Available online at www.sciencedirect.com
ScienceDirect
PERGAMON
Acta Astronautica 61 (2007) 763–768
ACTA ASTRONAUTICA
www.elsevier.com/locate/actaastro

First ACT global trajectory optimisation competition results found at CNES/CS

R. Bertrand[a,*], T. Ceolin[b], R. Epenoy[a]

[a] *Centre National d'Etudes Spatiales (CNES), 18 Avenue Edouard Belin, F31401 Toulouse Cedex 9, France*
[b] *CS Communication et Systèmes, Parc de la Plaine, rue Brindejonc des Moulinais, BP 15872, F31506 Toulouse Cedex 5, France*

Available online 2 May 2007

Abstract

This paper presents the solution of the CNES/CS joint team in response to the first ACT competition on global trajectory optimisation. The optimisation tools available at CNES and CS are based on a Lambert's problem formulation associated with a direct optimisation method, or on Pontryagin's maximum principle associated with decomposition–coordination and continuation–smoothing techniques. After a brief presentation of all these numerical methods, we propose a procedure for finding a solution of the ACT problem. The solution proposed by our team fulfills all the mission constraints and consists in a sequence of seven different swing-bys (three of the Earth, one of Venus, two of Jupiter and one of Saturn) before the encounter with asteroid 2001 TW229. All the characteristics of the trajectory are finally detailed in terms of duration, thrust arcs and altitude of pericentre.

Keywords: Interplanetary trajectory; Asteroid encounter; Parametric optimisation; Optimal control; Pontryagin's maximum principle

1. Introduction

This document details the solution obtained by the joint CNES/CS team in response to the first competition on global trajectory optimisation [1] organised by the European Space Agency's advanced concepts team (ACT). At the beginning, the numerical methods used are described. Then, the trajectory obtained is detailed by considering separately each heliocentric arc. We also highlight the important parameters of the overall trajectory and give the value of the objective function defined by the ACT.

* Corresponding author.
E-mail addresses: regis.bertrand@cnes.fr (R. Bertrand), thierry.ceolin@c-s.fr (T. Ceolin), richard.epenoy@cnes.fr (R. Epenoy).

0094-5765/$ - see front matter © 2007 Elsevier Ltd. All rights reserved.
doi:10.1016/j.actaastro.2007.03.004

2. General approach

The tools that we had at our disposal before the beginning of the competition require the knowledge of the sequence of bodies for flybys. So, the major difficulties of the ACT problem [1] are:

- To find the sequence of bodies to be visited.
- To find the launch window.

Once a sequence is chosen, the trajectory may be optimised with the tools available at CNES/CS and is described below. Therefore, the first step consists in finding an acceptable ballistic scenario by means of a direct optimisation method. Such a solution may be viewed as an initial guess for the solution of the optimal control problem derived from the ACT problem (step 2 of the solution procedure).

2.1. Optimisation method (ballistic scenario)

2.1.1. Modelling

Assuming patched conics and impulsive manoeuvres yields a solution mainly based on the well-known Lambert's problem. This approach reduces the number of unknown parameters. Indeed the objective function, which is related to the sum of each Delta-V, is determined by the knowledge of the departure and arrival velocities at the different bodies. These velocities are computed by solving each Lambert's problem which only requires the knowledge of the boundary points and duration of each arc. In case of transfers between celestial bodies, the solution of a Lambert's problem only depends on the departure and arrival dates. On the other hand, in the case of a transfer between a body and a point of the space (case of a deep space manoeuvre) five parameters are necessary: the dates and position components of the deep space manoeuvre. Finally, finding the optimum trajectory reduces to the solution of a non-linear parametric optimisation problem with equality and inequality constraints. This type of optimisation problem is well known in literature (see, for example, [2]) and several methods allow to solve it: gradient methods, Nelder and Mead non-linear simplex [3], etc. Basically the problem can be written as follows:

$$\underset{x \in \Re^n}{\text{Min}} \; f(x) \tag{1}$$

s.t. $h_j(x) = 0$ for $j = 1, \ldots, p$ with $p \leqslant n$, $g_i(x) \leqslant 0$ for $i = 1, \ldots, m$,

where f is the objective function, h_j ($j = 1, \ldots, p$), respectively, g_i ($i = 1, \ldots, m$), are the equality, respectively, inequality, constraints and x denotes the vector of the unknown variables.

