Chapter 11

Exercise 11A

1 **a** $\frac{x^7}{7} + c$

b $\frac{x^2}{2} + c$

c $-\frac{1}{2x^2} + c$

d $5x + c$

e $x^4 + c$

f $\frac{1}{2x^6} + c$

g $\frac{x^6}{8} + c$

h $\frac{32x^{\frac{3}{4}}}{3} + c$

i $\frac{2x^{\frac{5}{2}}}{5} + c$

j $\frac{3x^{\frac{7}{3}}}{7} + c$

k $2\sqrt{x} + c$

l $-\frac{5x^{\frac{3}{5}}}{3} + c$

m $4x^{\frac{3}{2}} + c$

n $\frac{1}{24x^4} + c$

2 **a** $x^3 + \frac{x^2}{2} - x + c$

b $\frac{x^5}{5} - \frac{5}{2}x^2 + 7x + c$

c $\frac{x^6}{4} - \frac{x^2}{8} - 4x + c$

d $\frac{4x^3}{9} - \frac{x^2}{10} - \frac{1}{x^5} + c$

3 **a** $\frac{2}{3}\sqrt{x}(x - 3) + c$

b $2x^{\frac{5}{2}}(2 - 3x) + c$

c $-4x^2 + \frac{2x^{\frac{5}{3}}}{5} - 5x^{\frac{4}{5}} + c$

d $3x^{\frac{5}{4}} - \frac{x^{\frac{4}{5}}}{10} + c$

4 **a** $\frac{k^5}{5} - \frac{3k^2}{2} - 5k + c$

b $\frac{2p^9}{3} + \frac{1}{3p^3} + c$

c $2\sqrt{t}(3t + 4) + c$

5 **a** $\frac{2}{3}x^3 - 5x + c$

b $\frac{-5x^{-3}}{3} + \frac{4}{3}x^{\frac{3}{2}} + c$

c $2x^{\frac{1}{5}} + c$

d $-\frac{3x}{4x^4} + 4x - \frac{x^3}{3} + c$

e $-\frac{3}{4}x^{-4} + 4x - \frac{x^3}{3} + c$

f $\frac{1}{6}\left(-\frac{1}{x^2} - \frac{3}{x} - \frac{96x^{\frac{5}{4}}}{5}\right) + c$

