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GTOC5: Results from the Jet Propulsion Laboratory

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Abstract. We present the methods and results of the Jet Propulsion Laboratory team in the 5$^{\text{th}}$ Global Trajectory Optimization Competition. Our broad-search strategy utilized several recently developed phase-free metrics for rapidly narrowing the search options. Two different, adaptive, branch-and-prune strategies were employed to build up asteroid sequences using a rendezvous-flyby-rendezvous building block, with a robust local optimizer in the loop. The best of these sequences were refined end-to-end using the same direct optimizer, to yield the winning 18-point, 18-asteroid solution.

1 Introduction

Several features of the GTOC5 problem make it uniquely challenging. In addition to the customary large timespan permitted for launch, there is a large number of asteroids (7075) to choose from, which makes the problem combinatorially challenging. Furthermore, a second combinatorial layer arises from the freedom to visit an arbitrary number of asteroids before returning to an asteroid for the impactor part.

Given the vast option space, it is necessary to use very rapid metrics for conducting a broad search. The broad search was conducted primarily using a branch-and-prune approach, where trajectories were built up chronologically by adding on new legs to trajectories in an initial set of single-leg trajectories. The question of which asteroids to consider for the new legs was addressed using phase-free metrics of the difficulty to reach a candidate asteroid's orbit. Using a local optimizer, MALTO, legs to each of the candidate asteroids were optimized individually for a combination of flight time and propellant usage. In the final step, the trajectories were optimized end-to-end, and then refined "by hand."

The final optimization included looking at shifting the flybys to occur at different parts of the trajectory in order to decrease the propellant mass consumed. In this manner, a trajectory was found which performed a rendezvous and subsequent flyby with eighteen asteroids, yielding an eighteen-point trajectory whose flight time was minimized by using all the available propellant as expeditiously as possible.

2 Broad Search

The primary broad search method was a chronological tree search in the branch-and-prune style. We did also briefly consider using a graph search over asteroids at discrete times, however it would have been difficult to incorporate the flyby requirement into our graph-search strategies. In the chronological tree search, it

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Table 1. *List of the eight lowest flight time Earth-asteroid legs, optimized for a weighted sum on flight time and propellant mass.*

AstID	TOF (days)	C3 km^2/s^2	M_f (kg)	Date of Launch (JD)
1712	53.9	6.3	3952.5	2459127.7064
5029	66.5	21.0	3941.4	2460657.0970
6259	67.2	25.0	3940.8	2460176.5848
5264	74.5	9.3	3934.4	2457396.4443
5778	76.2	21.6	3932.8	2458090.7222
5159	81.1	17.2	3928.5	2460853.9146
6112	89.7	13.1	3921.0	2460971.4692
4374	99.0	25.0	3912.8	2458107.7060

was quickly decided that a sufficiently simple, yet adequately representative, building block after launch from Earth would be a rendezvous-flyby-rendezvous (RFR) sequence, that is, flying by an asteroid (to drop the impactor) right after completing the rendezvous with that same asteroid and before proceeding to a rendezvous with the next asteroid. We termed this a return depth of zero for the flyby asteroid. Having a return depth of one or more, *i.e.* performing a rendezvous of one or more asteroids between the asteroid's rendezvous and flyby, was deemed too difficult to study in any depth, and so was relegated as an option to be used during the final solution refinement.

The first step, then, in the chronological tree search was to determine a good set of initial legs, from Earth to asteroid rendezvous, upon which RFR blocks could be added. This problem was sufficiently small as to be done exhaustively using the optimizer MALTO (described below). All possible launch dates and flight times (using a discretized grid) and all asteroids were considered. The optimization objective was to minimize a linear combination of propellant mass consumed on the leg and leg flight time. The weighting in the combination was chosen so that the estimated expiration of the flight time and propellant mass would be simultaneous once all the building blocks had been added on.

The Earth-asteroid (EA) legs were sorted by increasing flight time. The top eight legs are shown in Table 1. For the branch-and-prune process, up to the top two hundred legs were considered for the next stage. It is worth noting that some initial experiments suggested that the extra points allocated in the problem statement to asteroid Beletskij were not worth the added propellant and flight time needed to undertake the excursion to such a large semi-major axis and eccentricity, either at the start of the trajectory or at the end. Hence, asteroid Beletskij was not explicitly added either to the candidate EA legs or to the subsequent RFR sequences.

