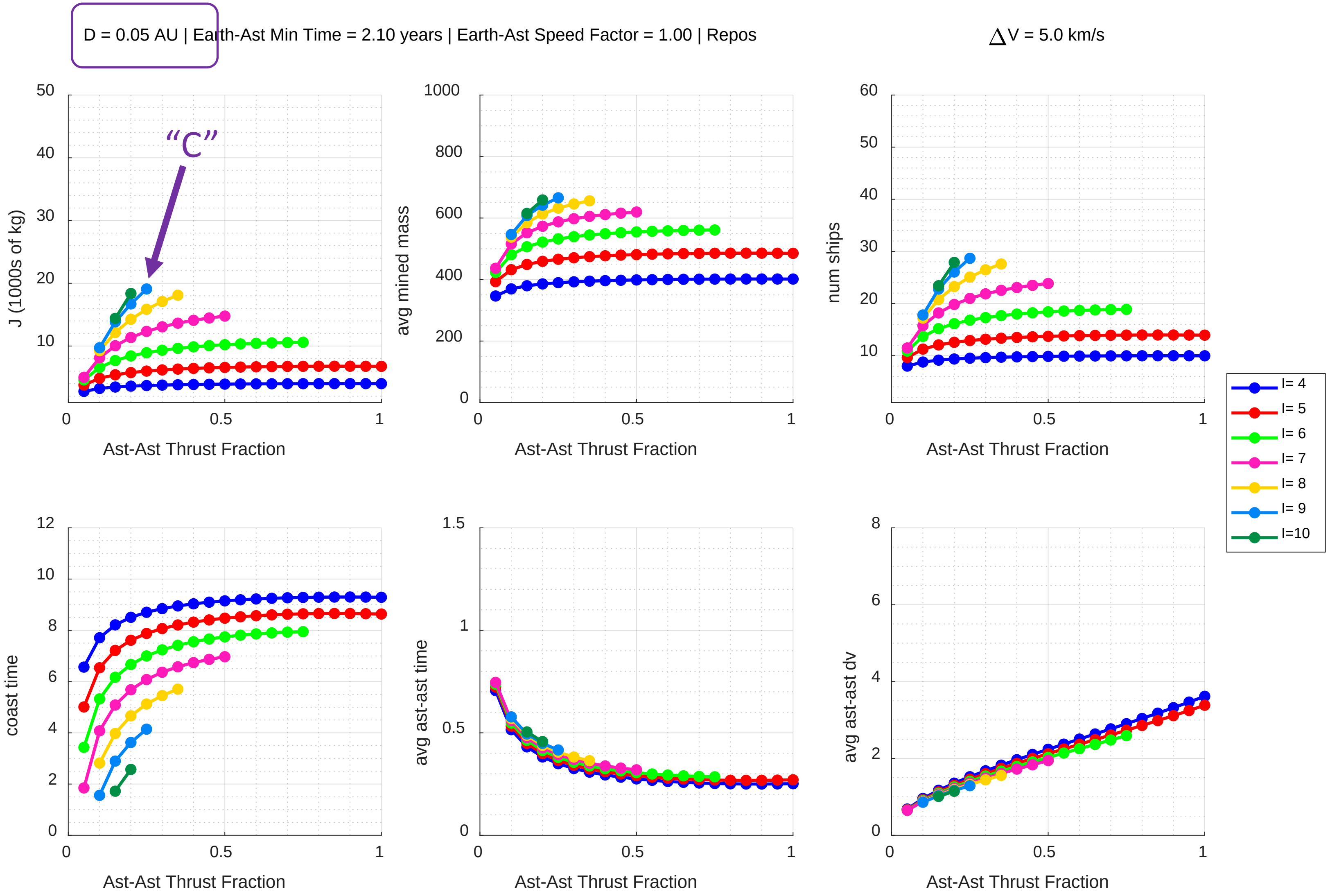
where ρ = 0.004 kg^{-1}, and $\bar{M}$ is the average collected mass transported back to the Earth per Mining Ship in this solution





