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1st ACT global trajectory optimization competition: Results found at GMV

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Abstract

We expose the rationale and methods that our team used to design the trajectory presented to the subject competition, and discuss our results. We identified a candidate EEVEEJSA trajectory that was comprised of (1) a priming EEV phase, (2) a VEE tour to prepare the departure to Jupiter, and (3) a EJSA phase that ended with an almost head-on collision at target perihelion. For every phase, a trajectory was designed that consisted in coast and thrust arcs, and swings by a planet. The design started with the final EJSA phase, and proceeded backwards to the other phases, with terminal conditions imposed by the initial conditions of the next phase. The merging of the three phases produced a feasible mission, which was refined with a local optimizer. The short mission duration (17 years) made possible to add one extra revolution in the Saturn-asteroid orbit to improve slightly the impact conditions and still satisfy the mission duration constraint. The resulting trajectory would impact the target on June 15th, 2039, at a closing speed of 49.1 km/s, after consuming 193.8 kg propellant.

1. Introduction

Were the target a near-Earth object (NEO), the problem proposed by the ACT would be of practical interest in the frame of a planetary defense against NEO threats. Kinetic deflection is one of the techniques proposed for mitigating NEO threats to the Earth [1]. The objective function proposed by ACT

$$J = m_f \cdot |(\vec{v} - \vec{v}_{\text{ast}})^{\mathrm{T}} \vec{v}_{\text{ast}}| \tag{1}$$

is a measure of the change in total mechanical energy of the target in the case of a perfectly inelastic impact, i.e., without ejecta, and would be a lower bound of the actual change in a hypervelocity impact. Changes in the specific total mechanical energy translate into semi-major axis changes, i.e., changes in the orbital period, and even the slightest change may cause a large Earth miss distance if the impactor arrives to its target well in advance of the date of closest approach of the NEO to the Earth. A more appropriate objective function in a case of planetary defense would have incorporated also the propagation time from spacecraft impact to the NEO encounter with the Earth.

The spacecraft itself is within the limits of current technologies. By combining nuclear power and electric propulsion, it is free to apply the same thrust at any heliocentric distance, and to deliver a large Δv in a mission of long duration. Although this offers great flexibility to design a trajectory, it also increases dramatically the complexity of the task.

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This paper summarizes the methods that we employed to solve the problem, and describes and discusses the results obtained at GMV.

2. Preliminary assessment

Considering the objective function to be maximized, it was clear that the preferred impact conditions were to have a frontal impact at the asteroid's perihelion, with the spacecraft in a retrograde orbit, coplanar with the orbit of the asteroid, with an aphelion as higher as possible, and impacting the asteroid at the spacecraft's perihelion. Since the final mass appears in the objective function, these impact conditions should be attained with minimum fuel expenditure. Hence, we discarded a solution based on a direct low-thrust trajectory (without planetary swingbys), since too much fuel is needed to reach retrograde motion exclusively using propulsion. On the other hand, low-thrust was surely required at least at the beginning of the trajectory, due to the low value of the maximum departure velocity from the Earth (2.5 km/s) that, with the exception of very special cases, does not allow a spacecraft to reach neither Venus nor Mars. Therefore, our choice was to look for a hybrid trajectory combining low-thrust and gravity assists.

We agreed on the need to make use of gravity assists with the giant planets, because of the large achievable gain in ΔV. In order to attain retrograde motion, a final swingby with Saturn is preferred because: (1) it is further from the Sun than Jupiter, (2) due to the much lower allowed minimum pericenter altitude, the deflection angle is higher. A retrograde orbit with an aphelion as high as the radius of the orbit of Saturn and a perihelion equal to the asteroid perihelion will produce an impact with a relative impact velocity of approximately 50 km/s. Since this velocity is greater than the characteristic velocity of the low-thrust engine ($g_0\ I_{sp}$), thrusting at the end of the last leg of the trajectory is not efficient, since the objective function would decrease more because of the fuel consumption than would increase because of the velocity gain.

