Chapter 9

Exercise 9A

1 **a** $8x^7$
b $-4x^3$
c 1
d $-\frac{1}{x^2}$
e $-\frac{6}{x^7}$
f $\frac{9}{x^{10}}$
g 0
h $12x^2$
i $6x^7$
j $-\frac{18}{x^3}$
k 0
l $\frac{1}{2x^4}$
m $\frac{3\sqrt{x}}{2}$
n $\frac{5x^{\frac{2}{3}}}{3}$
o $\frac{7x^{\frac{2}{5}}}{5}$
p $\frac{1}{2\sqrt{x}}$
q $\frac{1}{4x^{\frac{3}{4}}}$
r $\frac{2}{3x^{\frac{1}{3}}}$
s $-\frac{1}{2x^{\frac{3}{2}}}$
t $\frac{2}{3x^{\frac{5}{3}}}$
u $\frac{4}{\sqrt{x}}$
v $-\frac{2}{x^{\frac{4}{3}}}$
w $-\frac{5\sqrt{x}}{2}$
x 0
y $-\frac{3}{4x^{\frac{11}{4}}}$

2 **a** $2 + 2x$
b $-4x - 8$
c 3
d $3x^2 - 8x + 8$
e $x^2 + 4x - 12$
f $4x^3 - 6x^2 + 6x - 1$
g $\frac{12}{x^7} - 15x^2$
h $\frac{6}{x^7} - 2$
i $2x^3 + \frac{6}{x^3} + 2$
j $\frac{3x^4}{2} + \frac{1}{x^2}$

3 **a** $-\frac{1}{2x^{\frac{3}{2}}} + \frac{1}{2\sqrt{x}}$
b $18\sqrt{x} - \frac{5x^{\frac{3}{2}}}{2}$
c $18x^2 - \frac{1}{\sqrt{x}}$
d $\frac{1}{x^{\frac{4}{3}}} + \frac{x^{\frac{1}{4}}}{2}$
e $-\frac{2}{x^{\frac{3}{2}}} - \frac{8}{x^7} - 8$
f $\frac{2}{x^{\frac{3}{4}}} - \frac{4}{x^7}$
g $\frac{6}{x^{\frac{1}{4}}} + \frac{1}{5x^{\frac{3}{2}}}$
h $\frac{1}{x^{\frac{5}{4}}} - \frac{8}{x^7} - 8$
i $\frac{1}{5x^{\frac{3}{4}}} + \frac{1}{4x^{\frac{6}{5}}}$

4 **a** $3x^2 - 8x + 1$
b $\frac{4x^{\frac{1}{3}}}{3}$
c $\frac{3}{\sqrt{x}}$
d $4x - 3$
e $\frac{3}{4x^{\frac{1}{4}}} - 8$

5 **a** $4p^3 - 12p$
b $15p^4 + \frac{2}{p^3}$
c $3w^2 - 3$
d $5 - 30t^2$
e $12\sqrt{t} + \frac{2}{t^{\frac{3}{2}}}$
f $\frac{8}{3u^{\frac{1}{3}}} - 1$
g $-\frac{24}{t^5} - 8$

6 **a** $-\frac{8}{x^3} - 5$
c $\frac{12}{x^5} - 5$
d $x^{\frac{1}{3}}$
e $\frac{2}{x^2} + \frac{6}{x^3}$
f $\frac{2}{x^{\frac{1}{3}}} + \frac{2}{x^{\frac{4}{3}}}$

Exercise 9B

1 **a** $2x^2 + 5x - 12$
b $4x^2 - 7x + 3$
c $2x^3 - 11x^2 + 17x - 6$
d $x^3 - 5x^2 + 2x + 8$
e $x^3 - 3x + 2$
f $4x^3 - 4x^2 + x$
g $x^3 - 4x^2 + x$