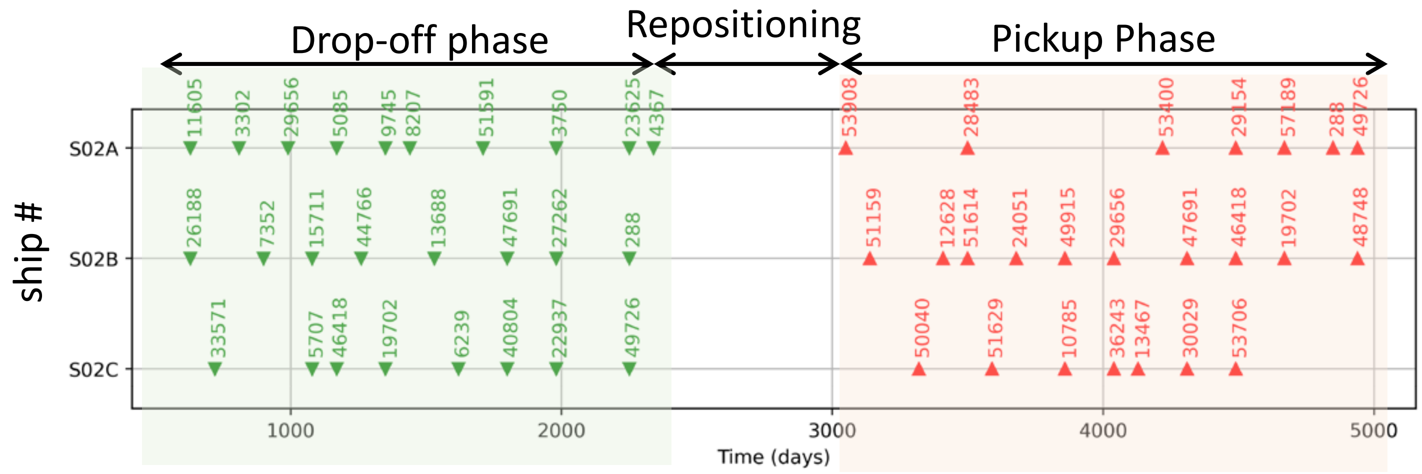
High-level modeling

- At the start of the competition, a high-level model was developed to build our intuition and guide our likely solution space
- Assume all ships perform the same
- Ast-Ast Transfers
  - Assume asteroids are all equally distributed at some distance (such as 0.05 AU)
  - Single parameter: thrust fraction (1 = min time, always thrusting)
- Fast earth-asteroid transfers are important
- Optimum number of miners depends on cluster density (unsurprising). Mostly likely range is 6 to 8 per ship
- Optimum number of ships around 30, which (unweighted) cost around 20,000 kg.

