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GTOC5: Results from the Politecnico di Torino and Università di Roma Sapienza

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Abstract. Problems that concern a large number of possible targets, such as those typical of Global Trajectory Optimization Competitions (GTOC), require a global search for the optimal solution and a local refinement. During the 5th edition of the Global Trajectory Optimization Competition, we used different approximated techniques to perform a global search of suitable asteroid sequences; a local optimization method based on an indirect approach was then used to verify the feasibility of the suggested legs and perform the optimization of multi-leg sequences. All the tools used by our team are presented in this paper. The best solution found is described and discussed.

1 Introduction

The problem proposed by GTOC5 organizers is quite challenging. Relevant trajectories encounter a great number of asteroids (18 in the best solution) selected from a very large list (7075 elements). The number of possible sequences is incredibly huge and it is necessary to individuate candidate solutions on the basis of a rough evaluation of the maneuvers that efficiently move the spacecraft from an asteroid to the following one. This approximate process aims at locating the global optimum region in the solution space.

The globally-optimal maneuver will be very close to running out of time and propellant. In particular, according to GTOC5 rules, more asteroids are reached, less propellant is available for the whole mission, due to the mass that is discharged at each rendezvous/flyby. The global search must be backed by a refinement, which improves the trajectory looking for the local optimum that spares either time or propellant (hopefully both). The local optimization is periodically carried out during the global search, in order to verify the feasibility of the suggested legs and to assess the spacecraft mass, which is quite important to define the available thrust-acceleration.

Validation of the most promising legs and improvement of assessed sequences were carried out by means of an indirect optimization method, based on the theory of optimal control. The time limits for the overall mission make a fast transfer (between an asteroid and the following one) really attractive. On the other hand, the available propellant is scarce, in particular when many asteroids appear in the sequence; mass can be saved at the expense of time, by introducing coast arcs. An optimal balance must be sought, and, in building the trajectory, both minimum-fuel legs (with fixed time of flight)

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and minimum-time legs were evaluated, according to the event sequence suggested by the global search methods.

Legs were periodically joined to perform a multi-leg optimization (up to five legs were considered) and determine the optimal intermediate times. The joined optimization of multi-leg segments is very important, as the previous propellant consumption determines spacecraft mass and available thrust-acceleration during the following legs. In comparison to the result of optimizing single legs, a greater propellant consumption is usually preferred in the initial legs, and coast arcs are pushed towards the final part of the segment. In this way, solutions up to 17 asteroids were found. On the competition last day, the fastest trajectory intercepting 17 asteroids has been searched for, in order to improve the secondary performance index.

2 Global Search and Asteroid Selection

A wise choice of the target asteroids and a control law, which guarantees a good balance between propellant consumption and flight time, are required to reach a large number of asteroids. Even though the latter task is mandatory, the former one is a necessary preliminary step; the decision on the next target is a complex problem as it requires, as in a chess match, to foresee some of the following targets in order to avoid to enter a dead end. Only some "global" or "synthetic" features of each transfer (i.e., estimated duration Δt and propulsive cost ΔV) are necessary to try a prediction of suitable sequences of asteroids. The global search is hence focused on finding a way to produce good estimates for possible maneuvers and on assembling sequences of maneuvers that could be useful to the maximization of the merit index.

2.1 Edelbaum approximation and sequential strategy

A way to estimate the cost of a maneuver is based on an elaboration of Edelbaum's formulas [1], which, under suitable hypotheses, permit to calculate the required ΔV to transfer a spacecraft between two orbits (subscripts 1 and 2); the flyby cost is neglected. The following relationship

$$\Delta V = \sqrt{(k_a \Delta a)^2 + (k_e \Delta e)^2 + (k_i \Delta i)^2} \tag{1}$$

considers the efforts to change the spacecraft energy (expressed by the semi-major axis change Δa), to modify shape and orientation of the ellipse (Δe), and to rotate the orbit plane (Δi); they are summed up taking into account the benefit of combining the three maneuvers. One has

$$\Delta a = a_2 - a_1 \tag{2}$$

$$\Delta e = \sqrt{(e_{p2} - e_{p1})^2 + (e_{q2} - e_{q1})^2} \tag{3}$$

$$\Delta i = \sqrt{(h_{x2} - h_{x1})^2 + (h_{y2} - h_{y1})^2 + (h_{z2} - h_{z1})^2} \tag{4}$$

