GTOC12: Sustainable Asteroid Mining

TheAntipodes

GTOC12 Workshop
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Team & Roles

Roberto

- Preliminary analysis
- Team coordination

Jack

- Asteroid groups
- Beam search
- Trajectory optimization
- Post-competition developments

Harry

- Mixing chains
- Interface

Minduli

- Trajectory optimization
- Mission processing

Andrea

- Beam search

Cristina

- Mixing chains
- Solution selection
- Asteroids groups

Xiaoyu

- HPC

Overview

Strategy

Trajectory Optimization

Asteroid Subsets & Chains

Solution Selection

Post-Competition Developments

Strategy

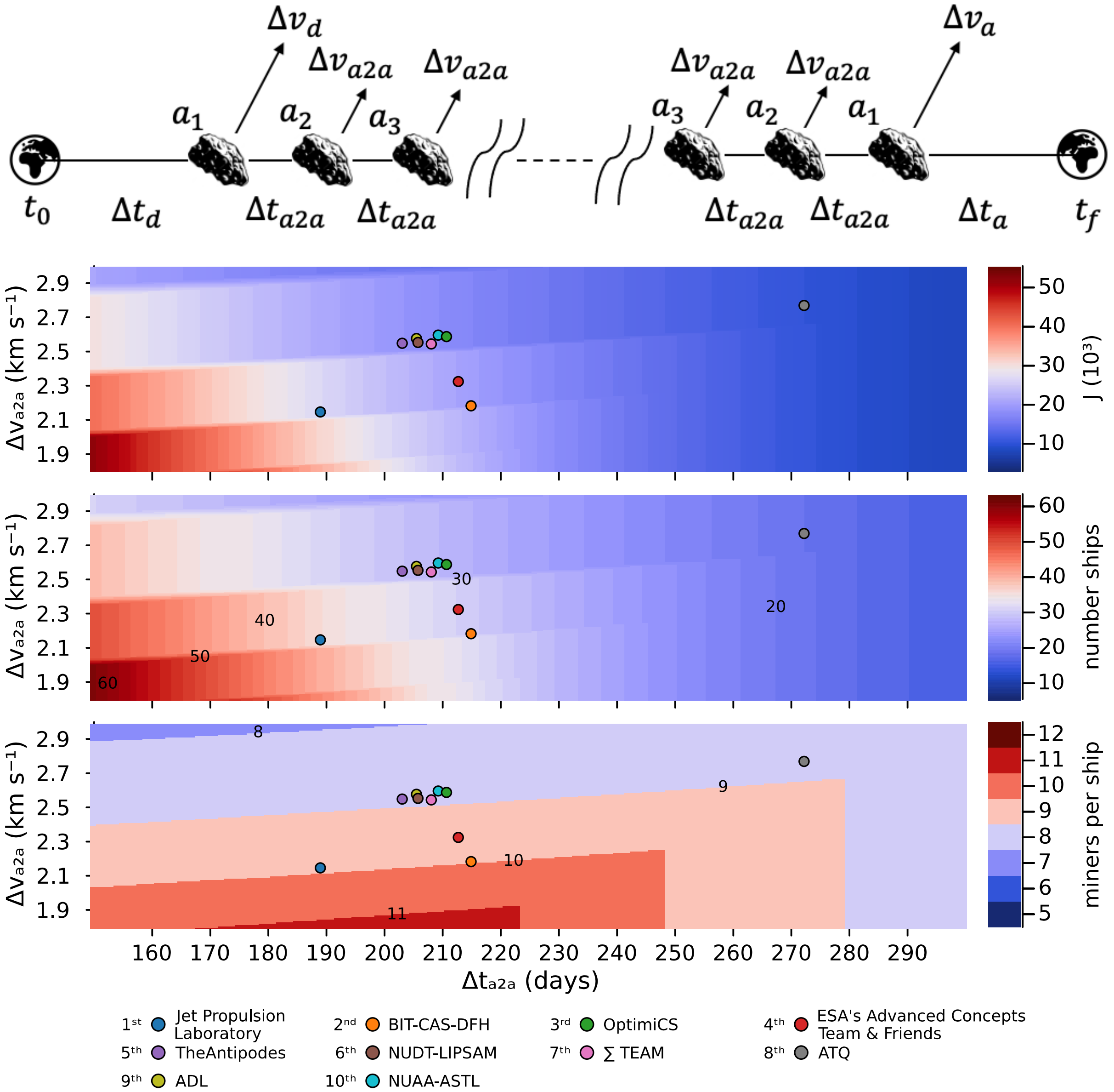
Preliminary Analysis

Simplifications

- All missions are the same
- Self-cleaning mission with collection in reverse order
- Departure and arrival legs have same $\Delta t_{d/a}$ and $\Delta v_{d/a}$
- All asteroid hops have the same Δt_{a2a} and Δv_{a2a}

Outcome

- Solutions with ~40 ships seemed achievable
- No more than 10 miners per ship
- Self-cleaning missions have some potentials (considering the small size of the groups)



Strategy: The Plan

Asteroid Pruning	Beam Search	Mixing	Refinement

• Reduce the asteroid population by **pruning** on
  • Orbital elements
  • Departure and arrival phasing
	• Independent **forward / backward** chains construction
	• Use of **low-thrust surrogate**
		• GA optimizer to build **mixed missions**
		• Score maximization **including bonus**
			• Low-thrust optimization with **indirect method**
			• Final manual removal/addition of asteroids

Strategy: The Reality

Asteroid Grouping

- Create **groups of asteroids** based on average minimum Lambert cost

Beam Search

- Forward and backward beam search to build **self-cleaning missions**
- Lambert cost with **heuristic penalty** (feasibility issues)

Trajectory Optimization

- Fixed-time **low-thrust** trajectory optimization using **successive convex optimization**
- Manual optimization of **rendezvous times** to increase returned mass

Solution Selection

- GA or integer programming to **select the subset of missions** to maximize the score (including bonus)