g $x - \frac{24}{5}x^{\frac{5}{6}} + c$

h $-\frac{1}{2}x - \frac{1}{6}x^{-2} - \frac{16}{5}x^{\frac{5}{4}} + c$

Exercise 11B

1 **a** $2x^2 - 7x + 3$

$\frac{2x^3}{3} - \frac{7x^2}{2} + 3x + c$

b $x^3 - 3x^2 - 4x$

$\frac{x^4}{4} - x^3 - 2x^2 + c$

c $x^3 + 5x^2 + 2x - 8$

$\frac{x^4}{4} + \frac{5x^3}{3}x^3 + x^2 - 8x + c$

d $5x^4 - 30x^3 + 45x^2$

$x^5 - \frac{15x^4}{2} + 15x^3 + c$

e $x^3 + 4x^2 - 3x - 18$

$\frac{x^4}{4} + \frac{4x^3}{3} - \frac{3x^2}{2} - 18x + c$

2 **a** $\frac{1}{5}x^{-3}$

b $\frac{3}{5}x^{\frac{-4}{3}}$

c $x^3 - 5x^2 + 2x - 8$

d $\frac{2}{3}x^2 - \frac{1}{3}^{x-2}$

e $\frac{3}{5}x^{\frac{-4}{3}}$

f $1 - 3x^{\frac{1}{2}}$

g $\frac{1}{6}x^{-2} - \frac{5}{3}x^2$

3 **a** $6x^{-3}$

$-\frac{3}{x^2} + c$

b $\frac{1}{5}x^{-4}$

$-\frac{1}{15}x^{-3} + c$

c $\frac{7}{3}x^{-8}$

$-\frac{1}{3x^7} + c$

d $4x^{-2} - x^2 + 5$

$-4x^{-1} - \frac{x^3}{3} + 5x + c$

4 **a** $3x^{\frac{1}{2}}$

$2x^{\frac{3}{2}} + c$

b $x^{\frac{4}{3}}$

$\frac{3x^{\frac{7}{3}}}{7} + c$

c $6x^{\frac{1}{5}}$

$5x^{\frac{6}{5}} + c$

d $4x^{\frac{-1}{2}}$

$8\sqrt{x} + c$

e $x^{\frac{-3}{2}}$

$-\frac{2}{\sqrt{x}} + c$

f $3x^{\frac{-1}{4}}$

$4x^{\frac{3}{4}} + c$

g $10x^{\frac{-5}{2}}$

$-\frac{20}{3x^{\frac{3}{2}}} + c$

h $\frac{1}{2}x^{\frac{-3}{4}}$

$2x^{\frac{1}{4}} + c$

5 **a** $x^4 - 4x^{-2}$

$\frac{4}{x} + \frac{x^5}{5} + c$

b $9x^{-3} - x$

$-\frac{9}{2x^2} - \frac{x^2}{2} + c$

c $1 - x^{-3} - 3x^{-4}$

$x + \frac{1}{2x^2} + \frac{1}{x^3} + c$

d $\frac{5}{3}x^{-2} - \frac{2}{3}x^2$

$\frac{1}{3}\left(-\frac{5}{x} - \frac{2x^3}{3}\right) + c$

e $x^{-2} + x^{-3} - 6x^{-4}$

$\frac{2}{x^3} - \frac{1}{2x^2} - \frac{1}{x} + c$

f $3x^{-2} + x^{-3} - \frac{8}{3}x^{-4} - \frac{4}{3}x^{-5}$

$\frac{1}{3x^4} + \frac{8}{9x^3} - \frac{1}{2x^2} - \frac{3}{x} + c$

6 **a** $-2x^2 + \frac{2x^{\frac{5}{2}}}{5} + c$

b $x^2 + \frac{2}{x} + c$

c $2\sqrt{x} + 2x - \frac{2}{3}x^3 - \frac{2}{5}x^{\frac{5}{2}} + c$

d $-\frac{25}{x} - 2x + \frac{x^3}{75} + c$

e $2\sqrt{x} - \frac{2x^{\frac{7}{2}}}{7} + c$

f $-\frac{4}{\sqrt{x}} + 2\sqrt{x} - \frac{2x^{\frac{3}{2}}}{3} + c$

7 **a** $-\frac{2}{7x^7} + c$

b $-\frac{1}{5t} + c$

c $\frac{p^4}{4} - p^3 - 3p^2 + 8p + c$

d $\frac{18x^{\frac{5}{3}}}{5} + c$

e $\frac{4\sqrt{w}}{3} + c$

f $-\frac{3}{2x^2} - \frac{x^2}{2} + c$

g $-2\left(\frac{t^5}{5} - t^4\right) + c$

h $-2u^{\frac{3}{2}} + 2u - \frac{1}{u} + c$

i $\frac{15x^{\frac{4}{5}}}{16} + c$

8,9 Qrsbc l r cvcpagqc

Exercise 11C

1 **a** $\frac{1}{6}(x + 1)^6 + c$

b $\frac{1}{9}(x - 3)^9 + c$

c $2(x - 2)^5 + c$

d $4\left(\frac{x^2}{2} + 6x\right) + c$

e $\frac{1}{10}(2x + 3)^5 + c$

f $\frac{1}{40}(5x - 2)^8 + c$

g $\frac{3}{28}(4x + 1)^7 + c$

h $-\frac{2}{9}(3x - 4)^9 + c$

2 a $-\frac{1}{2(x+2)^2} + c$
b $\frac{1}{7-x} + c$
c $-2(x-5)^{-4}$
d $-\frac{3}{5(x+8)^5} + c$
e $\frac{1}{9(1-3x)^3} + c$
f $-\frac{1}{2(2x+1)} + c$
g $-\frac{1}{8(4x-3)^4} + c$
h $\frac{1}{6(5-6x)^7} + c$