where components of eccentricity vector in the perifocal system and direction cosines of angular momentum in the heliocentric ecliptic system are used

$$e_p = e \cos(\Omega + \omega) \tag{5}$$

$$e_q = e \sin(\Omega + \omega) \tag{6}$$

$$h_x = \sin i \sin \Omega \tag{7}$$

$$h_y = -\sin i \cos \Omega \tag{8}$$

$$h_z = \cos i \tag{9}$$

The cost coefficients

$$k_a = \frac{V_0}{2a_0} \tag{10}$$

$$k_e = 0.649 V_0 \tag{11}$$

$$k_i = \frac{\pi}{2} V_0 \tag{12}$$

are obtained by averaging on a complete revolution the differential equations that relate the variations of orbital parameters to the propulsive effort, while using mean values for the spacecraft orbit

$$a_0 = (a_1 + a_2)/2 \tag{13}$$

$$V_0 = \sqrt{\mu / a_0} \tag{14}$$

The phase of the two asteroids is not directly taken into account, even though it has a major impact on the actual ΔV. However, given a departure date t_i and the estimated ΔV, the associated flight time Δt can be easily calculated from the propellant mass flow rate $\dot{m}_p$:

$$\Delta t = \Delta m / \dot{m}_p + \Delta t_{fb} \tag{15}$$

where the first term is related to the estimated ΔV

$$\Delta m = m_i \left(1 - e^{-\Delta V / c}\right) \tag{16}$$

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and the second term is a time penalty $\Delta t_{fb} = 2m_i$ which considers the time necessary to perform the flyby; this relationship between non-dimensional variables (see section 3) was empirically obtained from several test cases. The penalty diminishes with the spacecraft mass to reflect the thrust-acceleration increase. Note that flyby influence on ΔV is instead neglected.

The final position of the spacecraft is estimated as $\vartheta_f = \vartheta_1 + \dot{\vartheta}_0 \Delta t$, assuming a constant angular velocity $\dot{\vartheta}_0 = \sqrt{\mu/a_0^3}$. It corresponds to the desired position of the target asteroid, which is compared to its actual position at t_f; the difference is a measure of the correct phasing between the relevant asteroids.

This procedure to estimate transfer cost and asteroid phasing constitutes the core of the sequential strategy: assigned an asteroid, the time of its rendezvous is assumed as initial time; transfers to all the others asteroids, sorted on the basis of the estimated ΔV, according to Edelbaum's formulas, are analyzed. The maneuvers that have the target asteroid well phased, are optimized to verify their feasibility and actual cost, which depends also on the phase characteristics. A tree is created by propagating the trajectory up to five rendezvous by means of the same technique. Predictions are quite accurate for transfers with up to three rendezvous and become less reliable for longer sequences. Unfortunately the number of analyzed branches is limited by the computation capabilities and the necessity of verifying each leg. More branches are added only if this limited search does not produce interesting continuation.

This way of facing the global search is probably the most intuitive; however, it has some drawbacks: since the phase analysis depends on the initial time, in some cases a departure delay could avoid the discharge of an interesting branch. Moreover, less promising maneuvers are not examined, but their high cost might be offset by a very favorable continuation.

2.2 Chain strategy

The chain strategy assumes that the transfer between two asteroids is carried out in proximity of the most favorable position; all the opportunities that present adequate phasing are found in the whole allowed time domain. The features of each transfer, comprising estimated departure and arrival times, are recorded and constitute a catalog of elementary legs. Tentative chains are then created by connecting legs that present acceptable gaps (or overlaps) between estimated arrival and departure times.

The choice of the best position for the maneuver is the crucial point; experience acquired in GTOC2 suggests that a good transfer occurs where the asteroid orbits are close to tangency; therefore the nominal point of the maneuver is arbitrarily located at the orbit intersections (if any) or at the point of minimum distance, making reference to the orbit projections on the ecliptic plane. If the minimum distance overcomes an assigned value, maneuvers between the two asteroids are not considered worth of attention.