We also realized that a Jupiter swingby was very convenient to reduce the cost of reaching Saturn, and that a series of Earth, Venus, and/or Mars gravity assists were useful to gain the energy required to arrive at Jupiter.

3. Methodology

3.1. Approach

After having sketched a trajectory, the resulting problem was still complex due to the broadness of the search space: the launch window spans for up to 20 years, and the mission duration can be as long as 30 years, all this for a trajectory with multiple gravity assists and multiple low-thrust arcs. Considering the tools available, we decided to take a staged approach to produce a trajectory: first divide the problem in phases and design a good initial guess for each phase trajectory, and then merge the phases and optimize them together.

3.2. Numerical tools

We identified which in-house software tools were available at GMV and could be applied to the design of trajectory phases and to the optimization of the complete trajectory, after minor changes to use the exact data values imposed by the ACT. We describe them briefly next.

3.2.1. MITRADES

MITRADES (*MATLAB® Interactive TRAjectory DESign*) is an interactive environment for the design of interplanetary trajectories consisting of multiple heliocentric ballistic arcs, percussive swingbys and impulsive maneuvers. A mission is fully described by a sequence of events (such as launch, flyby, swingby, deep-space maneuver or DSM, and arrival to, or in-orbit injection around a body) and the set of parameters defining every event (e.g., date, mass, position, velocity, speed, angles, number of complete orbital revolutions in an arc, etc.). Additionally, a trajectory must satisfy some mission constraints, which are formulated here as bounds on mission parameter values or on the spacecraft state at mission events.

Being MITRADES an interactive tool, the human analyst is present in the design loop. The analyst initiates the design process by entering or retrieving:

- The sequence of events.
- Tentative values and bounds of the mission parameters.
- The selection of mission constraints.
- The merit function to be used to rank the trajectory trials.
- The mission parameters to be optimized
- The choice of an optimization method, currently restricted to either the exhaustive search in a grid or to a local optimizer.

The on-going results of the optimization are displayed in real time to the analyst, which may intervene at any time to redefine the problem (e.g., the sequence of events, the choice of optimizable mission parameters, or the set of constraints) or to veer the optimization (e.g. from a global scan to local refinement).

Some low-level functions are particularly relevant to trajectory optimality and accuracy, and to speed of execution: an efficient Lambert problem solver [2], a robust local large-scale non-linear programming solver (IPOPT [3]), a calculator of ΔV-optimized gravity-assist maneuvers, and utility routines to read JPL DE-405 ephemeris files.

3.2.2. GlOptImp

GlOptImp stands for *GLobal OPTimization of IMPulsive trajectories*. It performs a global search of ballistic trajectories in the Solar System, including multiple planetary swingbys, either ballistic or assisted, and impulsive deep-space maneuvers. GlOptImp makes use of a genetic algorithm to find the best trajectory in the search space, the ranking being in accordance with the selected objective function.

GlOptImp uses a trajectory structure similar to the one in MITRADES. The genetic algorithm Pikaia [4] optimizes the dates of the events (departure, arrival and intermediate swingbys) and the dates and positions of the deep space maneuvers. Due to the stochastic nature of the optimization with genetic algorithms, the "optimal" trajectory found is only a first guess for subsequent local optimization with a deterministic local optimizer. In fact, the past use of GlOptImp (see [5,6]) has been to quickly provide other tools with good initial guesses. In the ACT competition, GlOptImp has complemented MITRADES by allowing the analyst to search in optimization spaces of large dimensions, unsuited for the exhaustive search implemented in MITRADES.