3 a $\frac{2}{3}(x+1)^{\frac{3}{2}} + c$
b $\frac{3}{4}(x-4)^{\frac{4}{3}} + c$
c $10(x+6)^{\frac{6}{5}} + c$
d $4(x+4)^{\frac{5}{2}} + c$
e $2\sqrt{x-2} + c$
f $\frac{3}{2}(x+1)^{\frac{2}{3}} + c$
g $-\frac{8}{\sqrt{x-6}} + c$
h $3(x+4)^{\frac{1}{4}} + c$
i $\frac{2}{9}(3x+2)^{\frac{3}{2}} + c$
j $\frac{2}{5}(5x-3)^{\frac{1}{2}} + c$
k $\frac{1}{12}(6x+1)^{\frac{5}{4}} + c$
l $-\frac{1}{14(7x-4)^{\frac{3}{2}}} + c$
m $\frac{3}{2}(2x-5)^{\frac{1}{3}} + c$
n $\frac{1}{2}(4x-1)^{\frac{1}{4}} + c$
o $-\frac{1}{3(8x+3)^{\frac{1}{4}}} + c$
p $\frac{3}{56}(7x-1)^{\frac{8}{3}} + c$

4 a $\frac{1}{3-x} + c$
b $-\frac{3}{2(x+1)^2} + c$
c $-\frac{1}{15(x-2)^3} + c$
d $-\frac{1}{3(x+8)^4} + c$
e $\frac{1}{9(1-3x)^3} + c$
f $\frac{9}{15-25x} + c$
g $-\frac{1}{24(2x-7)^4} + c$
h $-\frac{3}{10(x+4)^5} + c$

5 a $\frac{2}{3}(x+4)^{\frac{3}{2}} + c$
b $\frac{16}{3}(x-1)^{\frac{3}{2}} + c$
c $2\sqrt{x-1} + c$
d $12\sqrt{x+3} + c$
e $\frac{3}{4}(x+4)^{\frac{4}{3}} + c$
f $\frac{4}{7}(x-1)^{\frac{7}{4}} + c$
g $\frac{2}{5}(x+5)^{\frac{5}{2}} + c$
h $-\frac{2}{\sqrt{x-2}} + c$
i $\frac{32}{5}(x-6)^{\frac{5}{4}} + c$
j $\frac{5}{9}(x+1)^{\frac{6}{5}} + c$
k $\frac{15}{2}(x-6)^{\frac{2}{3}} + c$
l $\frac{24}{5}(x-1)^{\frac{1}{6}} + c$

6 a $\frac{2}{9}(3x+2)^{\frac{3}{2}} + c$
b $\frac{1}{10}(4x-1)^{\frac{5}{2}} + c$
c $\frac{15}{14}(2x-3)^{\frac{7}{5}} + c$
d $\frac{8}{7}(7x-1)^{\frac{1}{6}} + c$
e $\frac{2}{9}(3x+2)^{\frac{3}{2}} + c$

7 a $\frac{1}{18}(3x-1)^6 + c$
b $-\frac{1}{2(x-2)^2} + c$
c $\frac{1}{5}(3-x)^5 + c$
d $-\frac{2}{3}(2-x)^{\frac{3}{2}} + c$
e $-\frac{1}{45}(1-5x)^9 + c$
f $\frac{2}{9}(3x-2)^6 + c$
g $\frac{1}{18(5-6x)^3} + c$
h $-\frac{2}{\sqrt{x-2}} + c$
i $-\frac{3}{8}(x-4)^4 + c$
j $-\frac{4}{7}(5-x)^{\frac{7}{4}} + c$
k $\frac{1}{8(1-3x)^4} + c$
l $\frac{3}{140}(4x-5)^5 + c$

Exercise 11D

1

$3\cos 3x$

$2\cos 2x$

$-4\cos 4x$

$6\cos(6x+5)$

$q\cos(qx+r)$

2 $\cos\left(x-\frac{\pi}{6}\right)$

$3\cos 3x$	$\sin 3x$	$\cos 3x$	$\frac{1}{3}\sin 3x$
$2\cos 2x$	$\sin 2x$	$\cos 2x$	$\frac{1}{2}\sin 2x$
$\cos\left(x-\frac{\pi}{6}\right)$	$\sin\left(x-\frac{\pi}{6}\right)$	$3\cos\left(x-\frac{\pi}{6}\right)$	$3\sin\left(x-\frac{\pi}{6}\right)$
$4\cos(4x-\pi)$	$-\sin 4x$	$\cos(4x-\pi)$	$-\frac{1}{4}\sin 4x$
$6\cos(6x+5)$	$\sin(6x+5)$	$\cos(6x+5)$	$\frac{1}{6}\sin(6x+5)$
$q\cos(qx+r)$	$\sin(qx+r)$	$\cos(qx+r)$	$\frac{\sin(qx+r)}{q}$

