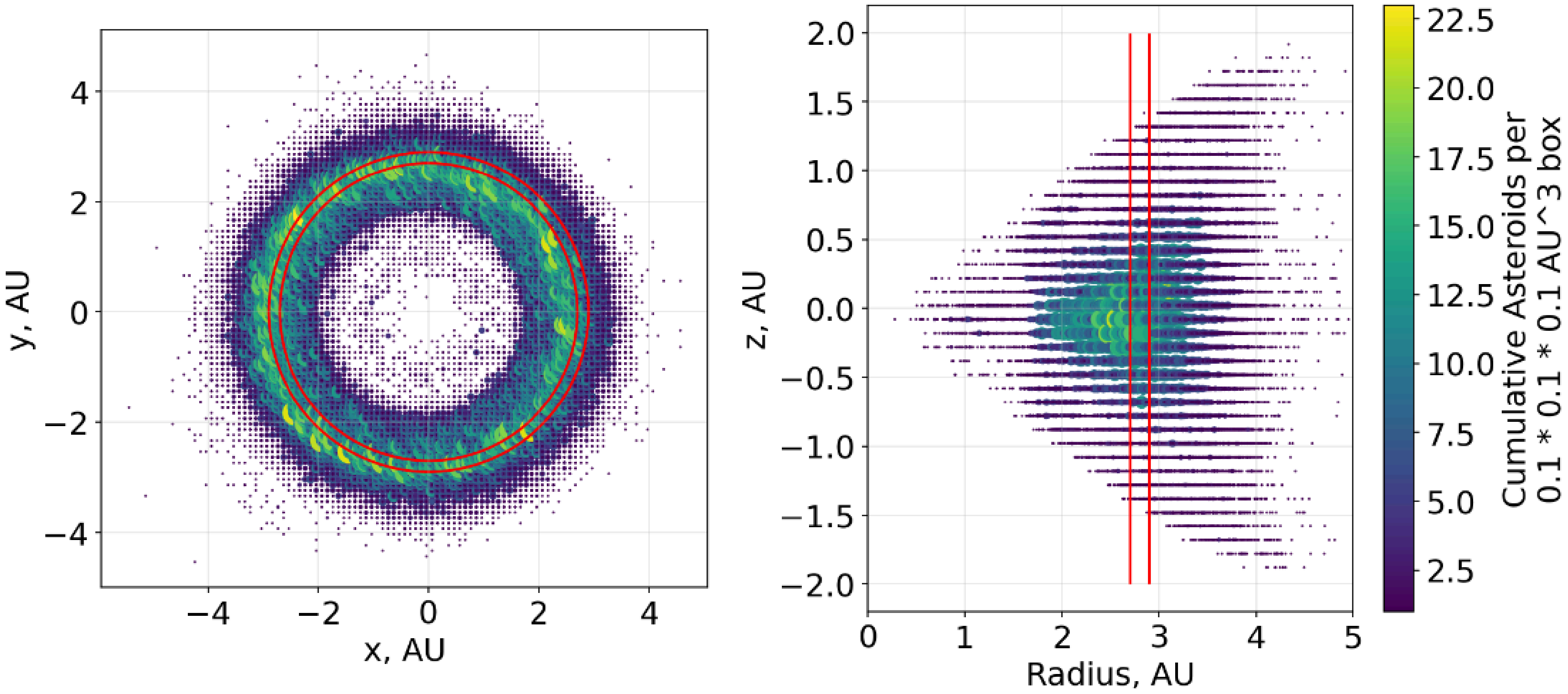
Asteroid Mining: ACT&Friends' Results for the GTOC12 Problem

The Team

The Overall Strategy

- **Leaving & Returning to Earth** – potentially using Mars & Venus fly-bys
- **Deploy and Mine** – this involves extensive search for low-thrust rendezvous transfers between asteroids



