GTOC8: Results and Methods of Team 3 – Tsinghua University

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GTOC8: RESULTS AND METHODS OF TEAM 3 – TSINGHUA UNIVERSITY

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In this paper the methods proposed by team 3 in the 8th Global Trajectory Optimization Competition (GTOC8) are introduced. The final formation of the three spacecraft is obtained by analyzing the major factors which affect the performance index. Then the optimal trajectories to construct the final formation are generated which are then used as nominal trajectories. All the possible chances for observing the radio sources are calculated using the nominal trajectories. Followed by a global search algorithm to determine the observing sequence and time, indirect methods are applied to obtain the transfers which satisfy all the constraints for observations.

INTRODUCTION

Compared with previous GTOCs, a more complex mission near the Earth is investigated in GTOC8 [*]. The same as GTOC7, three spacecraft with low-thrust propulsion are considered and they ought to cooperate and form different triangles in space to observe radio sources. The three spacecraft are moving in simplified dynamical environment: The Sun's gravity is excluded, Earth is modeled as a point mass, flybys of the Moon, with a minimum flyby altitude of 50 km and hyperbolic excess velocity of 0.25 km/s, are modeled as patched conics, and the Moon is assumed to follow a conic orbit around Earth. After an initial impulse, the low-thrust propulsion is the only propulsion to be used. Moon gravity assists can be applied to change the velocity of the spacecraft immediately.

The three spacecraft are initially collocated in a 400-km altitude circular orbit in the ecliptic plane around the Earth. The chemical propulsion with a specific impulse of 450 s and an impulsive capability of up to 3 km/s can be used only once. Then it has to rely on the low-thrust system with a specific impulse of 5000 s and low thrust whose maximum magnitude is to 0.1 N. The mission must end within three years when the last observation is made. Observations must be spaced at least 15 days apart.

The performance index to be maximized is

$$J = \Sigma Ph(0.2 + \cos^2 \delta) \tag{1}$$

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[*]http://sophia.estec.esa.int/gtoc_portal/wp-content/uploads/2015/05/gtoc8_problem_stmt.pdf

where h denotes the smallest of the three altitudes of the observing triangle and must be larger than 10000.0 km, δ is the declination of the source being observed, and P is a weighting factor for repeat observations that takes on the values of 1, 3, 6, or 0 according to the rules. The repeat observations of the same radio source with different h, if a specific ratio between them is satisfied, are favored as P becomes larger for later observations.

There is an upper limit of h for each observation which determines the upper bound of scores obtained from a single observation. The maximum distance between the spacecraft and the Earth is 1 million km which requires that the largest h for each observation, when an equilateral triangle is formed and three spacecraft and the Earth are in the same plane, is 1.5 million km, which is shown in Figure 1. Because repeat observations of a single radio source with different h may take the advantage of the increasing P, it is obvious that trying to observe every radio source three times with the required distances ratios is the most significant factor to take into consideration. Moreover, the third observation is supposed to be conducted with the maximum h because its multiplication with $P = 6$ contributes the most to the performance index. The declination δ also matters, as the factor concerning δ, i.e. $0.2 + \cos^2 \delta$, ranges between 0.2 and 1.2, which can lead to the maximum difference of 6 times. As a result, radio sources with small δ are preferred. If two spacecraft are symmetric about the ecliptic plane and the third keeps in the ecliptic plane, the triangle normal (i.e., the normal to the plane containing the triangle) always lies in the ecliptic plane, which helps observing radio sources with small δ. So we take advantage of the symmetry of two spacecraft. Since the thrust is low compared with the gravity from the Earth, it is not possible to significantly change the orbit of the spacecraft in just 15 days, which is exactly the least space between two observations. Consequentially the performance index is mainly determined by the nominal trajectory, while low-thrust propulsion is mainly used to generate the nominal trajectories and the remaining is used to fix the small errors between the nominal trajectories and actual trajectories. Thus the problem is transformed into the design of three nominal trajectories and the use of low-thrust propulsion to change them for observations. Here are a few requirements for the nominal trajectories

1. The final orbits of two spacecraft, denoted as S_1 and S_2 are symmetric about the ecliptic plane. The orbits are circular with an radius of 1×10^6 km and one of them has an inclination of $60°$.

2. The phase of the the third spacecraft, denoted as S_3, moves in the ecliptic plane. The orbit is circular with an radius of 1×10^6 km. The difference of its phase with S_1 and S_2 is π.

