



# Trajectory Optimization in China

## The GTOC11 Workshop

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Changsha, China, 2021.12.18



## Main Content:

- 1. Chinese Teams in the GTOC**
- 2. China Trajectory Optimization Competition (CTOC)**
- 3. Trajectory Optimization Techniques Related to the GTOC and CTOC**

# 1. Chinese Teams in the GTOC

- From GTOC1 to GTOC11, more and more Chinese teams participated in the competition and submitted feasible solutions.
- Some institutions and universities even have multiple teams participating in the competition.
- In GTOC11, as many as 33 Chinese teams registered, and the results submitted by the Chinese teams accounted for nearly 50%.



# 1. Chinese Teams in the GTOC

## Xi'an:

- XSCC
- NPU
- XINGYI

## Harbin:

- HIT
- HEU

## Beijing:

- Tsinghua
- CAS
- Beihang University
- Beijing Aerospace Control Center
- BIT
- AFDL
- SEU
- IDRLL
- BIACE
- BITTT
- .....

## Wuhan:

- Wuhan U

## Changsha:

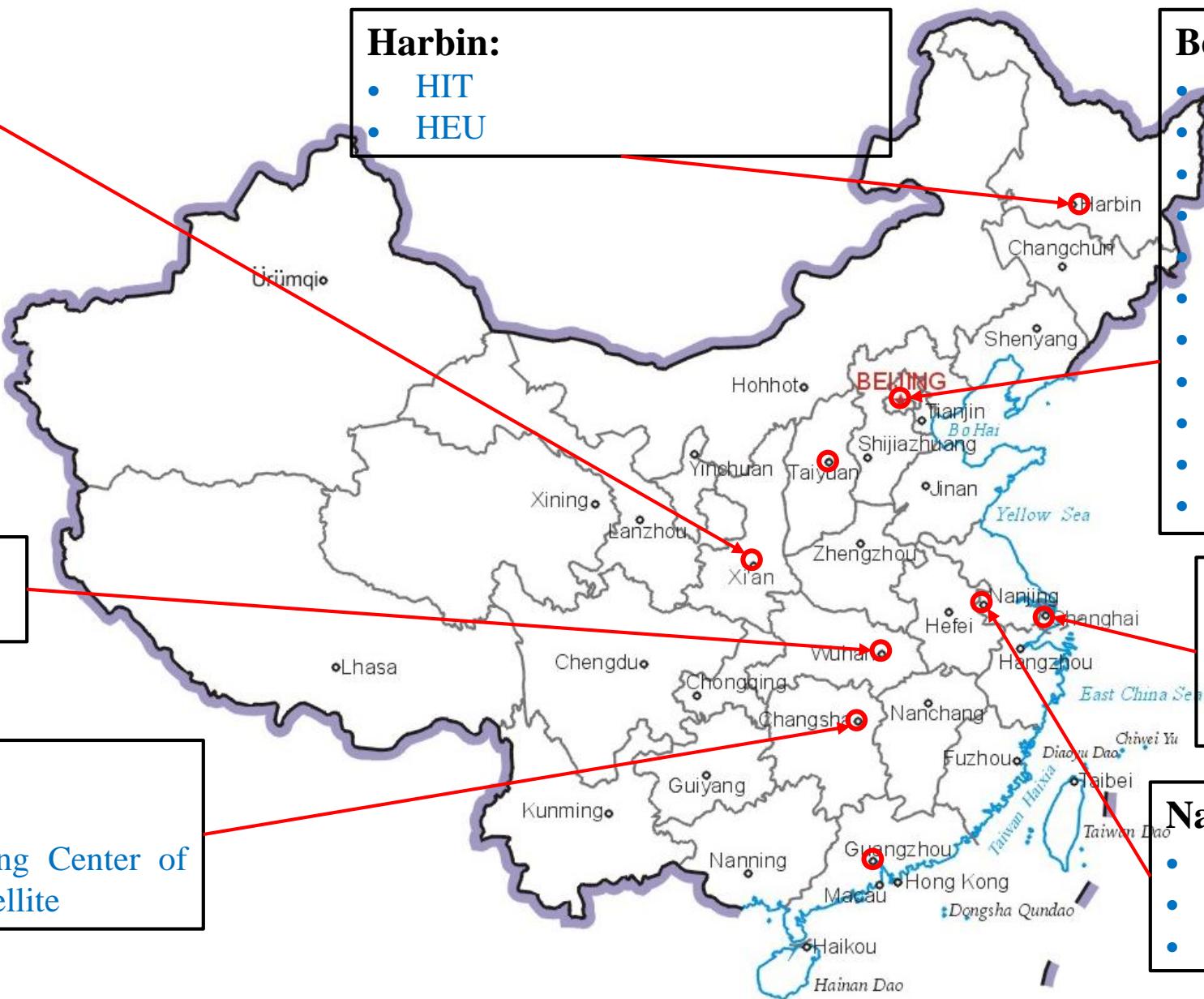
- NUDT
- Engineering Center of Nano-Satellite

## Shanghai:

- Shanghai Jiao Tong University
- Shanghai Institute of Satellite Engineering

## Nanjing:

- Nanjing U
- NUAA
- NUST





# 1. Chinese Teams in the GTOC

- Since GTOC5, more and more Chinese teams have got good rankings in the competition.
- So far, 14 Chinese teams have won the top five and 7 teams have won the top three in previous competitions.
- Chinese teams won the competition in the last two editions.

(NUDT&XSCC in GTOCX, TsinghuaLAD&509 in GTOC11)

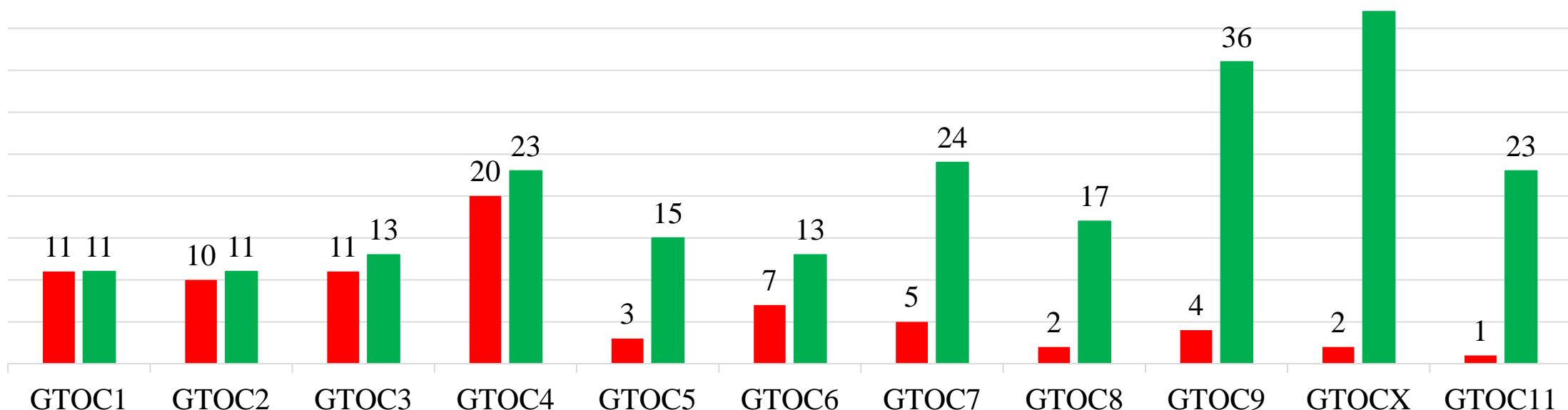
Team	Tsinghua	NUDT	XSCC	CAS	Beihang	BACC	Nanjing U	NUAA	HIT	BIT
GTOC1	11									
GTOC2	10									
GTOC3	11									
GTOC4	20				22					
GTOC5	3	13								
GTOC6	7		5	5	11					
GTOC7	5	13	12	4	11	15	16			
GTOC8	2		4	10		14	12			
GTOC9	4	2	3	15			12	15		
GTOCX	2	1	1	6	36	5		19	5	22
GTOC11	1	Organizer	Organizer	8	14			9	7	10

# 1. Chinese Teams in the GTOC

## Tsinghua University

- First Chinese team of the GTOC and participated in all the 11 competitions.
- From the bottom of GTOC1 to the top of GTOC11.
- A gold medal (GTOC11), two silver medals (GTOC8, GTOCX) and a bronze medal (GTOC5).

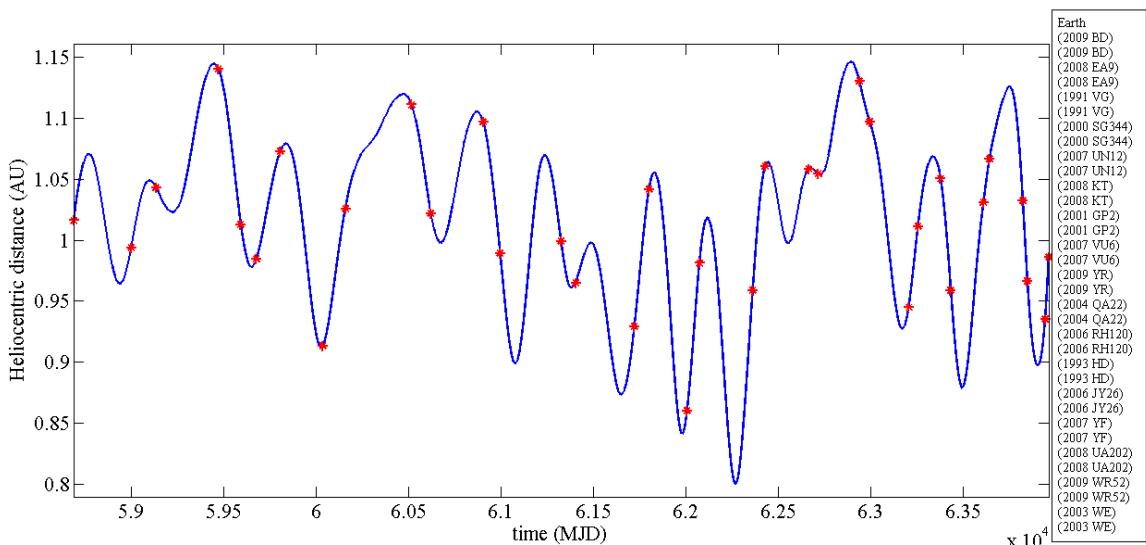
■ Tsinghua Ranking ■ Number of teams submitting results