2 **a** $3x^{-1}$
b $8x^{-3}$
c $5x^{-6}$
d $\frac{1}{3}x^{-2}$
e $\frac{1}{6}x^{-4}$
f $\frac{2}{5}x^{-1}$
g $\frac{4}{3}x^{-9}$
h $\frac{1}{2}x^{-7}$
i $3x^{-\frac{1}{2}}$
j $x^{-\frac{3}{4}}$
k $5x^{-\frac{4}{3}}$
l $\frac{4}{5}x^{-\frac{3}{2}}$
m $2x^{-\frac{5}{8}}$
n $2x^{\frac{1}{2}}$
o $5x^{-\frac{1}{2}}$
p $\frac{1}{2}x^{-\frac{1}{2}}$
q $\frac{3}{4}x^{\frac{1}{5}}$
r x^{-4}
s $x^{-\frac{1}{6}}$
t $\frac{5}{3}x^{-\frac{1}{4}}$
u $6x^2$
v $2x^{-3}$
w $\frac{3}{8}x^{-5}$
x $4x^{-\frac{1}{2}}$
y $\frac{1}{5}x^{\frac{4}{3}}$
z $\frac{7}{4}x^{-2}$

3 **a** $x^2 + 2x - 15$
$2x + 2$
b $8x^2 - 10x - 3$
$16x - 10$
c $x^3 + x^2 - 6x$
$3x^2 + 2x - 6$
d $x^3 + 5x^2 + 2x - 8$
$3x^2 + 10x + 2$
e $2x^4 - 4x^3 + 2x^2$
$8x^3 - 12x^2 + 4x$
f $x^3 - 5x^2 + 3x + 9$
$3x^2 - 10x + 3$
g $x^3 + 3x^2 - 6x - 8$
$3x^2 + 6x - 6$

4 **a** $5x^{-2}$
$-\frac{10}{x^3}$
b $7x^{-4}$
$-\frac{28}{x^5}$
c $\frac{1}{2}x^{-3}$
$-\frac{3}{2x^4}$
d $\frac{1}{6}x^{-2}$
$-\frac{1}{3x^3}$
e $4x^3 - 2x^{-5}$
$\frac{10}{x^6} + 12x^2$
f $\frac{4}{3}x^{-1}$
$-\frac{4}{3x^2}$
g $8x + 5 - x^{-2}$
$\frac{2}{x^3} + 8$
h $4x^{-3} - 3x^{-1}$
$-\frac{12}{x^4} + \frac{3}{x^2}$
i $\frac{3}{2}x^{-4} - 5x - 6$
$-5 - \frac{6}{x^5}$