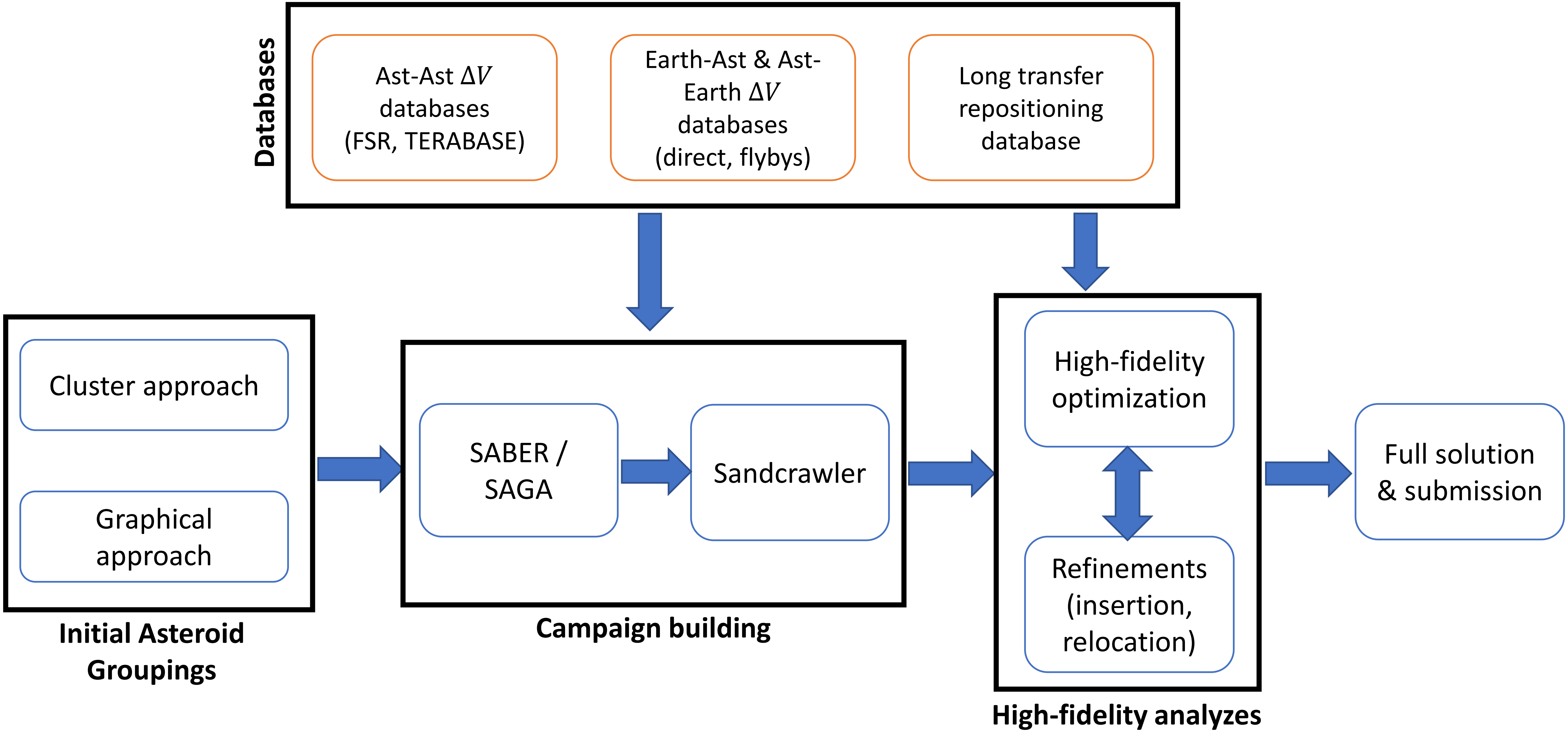


Solution structure

- From the beginning, we believed that it was crucial to employ a solution method using "collaborative" ships (those which drop-off and pickup miners from different asteroids) due to being able to take advantage of the natural phasing that results from orbital dynamics
- To enable this, we made two crucial "intentionally suboptimal" decisions in our solution structure based on intuition that they would speed up the process of finding solutions without too much penalty in the score.
  - Each ship would perform all of its drop-offs before performing any pickups
  - There would be no Earth flybys between the first drop-off and the last pickup
- This allowed solutions to be decomposed into two sets of "half-ships" which could be separately optimized to drop-off and pickup from the same set of asteroids and combined at the end. This was crucial to reducing the dimensionality of the problem.



Big Picture


Databases
Ast-Ast ΔV databases (FSR, TERABASE)
Earth-Ast & Ast-Earth ΔV databases (direct, flybys)
Long transfer repositioning database
Cluster approach
Graphical approach
Initial Asteroid Groupings
SABER / SAGA
Sandcrawler
Campaign building
High-fidelity optimization
Refinements (insertion, relocation)
High-fidelity analyzes
Full solution & submission