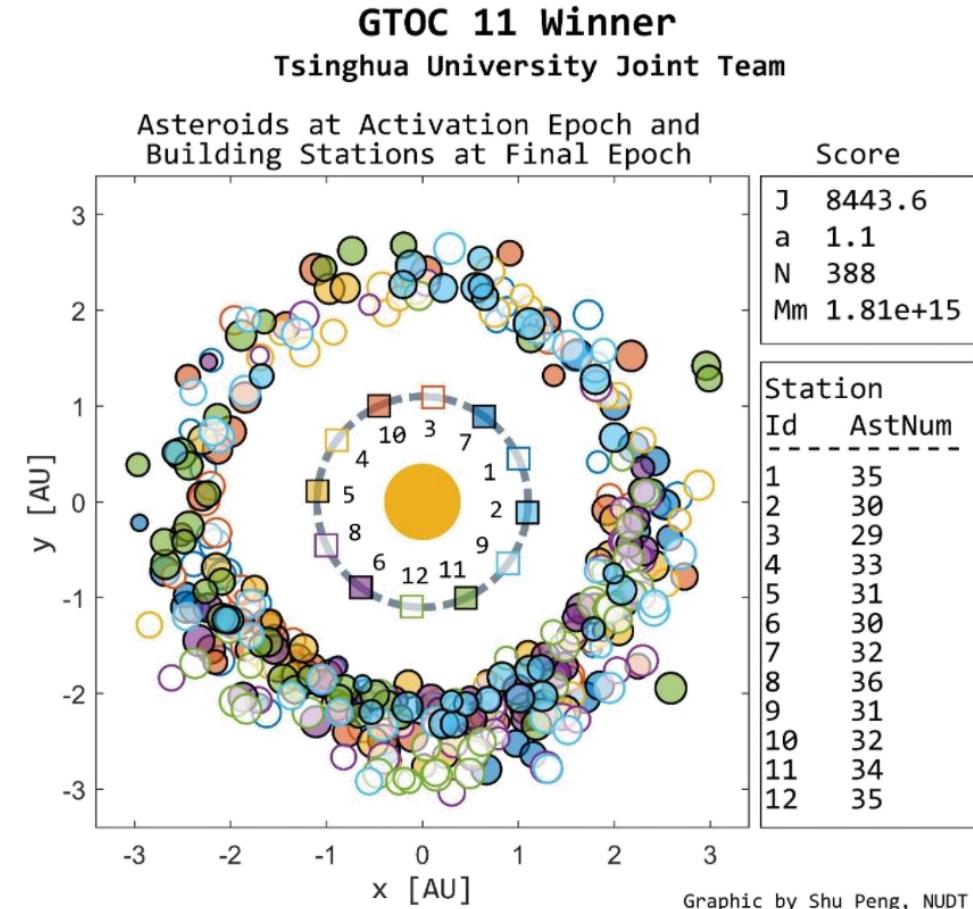
# 1. Chinese Teams in the GTOC

## Tsinghua University

- Adopt the indirect method and the homotopic approach for low-thrust trajectory optimization in GTOC5 and became the first Asian team to be ranked in the top three of the GTOC history.
- Implemented many combinatorial optimization methods (BS, GA, etc.) in GTOC11 and won its first championship with 8443 points.



GTOC5: variation curve of heliocentric distance of 17 asteroids landed and revisited within 15 years (Tsinghua University ranked third)

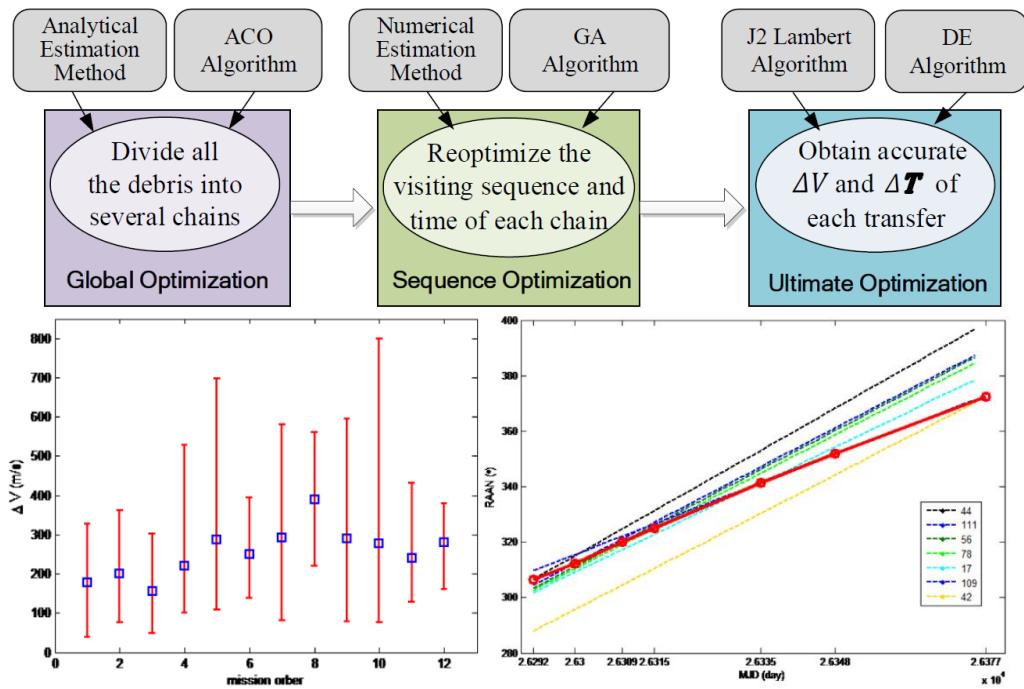


GTOC11: distribution of 388 asteroids visited and 12 solar-power stations built within 20 years (Tsinghua University ranked first)

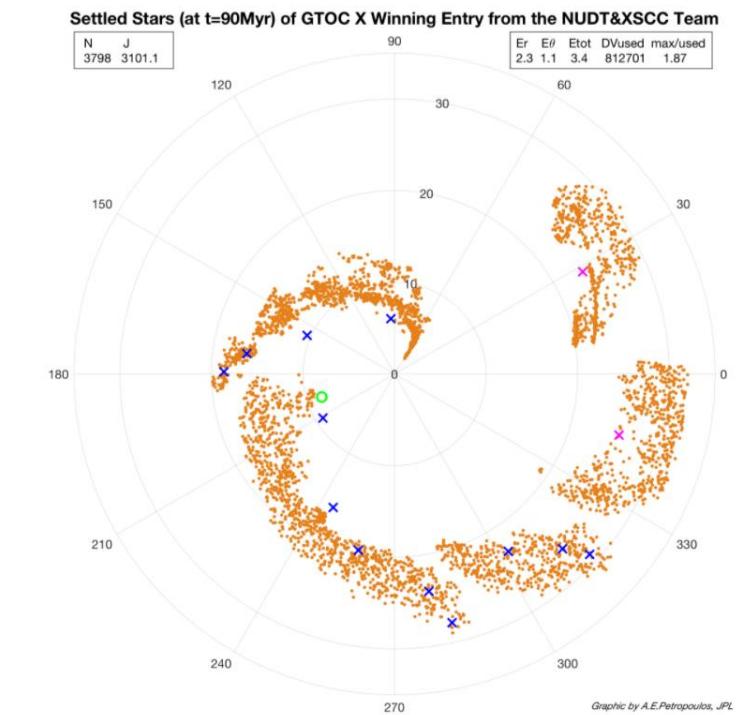
# 1. Chinese Teams in the GTOC

## National University of Defense Technology (NUDT) & Xi'an Satellite Control Center (XSCC)

- In GTOC9, NUDT ranked second and XSCC ranked third.
- In GTOCX, NUDT & XSCC joint team won the championship with 3101 points.
- By convention, the winning team of GTOCX, NUDT & XSCC joint team, organized the 11th edition of the Global Trajectory Optimization Competition (GTOC 11) at the end of 2021.