3. When forming the final orbits, the symmetry of orbits and the difference in phases is formed and kept.

4. When forming the final orbits, the radio sources that are observed after the final orbits are formed should be observed with increasing h.

The first impulse is not enough to raise the apogee of the spacecraft's orbit to the Moon's minimum height so that the low-thrust propulsion is required before the gravity assist. Instead of implementing indirect methods to optimize the low-thrust trajectory, which is extremely difficult because of the large numbers of revolutions and the high eccentricity of the orbit, an analytic control law is proposed which can be applied to raise the apogee without affecting the perigee. After this phase, the low-thrust propulsion is used to ensure Moon gravity assists, i.e., ensure the spacecraft and the Moon are at the same position at a certain moment. However, Moon gravity assists are applied for

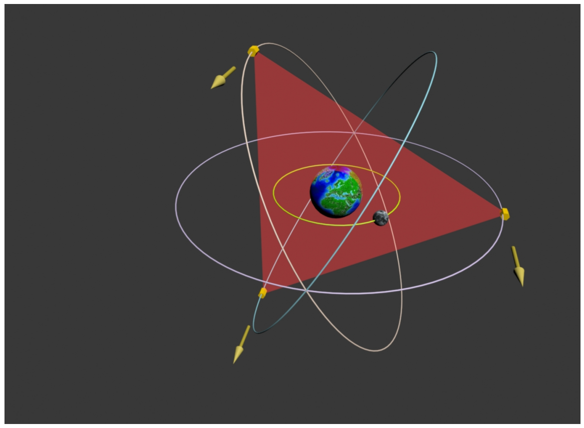

Figure 1. Final Formation of Three Spacecraft

different purposes for the three spacecraft. For the first two spacecraft, the inclination is changed while their symmetry about the ecliptic plane is maintained. Meanwhile, the third spacecraft always moves in the ecliptic plane and Moon gravity assist is used to increase the energy of its orbit significantly. After Moon gravity assists, the low-thrust propulsion is applied to guide the three spacecraft to their final orbits. In the first step, all the semi-major axises of three spacecraft's orbits are increased while the eccentricity are decreased simultaneously. Only a few revolutions are needed so indirect methods are capable of implementing such a purpose. The difference of the phase between S_1 and S_3 is set to π.

Then S_1 and S_2 change their inclinations to the required value with thrust being applied normal to the orbit plane. When every spacecraft has entered its own final orbit, the first two spacecraft move symmetrically in circular orbits whose semi-major axis are 0.95 million km and inclinations are 60° and 120°, respectively. The third spacecraft moves a circular orbit with semi-major axis of 0.95 million km and zero inclination. Such design is chosen because it always guarantees that the normal of the observing triangle always lies in the ecliptic plane which minimizes δ.

After the generation of nominal trajectories, all the possible observing moments and corresponding radio sources are easily found by propagating the trajectories and calculating the angles between the instant normal of the observing triangle with all the radio sources. Those moments when the minimum angle is below a certain threshold are termed as possible observing moments and the corresponding radio source and h are obtained. After obtaining all the possible observing chances, the problem is converted into how to choose the best sequence which maximizes the performance index. A branch-and-prune algorithm is applied to find the largest number of radio sources which can be observed three times with P =1, 3, and 6, respectively. Then the chances to observe radio sources twice with P =1 and 3, respectively are found. At last, those sources only observed once are added to the sequence.

The final step is to use the low-thrust propulsion to complete all the observations. The observing directions generated from the nominal trajectories have a small difference with the radio sources and the low-thrust propulsion are used to fix them. Indirect methods with homotopic approaches are applied. It is found that the maneuvers of S_1 and S_2 are enough to fix the errors. At every possible observing moment, the least deviations from the nominal trajectories of S_1 and S_2 are obtained by solving a quadratic programming problem with linear constraints. The third spacecraft coasts on its nominal trajectory. In most cases, the trajectories of the two spacecraft are solved separately. The trajectory of each spacecraft is divided into multiple segments by the observing moments. Between two observing moments, the problem to transfer from one state to another degenerates into a fuel-optimal low-thrust trajectory optimization problem with fixed initial and terminating states and fixed transfer time, which is solved with the indirect method. Actually, the second performance index, i.e., the final masses is not taken into consideration because it is unlikely that two teams have the same primary performance index. The perfect bang-bang control is not solved. Instead, an approximate result to the bang-bang control is obtained. Occasionally, one segment cannot be solved successfully because the deviation is too large, denoted as the failed moment. The trajectories of the two spacecraft are solved together and the observation is treated as an inner-point constraint. The conditions for optimality are easily derived and a multi-point boundary value problem (MPBVP) is built and then solved with a shooting method. The combination of these two methods is sufficient to obtain the fuel-efficient low-thrust trajectories in this step.