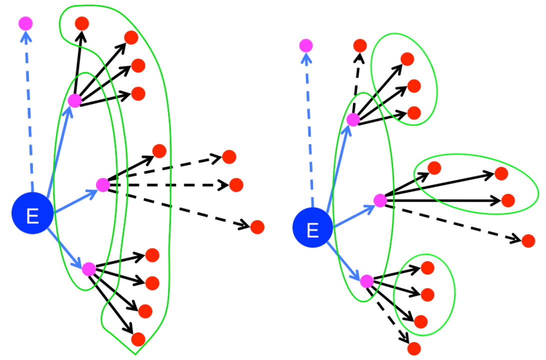

Figure 1. *Global- and local-rank pruning, left and right, respectively.*

Now the question arises of how to select the second asteroid for each of the EA legs in order to build up the trajectory to a sequence of Earth - asteroid 1 rendezvous - asteroid 1 flyby - asteroid 2 rendezvous. The candidate second asteroids were taken as the ones "closest" to the orbit of the first asteroid, as measured by the candidate Lyapunov function Q [1]. Q, called the proximity quotient, is also the basis of the Q-law feedback algorithm for phase-free, low-thrust orbit transfers — it is a highly non-linear, scaled function that may be thought of as the square of the "best-case" time-to-go. Other metrics of closeness were briefly considered, including a binning based on classical orbital elements, and a function of the angular momentum and eccentricity vectors. This set of candidate second asteroids, typically containing 600-1200 asteroids depending on the current stage in the trajectory build up, would be passed on to MALTO to optimize the candidate RFR blocks, again with a tuned linear combination of flight time and mass.

Once a set of optimized RFR blocks was obtained for appending to each trajectory, two different pruning approaches were taken before proceeding to tack on the next RFR block. Pruning was necessary because there were far too many optimized RFR blocks to carry forward. The two pruning strategies are depicted pictorially in Figure 1. In the "global-rank pruning," the trajectories with their newly added RFR blocks were ranked based on propellant mass and flight time; the top 100 or 200 new trajectories would be carried forward. In "local-rank pruning", the top few RFR block additions for each trajectory would be taken, up to a maximum of 100 or 200 new trajectories. In both cases, various di-

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versity preservation methods were used, for example, in local-rank pruning, only up to ten RFRs were permitted for each trajectory, so no single trajectory could hog the limelight in the next generation.

The pruning factors, diversity preservation strategies and the weighting factors in the optimisation were all changed adaptively after each generation (*i.e.*, RFR addition). This was not done automatically, but by making careful projections of how much longer the flight time and propellant mass might last and by examining the diversity characteristics of each generation before pruning.

3 Local Optimisation

The local optimization program, MALTO, also used in previous GTOCs, is based on the algorithm formulated by Sims and Flanagan [2]. In this formulation, the low-thrust arcs are modelled as a series of impulsive velocity increments (ΔVs). Each leg (*i.e.*, from one flyby body or control point to the next one in time) is split into a number of segments, as shown in Fig. 4. Targeting is done by means of a match point, which occurs after a specified number of segments from the first control point. The trajectory is propagated forward in time from the first control point to the match point, and backwards in time from the next control point to the match point. Trajectory propagation is conic, except that at the temporal midpoint of the segments a discontinuity in the velocity (ΔV) is allowed. The maximum magnitude of the ΔV is equal to the available thrust acceleration multiplied by the duration of the segment. From the rocket equation, the mass drop corresponding to the ΔV is computed. When a sufficient number of segments is used (for near-circular orbits, 20 to 30 segments per revolution is normally sufficient), this formulation provides an excellent approximation to an actual low-thrust arc. Other constraints, such as flyby altitude constraints, are applied as needed.