3.2.3. MerPro

MerPro optimizes finite-thrust exoatmospheric trajectories, either heliocentric or around a planet. The designer may define trajectories comprised of several phases, each phase being composed by thrust and coast arcs and ending at a body of the Solar System. Planetary swingbys connect the phases. Three control laws are available in the thrust arcs (see a general discussion on laws and numerical methods in [7])

- Thrust in an inertially fixed direction
- Explicit guidance, in which a bi-linear law defines the thrust direction in a local frame along time.
- Implicit guidance, where an adjoint or primer vector is propagated in parallel with the original state and used to compute the thrust direction along the arc. Following Lawden [8], the whole dynamics is governed by

$$\begin{aligned} \ddot{\mathbf{r}} &= \mathbf{g} + (T/m)\hat{\mathbf{p}}, \\ \ddot{\mathbf{p}} &= \nabla(\mathbf{p}\cdot\mathbf{g}), \end{aligned} \tag{2}$$

where $\mathbf{r}$ is the heliocentric position, $\mathbf{g}$ is the gravitational acceleration, T the applied thrust, m the spacecraft mass, and $\mathbf{p}$ is the primer vector defining $\hat{\mathbf{p}}$, the direction of thrust.

Equality and inequality constraints on the trajectory are incorporated in an augmented objective function with a penalty technique. There is a wide set of selectable constraints on the initial, intermediate or final state of any arc, and on the state change at a planet swingby. Two optimizers are available either to perform a global search with a genetic algorithm (Pikaia [4]) or to refine a first guess of the trajectory with a reliable derivative-free local optimizer (Nelder-Mead's simplex [9]).

In the frame of the ACT competition, MerPro used only the local optimizer, since MITRADES and GlOptImp produced the initial guess of the trajectory. Implicit guidance was the selected control law because of its optimality in an inverse square law field [8, pp. 79–80].

4. Solution

4.1. Feasible solution

4.1.1. From the inner Solar System to the asteroid via Jupiter and Saturn

As a result of the preliminary analysis, we had clear candidates for the required sequence of gravity assists in the final part of the trajectory. The spacecraft should leave the inner Solar System, swing by Jupiter, perform a gravity assist at Saturn in order to deflect the trajectory and get a retrograde orbit, and finally impact the asteroid close to its perihelion. The final gravity assist in the inner Solar System might be at Venus, the Earth, or Mars. From these three possibilities, the most promising one was a final swingby of the Earth (that might be followed by a swingby of Mars en-route to Jupiter in some cases).

GlOptImp was used to obtain an initial guess for the final part of the trajectory. Due to staff allocation constraints, only the two sequences mentioned before (E-J-S-Asteroid and E-M-J-S-Asteroid) were investigated. The optimization variables were the dates of the planetary swingbys, keeping the same objective function as in the original problem. Impulsive maneuvers

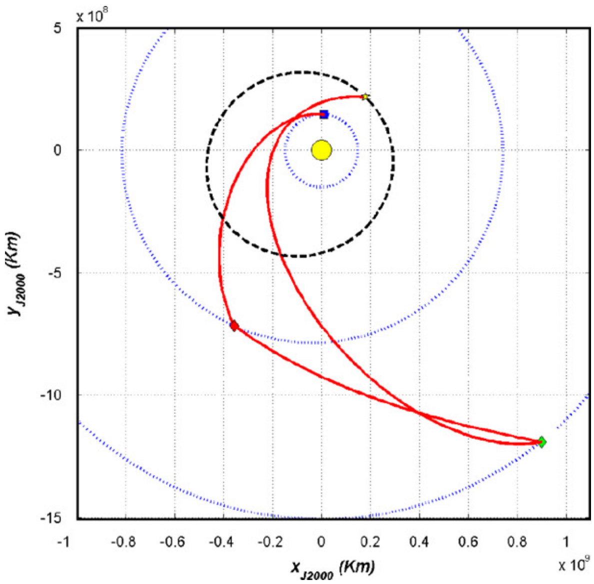

Fig. 1. Initial guess for the giant planets tour before impacting the asteroid (projection on the ecliptic plane). The sequence of encounters is Earth (December 2016) , Jupiter (August 2019), Saturn (August 2021) and impact at the asteroid (December 2026).

were inserted when the swingbys could not provide all the required deflection. The initial mass was fixed and equal to 1500 kg in all the cases, but the departure velocity from the Earth was constrained to be below 10.5 km/s.