This paper is organized as follows. The design of nominal trajectories is given in Sec. II which includes the details about how to raise the apogee to prepare for Moon gravity assist, the novel design of Moon gravity assist to ensure two spacecraft's symmetry, and the fuel-efficient control to guide the three spacecraft to their final orbit while maintaining the constraint concerning their phases. The method for obtaining observation opportunities with the nominal trajectories and the global search algorithm to find the optimal observing strategy are given in Sec. III. In Sec. IV indirect methods are applied to make the low-thrust transfers satisfy all the constraints. The conclusion and discussion are given in Sec. V.

NOMINAL TRAJECTORIES

The performance index is mainly determined by the nominal trajectories and the basic requirements of them are summarized in the above.

Raise to the Moon

The impulse of 3 km/s is not sufficient to raise the apogee of the spacecraft's orbit to the Moon's height even the direction is parallel with the spacecraft's velocity, so low-thrust propulsion must be used. Because the change of inclination is rather expensive, we might as well only investigate the motion in the ecliptic plane. As a result, the Moon flyby happens exactly when the Moon is at its ascending or descending node. The initial impulse is taken when the spacecraft crosses the node line of Moon's orbit. From then on the analytical control law is used to raise the apogee of the spacecraft's orbit.

Analytic Control Law The equinoctial elements are used to describe the motion of the spacecraft. The elements, denoted as $\boldsymbol{x}$, compose (p, f, g, h, k, L) and the dynamical equation is

$$\begin{cases} \dot{\boldsymbol{x}} = \boldsymbol{f}_0 + \dfrac{T}{m}\boldsymbol{M}\boldsymbol{\alpha} \\ \dot{m} = -T/c \end{cases} \tag{2}$$

where T is the thrust magnitude; m is the instantaneous mass of the spacecraft; $\boldsymbol{\alpha}$ is the thrust direction;the constant $c = I_{\text{sp}} g_0$ where I_{sp} is the specific impulse and g_0 the gravitational acceleration at sea-level. The details of vector fields $\boldsymbol{f}_0$ and M can be found at Ref.[1] Through trial and error we find the control law which maximizes the apogee change rate is the most efficient to reach the

Moon. The optimal direction to change the apogee is $\boldsymbol{\alpha}/\|\boldsymbol{\alpha}\|$ where the components of $\boldsymbol{\alpha}$ are

$$\begin{cases} \boldsymbol{\alpha}_1 = f\sin(L) - g\cos(L) \\ \boldsymbol{\alpha}_2 = \dfrac{2e(1-e)}{w} + \left(f\left(\cos(L) + \dfrac{f+\cos(L)}{w}\right) + g\left(\sin(L) + \dfrac{g+\sin(L)}{w}\right)\right) \\ \boldsymbol{\alpha}_3 = 0 \end{cases} \tag{3}$$

where $w = 1 + f\cos L + g\cos L$ and $e = \sqrt{f^2+g^2}$ is the eccentricity.

However, in order to better detect the terminating moment of the analytic control phase, it is worthwhile to divide the orbits by revolutions. The dynamical equations are written with respect to L, it is obtained by

$$\begin{cases} \dfrac{\mathrm{d}\boldsymbol{x}_i}{\mathrm{d}L} = \dfrac{\dot{\boldsymbol{x}}_i}{\dot{L}}, i = 1, ..., 5 \\ \dfrac{\mathrm{d}t}{\mathrm{d}L} = \dfrac{1}{\dot{L}} \\ \dfrac{\mathrm{d}m}{\mathrm{d}L} = \dfrac{\dot{m}}{\dot{L}} \end{cases} \tag{4}$$

