The Problem of the 11th Global Trajectory Optimization Competition
—*"Dyson Sphere" Building*

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1. Background

"Dyson Sphere", originally proposed by Freeman Dyson in 1960*, is a hypothetical giant artificial structure composed of numerous solar-power satellites. It is designed to surround the Sun such that most energy produced by the Sun can be captured. We suppose that the propulsion technology will be greatly improved in the next century, and a multifunctional asteroid transfer device (ATD) will be invented. The ATD can be deployed on an asteroid to covert its materials into propellants and provide a continuous thrust for it to maneuver in space. With these technological advances, it is expected to build a preliminary "Dyson Sphere", namely the "Dyson Ring", which consists of a number of solar-power stations placed in the same orbit around the Sun, using materials of asteroids for construction. In particular, the plan is to build 12 stations in a "Dyson Ring" orbit, using asteroids selected from over 83000 candidates and transferred to those stations via ATDs. The task in GTOC-11 is to design the "Dyson Ring" orbit, placements of 12 stations in it, and a series of missions to transfer asteroids to these stations while maximizing the transferred asteroids mass and minimizing the cost (propellant) to accomplish these missions.

2. Problem Description

A maximum of 10 Mother Ships are launched from the Earth, each with a hyperbolic excess velocity $\boldsymbol{v}_\infty$ between 0 and 6 km/s and an unconstrained direction. These Mother ships can be operated in parallel. The launch can happen on any date between January 1, 2121 00:00:00 UT (modified Julian date MJD 95739) and January 1, 2141 00:00:00 UT (MJD 103044) inclusive. After departing from the Earth, each Mother Ship can execute impulsive maneuvers and flyby asteroids, with a relative velocity to the flyby asteroid smaller than 2 km/s. A maximum of 4 impulsive velocity changes is allowed during each transfer between two successive asteroid flybys.

At each moment when flying by an asteroid, a Mother Ship will release an ATD, and the ATD will take an instantaneous impulse to rendezvous with the asteroid. This rendezvous process and the corresponding propellant consumption is not of concern in this problem formulation. After that, the ATD can be activated at a proper time to transfer the asteroid to a building station. Once the ATD is activated on the asteroid, it

* Freeman J. Dyson. Search for Artificial Stellar Sources of Infrared Radiation, *Science*, 1960, 131(3414):1667-1668.

cannot be shut down until the asteroid rendezvouses with a station position. The asteroid obtains a fixed magnitude of acceleration (i.e., 1e-4 m/s^2) once the ATD is activated. The ATD is able to generate thrust through in-situ resource utilization of the asteroid and hence does not need to consume extra power to transfer the asteroid.

A total of 12 stations are expected to be built, which are uniformly distributed on a designed circular "Dyson Ring" orbit. Suppose the argument of latitude (i.e., phase) of the first building station is φ_1, then the phases of the remaining 11 building stations can be determined by $\varphi_j = \varphi_1 + 30 \cdot (j-1)$, $j = 2,3,...,12$. An asteroid can be transferred to any building station as long as the time is sufficient. The four orbital elements (i.e., semi-major axis a, inclination i, right ascension of ascending node Ω, and the phase of the first building station φ_1) are freely designed, as long as the semi-major axis a is not smaller than 0.65 AU. In order to avoid conducting different building missions in parallel, the 12 building stations must be built one by one, which means that the asteroids are required to arrive at these building stations group by group, while the building sequence and transfer sequence are not limited. The duration between arrival of the last asteroid of one building station and arrival of the first asteroid of the next building station should be at least 90 days.

Figure 1 illustrates a schematic view of Mother Ship flying by asteroids which are then transferred to the building stations.