The best trajectory found in the first runs of GlOptImp is represented in Fig. 1 and summarized below

- Earth departure in December 2016, $V_\infty = 9.2\,\mathrm{km/s}$.
- DSM between Jupiter and Saturn, and immediately after Saturn swingby.
- Asteroid impact in December 2026, close to the asteroid perihelion (true anomaly: 5°).
- Relative impact velocity: 48.9 km/s (45.0 km/s along the asteroid velocity).

We selected this trajectory as a first guess for the final part of the trajectory, but it turned out to be far from ideal because

- The relative velocity at impact is not tangent to the orbit of the asteroid (it is contained in the orbital plane of the asteroid, but it deviates by more than 20° from the tangent direction).
- Two DSMs are required. This implies that the spacecraft will arrive at the asteroid with less mass than in a completely ballistic transfer. The Jupiter swingby takes place early within the valid launch window period: less than 8 years are available to reach Jupiter.

After examination of the results of the ACT contest, we have analyzed why we did not find the best solution. There were mainly two reasons: (1) the constraint that we imposed on the maximum departure velocity from the Earth (10.5 km/s) prevented us from finding many interesting trajectories, (2) we did not investigate trajectory sequences including an additional swingby of Jupiter as the spacecraft falls towards the asteroid.

4.1.2. Strategy to reach Jupiter

At this stage, we needed to find a tour of planets in the inner Solar System so as to arrive at Jupiter with the conditions required to match the previously computed trajectory (Jupiter swingby in December 2018 with 7.75 km/s of relative velocity). MITRADES was used to obtain such a trajectory. The different "classical" trajectory types were investigated (ΔV-EGA, VEGA, VEEGA, etc., see [10]). It is important to notice that it was challenging to find a trajectory arriving at Jupiter with the required conditions since, due to the constraint on the earliest allowed launch date (January 2010), there was no time to wait for a more appropriate phasing of the planets.

The best trajectory found is presented in Fig. 2.

- Earth departure in March 2012, $V_\infty = 4.3\,\mathrm{km/s}$.
- Trajectory sequence: Earth-Venus-Earth-DSM-Earth.
- After the first Earth swingby, the spacecraft performs a 3:1 Delta-V Earth Gravity Assist, with a 425-m/s impulsive maneuver at aphelion.

4.1.3. Departure from the Earth with low-thrust

MerPro designed the low-thrust trajectory that leaves the Earth with the 2.5-km/s hyperbolic excess imposed on the problem statement, and returns to the Earth in March 2012 with the 4.475-km/s hyperbolic excess necessary to start the EVEE tour. I.e., the vehicle has to gain 1.975 km/s in the synodic frame.

The most effective low-thrust Earth–Earth transfer was a planar trajectory in the ecliptic plane defined by

- Radial launch, with the escape asymptote either in Sun or anti-Sun direction from the Earth.
- Optimal, implicit guidance of the thrust direction, also in the ecliptic plane.
- Return to the Earth after an integer number of years, with the Earth in approximately the same position as at departure.