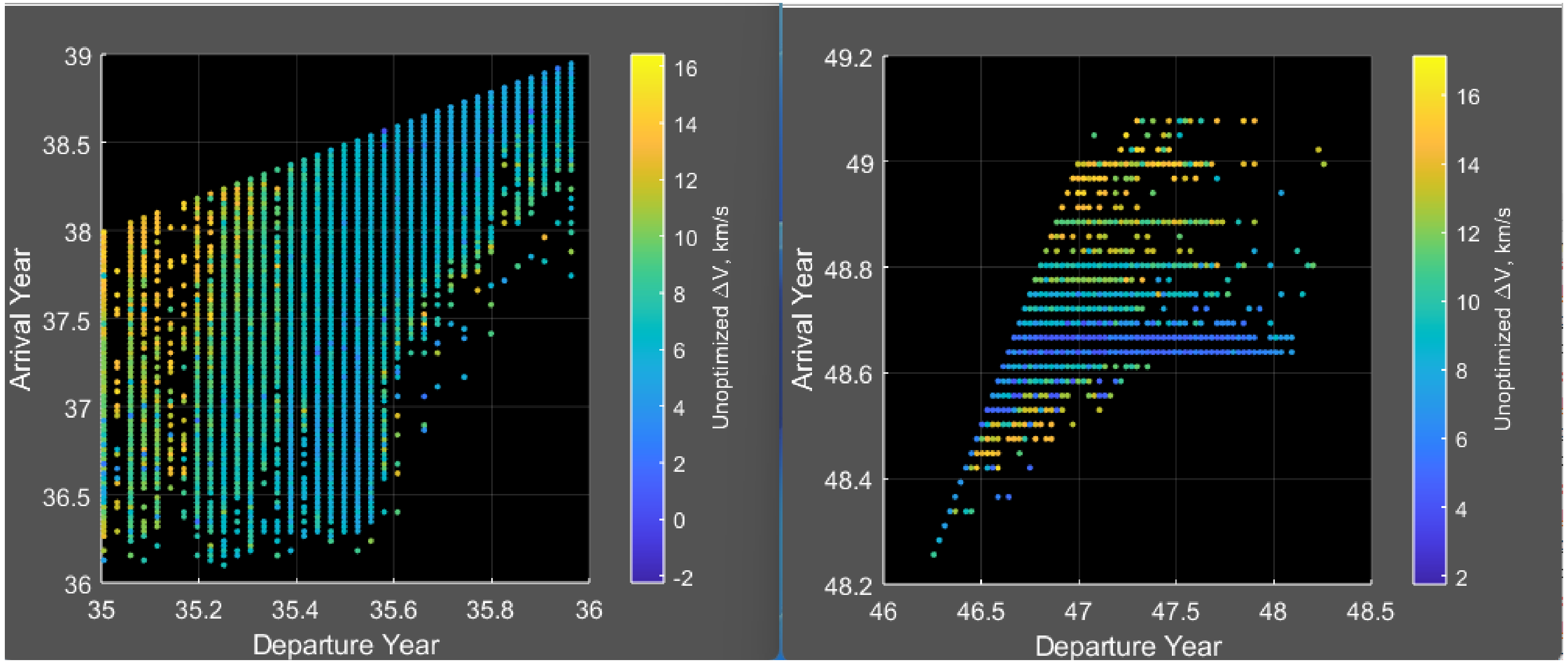
Pre-computed
ΔV databases

Ast-Ast Transfer ΔV databases

- This problem was combinatorially too large to do a full factorial database of 60k * 60k * start time grid * transfer time grid * mass grid with optimized low-thrust transfers!!
- Free space rendezvous (FSR) database
  - Analytic approximation (zero gravity) of any ast-ast low-thrust transfers
  - Can cover arbitrary flight times via interpolation
  - After refinements, the ΔV estimates from FSR were found to be within 10% of the actual optimized transfer cost for most of the cases
- TERABASE: low-thrust Lambert approximation
  - Approximated mass optimal low-thrust transfers from impulsive Lambert solutions
  - Database of ast-ast transfers (after initial filtering) with 30-day grid of starting time and flight time (<1 year)
  - ~100 billion transfers
- Repositioning database for long transfers (>2 years): optimized low-thrust transfers via Tycho (collocation tool) and MALTO

Earth-Ast and Ast-Earth Transfer ΔV databases

- Database of low-thrust transfers between Earth and each asteroid
  - Venus, Earth, Mars flyby sequences were considered
  - STAR combinatorial tool was used to generate impulsive initial guesses for good flyby sequences
  - Tycho collocation tool was used for high-fidelity trajectories



STAR EMX departure (left) and return (right) results

Campaign building methodology

Low/Medium Fidelity Campaign Building

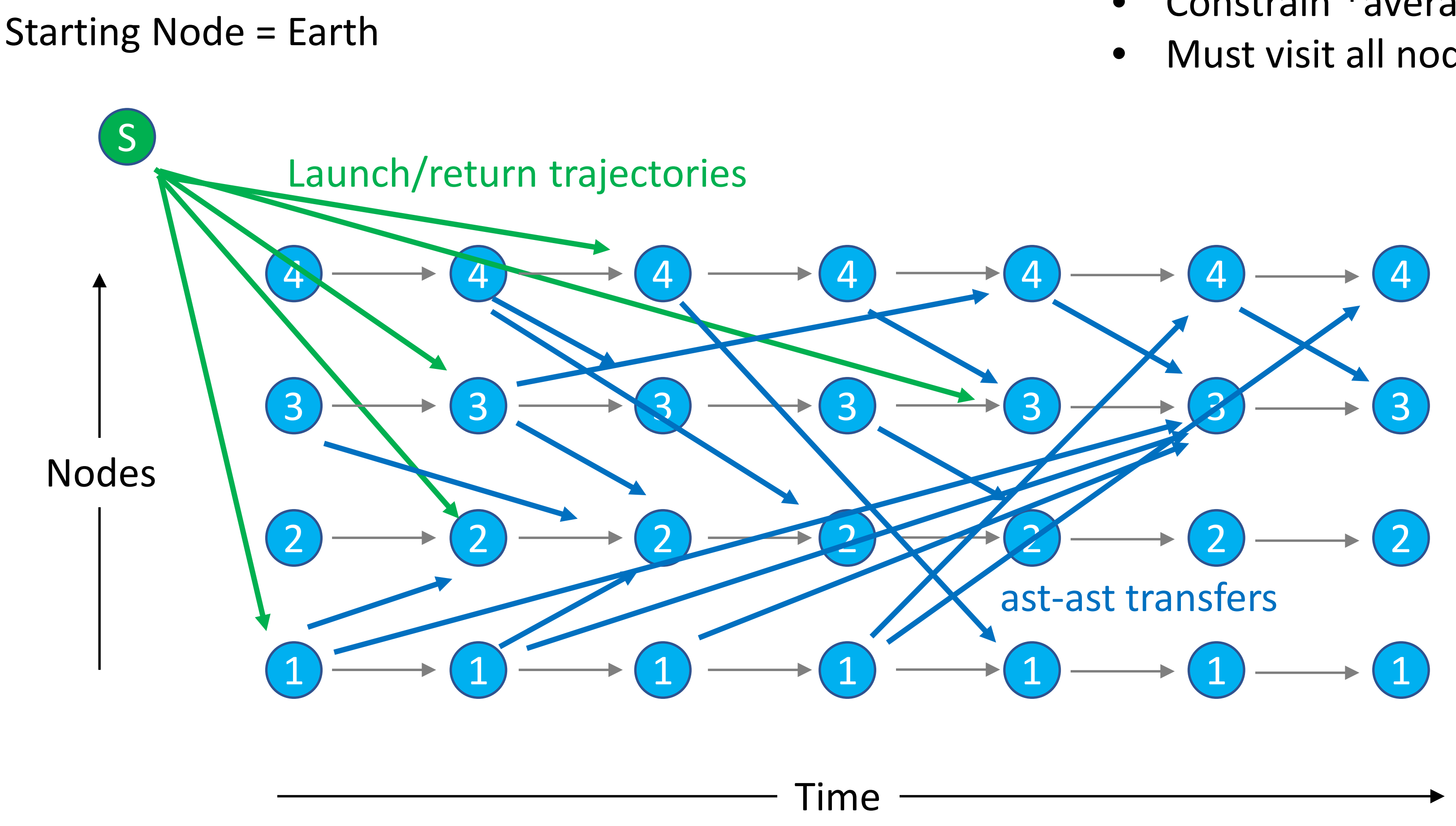
- Two combinatorially gigantic global optimization problems to be solved at the "campaign building" step:
  - How to assign 10-40 ships to visit each of a given subset of asteroids twice
  - How to pick a subset of 100-400 asteroids from the 60k possible asteroids