GTOC9: Optimization framework and some results (NUDT team ranked second)

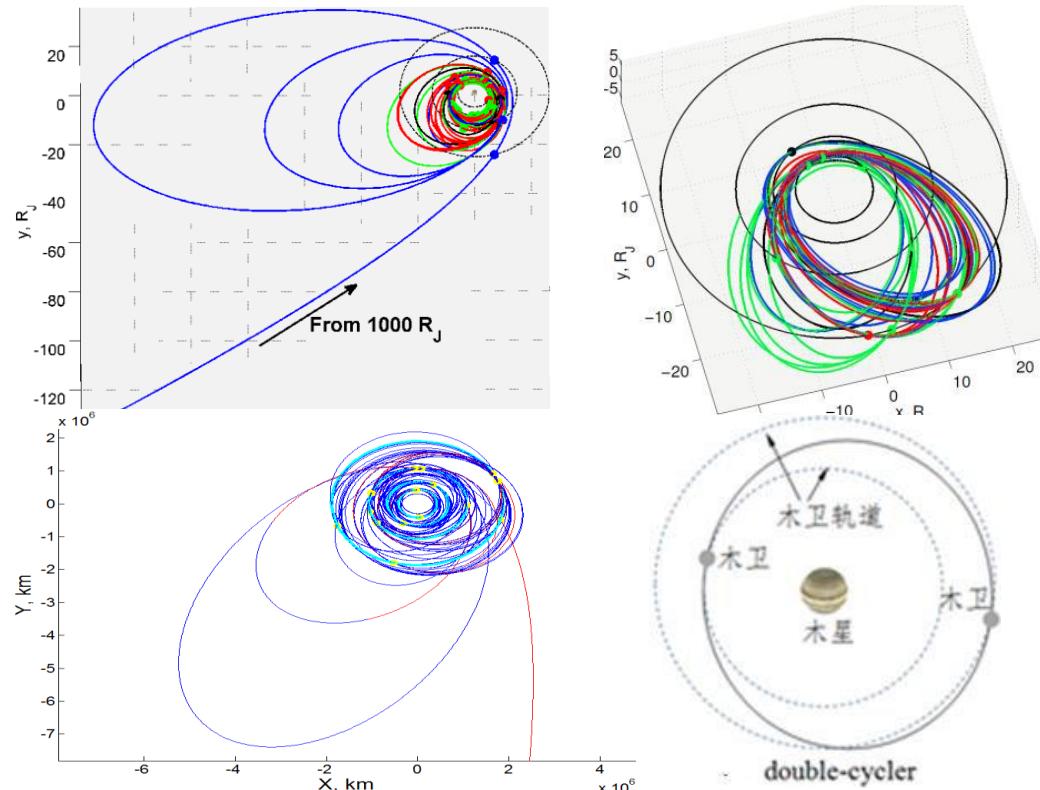


GTOCX: Final results (NUDT & XSCC ranked first)

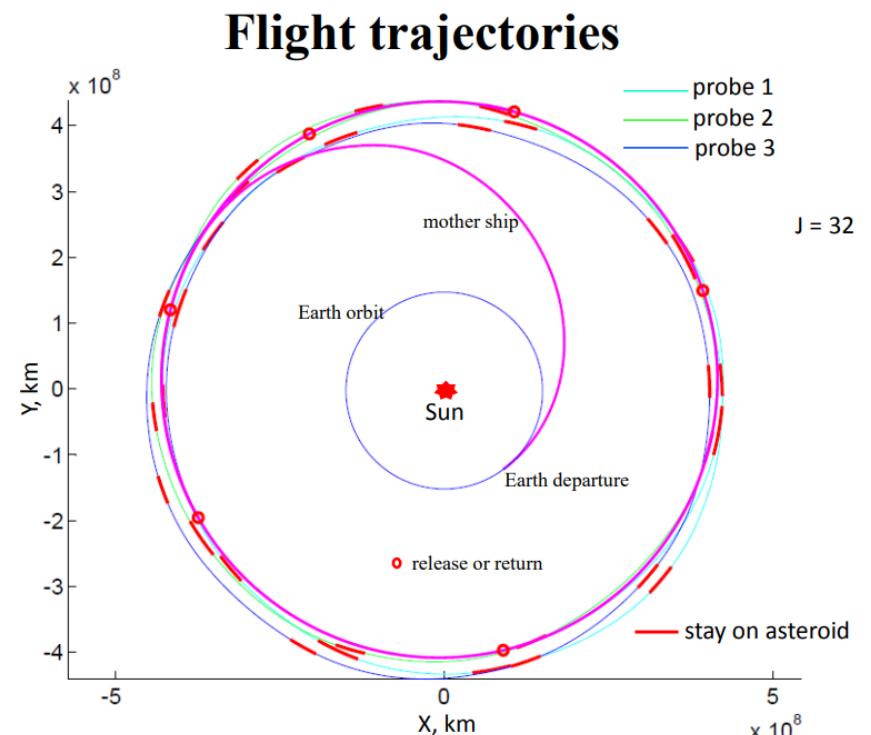
# 1. Chinese Teams in the GTOC

## Technology and Engineering Center for Space Utilities, Chinese Academy of Sciences (CAS)

- Participated in all the 6 competitions from GTOC6 to GTOC11.
- Ranked fifth in GTOC6 and ranked fourth in GTOC7.



**GTOC6: The result is improved to 320 after the competition  
(CAS ranked 5)**



**GTOC7: Flight trajectory diagram of mother ship and three probes (CAS ranked 4)**



# 1. Chinese Teams in the GTOC

## Other Chinese Teams (HIT, NUAA, BIT, etc.)

- In the last three competitions (GTOC9, GTOCX, GTOC11), the Chinese teams accounted for nearly 50% of the top ten.
- Some Chinese teams, such as **Harbin Institute of Technology (HIT)**, **Nanjing University of Aeronautics and Astronautics (NUAA)**, **Beijing Institute of Technology (BIT)**, etc., showed their competitiveness in recent competitions.

Rank	Team Name	Missions	Removed	$J$ in MEUR
1	Jet Propulsion Laboratory	10	123	731.2756
2	NUDT Team	12	123	786.21452
3	XSCC-ADL	12	123	821.37966
4	Tsinghua-LAD	12	123	829.57987
5	NPU	13	123	878.99821

Rank	Team Name	N	DVused	DVmax	Best Score
1	NUDT&XSCC	3798	0.812701E+06	0.152170E+07	3101.14604
2	Tsinghua LAD-XINGYI	2806	0.624757E+06	0.112500E+07	2070.53723
3	ESA-ACT	2652	0.660946E+06	0.106350E+07	1996.11072
4	The Aerospace Corporation	2435	0.769120E+06	0.976800E+06	1559.28104
5	HIT_BACC	2855	0.714294E+06	0.114520E+07	1167.423
6	CSU	1246	0.181974E+06	0.501200E+06	1111.00887

Name	Submissions	Mmin, N	Best Score
TsinghuaLAD&509	11	1.81364e+15, 388	8443.630600
ACT&Friends	5	2.0125e+15, 301	6359.724900
theAntipodes	44	1.27672e+15, 293	5992.298400
UT Austin	5	1.13283e+15, 235	5885.469300
ASRL	34	1.10046e+15, 209	5525.388800
The Eccentric Anomalies	12	1.89224e+15, 346	5487.543400
HIT	21	1.08546e+15, 250	5208.346300
GHWZZ	1	1.50229e+15, 294	4794.468600
ASTL-NUAA	7	1.03068e+15, 213	3735.160200
Team_BIT&ITNS	6	8.00267e+14, 199	3532.704400



## 1. Chinese Teams in the GTOC

Chinese Teams made great progress in the GTOC!

- G1 (11/11), G2 (10/11), G3 (11/13), G4 (20, 22/23)
- GTOC5 (3, 13/15), GTOC6 (5, 7/13)
- GOTC7 (4, 5/24), GTOC8 (2, 4, 10/17)
- GTOC9 (2, 3, 4, 5 / 36)
- GTOCX (1, 2, 5, 6 / 42)
- GTOC11 (1, 7, 8, 9, 10 / 23)

	Tsinghua	NUDT	XSCC	CAS	Beihang	BACC	N U	NUAA	HIT	BIT
G1	11									
G2	10									
G3	11									
G4	20				22					
G5	3	13								
G6	7		5	5	11					
G7	5	13	12	4	11	15	16			
G8	2		4	10		14	12			
G9	4	2	3	15			12	15		
GX	2	1	1	6	36	5		19	5	22
G11	1			8	14			9	7	10

# 1. Chinese Teams in the GTOC

## What GTOC brought to China:

- Improvement of Overall Teams' Level:

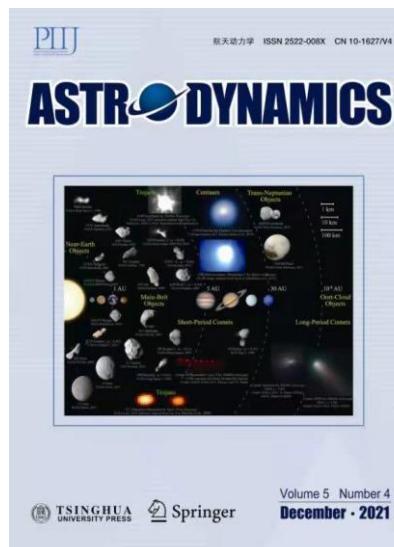
The capability of complex space mission design in China is largely improved through GTOCs, along with the development of China Space.

- Growth of Researchers and Engineers :

Many young researchers and engineers grow with GTOC and now become the backbone in their field.

- More International Friends:

A deep friendship with friends who love this competition worldwide is built, and the GTOCs bring many opportunities for international cooperation.





## **2. China Trajectory Optimization Competition (CTOC)**



## 2. China Trajectory Optimization Competition (CTOC)

2009

Up to 2009, Tsinghua participated in the GTOC 1-3, while the proposed solutions were unsatisfactory. To improve the trajectory design ability of Chinese teams, the CTOC was initiated, and 33 teams participated in the first edition.

2014

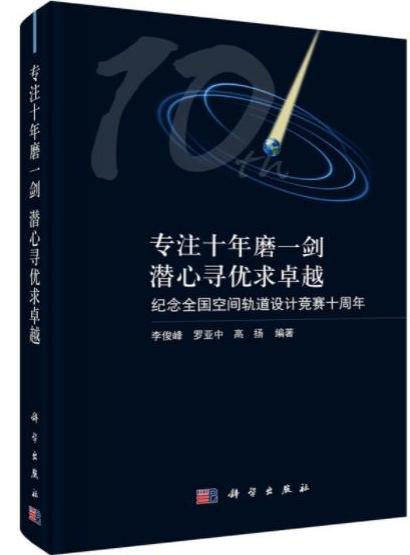
In 2014, to attract more teams, CTOC 6 released A and B problems. Each team can solve one or both of the problems, depending on their own ability and time.

2017

In 2017, CTOC 9 began to release the English translation of the problems, and 52 teams (3 of them are international teams) participated in this time.

2019

The CTOC 10 (2019) and 11 (2020) released one problem, the difficulty is similar to the GTOC, and more and more teams from all over the world participated and submitted excellent results.