The optimisation variables are the magnitude and direction of the impulses; the launch, flyby and arrival times; the incoming and outgoing v_∞ vectors at the flyby bodies; the outgoing/incoming v_∞ vectors at the launch/arrival body; and also the initial spacecraft mass (for the outside chance that the benefit of increased thrust acceleration could outweigh the penalty of reduced mass; initial mass would be reduced by dumping some propellant instantaneously). The optimisation engine used is SNOPT [3], which is based on the sequential quadratic programming method. SNOPT finds locally optimal solutions which satisfy the non-linear constraints. Appropriate scaling is used for the variables and analytic derivatives are used.

While the replacement of the low-thrust arcs with a series of impulsive ΔVs is a reasonable approximation, the question arises of how to obtain a smoothly varying state for the spacecraft which adequately represents the motion the spacecraft would have under low thrust. The approach taken here is simple. Essentially, a weighted average state is computed for intermediate times on any given segment. The known spacecraft state at the start of the segment is propagated forward conically up to the end time of the segment, and the known spacecraft state at the end of the segment is propagated conically backwards up to the start time of the segment. At any interior time point on the segment, a linearly weighted average is taken of the states at that time point as taken from the forward-propagated state history and the backwards-propagated state history. The direction of the thrust vector is taken as inertially fixed for the duration of the segment and equal to the direction of the impulsive ΔV on that segment. The magnitude of the thrust is taken as that magnitude which yields a characteristic velocity (from the rocket equation) equal to the impulsive ΔV. At higher acceleration levels, this approximation becomes less accurate. To overcome this effect, more segments can be used. For the submitted solution, around 100 segments were used per revolution at the end of the trajectory when the thrust acceleration had gotten substantially higher.

4 Results

The global-rank pruning and the local-rank pruning were yielding comparable results up to about the 15-asteroid mark, that is, 15 consecutive rendezvous-flyby-rendezvous blocks. Beyond that, the local-rank pruning method started out-pacing the global-rank method. Both approaches reached 17 asteroids, however, only the trajectories from the local-rank method had sufficient flight time and propellant left to reach an 18^{th} asteroid. All of the 18-asteroid solutions we obtained happened to share the same itinerary up to the 17^{th} asteroid, indicating that the number of possible 18-asteroid solutions is probably rather limited, and far smaller than the number of 17- asteroid possibilities. Table 3 lists the first asteroids in all of the trajectories we found with 16 or more asteroids.

