Some Methods for Global Trajectory Optimisation

used in the

First ACT Competition on Global Trajectory Optimisation
European Space Agency

Team 11: Jet Propulsion Laboratory
California Institute of Technology
Pasadena, California, USA

Presented by Anastassios E. Petropoulos

ACT Global Trajectory Optimisation Workshop
Noordwijk, The Netherlands, 02 February 2006

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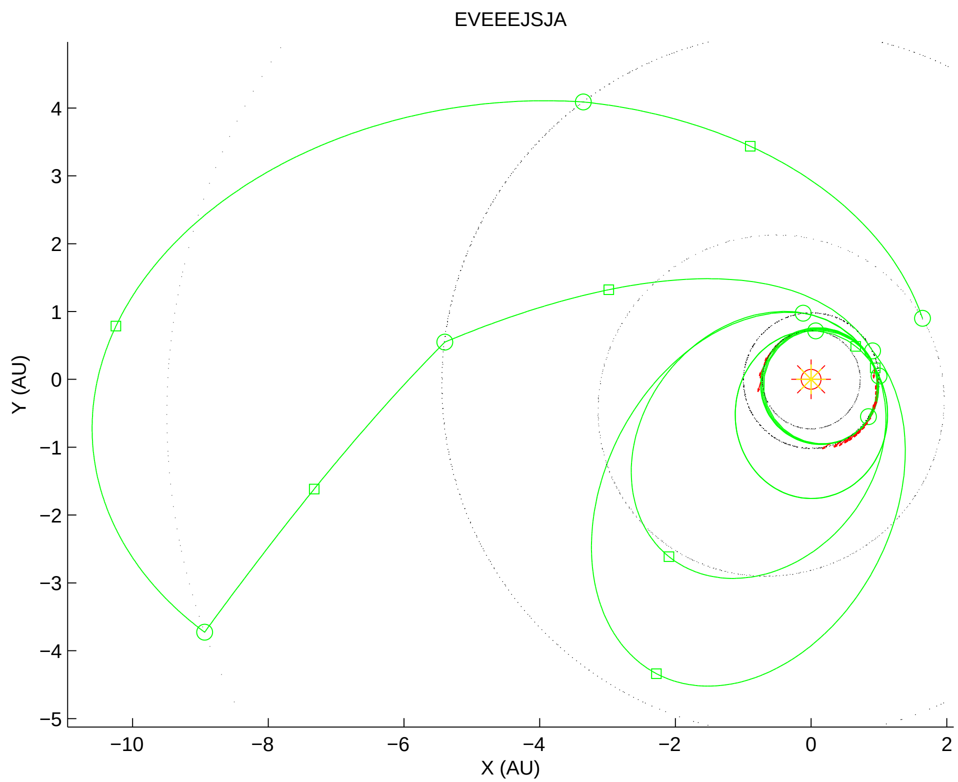