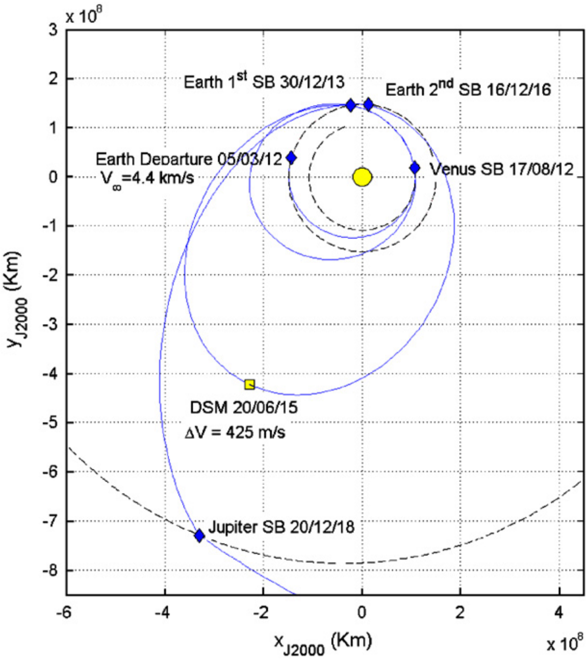

Fig. 2. Initial guess for the trajectory from the Earth to Jupiter (projection on the ecliptic plane).

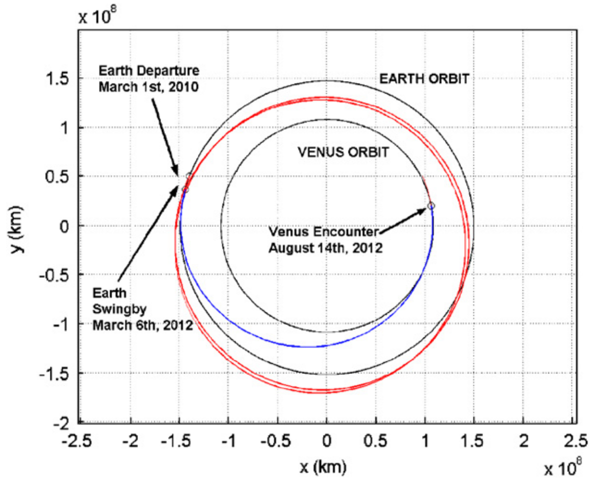

Fig. 3. Earth–Earth transfer before the departure to Venus (projection on the ecliptic plane).

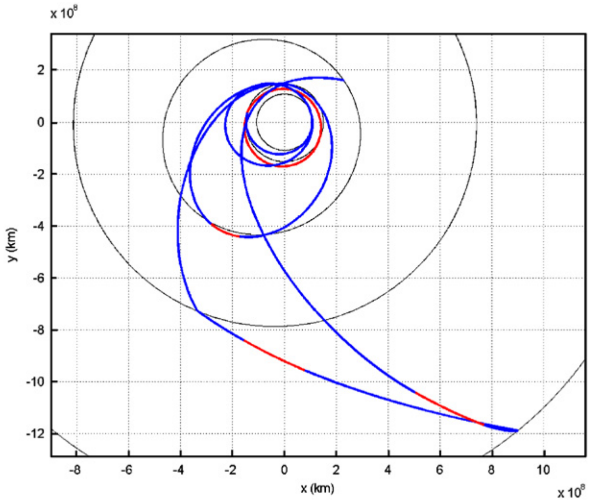

Fig. 4. Optimized trajectory impacting in December 2026 (projection on the ecliptic plane).

This transfer allows the vehicle to gain up to approximately 1.35–1.40 km/s in the first year, the exact value depending upon the departure date. Therefore, a one-year transfer was not long enough to get the 1.975-km/s gain, and at least two years were needed. The one feasible solution was a two-year transfer with Earth departure in March 2010, close to the opening of the 20-year launch window in January 2010. Coast arcs were inserted in the Earth–Earth transfer to save propellant. Fig. 3 presents the resulting trajectory.

4.2. Improvement

4.2.1. Phase merging and optimization

The merging of the three trajectories described in Section 4.1 produced the first guess of a single trajectory spanning from launch to impact. The resulting trajectory contained impulsive maneuvers that have to be smeared with MerPro into low-thrust arcs compliant with the maximum thrust level of the propulsion system. The value of the objective function corresponding to this first feasible trajectory was $1\,283\,000\,\mathrm{km}^2\,\mathrm{kg/s}^2$.