The cost in terms of ΔV is evaluated on the basis of an impulsive maneuver

$$\Delta V = \Delta V_{ip} + \Delta V_{op} \tag{17}$$

which is sum of the in-plane maneuver

$$\Delta V_{ip} = \sqrt{(V_{r2} - V_{r1})^2 + (V_{t2} - V_{t1})^2} \tag{18}$$

and the out-of-plane maneuver

$$\Delta V_{op} = \Delta i (V_{t1} + V_{t2})/2 \tag{19}$$

with radial and tangential velocity components given by

$$V_r = \sqrt{\mu/p}\; e \sin\nu \tag{20}$$

$$V_t = \sqrt{\mu/p}(1 + e\cos\nu) \tag{21}$$

The impulsive approximation does not hold if the impulsive ΔV is beyond a threshold related to the available thrust-acceleration; in these circumstances ΔV is evaluated using Edelbaum's formulae, presented in the previous subsection.

The impulsive approximation requires that the maneuver point is close to the line of nodes between the orbit planes of the relevant asteroids. The maneuver point is rejected if its angular distance from either node is greater than

$$\Delta\vartheta_{max} = \tan^{-1}(0.2/\sqrt{\Delta i}) \tag{22}$$

This dependence on Δi was introduced to make the condition tight when the plane rotation is large, and, on the contrary, weak in the case of almost coplanar orbits. This condition does not apply to Edelbaum's approximation.

To ensure a proper phasing between the two asteroids, for each passage of the departure asteroid at the nominal point during the competition window, the actual angular distance between the asteroids is considered

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and a maneuver opportunity only occurs if the phase angle is less than a prescribed value (difference above 4 or 8 degrees are not accepted in the case of impulsive or Edelbaum approximation, respectively). Start and end times of the maneuver are evaluated by splitting the time-length, evaluated using the required velocity change and the available thrust magnitude, symmetrically around the orbit intersection or the minimum-distance point (i.e., the nominal point of the maneuver).

A reduced set of asteroids (containing about one thousand elements) has been considered by discharging the asteroids with eccentricity larger than 0.4 and inclination greater than 10 degrees. Using the previous approximations, a database of estimated usable maneuvers has been created. It collects a list of favorable transfers for the whole range of permitted dates. A transfer is described by initial date, final date, propellant consumption. The maneuvers are aggregated to form a series of possible sequences. The time between arrival to and departure from the same asteroid cannot be smaller or greater than assigned quantities; in the former case (small overlap) the maneuver probably could not be carried out; in the latter (large gap) the sequence would waste too much time.

In the last days of GTOC5 the procedure has been improved by creating different databases for suitable mass values from the initial to the final one, as the leg time-lengths depend on the available thrust-acceleration and therefore the current mass. Additional databases were created using the same mass for up to five legs; eventually, a different value was adopted for each leg.

2.3 First asteroid

The mission departs from the Earth, and an extensive search of the solution space must be performed to identify a suitable first asteroid for the sequence. Asteroids with semimajor axis close to 1 AU, small eccentricity and inclination were selected; among them preference was given to the asteroids well phased for a ballistic mission that left the Earth from the line of nodes and reached the target in correspondence of perihelion or aphelion. A local optimization of the rendezvous mission to these asteroids was carried out assuming optimal phasing: free hyperbolic excess velocity was initially considered, but the relevant constraint was introduced when required. Comparison of actual and required asteroid positions on the possible arrival dates allowed to find favorable legs to a set of first asteroids, which were used as starting points for the sequential strategy and whose presence in the chain strategy database was checked. This strategy proved to be very efficient as good initial sequences (comprising the one of the best solution found by this team) were found. On the contrary, a reverse strategy, which tried to find suitable legs from Earth to the initial asteroid of long chains suggested by the chain strategy, did not provide good results.

3 Indirect Optimization

Accurate integration and adequate optimization to assure actual feasibility with suitable values of propellant consumption and time-of-flight are required for each trajectory leg, which joins two consecutive asteroids. The authors have been applying indirect optimization to space trajectories for a long time. Their indirect approach is based on the split of the trajectory into arcs, which join at relevant points. An almost mechanical application of optimal control theory (OCT) determines the boundary conditions for optimality at the arc junctions. This modular approach is very useful for the present problem, as the trajectory is made of similar repeating legs.