3

$6\sin 3x$	$-2\cos 3x$
$5\sin 2x$	$-\frac{5}{2}\cos 2x$
$3\sin\left(x-\frac{\pi}{6}\right)$	$-3\cos\left(x-\frac{\pi}{6}\right)$
$\frac{1}{2}\sin(4x-\pi)$	$\frac{1}{8}\cos 4x$
$-3\sin(6x+5)$	$\frac{1}{2}\cos(6x+5)$
$p\sin(qx+r)$	$-\frac{p\cos(qx+r)}{q}$

4

$\cos 2x$	$\frac{1}{2}\sin 2x$
$5\cos 2x$	$\frac{5}{2}\sin 2x$
$2\cos\left(x+\frac{\pi}{2}\right)$	$2\cos(x)$
$\frac{3}{4}\cos\left(\frac{1}{2}x-2\pi\right)$	$\frac{3}{2}\sin\frac{x}{2}$
$-2\sin(4x-1)$	$\frac{1}{2}\cos(4x-1)$
$p\cos(qx+r)$	$\frac{p\sin(qx+r)}{q}$

Exercise 11E

1 **a** $8\sin x + c$

b $-3\cos x + c$

c $4\cos x + c$

d $2\sin x + c$

e $\frac{3}{2}\sin x + c$

f $\frac{5}{4}\cos x + c$

g $4\sin\left(x - \frac{\pi}{3}\right)$

h $-5\cos(x - 2) + c$

i $\frac{1}{5}\sin 5x + c$

j $-\frac{1}{4}\cos 4x + c$

k $4\sin 2x + c$

l $-\frac{1}{6}\sin 3x + c$

m $-2\cos\frac{1}{2}x + c$

n $4\sin\left(\frac{3x}{2}\right) + c$

o $-\frac{9}{5}\cos 5x + c$

p $-\frac{1}{2}\sin 4x + c$

2 **a** $\frac{5x^3}{3} - 3\cos x + c$

b $-\frac{3}{x} + 2\sin x + c$

c $\frac{8x^{\frac{3}{2}}}{3} + \frac{1}{2}\cos 2x + c$

d $\frac{(x-3)^6}{6} - \cos\left(x - \frac{\pi}{6}\right) + c$

e $\frac{1}{24}(4x + 1)^6 - \frac{1}{3}\sin 3x + c$

f $\frac{1-2x}{2x^2} - 4\cos(x - 1) + c$

g $4\sqrt{x} - \frac{5}{3}\sin 3x + c$

h $-\frac{5}{8x^2} + 8\cos\frac{1}{2}x + c$

i $-\frac{1}{3(x-5)^3} - \frac{1}{8}\sin 8x + c$

j $-\frac{1}{6}(1 - 4x)^{\frac{3}{2}} - \frac{2}{3}\cos\left(3x + \frac{\pi}{4}\right) + c$

3 **a** $1 + \cos 2x = 2(\cos x)^2$

$(\cos x)^2 = \frac{1}{2}(1 + \cos 2x)$

$= \frac{1}{2} + \frac{1}{2}\cos 2x$

b $\int\left(\frac{1}{2} + \frac{1}{2}\cos 2x\right)dx$

$= \frac{1}{2}x + \frac{1}{2}\int\cos 2x\,dx$

$= \frac{1}{2}x + \frac{1}{4}\sin 2x + c$

4 **a** $1 - 2(\sin x)^2 = \cos 2x$

$-2(\sin x)^2 = (\cos 2x - 1)$

$(\sin x)^2 = \frac{1}{2} - \frac{1}{2}\cos 2x$

b $\int\left(\frac{1}{2} - \frac{1}{2}\cos 2x\right)dx$

$= \frac{1}{2}x - \frac{1}{2}\int\cos 2x\,dx$

$= \frac{1}{2}x - \frac{1}{4}\sin 2x + cc$

5 **a** $2x - \sin 2x + c$

b $\frac{1}{4}(x + \sin x\cos x) + c$

c $\frac{1}{4}(2x - 2\cos 2x - \sin 2x) + c$

d $\frac{x}{2} + \frac{1}{4}\sin 2x - \frac{2x^3}{9} + c$

e $-\frac{1}{2}\sin 2x + c$

f $\frac{2}{5}x + \frac{1}{10}\sin 2x + c$

6 $\int\frac{1}{2} - \frac{1}{2}\cos 2x + \frac{1}{2} + \frac{1}{2}\cos 2x\,dx$

$= x + c$

$\int\left((\sin x)^2 + (\cos x)^2\right)dx = \int 1\,dx = x + c$

Challenge

a $(x + y)\left(x^2 - xy + y^2\right) = x^3 + y^3$ and

$(x - y)\left(x^2 + xy + y^2\right) = x^3 - y^3$

b **i**

$(\sin x)^3 + (\cos x)^3 = (\sin x + \cos x)\left((\sin x)^2 + (\cos x)^2 - \sin x\cos x\right)$