From this first feasible trajectory, MerPro proceeded to optimize the whole trajectory while satisfying mission constrains. Apart from solving a highly non-linear problem, MerPro had to cope with a sizeable set of 87 optimization variables, consisting in all the dates, arc durations, state adjoints at the beginning of thrust arcs, and velocities in the outgoing asymptotes of launch or planet swingbys, as well as with a few auxiliary variables. In the optimized trajectory, presented in Fig. 4, the spacecraft impacts the asteroid in December 2026 after less than 17 years of flight. The rise of the objective function to $1\,417\,000\,\mathrm{km}^2\,\mathrm{kg/s}^2$ is remarkable.

4.2.2. Addition of a final complete revolution after Saturn swingby

The flight time of the optimized trajectory presented in Section 4.2.1 is more than 13 years shorter than the maximum mission duration of 30 years. This and the

Table 1
Description of the main events of GMV's trajectory

Event	Event date/arc dates (MJD2000)	Details
Launch	3712.141	2.5-km/s hyperbolic excess velocity 1500-kg mass
Thrust arc #1	3712.141 to 4141.740	429.6 days 60.6-kg propellant consumption
Thrust arc #2	4336.181 to 4448.079	111.9 days 15.8-kg propellant consumption
Earth swingby #2	4448.079	1423.6-kg mass 4.475-km/s hyperbolic excess velocity 9571-km perigee radius
Venus swingby	4612.737	1423.6-kg mass 5.917-km/s hyperbolic excess velocity 9992-km pericenter radius
Thrust arc #3	4612.737 to 4621.580	8.8 days 1.2-kg propellant consumption
Earth swingby #3	5112.221	1422.4-kg mass 10.932-km/s hyperbolic excess velocity 7693-km perigee radius
Thrust arc #4	5595.634 to 5721.621	126.0 days 17.8-kg propellant consumption
Earth swingby #4	6194.994	1404.6-kg mass 9.219-km/s hyperbolic excess velocity 7497-km pericenter radius
Jupiter swingby	6924.388	1404.6-kg mass 7.735-km/s hyperbolic excess velocity 60 0000-km pericenter radius
Thrust arc #5	7052.574 to 7214.180	161.6 days 22.8-kg propellant consumption
Saturn swingby	7797.944	1381.8-kg mass 11.889-km/s hyperbolic excess velocity 85941-km pericenter radius
Thrust arc #6	11 841.261 to 12 377.681	536.42 days 75.7-kg propellant consumption
Impact	14 410.244	1306.1-kg mass 49.099-km/s relative to the asteroid $1\,455\,910\,\mathrm{kg\,km^2/s^2}$ merit function

trajectory shape suggested a way to improve the mission merit figure. It was possible to wait one extra revolution in the Saturn-asteroid orbit and split the aphelion maneuver after Saturn swingby into two near-aphelion maneuvers 13 years apart. The intended effect of this change was to rotate the apsidal line to improve impact geometry, not to change the energy of the orbit. MerPro was again used to optimize a trajectory that impacted the target asteroid in June 15th, 2039. Details are given in Table 1 and 2 and Figs. 5 and 6. The final value of the objective function, $1\,456\,000\,\mathrm{km^2\,kg/s^2}$, is a moderate 2.75% gain with respect to the original trajectory. Although this meager rise would not pay back the longer flight time in a real mission, this trajectory offered a higher merit value, and was therefore the one presented to the ACT competition.

Table 2
Mission summary

Earth launch	March 1st, 2010 2.5-km/s hyperbolic velocity excess
Propellant consumption	193.8 kg
Total thrust duration	1375.3 days
Equivalent delta-V	3.393 km/s
Asteroid impact	June 15th, 2039 49.1-km/s hyperbolic excess velocity
Merit function	$1.455910 \times 10^6\,\mathrm{kg\,km^2/s^2}$

5. Conclusions

We have presented the work carried out at GMV in November 2005 for the 1st ACT Global Trajectory Competition, along with the details of the solution