Highly-dense region in the [2.7,2.9] AU range

Leaving & Returning to Earth

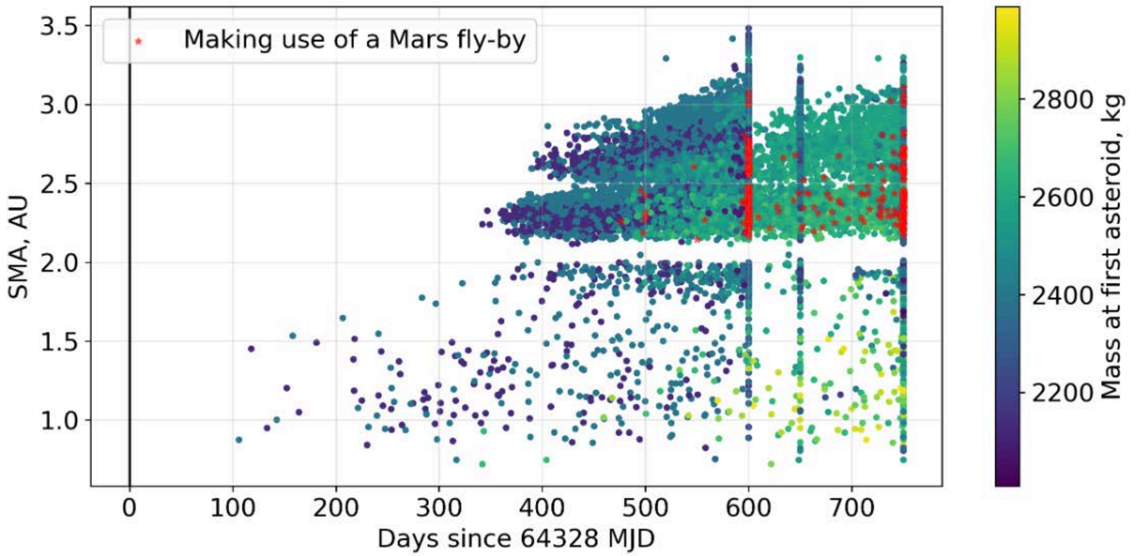
Database with Departure Options

- For each asteroid, we introduce and solve **two** distinct **OCPs** (**min** arrival **time**, **max** arrival **mass**):

$$\mathcal{P}_1 : \begin{cases} \text{min:} & t_f \\ \text{subject to:} & m_f \geq m_{f\,\text{min}} \\ & t_s \geq 64328 \quad MJD \\ & \dots \end{cases}$$

$$\mathcal{P}_2 : \begin{cases} \text{max:} & m_f \\ \text{subject to:} & t_f \leq t_{f\,\text{max}} \\ & t_s \geq 64328 \quad MJD \\ & \dots \end{cases}$$

- Venus & Mars flybys: initial pruning based on **single DSM** model



→ we reduced to 18,000 out of the 60,000

→ 665 were **Mars** flyby, and none **Venus** flyby

→ all results were stored in a **database** of departure legs

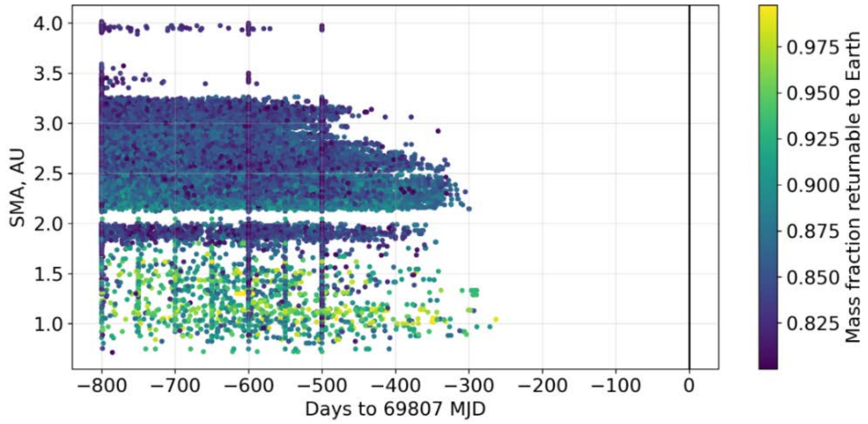
→ the database consists of **asteroid ID**, **time** of **arrival**, and **remaining mass**

Database of Return Legs

- For each asteroid, we solve an OCP (maximum final mass aka):

$$
\mathcal{P}_3 : \begin{cases} \text{max:} & m_f \\ \text{subject to:} & t_s \geq 69807 - \Delta t_{\max} \\ & t_f \leq 69807 \\ & \dots \end{cases}
$$

- Effectively, this means **maximizing** the amount of **material** that can be returned from each asteroid



→ the **OCP** was solved for different **maximum time** of **flights** (ranging from 500 to 800)

→ **starting mass** is fixed at 1,200 kg

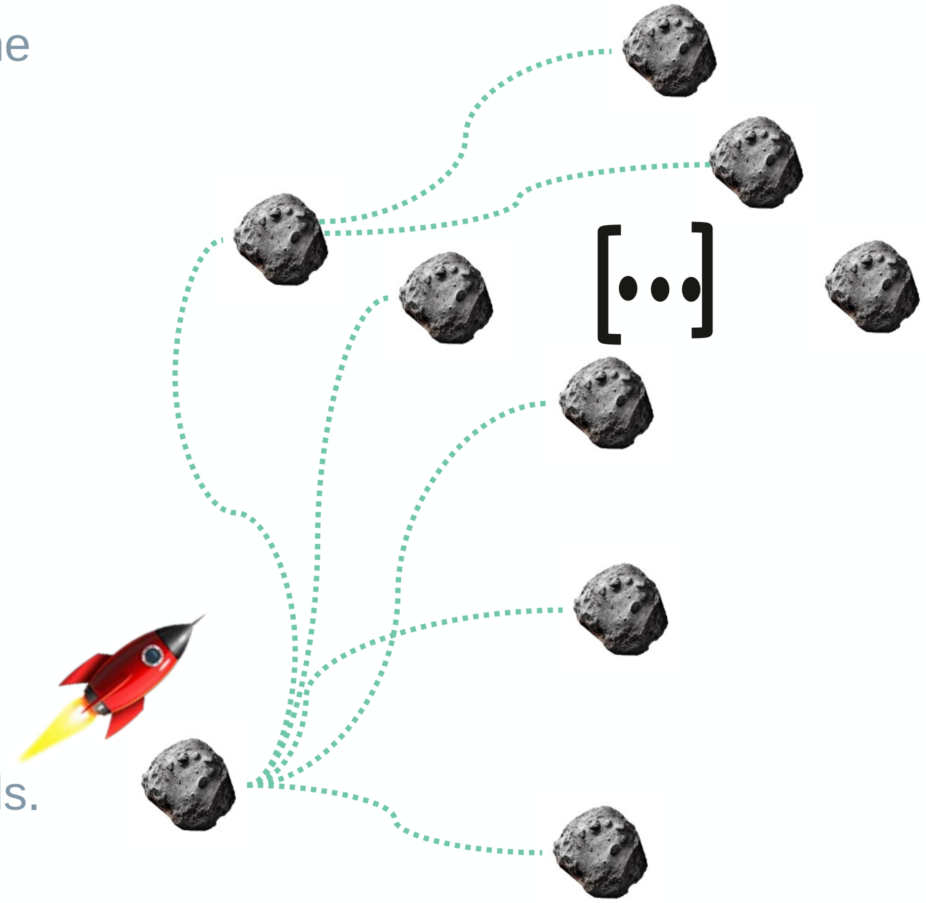
→ all results were stored in a database of **return legs**

