Chapter 2

Exercise 2A

1 a $\frac{\sqrt{3}}{2}$

   b $\frac{1}{2}$

   c $\frac{\sqrt{3}}{2}$

   d 0

   e $-\frac{\sqrt{3}}{2}$

   f $-\frac{1}{\sqrt{2}}$

   g 1

   h $\frac{1}{2}$

   i 0

   j $-\frac{\sqrt{3}}{2}$

   k $-\frac{1}{\sqrt{3}}$

   l $\frac{\sqrt{3}}{2}$

   m 0

   n $\frac{1}{\sqrt{3}}$

   o $\frac{1}{2}$

   p 0

2 $\sin A = \frac{3}{\sqrt{13}}$ $\cos A = \frac{2}{\sqrt{13}}$

3 $\cos P = \frac{\sqrt{11}}{6}$ $\tan P = \frac{5}{\sqrt{11}}$

4 a $\cos X = \frac{12}{13}$ $\tan X = \frac{5}{12}$

   b $\cos X = -\frac{12}{13}$ $\tan X = -\frac{5}{12}$

Exercise 2B

1 a $\frac{\pi}{6}$

   b $\frac{5\pi}{6}$

   c $\frac{5\pi}{3}$

   d $\frac{3\pi}{4}$

   e $\frac{7\pi}{4}$

   f $\frac{2\pi}{3}$

   g $\frac{4\pi}{3}$

   h $\frac{5\pi}{2}$

   i $-\frac{\pi}{3}$

   j 4π

2 a 45°

   b 15°

   c 36°

   d 270°

   e 225°

   f 135°

   g 210°

   h 420°

   i −30°

   j 540°

3 a **(i)** 0.82   **(ii)** 5.65

   b **(i)** 249.8   **(ii)** 90.0

4 a $\frac{1}{\sqrt{2}}$

   b $-\frac{1}{\sqrt{2}}$

   c $-\frac{\sqrt{3}}{2}$

   d $-\frac{1}{\sqrt{3}}$

   e -1

   f $\sqrt{3}$

   g $\frac{1}{2}$

   h -1

Exercise 2c

1 a $\sin(P + Q)$

   b $\sin(M - N)$

   c $\cos(105°)$

   d $\sin(-15°) = -\sin(15°)$

   e $\sin\left(\frac{2\pi}{15}\right)$

   f $\cos(205°)$

   g $\cos(-15°) = \cos(15°)$

   h $\cos\left(\frac{11\pi}{12}\right)$

2 a $\sin P \cos Q + \cos P \sin Q$

   b $\cos R \cos S + \sin R \sin S$

   c $\cos 48 \cos A - \sin 48 \sin A$

   d $\cos 15 \sin B - \cos B \sin 15$

   e $\frac{1}{2}\left(\cos x + \sqrt{3} \sin x\right)$

   f $\frac{1}{\sqrt{2}}\left(\cos x + \sin x\right)$

g $-\frac{1}{2}\left(\sin t + \sqrt{3}\cos t\right)$

h $-\frac{1}{2}\left(\sqrt{3}\cos x + \sin x\right)$

3 **a** $\sin 30\cos t + \cos 30\sin t$

$= 2(\sin 30\cos t - \cos 30\sin t)$

$\tan 30\cos t + \sin t$

$= 2\tan 30\cos t - 2\sin t$

$3\sin t = \tan 30\cos t$

$\tan t = \frac{1}{3}\tan 30$

$\tan t = \frac{1}{3\sqrt{3}}$

b $2\cos x\cos\left(\frac{\pi}{4}\right) - 2\sin x\sin\left(\frac{\pi}{4}\right)$

$= \cos\left(\frac{\pi}{4}\right)\cos x + \sin x\sin\left(\frac{\pi}{4}\right)$

$2\cos x - 2\sin x = \cos x + \sin x$

$3\sin x = \cos x$

$\tan x = \frac{1}{3}$

Exercise 2D

1 **a** 0

b $\frac{\sqrt{3}}{2}$

c $\frac{1}{2}$

d $-\frac{\sqrt{3}}{2}$

e $\frac{1+\sqrt{3}}{2\sqrt{2}}$

f $\frac{1-\sqrt{3}}{2\sqrt{2}}$

2 0

3 $\frac{16}{65}$

4 $\sin P = \frac{3}{5}$

$\cos P = \frac{4}{5}$

$\sin Q = \frac{1}{\sqrt{50}}$

$\cos Q = \frac{7}{\sqrt{50}}$

$\cos P\cos Q - \sin P\sin Q = -\frac{3}{25\sqrt{2}} + \frac{14\sqrt{2}}{25}$

$= \frac{1}{\sqrt{2}}$

5 $\cos X = \frac{3}{5}$

$\sin X = \frac{4}{5}$

$\sin X\cos 30 + \cos x\sin 30 = \frac{3}{10} + \frac{2\sqrt{3}}{5}$

$= \frac{1}{10}\left(3 + 4\sqrt{3}\right)$

6 $\sin A = \frac{2}{3}$

$\cos A = \frac{\sqrt{5}}{3}$

$\cos A\cos\left(\frac{3\pi}{2}\right) - \sin A\sin\left(\frac{3\pi}{2}\right) = \frac{2}{3}$