We can obtain the change of equinoctial elements during one revolution by integration Eq. 4 from 0 to 2π. After every revolution, the apogee is checked and the analytic control phase terminates when it exceeds the height of the Moon. The time consumption, denoted as T_a, is recorded. Such a choice increases the robustness of our algorithm. When the analytic control phase terminates, the final state of the spacecraft is used as the initial state for the next phase where the spacecraft flies to the Moon for the flyby. The terminating moment, denoted as t_f, is chosen to be the moment when the Moon reaches the ascending or descending node. The length of this phase, denoted as T_f, is unknown unless the moment when the first impulse is applied, denoted as t_0, is determined. Because of the periodic motion of the spacecraft at the initial orbits, t_0 can be added by its period, denoted as T_0, any times. The length of this phase is guessed to be half of the period of the spacecraft's current orbit. Because t_f is fixed and T_a does not depend on t_0 or T_f, we guess $t_0' = t_f - T_f - T_a$ and then adjust it so that

$$t_0' + \delta t_0 = t_0$$

where t_0 is the moment when the spacecraft reached the desired phase and $\delta t_0 < T_0$. Then we obtain $T_f = t_f - t_0 - T_a$. After obtaining T_f, we can use indirect methods to calculate the low-thrust trajectory to reach the Moon.

Indirect Methods Indirect methods are widely used to optimize low-thrust trajectories. They are used in the final phase to reach the Moon because it is rather difficult to design the analytic control law so the spacecraft can reach the Moon at the given time. Indirect methods are briefly introduced here and more details can be found in Ref.[2]

After introducing a positive numerical multiplier[2] λ_0 and adding the logarithmic barrier,[3] the performance index is chosen as

$$J = \int_{t_0}^{t_f} J_L \mathrm{d}t = \lambda_0 \frac{T}{c} \int_{t_0}^{t_f} u - \varepsilon \ln[u(1-u)]\mathrm{d}t \tag{5}$$

where J_L is the Lagrangian form of performance index; $\varepsilon \in [0,1]$ gradually inclines to 0 to better approximate the bang-bang control. By introducing adjoint variable $\boldsymbol{\lambda} \triangleq [\boldsymbol{\lambda}_{\boldsymbol{x}}; \lambda_m]$, the Hamiltonian is built as

$$H = \boldsymbol{\lambda}_{\boldsymbol{x}}^{\mathrm{T}} \dot{\boldsymbol{x}} + \lambda_m \dot{m} + J_L \tag{6}$$

and the optimal control, which minimizes H, is

$$\begin{cases} \boldsymbol{\alpha} = -\dfrac{\boldsymbol{M}^{\mathrm{T}}\boldsymbol{\lambda}_{\boldsymbol{x}}}{\left\|\boldsymbol{M}^{\mathrm{T}}\boldsymbol{\lambda}_{\boldsymbol{x}}\right\|} \\ u = \dfrac{2\varepsilon}{\rho + 2\varepsilon + \sqrt{\rho^2 + 4\varepsilon^2}}. \end{cases} \tag{7}$$

where ρ is the switching function and defined as

$$\rho = 1 - \frac{\lambda_m}{\lambda_0} - \frac{c}{m\lambda_0}\left\|\boldsymbol{M}^{\mathrm{T}}\boldsymbol{\lambda}_{\boldsymbol{x}}\right\|. \tag{8}$$

The differential equations of the adjoint variables that are termed as Euler-Lagrange equations are given as

$$\begin{cases} \dot{\boldsymbol{\lambda}}_{\boldsymbol{x}} = -\dfrac{\partial H}{\partial \boldsymbol{x}} \\ \dot{\boldsymbol{\lambda}}_m = -\dfrac{\partial H}{\partial m} \end{cases} \tag{9}$$

where the detailed formulations in equation (9) can be found in Ref. 4.

In this phase to reach the Moon, the initial states, initial moment and final moment are already given. The final position of the spacecraft is constrained to be the same with the Moon's, while the final velocity, denoted as $\boldsymbol{v}_f$, are determined by the next step where Moon gravity assists are designed. However, the design of Moon gravity assists requires $\boldsymbol{v}_f$. A trial and error approach has to be taken. We first treat the transfer without any constraints on the velocity so we can estimate the approximate value of $\boldsymbol{v}_f$. The velocity $\boldsymbol{v}_f$ is then used to design the Moon gravity assists after which the required $\boldsymbol{v}_f$ is obtained. Then the transfer is treated with the constraints on the velocity $\boldsymbol{v}_f$ and solved again. Different constraints require different boundary conditions which are summarized as follows.