Preliminary analysis showed that performing the flyby immediately after the rendezvous of the same asteroid is, in general, not very expensive; this event sequence has been assumed to limit the search space, even though some interesting opportunities may be missed. Therefore, the basic maneuver consists of a transfer between two asteroids (rendezvous conditions are enforced) with intermediate flyby of the departure one; in addition, the prescribed minimum velocity at flyby must be enforced.

Position $\mathbf{r}$, velocity $\mathbf{v}$ and mass m of the spacecraft are the problem state variables. They are made non-dimensional by using the radius of Earth's orbit, the corresponding circular velocity and the spacecraft mass at the start of the whole mission (4000 kg) as reference values. The spacecraft state is described by differential equations

$$d\mathbf{r}/dt = \mathbf{v} \tag{23}$$

$$d\mathbf{v}/dt = -\mathbf{r}/r^3 + \mathbf{T}/m + \mathbf{a}_p \tag{24}$$

$$dm/dt = -T/c \tag{25}$$

The trajectory is controlled by the thrust vector $\mathbf{T}$ (effective exhaust velocity c is constant).

 DOI: 10.2420/AF08.2014.29

At departure from asteroid A1 (subscript 0) one has

$$\mathbf{r}_0 = \mathbf{r}_{A1}(t_0) \tag{26}$$
$$\mathbf{v}_0 = \mathbf{v}_{A1}(t_0) \tag{27}$$
$$m_0 = k \tag{28}$$

where k is a fixed value, which derives from the previous maneuvers. At flyby of A1 (subscripts 1- and 1+ denote values just before and after the flyby)

$$\mathbf{r}_{1+} = \mathbf{r}_{1-} = \mathbf{r}_{A1}(t_1) \tag{29}$$
$$m_{1+} = m_{1-} - \Delta m_{FB} \tag{30}$$
$$t_{1+} = t_{1-} \tag{31}$$
$$\mathbf{v}_{1+} = \mathbf{v}_{1-} \tag{32}$$
$$[\mathbf{v}_{1-} - \mathbf{v}_{A1}(t_1)]^2 = v_\infty^2 \tag{33}$$

and at rendezvous with asteroid A2 (subscript 2)

$$\mathbf{r}_2 = \mathbf{r}_{A2}(t_f) \tag{34}$$
$$\mathbf{v}_2 = \mathbf{v}_{A2}(t_f) \tag{35}$$

The adjoint variables are introduced and the Hamiltonian is defined as

$$H = \boldsymbol{\lambda}_r^T \mathbf{V} + \boldsymbol{\lambda}_v^T (\mathbf{g} + \mathbf{T}/m) - \lambda_m T/c \tag{36}$$

OCT provides the Euler-Lagrange equations for the adjoint variables

$$\frac{d\boldsymbol{\lambda}_r}{dt} = -\left[\frac{\partial \mathbf{g}}{\partial \mathbf{r}}\right]^T \boldsymbol{\lambda}_v \tag{37}$$
$$\frac{d\boldsymbol{\lambda}_v}{dt} = -\boldsymbol{\lambda}_r \tag{38}$$
$$\frac{d\lambda_m}{dt} = \frac{\lambda_v \mathbf{T}}{m^2} \tag{39}$$

Thrust magnitude and direction are the problem control variables. According to Pontryagin's Maximum principle (PMP), thrust must be parallel to the velocity adjoint vector $\boldsymbol{\lambda}_v$, i.e., the primer vector [2]. The Hamiltonian becomes

$$H = \boldsymbol{\lambda}_r^T \mathbf{V} + \boldsymbol{\lambda}_v^T \mathbf{g} + T S_F \tag{40}$$

where the switching function $S_F = \lambda_v/m - \lambda_m/c$ is introduced. Thrust has its maximum value when $S_F > 0$, whereas it is zero when $S_F < 0$, in agreement with PMP.