5 **a** $8x^{\frac{1}{2}}$

$\frac{4}{\sqrt{x}}$

b $x^{\frac{2}{3}}$

$\frac{2}{3\sqrt[3]{x}}$

c $12x^{\frac{3}{4}}$

$\frac{9}{\sqrt[4]{x}}$

d $4x^{\frac{5}{2}}$

$10\sqrt[2]{x^3}$

e $x^{\frac{7}{2}}$

$\frac{7}{2}\sqrt{x^5}$

f $x^{-\frac{1}{2}}$

$-\frac{1}{\sqrt{x^3}}$

g $x^{-\frac{3}{2}}$

$-\frac{3}{2\sqrt[2]{x^5}}$

h $4x^{-\frac{1}{4}}$

$-\frac{1}{\sqrt[4]{x^5}}$

i $10x^{-\frac{3}{2}}$

$-\frac{15}{\sqrt[2]{x^5}}$

j $9x^{-\frac{5}{6}}$

$-\frac{15}{2\sqrt[6]{x^{11}}}$

k $\frac{3}{2}x^{\frac{4}{9}}$

$\frac{2}{3\sqrt[9]{x^5}}$

l $\frac{1}{8}x^{-\frac{6}{5}}$

$-\frac{3}{20\sqrt[5]{x^{11}}}$

m $\frac{5}{2}x^{-\frac{2}{3}}$

$-\frac{5}{3\sqrt[3]{x^5}}$

6 **a** $x - 4x^{-1}$

$\frac{4}{x^2} + 1$

b $3 - 5x^{-1}$

$\frac{5}{x^2}$

c $4x^{-1} - x^2$

$-\frac{4}{x^2} - 2x$

d $1 + 5x^{-1} - 3x^{-2}$

$\frac{6}{x^3} - \frac{5}{x^2}$

e $\frac{1}{2}x^{-3} - \frac{1}{2}x$

$-\frac{1}{2} - \frac{3}{2x^4}$

f $-\frac{1}{3}x^{-3} - \frac{1}{2}x^{-2} + \frac{1}{6}x^{-1}$

$\frac{1}{x^4} + \frac{1}{x^3} - \frac{1}{6x^2}$

g $1 + 4x^{-2} + 5x^{-1}$

$-\frac{8}{x^3} - \frac{5}{x^2}$

h $5x^{-1} + 3x^{-2} - 2$

$-\frac{6}{x^3} - \frac{5}{x^2}$

i $4 - 4x^{-1} - 7x^{-2} - 2x^{-3}$

$\frac{6}{x^4} + \frac{14}{x^3} + \frac{4}{x^2}$

7 **a** $-\frac{3}{2\sqrt{x}} + \frac{3\sqrt{x}}{2}$

b $\frac{9}{x^4}$

c $\frac{1}{2\sqrt{x^3}} - \frac{1}{\sqrt{x}}$

d $-\frac{1}{x^{\frac{3}{2}}} + \frac{4}{x^2} - \frac{3}{x^{\frac{5}{2}}}$

e $-\frac{3}{2x^{\frac{5}{2}}} + 3\sqrt{x}$

f $-\frac{18}{x^3} + \frac{2x}{9}$

8 a $-\frac{1}{x^{\frac{3}{2}}} - \frac{5x^{\frac{3}{2}}}{2}$

b $-\frac{1}{2x^{\frac{3}{2}}} - \frac{1}{2\sqrt{x}} + \frac{3\sqrt{x}}{8}$

c $-\frac{4}{x^5} + \frac{9x}{2x^4} - \frac{1}{x^3}$

d $\frac{-5}{2x^{\frac{3}{2}}} + \frac{2}{x^2} + \frac{15}{x^{\frac{5}{2}}}$

e $-\frac{25}{6x^{\frac{11}{3}}}$

Exercise 9C

1 $\lim\limits_{h \to 0} \frac{f(x+h)-f(x)}{h}$

$\lim\limits_{h \to 0} \frac{x+h-x}{h}$

$\lim\limits_{h \to 0} \frac{h}{h}$

1

2 $\lim\limits_{h \to 0} \frac{f(x+h)-f(x)}{h}$

$\lim\limits_{h \to 0} \frac{cx+ch-cx}{h}$

$\lim\limits_{h \to 0} \frac{ch}{h}$

c

3 a $\lim\limits_{h \to 0} \left(\frac{f(x+h)-f(x)}{h}\right)$

$\lim\limits_{h \to 0} \frac{(x+h)^3 - x^3}{h}$

$\lim\limits_{h \to 0} \frac{x^3+3x^2h+3xh^2+h^3-x^3}{h}$

$\lim\limits_{h \to 0} 3x^2 + 3hx + h^2$

$= 3x^2$

b $\lim\limits_{h \to 0} \left(\frac{f(x+h)-f(x)}{h}\right)$

$\lim\limits_{h \to 0} \frac{(x+h)^4 - x^4}{h}$

$\lim\limits_{h \to 0} h^3 + 4h^2x + 6hx^2 + 4x^3$

$= 4x^3$

4 $(x+h)^n = x^n + nx^{n-1}h + (n-1)x^{n-1}h^2 + (n-2)x^{n-2}h^2 + \dots 1x^0h^n$

numerator of first principle quotient:

$(x+h)^n - x^n = x^n + nx^{n-1}h + (n-1)x^{n-1}h^2 + (n-2)x^{n-2}h^2 + \dots 1x^0h^n - x^n$

denominator = h

quotient:

$nx^{n-1} + (n-1)x^{n-1}h + (n-2)x^{n-2}h^2 + \dots h^{n-1}$

$\lim\limits_{h \to 0}\left(nx^{n-1} + (n-1)x^{n-1}h + (n-2)x^{n-2}h^2 + \dots h^{n-1}\right) = nx^{n-1}$

6 a $\frac{x-(x+h)}{x(x+h)} = -\frac{h}{x(x+h)}$

b $\frac{\frac{1}{x+h}-\frac{1}{x}}{h} = \frac{\frac{-h}{x(x+h)}}{h} = -\frac{1}{x(x+h)}$

c $\lim\limits_{h \to 0}\left(-\frac{1}{x(x+h)}\right) = \frac{-1}{x^2}$

d $\frac{\frac{k}{x+h}-\frac{k}{x}}{h} = \frac{\frac{-kh}{x(x+h)}}{h} = -\frac{k}{x(x+h)}$