When the final velocity has no constraint, it is easily derived that

$$\begin{cases} \boldsymbol{r}(t_f) = \boldsymbol{r}_M \\ \boldsymbol{\lambda}_{\boldsymbol{v}}(t_f) = \boldsymbol{0} \\ \lambda_m(t_f) = 0 \end{cases} \tag{10}$$

where $\boldsymbol{r}_M$ is the position of the Moon at t_f and when the final velocity is constrained, the second component of Eq. 10 is replaced by

$$\boldsymbol{v}(t_f) = \boldsymbol{v}_f \tag{11}$$

With all the boundary conditions obtained, we might as well use shooting methods to solve the optimal control problem and obtain the optimal control to reach the required state. It should be noted that equinoctial elements are used while the boundary condition in Eq. 10 is expressed by $\boldsymbol{\lambda}_{\boldsymbol{v}}$. The canonical transformation of the adjoint variables are applied.

An illustration of the final orbit of this phase is shown in Figure 2. It should be noted that the trajectory in red is the phase when the optimal control is applied to reach the Moon.

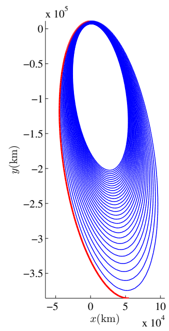

Figure 2. Raise the Apogee and Reach the Moon

Gravity Assists

The gravity assist of S_1 takes place first at the descending node of the Moon. The goal is that after the first gravity assist the period of S_1 is the same with the Moon's and it can intersect with the Moon at the next ascending point where S_2 performs a flyby and cooperate with S_1. We manually design the flybys to make sure that S_1 and S_2 move symmetrically with respect to the ecliptic plane after the gravity assists. Such a design requires different $\boldsymbol{v}_\infty$ (with respect to the Moon) for S_1 and S_2, which is satisfied by the first step. After the gravity assists we might as well only investigate the trajectories of the one with larger mass which is found to be S_1 because S_2 can always keep the symmetry.

The gravity assist changes the inclination, semi-major axis, and eccentricity of S_1. There is a trade-off between the change of semi-major axis and the inclination. The inclination shall not be too low otherwise the minimum h, which is 10000 km, cannot be obtained. On the other hand, if the apogee of the orbit after the flyby is too low, a longer time has to be spent to circularize the orbit. Too aggressive choice of them might make the problem infeasible, i.e. the gravity assist is insufficient to change the orbit of S_1 to the desired one. It is through trial and error that we set the apogee 8×10^5 km and the inclination $2°$.

The flyby of S_3 takes place at the next descending node. This flyby is used to increase the energy of the orbits so the relative velocity after and before the flyby are all in the ecliptic plane. The apogee is changed to 8×10^5 km.

Circularize the Orbits

We choose to circularize the orbits of S_1 and S_2 immediately after the flyby for two reasons: 1) we want to make the difference of phases between S_1 and S_3 become π as soon as possible; and 2) it is easier to change the inclination with larger semi-major axis. In order not to violate the constraint which requires the distance of any spacecraft from the Earth be less than 1 million km, the final semi-major axis is chosen to be 0.99 million km. First we use the indirect methods to calculate the time-optimal trajectories. The optimal control thrust direction is the same with Eq. 7 while the engine is always on. The final phase has no constraint. However, numerical results show that such an aggressive choice might violate the maximum distance constraint. Because our algorithm is rather difficult to handle state-variable inequality constraints, we choose to use fuel-optimal control. We preset the transfer time and calculate the fuel-optimal trajectory to reach the final circular orbit

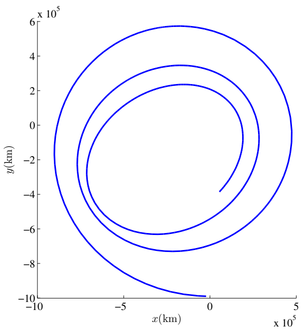

Figure 3. Trajectory of S_1 to Circularize its Orbit

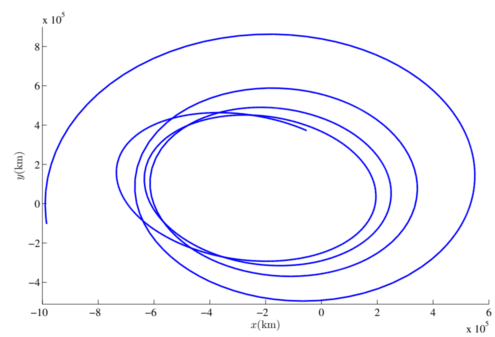

Figure 4. Trajectory of S_3 to Circularize its Orbit

without any constraint about L. Thus the boundary conditions about λ_L at the final moment is

$$\lambda_L = 0 \tag{12}$$

We check a posteriori whether the constraint is violated and adjust the length of this phase to avoid the violation. After the design of S_1, the trajectory of S_3 is determined so that the final L is π later than S_1. After choosing the transfer time manually, the design of the trajectory of S_3 in this phase is converted into a fuel-optimal control problem with fixed final state and fixed transfer time. It should be noted that when designing the nominal trajectories, the maximum thrust magnitude is chosen to be 0.8 times the actually maximum thrust magnitude. Such a choice makes it possible to fix the errors of the nominal trajectories in the last step. The trajectories of S_1 and S_3 are shown in Fig. 3 and 4.