- Core Problem Construction:
  - Drop-off Vehicle Routing Problem (Earth → Dropoff1 → Dropoff2 → … → DropoffN)
  - Pickup Vehicle Routing Problem (PickupN → PickupN-1 → … → Pickup1 → Earth)
  - Matchmaking drop-off and pickup half-ships via repositioning ΔV

- Vehicle routing problem (VRP) is…
  - **Time Dependent** – edge costs depend on the departure time
  - **Multiple** – there are multiple vehicles serving a set of nodes
  - **Open** – vehicles are not required to return to the starting node (this is handled by matchmaking)
  - **Constrained** – vehicles have a limited ΔV capacity and time available

Vehicle Routing Problem Overview

- Objective function: min arrival time
- Constrain *average* ΔV of all edges
- Must visit all nodes



Low Fidelity Campaign Building (SAGA/SABER)

- Input: subset of nodes (unordered)
- Output: improved subset of nodes and approximate mass per ship
- Steps:
  - 2x VRP solved approximately with unconstrained number of ships, high cost function per ship and low convergence tolerance
  - Mass per ship is averaged from the two half-ships, matchmaking is ignored
  - Iterate to add/remove nodes from the subset and improve mass/ship
    - SAGA = Sandcrawler Approximating Genetic Algorithm
      - Use a simple GA to add/remove nodes with mutation operators
    - SABER = Sandcrawler Approximating Brute-force Expansion and Reduction
      - Look at a set of neighboring nodes (~100s) and evaluate the LP for adding each node. Add the node which improves the cost function the most. After reach a max number of nodes, look at subtracting each node and remove the one which improves the cost function the most. Repeat!

Medium Fidelity Campaigns (SANDCRAWLER)

- Input: subset of nodes (unordered), typically from SAGA/SABER
- Output: full campaign of multiple ships visiting the entire subset of asteroids with approximate transfer times and ΔVs
- Steps:
  - “Coarse” half-ship solutions – solve for number of ships
    - 2x VRP with unconstrained number of ships, high cost function per ship
  - “Fine” half-ship solutions – solve for best assignment of asteroids to halfships
    - Same VRP as above, but with constrained number of ships and zero cost function per ship
  - Matchmaking – assign drop-off halfships to pickup halfships so as to minimize the mass infeasibility of drop-off + repositioning + pickup ΔVs

Asteroid groupings

Campaign Building / Solution Synthesis

- Due to computational constraints of Sandcrawler, we could not optimize more than about 180 asteroids together (sweet spot was 50-100 asteroids)