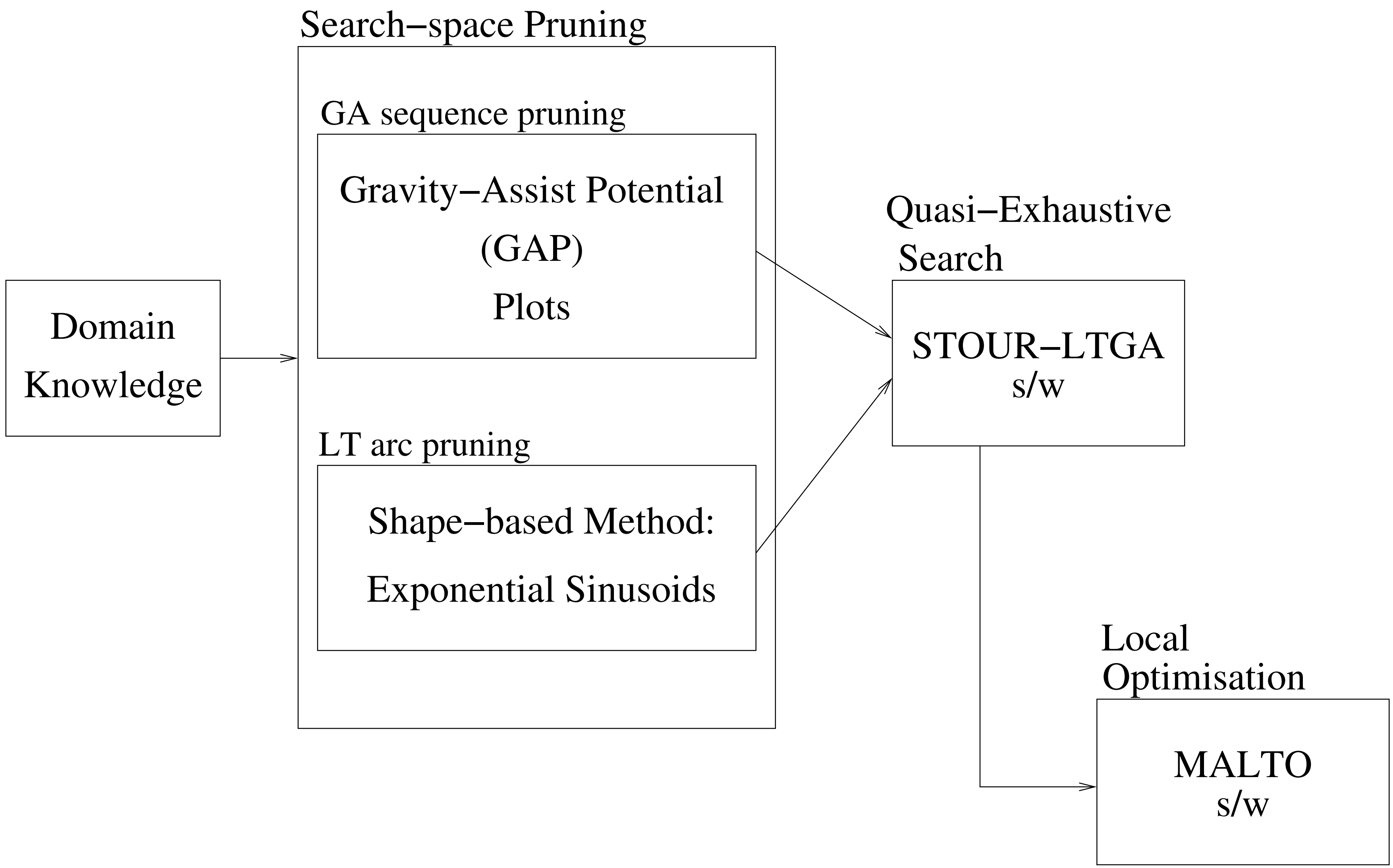
‡ *at workshop*

The Challenge

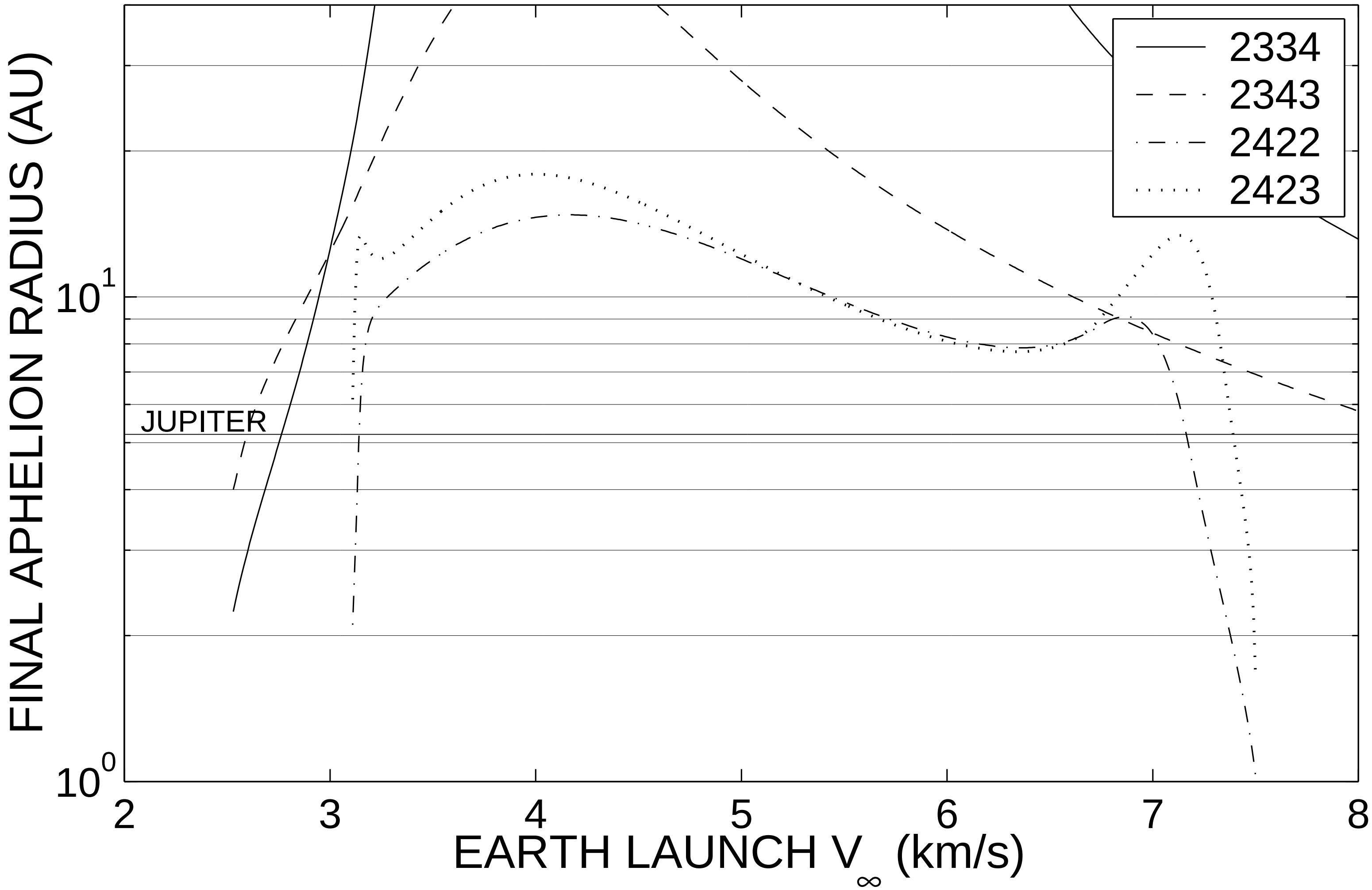
- **Large Search Space**
  - **20-yr launch window**
  - **TOF up to 30 yrs**
  - **5 reasonable flyby bodies for GA**
  - **Low-thrust arcs**
- **Many local optima: How many? 10^3?, 10^6?, 10^9?**

Domain knowledge essential: Even with "good" initial guess, local optimisation can take seconds/minutes/hours of CPU time.