$\lim\limits_{h \to 0}\left(-\frac{k}{x(x+h)}\right) = \frac{-k}{x^2}$

e $\frac{\frac{1}{(x+h)^2}-\frac{1}{x^2}}{h} = \frac{\frac{-2xh-h^2}{x^2(x+h)^2}}{h} = \frac{-2x-h}{x^2(x+h)^2}$

$\lim\limits_{h \to 0}\left(-\frac{-2x-h}{x^2(x+h)^2}\right) = \frac{-2x}{x^4} = -\frac{2}{x^3}$

$\frac{\frac{1}{(x+h)^3}-\frac{1}{x^3}}{h} = \frac{\frac{x^3-(h^3+3h^2x+3hx^2+x^3)}{x^3(x+h)^3}}{h} = \frac{-(h^2+3hx+3x^2)}{x^3(x+h)^3}$

$\lim\limits_{h \to 0}\left(-\frac{(h^2+3hx+3x^2)}{x^3(x+h)^3}\right) = \frac{-3x^2}{x^6} = -\frac{3}{x^4}$

7 a $\left(\sqrt{x+h} - \sqrt{x}\right)\left(\sqrt{x+h} + \sqrt{x}\right)$

$= \left(\sqrt{x+h}\right)^2 - \left(\sqrt{x}\right)^2$

$= h$

b $\lim\limits_{h \to 0}\left(\frac{\sqrt{x+h}-\sqrt{x}}{h}\right)$

$\lim\limits_{h \to 0}\left(\frac{\frac{h}{\sqrt{x+h}+\sqrt{x}}}{h}\right)$

$\lim\limits_{h \to 0}\left(\frac{1}{\sqrt{x+h}+\sqrt{x}}\right)$

c $\frac{1}{2\sqrt{x}}$

8 $\dfrac{\frac{1}{\sqrt{x+h}}-\frac{1}{\sqrt{x}}}{h} = \dfrac{\sqrt{x}-\sqrt{x+h}}{h\sqrt{x}\sqrt{x+h}}$

$\underset{h\to 0}{Lim}\left(\dfrac{-h}{\sqrt{x+h}+\sqrt{x}} \times \dfrac{1}{h\sqrt{x}\sqrt{x+h}}\right)$

$= \underset{h\to 0}{Lim}\left(\dfrac{-1}{\sqrt{x}(h+x\sqrt{x}\sqrt{h+x})}\right) = \dfrac{-1}{2x^{\frac{3}{2}}}$

9 $\underset{h\to 0}{Lim}\ \dfrac{g(x+h)+f(x+h)-(g(x)+f(x))}{h}$

$\underset{h\to 0}{Lim}\ \dfrac{g(x+h)-g(x)}{h} + \dfrac{f(x+h)-f(x)}{h}$

$= f'(x) + g'(x)$

10 a Taking a larger Delta about centre of area of interest.

b $\dfrac{(x+h)^2-(x-h)^2}{2h} = \dfrac{x^2+2xh+h^2-(x^2-2xh+h^2)}{2h}$

$= \dfrac{4xh}{2h}$

$\underset{h\to 0}{Lim} = 2x$

$\dfrac{(x+h)^3-(x-h)^3}{2h} = \dfrac{x^3+3xh^2+3x^2h+h^3-(x^3+3xh^2-3x^2h-h^3)}{2h}$

$= \dfrac{2h^3+6hx^2}{2h}$

$\underset{h\to 0}{Lim} = 3x^2$

Exercise 9D

1 a 4
b −2
c 2
d 3

2 a 10
b 5
c 0
d 1
e −6
f 4

3 a −2
b $-\frac{2}{9}$
c −8

4 a $\frac{3}{4}$
b $\frac{1}{2}$
c 6

5 −1

6 $\frac{1}{400}$

7 a −3
b −3

8 $(-1,8)$

9 −2

10 a $x = 2$
$x = 4$
b $x = -2$
$x = \frac{2}{3}$

11 $k = -4$
(-2)

12 4

13 $a = 38$
$b = 9$

14 $(-5,-12)$

15 a 2
b Straight line gradient $= -\frac{1}{2}$, $m_1m_2 = -1$, so they are perpendicular.

16 a $(6,5)$
b $(2,5)$
c $(4,0)$

17 a $3(x-1)^2 + 1$
b $3x^2 - 6x + 2 = 3(x-1)^2 - 1$
Minimum value of expression is −1.