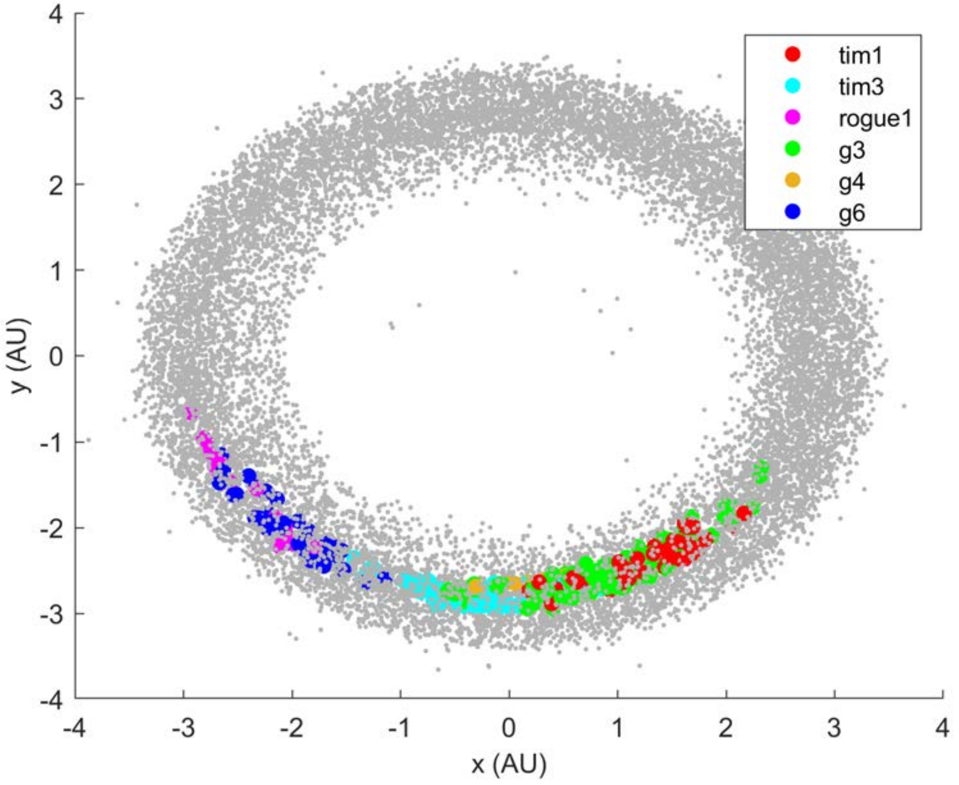
→ subdivide the problem into fully independent “groupings” of asteroids and ships which were optimized separately and concatenated to form a full solution

- We focused development on permutations of ~10 core groupings
- We used 6 groupings in final submission (see table on the right)

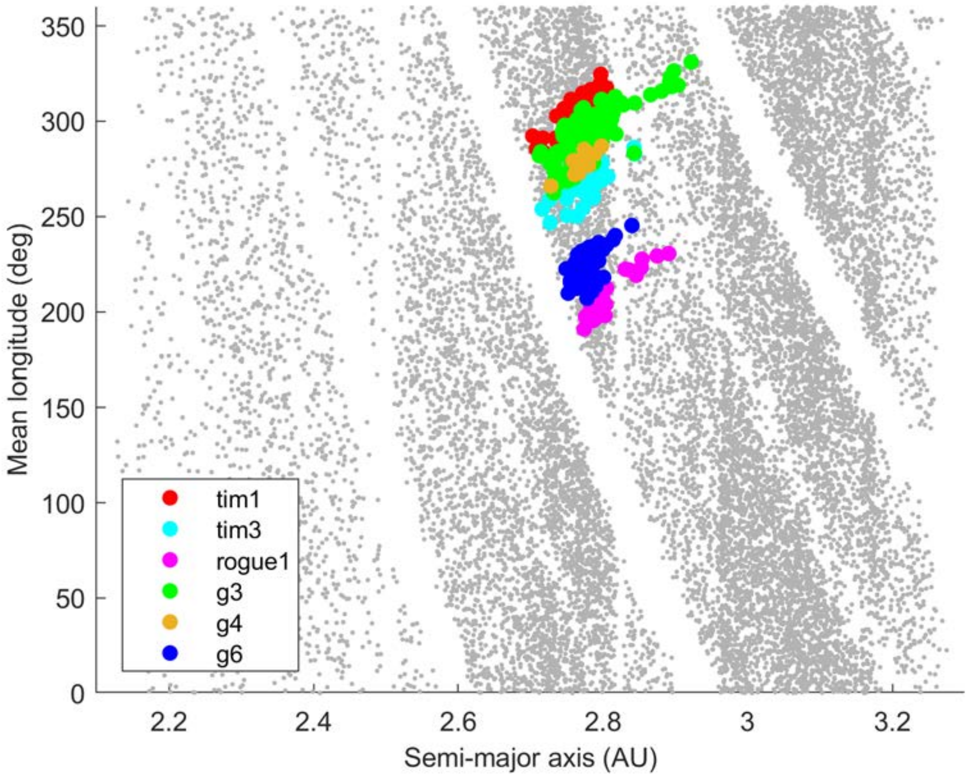
Groupings	# of ships	# of asteroids
g3	14	130
g4	2	17
g6	5	44
rogue1	2	19
tim1	6	51
tim3	6	52

Finding asteroid groupings

- Two main methods were used to find the core asteroid groupings
  - Clusters of mutually-good drop-off nodes (in years 2-6) and pickup nodes (in years 10-13), then groupings are made of asteroids belonging to clusters in both families
  - Graphical "lasso" approach in phase space (bottom right plot)