Asteroid
Subsets & Chains

Subset Generation

1. Apply loose pruning cuts to asteroid dataset to reduce size

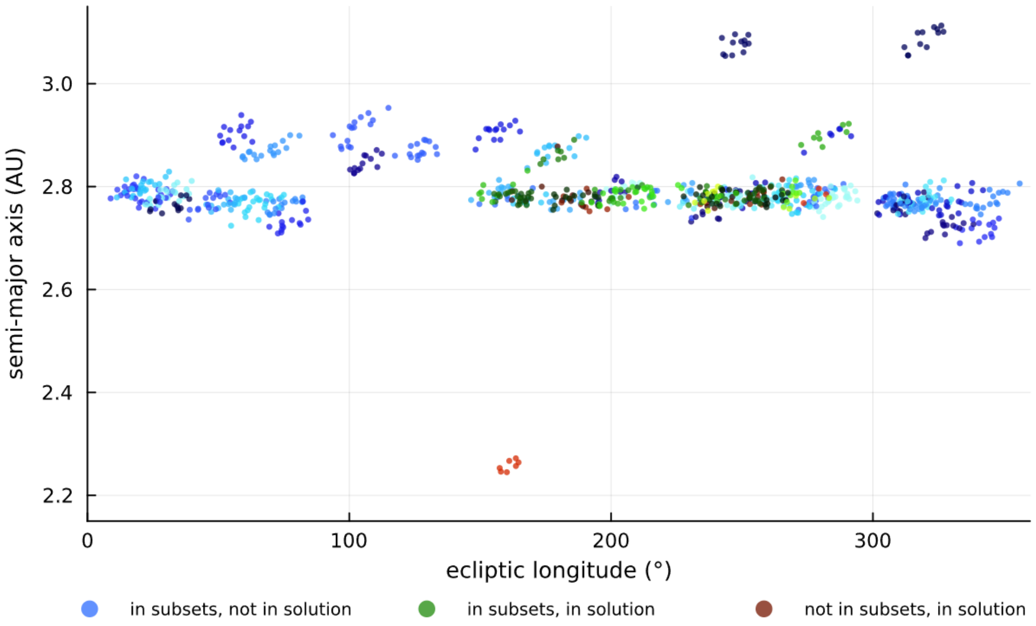
20% of asteroids cut

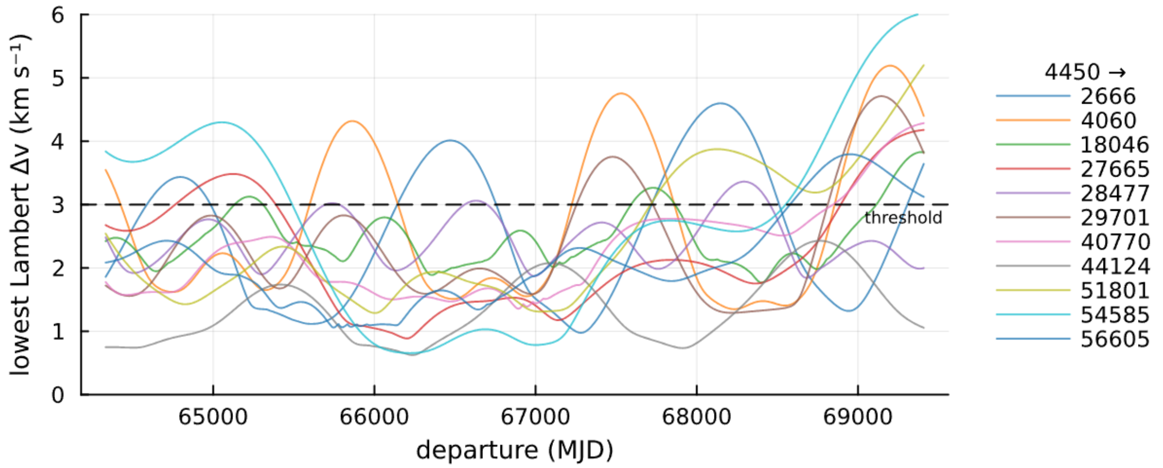
2. Calculate the *average lowest* Lambert Δv between each asteroid and its close neighbours throughout the competition timeframe with transfer time-of-flight from 150 to 400 days

3. Create subsets of 'near' asteroid based on a threshold

Subsets are generally **not** independent

- Combination strategies for independent subsets were explored but not extensively used in the submitted solution





Beam Search for Chain Generation

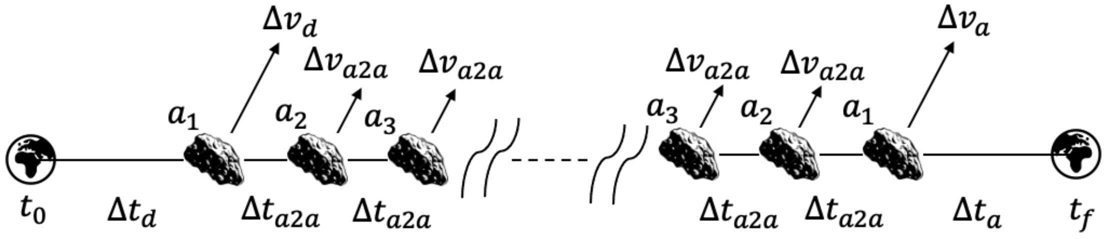
Construction is performed on each subset **one asteroid at a time** until the ship returns to Earth

- Branching new trajectory legs
- Selecting promising options for further branching
- Search also performed with reversed collection

Lambert Δv is used to cost each arc with time-of-flights between

- 500-700 days for initial and terminal transfers (adjusted Δv based on the 6 $\mathrm{km\,s^{-1}}$ provided)
- 170-250 days for intermediate transfers

Heuristic is introduced to try **improve feasibility** in low-thrust refinement



Asteroids	Beam Search	Direct Solution	
		Mined Mass (kg)	Fuel Remaining (kg)
7	lambert	609.6	−195.4
	heuristic	571.9	126.2
8	lambert	632.6	−258.6
	heuristic	620.5	−50.1
9	lambert	671.6	−391.2
	heuristic	653.6	−163.2

$$\text{tof} \geq \begin{cases} c_1 \sqrt[3]{\Delta v} & \text{(initial transfer)} \\ c_2 \sqrt[3]{\Delta v} & \text{(terminal transfer)} \\ c_3 \Delta v (1 + m_{\text{current}}/m_{\max}) + h c_4 & \text{(deploying)} \\ c_5 \Delta v + h c_6 & \text{(collecting)} \end{cases}$$

Trajectory Optimization

Sequential Convex Programming

Sequential Convex Programming (SCP) can be used to solve the low-thrust trajectory optimization problems in GTOC12 **efficiently**

- Linearization around a reference trajectory using the State Transition Matrix (STM)
  - Using Lambert-based reference
  - Drawback: limited accuracy of this approximation

- Allows for a convex formulation of the trajectory optimization problem
- The key step is to **repeatedly linearize and resolve** until the dynamics are accurately expressed

$$\mathbf{x}_{n+1} = A_n \mathbf{x}_n + B_n \mathbf{u}_n + C_n$$

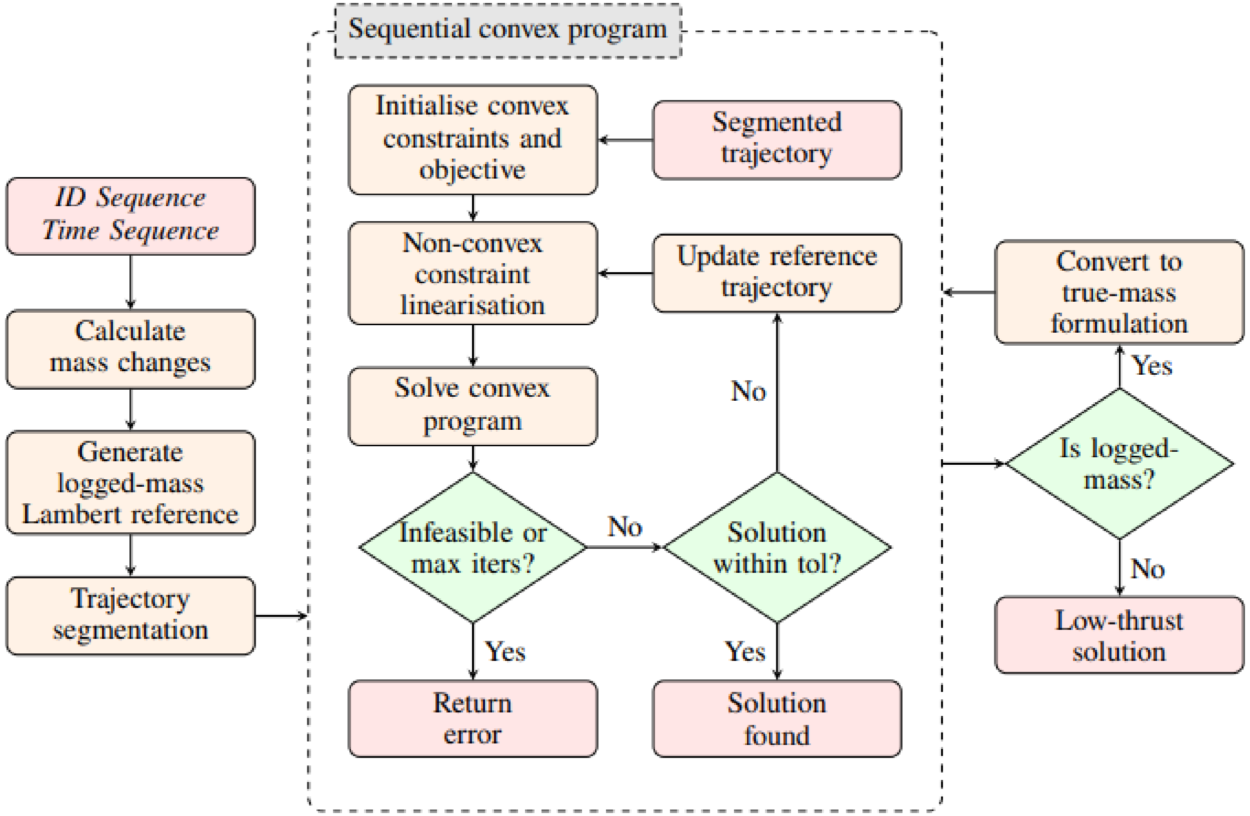
$$A_n = \frac{\partial}{\partial \mathbf{x}} \int_{t_n}^{t_{n+1}} \dot{\mathbf{x}}\,\mathrm{d}t \bigg|_{(\bar{\mathbf{x}}_n, \bar{\mathbf{u}}_n)}$$

$$B_n = \frac{\partial}{\partial \mathbf{u}} \int_{t_n}^{t_{n+1}} \dot{\mathbf{x}}\,\mathrm{d}t \bigg|_{(\bar{\mathbf{x}}_n, \bar{\mathbf{u}}_n)}$$

$$C_n = \bar{\mathbf{x}}_n - A_n \bar{\mathbf{x}}_\mathbf{n} - B_n \bar{\mathbf{u}}_\mathbf{n}$$