2.1.2. Taking constraints into account

There are two ways for taking into account constraints in an optimisation problem. The first one is based on duality: the number of unknowns of the initial problem is increased by introducing Lagrange's parameters and the method consists in solving the Karush–Kuhn–Tucker necessary optimality conditions. The second approach consists in introducing constraints directly in the cost function by means of penalty terms. In this way, the problem can be written as follows:

$$\underset{x \in \Re^n}{\text{Min}} \; f(x) + \beta.P(x), \tag{2}$$

where for exact penalty functions ($x_{\min} \leqslant x \leqslant x_{\max}$):

$$P(x) = \text{Max}[\beta.(x - x_{\max}), \beta.(x_{\min} - x), 0]. \tag{3}$$

In that case, the problem dimension remains unchanged. We have chosen this way for dealing with our constraints. Thereby, the resulting unconstrained problem can be solved by means of a classical optimisation algorithm. Of course, we have to ensure that the solution of the new problem (with penalty terms) is also solution of the initial problem. Fortunately, there exists some specific optimality conditions in the framework of constraints treated through exact penalty terms in the objective function [4]. The main condition is

$$\beta > \sum_{i=1}^{m} |\lambda_i^*| + \sum_{j=1}^{p} |\mu_j^*|. \tag{4}$$

β is a multiplicative coefficient associated with the penalty functions, the three vectors x^*, λ^*, μ^* are the Kuhn–Tucker triplet satisfying the following first order optimality conditions:

$$\nabla_x L(x^*, \lambda^*, \mu^*) = 0, \tag{5}$$

$$\lambda_i^* \geqslant 0, \quad g_i(x^*) \leqslant 0, \quad \lambda_i g_i(x^*) = 0, \quad i = 1, \ldots, m, \tag{6}$$

$$h_j(x^*) = 0, \quad j = 1, \ldots, p, \tag{7}$$

where $L(x, \lambda, \mu)$ is the Lagrangian function defined as follows:

$$L(x, \lambda, \mu) = f(x) + \sum_{i=1}^{m} \lambda_i g_i(x) + \sum_{j=1}^{p} \mu_j h_j(x). \tag{8}$$

2.1.3. Optimisation algorithm

By using penalty terms, the first derivative of the new cost function is not available everywhere. Moreover, the problem may present several local solutions. In general, the number of bodies to be considered (so the number of unknowns) is small. Due to the above reasons, we have chosen a "direct" optimisation method, which does not use the gradient of the cost function: the non-linear simplex of Nelder–Mead [3]. This method is robust and widely used in several engineering domains. We have also used it for the optimisation of a planetary station acquisition strategy [5]. Although it is a local optimisation method, it has global convergence properties (so is able in some cases to find the global solution). Results obtained in interplanetary trajectories optimisation with this approach can be found in [6,7].

2.2. Optimisation method (low-thrust case)

2.2.1. Definition of the optimal control problem

This first approach described above provides us with an initial guess for solving the optimal control problem resulting from the ACT problem. Using the classical equations of the two-body problem, the space dynamics equations of the probe can be written as follows:

$$\begin{cases} \dot{r}(t) = v(t), \\ \dot{v}(t) = -\mu \dfrac{r(t)}{\|r(t)\|^3} + \dfrac{U(t)}{m(t)}, \\ \dot{m}(t) = -\dfrac{\|U(t)\|}{c}, \end{cases} \tag{9}$$

where, at time t, $r(t)$ denotes the heliocentric position vector of the probe, $v(t)$ denotes its heliocentric velocity vector, $m(t)$ denotes its mass and the control variable is $U(t)$ corresponding to the thrust vector (with $\|U(t)\| \leqslant F_{\max} = 0.04\,\mathrm{N}$). Moreover, μ denotes the Sun gravitational constant and $c = g_0.Isp$ is the engine exhaust velocity. In addition, in case of a gravity assist we use the classical model associated with the two-body problem. At the date of the swing-by, the mass of the probe and its energy are preserved but the swing-by induces a instantaneous deviation of its velocity vector. Furthermore, the objective function is defined by a function J (see [1]) of the trajectory and the optimisation parameters are:

- The departure date from the Earth.
- The date of the encounter with asteroid 2001 TW229.
- The number of planetary swing-bys.
- The planets selected for swing-bys.
- The date and the pericentre radius for each planetary gravity assist.