18 note || means OR
$x < -2 \;||\; x > \frac{4}{3}$

19 a 0.3162
b 3
c $(5.5, 4.25)$

Exercise 9E

1 **a** $8\cos x$

b $-3\sin x$

c $-\cos x$

d $\frac{1}{2}\cos x$

e $-\frac{2}{3}\sin x$

f $\frac{5}{8}\sin x$

g $12x + 7\cos x$

h $3\cos x - 7\sin x$

i $6\cos x - \sin x$

j $\cos x + \sin x$

k $\sin x - \frac{6}{x^3}$

l $\frac{4\cos x}{5} - \frac{3}{\sqrt{x}}$

m $5\sin x - \frac{3}{4x^2}$

n $9\cos x + 15x^2 + \frac{5}{3x^{\frac{8}{3}}}$

o $3\sin x - \frac{2}{3x^3}$

p $-\frac{12}{x^4} + \frac{1}{x^2} + \frac{1}{5}\sin x$

q $-4\cos x - \frac{3}{x^{\frac{3}{2}}}$

r $\frac{5}{6}\sin x + \frac{10}{x^3} - \frac{3}{2x^{\frac{5}{2}}}$

s $\frac{1}{5}(\cos x + 3\sin x)$

2 **a** 3

b -1

c $\frac{1}{2\sqrt{2}}$

d $2\sqrt{3}$

3 **a** 2

b $\frac{5}{2}$

c $-\frac{3}{8}$

d $3\sqrt{2}$

4 -1

5 **a** -1.25

b -3.03

c -0.08

d 4.01

6 $x = \frac{\pi}{3}$

$x = \frac{5\pi}{3}$

7 $x = \frac{\pi}{6}$

$x = \frac{5\pi}{6}$

8 $\frac{2\pi}{3}$

9 $\left(\frac{\pi}{4}, \frac{1}{2\sqrt{2}}\right)$

10 $\left(\frac{2\pi}{3}, 2\sqrt{3}\right)$

$\left(\frac{4\pi}{3}, -2\sqrt{3}\right)$

11 $2\sin\left(\frac{5\pi}{6}\right) - \frac{1}{\left(\frac{5\pi}{6}\right)^2}$

$1 - \frac{1}{\left(\frac{5\pi}{6}\right)^2}$

$1 - \frac{36}{25\pi^2}$

$\frac{25\pi^2 - 36}{25\pi^2}$

$\frac{(5\pi + 6)(5\pi - 6)}{25\pi^2}$

12 **a** $3\cos x - 2$

b $x = -\frac{5\pi}{6}$

$x = -\frac{\pi}{6}$

13 $\frac{5\pi}{6}$

14 (4.07, 4.14)

15 $\left(\frac{4\pi}{3}, 4\right)$

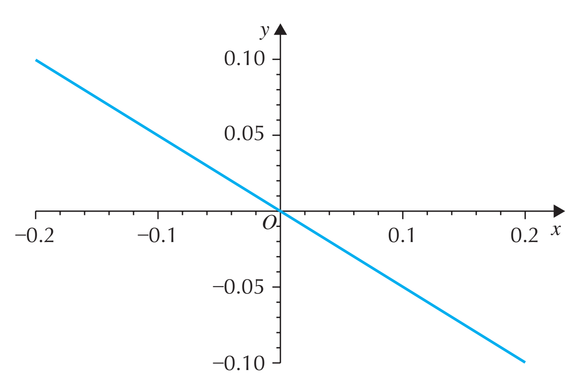
$\left(\frac{5\pi}{3}, 2\right)$

16 $\left(\frac{5\pi}{12}, 5\right)$

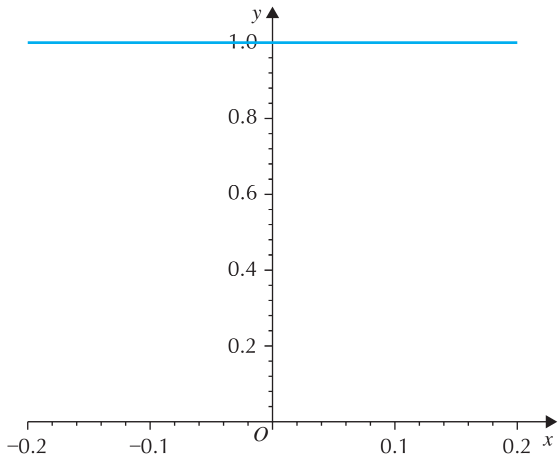
Exercise 9F

1 $\sin(x + h) = \sin x \cos h + \cos x \sin h$.