`DifferentialEquations.jl`
`+ ForwardDiff.jl`
= **Fast and flexible STM calculation**

Convex Program

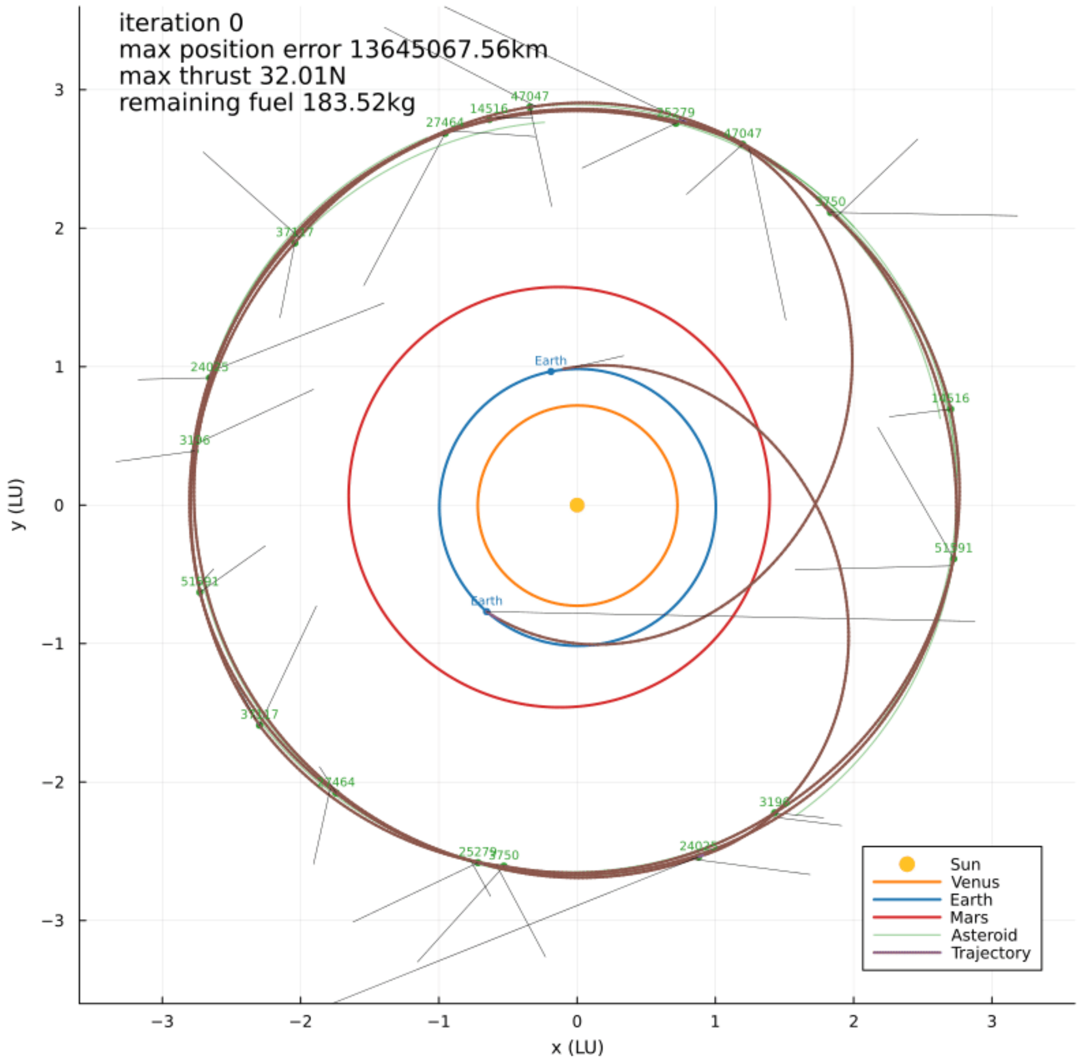


- All legs are linked via initial and terminal constraints with a fixed time sequence so that the **entire trajectory can be solved at once**
- Dynamics expressed in a **logged-mass form** helps to improve convergence
- The Δv provided in the initial and terminal Earth legs can be included in the state constraints. GA are included in a similar manner but were not used

Refer to paper (in review) for implementation specifics

Performance in GTOC12

ID Sequence	[0, 25279, 27464, 24025, 37117, 3750, 3196, 51591, 47047, 14516, 14516, 3750, 47047, 37117, 3196, 51591, 27464, 25279, 24025, -3]
Time Sequence (MJD)	[64458.0, 64983.0, 65148.0, 65388.0, 65643.0, 65848.0, 66043.0, 66258.0, 66588.0, 66773.0, 68023.0, 68188.0, 68448.0, 68648.0, 68808.0, 68908.0, 69083.0, 69193.0, 69353.0, 69807.0]



Example

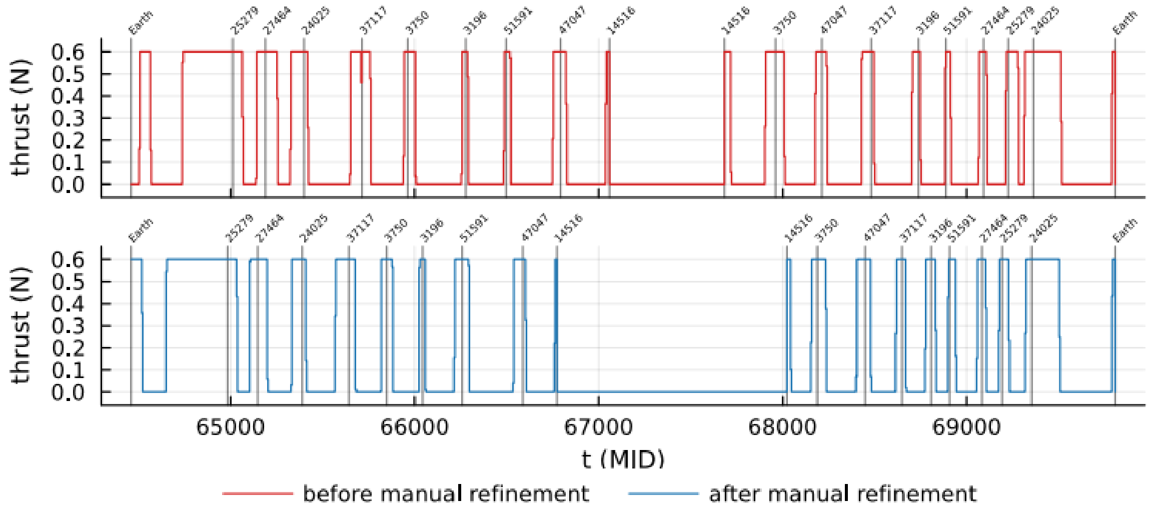
- Best (self-cleaning) ship included in our solution
- 711.29kg returned mass 9-asteroid chain
- 5-day discretization used with zero-order-hold control interpolation

~0.5 seconds for the entire low-thrust solve process per ship

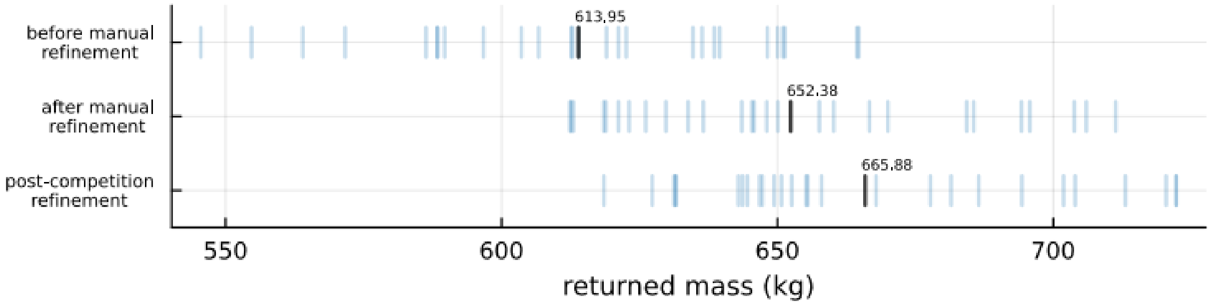
Manual Refinement

Trajectories from the beam search tended to have excess/missing fuel at termination, so we **manually edited the rendezvous times** to increase mined mass whilst remaining feasible.