Approach Used to Search for Global Optimum



Gravity-Assist-Potential Plots



J. Spacecraft and Rockets, v.37, no.6, Nov-Dec 2000, "Trajectories to Jupitervia Gravity Assists from Venus, Earth, and Mars," Petropoulos, Longuski, and Bonfiglio.

Gravity-Assist Sequences Considered

- **Inner Solar System (Y,V,E,M): GAP plots and intuition**
- **Outer Solar System (J,S): v_∞ estimates, plots of v_∞-turning, intuition.**

EVEEJSA
EVEEJSJA
EVEESJA
EVEMEJSJA
EVEMEMJSJA
EVEEESA
EVEEEJSA
EVEEESJA
EVEEEJSJA
EVEEJESJA
EVEEJVESA
EVEEJVESJA
EVEMJJA
EVEMJSA
EVEMJSJA

STOUR-LTGA

Satellite Tour design program - Low Thrust, Gravity Assist

Two Parts:

1. **Conic**
   - **Developed at JPL for Galileo tour of Jovian system**
   - **Automated by Steven Williams (except ΔV capability)**
   - **Exhaustively finds all ballistic trajectories**
2. **Low-Thrust**
   - **Developed at Purdue University**
   - **Uses the shape method with exponential sinusoids**

"Automated Design of Multiple Encounter Gravity-Assist Trajectories," Steven N. Williams, M.S. Thesis, Purdue Univ., W. Lafayette, IN, USA, Aug 1990.

"A Shape-Based Approach to Automated, Low-Thrust, Gravity-Assist Trajectory Design," PhD Dissertation, Purdue Univ., W. Lafayette, IN, USA, April 2001.

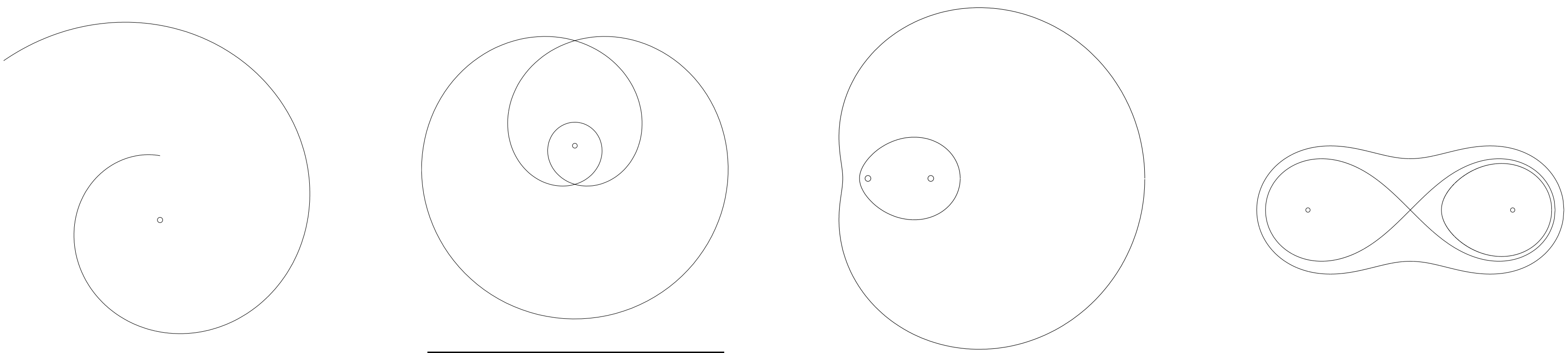
STOUR-LTGA : Conic Part

- **Lambert Problem Solver**
  - *E.R. Lancaster, R.C. Blanchard, and R.A. Devaney, "A Note on Lambert's Theorem," J Spacecraft and Rockets, v.3, no.9, Sept 1966.*
  - *E.R. Lancaster and R.C. Blanchard, "A Unified Form of Lambert's Theorem," Goddard Space Flight Center, Greenbelt, MD, USA, NASA TN D-5368, 1969.*

- **C_3-matching algorithm**
  - **Developed at JPL and refined at Purdue by Williams.**

STOUR-LTGA : Low-Thrust, Assumed-Shape Method

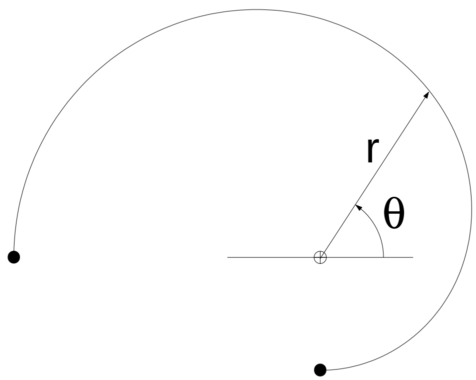
- **Trajectory shape assumption efficiently replaces conics**