At the end of the interplanetary cruise, the heliocentric position of the spacecraft and the heliocentric position of asteroid 2001 TW229 have to be equal. Finally, if we denote by $x(t)=[x_1(t)=m(t), x_2(t)=r(t), x_3(t)=v(t)]$ the state vector of our optimal control problem, we can write down the problem to solve under the following compact form:

$$\begin{cases} \min\limits_{\|U(t)\| \leqslant F_{\max}} J(t_f, x(t_f)), \\ \dot{x}(t) = f(t, x(t), U(t)), \quad t \in [t_0, t_f], \\ \psi_0(t_0, x(t_0)) = 0, \quad \psi_1(t_f, x(t_f)) = 0, \\ x_1(t_i^+) = x_1(t_i^-), \quad x_2(t_i^+) = x_2(t_i^-) = \alpha(t_i), \\ \eta_i(t_i, x_3(t_i^-), x_3(t_i^+)) = 0, \quad t_i \in]t_0, t_f[\text{free}, \\ i = 1, \ldots, n, t_0 \text{ given} \quad t_f \text{ free}, \end{cases} \tag{10}$$

where t_i^- and t_i^+ $(i = 1, \ldots, n)$ are the dates just before and just after swing-by number i.

2.2.2. Numerical methods

The optimal control problem is solved by means of Pontryagin's maximum principle which is a first order necessary optimality condition for the problem. This principle drives to a multi-point boundary value problem (MPBVP) characterised by a great number of unknown costate variables. To overcome numerical issues associated with the solution of an MPBVP, a decomposition–coordination technique is applied in order to split the global problem into several two points boundary value problems (TPBVP) [8]. Each TPBVP corresponds to a trajectory between two bodies. Coordination parameters are then introduced into the objective function and/or into the bound conditions in order to "link" each arc of the whole trajectory. The optimal values of the coordination parameters are obtained thanks to a fixed point algorithm: these parameters are updated at each step of the algorithm until the necessary optimality conditions of the global problem are fulfilled.

Other techniques are also used to find the solution of each TPBVP such as the continuation–smoothing technique for bang-bang controls. In fact, in the case of bang-bang controls, Newton's method used for finding a zero of the shooting function is extremely sensitive to the initial guess. This is the reason why smoothing the control function by adding a perturbing term (logarithmic barriers, for instance) into the objective function of each sub-problems allows to decrease this sensitivity. The bang-bang solution is then obtained by the use of a continuation procedure in which the value of the perturbing term tends progressively to 0. One can refer to [9] for more details about this continuation–smoothing technique.

Finally, the combined use of decomposition–coordination methods and continuation–smoothing techniques leads to a very powerful tool for solving very difficult optimal control problems such as the ACT one.

3. Application of the methods to the problem

The solution of the ACT problem cannot be found directly by our optimisation or optimal control methods because of the non-standard optimisation criterion. So, the solution proposed in Section 4 has been obtained by applying the following 7-step procedure:

(a) *Step one*: *software customisation*: Our tools have been modified in order to take into account the ACT problem characteristics: implementation of the ACT criterion, setting of ACT model parameters (gravitational constants, planet's radius, etc.) and integration of the asteroid ephemeris.

(b) *Step two*: *intuitive understanding of the ACT problem*: An intuitive understanding of the ACT criterion gives the main outlines of a solution:

- Opportunities for the encounter with the asteroid: one has to reach the asteroid at its perihelion, the S/C relative velocity must be almost collinear to the asteroid velocity, the modulus of this velocity has to be maximised (as on a retrograde orbit, for instance).
- The heliocentric trajectory must minimise the propellant consumption in order to maximise the final S/C mass.

(c) *Step three*: *validation of the customisations*: The validation of the customisations is done on direct (without any gravity assist manoeuvre) Earth–2001 TW229 trajectories.

(d) *Step four*: *improving the cost function value with several swing-bys*: After finding direct trajectories, several swing-bys are introduced into the mission scenario in order to increase the cost function value. In fact, gravity assist manoeuvres allow to save propellant mass and to change efficiently the S/C orbit.

(e) *Step five*: *How to reach a retrograde orbit?*: Two possibilities are offered to reach a retrograde orbit in the solar system: the use of the giant planets for gravity assist manoeuvres or the use of the Sun for a close flyby. This second solution cannot be envisaged here because of the lower bound of the solar distance defined by the ACT [1].

(f) *Step six*: *the trajectory is split into two parts*: In order to make the solution of the problem easier, the overall trajectory is split into two parts: the first one consists in transferring the S/C from the Earth to Jupiter by means of several inner planets' swing-bys. The goal of this first part is to reach Jupiter with an S/C maximum mass. The second part is dedicated to the encounter with the target asteroid. Therefore Jupiter and Saturn swing-bys are used to maximise the value of the ACT criterion.

(g) *Step seven*: *coordination of the two parts*: The last step of the procedure consists in linking the two main trajectory parts in order to build the overall trajectory from the Earth to asteroid 2001 TW229. This solution fulfills all the mission constraints defined in [1].