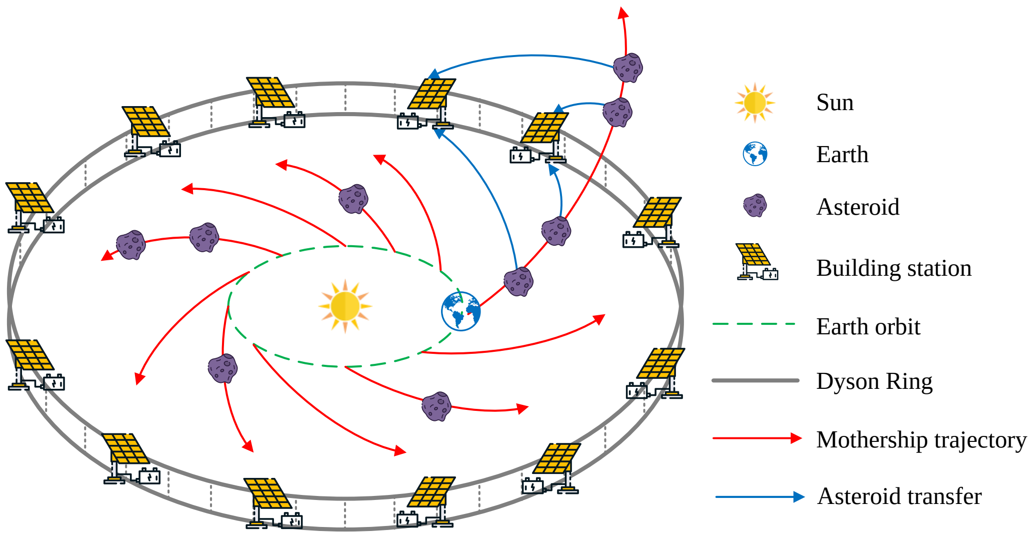

Figure 1 Illustration of the construction of the "Dyson ring".

The mission is limited within 20 years. The performance index of the task is described in Section 4, and the detailed constraints are given in Section 5. Participants are required to select a number of asteroids from the candidate pool and design flyby trajectories for Mother Ships and transfer trajectories for asteroids, so as to maximize the final performance index. More than 83000 asteroids are considered as candidates and their parameters (including ID, epoch (MJD), a (AU), e, i (deg), Ω (deg), ω (deg), M (deg) and mass (kg)) are given in the file Candidate_Asteroids.txt.

3. Dynamic Model and Maneuvers

The motion of the ships and asteroids are both simplified to Keplerian motion. Denote by $\boldsymbol{r}$ and $\boldsymbol{v}$ the position and velocity of a ship (asteroid) in the J2000 heliocentric ecliptic coordinates frame. The dynamic model considered in this problem is as follows:

$$\begin{cases} \dot{\boldsymbol{r}} = \boldsymbol{v} \\ \dot{\boldsymbol{v}} = -\dfrac{\mu}{r^3}\boldsymbol{r} \end{cases} \tag{1}$$

where μ is the gravitational parameter of the sun.

Maneuvers of the Mother Ships are assumed to be impulsive, which means the velocity is changed instantaneously.

Maneuvers of the asteroids (provided by the ATD) are modeled as continuous-thrust maneuvers with a fixed magnitude of acceleration Γ_{ATD} =1e-4 m/s^2. Due to the use of ATD, the mass of the asteroid decreases with a mass flow, which is proportional to the initial mass of the asteroid m_0^{ast}:

$$\dot{m} = \alpha \cdot m_0^{ast} \tag{2}$$

where α = 6e-9 s^{-1} is the proportion coefficient. Then, the remaining mass of the transferred asteroid when arriving at a building station is calculated as:

$$m^{ast}(\Delta t) = m_0^{ast} - \dot{m} \cdot \Delta t \tag{3}$$

where Δt (unit s) refers to the transfer duration from the ATD activation time to the arrival time at the building station.

4. Performance Index

The flyby asteroids are required to transfer to the 12 building stations group by group. Suppose m_{ij}^{ast} is the remaining mass of the i-th asteroid at the j-th building station, the sum of the remaining mass of all the asteroids at the j-th building station is expressed as:

$$M_j = \sum_{i=1}^{n_j} m_{ij}^{ast} \quad j = 1, 2, \ldots, 12 \tag{4}$$

where n_j is the number of the asteroids at the j-th building station. The minimal mass among all building stations is defined as:

$$M_{\min} = \min\{M_j \mid j = 1, 2, \ldots, 12\} \tag{5}$$

Moreover, solutions that are submitted earlier in the submission period will be rewarded an additional bonus. The bonus factor is set as:

$$B = 2 - \sqrt{1 - \left(1 - \frac{t_{submission} - t_{start}}{t_{end} - t_{start}}\right)^3} \tag{6}$$

where t_{start} is the time when the leaderboard starts ranking, $t_{submission}$ is the time when the solution file is received at the website and t_{end} is the end time of the competition.