→ **no flyby** seemed to help

Deploy and Mine

Design of Multiple Asteroid Rendezvous Missions

- We assume that each ensemble is **self-sufficient**: it contains only one ship → sub-optimal but **simplifies**
- We divide the task into **two** nested problems:
  1) **Outer** combinatorial problem that seeks the **optimal sequence** of asteroids to visit
  2) **Inner** continuous optimization problem to find **feasible** and **optimal low-thrust** transfers between asteroids
- 1) → **trajectory scaffolding** and **beam search**
- 2) → **astrodynamics manipulations** and **machine learning** methods. The objective: have **fast** and **reliable** ways to approximate low-thrust transfers → **without** solving OCPs



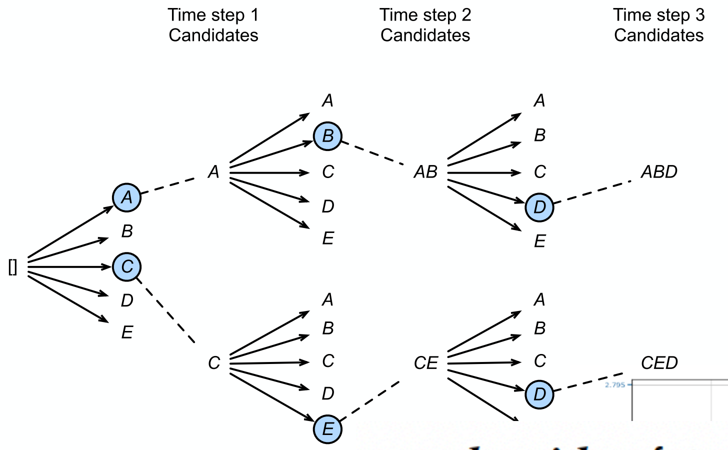
Deploy and Mine – Part 1

Design of Self-Sufficient Ships

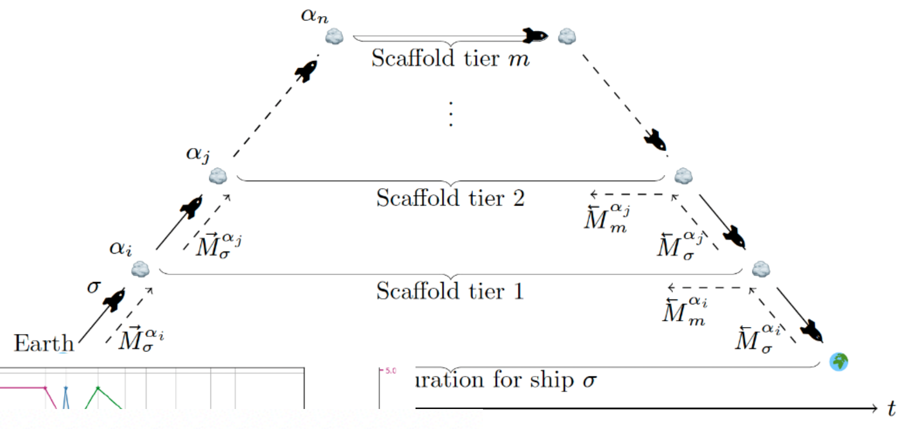
Combinatorial Chain Construction

- We used two strategies to approach the **combinatorial** part of the problem:

Beam search with lookahead



Trajectory scaffolding



algorithm future-score(α_i, t_i, m_i)

$h \leftarrow$ *the hop that lead the beam search to* (α_i, t_i, m_i)

