Chapter 7

Exercise 7A

1 **a** yes
b yes
c no
d yes
e no
f yes

2 **a** 3
b 1
c 7
d 5
e 2

3 yes

4 no

5 no

6 yes

7 no

8 no

9 yes

Exercise 7B

1 $(x-2)(x-1)(x-4)$

2 $(x+1)(x-1)(x+4)$

3 $(x-2)(x-3)(x-1)$

4 $(x+1)(x+8)(x-1)$

5 $(x+3)(x+1)(x-4)$

6 $(x+2)(x-1)(x+4)x$

7 $(x-3)(x+3)(x-2)$

8 $(x+5)(x-2)(x-2)x$

Exercise 7C

1 **a** $(x+1)(x-1)(x-1)$
b $(x+1)(x+1)(x-1)$
c $(x+3)(x+1)(x-1)$
d $(x+5)(x-1)(x-1)$
e $(x+2)(x-2)(x-2)$
f $(x-3)(x-2)(x-1)$

2 **a** $(x-3)(x+2)(x+3)$
b $(x-1)(x+2)(x+4)$
c $(x-3)(x-2)(x+1)$
d $(x-3)(x-1)(x+4)$
e $(x-4)(x+1)(x+5)$
f $(x-3)(x-2)(x+5)$

3 **a** $(x-1)(x-3)x$
b $(x-1)x(x+1)$
c $(x-4)(x-1)(x+2)x$
d $(x+1)(x+1)(x-1)(x-1)$

Exercise 7D

1 **a** $2\left(\frac{1}{2}\right)^3+11\left(\frac{1}{2}\right)^2+4\left(\frac{1}{2}\right)-5=0$
b $(x+1)(x+5)(2x-1)$

2 **a** $(x-1)(x+1)(2x+1)$
b $(x-2)(x-1)(3x+1)$
c $(x-3)(x-2)(2x-1)$
d $(x+1)(x-1)(4x+3)$
e $(5x+2)(x+2)(x-3)$
f $(4x+1)(x-4)(x+3)$
g $(x-1)(2x+1)(3x+2)$
h $(2x-1)(x+2)(3x+1)$
i $(x+2)\left(x^2-x+3\right)$
j $(x-1)\left(2x^2-3x+3\right)$

3 **a** $(x-2)(x-2)(x+2)(x+2)$
b $(x-1)(x-1)(x-1)(x+1)$
c $(x-2)(x-1)(x+1)(x+1)$
d $(x-3)(x-3)(x-1)(x+2)$

4 **a** $3x^2(x-1)(x+1)$
b $(x-3)(x-2)(x-1)x$
c $(x-2)(x-1)(x+1)(x+2)$
d $(x-3)(x-2)(x-1)(x+3)$
e $(x+1)(x-1)(x+1)(x+5)$
f $(2x+1)(x-4)\left(x^2-x+1\right)$
g $(x-2)(x+2)\left(x^2+3\right)$
h $(2x-1)(x-1)(2x+1)(x+2)$

i $2(x-2)(x-4)(x+1)(x+4)$

j $(x-3)(x+1)(x^2-3)$

5 $(x-8)(x+1)(x-3)$

6 5

7 **a** $x-3$

b x min = 4.

Area = $\frac{13}{3}cm^3$

Exercise 7E

1 $p = 5$

2 $q = 20$

3 $k = 1$

$(x-1)(x+1)(2x+1)$

4 $a = 7$

$(x-2)(x-1)(x+3)(x+7)$

5 $k = 3$

$(x+1)(x-1)(2x+3)$

6 $a = 2$

$b = 2$

7 $p = -8$

$q = 12$

$(x-2)(x-1)(x+2)(x+3)$

Exercise 7F

1 **a** 2

b –10

c 85

d $\frac{-3}{4}$

e $\frac{8}{3}$

2 **a** 2

b –7

c 8

3 $a = 3$

$b = -1$

4 $a = -1$

$b = 1$

5 **a** $x^2 + 7x + 11$

23

b $x^2 - x$

4

c $2x^2 - 3x + 6$

0

d $4x^2 + 4x + 2$

3

Exercise 7G

1 **a** (1, 0)

(2, 0)

b (–5, 0)

(–1, 0)

(3, 0)

c (–5, 0)

(–1, 0)

(5, 0)

d (–1, 0)

(–3, 0)

(–8, 0)

e (6, 0)

(–3, 0)

(2, 0)

f (2, 0)

(–0.5, 0)

(1, 0)

g (–2, 0)

(–1, 0)

(1, 0)

(2, 0)