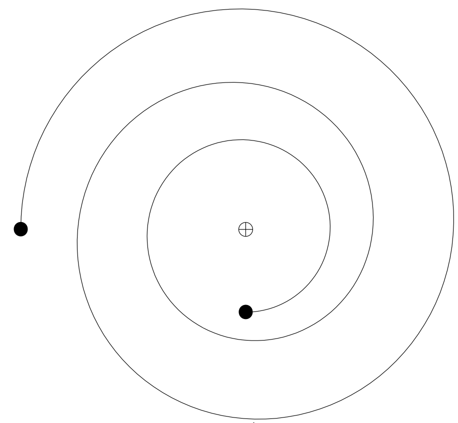
Logarithmic spiral | **Exponential sinusoid** | **Cartesian Oval** | **Cassini Oval**

- **Shape equation + EOMs $\Longrightarrow$ one DOF available**
- **Use DOF to specify, for example, one of:**
  - **Thrust angle**
  - **Thrust acceleration**
  - **Specific angular momentum**

Replace Conic with Exponential Sinusoid Shape



$k_2 = 2/3$



$k_2 = 2/11$

- **Exponential Sinusoid:**

$$r = k_0 \exp(k_1 \sin k_2 \theta)$$

flexibility in geometry with only 3 parameters (k_0, k_1, k_2)

- **Conic:** $r = a(1 - e^2)/(1 + e \cos \theta)$, **has 2 parameters** (a, e)

Tangential Thrust

- **Makes v and g tractable, periodic functions of θ:**

$$\dot{\theta}^2 = f(\theta;\, k_0, k_1, k_2)$$

$$g \equiv \frac{F}{\mu/r^2} = g(\theta;\, k_0, k_1, k_2)$$

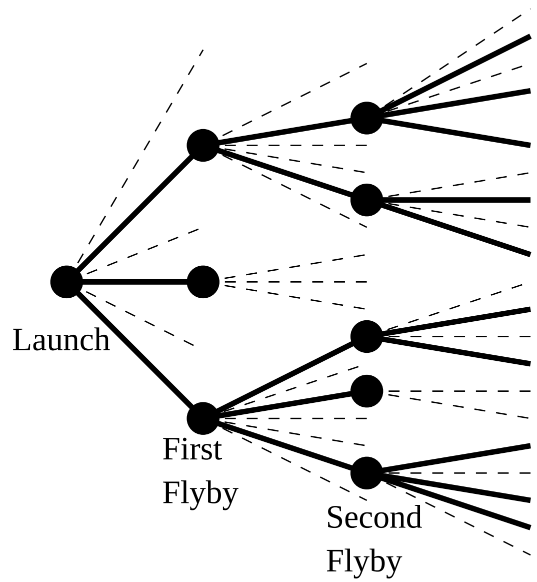
- **TOF available through quadrature**

$$t = \int f^{-1/2}\, \mathrm{d}\theta$$

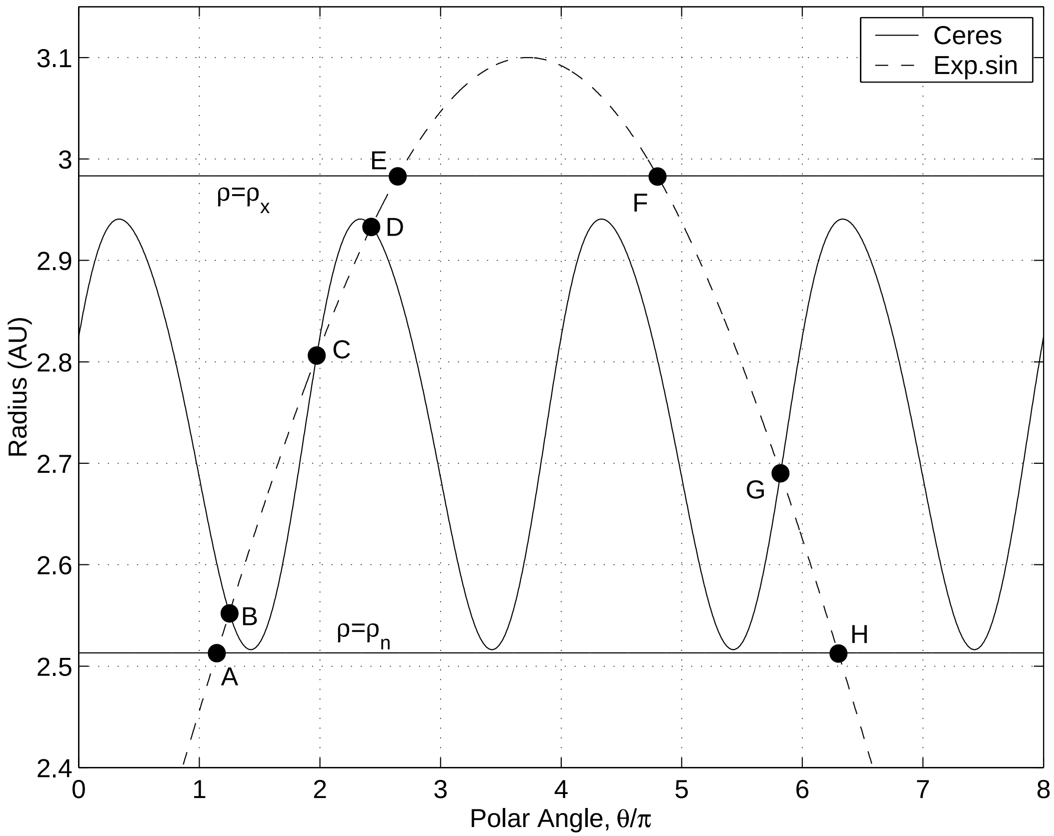
Automation Approach

For each launch v_∞/date:

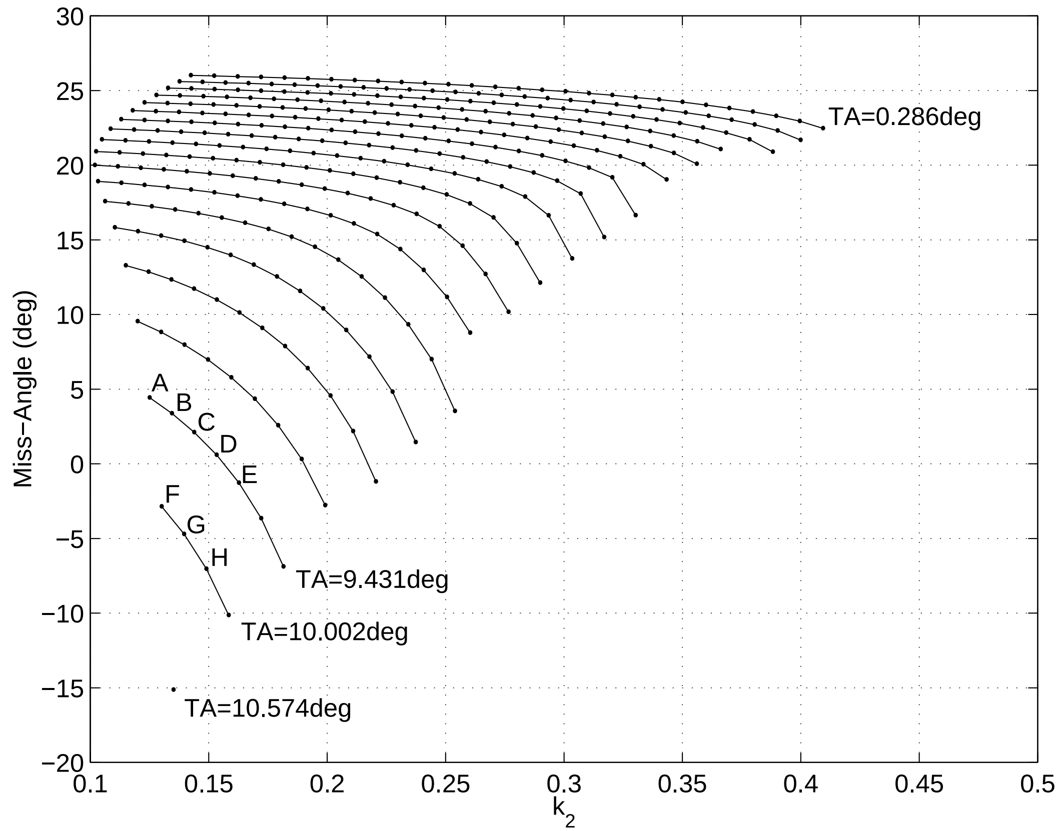
- **Compute exp sin options for first leg**
- **Select options and for each compute options for next leg**