$q_1 \leftarrow f(h)$

$q_2 \leftarrow min\{f(\mathrm{rev}(h) \oplus T \cdot 365.25) \mid T \in \{3, \dots, 9\}\}$

return $q_1 + q_2$

Deploy and Mine – Part 2

Computing Low-Thrust Hops

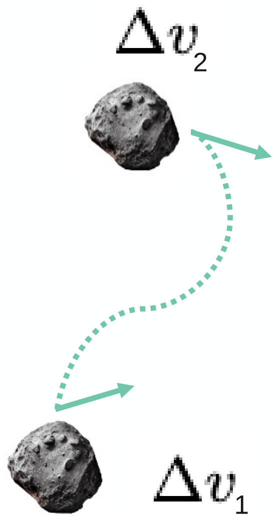
Asteroid Jumps

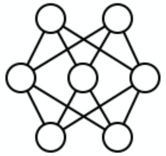
- Ideally, **every time** we assess a combination:
    - we should solve the associated low-thrust OCP **for different time of flights**
    - this is not feasible → hundreds of ms each!! (needs to be done billion times)
    - The control problems to solve are:
        - First find the **minimum** possible **time** for the jump (i.e., time-optimal control problem)
        - Then solve the **minimum propellant** associated control problems, starting with that time-optimal time of flight

$$
\mathcal{P}_{\text{MIM}}:\left\{\begin{aligned}
\text{given:}\quad & \alpha_s, \alpha_f, t_s, t_f \\
\text{find:}\quad & \mathbf{u}(t) \\
\text{to maximize:}\quad & m_s \\
\text{subject to:}\quad & \mathbf{x}(t_s) - \mathbf{x}_{\alpha_s}(t_s) = \mathbf{0}\ , \\
& \mathbf{x}(t_f) - \mathbf{x}_{\alpha_f}(t_f) = \mathbf{0} \\
& \dot{\mathbf{y}} = \mathbf{f}(\mathbf{y}, \mathbf{u}) \\
& |\mathbf{u}| \leq T_{\max}
\end{aligned}\right.
\qquad
\mathcal{P}_{\text{MINT}}:\left\{\begin{aligned}
\text{given:}\quad & \alpha_s, \alpha_f, t_s, m_s \\
\text{find:}\quad & \mathbf{u}(t) \\
\text{to minimize:}\quad & t_f \\
\text{subject to:}\quad & \mathbf{x}(t_s) - \mathbf{x}_{\alpha_s}(t_s) = \mathbf{0}\ , \\
& \mathbf{x}(t_f) - \mathbf{x}_{\alpha_f}(t_f) = \mathbf{0} \\
& \dot{\mathbf{y}} = \mathbf{f}(\mathbf{y}, \mathbf{u}) \\
& |\mathbf{u}| \leq T_{\max}
\end{aligned}\right.
$$

Low-Thrust Approximations

- We spent quite some time to figure out ways to compute the low-thrust solution *without* solving **OCP** (for both time and fuel optimal ones)
- We used for this:
  - **Astrodynamics manipulations**
  - **Machine learning**





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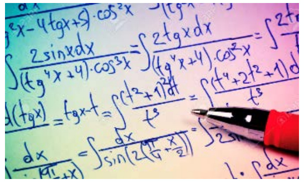
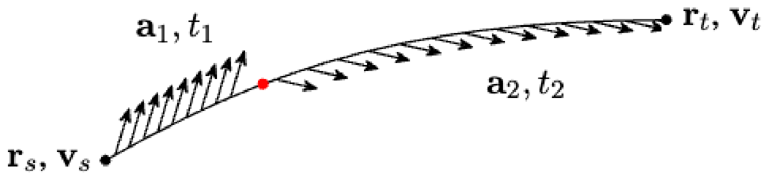


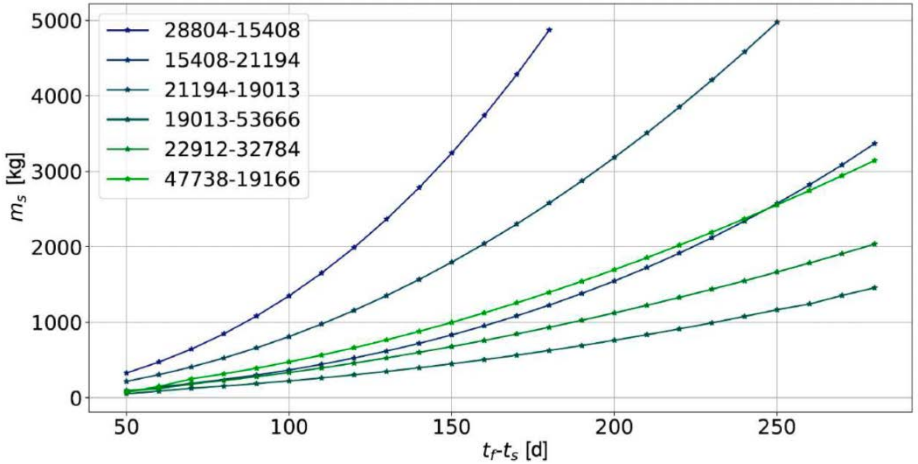


High-thrust solution (Lambert)

Low-thrust solution

MIMA

- Given a fixed-time transfer between two asteroids (a hop), two important questions:
  - What is the **maximum initial mass** (i.e., is the hop possible)?
  - What is the **minimum propellant cost** of the hop given a time of flight?