$= (\sin x + \cos x)(1 - \sin x\cos x)$

$= (\sin x + \cos x)\left(1 - \frac{1}{2}\sin 2x\right)$

ii

$(\sin x)^3 - (\cos x)^3 = (\sin x - \cos x)\left((\sin x)^2 + (\cos x)^2 + \sin x\cos x\right)$

$= (\sin x - \cos x)(1 + \sin x\cos x)$

$= (\sin x - \cos x)\left(1 + \frac{1}{2}\sin 2x\right)$

c

$$(\sin x)^6 - (\cos x)^6 = \left((\sin x)^3 + (\cos x)^3\right)\left((\sin x)^3 - (\cos x)^3\right)$$

$$(\sin x + \cos x)\left(1 - \tfrac{1}{2}\sin 2x\right)(\sin x - \cos x)\left(1 + \tfrac{1}{2}\sin 2x\right)$$

$$\left((\sin x)^2 - (\cos x)^2\right)\left(1 - \tfrac{1}{4}(\sin 2x)^2\right)$$
$$= -\cos 2x\left(1 - \tfrac{1}{4}(\sin 2x)^2\right)$$
$$= -\cos 2x\left(\tfrac{3}{4} + \tfrac{1}{4}(\cos 2x)^2\right)$$

now

$$\tfrac{3}{4} + \tfrac{1}{4}(\cos 2x)^2 =$$
$$\tfrac{7}{8} + \tfrac{1}{8}\left(2(\cos 2x)^2 - 1\right) =$$
$$\tfrac{7}{8} + \tfrac{1}{8}\left((\cos 2x)^2 - (\sin 2x)^2\right) =$$
$$\tfrac{7}{8} + \tfrac{1}{8}\cos 4x$$

so,
from above,

$$-\cos 2x\left(\tfrac{3}{4} + \tfrac{1}{4}(\cos 2x)^2\right) =$$
$$-\cos 2x\left(\tfrac{7}{8} + \tfrac{1}{8}\cos 4x\right)$$

i

$$\cos(A + B) = \cos A\cos B - \sin A\sin B$$
$$\cos(A - B) = \cos A\cos B + \sin A\sin B$$
$$\Rightarrow \cos(A + B) + \cos(A - B) = 2\cos A\cos B$$

ii

$$\cos 2x\cos 4x = \tfrac{1}{2}(\cos 6x + \cos 2x)$$

so

$$-\tfrac{7}{8}\cos 2x - \tfrac{1}{8}\left(\tfrac{1}{2}(\cos 6x + \cos 2x)\right)$$
$$= -\tfrac{1}{16}\cos 6x - \tfrac{15}{16}\cos 2x$$

e $\int\left(-\tfrac{1}{16}(\cos 6x + 15\cos 2x)\right)dx$

$= -\tfrac{15}{32}\sin 2x - \tfrac{1}{96}\sin 6x + c$

Exercise 11F

1 solution depends on integration constant c

2 **a** Raise the graph of $y = x^2$ up the y axis by the amount c.

b There is no reason for the gradients to be different, the change in divided by the change in x will be the same $y'(P) = y'(Q) = 2a$.

3 **a** A particular solution will be one among an infinite set of possible solutions depending on c.

b the x^1 term would give a different value for the gradient/derivative of y.

4 $f(x) = x^2 + c$

but

$f(3) = 4$

so

$4 = 3^2 + c$

$c = -5$

$f(x) = x^2 - 5$

5 A data point on the curve.

Exercise 11G

1 $x^3 - x^2 + 4x + 25$

2 $4x^{\frac{3}{2}} - 24$

3 $3 - 2\cos 2x$

4 **a** $x^3 + 5x^2 - 2x - 24$

b **i** $(-3)^3 + 5(-3)^2 - 2(-3) - 24 = 0$

ii $(x - 2)(x + 4)(x + 3)$

c $(2, 0)$

$(-4, 0)$

$(-3, 0)$

5 a $4\left(\frac{x^2}{2} - x\right) + 7 = 2(x-1)^2 + 5$

b By completing the suare you can see that the minimum value is 5 so no roots.