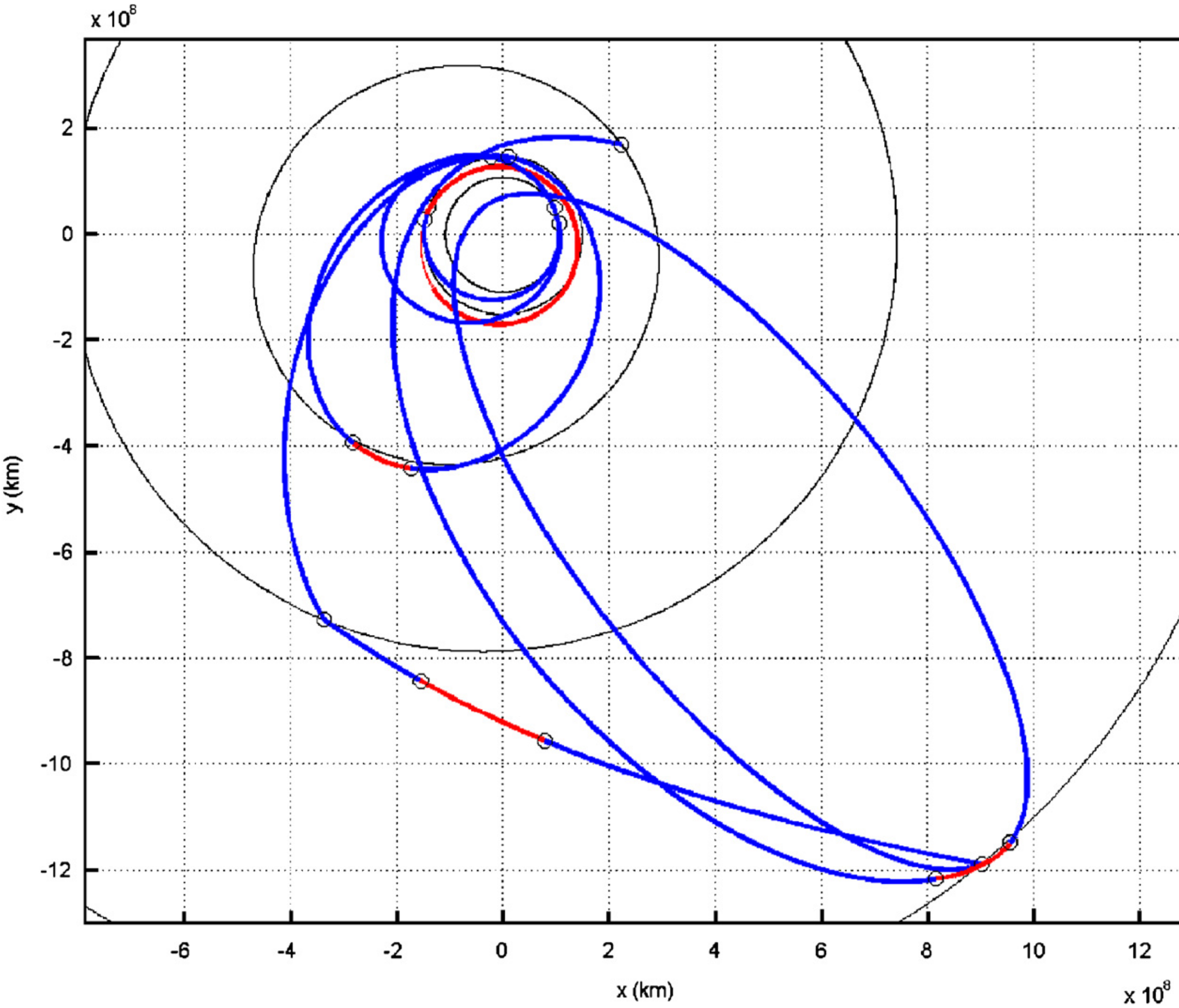

Fig. 5. Ecliptic projection of the final trajectory.

obtained. The problem posed by the ACT was, in principle, far too complex to be solved automatically. The search space is so vast (low thrust arcs, as many gravity assists as desired, very long launch interval and time of flight, etc.) that global optimization methods cannot be directly applied, unless an experienced analyst introduces certain assumptions and guesses. This was the approach followed in GMV to find a good solution: that is, to combine the knowledge and intuition of the members of the team with powerful tools for trajectory design and optimization. Note that, for this particular problem, the best solution happened to be a ballistic trajectory following the Venus swingby. Having known this in advance would have reduced considerably the complexity of the problem and therefore the dimension of the search space.