- Maximum initial mass approximation (**MIMA**) could be used to estimate the maximum mass (GTOC7):

$$m^*_{\text{MIMA}} = 2\frac{T_{\max}}{a_D}\left(1 + \exp\left(\frac{-a_D T}{I_{sp} g_0}\right)\right)^{-1}$$

- Can we do **better**? … it turns out we can

→ THE EUROPEAN SPACE AGENCY

Analytical Approximation - MIMA2

$$\dot{\mathbf{x}} = \mathbf{F}(\mathbf{x}) + \tilde{\mathbf{a}}(t)$$

We expand the dynamics at first order

$$\delta\dot{\mathbf{x}} = \mathbf{F}'(\mathbf{x}_K)\delta\mathbf{x} + \tilde{\mathbf{a}}(t)$$

$$\delta\mathbf{x}(t) \;=\; \mathbf{M}(t)\delta\mathbf{x}_0$$

The Keplerian state transition matrix appears

Introducing z(t) = dx0(t)

(variation of parameters)

$$\dot{\mathbf{z}} = \mathbf{M}^{-1}(t)\tilde{\mathbf{a}}(t)$$

$$\delta\mathbf{z}_T = \delta\mathbf{z}_0 + \int_0^T \mathbf{M}^{-1}(s)\tilde{\mathbf{a}}(s)\, ds$$

The final solution can be expressed in this form

Analytical Approximation - MIMA2

- Then, similarly to the MIMA, we impose that the thrust profile is made of **two arcs**, with constant thrusts, of the **same magnitude**
- By substituting the boundary conditions leads to the following solution (e.g. via **Simpson** rule):

$$\delta \mathbf{x}_T - \mathbf{M}_T \delta \mathbf{x}_0 = \frac{\mathbf{M}_T}{6} \left(\left(\mathbf{M}_0^{-1} + 4\mathbf{M}_{t_1/2}^{-1} + \mathbf{M}_{t_1}^{-1} \right) \Delta \mathbf{v}_1^* + \left(\mathbf{M}_{T-t_2}^{-1} + 4\mathbf{M}_{T-t_2/2}^{-1} + \mathbf{M}_T^{-1} \right) \Delta \mathbf{v}_2^* \right)$$

with: $\Delta \mathbf{v}_1^* = [\mathbf{0}, \mathbf{a}_1 t_1]^\mathsf{T}$ & $\Delta \mathbf{v}_2^* = [\mathbf{0}, \mathbf{a}_2 (T - t_1)]^\mathsf{T}$

- This is a system of 6 equations in 7 unknowns (the 2 DVs, and the switching time). By imposing **equal magnitude** of the two accelerations, we add an extra equation and solve the system

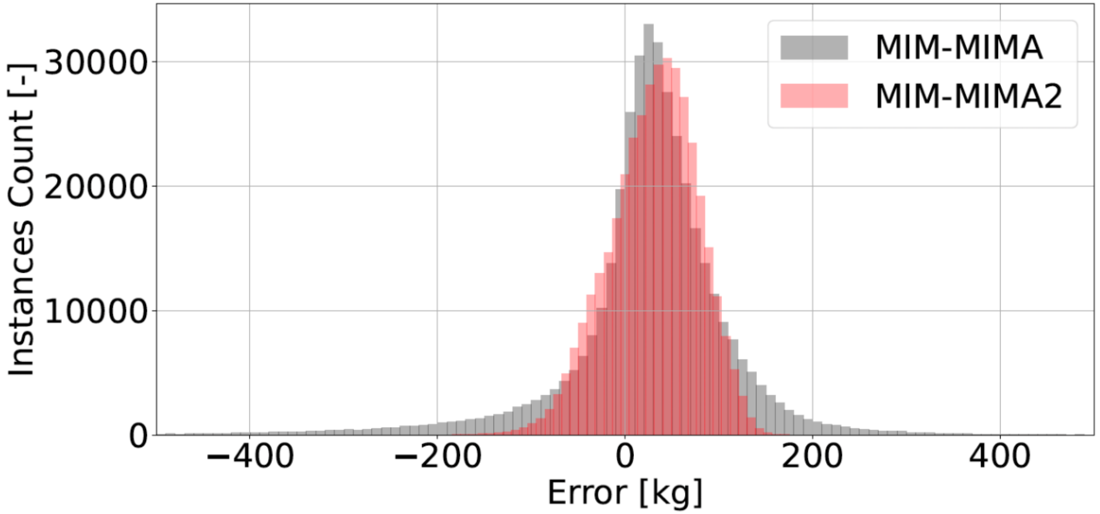
$$(T - t_1)|\Delta \mathbf{v}_1^*| = t_1 |\Delta \mathbf{v}_2^*|$$

- Et voilà! We have obtained an **approximation** of the maximum initial mass, only at the cost of computing the STM for a few points on the **ballistic transfer**!

$$m^*_{\text{MIMA2}} = \frac{T_{\max}}{a}$$

MIMA vs MIMA2

- How does **MIMA2** perform **vs MIMA**?

