

Chapter 10

Exercise 10A

1 a $y = 5x + 5$

b $y = -5$

c $y = -x - 7$

d $y = 22x - 24$

e $y = 25x + 54$

f $y = -7x + 8$

2 a $y = -\frac{1}{2}x + \frac{1}{12}(\pi + 6\sqrt{3})$

b $y = \frac{3}{2}x + \frac{1}{6}(3\sqrt{3} - \pi)$

c $y = 2\sqrt{2}x + 2\sqrt{2}(1 - \frac{\pi}{4})$

d $y = -\sqrt{3}x + \frac{1}{12}(12 + 7\sqrt{3}\pi)$

e $y = \frac{5}{2}x + \frac{5}{2}(\sqrt{3} - 1)$

f $y = \frac{\sqrt{3}}{4}x - \frac{1}{4}(1 + \sqrt{3})$

3 a $y = 7x$

b $y = 5x - 3$

c $y = -4x - 5$

d $y = 20x - 72$

e $y = -27x - 46$

f $y = 5x + 8$

4 a $y = -\frac{\sqrt{3}}{2}x + \frac{1}{6}(3 + \sqrt{3}\pi)$

b $y = x + \frac{1}{6}(3\sqrt{3} - \pi)$

c $y = -4\sqrt{3}x + \frac{4}{3}(2\sqrt{3}\pi - 3)$

d $y = 6x - \frac{3\pi}{2}$

e $y = 2x - \frac{2}{3}(\pi + 3\sqrt{3})$

f $y = -\sqrt{2}x + \frac{1}{8}(8\sqrt{2} + 5\sqrt{2}\pi)$

5 a $y = -6x - 31$

b $y = 5x + 1$

c $y = -6x + \frac{7}{4}$

d $y = 4x + 4(\sqrt{3} - 1)$

e $y = -2x + \frac{1}{2}(\sqrt{3} + \frac{\pi}{6})$

f $y = \sqrt{3}x + \frac{1}{3}(3 - \sqrt{3}\pi)$

6 a $y = 11x - 26$

b $y = 18x + 10$

c $y = -2x + 1$

d $y = 0$

e $y = \frac{1}{4}x + 3$

f $y = \frac{1}{3}x + 12$

7 $y = -8x + 48$

8 a $a = -2$

$b = -5$

b $y = 10x - 32$

c $x = \frac{2+\sqrt{31}}{3}$

$x = \frac{2-\sqrt{31}}{3}$

9 a $y = 3x - 22$

b $(-4, 74)$

10 $y = 6\sqrt{2}x - \frac{1}{2}(4\sqrt{2} + 3\sqrt{2}\pi)$

11 $y = 2x - \frac{1}{3}(3\sqrt{3} + 2\pi)$

12 a $y = 10x - 1$

13 a $-\frac{2}{49}$

b $y = -\frac{2}{121}x + \frac{17}{121}$

14 $y = 5x - 11$

15 $y = 15x + 14$

16 $y = -2x + \frac{13}{16}$

17 $y = 96x - 272$

18 a $x = \frac{3}{4}$

b $y = x + \frac{3}{4}$

19 $y = -\frac{3\sqrt{3}}{8}x + \frac{1}{16}(7\sqrt{3}\pi - 2)$

$y = -0.6496x + 2.2556$

20 $-\frac{3}{4}x + 9$

21 $y = -\frac{1}{4}x + \frac{19}{4}$

22 a $f(g(x)) = \sqrt{5 + 8\sin x}$

b i $h'(x) = \frac{4 \cos x}{\sqrt{5 + 8\sin x}}$

ii $\frac{4 \cos x}{\sqrt{5 + 8\sin x}} = \frac{2\sqrt{3}}{3}$

$$12\cos x = 2\sqrt{3}\sqrt{5 + 8\sin x}$$

$$12^2(\cos x)^2 = 12(5 + 8\sin x)$$

$$12(\cos x)^2 - 8\sin x - 5 = 0$$

c $y = \frac{2}{\sqrt{3}}x + \frac{1}{9}(27 - \sqrt{3}\pi)$

$$y = 1.1547x + 2.3954$$

23 a $x \neq -3$ & $x \neq 5$ where & means AND [|| means OR]

b $y = -0.4938x + 3.4074$

24 a $y = -4x + 9$

b $y = 2x, B = (3, 6)$

c $(4.2, 8.4)$

25 a $y = 2x - 2$

b $\frac{12}{\sqrt{145}}$

c $(5.5, 11.25)$

Challenge

1 $-k^3$

2 Half turn because $f(-x) = -f(x)$.
Centre $(0, 0)$

3 "a" does not affect any shift in x -axis so ax^3 has same symmetry as x^3 .