The boundary conditions for optimality of the baseline leg derive from a general formulation of OCT [3, 4], which is omitted for the sake of conciseness. If the initial time is free and the final mass m_2 is maximized, the transversality condition at departure is

$$S_{F0} = 0 \tag{41}$$

At flyby, the position adjoint vector has a free discontinuity, the primer vector is continuous and parallel to relative velocity

$$\begin{aligned}\boldsymbol{\lambda}_{v1+} &= \boldsymbol{\lambda}_{v1-} = \\ &= [\mathbf{v}_{1-} - \mathbf{v}_{A1}(t_1)]\lambda_{v1-}/v_\infty\end{aligned} \tag{42}$$

and the transversality condition is

$$\begin{aligned}&\left[\boldsymbol{\lambda}_r^T(\mathbf{v} - \mathbf{v}_{A1}) + T S_F\right]_{1+} = \\ &= \left[\boldsymbol{\lambda}_r^T(\mathbf{v} - \mathbf{v}_{A1}) + T S_F\right]_{1-}\end{aligned} \tag{43}$$

One also has $\lambda_{m2} = 1$ and $S_{F2} = 0$ at the final point.

Departure time may be specified (e.g., forcing an immediate departure after the asteroid has been reached), replacing Eq. 41. Minimum-time legs have also been considered; the boundary conditions at the final point become $\lambda_{m2} = 0$ and $H_2 = 1$. The first leg departing from Earth is different; v_∞ is not zero and subsequent Earth flyby is not required. Either optimal or maximum v_∞ departures are considered, and slightly different sets of boundary conditions pertain to the first leg.

Several baseline legs are joined together to optimize a multi-rendezvous, multi-flyby trajectory. Boundary conditions for optimality are slightly modified at intermediate rendezvous, where time continuity and mass discontinuity (Δm_{RV}) are enforced. When multiple legs are joined, an improved accuracy is required to obtain convergence; in these cases, instead of deciding the thrust magnitude according to S_F during integration, the trajectory is split into burn and coast arcs on the basis of the single-leg solutions; $S_F = 0$ is imposed where the engine is switched on or off, in order to determine the corresponding times.

From the numerical point of view, the optimization problem is transformed into a multi-point boundary value problem (MPBVP). An iterative procedure [5] based on Newton's method is employed to solve the MPBVP.

4 Results

Both global search strategies have been used by Team13 and both suggested useful legs and favorable asteroid sequences. With some exceptions, sequential strategy and

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TABLE 1. *Trajectory events*

Event	Date, MJD	Body	mass, kg	v_∞, km/s
departure	59029.82	Earth	4000.00	1.92
rendezvous	59182.60	2007 UN12	3959.66	0.00
flyby	59325.80	2007 UN12	3843.12	0.40
rendezvous	59564.06	2001 GP2	3718.02	0.00
flyby	59696.06	2001 GP2	3593.18	0.40
rendezvous	59967.18	2006 JY26	3377.17	0.00
flyby	60076.73	2006 JY26	3249.29	0.40
rendezvous	60288.52	2010 JR34	3076.33	0.00
flyby	60398.01	2010 JR34	2969.70	0.40
rendezvous	60856.82	2009 BD	2688.00	0.00
flyby	60982.30	2009 BD	2602.32	0.40
rendezvous	61403.94	2008 JL24	2492.96	0.00
flyby	61494.29	2008 JL24	2401.14	0.40
rendezvous	61704.81	2000 SG344	2318.36	0.00
flyby	61797.21	2000 SG344	2234.24	0.40
rendezvous	62036.91	2008 UA202	2091.41	0.00
flyby	62174.34	2008 UA202	2019.10	0.40
rendezvous	62357.31	2006 RH120	1958.36	0.00
flyby	62440.89	2006 RH120	1877.76	0.40
rendezvous	62699.23	2004 QA22	1712.13	0.00
flyby	62762.93	2004 QA22	1631.45	0.40
rendezvous	62891.52	2003 WT153	1562.39	0.00
flyby	62978.70	2003 WT153	1490.33	0.40
rendezvous	63174.77	2006 BZ147	1366.50	0.00
flyby	63234.79	2006 BZ147	1296.01	0.40
rendezvous	63422.79	2009 YR	1192.48	0.00
flyby	63464.58	2009 YR	1119.60	0.40
rendezvous	63681.18	2006 WB	975.84	0.00
flyby	63723.64	2006 WB	915.42	0.40
rendezvous	63867.61	1991 VG	787.57	0.00
flyby	63903.95	1991 VG	732.34	0.40
rendezvous	63999.76	1993 HD	700.25	0.00
flyby	64028.08	1993 HD	645.33	0.40
rendezvous	64211.06	2009 CV	553.95	0.00
flyby	64231.41	2009 CV	501.34	0.40

Edelbaum's approximation, due to its continuous-thrust nature, proved more reliable for the initial legs of the trajectory, when the large spacecraft mass causes a small acceleration. On the contrary, impulsive approximation and chain strategy were more reliable in the mission final phases, which exhibit shorter burn times due to the larger thrust-acceleration. The chain search strategy can update its database by including legs between asteroids that have already been verified and optimized. It proved useful in the final days of the competition, suggesting alternative sequences in replacement of segments of the base trajectory.