GMV's trajectory placed 3rd out of the 12 trajectories presented. Because of the limited time and manpower available for the competition, our team focused on obtaining a feasible, good, local optimum trajectory, rather than in a thorough investigation on the global optimum. Hence, the search for the initial guess was not exhaustive: we did not consider all the possible sequences of swingbys, and introduced some constraints to limit the search space. Although the initial guess was not the global optimum, the method to build and improve the complete trajectory worked successfully. This is remarkable taking into account that the final trajectory combined 7 gravity assists and 6 low-thrust arcs, and considering the difficulty of satisfying the constraint in the launch date due to the early Jupiter swingby in our initial guess.

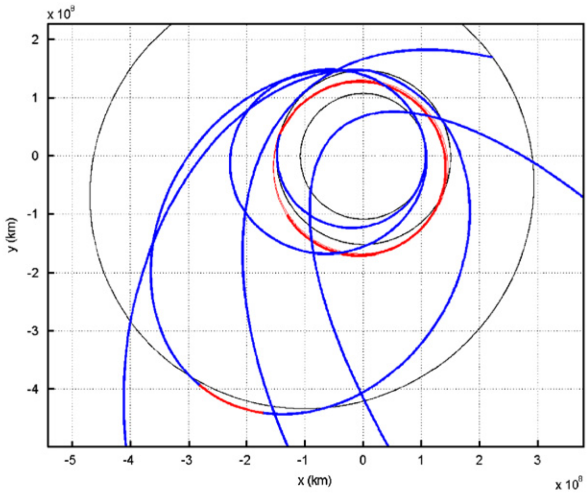

Fig. 6. Zoom of the trajectory (detail of the EVEE tour and impact).

Acknowledgments

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References

[1] R.B. Adams, et al., Survey of technologies relevant to defense from Near-Earth objects NASA/TP-2004-213089, July 2004, pp. 93–101.

[2] C. Simó, Solución del problema de Lambert mediante regularización, Collectanea Matemática XXIV (1973).

[3] A. Wächter, L.T. Biegler, On the Implementation of a primal-dual interior point filter line search algorithm for large-scale nonlinear programming, Mathematical Programming 106 (1) (2006) 25–27.

[4] P. Charbonneau, Genetic algorithms in astronomy and astrophysics, The Astrophysical Journal 101 (Suppl.) (1995) 309–334.

[5] P. Rogata, E. Di Sotto, M. Graziano, F. Graziani, Guess value for interplanetary transfer design through genetic algorithms, AAS Paper 03-140, February, 2003.

[6] C. Corral van Damme, R. Cadenas, M. Graziano, A survey of rendezvous and impact ballistic trajectories to NEOs, AAS Paper 06-213, January, 2006.

[7] J.T. Betts, Survey of numerical methods for trajectory optimisation, Journal of Guidance, Control, and Dynamics 21 (2) (1998) 193–207.

[8] D.F. Lawden, Optimal Trajectories for Space Navigation, Butterworth & Co. Ltd., London, 1963.

[9] W.H. Press, et al., Numerical Recipes in C: The Art of Scientific Computing, second ed., Cambridge University Press, 1992 pp. 408–412.

[10] A.E. Petropoulos, J.M. Longuski, E.P. Bonfiglio, Trajectories to Jupiter via gravity assists from Venus, Earth, and Mars, Journal of Spacecraft and Rockets 37 (6) (2000) 776–783.