4 a $g(x) = ax^3 + cx$

$$g(-x) = -ax^3 - cx = -g(x)$$

so half turn symmetry about origin.

b Half turn about $(0, d)$

5 expand $a\left(x - \frac{b}{3a}\right)^3$ to get

$$ax^3 - bx^2 + \frac{xb^2}{3a} - \frac{b^3}{27a^2}$$

expand $b\left(x - \frac{b}{3a}\right)^2$ to get

$$\frac{b^3}{9a^2} - \frac{2b^2x}{3a} + bx^2$$

expand $c\left(x - \frac{b}{3a}\right)$ to get

$$cx - \frac{bc}{3a}$$

adding up above three terms plus additional term d to get

$$ax^3 + cx - \frac{b^2x}{3a} + d - \frac{bc}{3a} + \frac{2b^3}{27a^2}$$

6 Combining in polynomial of x format:

$$ax^3 + \frac{(3ac - b^2)}{3a}x + \frac{27a^2d + 2b^3 - 9abc}{27a^2}$$

$$p = \frac{3ac - b^2}{3a}$$

$$q = \frac{27a^2d + 2b^3 - 9abc}{27a^2}$$

q is the y -axis origin offset. If subtract q then brings back rotation to centre.

7 $\frac{b}{3a}, q$

Shift by $\frac{b}{3a}$ on x axis and shift by q on y axis.

Exercise 10B

1 a $f'(1) = 5$ so increasing

b $f'(-1) = -4$ so decreasing

c $f'(-1) = -6$ so decreasing

d $f'(-2) = 38$ so increasing

e $f'(-1) = -1$ so decreasing

2 a $x < -1$ || $x > 3$ note || means OR

b $-3 < x < -1$

c $x < -\sqrt{2}$ || $x > \sqrt{2}$

d $x < -2$

e $0 < x < 4$

3 $f'(-1) = 1$ so increasing.

4 $f\left(\frac{1}{3}\right) = \frac{-1}{3}$ so decreasing.

5 a $h'\left(\frac{1}{2}\right) = \frac{3}{2}$ so increasing

b $x < \frac{1}{3}$

6 $g'(2) = -4$ so decreasing

7 $k'(-1) = -48$ so decreasing

8 $f'\left(\frac{\pi}{6}\right) = \frac{\sqrt{3}}{2}$ so increasing

9 $g'\left(\frac{\pi}{6}\right) = 0.5359$ so increasing

10 $p'(\frac{\pi}{12}) = 0.389$ so increasing

11 $f'(x) = x^2 - 6x + 10$

Completing the square gives

$(x - 3)^2 + 1$ - which is always > 0 therefore increasing.

12 $h'(x) = 2x^2 + 4x + 2$

Completing the square gives

$2(x + 1)^2$ which is always ≥ 0 ie never negative.

13 a $x > \frac{1}{4}$

b $f'(x) = -\frac{2}{(4x - 1)^{\frac{3}{2}}}$

$f'(\frac{5}{16}) = -16$

c Numerator and denominator (because of $\frac{3}{2}$ power) are both positive. External negative sign means negative overall and so decreasing.

14 2

15 $f'(\frac{2\pi}{3}) = \frac{-9}{4}$.

16 $g'(x) = \frac{x^2 + 6}{x^2}$ which is always positive, so always increasing.

17 a i $g(f(x)) = \frac{3}{x^2 + 4x + 9}$

ii $(x+2)^2 + 5$ Denominator has no roots; ie is never zero so expression is defined for all x .

b $h'(x) = -\frac{6x+12}{(x^2+4x+9)^2}$

$h'(-3) = \frac{1}{6}$

so increasing

18 A, E

19 a $g'(x) = 9 - \frac{1}{(x-1)^2}$

b $g'(x) = \frac{9(x^2 - 2x + 1) - 1}{(x-1)^2}$

Denominator is > 0 so focus on numerator:

$9x^2 - 18x + 8 > 0$

c $x < \frac{2}{3} || x > \frac{4}{3}$ where $||$ means OR

20 3

21 a $p(\sin(ax + b))^2 + p(\cos(ax + b))^2$

$= p(1) = p$

so gradient is zero.

b see (a)

Exercise 10C

1 a $(-1, \frac{8}{3})$ maximum

$(1, \frac{4}{3})$ minimum

c $(3, 5)$ inflexion.