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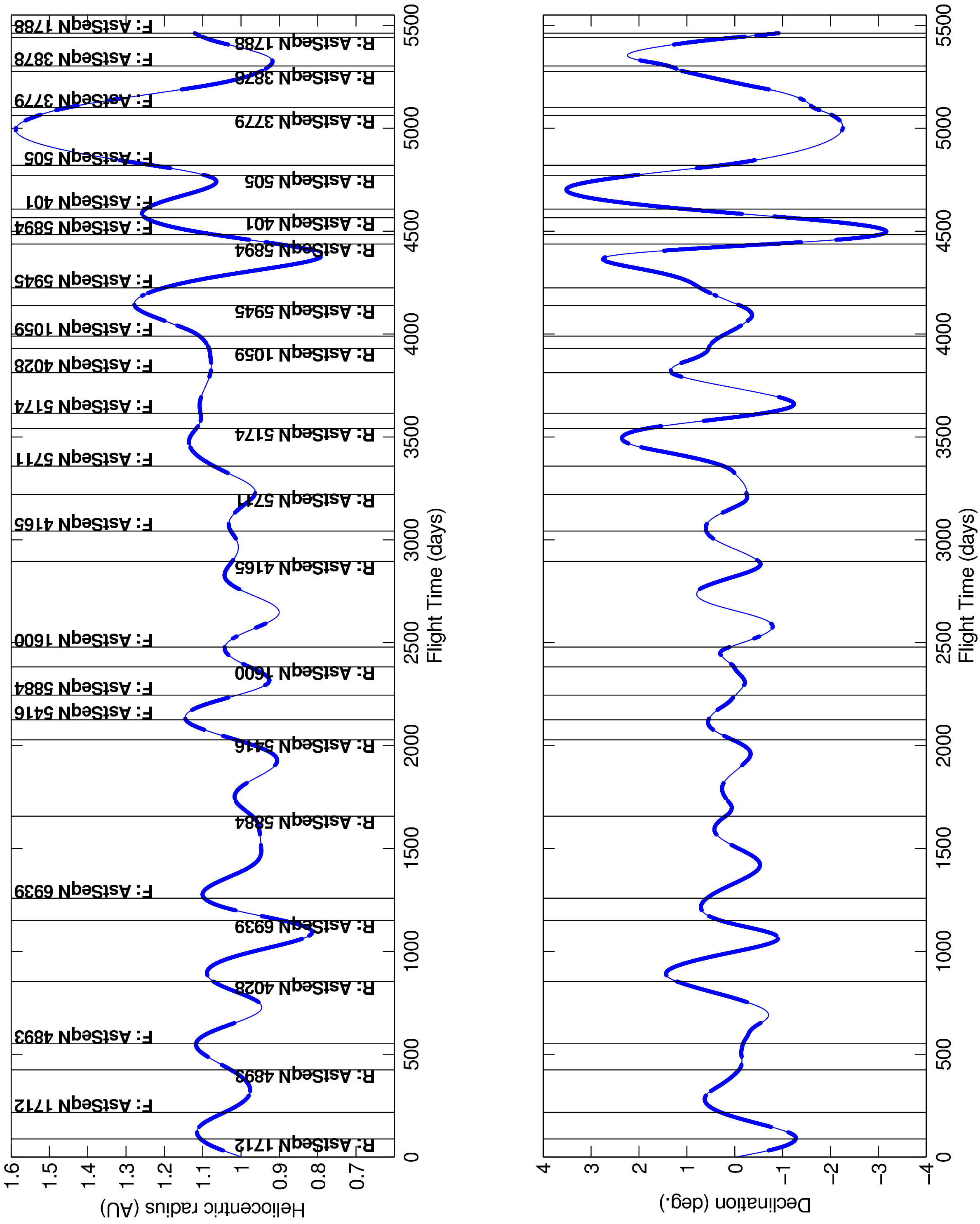