2



3



4 **a** $circ = \pi(1)^2$

$fraction = \frac{h}{360}\pi(1)^2 = \frac{\pi h}{360}$

b $area = \frac{\pi h}{2\pi} = \frac{h}{2}$

c $OA = 1$

$OB = 1\cos(h)$

$area = \frac{1}{2}(OA)(AB)\sin(h)$

$= \frac{1}{2}\cos(h)\sin(h)$

d $area = \frac{1}{2}(base)(height)$

$= \frac{1}{2}(1)1\tan(h) = \frac{1}{2}\tan(h)$

5 **a** From above

$\frac{1}{2}\sin(h)\cos(h) < \frac{h}{2} < \frac{1}{2}\tan(h)$

b $\frac{2}{\sin(h)\cos(h)} > \frac{2}{h} > \frac{2}{\tan(h)}$

$\frac{1}{\cos(h)} > \frac{\sin(h)}{h} > \cos(h)$

c 1

6 **a** $\frac{\cos(h)-1}{h} \times \frac{\cos(h)+1}{\cos(h)+1} = \frac{(\cos(h))^2-1}{h(\cos(h)+1)}$

$= \frac{-(\sin(h))^2}{h(\cos(h)+1)}$

b Yes

$\lim_{h\to 0}(\sin(h)) = 0$ for obvious reasons. Therefore

$\lim_{h\to 0}\left(\frac{-(\sin(h))^2}{h(\cos(h)+1)}\right) = 0$

7 $\frac{\sin x(\cos(h)-1)+\cos x\sin(h)}{h}$

$= \sin x \lim_{h\to 0}\left(\left(\frac{\cos(h)-1}{h}\right)\right) + \cos x \lim_{h\to 0}\left(\left(\frac{\sin(h)}{h}\right)\right)$

$= \sin x \times 0 + \cos x \times 1 = \cos x$

8 $\lim_{h\to 0}\left(\frac{\cos(x+h)-\cos x}{h}\right)$

$= \lim_{h\to 0}\left(\frac{\cos x\cos(h)-\sin x\sin(h)-\cos x}{h}\right)$

$= \lim_{h\to 0}\left(\frac{\cos x(\cos(h)-1)}{h} - \frac{\sin x\sin(h)}{h}\right)$

$= \cos x \lim_{h\to 0}\left(\frac{\cos(h)-1}{h}\right) - \sin x \lim_{h\to 0}\left(\frac{\sin(h)}{h}\right)$

$= -\sin x \lim_{h\to 0}\left(\frac{\sin(h)}{h}\right)$

$= -\sin x$

9 $\frac{d}{dx}\sin x^\circ = \frac{\pi}{180}\cos x^\circ$

Exercise 9G

1
- **a** $3(x+4)^2$
- **b** $6(x-2)^5$
- **c** $9(x+3)^8$
- **d** $5(x-1)^4$
- **e** $20(x+1)^3$
- **f** $48(x-3)^5$
- **g** $4(x+5)^7$
- **h** $2(x-5)^6$
- **i** $-\frac{1}{(x+2)^2}$
- **j** $-\frac{4}{(x-5)^5}$
- **k** $-\frac{7}{(x+6)^8}$
- **l** $-\frac{4}{(x-3)^5}$
- **m** $-\frac{12}{(x-2)^4}$
- **n** $-\frac{18}{(x-7)^3}$
- **o** $-\frac{6}{(x+1)^9}$