		Tsinghua	XSCC	NU
G1	2005	11		
G2		10		
G3		11		
G4		20		
G5	2010	3		1
G6	2012	7	5	1
G7		5	12	1
G8		2	4	
G9		4	3	

## 2. China Trajectory Optimization Competition (CTOC)

- CTOC 1: Low-thrust sample return of a target near-Earth asteroid (Asteroid 2001 GP2)

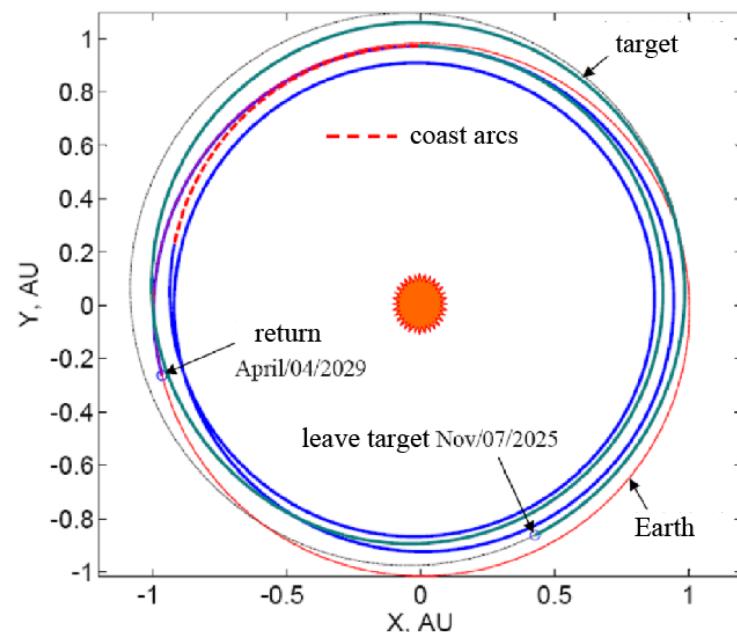
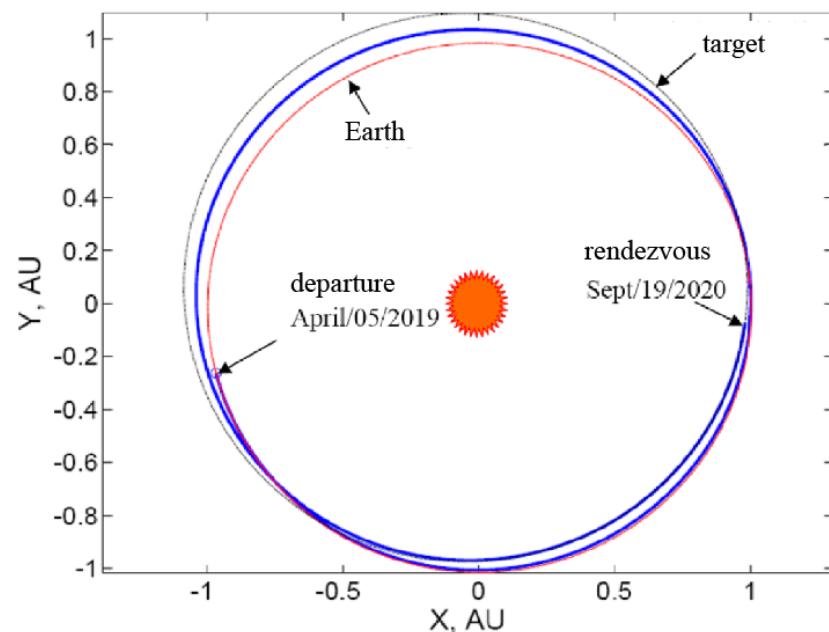
1<sup>st</sup>: Chinese Academy of Sciences

2<sup>nd</sup>: Xi'an Satellite Control Center (Control Team)

3<sup>rd</sup>: National University of Defense Technology

Only 3 feasible solution submitted:  
33→6→3

- The CTOC 1 winner's solution





## 2. China Trajectory Optimization Competition (CTOC)

- In 2009, China Trajectory Optimization Competition, CTOC, was initiated by
- The CTOC series aims to:

- I. provide a communication platform for the Chinese universities and space mission research institutions
- II. explore the basic optimization theories, develop new techniques, sophisticated algorithms, and new mission design concepts
- III. Incubate young scholars, to improve the ability of Chinese teams in the trajectory optimization
- IV. support the on-going or future space exploration missions

- CTOC takes the similar rules as the GTOC:

- I. The winner of each edition automatically be the organizer of the next edition of competition
- II. The competition is organized every one or two years
- III. The organizers are free to design the schedule, which is limited to 30-60 days
- IV. The Chinese Society of Theoretical and Applied Mechanics (STAM) is the supporting institution



Supporter:





## 2. China Trajectory Optimization Competition (CTOC)

- 11 editions have been organized, releasing 15 interesting problems

CTOC	Time	Problem	New topic
CTOC 1	2009	Asteroid sample return	
CTOC 2	2010	Mars and asteroids exploration	Multiple targets
CTOC 3	2011	Planets exploration with asteroid flyby	
CTOC 4	2012	Multitask Asteroid exploration	
CTOC 5	2013	Manned near-Earth asteroid exploration	
CTOC 6	2014	Near-Earth asteroid sample return	Multibody gravitational field, Multiple GA
		Hurtling to the edge of the solar system	
CTOC 7	2015	Cruising on the surface of irregular asteroid	Polyhedral gravitational field
		Formation reconstruction in low Earth orbit	
CTOC 8	2016	Sun-synchronous orbit space debris removal	Multiple rendezvous with $J_2$ perturbation
		Satellite scheduling for ground point targets observation	
CTOC 9	2017	Geostationary orbit resource formation detection	Formation flying design
		Low Earth orbit navigation enhancement	
CTOC 10	2019	Jupiter's magnetic field and its moons exploration	Magnetic field measurements
CTOC 11	2020	Orbit design for complex observation of ground targets	Complex Earth observation



## 2. China Trajectory Optimization Competition (CTOC)

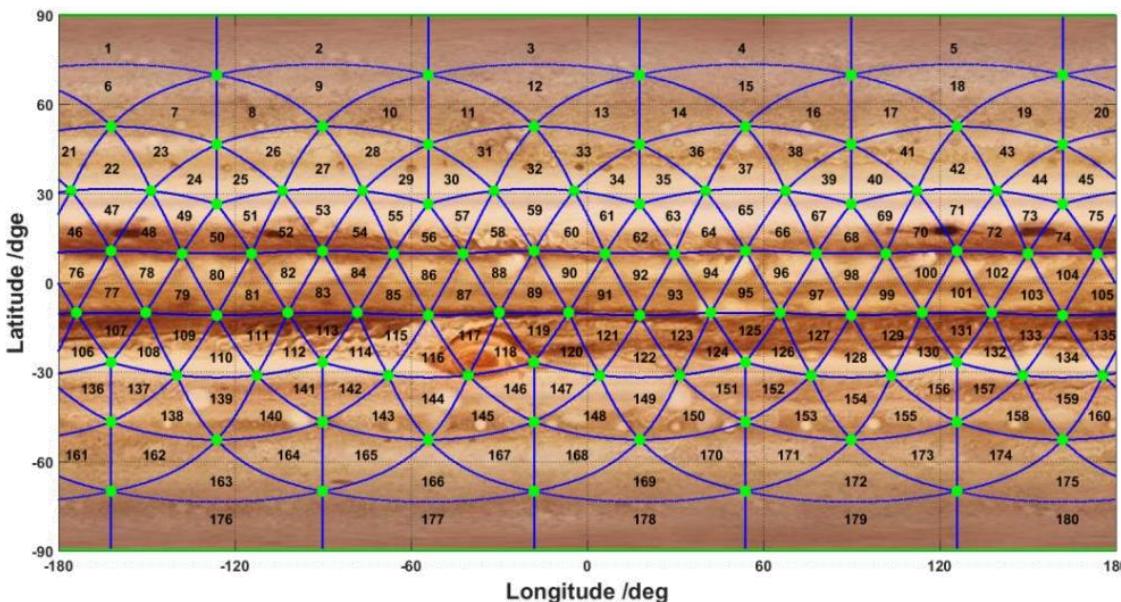
- Comparisons between the GTOC and CTOC

- I. CTOC drew on similar and simpler topics in the early stage and tried different types of topics from the GTOC recently.
- II. The real-time ranking has also been achieved in CTOC 11
- III. CTOC can be organized with a longer competition time

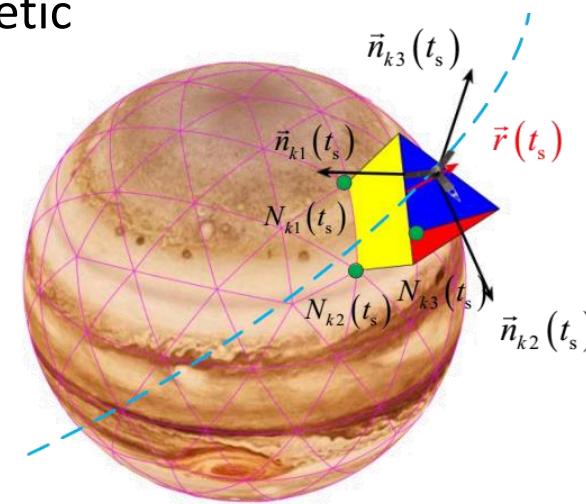
	GTOC	CTOC
Frequency	1-2 years since 2005	1-2 years since 2009
Dynamic model	Two-body, $J_2$ perturbation, Galaxy orbital dynamics	Two-body, Multi-body, C-W equations, $J_2$ perturbation, Polyhedral gravity
Edition	11 (with 11 problems released)	11 (with 15 problems released)
Real-time ranking	GTOC 9, X, 11	CTOC 11
Length	28 days	30-60 days
Reward	The trophy	Certificates and bonuses
Portal	GTOC Portal	Announce the schedule by email