Many sequences of several asteroids have been suggested by the global search and analyzed by means of local optimization. In the last week of competition, attention was exclusively dedicated to the most promising trajectory, with adjustments and asteroid replacements to improve the solution, which eventually reached 17 asteroids. On the competition last day, efforts were made to find the fastest 17-asteroid mission, which is the best solution found by this team and summarized hereafter:

- departure on June 19, 2020 (59029.82 MJD), $v_\infty = 1.92$ km/s;

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- arrival (flyby of asteroid 2009 CV) on Sept. 26, 2034 (64231.41 MJD);
- final mass after release of penetrator mass 500.34 kg;
- total time of flight $t_f - t_0 = 5201.58$ days;
- performance index $J = 17$ (17 rendezvous and 17 penetrations).

Details of the event sequence are given in Table 1; a graphical representation is shown in Fig. 1.

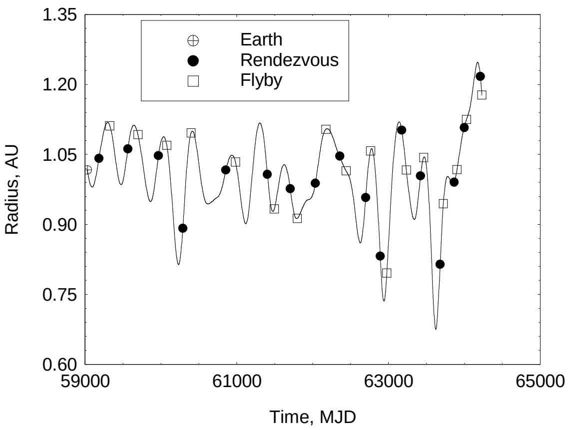

Figure 1. *Graphical representation of the solution.*

A typical leg between two asteroids is presented in Fig. 2; one should note that the maneuver from flyby to rendezvous occurs where the orbits are almost tangent. Positions of both asteroids at initial, flyby, and final time are shown: requirements in terms of phasing appear very tight to perform an efficient rendezvous. In unfavorable circumstances, thrust is used to move the spacecraft inward to increase its angular velocity and reach a leading asteroid. The opposite maneuver is performed to wait for a trailing one.

Table 2 presents the actual ΔV to perform each leg of the final trajectory; it includes the flyby cost (velocity change from asteroid departure to its flyby), which results to be almost the same for all the legs. The estimated ΔV is also shown; usually it has been provided by Edelbaum's formulas (Eq. 1); in two occurrences (denoted by asterisks) the leg was suggested by the impulsive approximation, and the corresponding ΔV (Eq. 17) is shown. The former approximation overvalues the propellant consumption, whereas the latter underestimates the needed ΔV, but it is more effective in positioning the maneuver and analyzing the asteroid phasing. As a

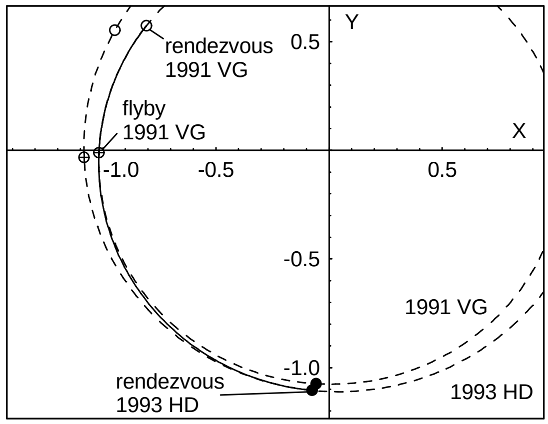

Figure 2. *Leg from asteroid 1991 VG to asteroid 1993 HD.*

matter of fact, Edelbaum's formulas give a reliable guess of the overall ΔV needed for the entire leg: differences are quite low, usually ranging from -18% to $+7\%$, as the benefit of the synergistic use of propulsion offsets the flyby cost. Noticeable exceptions concern the leg departing from 2006 BZ147 and the last leg from 1993 HD. These transfers are very fast and Edelbaum's approximation is less reliable; moreover the flyby cost becomes significant for low-ΔV transfers.