- Large increase in average mined mass per ship (613.95kg to 652.38kg)
- NOT optimal in **two ways**: our time, and the solution



649.99kg refined to 711.29kg



Post-Competition refinement later

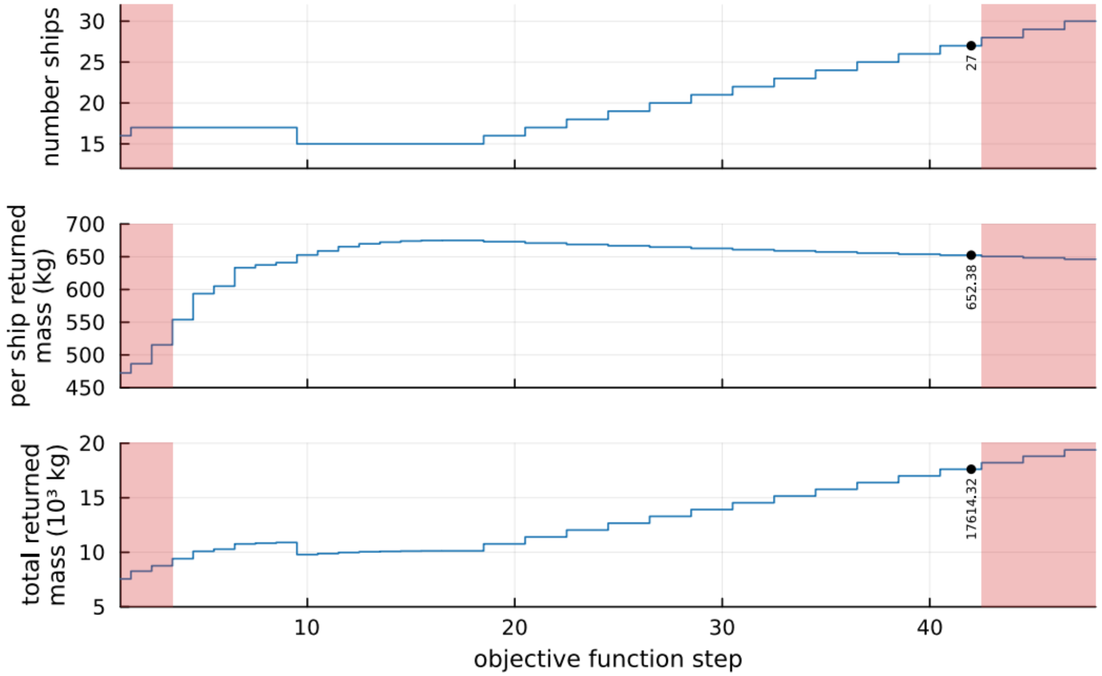
Solution Selection

Selection Approach

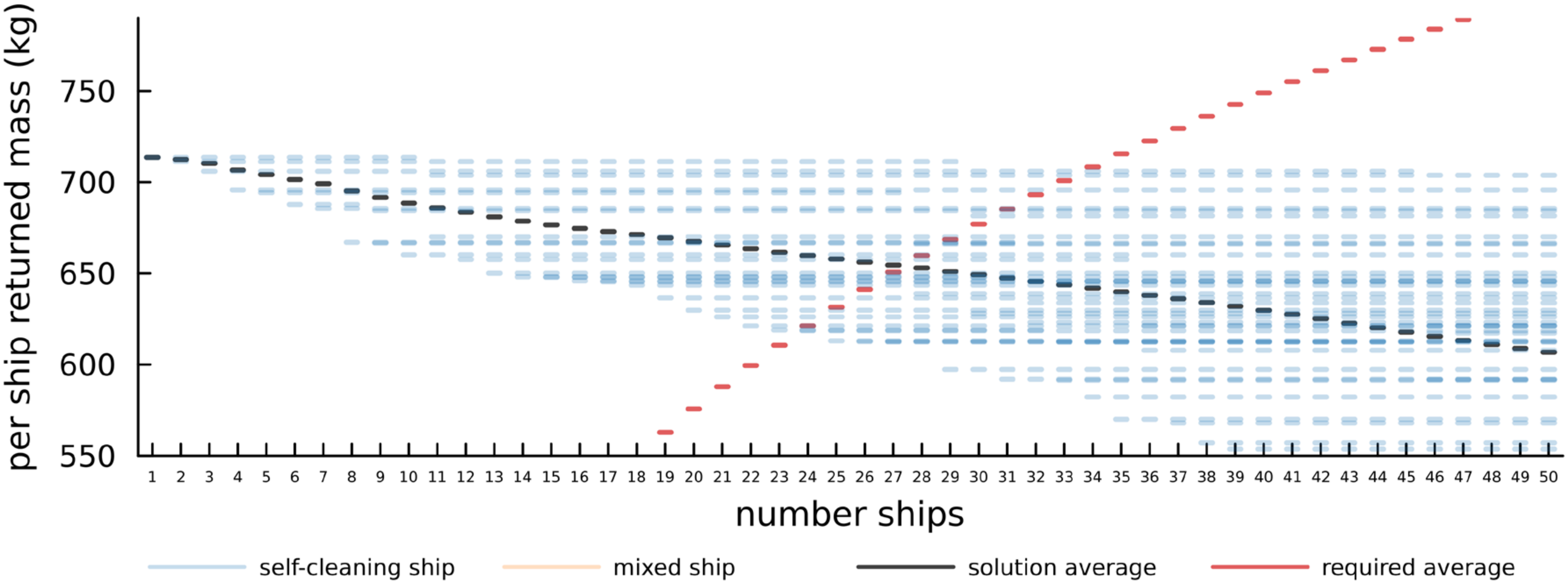
Methodology

- GA optimizer to select the mining ships from the pool of solutions
- Half of the ships are randomly selected initially
- Repeated cycles of adding and swapping ships based on the average mass as a ranking function
- Objective function is the total mined mass

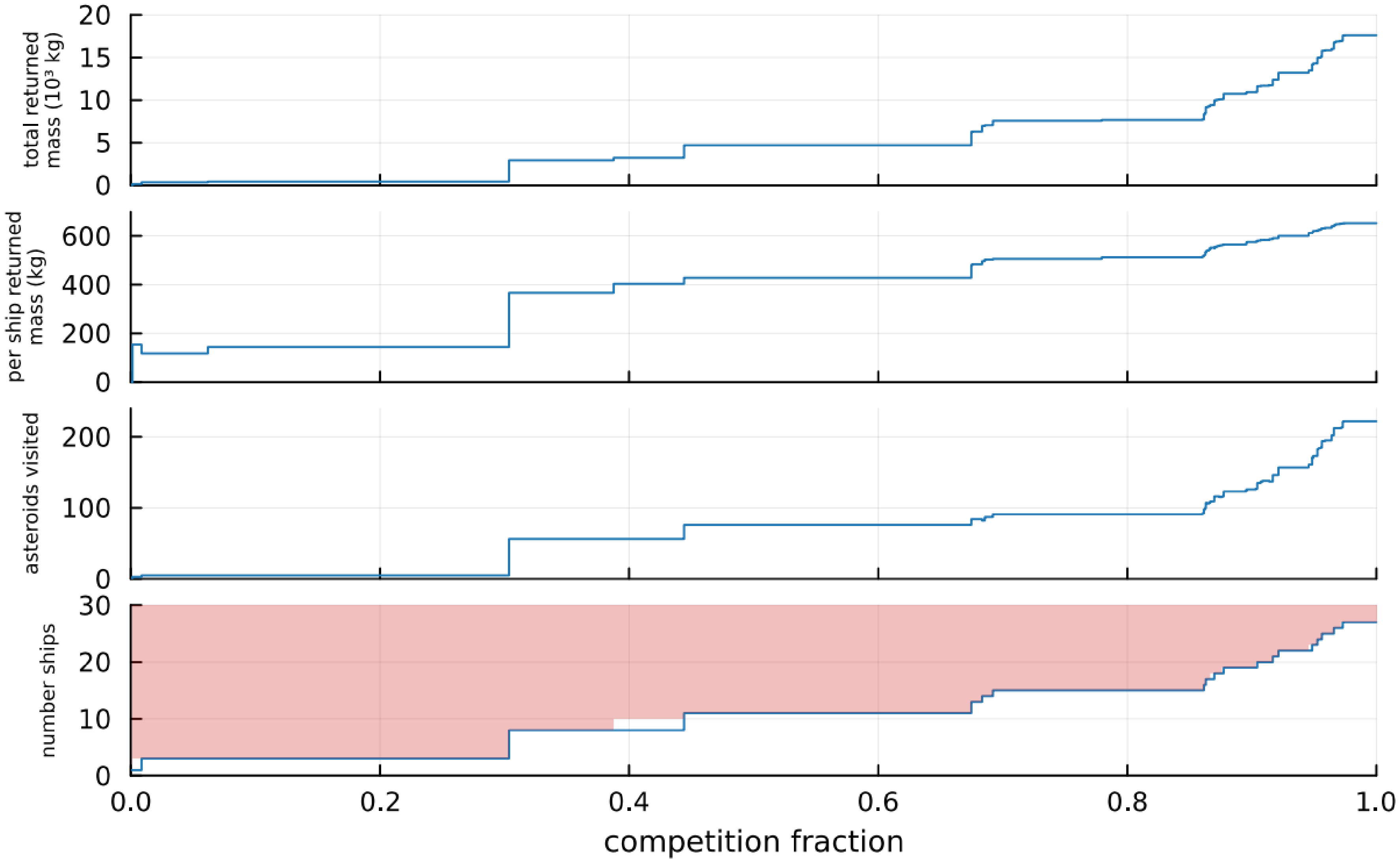
- Algorithm initially developed for combining deployment and collector sequences
- Mixed-integer programming provides superior performance and guarantees of optimality