## 2. China Trajectory Optimization Competition (CTOC)

- CTOC 10: Trajectory design and optimization for the exploration of the Jovian magnetic field and Galilean moons. (2019.03.20 – 2019.04.20)
  - I. Main task: full coverage measurement of Jovian magnetic field
  - II. Subtask 1: fast coverage measurement of Jovian magnetic field
  - III. Subtask 2: secular measurement of Jovian magnetic field
  - IV. Subtask 3: Galilean moons observation



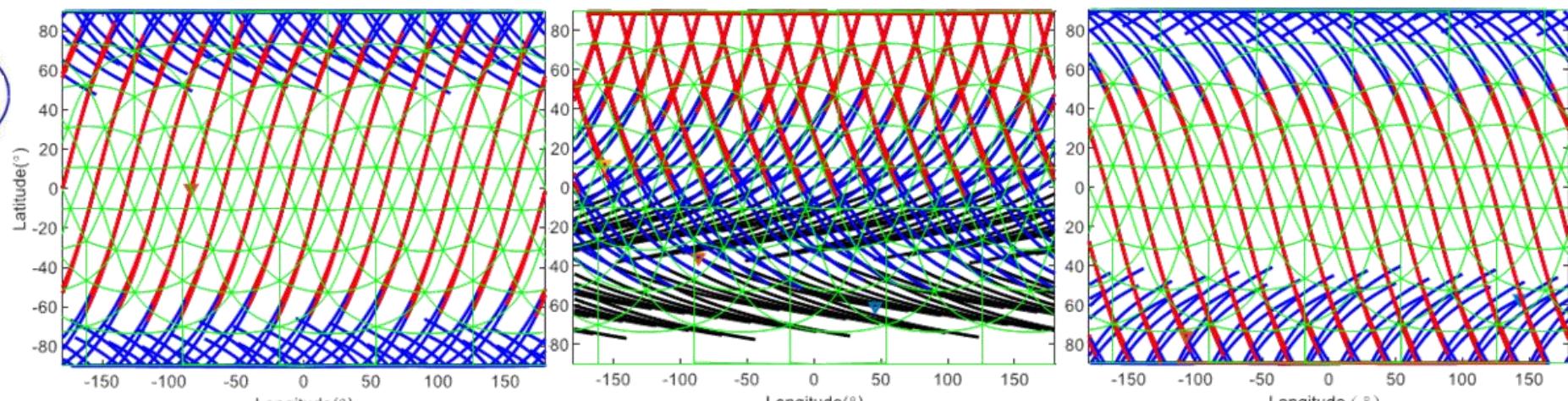
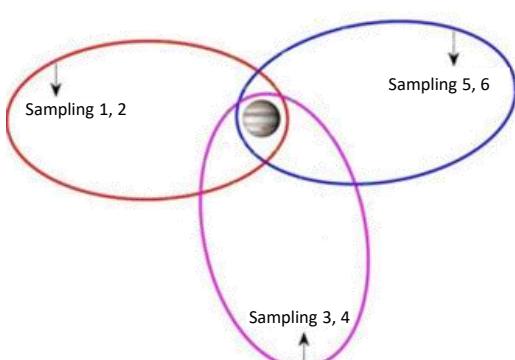
- Dynamic equation:  $\ddot{\mathbf{r}} = -\frac{\mu}{r^3} \mathbf{r} + \mathbf{a}_{J_2}$
  - Impulsive maneuver:  $\mathbf{v}(t^+) = \mathbf{v}(t^-) + \Delta \mathbf{v}$
  - Gravity assist is allowed
- Constraints on the task duration, initial orbit, radiation protection, and sampling conditions are detailed in the problem description file.



## 2. China Trajectory Optimization Competition (CTOC)

- CTOC 10: Trajectory design and optimization for the exploration of the Jovian magnetic field and Galilean moons. (2019.03.20 – 2019.04.20)-Presented by NUAA
- The ranking is announced after the competition:
  - 1<sup>st</sup>: Harbin Institute of Technology
  - 2<sup>nd</sup>: National University of Defense Technology (led by Luo Yazhong)
  - 3<sup>rd</sup>: National University of Defense Technology (led by Zhang Hongbo)
  - 4<sup>th</sup>: Xi'an Satellite Control Center
- The CTOC 10 winner's result

	Time (day)	Score
Main task	1094.8	1523.9
Subtask 1	190.5	186.8
Subtask 2	677.2	20.8
Subtask 3	\	0



## 2. China Trajectory Optimization Competition (CTOC)

- CTOC 11: Orbit design and maneuver optimization of two satellites for complex observation tasks with a large number of targets on the Earth. (2020.09.20 – 2020.11.09) -Presented by HIT

- I. Two observation satellites, the initial orbit elements and maneuvers can be optimized.
- II. Subtask 1: fast observation of 200 stationary targets
- III. Subtask 2: accurate revisit observation of stationary targets
- IV. Subtask 3: multiple uniform observation of moving targets

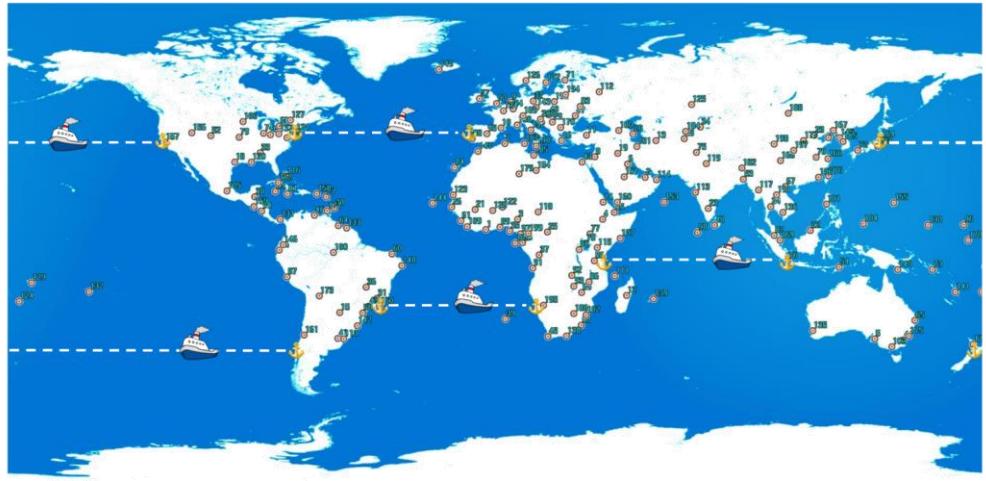
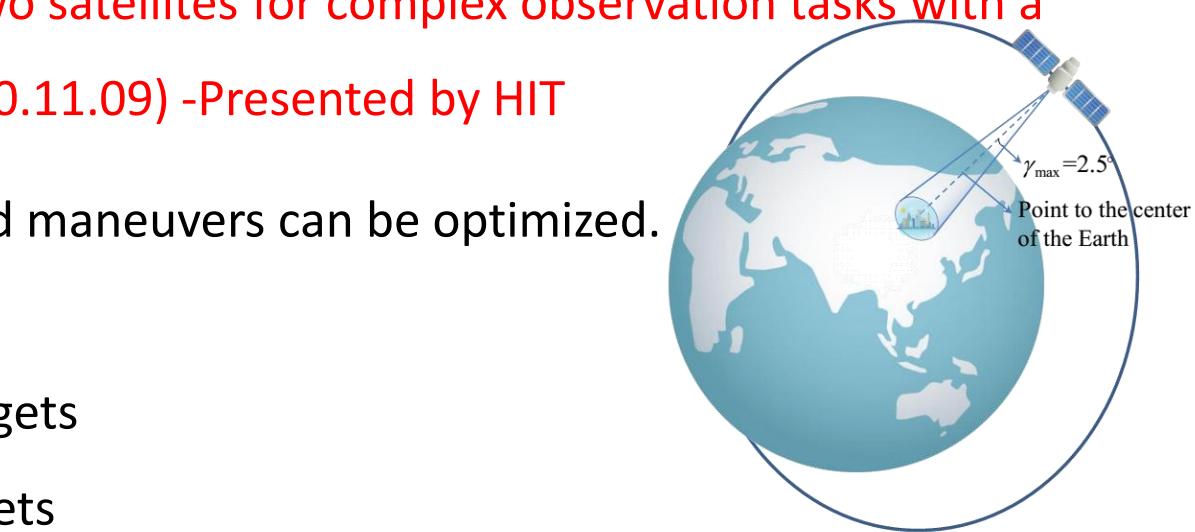


Figure 1: Target distribution diagram.



- Dynamic equation:  $\ddot{\mathbf{r}} = -\frac{\mu}{r^3} \mathbf{r} + \mathbf{a}_{J_2} + \mathbf{a}_{\text{darg}}$
  - Impulsive maneuver:  $\mathbf{v}(t^+) = \mathbf{v}(t^-) + \Delta \mathbf{v}$
  - Data Transmission with four given relay satellites
- Constraints on the task duration, maneuverability, orbit height, field of view, and the transmission conditions are detailed in the problem description file.