Change the Inclination

After circularizing the orbit of S_1 and S_3, the orbit of S_3 is not changed. The spacecraft S_1 coasts for a short time and starts changing its inclination. The final inclination is $60°$ which is changed to in two steps. In the first step, the inclination is changed to $13°$ with the low-thrust system. Then it coasts for a short period which is manually chosen to be 1.5 times the period of the current orbit. Finally the low-thrust system turns on to propel the spacecraft to its final orbit. It is found that such a control only changes the relative phase between S_1 and S_3 a little. The reason why we do not propel the spacecraft to the final inclination as soon as possible is that otherwise there is not enough time for observing radio sources with smaller h. During the first coast we can observe radio sources with the minimum h, and the second coast provides more chances to observe with medium h, which will be explained later. In this step, the thrust magnitude is chosen 0.8 times the maximum for the same reason with the last step. The trajectory is shown in Fig. 5 where the blue and red trajectory denote the first and second phase of inclination changing.

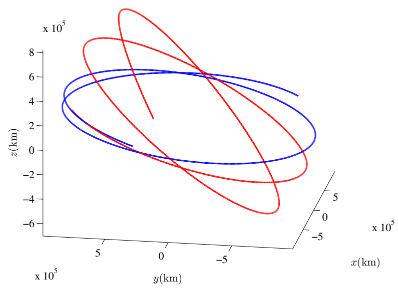

Figure 5. Trajectory of S_1 to Change its Inclination

GLOBAL SEARCH OF OBSERVING OPPORTUNITIES

Finding all Possible Observing Chances

After obtaining the nominal trajectories of the three spacecraft, we can find all the possible observing opportunities. Because adaptive-step integrator might choose long integration step, we use cubic spline to interpolate the states of spacecraft. A small step such as 1 hour is chosen and the states of spacecraft at every step are obtained. The triangle is calculated and its normal is obtained. Because the normal of our triangle is always in the ecliptic plane, we only record its right ascension. Every two steps form an interval and any radio source whose right ascension is within the interval is termed as a possible observing opportunity. With such a method all the possible observing opportunities are picked out and ranked by the corresponding moment of observation.

Finding Optimal Opportunities

The next step is to obtain the optimal choice of observing opportunities which maximize the performance index. All the observing opportunities together with the minimum h are plotted in Fig. 6 where the red dots denote all the possible observing opportunities and the blue curves denote the h of the triangle. They are divided into four regions according to the maximum h. It is easy to find that the maximum h in region I, II, and IV satisfy the 1:3:9 condition. Because the period of the orbit changes a little after circularizing and the node line changes little when the inclination of S_1 is changed, it is easy to verify that the moments of observing a certain radio source has the same period when the spacecraft are in region II, III, and IV. Although in region I the moments of observing radio sources are not as periodic as others because of the change of semi-major axis, it is safe to say that almost all radio sources that are observed in other regions are also observed in region I. It is easy to find that our design of nominal trajectories takes advantage of the change of inclination to raise h when the radio sources are observed repeatedly.

Region III is the least favorable part because it takes a significant proportion of time and the h in this region cannot cooperate with others to satisfy the 1:3:9 condition. However, it is unavoidable

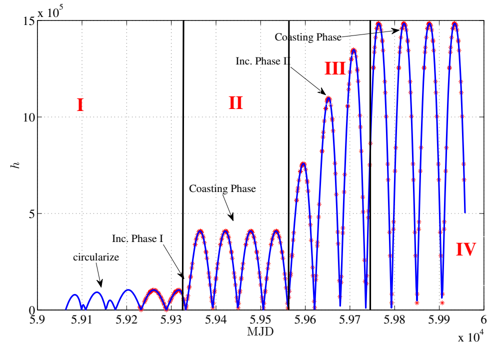

Figure 6. All the Observing Opportunities

because the change of inclination is difficult and consumes too much time. Our design also explains why the two coasting segments are important in the phase of changing the inclination. The two coasting segments extends the length of region I and II so that more radio sources can be observed under the 1:3:9 condition. The length of the two coasting segments are thus adjusted by trial and error.