Selection Result



Solution Timeframe

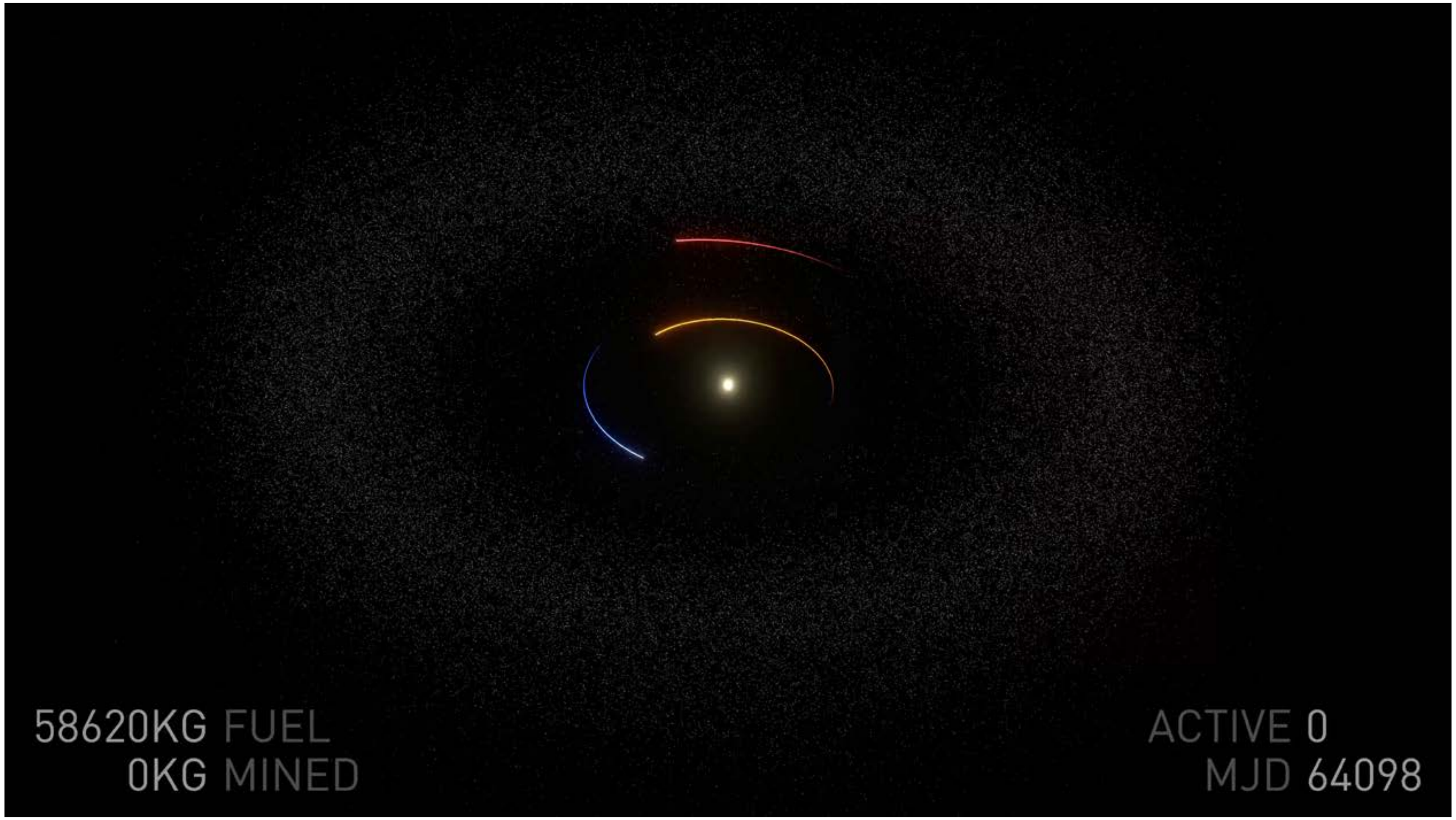


Submitted Solution

- **5th in competition**
- 27 self-cleaning ships
- 222 asteroids mined
- 8.22 ratio (222/27)
- Average mass returned per ship was **652.38kg**
- Very limited HPC use; majority of calculations using our home machines

Time of submission:
July 16, 2023 01:23 NZT

J = 15488.896



Post-Competition Developments

Problems in Approach Taken

Ship Selection

- algorithm slow and not necessarily optimal
- tends naturally to a mixed-integer programming approach

improved ***first***

Beam Search

- only a local optimization routine
- no support for mixed solutions
- reliant on custom heuristics

improved ***last***

Manual Refinement

- extremely time consuming and not at all optimal
- some ideas with convex

improved ***next***

Strategy: Post-Competition

Asteroid Grouping

- Create **large groups of asteroids** based on a **range** of statistics

MIP-based Chain Optimization

- Custom MIP formulation similar to the **optimal flow problems**
- Provides the best n chains **based on Lambert Δv**
- Able to generate mixed chains

Trajectory & Rendezvous Optimization

- Variable-time **low-thrust** trajectory optimization using **successive convex optimization**
- Finds optimal rendezvous times that **maximize returned mass**

Solution Selection

- MIP-based solution selection to handle **large amounts** of self-cleaning and mixed ships

Ship Selection with MIP

Mixed Integer Formulation

- Binary variable for each possible ship
- Constraints based on asteroids

- Benchmark: obtain all optimal ship combinations of size **1-50** from **580** mixed and self-cleaning ships
  - Gurobi (Commercial) = 0.95s
  - HiGHS (Open-Source) = 1.40s

Either case: optimal ship selection is fast

$$\text{maximize } \sum ms$$

$$\sum s_{s\in g} = \mathrm{len}(g) \quad \forall g$$

$$\sum c_{a,s} s \leq 2 \quad \forall a$$

$$\sum s \leq N$$

$s =$ ships (binary variable)

$m =$ returned mass per ship

$a =$ asteroids (1:60000)