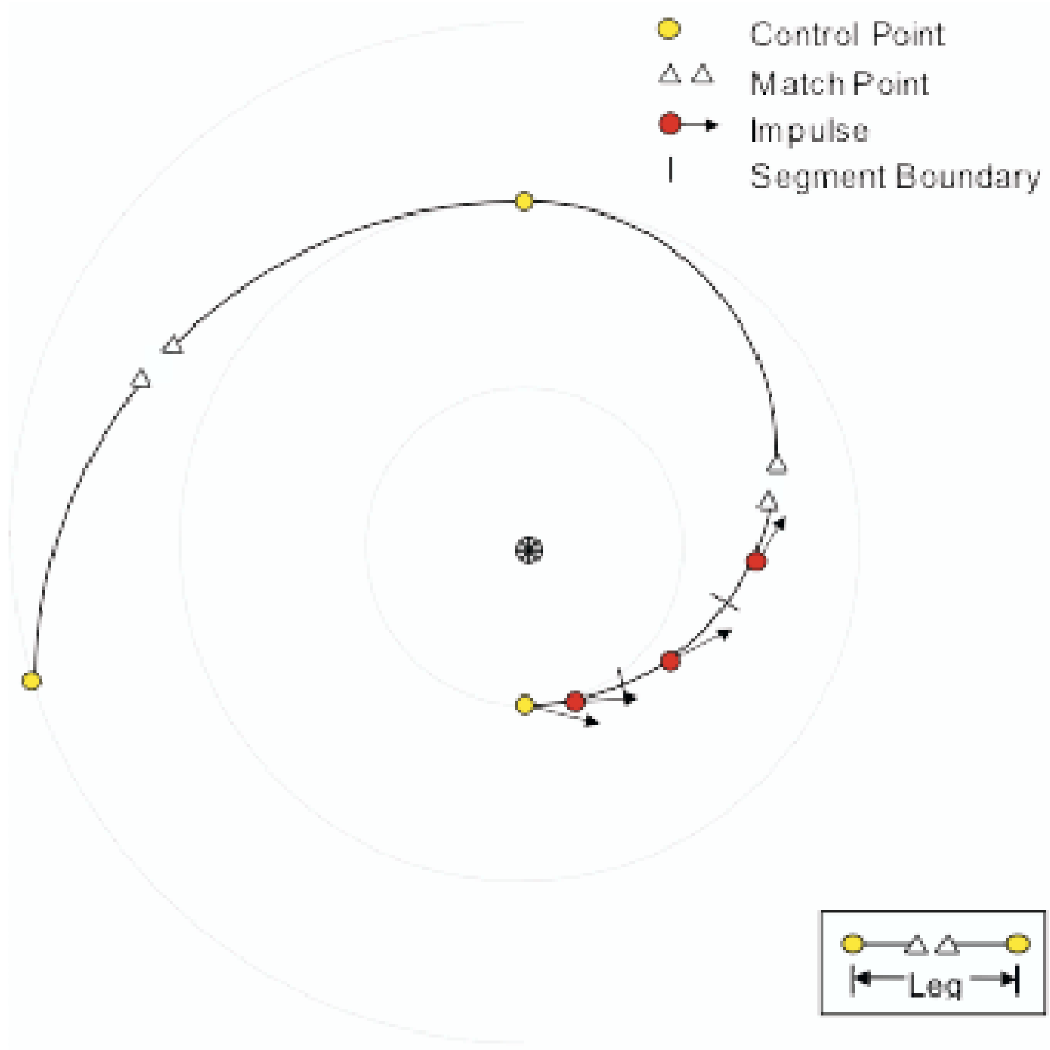
Root-Finding (1) : Orbit Crossing



Root-Finding (2) : Miss-Angle



Local Optimisation : MALTO s/w



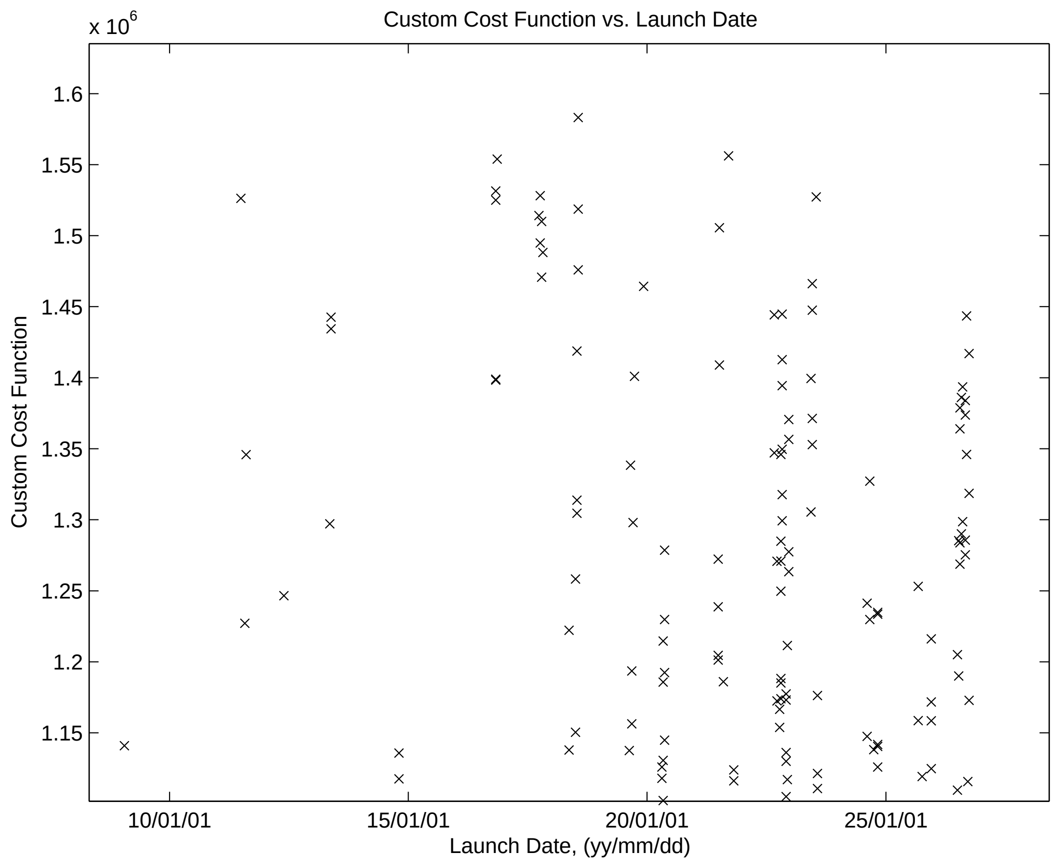
Sims, J. A., and Flanagan, S. N., "Preliminary Design of Low-Thrust Interplanetary Missions," American Astronautical Society, AAS Paper 99-338, AAS/AIAA Astrodynamics Specialists Conference, Girdwood, Alaska, USA, Aug 1999.