The task of this competition is to maximize $M_{\min}$ (unit kg) while minimizing the sum of ΔV used by each Mother Ship and the semi-major axis of the designed "Dyson Ring" orbit. Finally, the performance index to be maximized is as follows:

$$J = B \cdot \frac{10^{-10} \cdot M_{\min}}{a_{Dyson}^2 \sum_{k=1}^{10} \left(1 + \Delta V_k^{Total} / 50\right)^2} \tag{7}$$

where a_{Dyson} (unit AU) refers to the semi-major axis of the "Dyson Ring" orbit and ΔV_k^{Total} (unit km/s) refers to the total velocity increments of the *k*-th Mother Ship. Note that the hyperbolic excess velocity $\boldsymbol{v}_\infty$ is not included in ΔV_k^{Total}.

5. Constraints and Parameters

5.1 Time Constraints

- All the mission events (including the departures, maneuvers and flybys of Mother Ships and the maneuvers of ATDs on the asteroids) must take place between 1st January 2121 and 1st January 2141, which means the epoch of any event t_{event} is constrained to $95739 \text{ MJD} \le t_{event} \le 103044 \text{ MJD}$;
- The time interval between arrival of the last asteroid at one building station and arrival of the first asteroid at the next building station cannot be shorter than 90 days: $t_{FirstArrival}^{\mathrm{station}(i+1)} - t_{LastArrival}^{\mathrm{station}(i)} \ge 90 \text{ days}$;
- An ATD must stay on the reached asteroid for a minimum duration of 30 days after releasing before it is activated to transfer the asteroid.

5.2 Position and Velocity Constraints

- The Mother Ship's escaping velocity from the earth cannot exceed 6 km/s: $\|\boldsymbol{v}_\infty\| \le 6 \text{ km/s}$;
- The ATD can be released by the Mother Ship and rendezvous with the asteroid only when their relative distance is smaller than 1 km and their relative speed is smaller than 2 km/s at the flyby time t: $\|\boldsymbol{r}^{mother}(t) - \boldsymbol{r}^{ast}(t)\| \le 1 \text{ km}$, $\|\boldsymbol{v}^{mother}(t) - \boldsymbol{v}^{ast}(t)\| \le 2 \text{ km/s}$.

5.3 Other Constraints

- The asteroid obtains a fixed magnitude of acceleration once the ATD is activated (1e-4 m/s^2). The ATD keeps on until the asteroid is transferred to a building station.
- The semi-major axis of the "Dyson Ring" orbit cannot be smaller than 0.65 AU, and the eccentricity is fixed with 0. Other orbit elements are freely designed.
- Gravity assists are not allowed during the whole flight for both Mother Ships

and asteroids.

- The distance to the Sun of a Mother Ship or an asteroid cannot be smaller than 0.4 AU.
- The same asteroid can only be transferred to a building station once. An ATD can only be deployed on one asteroid and it will be invalid once the asteroid reaches the building station.
- A maximum of 4 impulses is allowed for a Mother Ship during the flight between the Earth and the first flyby asteroid and between any two consecutive flyby asteroids.

5.4 Tolerances

- Errors in position, velocity, and mass are 10 km, 0.01 m/s, and 1 kg, respectively.

5.5 Constants and Conversions

The Earth's orbital elements at the Epoch of $t = 59396$ MJD are given in Table 1. The values of necessary constants and unit conversions are given in Table 2.

Table 1 Earth's orbital elements in the J2000 heliocentric ecliptic reference frame

Elements	Value	Units
a	9.998012770769207e-1	AU
e	1.693309475505424e-2	
i	3.049485258137714e-3	deg
Ω	1.662869706216879e2	deg
ω	2.978214889887391e2	deg
M	1.757352290983351e2	deg
t	59396	MJD

Table 2 Constants and unit conversions

Constant	Value	Units
μ	1.32712440018e11	km^3/s^2
AU	1.49597870691e8	km
Γ_{ATD}	1e-4	m/s^2
α	6e-9	s^{-1}
Day	86400	s
Year	365.25	days
00:00:00 1st January 2121	95739	MJD
00:00:00 1st January 2141	103044	MJD

6. Submission

The website https://gtoc11.nudt.edu.cn/ is used to submit solutions. Only registered users can submit solutions and the score will be calculated automatically to the corresponding team. For each team, the optimal solution among all submitted solutions will be ranked and displayed on the Leaderboard. Besides the best score, some other information, including $M_{\min}$ (minimal mass among 12 building stations), and N (total number of the transferred asteroids) will also be displayed.