The method for finding the optimal observing opportunities is summarized as follows

1. Search radio sources that will be repeatedly observed three times ($P = 1, 3, 6$)

   (a) Search the radio sources with $P = 6$ in region IV and region III. The first radio sources in the sequence is chosen as one of the last 30 red points in region IV. From right to left, the new radio source can be added into a sequence if the following conditions are satisfied: 1) $h > 7.0 \times 10^5$ km; and 2) the observation time of the new radio source is less than the one of the last radio source in the current sequence. A termination criterion of increasing sequence is either no new radio source can be found or a sequence's length reaches 15.

   (b) Search the radio sources with $P = 3$ in region II. Starting from the last red point of this region, a search from right to left is carried out. A point will be kept if the corresponding radio source has been selected in one of sequences with $P = 6$ and its height is less than 1/3 of the one with $P = 6$. After the first ones in the sequences of $P = 3$ has been selected, new points will be added to a sequences if the following conditions are satisfied: 1) the new radio source is different with all other radio source in current sequences of $P = 3$; 2) the observation time of the new radio source is less than the one of the last radio source in the current sequence; and 3) this radio source has been selected in one of sequences with $P = 6$ and its height is less than 1/3 of the one with $P = 6$. When no more eligible new radio source can be found, this step is end and we will get sequences with $P = 3$.

   (c) Search the radio sources with $P = 3$ in region I. Similar with step (b) but the compared sequences is the sequences with $P = 3$, sequences with $P = 1$ can be found.

   (d) After the previous steps, sequences which will be repeatedly observed three times can be obtained. The radio sources are the ones of the sequences with $P = 1$. Also, we can obtain the information of the whole sequences with P =1, 3 and 6. The maximum of the number of the different radio sources is found to be 12.

2. Search radio sources that will be repeatedly observed two times ($P = 1, 3$). This searching is carried as the previous searching but the regions are region III ($P = 3$) and region I ($P = 1$) and every radio source selected cannot be the same as anyone found in the searching 1.

3. Search all other radio sources that can be observed ($P = 1$). Region I will be searched for this searching. Similar as the previous searching, but every radio source selected cannot be the same as anyone found in the previous searched sequences.

4. Select the best sequence. With the previous searching steps, we can get whole sequences with the subsequences with $P = 1$, 3 and 6, $P = 1$ and 3, and $P = 1$. And then we compute the J of each sequence. The sequence with the highest J is selected.

LOW-THRUST IMPLEMENTATION

The errors between the normal direction of nominal trajectories should be fixed in order to finish the observation. This step can be divided into multiple low-thrust trajectory optimization problems. Because of the large number of revolutions, large number of observations which require the cooperation of three spacecraft, it is almost impossible to optimize the trajectory in whole from when the observation starts to the last observation. Instead, it is divided into multiple segments by the observing moments. The spacecraft have to start from a triangle observing one radio source, cooperate, and form the next triangle to observe the next radio source. Denote $\boldsymbol{r}_0^{(i)}$, $\boldsymbol{v}_0^{(i)}$ the position and velocity of the i-th spacecraft at the initial moment, denoted as t_0 of a segment. The nominal position and velocity are denoted as $\boldsymbol{r}_N^{(i)}$, $\boldsymbol{v}_N^{(i)}$ at the final moment of this segment, denoted as t_f. Denote $\delta\boldsymbol{r}^{(i)}$ the deviation of position for the i-th spacecraft and $\boldsymbol{n}$ the direction of the radio source. In order to finish an observation, it should be satisfied that

$$\begin{cases} (\boldsymbol{r}_0^{(1)} + \delta\boldsymbol{r}^{(1)} - \boldsymbol{r}_0^{(3)} - \delta\boldsymbol{r}^{(3)}) \cdot \boldsymbol{n} = 0 \\ (\boldsymbol{r}_0^{(2)} + \delta\boldsymbol{r}^{(2)} - \boldsymbol{r}_0^{(3)} - \delta\boldsymbol{r}^{(3)}) \cdot \boldsymbol{n} = 0 \end{cases} \tag{13}$$

The simplest method is to calculate $\delta\boldsymbol{r}^{(i)}$ with Eq. 13 being satisfied. However, it is rather difficult to estimate the relation between $\delta\boldsymbol{r}^{(i)}$ and the fuel consumption. We choose a quadratic performance index, i.e.

$$J_O = \Sigma_{i=1}^{3} \delta\boldsymbol{r}^{(i)} \cdot \delta\boldsymbol{r}^{(i)} \tag{14}$$

The solution of quadratic programming with linear constraint is rather mature and such a problem is easily solved. However, we find that $\delta\boldsymbol{r}^{(3)}$ is usually small so we might as well set it null and reduce the difficulty of the problem. By finding $\delta\boldsymbol{r}^{(i)}$ first and we convert the complex optimal control problem which requires the cooperation of spacecraft into separate easy problems.