Figure 2. *Submitted, 18-point solution, J2000 heliocentric ecliptic frame.*

Table 2. *Submitted, 18-point solution.*

Enc #	AstSeqN	GTOC5 name	Encounter Date (yyyy.mm.dd)	TOF from launch (days)	TOF from last Enc (days)	s/c Mass before drop (kg)	Drop mass (kg)	v_∞ (km/s)
1	-	Earth	2020.10.15	0.00	0.00	4000.00	0.00	3.231
2	1712	(2001 GP2)	2021.01.12	88.84	88.84	3951.33	40.00	0.000
3	1712	(2001 GP2)	2021.05.22	218.29	129.45	3823.08	1.00	0.400
4	4893	(2007 UN12)	2021.12.13	424.02	205.73	3670.91	40.00	0.000
5	4893	(2007 UN12)	2022.04.20	551.35	127.33	3553.32	1.00	0.400
6	4028	(2006 JY26)	2023.02.16	853.72	302.38	3405.66	40.00	0.000
7	6939	(2010 JR34)	2023.12.10	1150.21	296.49	3146.84	40.00	0.000
8	6939	(2010 JR34)	2024.03.27	1258.97	108.76	3035.81	1.00	0.400
9	5884	(2009 BD)	2025.04.30	1657.39	398.42	2744.99	40.00	0.000
10	5416	(2008 JL24)	2026.05.06	2028.32	370.93	2533.23	40.01	0.000
11	5416	(2008 JL24)	2026.08.11	2125.37	97.05	2435.77	1.00	0.400
12	5884	(2009 BD)	2026.12.09	2245.41	120.05	2402.63	1.00	1.972
13	1600	(2000 SG344)	2027.04.25	2382.60	137.19	2323.82	40.00	0.000
14	1600	(2000 SG344)	2027.07.30	2478.87	96.27	2239.36	1.00	0.400
15	4165	(2006 RH120)	2028.09.18	2894.91	416.03	2151.60	40.00	0.000
16	4165	(2006 RH120)	2029.02.13	3042.33	147.42	2079.32	1.00	0.400
17	5711	(2008 UA202)	2029.08.11	3221.29	178.96	2016.40	40.00	0.000
18	5711	(2008 UA202)	2029.12.26	3358.63	137.34	1945.74	1.00	0.400
19	5174	(2008 CX118)	2030.06.27	3541.30	182.67	1824.74	40.00	0.000
20	5174	(2008 CX118)	2030.09.08	3614.81	73.51	1746.25	1.00	0.400
21	4028	(2006 JY26)	2031.03.25	3812.19	197.38	1675.47	1.00	1.794
22	1059	(1993 HD)	2031.07.20	3929.83	117.64	1616.19	40.00	0.000
23	1059	(1993 HD)	2031.09.18	3989.53	59.70	1536.86	1.00	0.400
24	5945	(2009 CV)	2032.02.14	4138.55	149.02	1438.30	40.00	0.000
25	5945	(2009 CV)	2032.05.10	4224.29	85.75	1373.16	1.00	0.400
26	5894	(2009 BK2)	2032.12.08	4437.11	212.82	1214.14	40.00	0.000
27	5894	(2009 BK2)	2033.01.24	4483.52	46.41	1145.08	1.00	0.400
28	401	(2001 CC21)	2033.04.15	4565.03	81.51	1072.27	40.00	0.000
29	401	(2001 CC21)	2033.05.27	4606.91	41.88	1007.93	1.00	0.400
30	505	(2001 WC47)	2033.11.08	4771.64	164.73	867.08	40.00	0.000
31	505	(2001 WC47)	2033.12.27	4820.55	48.91	812.03	1.00	0.400
32	3779	(2005 YA37)	2034.08.25	5061.75	241.20	782.74	40.00	0.000
33	3779	(2005 YA37)	2034.10.04	5101.33	39.58	728.31	1.00	0.400
34	3878	(2006 BZ147)	2035.03.27	5275.62	174.28	651.01	40.00	0.000
35	3878	(2006 BZ147)	2035.04.23	5302.33	26.71	597.32	1.00	0.400
36	1788	(2001 QJ142)	2035.09.09	5441.56	139.23	553.70	40.00	0.000
37	1788	(2001 QJ142)	2035.09.29	5461.82	20.27	501.00	1.00	0.400

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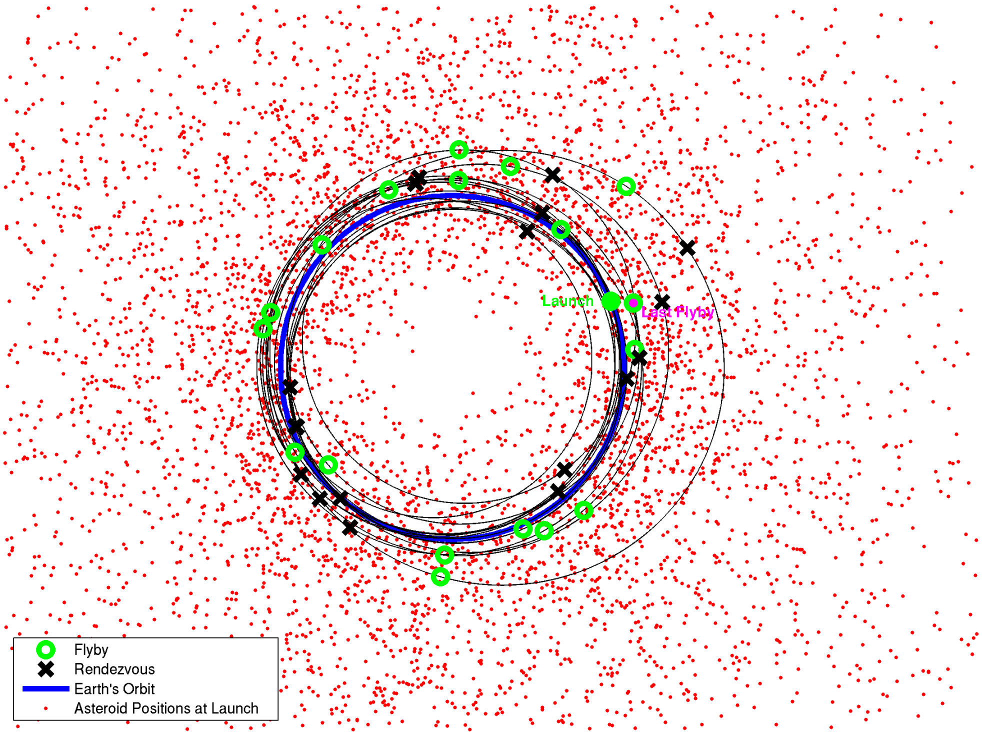