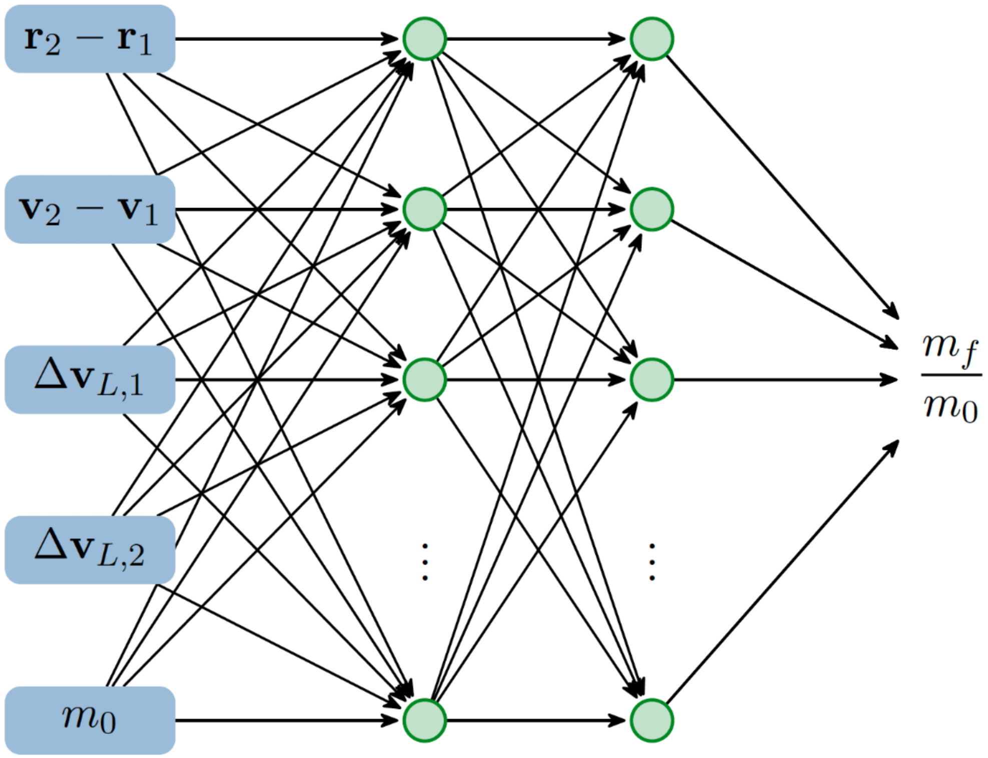


- With similar reasoning, we can also approximate the DV cost of a hop with mass below the MIM - this time the initial mass is known. but the switching time are both unknown (since there is intermediate **coasting**). So we add: $a = \frac{T_{\max}}{m_s}$ which leads to: $|\Delta \mathbf{v}_1^*| = t_1 a, \ |\Delta \mathbf{v}_2^*| = t_2 a$

- Finally, from MIMA2 we can also extract the **MINTA2** approximation (minimum initial time): inverting the curve (e.g. via root solver)

ML Approximation

- In the same spirit, we sought to solve the problem with **ML**
- We used the **Lambert** transfer DVs as **attributes**, together with the **relative position & velocity** of the two asteroids, and the **initial mass**
- We made sure that the NN treats in the same way hops with the same geometry and distance from the Sun → hence why we used their **relative** positions as attributes
- We trained on **800,000** hops computed solving the OCP