4. Competition results

The solution obtained by our numerical optimisation methods includes seven different planetary swing-bys. The sequence of the visited bodies is Earth–Earth–Venus–Earth–Earth–Jupiter–Saturn–Jupiter– asteroid 2001 TW229.

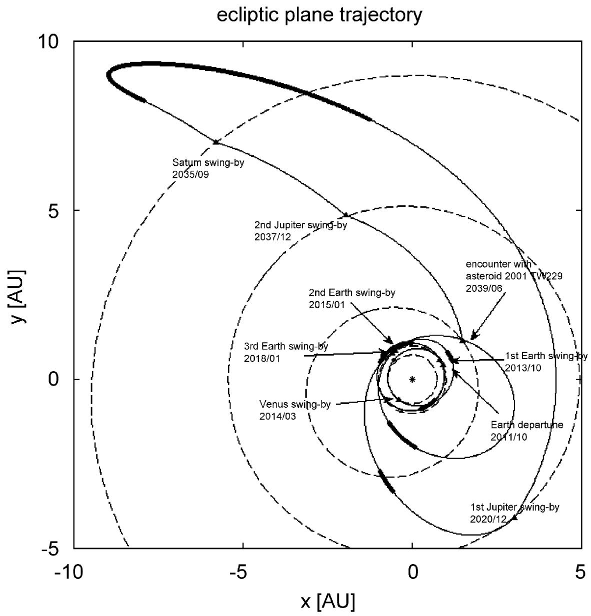

Fig. 1. Overall trajectory from Earth to asteroid 2001 TW229 (ecliptic view).

The overall trajectory is depicted in Fig. 1. We detail hereafter each phase of the mission.

Phase #1: *Earth to Earth*: The spacecraft leaves the Earth on 19 October 2011 with a relative velocity equal to 2.5 km/s. After three thrusting phases the spacecraft flies by the Earth on 29 October 2013 with a relative velocity equal to 3.22 km/s. During the planetary phase the spacecraft is on a hyperbolic trajectory with a pericentre radius of 14,618 km. The thrusting periods are given in bold lines in Fig. 1.

Phase #2: *Earth to Venus*: The duration of this Earth to Venus trajectory is equal to 151.5 days. A very short thrusting phase adjusts the trajectory in order to fly by Venus on 29 March 2014 with a 5.34 km/s relative velocity. The planetary phase is characterised by a pericentre radius equal to 23528 km.

Phase #3: *Venus to Earth*: The duration of this phase is about 297 days. After 22 days of thrust the probe flies by the Earth for a second time on 21 January 2015 with a relative velocity equal to 9.24 km/s. The pericentre radius of the hyperbolic trajectory around the Earth is equal to 6683 km.

Phase #4: *Earth to Earth*: During this phase the spacecraft is on a 1:3 resonant orbit with the Earth orbit. Then the third Earth's swing-by occurs on 27 January 2018 and the relative velocity is equal to 8.71 km/s. The swing-by conditions are adjusted by a 76-day thrusting phase. During the planetary phase the spacecraft is on a hyperbolic trajectory with a 6688 km pericentre radius.

Table 1
Details of the mission events (launch, swing-bys and encounter)

Date	Event	Relative velocity v_∞ (km/s)	Pericenter radius r_p (km)	Sun distance (AU)	Spacecraft mass (kg)
19 October 2011	Launch	2.5	—	0.996	1500
29 October 2013	First Earth swing-by	3.218	14,618	0.993	1478
29 March 2014	Venus swing-by	5.338	23,528	0.724	1478
21 January 2015	Second Earth swing-by	9.236	6683	0.984	1475
27 January 2018	Third Earth swing-by	8.708	6688	0.985	1464
27 December 2020	First Jupiter swing-by	5.804	0.95e + 06	5.085	1453
03 September 2035	Saturn swing-by	14.74	1.60e + 06	9.114	1003
18 December 2037	Second Jupiter swing-by	23.06	1.84e + 06	5.223	1003
14 June 2039	Encounter with asteroid 2001 TW229	50.154	—	1.882	1003

Phase #5: *Earth to Jupiter*: Then the spacecraft moves towards the outer planets. A 74-day thrusting period allows to reach Jupiter on 27 December 2020 with a relative velocity of 5.80 km/s. The hyperbolic trajectory has a pericentre radius roughly equal to 950,000 km and the spacecraft leaves Jupiter with a mass of 1453 kg.