2 Months 0, 3 and 5

3 **a** $A(2, 0)$

$B(3, 0)$

b 72 m

Exercise 7H

1 **a** $f(x) = x^3 - 6x^2 + 11x - 6$

b $f(x) = 2x^3 + 4x^2 - 10x - 12$

c $f(x) = 3x^3 - 12x^2 - 12x + 48$

d $f(x) = x^3 + x^2 - 8x - 12$

e $f(x) = 2x^3 - 2x^2 - 16x + 24$

f $f(x) = -x^3 - 4x^2 + x + 4$

g $f(x) = -2x^3 + 4x^2 + 10x - 12$

h $f(x) = x^4 - 5x^2 + 4$

i $f(x) = x^4 - 2x^2 + 1$

2 $f(x) = \frac{1}{2}x^3 - \frac{3}{4}x^2 - \frac{11}{4}x + \frac{3}{2}$

3 $\frac{1}{32500} = \left(-x^4 + 150x^3 - 6875x^2 + 93750x\right)$

4 $f(x) = 2x^3 - 8x^2 + 2x + 12$

$a = -1$

Exercise 7I

1 a $x = -4, -1, 2$

b $x = -4, -2, -1$

c $x = -1, 3, 5$

d $x = -4, 2$

2 a $x = -3, -1, 1$

b $x = -3, -2, 0, 5$

c $x = -\frac{5}{2}, -1, 2$

d $x = -2, 1, 2, 3$

e $-2, -1, \frac{1}{2}, 3$

f $-2, -\frac{1}{2}, \frac{1}{3}, 2$

3 a $x = -4, -3, 1$

b $x = -6, -2, 1$

c $x = -2, 3, 7$

d $x = -1, -\frac{2}{3}, 3$

e $x = -5, -1, 1, 4$

f $x = -\frac{1}{2}, \frac{1}{2}, 1$

4 $a = -3$

$x = -2, -\frac{1}{2}, 1$

5 $k = 3$

$x = -3$

Factorising gives $(x + 3)\left(x^2 + 3\right)$ and we can see second term cannot be zero.

6 a $h(1) = 0$

b $-(t - 8)(t - 6)(t - 3)(t - 1)$

c $t = 3$

7 a 4 months

b Factors are $(t - 4)\left(2t^2 - 8t + 11\right)$ and second term has no roots.

c £786

8 a $v(x) = (x + 18)(x + 12)(x + 8)$

b $+1728 + 456x + 38x^2 + x^3 = 3456$

$x^3 + 38x^2 + 456x - 1737 = 0$

c Factor above exporession to get

$(x - 3)\left(x^2 + 41x + 579\right)$

Second term has no roots so dimensions are (with $x = 3$) 11, 15, 21

9 $4 \times 4 \times 8$

10 $x < -1 \,||\, -1 < x < 1 \,||\, x > 3$

where || means OR

Exercise 7J

1 $(-2, -8), (3, 7), (4, 10)$

2 a $(-6, -241), (-3, -52), (2, -17)$

b $(-7, 72), (-3, 40), (1, 8)$

c $(-3, 19), (1, 3), (3, 55)$

3 a $(-1, -20), (4, -10), (7, -4)$

b $(-2, -19), (-1.5, -9.875), (1, 2)$

c $(1, 0), (2, -10), (3, 30), (4, 180)$

d $(-1, 3), (-0.5, 0.875), (0.5, 1.125), (1, 5)$

e $\left(+\frac{1}{3}, \frac{284}{27}\right), \left(-\sqrt{5}, 15 - 11\sqrt{5}\right), \left(\sqrt{5}, 15 + 11\sqrt{5}\right)$

Exercise 7K

1 a $k = \{-8, 8\}$

b $k = 40$

c $k = \{0, 5\}$

d $k = \{-1, 4\}$

e $k = \{-1, 3\}$

2 a $k = 4$

b $x = \left(-\frac{2}{3}\right)$

3 a $x < -1 \,||\, x > 3$

b $-3 \leq x \leq \frac{1}{2}$

c $-1 < x < 4$

d $x < -2 \,||\, x > 8$

e $x \leq -\frac{1}{3} \,||\, x \geq \frac{1}{3}$

f $x \leq \frac{1}{3} \,||\, x \geq \frac{1}{2}$

4 a $k \leq \frac{1}{3}$

b $k \leq 3$

c $k \leq 0 \,||\, k \geq \frac{1}{3}$

5 $k > 0$

6 $-4 \leq k \leq 1$

7 Descriminant ≥ 0

$(3k-2)^2 - 4 \times 2k \times (k-2) \geq 0$

$k^2 + 4k + 4$

$= (k+2)^2 \geq 0$

So roots are always real.

8 $k < 0 \,||\, k > 4$

9 a 2

b $(x-1)\left(3x^2 + 4x + 10\right)$

second term has no real roots

$(1, -8)$

10 a $a = -1$

$b = -2$

b roots are

$1, \frac{1+\sqrt{5}}{2}, \frac{1-\sqrt{5}}{2}$

only rational root is 1.

$(1, 0)$