$g =$ ship groups

$N =$ number ships

$c_{a,s} =$ visited count of a by s

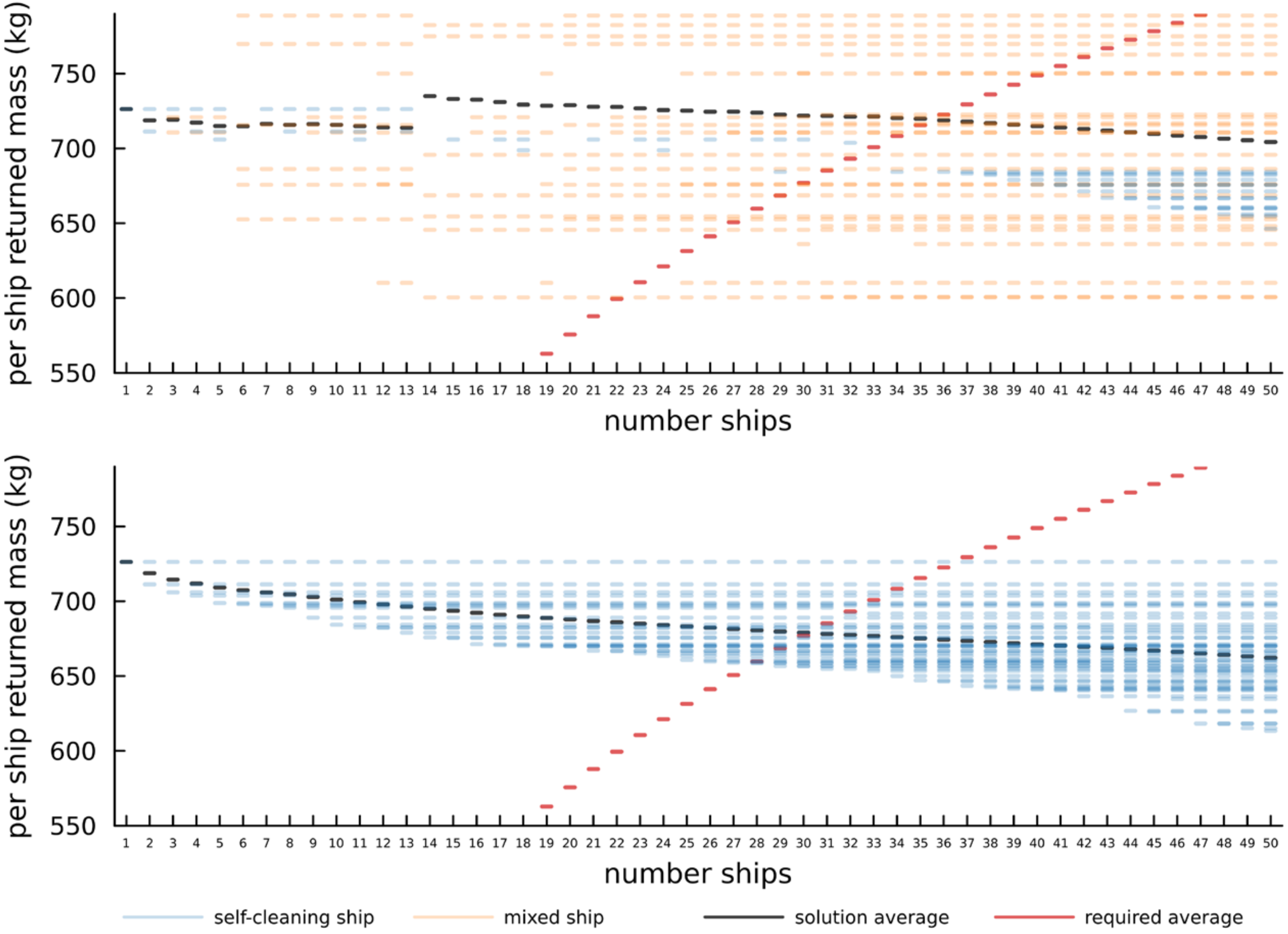
Combined Top 10 Solutions

All solutions

- 35 ships (NC)
- 719.79kg average
- Optimal combination:
    - 35x JPL

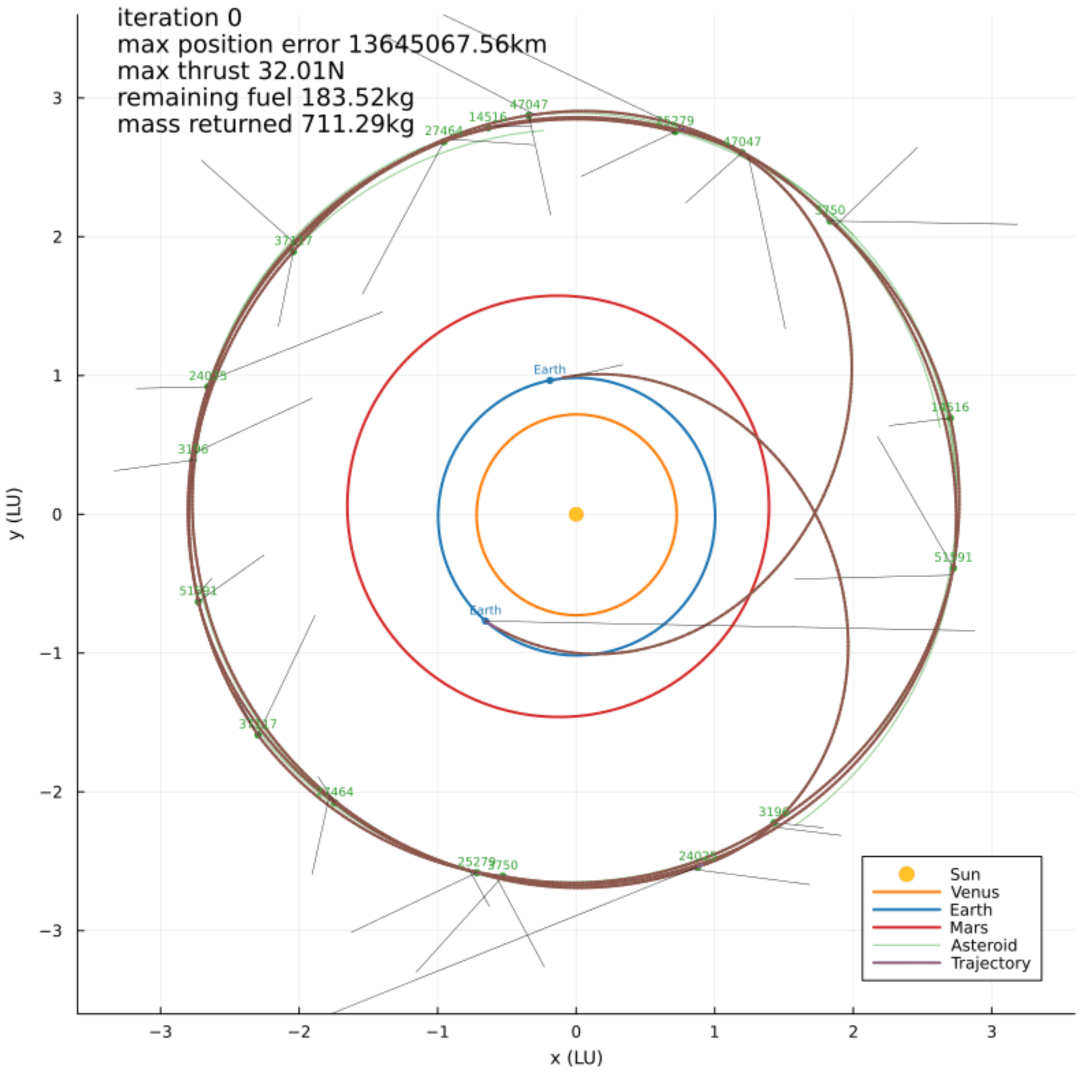
Only self-cleaning

- 30 ships (+2)
- 679.03kg average
- Optimal combination:
    - 10x OptimiCS
    - 8x ESA
    - 7x TheAntipodes
    - 2x NUDT-LIPSAM
    - 3x ∑ TEAM