p $-\frac{15}{2(x-4)^{10}}$

q $4(x-1)^3$

r $-5(x+4)^4$

s $18(x+2)^8 - 12x^2$

t $20(x-1)^3 + \frac{3}{\sqrt{x}}$

u $\frac{10}{x^3} - \frac{3}{(x+4)^2}$

v $-\frac{12}{7x^4} - \frac{16}{(x-4)^3}$

2 **a** $18(3x+1)^5$

b $20(5x-2)^3$

c $10(2x-7)^4$

d $36(4x+1)^8$

e $42(3x-4)^6$

f $180(6x+2)^2$

g $320(5x-4)^7$

h $168(7x-1)^3$

i $-\frac{8}{(4x-1)^3}$

j $-\frac{16}{(2x+5)^9}$

k $-\frac{9}{(9x-2)^2}$

l $-\frac{30}{(5x+4)^7}$

m $-\frac{24}{(2x-1)^5}$

n $-\frac{70}{(7x+1)^3}$

o $-\frac{6}{(2x+5)^4} - 8$

p $-\frac{5}{4x^{\frac{9}{4}}} + \frac{12}{(3x-1)^2}$

q $4 - \frac{6}{(x-4)^7} + \frac{1}{x^2}$

r $\frac{2}{x^{\frac{3}{2}}} - \frac{48}{(8x-1)^3}$

s $\frac{3}{2x^{\frac{5}{2}}} + 3\sqrt{x} - \frac{48}{(4x-1)^3}$

t $\frac{4}{x^{11}} + \frac{30}{(3x+4)^6}$

3 **a** $-5(1-x)^4$

b $-\frac{3}{(x+5)^4}$

c $28(7x+3)^3$

d $4\left(\frac{2x}{3} - 4\right)^5$

e $-30(2-5x)^5$

f $6\left(2 + \frac{3x}{5}\right)^9$

g $-\frac{12}{5}(2-3x)^3$

h $\frac{1}{(6-x)^2}$

i $\frac{30}{(1-2x)^4} - 14x$

j $9\sqrt{x} - \frac{1}{(1-2x)^5}$

4 **a** $-\frac{4}{(x+1)^5}$

b $-\frac{2}{(x-5)^3}$

c $-\frac{20}{(4x+1)^6}$

d $-\frac{3}{(3x-4)^2}$

e $-\frac{5}{(x-3)^2}$

f $-\frac{6}{(x+1)^4}$

g $-\frac{24}{(2x-5)^3}$

h $-\frac{240}{(5x-1)^7}$

i $-\frac{1}{3(x+2)^2}$

j $-\frac{1}{2(x-1)^3}$

k $-\frac{6}{(x+1)^5}$

l $-\frac{10}{3(3x-2)^6}$

m $\frac{1}{(2-x)^2}$

n $\frac{24}{(5-3x)^2}$

o $\frac{16}{(5-8x)^3}$

p $\frac{9}{(1-2x)^3}$

5 **a** $\frac{4}{3}(x+5)^{\frac{1}{3}}$

b $\frac{3\sqrt{x-1}}{2}$

c $10(4x+1)^{\frac{3}{2}}$

d $\frac{35}{4}(7x-2)^{\frac{1}{4}}$

e $\frac{1}{3(x-2)^{\frac{2}{3}}}$

f $\frac{1}{2(2x-5)^{\frac{3}{4}}}$

g $-\frac{1}{3(x+4)^{\frac{4}{3}}}$

h $-\frac{15}{2(5x+6)^{\frac{5}{2}}}$

i $-\frac{2}{3(x-1)^{\frac{5}{3}}}$

j $-\frac{3}{8(x+2)^{\frac{5}{2}}}$

k $\frac{2}{3(x-4)^{\frac{1}{3}}}$

l $15(6x+1)^{\frac{3}{2}}$

m $-\frac{1}{2(x+2)^{\frac{3}{2}}}$

n $-\frac{6}{(x-3)^{\frac{7}{4}}}$

o $\frac{1}{5(4-x)^{\frac{6}{5}}}$

p $-45(2-5x)^{\frac{1}{2}}$

q $\frac{6}{(5-2x)^{\frac{7}{4}}}$

6 **a** $8x\left(x^2-3\right)^3$

b $5\left(3x^2-4x\right)\left(x^3-2x^2+1\right)^4$

c $3\left(4x^3-5\right)\left(x^4-5x-2\right)^2$

d $-36x\left(4-3x^2\right)^5$

e $-\frac{4x+5}{\left(2x^2+5x-3\right)^2}$

f $-\frac{4\left(-2-3x^2\right)}{\left(-x^3-2x+3\right)^5}$

g $-\frac{2x}{\left(x^2-5\right)^2}$

h $\frac{4x-1}{2\sqrt{2x^2-x+5}}$

i $\frac{9x^2}{2\left(2-3x^3\right)^{\frac{3}{2}}}$

j $6x^3\sqrt{x^4-1}$

k $\frac{3x^2+2x+1}{3\left(x^3+x^2+x+1\right)^{\frac{2}{3}}}$

l $\frac{3\left(\sqrt{x}-2\right)^5}{\sqrt{x}}$

7 24

8 $-\frac{1}{9}$

9 -192

10 **a** $x < \frac{1}{4}$

b $-\frac{2}{3}$

c $\frac{3}{16}$

11 $x=-2$

$x=1$

$x=4$

12 $(-3,2)$

13 **a** $x^2-x+3=\left(x\frac{-1}{2}\right)^2+\frac{11}{4}>0$
therefore square root is real for all x.