MALTO

- **SNOPT optimisation engine**
- **Analytic derivatives supplied**
- **Scaling of variables used**

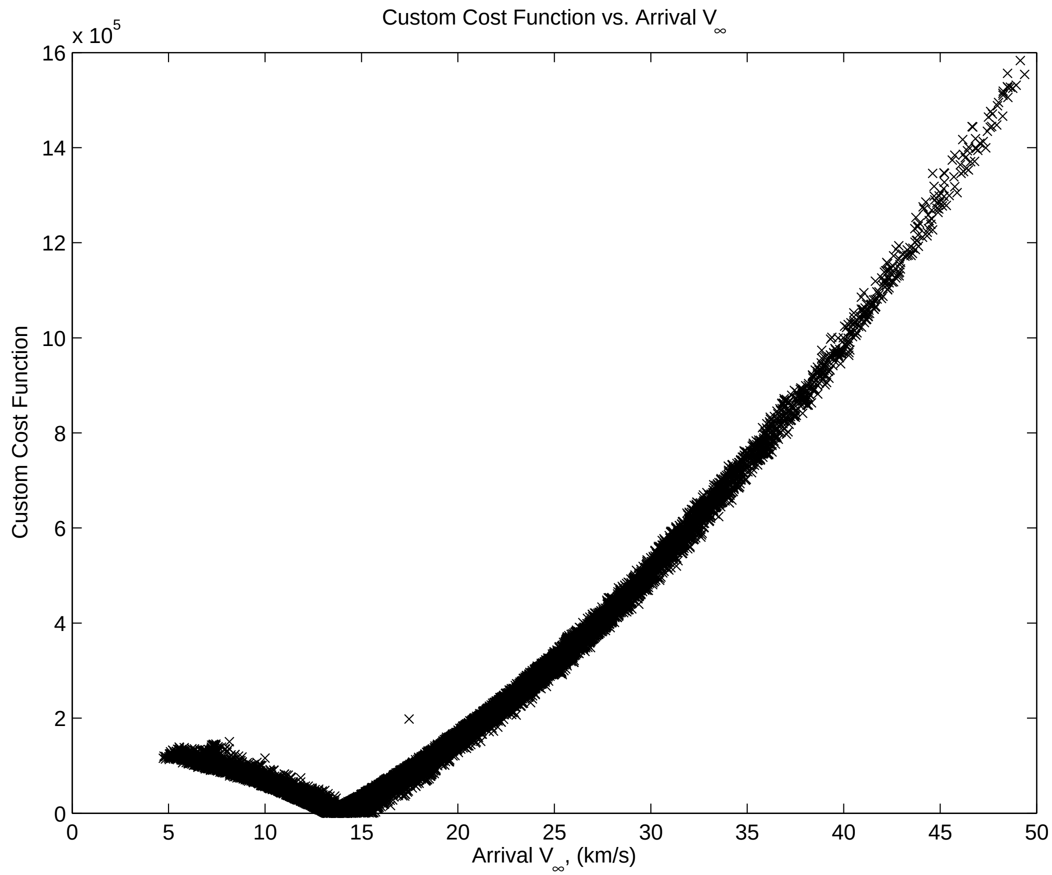
Results : Example of Global Search (1)

EVEEJSA path : Objective Function vs. Launch Date



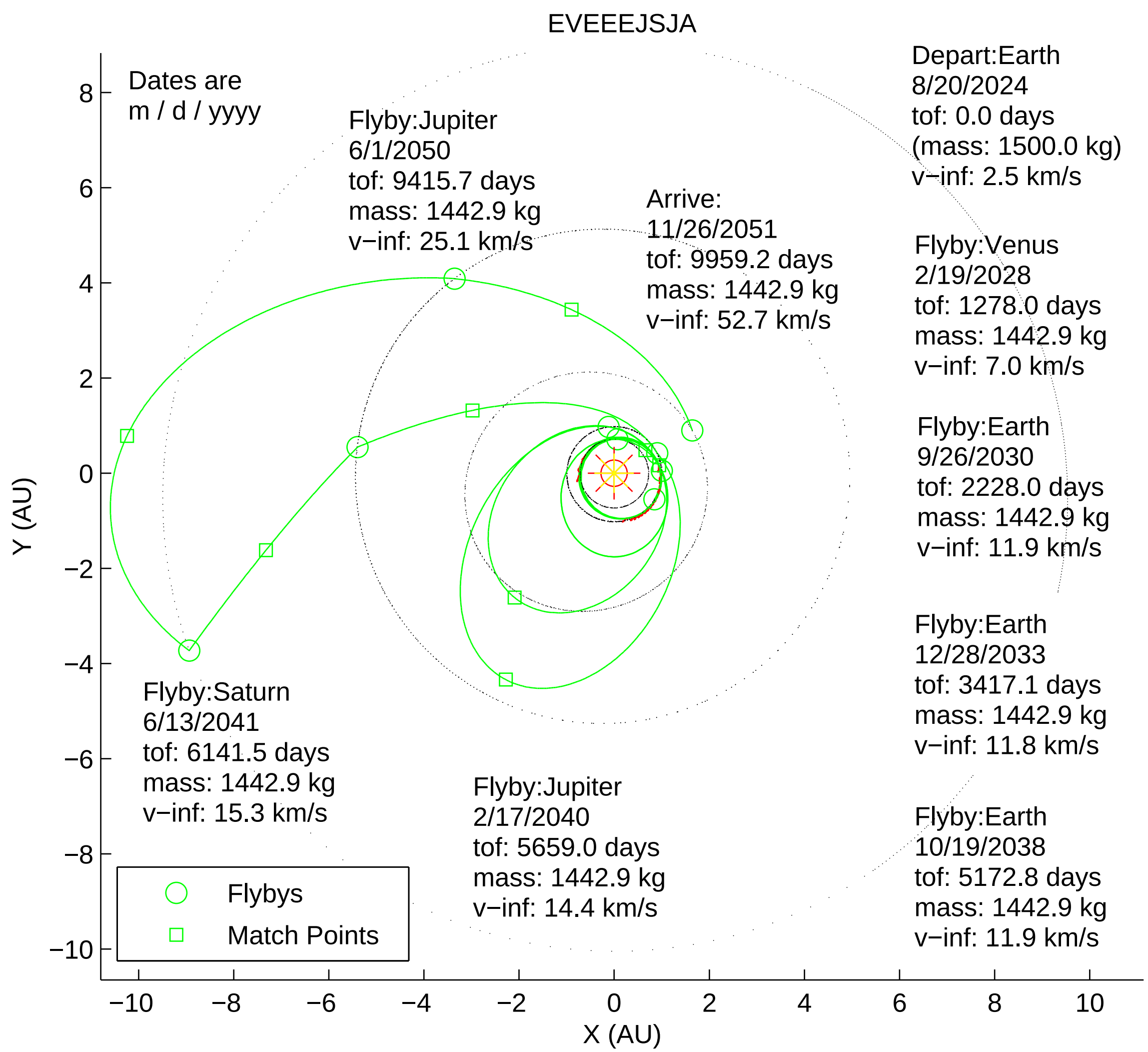
Results : Example of Global Search (2)