Estimation of transfer times is more complex, also because coast arcs are introduced to reduce the propellant consumption but cannot be foreseen. Results prove that the relationship providing the flyby penalty is accurate, even though a 25% greater coefficient appears more appropriate. Table 2 shows that Eq. 15, which includes the flyby maneuver, is in substantial agreement with the leg actual time-length. This kind of analysis is however difficult and, at a certain extent, questionable. According to the global strategy of the mission, some legs are flown to reach the asteroid as soon as possible without coast arcs; other legs have plenty of available time which is exploited to reduce the propellant consumption.

Adjustments have been tried to increase the accuracy of the estimations obtained by means of Edelbaum's approximation, which assumes almost circular orbits, whereas orbit eccentricities are often not negligible. In particular, tangential thrust has effects on both energy and eccentricity; when the transfer is short (less than one revolution), performance depends on the spacecraft position along its orbit. In a favorable position (e.g., close to the perihelion if both energy and eccentricity must be increased), the actual ΔV may be lower than the value provided by Edelbaum's approximation; unfa-

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TABLE 2. *Leg actual and estimated characteristics*

	Whole Leg		Flyby		Estimation			Ratio	
Departure Asteroid	ΔV km/s	Δt days	ΔV_{fb} km/s	Δt_{fb} days	ΔV_{est} km/s	Δt_{est} days	Δt_{fb} days	$\frac{\Delta V_{est}}{\Delta V}$	$\frac{\Delta t_{est}}{\Delta t}$
2007 UN12	1.547	381	0.580	143	1.290	306	115	0.83	0.80
2001 GP2	2.503	403	0.687	132	2.200	409	118	0.88	1.01
2006 JY26	2.386	321	0.785	110	2.067	355	98	0.87	1.11
2010 JR34	3.576	568	0.653	109	3.772	504	89	1.05	0.89
2009 BD	1.764	547	0.512	125	1.468	224	78	0.83	0.41
2008 JL24	1.649	301	0.628	90	1.672	226	72	1.01	0.75
2000 SG344	2.507	332	0.576	92	2.640	289	67	1.05	0.87
*2008 UA202	1.352	320	0.467	137	2.249	232	61	1.66	0.72
2006 RH120	3.331	342	0.629	84	3.142	277	57	0.94	0.81
2004 QA22	1.980	192	0.725	64	1.636	152	50	0.83	0.79
2003 WT153	3.160	283	0.626	87	3.149	221	45	1.00	0.78
2006 BZ147	3.112	248	0.684	60	1.719	125	40	0.55	0.50
2009 YR	4.870	258	0.852	42	4.322	213	35	0.89	0.83
*2006 WB	5.044	186	0.649	42	4.202	170	28	0.83	0.91
1991 VG	1.884	132	0.606	36	2.025	79	22	1.07	0.60
1993 HD	5.121	211	0.673	28	2.210	75	20	0.43	0.35

vorable situations may also arise. Moreover, the actual angular velocity may exhibit relevant changes with respect to the average value $\dot{\vartheta}_0$, modifying the final longitude ϑ_f. Some improvements have been investigated, for instance, by considering the maneuver actual radius, but insufficient time precluded a complete and successful analysis.

5 Conclusion

Team13 employed a set of tools which were sufficient to find a good solution to GTOC5 problem. The use of formulas based on Edelbaum's approximation permitted a satisfactory reduction of the options to be analyzed by the indirect optimization code, which was used to assess the actual feasibility of the trajectory legs. An estimation based on impulsive approximation suggested some additional legs, that were actually included in the final trajectory. The chain strategy supported the search for improvements to the provisional solution. A major enhancement would have been a technique which provided an affordable estimate of the cost related to incorrect phasing between asteroids. Finally, Team13 did not consider the possibility of delayed flybys and this point may have been crucial for the victory.

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