6 a $x^3 + x^2 - 5x + 3$

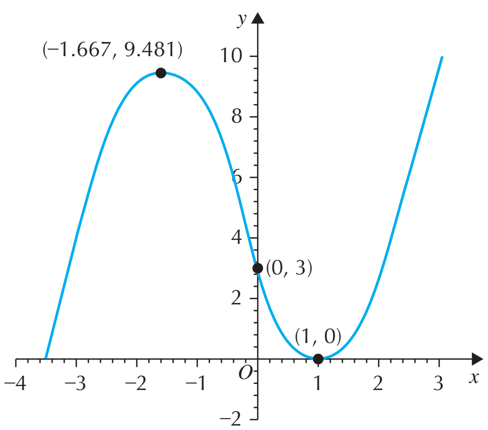
b i $(-3)^3 + (-3)^2 - 5(-3) + 3 = 0$

ii $x = 1$

c i ii $(1, 0)$ minimum

$(-1.667, 9.481)$ maximum

d



7 a $x^3 + 2x^2 - 4x - 8$

b i $(2)^3 + 2(2)^2 - 4(2) - 8 = 0$

ii $(x+2)(x+2)(x-2)$

c t.p. are at $x = -2$ and $x = \frac{2}{3}$.

Since $x = -2$ is both a root $(-2, 0)$ and a turning point the x-axis must be a tangent at $x = -2$.

8 a $\frac{2}{3}$

b $y = mx + c$

$f'(x) = \frac{2}{3}x - 4$

c $= \frac{1}{3}x^2 - 4x + 15$

9 a $-2x + 6$

b $-x^2 + 6x - 9$

10 a $a = 3$

$b = 4$

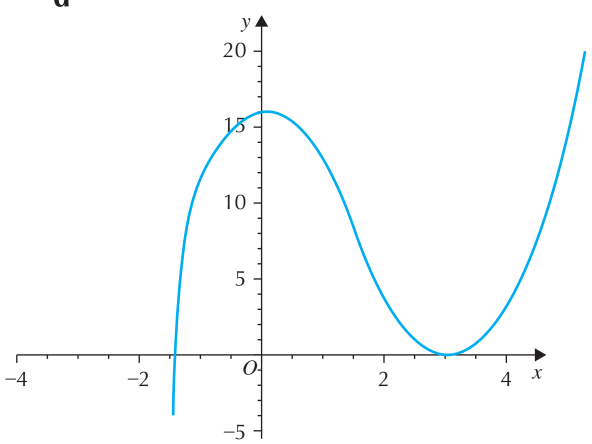
b $x^3 - 6x^2 + 16$

c $(x-2)(x^2 - 4x - 8)$

or $(x-2)\left(x - 2 - 2\sqrt{3}\right)\left(x - 2 + 2\sqrt{3}\right)$

or $(x-2)(x - 5.464)(x + 1.464)$

d



tp are

$(0, 16)$

$(4, 56)$

root is

$(-1.464, 0)$, $(2, 0)$, $(5.464, 0)$

y-intercept is $(0, 16)$

11 a $a = 4$

$b = 2$

b $2\sin 2x + 1$

c $\left(\frac{7\pi}{12}, 0\right)$

$\left(\frac{11\pi}{12}, 0\right)$

12 a $3(2x-1)^5$

b $\frac{1}{4}(2x-1)^6 - 36$

c i $\frac{2}{3}x + 5$

ii $\left(-\frac{15}{2}, 0\right)$

13 a 10

b At intersection between curve and its tangent;

$px^2 + 12x + p - 5 = \frac{p^2 - 5p - 36}{p}$

re-arranging:

$px^2 + 12x + \frac{36 + 5p - p^2 + p^2 - 5p}{p} = 0$

$px^2 + 12x + \frac{36}{p} = 0$

examine descriminant:

$b^2 - 4ac = 0$ for tangent

$144 - 4 \times 36 = 0$

so it is a tangent.

14 a $1.5x - 4$

b $r > \frac{16}{3}$