b $x=3$

14 **a** $3\left(px^2-4x+p\right)^2(2px-4)$

b If $p=2$ then root $x=1$

If $p=-2$ then root $x=-1$

15 **a** $g'(x)=-\frac{2(p-5)x}{\left((p-5)x^2-9p+2\right)^2}$

b $p=5$

16 **a** $(-1)^3+(-1)^2-4(-1)-4=0$

$(x+2)(x-2)(x+1)$

b $-\frac{12x^3+12x^2-48x-48}{(3x^4+4x^3-48x)^2}$

c $-2, -1, 2$

Exercise 9H

1 **a** $2\cos 2x$
b $-5\sin 5x$
c $12\cos 4x$
d $-18\sin 3x$
e $-2\sin\left(2x+\frac{\pi}{6}\right)$
f $\cos(6x-\pi)$
g $6\cos(4x)$
h $-6\sin(10x)$
i $2\cos(6x+2)$
j $2\sin\left(\frac{x}{4}\right)$
k $-\cos(2-x)$
l $3\cos(1-9x)$
m $-6\cos(3x)-5\sin(5x)$
n $\cos x-24\sin(3x)$
o $3\cos 3x+2\sin x$
p $\frac{5}{2}\cos(2x-\pi)+\frac{3}{2}\sin\left(\frac{3x}{2}\right)$

2 **a** 1
b -1
c $-\frac{3\sqrt{3}}{2}$
d $\sqrt{2}$
e 6

3 **a** 1.42
b -8.16
c -5.75
d 1.68

4 **a** $2\cos x\sin x$
b $-3\sin x(\cos x)^2$
c $6\cos x(\sin x)^2$
d $-30\sin x(\cos x)^5$
e $-2\cos x(\sin x)^3$
f $-6\sin x(\cos x)^4$
g $3(\cos x-\sin x)(\cos x+\sin x)^2$
h $10\cos(1-2x)\left(\sin(2x-1)\right)^4$

5 1

6 **a** $-\frac{\cos x}{\sin x}\frac{1}{\sin x}=-\frac{\cos x}{(\sin x)^2}$
b $3\frac{\tan x}{\cos x}$
c $\frac{\cos x}{2\sqrt{\sin x}}$
d $2\frac{\tan x}{(\cos x)^2}$
e $-3\frac{\tan 3x}{\cos 3x}$
f $\frac{\cos x}{2\sqrt{\sin x}}$
g $\cos x\cos(\sin x)$
h $-\cos x\sin(\sin x)$

7 $y=\sin x^\circ=\sin\left(\frac{\pi}{180}x\right)$
$\frac{dy}{dx}=\frac{\pi}{180}\cos x^\circ$

8 $y=\cos x^\circ=\cos\left(\frac{\pi}{180}x\right)$
$\frac{dy}{dx}=-\frac{\pi}{180}\sin x^\circ$

9 $x=\frac{2\pi}{9}$
$x=\frac{4\pi}{9}$
$x=\frac{8\pi}{9}$

10 $\left(\frac{\pi}{12},2\sqrt{3}\right)$
$\left(\frac{5\pi}{12},-2\sqrt{3}\right)$

11 $x=\frac{7\pi}{18}$
$x=\frac{11\pi}{18}$

12 $x=\frac{\pi}{2}$
$x=\frac{7\pi}{6}$
$x=\frac{3\pi}{2}$
$x=\frac{11\pi}{6}$

13 **a** $\cos\left(3x^2-1\right)$
b $6x$
c does equal $\frac{3}{2}(\cos 2x+1)$
d $y=3(\cos x)^2-1$
also
$y=\frac{a}{\sqrt{3}}x$
so a = 4.5

14 $\frac{\pi}{6}$