The solution file must be an ASCII text file. An example file Ex_Solution.txt is attached on the website. The file format is summarized in Table 3 and described below.

- **Mother Ship**

Basic information of the first Mother Ship, including Mother Ship ID, total number of impulses used by the first Mother Ship and total number of asteroids visited by the first Mother Ship should be given in Line 1. Note that the ID of the *i*-th Mother Ship should be recorded as *i*.

Detailed maneuvers and flybys information of the first Mother Ship, including epochs (MJD), positions (km), velocities (km/s), impulses (km/s) and target ID should be given from Line 2 to Line *m*-1. Note that if the current line is for departing from the Earth, the target ID should be recorded as -1. If for maneuvers, the target ID should be recorded as 0. If for flybys, the target ID is the asteroid ID that is given in the file Candidate_Asteroids.txt. dv_x, dv_y and dv_z should be filled with 0 for flybys.

The basic information, as well as detailed maneuvers and flybys information of other Mother Ships should be given from Line *m* to Line *n*-1. The data formats are the same as the format from Line 1 to Line *m*-1.

- **Building Station**

The four orbit elements of the first building station at the epoch of $t = 95739$ MJD, including semi-major axis (AU), inclination (deg), right ascension of ascending node (deg), and phase (deg), should be given in Line *n*.

- **Asteroid**

The type of continuous-thrust trajectories (0 or 1) should be given in Line *n*+1. If the acceleration direction is fixed in each integration step, the type should be recorded as 0, then the solution will be verified by checking the position and velocity line by line. If the acceleration direction changes all the time, the type should be recorded as 1, then the solution will be verified by checking the whole piece of trajectory.

The basic information of the first transferred asteroid, including asteroid ID, building station ID and total number of following maneuver lines for this asteroid should be given in Line *n*+2. Note that the ID of the *j*-th building station should be recorded as *j*.

The detailed maneuvers information of the first transferred asteroid, including epochs (MJD), positions (km), velocities (km/s), acceleration (m/s^2) and remaining mass (kg) should be given from Line *n*+3 to Line *n*+*p*. Note that the step (time interval between any two consecutive lines) cannot be longer than 1 day.

The basic and detailed maneuvers information of other transferred asteroids should be given after Line *n*+*p*. The data formats are the same as the format from Line *n*+3 to Line *n*+*p*.

Table 3 Solution File format

	Mother Ship ID	Impulses number	Visited asteroids number								
Line 1	1	int	int								
	Epoch	x	y	z	v_x	v_y	v_z	dv_x	dv_y	dv_z	Target ID
Line 2	float	float	float	float	float	float	float	float	float	float	-1
…	…	…	…	…	…	…	…	…	…	…	…
Line m-1	float	float	float	float	float	float	float	float	float	float	int
Line m	2	int	int								
	float	float	float	float	float	float	float	float	float	float	int
	…	…	…	…	…	…	…	…	…	…	…
	float	float	float	float	float	float	float	float	float	float	int
…	…	…	…	…	…	…	…	…	…	…	…
	10	int	int								
	float	float	float	float	float	float	float	float	float	float	int
	…	…	…	…	…	…	…	…	…	…	…
Line n-1	float	float	float	float	float	float	float	float	float	float	int
	a	i	Ω	φ_1							
Line n	float	float	float	float							
	Continuous-thrust Type										
Line n+1	bool										
	Asteroid ID	Building station ID	Following lines number								
Line n+2	int	int	int								
	Epoch	x	y	z	v_x	v_y	v_z	a_x	a_y	a_z	m_f
Line n+3	float	float	float	float	float	float	float	float	float	float	float
…	…	…	…	…	…	…	…	…	…	…	…
Line n+p	float	float	float	float	float	float	float	float	float	float	float
Line n+p+1	int	int	int								
Line n+p+2	float	float	float	float	float	float	float	float	float	float	float
…	…	…	…	…	…	…	…	…	…	…	…