GTOC12 asteroid position on Jan 1, 2035



GTOC asteroid phase on Jan 1, 2035
(inclination < 15 deg, eccentricity < 0.1)

High-fidelity optimization & refinements

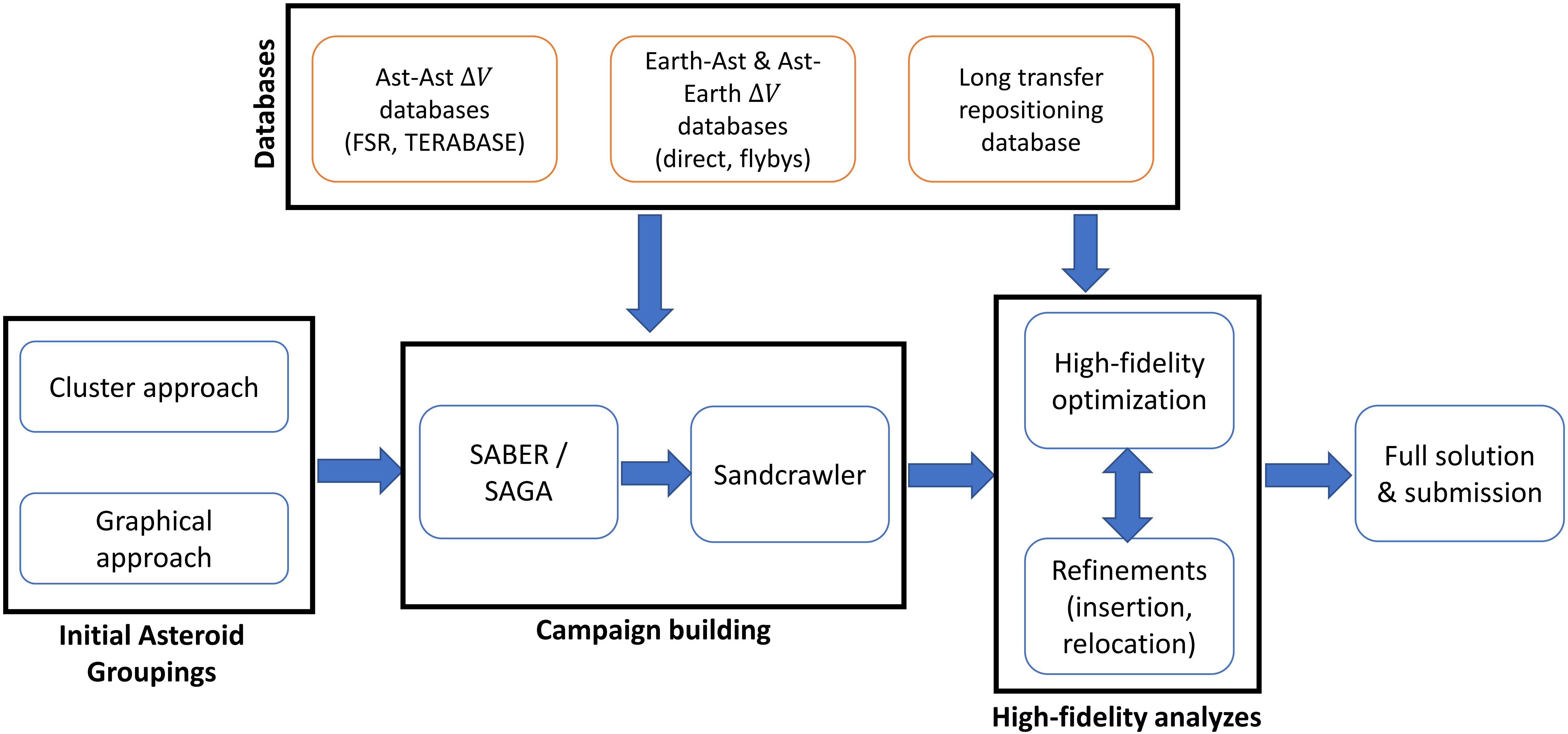
High-fidelity optimization & refinements

- Sandcrawler solutions are then ingested by Gtool high-fidelity optimizer
  - Multiple shooting, nonlinear optimizer with nodes at flybys, drop-off & pickup
  - Piecewise constant thrust modeling
  - Custom objective function: total collected mass
  - Multiple spacecraft, coupled optimization of entire groupings
    - Necessary because drop-off/pickup times are dependent on multiple spacecraft
    - g3 campaign: 14 spacecraft, 25261 optimization variables, 20699 constraints

    → huge nonlinear optimization problem!