7 $\tan P = \frac{2}{\sqrt{5}}$

$\tan Q = \frac{\sqrt{5}}{3}$

$\tan(P + Q) = \frac{\tan P + \tan Q}{1 - \tan P\tan Q}$

$= 3\left(\frac{2}{\sqrt{5}} + \frac{\sqrt{5}}{3}\right) = \frac{11}{\sqrt{5}}$

OR

$\sin P = \frac{2}{3}$

$\cos P = \frac{\sqrt{5}}{3}$

$\sin Q = \frac{\sqrt{5}}{4}$

$\cos Q = \frac{3}{4}$

$\frac{\sin(P + Q)}{\cos(P + Q)} = \frac{\sin P\cos Q + \cos P\sin Q}{\cos P\cos Q - \sin P\sin Q} = \frac{11}{\sqrt{5}}$

8 $\sin A = \frac{2}{\sqrt{13}}$

$\sin B = \frac{\sqrt{3}}{4}$

$\cos A = \frac{3}{\sqrt{13}}$

$\cos B = \frac{\sqrt{13}}{4}$

$\sin(A + B) = \sin A\cos B + \cos A\sin B$

$= \frac{1}{52}\left(26 + 3\sqrt{39}\right)$

9 Mark X vertically below B.

$AX = 6$
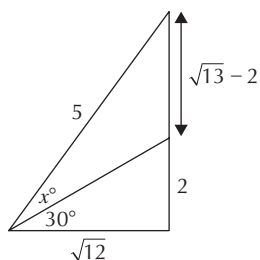
$XC = 15$

$AC = 21$

$21^2 = 10^2 + 17^2 - 2 \times 10 \times 17\cos ABC$

$\Rightarrow \cos ABC = -\frac{13}{85}$

10

$$\cos x = \frac{5^2+4^2-\left(\sqrt{13}-2\right)^2}{2\times 4\times 5}$$

$$= \frac{6+\sqrt{13}}{10}$$

11 $\frac{2ab}{a^2+b^2}$

Exercise 2E

1 a $\frac{\sqrt{3}}{2}$

  b $\frac{1}{\sqrt{2}}$

  c $\frac{1}{2}$

  d $-\frac{1}{2}$

2 a $\frac{4\sqrt{21}}{25}$

  b $-\frac{17}{25}$

  c $-\frac{4\sqrt{21}}{17}$

3 $-\frac{120}{169}$

4 $-\frac{11}{5\sqrt{5}}$

5 $\frac{1}{50}\left(24-7\sqrt{3}\right)$

6 $2\cos^2 A - 1 = \frac{3}{5}$

  $2\cos^2 A = \frac{8}{5}$

  $\cos^2 A = \frac{4}{5}$

  $\cos A = \frac{2}{\sqrt{5}} = \frac{2\sqrt{5}}{5}$

7 $\frac{3}{5}$

8 $\tan A = \frac{3}{4}$

  $\tan 2A = \frac{2\tan A}{1-\tan^2 A} = \frac{24}{7}$

9 $-\frac{12}{13}$

10 $7.937\,\text{m}^2$

11 a $180 = x + 2y$

  $y = 90 - \frac{x}{2}$

  $\frac{b}{\sin X} = \frac{a}{\sin\left(90-\frac{x}{2}\right)}$

  b $b = \frac{2a\sin\left(\frac{x}{2}\right)\cos\left(\frac{x}{2}\right)}{\cos\left(\frac{x}{2}\right)} = 2a\sin\left(\frac{x}{2}\right)$

Exercise 2F

1 a -1

  b $\frac{1}{7}$

  c $-\frac{3}{4}$

2 a 1

  b $\frac{1}{7}$

  c $\frac{4}{3}$

Exercise 2G

1 a $\sin(x + 45) = \frac{1}{\sqrt{2}}(\cos x + \sin x)$

  $\cos(x + 45) = \frac{1}{\sqrt{2}}(\cos x - \sin x)$

  adding gives $\sqrt{2}\cos x$

  b $2\cos(x + 30) = \left(\sqrt{3}\cos x - \sin x\right)$

  $-\sin x = \sqrt{3}\cos x - 2\sin x$

  c $\sin(x - 60) = -\frac{1}{2}\sqrt{3}\cos x + \frac{1}{2}\sin x$

  $\cos(x + 30) = \frac{1}{2}\sqrt{3}\cos x - \frac{1}{2}\sin x$

  adding gives 0

  d $\sin(x + 225) = -\frac{1}{\sqrt{2}}(\cos x + \sin x)$

  $\cos(x + 135) = -\frac{1}{\sqrt{2}}(\cos x + \sin x)$

  subtracting gives 0

  e $a = \sin\left(\theta + \frac{\pi}{3}\right) = \frac{1}{2}\left(\sqrt{3}\cos\theta + \sin\theta\right)$