Phase #6: *Jupiter to Saturn*: This phase is probably the most important and particular phase of the overall mission. Indeed an extremely long thrusting period (duration of 3190 days) allows to transfer the spacecraft on a heliocentric retrograde orbit. The 180° change of the inclination is exclusively performed by means of the probe engine. This is done when the semi-major axis of the spacecraft orbit reaches its maximum value in order to minimise the propellant consumption. Saturn is reached on 03 September 2035 and the swing-by is performed with a pericentre approximately equal to 1,600,000 km.

Phase #7: *Saturn to Jupiter*: The spacecraft trajectory is now purely ballistic. The departure conditions at Saturn allow to reach Jupiter one more time after a 837-day cruise (on 18 December 2037). The associated relative velocity is equal to 23.06 km/s. The spacecraft flies by Jupiter at minimum distance roughly equal to 1,840,000 km.

Phase #8: *Jupiter to asteroid* 2001 $TW229$: This last ballistic phase drives the probe to the impact with asteroid 2001 TW229 on 14 June 2039 when the target asteroid is located close to its perihelion. The retrograde trajectory allows to reach a very high relative velocity with respect to the asteroid. This last one is equal to 50.15 km/s. The angle between the spacecraft relative velocity and the asteroid heliocentric velocity is about 166°. This means that the relative velocity is closely opposite to the asteroid one.

Table 2
Summary of the thrusting periods

Thrust phase	Begin (MJD 2000)	End (MJD 2000)	Duration (days)
#1	4378.65	4452.01	73.36
#2	4756.56	4808.43	51.87
#3	4952.93	4978.86	25.935
#4	5140.75	5141.05	0.303
#5	5351.74	5373.45	21.72
#6	5616.96	5693.03	76.07
#7	6849.61	6924.17	74.55
#8	9184.06	12374.75	3190.69

Summary of the mission: All the important mission characteristics (launch and swing-by dates, thrusting periods, etc.) are summarised in Tables 1 and 2.

The final spacecraft mass is equal to 1002.78 kg and the value of the objective function J is equal to

$$J = 1,194,884.11\,\mathrm{kg\,km^2/s^2}. \tag{11}$$

The duration of the proposed mission is 10,100 days (or 27.65 years). This value fulfills the mission constraints.

5. Conclusions

The problem proposed by ACT was very interesting but also very difficult to solve. Nevertheless all the numerical tools available in CNES and CS have been used with success and a solution fulfilling all the mission constraints has been found. The main drawback of the proposed solution trajectory comes from the inclination change needed to reach a final retrograde orbit. In our solution this is done by means of the S/C engine (see the solution of Moscow Aviation Institute) whereas other teams have found solutions for which the inclination change is performed through a Jupiter (see the solution of Politecnico di Torino) or a Saturn gravity assist (see solutions of GMV, Deimos and JPL teams). This mainly explains the difference between the values of the objective functions obtained by the different teams. Moreover, the procedure we have used to select the sequence of flyby is efficient because the proposed sequence is very close to the best sequence found (see solution of JPL).

References

[1] D. Izzo, 1st ACT global trajectory optimisation competition: problem description and summary of the results, Acta Astronautica (2007) this issue, doi: 10.1016/j.actaastro.2007.03.003.

[2] J. Nocedal, S.J. Wright, Numerical Optimization, Springer, New York, 1999.

[3] J.A. Nelder, R. Mead, A simplex method for function minimization, Computer Journal 7 (1965) 308–313.

[4] D.P. Bertsekas, Constrained Optimization and Lagrange Multiplier Methods, Academic Press, New York, 1982.

[5] F. Dufour, T. Ceolin, J.M. Garcia, J. Bernussou, J. Bernard, D. Delobette, Optimization of the target orbit acquisition segment of an interplanetary mission, IAF-94-A.2.016, Jerusalem, Israel.

[6] T. Ceolin, J.M. Garcia, J.M. Enjalbert, C. Brochet, J. Bernard, A general method for interplanetary trajectory optimization, in: 11th IAS, Gifu, Japan, 19–25 May 1996.

[7] T. Ceolin, Optimization of impulsive thrusts for the calculation of heliocentric trajectories and planetary station acquisition, Ph.D. Thesis, Paul Sabatier University, Toulouse III, France, 1998 (in French).

[8] R. Bertrand, Optimization of low-thrust interplanetary trajectories, Ph.D. Thesis, Paul Sabatier University, Toulouse III, France, 2001 (in French).

[9] R. Bertrand, R. Epenoy, New smoothing techniques for solving bang-bang optimal control problems—numerical results and statistical interpretation, Optimal Control Application and Methods 23 (2002) 171–197.