Adaptive-Mesh Convex Optimization

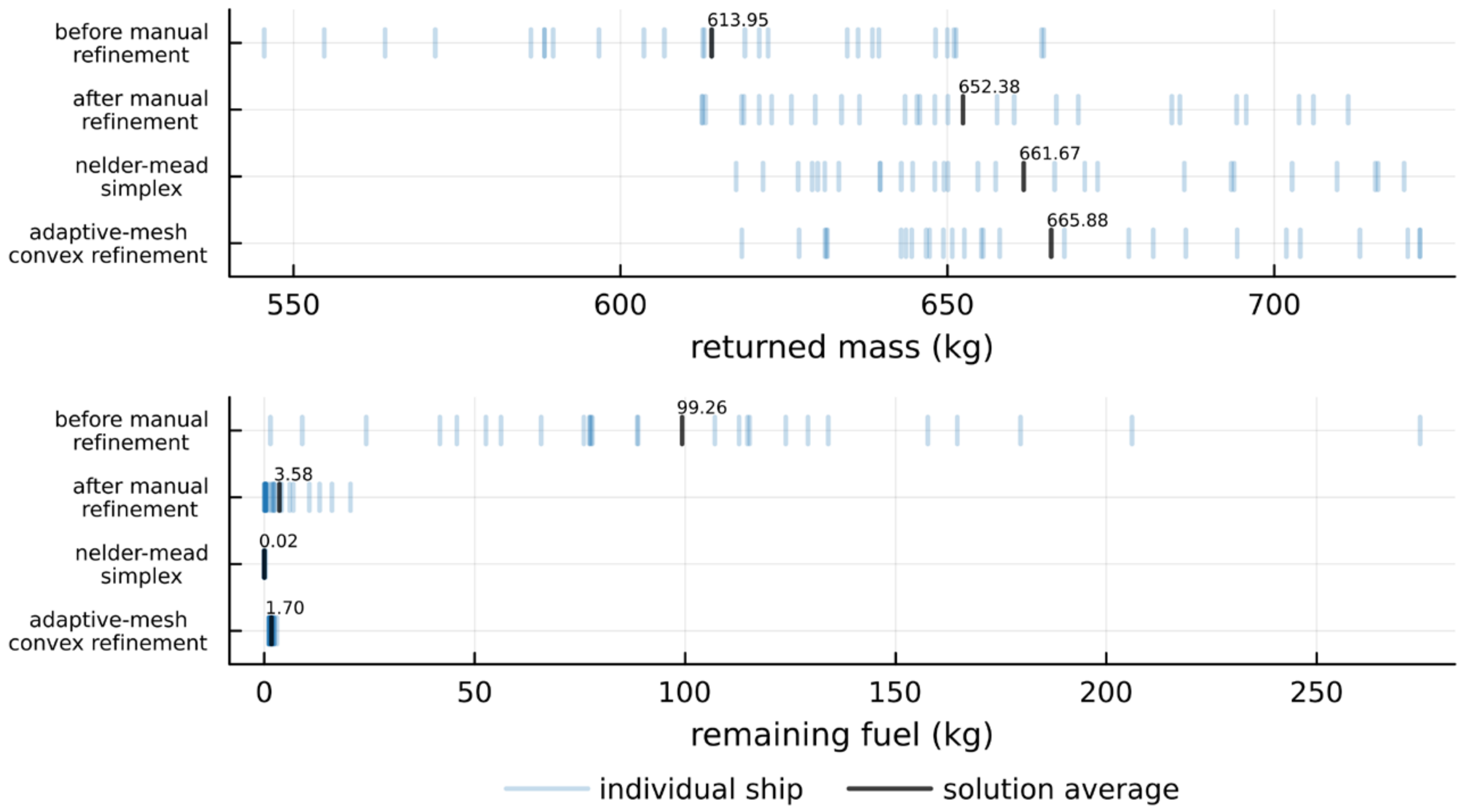
- The SCP algorithm is modified to add the time discretization as a decision variable in the optimization, allowing the SCP to **operate with an adaptive mesh and non-fixed rendezvous times**
- Achieved by adding a ‘time delation’ parameter into the STM and linearizing the rendezvous position constraints
- Convergence is more problematic (needing more iterations) due to increased nonlinearity but can be achieved using trust regions



SCP can optimize multi-target rendezvous timing

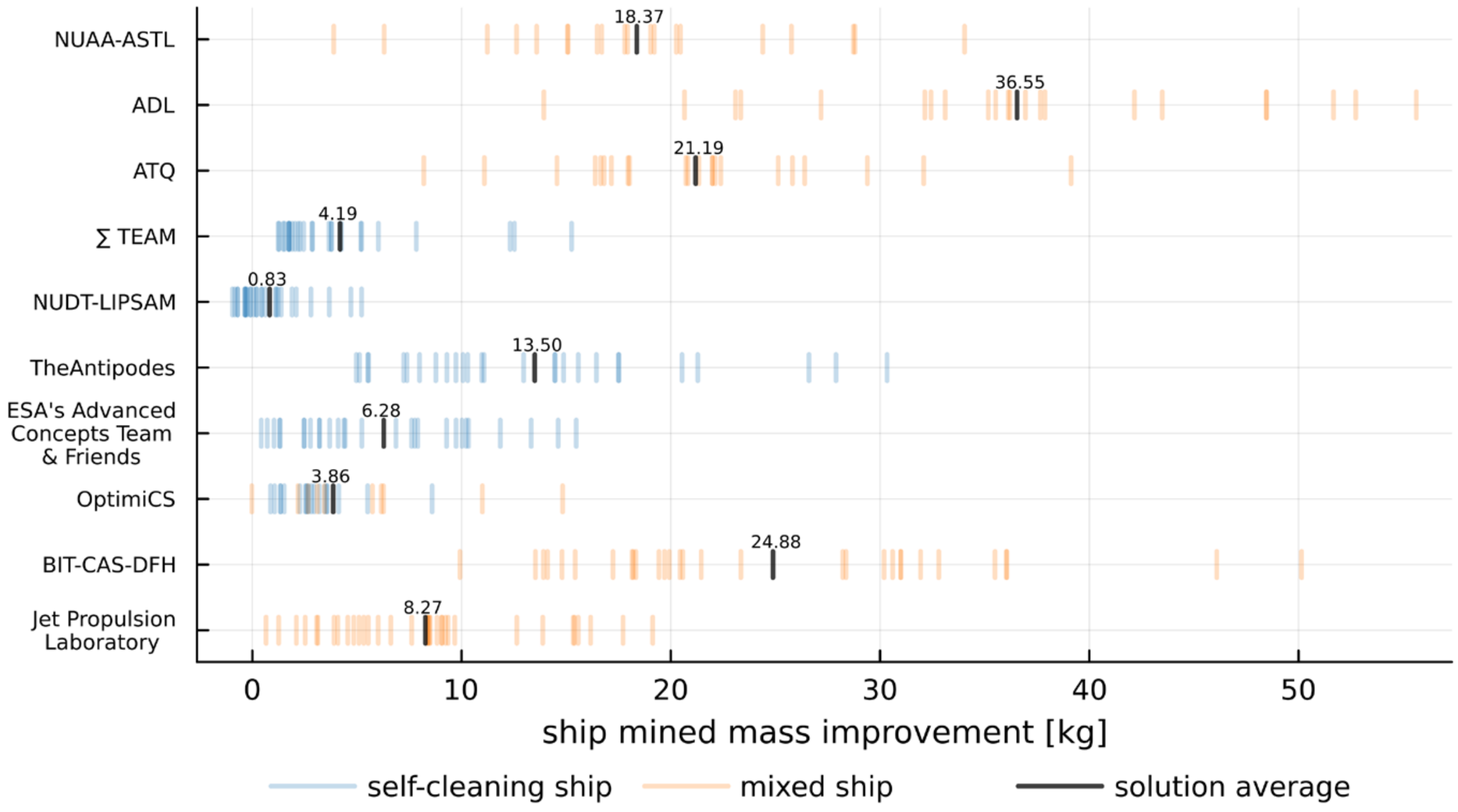
Application to Our Solution

- Nelder-mead simplex local optimizer for comparison, with decision variables of rendezvous times
- **~1h run time** for Nelder-mead per ship
- **<20sec run** time per ship for 20-day convex dynamic discretization



Application to Top 10 Solutions

- All optimization performed with **20-day** dynamic discretization
- Most teams have significant available improvements
- Mixed solutions tended to have greater improvements
- Further improvement (~0.8kg/ship) available with change to 5-day adaptive discretization

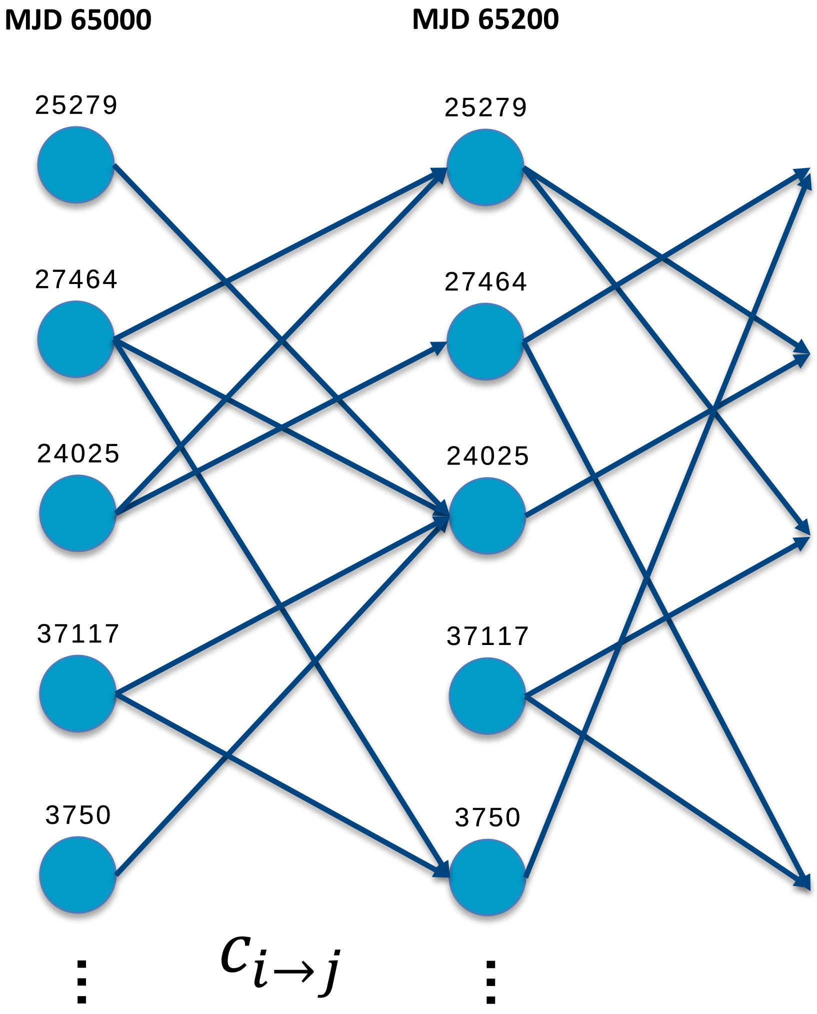


MIP Process for Chain Creation

MIP for chain creation is based on the design of **binary optimal flow problems**