d $(-\frac{2}{3}, \frac{67}{27})$ Maximum

$(2, -7)$ Minimum

e $(-1, \frac{15}{2})$ Maximum

$(1.3333, 1.148)$ minimum

f $(-\frac{3}{2}, \frac{53}{8})$ Maximum

$(\frac{1}{3}, -\frac{25}{54})$ minimum

2 a $(-\frac{5}{3}, -\frac{121}{27})$ Minimum

$(1, 5)$ Maximum

b $(\frac{2}{3}, -\frac{14}{27})$ Minimum

$(4, 18)$ Maximum

c $(-2, 64)$ Maximum

$(2, -64)$ Minimum

d $(0, 0)$ Minimum

$(\frac{4}{3}, \frac{32}{27})$ Maximum

e $(-3, -18)$ Minimum

$(3, 18)$ Maximum

f $(0, 0)$ Minimum

$(\frac{5}{2}, \frac{125}{24})$ Maximum

3 a $(0, 0)$ Inflexion

$(\frac{3}{2}, -\frac{27}{16})$ Minimum

b $(0, 0)$ inflexion

$(\frac{9}{2}, -\frac{2187}{16})$ Maximum

c $(-4, -256)$ Minimum

$(0, 0)$ Maximum

$(4, -256)$ Minimum

- d** $(-1, \frac{1}{2})$ Maximum
 $(0, 0)$ Minimum
 $(1, \frac{1}{2})$ Maximum
- e** $(-1, -5)$ Inflexion
 $(0, -6)$ Minimum
- 4 a** $(\frac{1}{4}, 0)$
b Inflexion
- 5 a** $3(x^2 - 2x - 8)^2(2x - 2)$
b i, ii
 $(-2, 0)$, Inflexion
 $(1, -729)$, minimum
 $(4, 0)$, Inflexion
- 6 a** $\frac{1}{x(x-6)}$
b $x = 0$
 $x = 6$
c i $\frac{-(2x-6)}{x^2(x-6)^2}$
ii $(3, -\frac{1}{9})$ Maximum
- 7** There are no stationary points.
- 8 a** This can come in many forms, one of which is:
 $4\sin x \cos x - 1$
b $(0.2618, -0.1278)$ Minimum
 $(1.309, 0.557)$ Maximum
- 9 a** $3(x-2)^2 - 5$
b The curve has two stationary points.
- 10 a** $y'(x) = 3(x^2 - 5x + 8)^2(2x - 5)$
 Only the factor $(2x - 5)$ has roots:
 at $x = \frac{5}{2}$
b $(\frac{5}{2}, 5.360)$ minimum
- 11 a i** $2^3 - 4(2)^2 + 2 + 6 = 0$
ii $(x-2)(x+1)(x-3)$

- b c**
 $x = -1$, min
 $x = 2$, max
 $x = 3$, min

12 a i $(-1)^3 - 4(-1)^2 + (-1) + 6 = 0$
ii $x = 2$
 $x = 3$

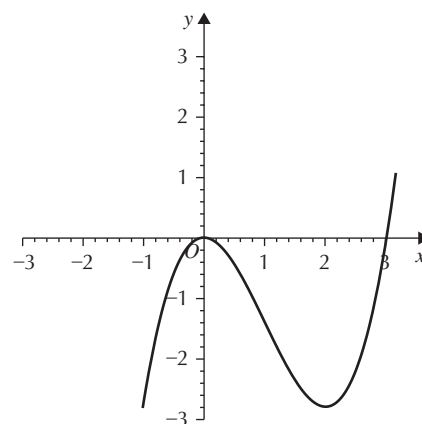
- b** $(0, 72)$, min
 $(0.255436, 72.129)$, max
 $(11.74456, -5994.1)$, min

13 a $f(g(x)) = (x+2)^3 - 7(x+2)^2$
 $= x^3 - x^2 - 16x - 20$

- b i** $(-2)^3 - (-2)^2 - 16(-2) - 20 = 0$
ii $(x-5)(x+2)(x+2)$
c $(-2, 0)$ Max
 $(2.6667, -50.81)$ Min

Exercise 10D

- 1 a** roots:
 $(0, 0)$
 $(3, -0)$
 y-axis:
 $(0, 0)$
 Stationary points:
 $(0, 0)$ Max
 $(2, -4)$ Min



b roots:

$(-3.464, 0)$

$(0, 0)$

$(3.464, 0)$

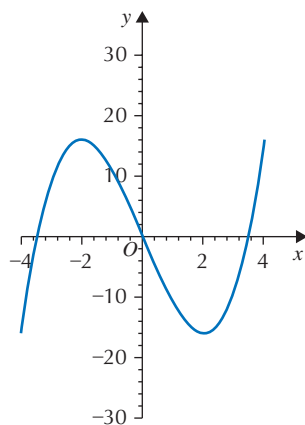
y-axis:

$(0, 0)$

Stationary points:

$(-2, 16)$ Max

$(2, -16)$ Min



c roots:

$(-1.732, 0)$

$(0, 0)$

$(1.732, 0)$

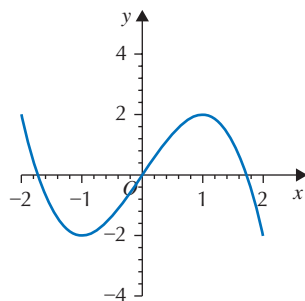
y-axis:

$(0, 0)$

Stationary points:

$(-1, -2)$ Min

$(1, 2)$ Max



d roots:

$(-2, 0)$

$(1, 0)$

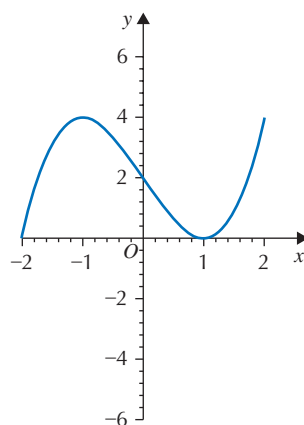
y-axis:

$(0, 2)$

Stationary points:

$(-1, 4)$ Max

$(1, 0)$ Min



e roots:

$(0.5, 0)$

$(2, 0)$

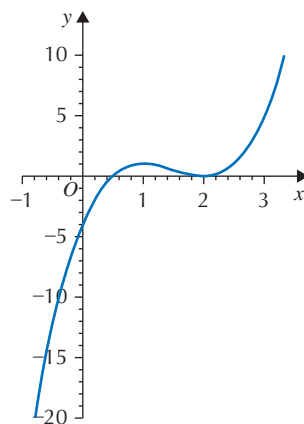
y-axis:

$(0, -4)$

Stationary points:

$(1, 1)$ Max

$(2, 0)$ Min

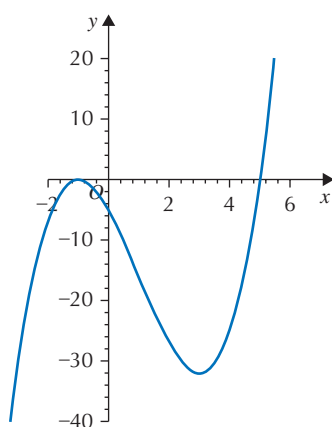


f roots: $(-1,0)$ $(5,0)$

y-axis:

 $(0,-5)$

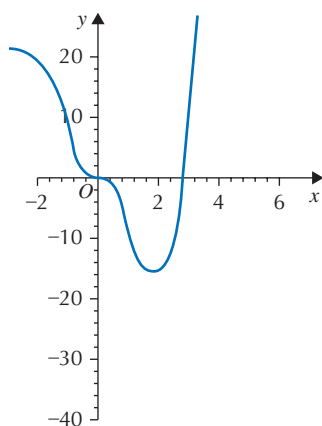
Stationary points:

 $(-1,0)$ Max $(3,-32)$ Min**g** roots: $(0,0)$ $(2.667,0)$

y-axis:

 $(0,0)$

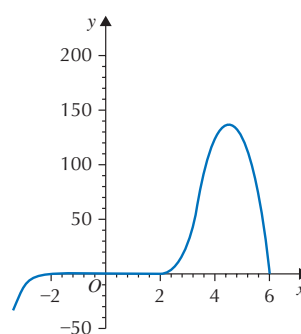
Stationary points:

 $(0,0)$ Inflexion $(2,-16)$ Min**h** roots: $(0,0)$ $(6,0)$

y-axis:

 $(0,0)$

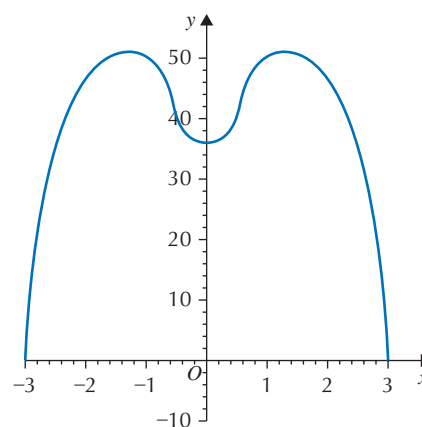
Stationary points:

 $(0,0)$ Inflexion $(4.5,136.688)$ Max**i** roots: $(-3,0)$ $(3,0)$

y-axis:

 $(0,36)$

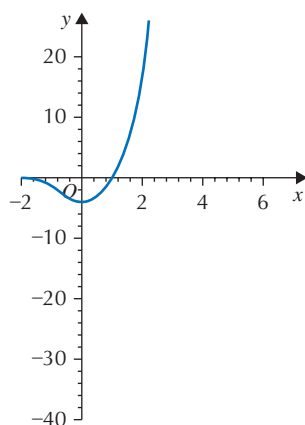
Stationary points:

 $(-1.581,42.25)$ Max $(0,36)$ Min $(1.581,42.25)$ Max

2 a i $(1)^3 + 3(1)^2 - 4 = 0$
 ii $(x-1)(x+2)(x+2)$

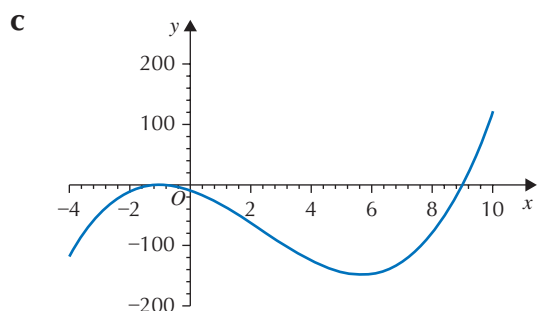
b $(-2, 0)$ Max
 $(0, -4)$ Min

c roots:
 $(1, 0)$
 $(-2, 0)$
 y-axis:
 $(0, -4)$



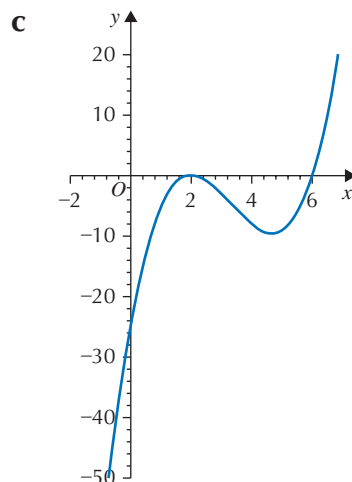
3 a i $(-1)^3 - 7(-1)^2 - 17(-1) - 9 = 0$
 ii $x = 9$

b $(-1, 0)$ Max
 $(5.667, -148.148)$ Min



4 a $(x-2)(x-2)(x-6)$

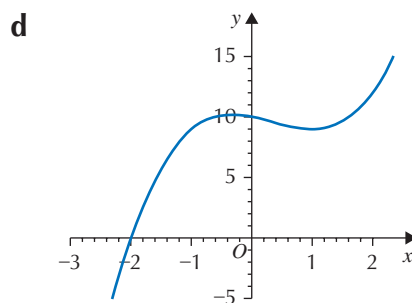
b i $(2, 0)$
 $(6, 0)$
 ii $(2, 0)$ Max
 $(4.667, -9.481)$



5 a i $(-2)^3 - (-2)^2 - (-2) + 10 = 0$
 ii $(x+2)(x^2 - 3x + 5)$

b i $(-0.3333, 10.185)$ Max
 $(1, 9)$ Min

c Only one root – and to left of stationary points.

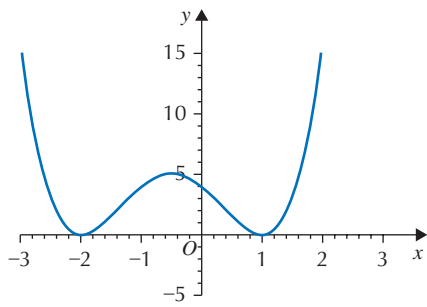


6 a i $(-2)^4 + 2(-2)^3 - 3(-2)^2 - 4(-2) + 4 = 0$

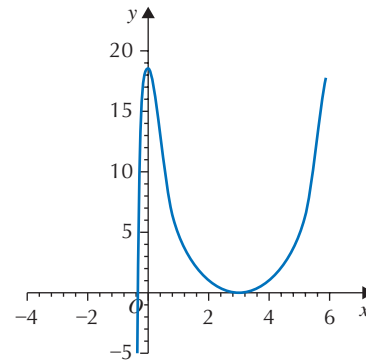
ii $(x-1)(x-1)(x+2)(x+2)$
 $= (x-1)^2(x+2)^2$

b Product of two squares so cannot be negative.

c i $(-2, 0)$ Min
 $(-0.5, 5.0625)$ Max
 ii $(1, 0)$ Min

d

c $(-0.3333, 18.5185)$ Max
 $(3, 0)$ Min

d

7 a $k = -4$

b i $(x + 2)(x - 3)^2$

ii There is a root **and** sp at $x = 3$ to y axis must be tangent.