FIGURE 3. *Submitted, 18-point solution, J2000 heliocentric ecliptic projection.*

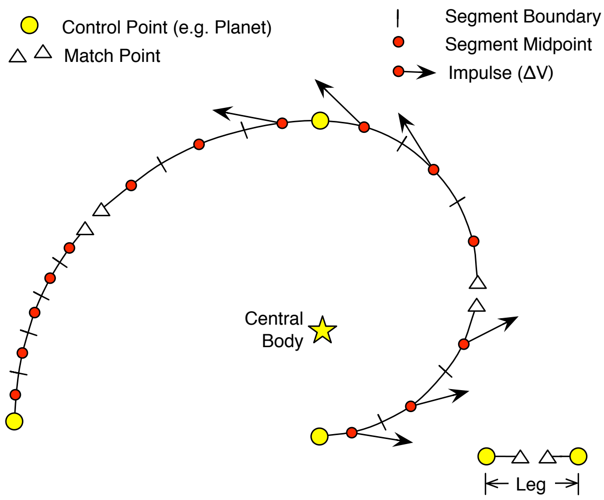

FIGURE 4. *Trajectory Modeling in Local Optimizer, MALTO (after Sims and Flanagan [2])*

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Table 3. *List of first asteroids visited for solutions found with 16 or more asteroids*

AstID	AstName	a (AU)	e	i (deg.)	ω (deg.)	Ω (deg.)	Max J
1043	(1991 VG)	1.027	0.049	1.45	24.5	74.0	16
1712	(2001 GP2)	1.038	0.074	1.28	111.3	196.9	18
2679	(2003 WT153)	0.894	0.178	0.37	148.5	56.0	16
5249	(2008 EA9)	1.059	0.080	0.42	335.9	129.4	17
5416	(2008 JL24)	1.038	0.107	0.55	281.9	225.8	17
5884	(2009 BD)	1.002	0.047	0.38	100.2	53.1	16
5945	(2009 CV)	1.112	0.150	0.96	178.9	24.1	17
6112	(2009 HC)	1.039	0.126	3.78	269.8	203.8	16
6979	(2010 KV7)	1.215	0.219	0.31	36.9	255.9	16

4.1 Submitted solution

The best of the 18-asteroid solutions was further refined systematically. First, we attempted to add a rendezvous of a new asteroid, anywhere along the trajectory, by looking for close, low-speed approaches with any of the asteroids that were not already on the itinerary. Neither sufficient propellant nor sufficient flight time remained to realize this option. Second, we tried to minimize the flight time by trying to beneficially increase the return depth for some asteroids and by consuming all the available propellant. The return-depth increase was found by looking for close approaches at later times. Two increases in return depth were made: Asteroid 4028 went to a return depth of 7, and asteroid 5884 to 1.

The final, submitted 18-asteroid solution, since it performs a rendezvous and subsequent flyby with each asteroid, has a performance index of $J = 18$. The flight time is 5461.82 days. The thruster is "on" for 3134.95 days, and the final spacecraft mass is 500 kg (after the impactor mass of 1 kg is released at the final asteroid). Our submitted solution is shown in Table 2 and in Figures 3 and 2.

5 Acknowledgements

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6 Conclusions

Having a good local optimizer was essential in our effort, both in terms of speed and of robustness. Also, the Q-law screening metric was indispensable for trimming down the number of candidates passed to the optimizer. The local-rank-pruning strategy appeared considerably better than the global-rank-pruning strategy, however, this may simply have been due to not preserving enough diversity in the latter. Lastly, the adaptive tuning of the optimizer weightings and branch-and-prune parameters was found to be very important as trajectories were built up.

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