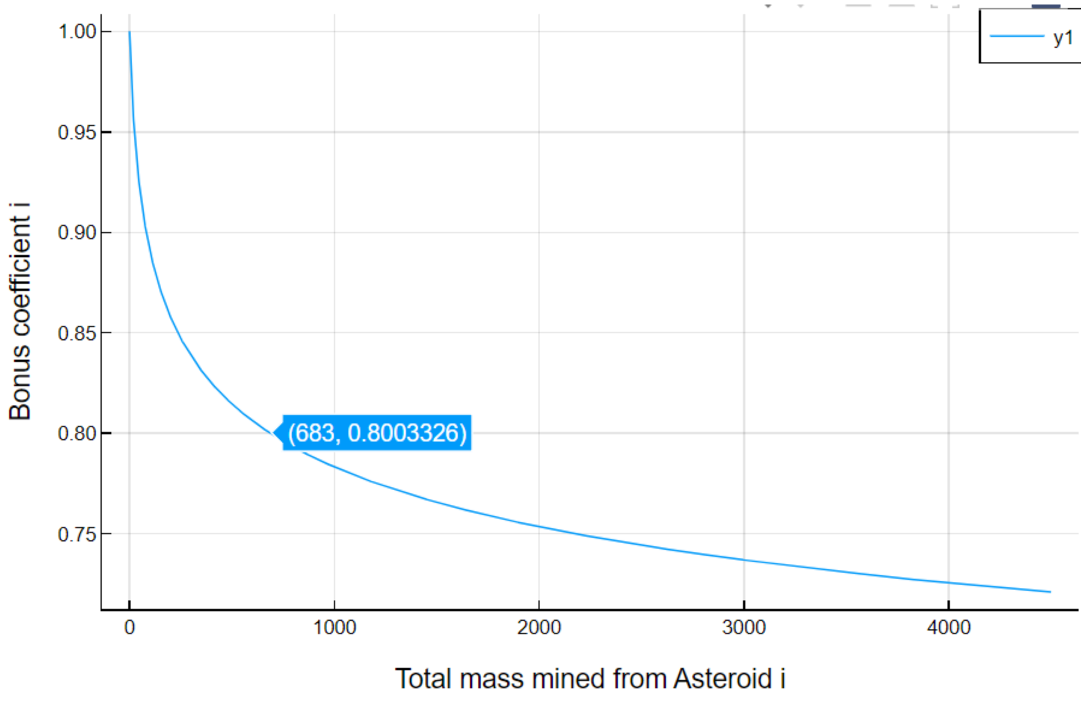
- Genetic algorithm was developed to swap/add asteroids in fully optimized campaigns and improve performance or remove infeasibilities
- Matchmaking was sometimes redone based on optimized repositioning ΔVs

Putting it All Together



Game Theory Strategy

- Submit only one solution, once objective improvements are flattening out (e.g. solution can no longer be improved by more than 20% or so)
  - Submitting multiple solutions could only hurt ourselves, by devaluing our own asteroids
  - This particular aspect led us (and perhaps others) to hide our solutions until the end
  - A test solution was submitted on day 7, to the worst asteroid of the whole GTOC set
- Avoid submitting the solution on the last day, as many teams are likely to submit their final solution
- Only consider bonus coefficient for selecting the campaigns that are integrated in the final solution (like a tiebreaker between different campaigns with similar average collected mass)



Checker Thoughts

- Given the complexity of the problem, the checker was very useful.
- Some tolerances were set too tight, making it very challenging to pass trajectories.
  - Mass tolerance of 1g was too tight compared to position/velocity tolerances
  - Many constraints were checked with tolerances tighter than numerical precision (e.g. 1e-30)
  - In practice, there is no need to be so accurate in a preliminary design

Animation: submitted solution, 35 ships

Post-competition 36-ship solution

- After the competition, a 36-ship solution was found with significantly higher collected mass
- However, even if we had come up with the 36-ship solution within the competition time, it would have had a lower score due to the bonus coefficient penalty (unless we had not submitted the 35-ship solution in the first place)

	35-ship solution (submitted)	36-ship solution (post-competition)
Number of ships	35	36
Number of asteroids	313	320
Number of Mars flybys	0	1 (September 7, 2035)
Total collected mass	25192.7 kg	26062.6 kg
Average collected mass	719.8 kg / ship	724.0 kg / ship
Total weighted collected mass (score)	22532.67 kg (89.44%)	21904.51 kg (84.05%) (bonus coefficients at the end of competition)

Summary

- Multiple fidelity approaches key to moving from global to local optimization in a highly dimensional problem
- Collaborative ships were key to increase collected mass/ship
  - Robust multiple spacecraft, high-fidelity optimization capability was useful
- In addition to clustering algorithms, simple manual/graphical approach was important to find useful groupings
- **Submitted solution**: 35 ships, 719.8 kg/ship, 22532.67 kg (89.44%)
  - Submitted on the penultimate day of the competition, to avoid severe bonus coefficient hit from other team's submissions on the last day
- After the competition ended, the team found 36-ship solution with 724 kg/ship
- Thanks to the GTOC12 organizers for a great GTOC!