EVEEJSA path : Objective Function vs. Arrival v_∞



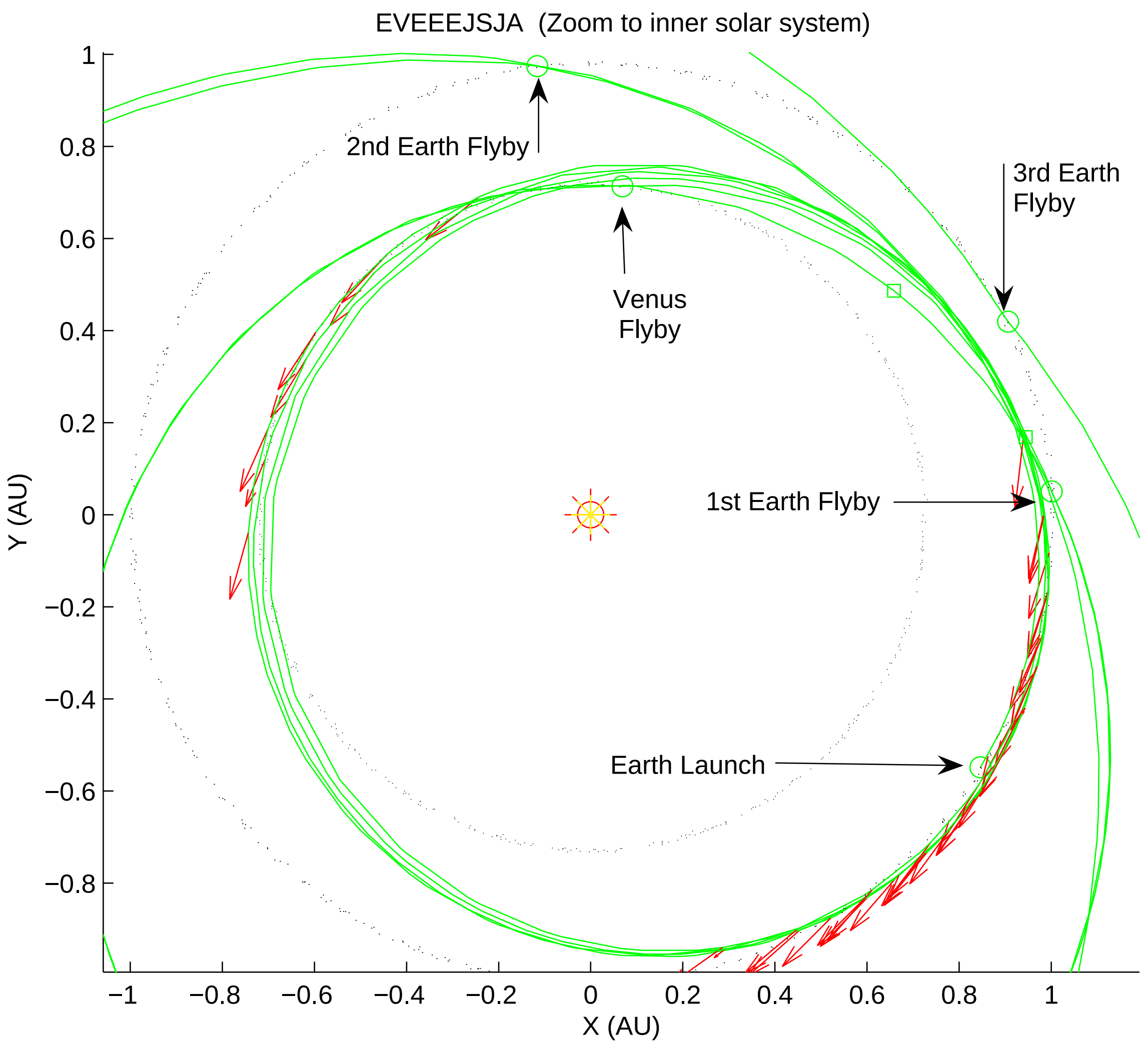
Results : Best Trajectory Found

EVEEEJSJA path



Results : Best Trajectory Found (zoom)

EVEEEJSJA path



Thank You **to the competition organisers**
for the intellectual challenge and
for the invitation to the Workshop!