  $b = \cos\left(\theta + \frac{\pi}{6}\right) = \frac{1}{2}\left(\sqrt{3}\cos\theta - \sin\theta\right)$

  $\sin\theta - a + b = 0$

2 a

$x = \cos(\alpha + \beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$

$y = \cos(\alpha - \beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta$

$x + y = 2\cos\alpha\cos\beta$

b

$x = \sin(\alpha + \beta) = \cos\beta\sin\alpha + \cos\alpha\sin\beta$

$y = \sin(\alpha - \beta) = \cos\beta\sin\alpha - \cos\alpha\sin\beta$

$x - y = 2\cos\alpha\sin\beta$

3 $\cos(2x) = 2\cos^2 x - 1$

$\cos^2 x = \frac{1}{2}(1 + \cos(2x))$

4 $\frac{\cos x\cos y - \sin x\sin y}{\cos x\cos y}$

divide top and bottom by $\cos x\cos y$

$= 1 - \tan x\tan y$

5 a $(\cos x - \sin x)(\cos x + \sin x)$

$= \cos^2 x - \sin^2 x = \cos(2x)$

b $\cos(2x) = 2\cos^2 x - 1$

$2\cos^2 x - 1 + \cos x$

$= 2\cos^2 x + \cos x - 1$

$= (2\cos x - 1)(\cos x + 1)$

6 $2(\cos^2 x - \sin^2 x) - \cos^2 x$

$\cos^2 x - 2\sin^2 x$

$1 - \sin^2 x - 2\sin^2 x = 1 - 3\sin^2 x$

7 $\sin(2a + a)$

$\sin(2a)\cos a + \cos(2a)\sin a$

$(2\sin a\cos a)\cos a + (\cos^2 a - \sin^2 a)\sin a$

$\sin a(2\cos^2 a + \cos^2 a - \sin^2 a)$

$\sin a(3\cos^2 a - \sin^2 a)$

$\sin a(3(1 - \sin^2 a) - \sin^2 a)$

$\sin a(3 - 4\sin^2 a)$

$3\sin a - 4\sin^3 a$

8 $\cos 4\theta = \cos(2\theta + 2\theta)$

$= 2\cos^2(2\theta) - 1$

$= 2(2\cos^2\theta - 1)^2 - 1$

$= 2(4\cos^4\theta - 4\cos^2\theta + 1) - 1$

$= 8\cos^4\theta - 8\cos^2\theta + 1$

9 $(\cos^2 A + \sin^2 A)(\cos^2 A - \sin^2 A)$

$= \cos^2 A - \sin^2 A = \cos(2A)$

10 a $\sin(x + y) = \cos y\sin x + \cos x\sin y$

$\sin(x - y) = \cos y\sin x - \cos x\sin y$

adding gives $2\sin x\cos y$

b $x + y = A \Rightarrow x = \frac{A + B}{2}$

$x - y = B \Rightarrow y = \frac{A - B}{2}$

$2\sin\left(\frac{A + B}{2}\right)\cos\left(\frac{A - B}{2}\right)$

c $\cos A + \cos B = 2\cos\left(\frac{A + B}{2}\right)\cos\left(\frac{A - B}{2}\right)$

d $\frac{2\sin(4x)\cos x}{2\cos(2x)\cos x} = \frac{\sin(4x)}{\cos(2x)}$

$= \frac{2\sin(2x)\cos(2x)}{\cos(2x)} = 2\sin(2x)$

Exercise 2H

1 a $k = 5$, $\alpha = 36.87°$

b $k = \sqrt{34}$

$\alpha = 329.04°$

c $k = \sqrt{5}$

$\alpha = 116.57°$

2 a $k = \sqrt{2}$

$\alpha = \frac{\pi}{4}$

b $k = 2$

$\alpha = \frac{7\pi}{6}$

c $k = \sqrt{41}$

$\alpha = 2.246$

Exercise 2I

1   **a** $\sqrt{5}\cos(x-63.43)$
    b $\sqrt{5}\cos(x+296.57)$
    c $\sqrt{5}\sin(x+26.57)$
    d $\sqrt{5}\sin(x-333.43)$

2   $\sqrt{41}\cos(x+38.66)$

3   $2\sin\left(\theta-\frac{7\pi}{4}\right)$

4   $\sqrt{2}\sin\left(x-\frac{\pi}{4}\right)$

5   $2\cos\left(x+\frac{\pi}{6}\right)$

6   $\sqrt{18}\sin\left(x+\frac{\pi}{4}\right)$

7   $5\sin(2x-36.87)$

8   $\sqrt{13}\cos(3x-33.69)$

9   $2\cos(x-30)$