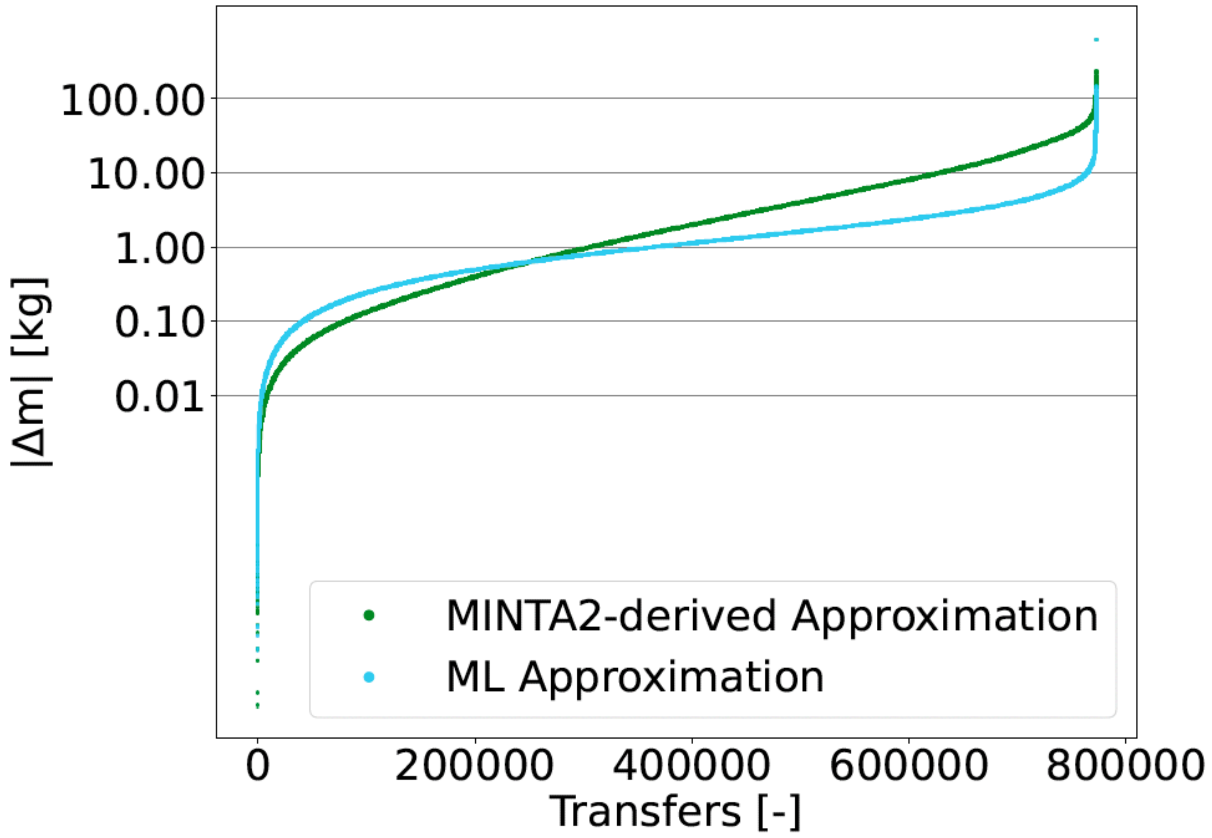
Humans vs Machines

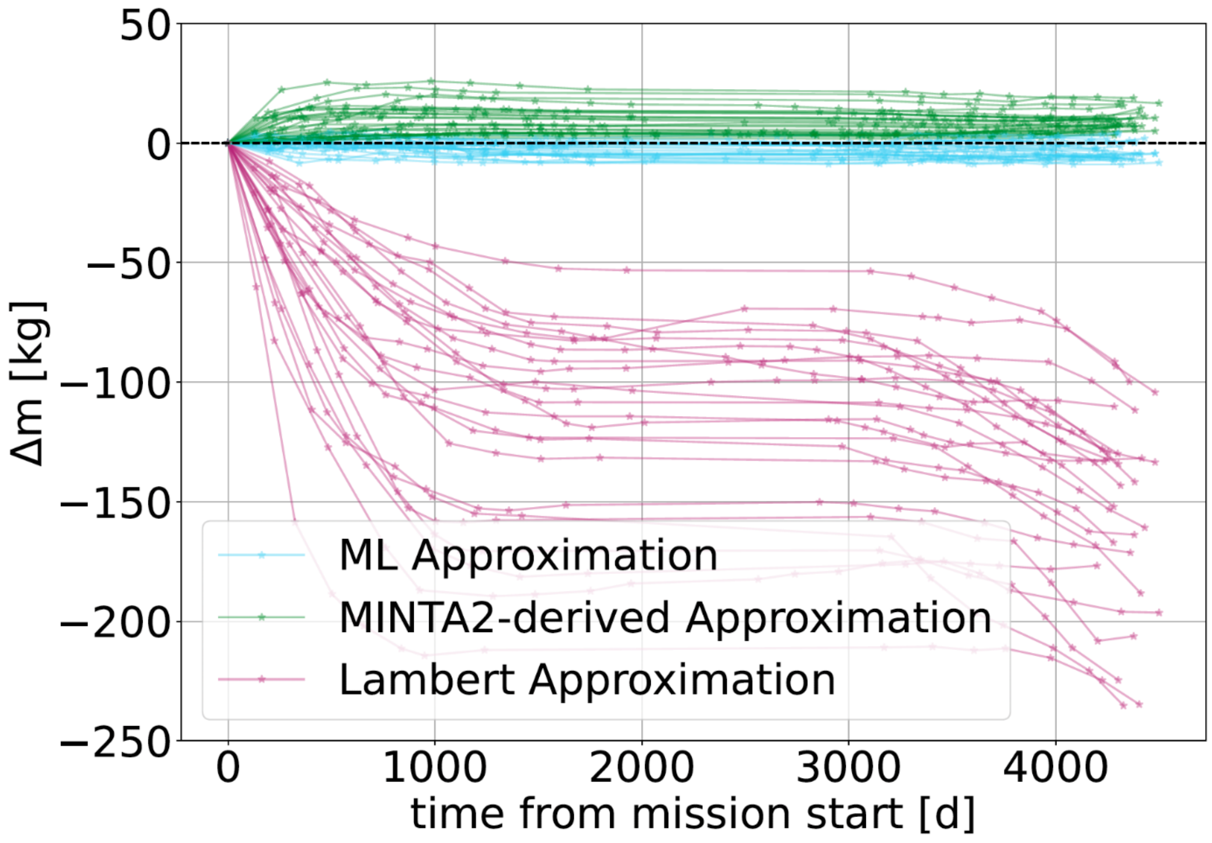
- We now obtained **analytical** & **ML** approximations
- The timings are good…. More than **100,000x** computational gain from the OCP solution
- The ML approximation is **3x** faster
- And in terms of **accuracy**? Do machines have an edge vs humans?



Humans vs Machines

- Errors remain for most cases below 1kg!
- They even maintain error ranges below 5/10 kg across 20 and more consecutive hops!