1. Select subset to operate within
2. Choose number of deployments and collections, and how many chains to create
3. Choose a fixed time schedule for rendezvous
4. Calculate the transfer Δv for all possible transfers at each stage of the trajectory (used Lambert Δv but can be low-thrust)
5. Initialise a binary variable for the transfers that are ‘cheap’ (below some limit)
6. Setup constraints on binary variables (e.g. self-cleaning, deploy + collect) and solve MIP
7. Optimize time schedule of solution with time-refinement, and repeat process from step 4

Repeated across many subsets to develop new solution sets

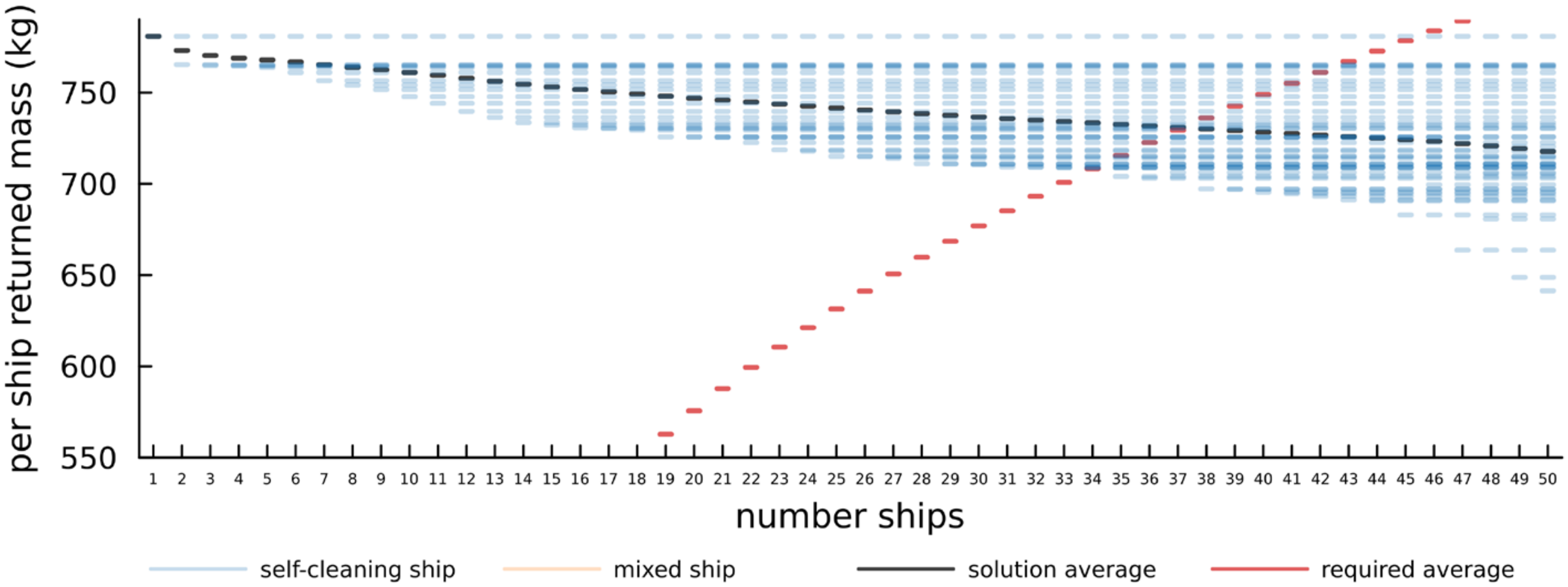


Best Self-Cleaning Solution Found

- 37 ships, 338 asteroids (9.14 ratio)
- 730.95kg average (729.44kg needed for 37 ships)
  - Mixture of 8, 9 and 10 asteroid chains
  - Highest self-cleaning chain 780.84kg (10 asteroid)
  - All ships return >700kg

```
Check successfully!
The number of mining ships is 37 ;
The number of mined asteroids is 338 ;
The total resource mass is 27045.3 kg.
Press any key to continue . . .
```

J = 22847.922

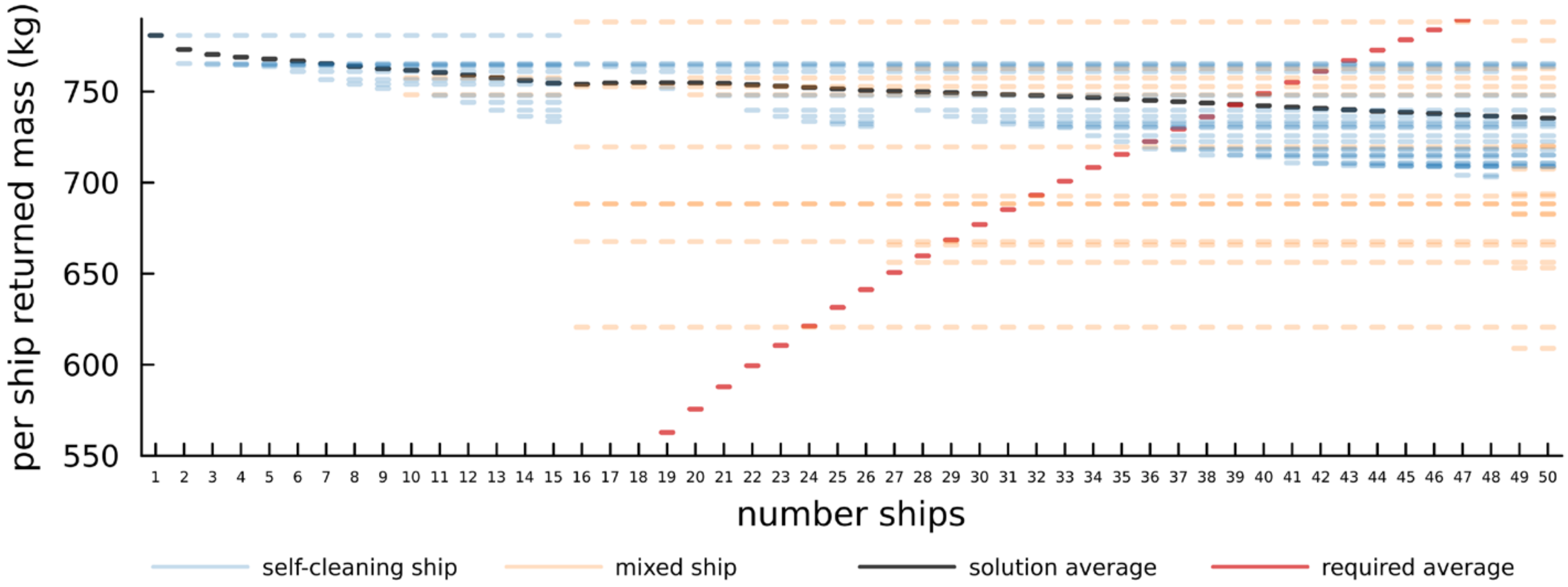


Best Solution Found

- 39 ships, 356 asteroids (9.13 ratio)
- 742.95kg average (742.60kg needed for 39 ships)
    - JPL group 1 (14 mixed) modified (average 752.52kg)
    - JPL group 2 (6 mixed) modified (average 735.54kg)
    - Custom 2 mixed (average 752.90kg)
    - 17 custom self-cleaning (average 736.52kg)

```
Check successfully!
The number of mining ships is 39 ;
The number of mined asteroids is 356 ;
The total resource mass is 28975.1 kg.
Press any key to continue . . .
```

J = 24474.156



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Independent Subset Generation

Methodology

- Using graph theory to create independent subsets
- Adjacency matrix is sparse but can be reordered so that connected asteroids are near to each other
- Asteroid set can then be pruned based on this criterion
- Post-competition we found that the MATLAB function `symrcm` provided a superior result