## 2. China Trajectory Optimization Competition (CTOC)

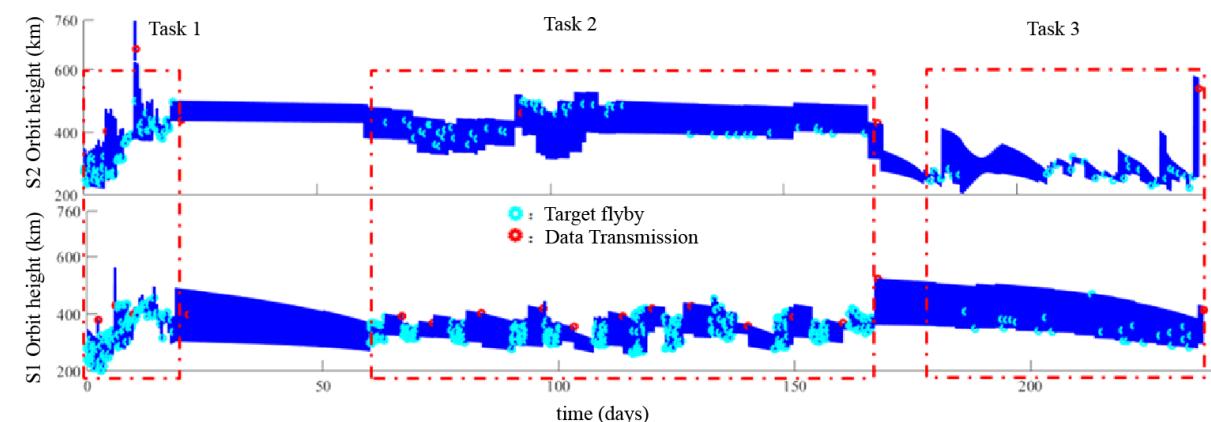
- CTOC 11: Orbit design and maneuver optimization of two satellites for complex observation tasks with a large number of targets on the Earth. (2020.09.20 – 2020.11.09)

**Remaining time to the end of the competition: 00 days 00 hours 00 minutes 00 seconds**

Name	Submissions	Last Submission	Best Submission	Best Score
空间应用中心	6	2020-11-09 11:25:24	2020-11-09 11:25:24	602.16278
Tsinghua-Lad	22	2020-11-09 09:23:39	2020-11-09 09:23:39	483.81506
飞天梦之队	15	2020-11-09 04:43:29	2020-11-09 04:43:29	438.57378
Jena & Wuhan	20	2020-10-22 11:01:00	2020-10-22 11:01:00	344.45916
Lonely Voyager	25	2020-11-09 11:57:06	2020-11-09 11:57:06	264.24863
ASTL	12	2020-11-09 11:46:01	2020-11-09 11:46:01	257.12258
Space_Omega	18	2020-11-09 07:57:26	2020-11-09 07:57:26	235.30303
微小的工作	5	2020-11-09 11:27:13	2020-11-09 11:27:13	57.311257
上网不涉密	7	2020-11-02 22:30:57	2020-11-02 22:30:57	56.273245

- The CTOC 11 winner's result

	Task 1	Task 2	Task 3	Total
Score	352.49	122.39	127.18	602.16
Time/day	19.19	120	60	240
Fuel/kg (S1/S2)	128/141	44/103	61/5	233/249





### **3. Trajectory Optimization Techniques Related to the GTOC and CTOC**

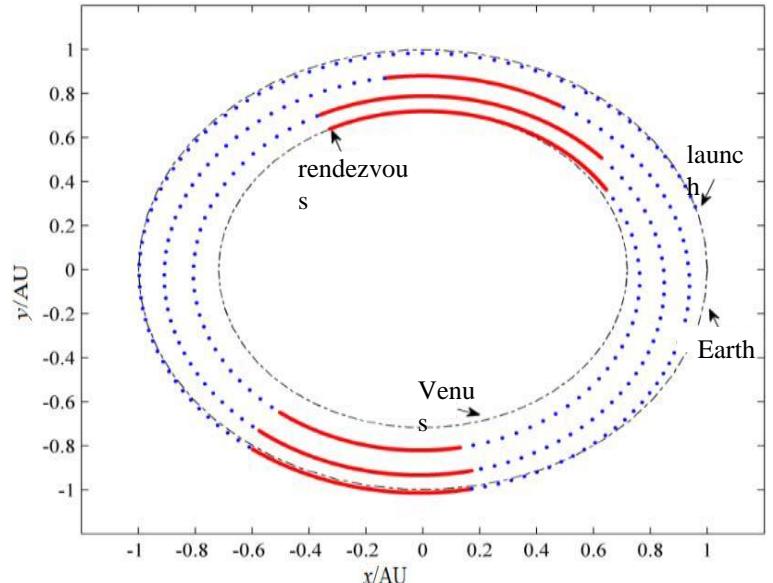
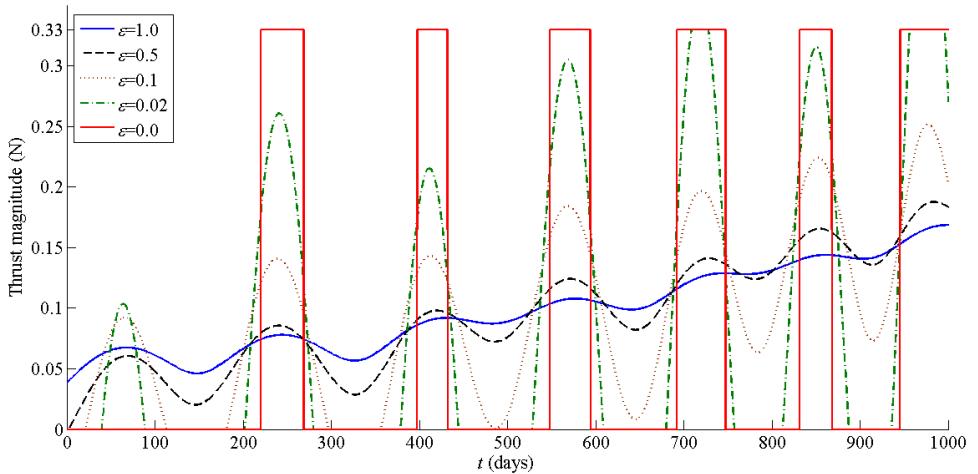
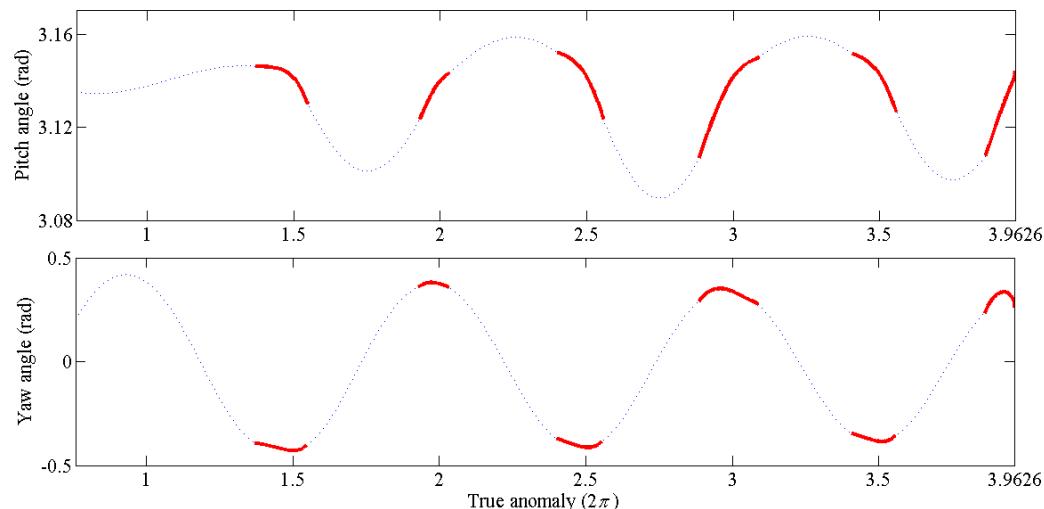
### 3. Trajectory Optimization Techniques Related to the GTOC and CTOC

#### Low-Thrust Trajectory Optimization

GTOC1~GTOC8, GTOC11

CTOC1~CTOC8

- Most Chinese teams have overcome the difficulty of low-thrust trajectory optimization (low-thrust lambert problem)
- A series of low-thrust optimization programs have been developed based on direct method, indirect method, and hybrid method.
- Tsinghua's method: the computation time of a solution reduces from several days to a few milliseconds.