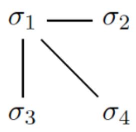




Choosing our Ships

- Having a **pool** of promising ships, we had to select a **subset** to make our final solution
- The objective was to **maximize** the cumulative **collected mass**, accounting for **penalties**, while adhering to the crucial **non-overlap condition**
- We mapped this as an **integer linear programming (ILP)** problem:

$$
\begin{aligned}
\mathcal{S} = \{ \\
&\sigma_1 = (300, 1, \{a, b, c\}), \\
&\sigma_2 = (75, 75, \{a, d\}), \\
&\sigma_3 = (75, 75, \{b, e\}), \\
&\sigma_4 = (100, 100, \{c, f\}) \\
\}
\end{aligned}
$$



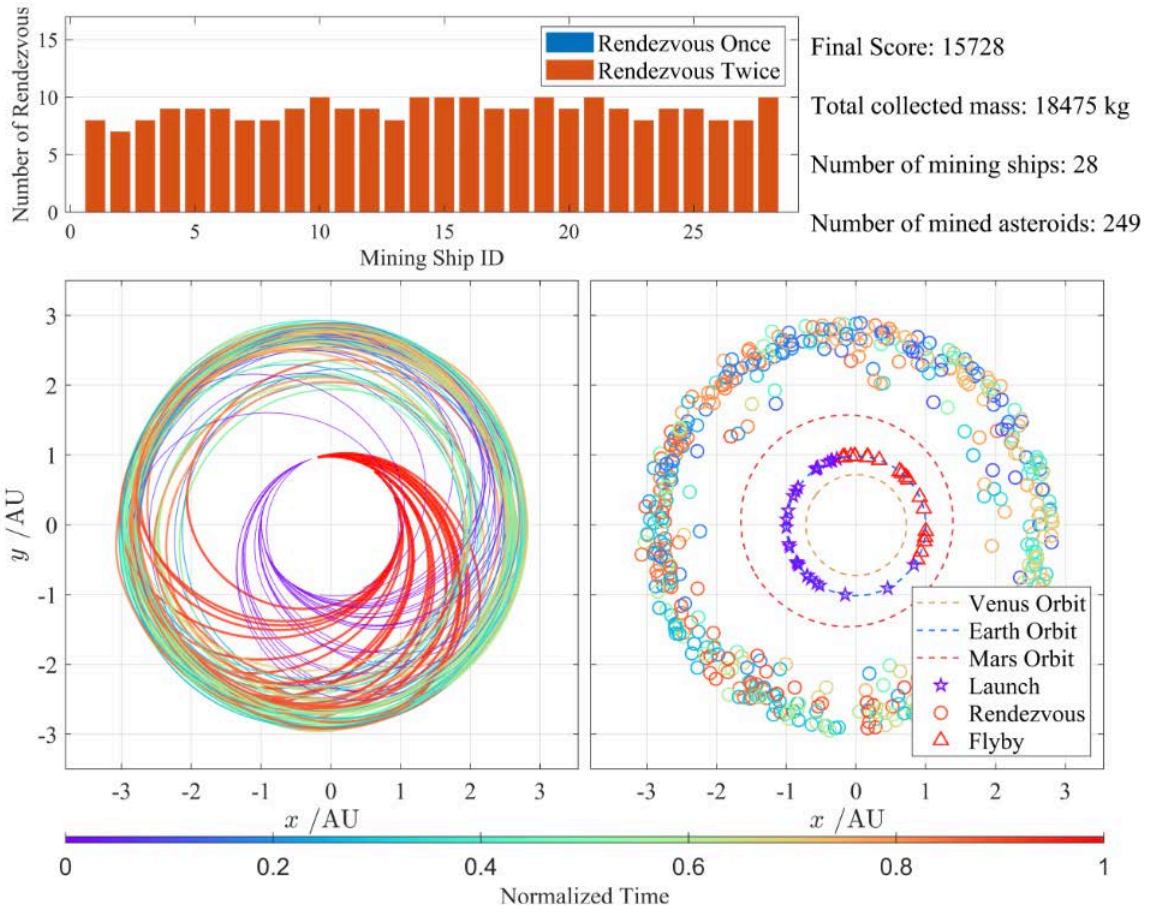
Maximize

$$1x_1 + 75x_2 + 75x_3 + 100x_4$$

Subject To

$$x_1 + x_2 \leq 1$$
$$x_1 + x_3 \leq 1$$
$$x_1 + x_4 \leq 1$$
$$x_1, x_2, x_3, x_4 \in \{0, 1\}$$

- We solved this problem using a two-pass algorithm utilizing **0-1 linear programming**
- We also added the possibility to automatically **remove** an **asteroid**
- In this way we constructed our **final solution**



Number of Rendezvous
Rendezvous Once
Rendezvous Twice
Mining Ship ID
Final Score: 15728
Total collected mass: 18475 kg
Number of mining ships: 28
Number of mined asteroids: 249
y /AU
x /AU
Venus Orbit
Earth Orbit
Mars Orbit
Launch
Rendezvous
Flyby
Normalized Time

Thank you!