The proper choice of the final velocity also matters. It is straightforward to set it free at every t_f. However, we find that such a choice might leads to problems. Suppose the i-th segment is solved with free final velocity, the final velocity might be much different from the nominal velocity in order to save fuel. Consequentially the $i+1$-th segment might be impossible to solve because the improper initial velocity. As a result, the final velocity is chosen to be the velocity of the nominal trajectory. Such a choice might leads to excess consumption of fuel but it does not matter in this problem because the fuel is enough and the primary index is unlikely to be the same. Besides, more fuel consumption during the first few segments will make the later segments easier to implement because lower mass leads to larger acceleration.

Nevertheless, it is still possible that one segment cannot be solved directly. The problem is well solved by solving this segment with the next together and treating the constraint for observing as an inner-point constraint. The moment when the observation is performed, denoted as t_m is not changed in order to simplify the problem. After introducing two adjoint multipliers μ_1 and μ_2 to two constraints in Eq. 13. The boundary conditions for observing is

$$\begin{cases} (\boldsymbol{r}^{(1)}(t_m) - \boldsymbol{r}_0^{(3)}) \cdot \boldsymbol{n} = 0 \\ (\boldsymbol{r}^{(2)}(t_m) - \boldsymbol{r}_0^{(3)}) \cdot \boldsymbol{n} = 0 \end{cases} \tag{15}$$

After introducing numerical multipliers μ_1 and μ_2 to Eq. 15, the boundary conditions for optimality are derived as

$$\begin{cases} \boldsymbol{\lambda_r}^{(1)}(t_m^-) - \boldsymbol{\lambda_r}^{(1)}(t_m^+) + \mu_1 \boldsymbol{n} = 0 \\ \boldsymbol{\lambda_r}^{(2)}(t_m^-) - \boldsymbol{\lambda_r}^{(2)}(t_m^+) + \mu_2 \boldsymbol{n} = 0 \end{cases} \tag{16}$$

which indicates $\boldsymbol{\lambda_r}$ at the observing moment has a discontinuity which is parallel to $\boldsymbol{n}$. With all the boundary conditions derived, a MPBVP is built and solved through shooting methods. All the observing chances determined in the last step can be accomplished with these two methods.

RESULTS

By the methods introduced above, we got the best feasible solution. 22 different radio sources are observed 50 times during the whole mission. The number of the radio sources with P =1, 3 and 6 is 12. The number of the radio sources with $P = 1$, 3 and 6 is 3. The final result of the performance index is

$$J = 128286317.0\text{km}$$

The trajectories of the three spacecrafts (white lines) in the whole mission are shown in Fig. 7. The red line represents the orbit of the moon.

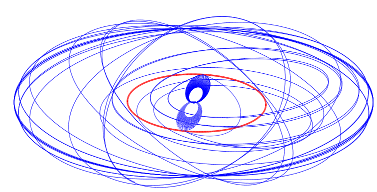

Figure 7. Trajectories during the Whole Mission

CONCLUSION

In this paper, the methods that Team 3, Tsinghua University used for GTOC8 are introduced. With the preliminary analysis, we proposed the final formation, by which maximal h and $\cos\delta$ can be achieved, of the three spacecrafts. The flight of each spacecraft is divided into two phases: 1) reaching the moon; and 2) construct the nominal trajectories after the moon's gravity assist. In the first phase, a rapid method to design low-thrust trajectories with multiple revolutions is proposed.

The second phase is divided into several sub-phases (see Fig. 6) in order to realize desired phase difference, potential to find chances of P =1, 3 and 6, and desired final formation. A global method to search the best observation sequences with the nominal trajectories is then proposed. An indirect method is then proposed to solve the fuel optimal trajectories between every other observation chances based the nominal trajectories for obtaining accurate trajectories. With the proposed methods, we got the best feasible solution with $J = 128286317.0$ km.

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