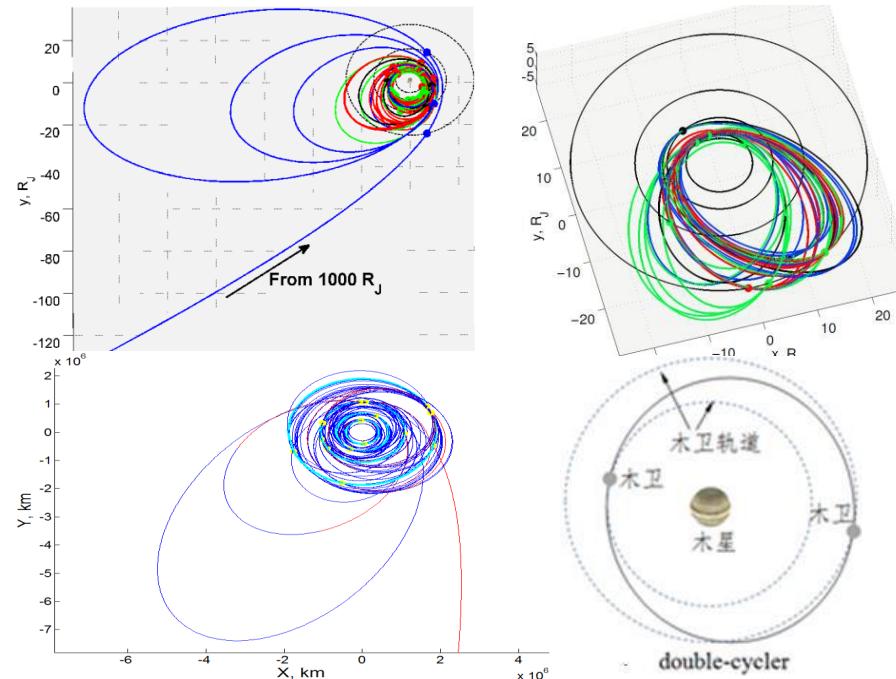


### 3. Trajectory Optimization Techniques Related to the GTOC and CTOC

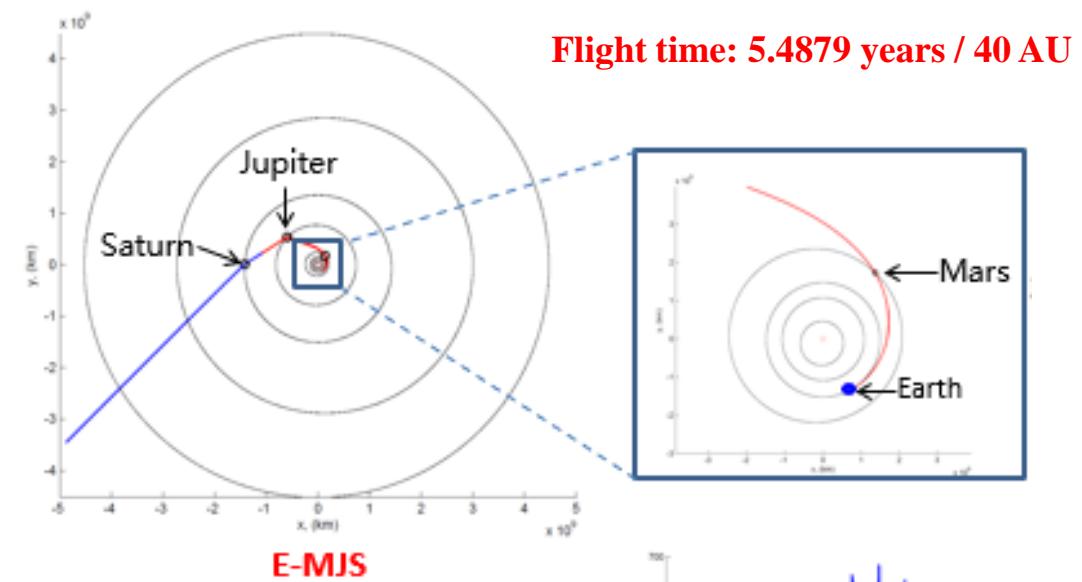
#### Multiple Gravity Assists

GTOC1, GTOC3, GTOC6, GTOC8  
 CTOC2~CTOC4, CTOC6A, CTOC6B, CTOC10

- Multiple gravity-assist trajectory optimization is a complex mixed-integer optimization problem.
- Hundreds of gravity-assist trajectories are still challenging to design and optimize.
- Resonant gravity assist is an effective strategy to design some specific missions.



GTOC6: Hundreds of gravity Assists using Galilean moons of Jupiter



CTOC6: Hurting to the edge of the solar system

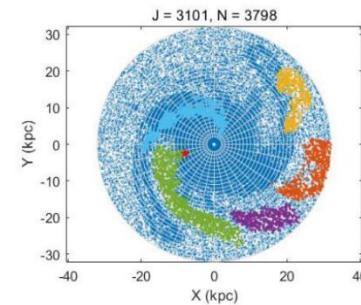
### 3. Trajectory Optimization Techniques Related to the GTOC and CTOC

#### Sequence Optimization

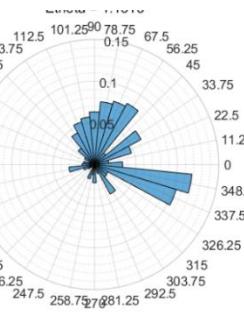
GTOC2~GTOC5, GTOC7~GTOC11

CTOC2~CTOC4, CTOC8A, CTOC8B, CTOC9A, CTOC11

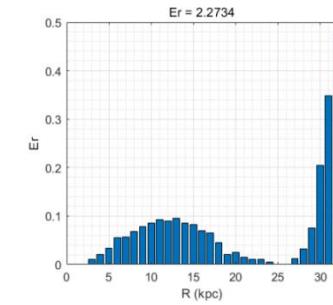
- In recent competitions, sequence optimization algorithms (GA, ACO, BS) have been widely used and developed.
- Many studies focused on the fast estimation to orbit transfer consumption.



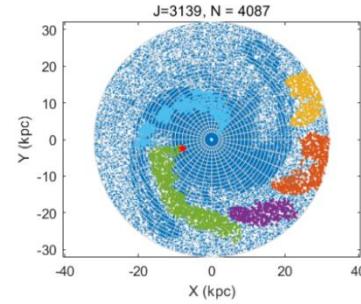
(a) Targets distribution



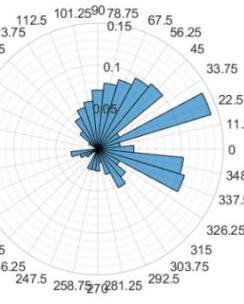
(b) Etheta



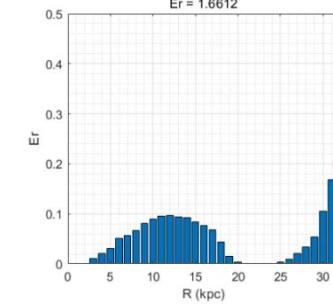
(c) Er



(d) Targets distribution

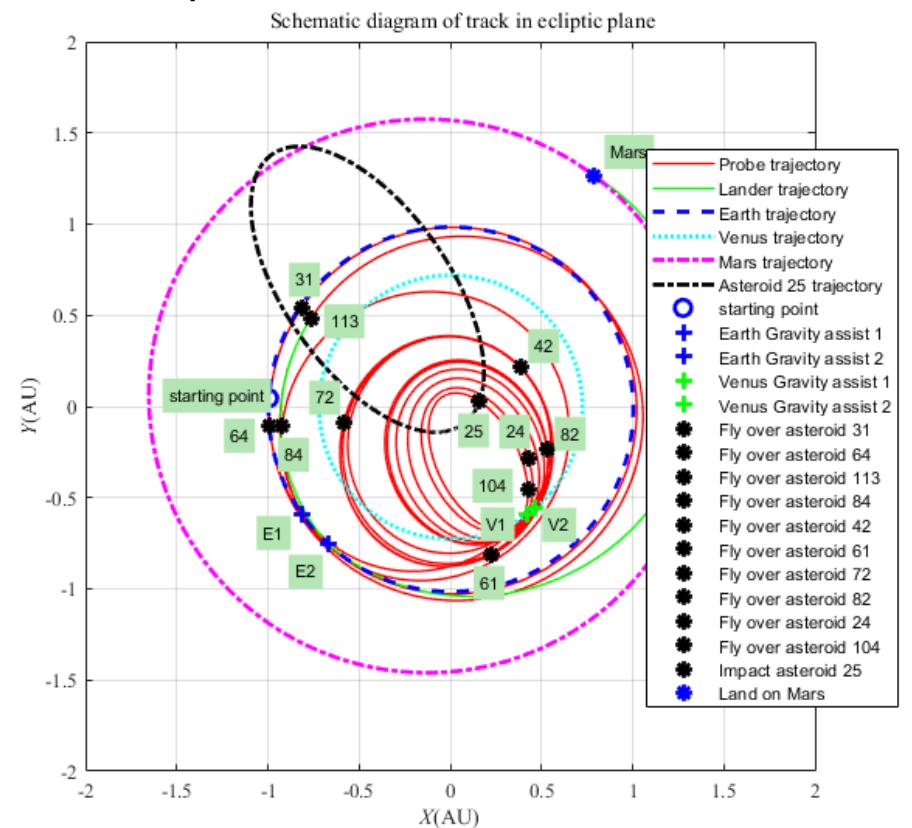


(e) Etheta



(f) Er

GTOCX: Spatial distribution for solution 3101 (NUDT & XSCC)



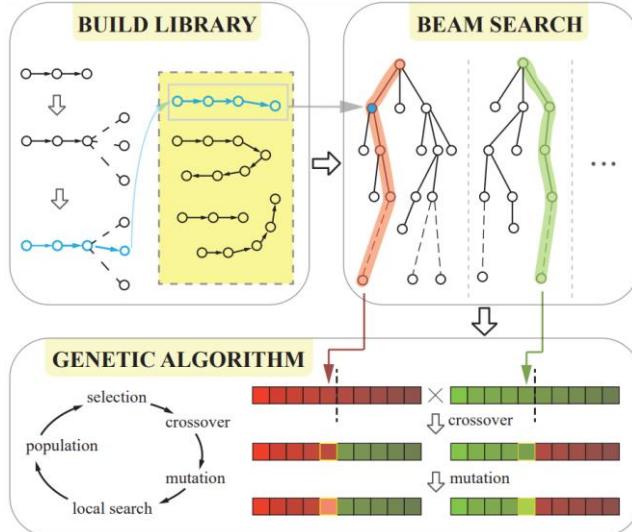
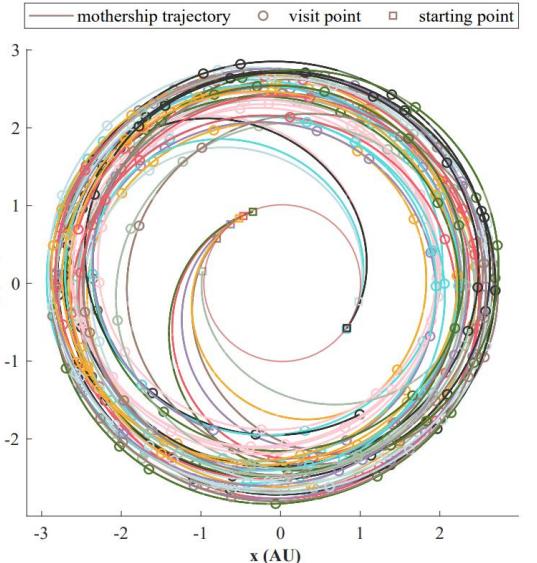
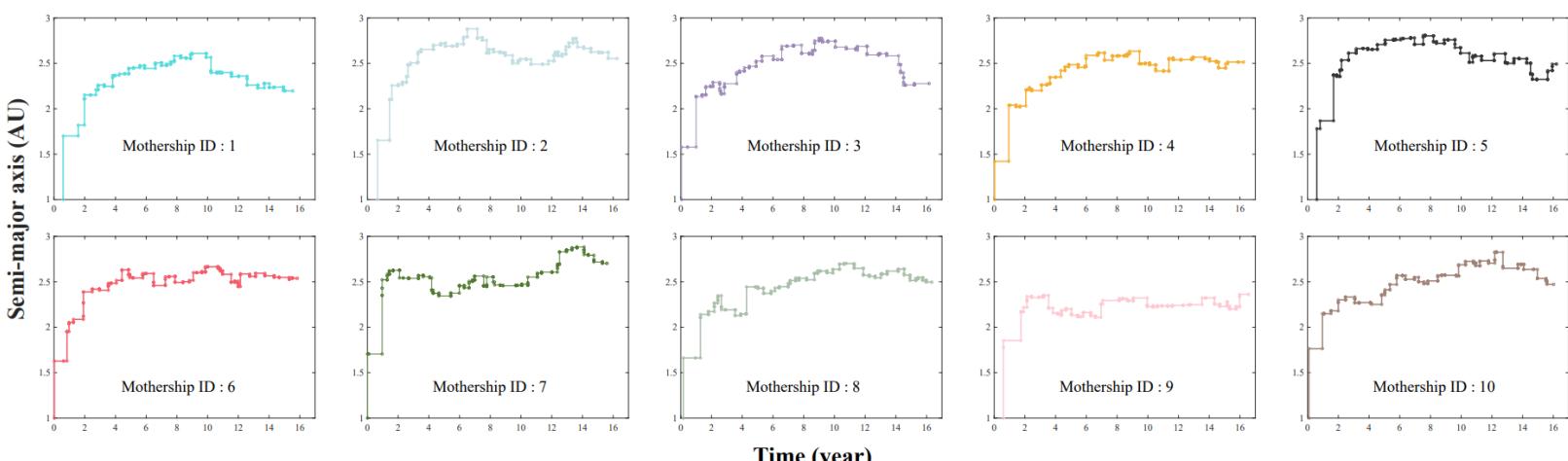
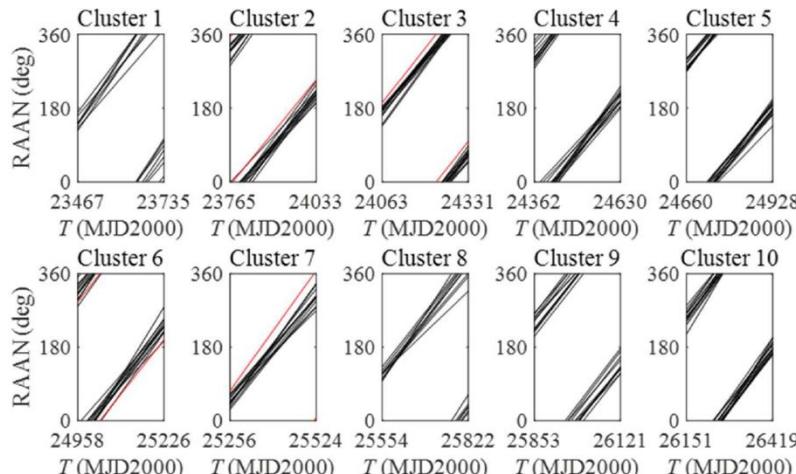
CTOC2: Schematic diagram of the projection of the whole task trajectory to the ecliptic plane (Tsinghua University)

### 3. Trajectory Optimization Techniques Related to the GTOC and CTOC

#### Multiple satellites planning

GTOC7~GTOC11, CTOC7B, CTOC9A, CTOC11

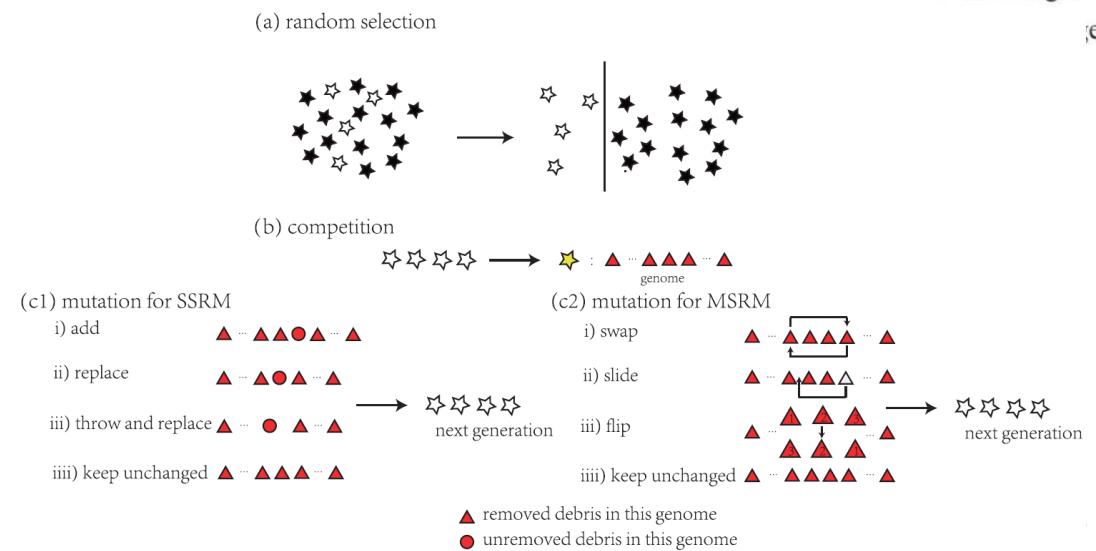
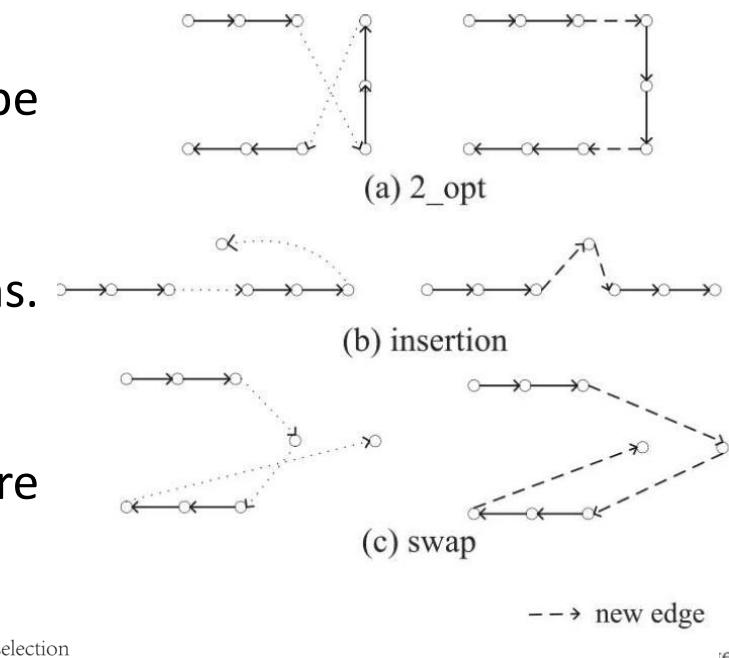
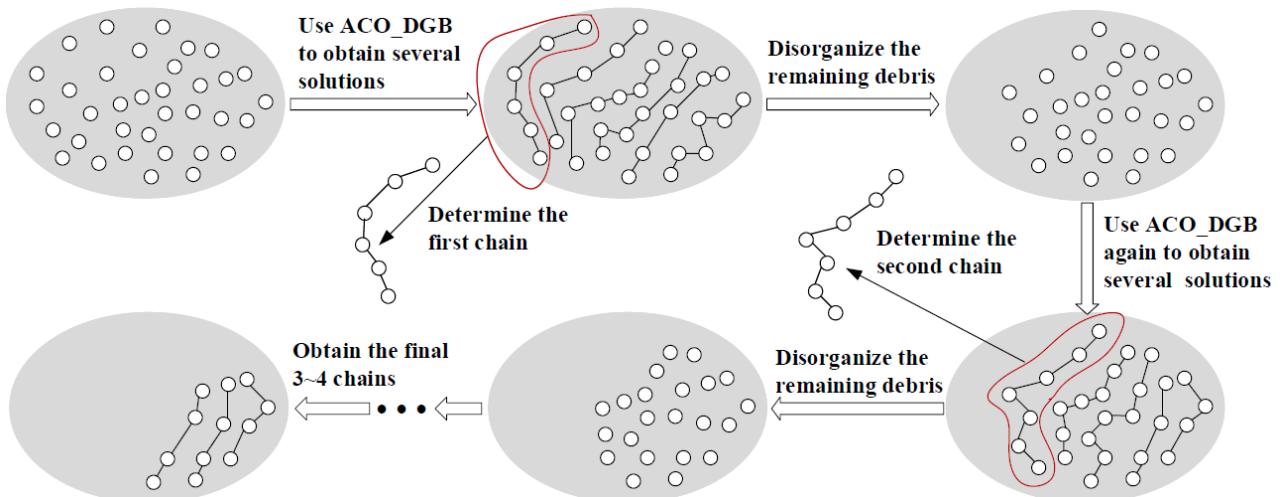
- The design of multi-satellite mission began to be involved in the recent competitions.
- Such as formation cooperation, target assignment, constellation design, etc.
- General strategies to deal with multiple satellites planning are still worth studying.



### 3. Trajectory Optimization Techniques Related to the GTOC and CTOC

#### Global Optimization and Local Optimization

- Generally, the top-level global strategy is the most critical, but it needs to be designed artificially.
- There is no obvious boundary between the local and the global optimizations. Local optimization can make up for the shortcomings of global optimization.
- The techniques of intelligent optimization and machine learning may be more widely used in the future.





# Summary

- Trajectory optimization technique has got profound development under the competition events.
- I think, At this point, we can solve all kind of real engineering mission planning problems somewhat easily. We had gone much further than what engineering needs in optimization.
- What will be next?



Looking forward to seeing  
you in the GTOC12!