Chapter 5

Exercise 5A

1 **(i)** **a** $\begin{pmatrix} -3 \\ 6 \\ 5 \end{pmatrix}$ **b** $\begin{pmatrix} 0 \\ 4 \\ -5 \end{pmatrix}$ **c** $\begin{pmatrix} 12 \\ 18 \\ 4 \end{pmatrix}$

(ii) **a** $\sqrt{70}$ **b** $\sqrt{41}$ **c** $\sqrt{34}$

2 **a** $3\underset{\sim}{i} + 5\underset{\sim}{j}$

b $7\underset{\sim}{i} - 9\underset{\sim}{j}$

c $3\underset{\sim}{i} - 2\underset{\sim}{j} - 6\underset{\sim}{k}$

d $1\underset{\sim}{i} + 0\underset{\sim}{j} - 4\underset{\sim}{k}$

e $8\underset{\sim}{i} + 2\underset{\sim}{j} - 1\underset{\sim}{k}$

3 **a** $\begin{pmatrix} 2 \\ 5 \\ -1 \end{pmatrix}$ **b** $\begin{pmatrix} 7 \\ -3 \\ 9 \end{pmatrix}$ **c** $\begin{pmatrix} 6 \\ 0 \\ -5 \end{pmatrix}$ **d** $\begin{pmatrix} 0 \\ 8 \\ 5 \end{pmatrix}$

4 **(i)** **a** $4\underset{\sim}{i} - 4\underset{\sim}{j} + 11\underset{\sim}{k}$

b $-10\underset{\sim}{i} + 5\underset{\sim}{j} + 16\underset{\sim}{k}$

c $-9\underset{\sim}{i} + 7\underset{\sim}{j} + 10\underset{\sim}{k}$

(ii) **a** $3\sqrt{17}$

b $\sqrt{381}$

c $\sqrt{230}$

5 $x = 5$

$y = 2$

6 $x = 3$

b $y = -5$

7 $x = 4$

$y = 3$

$z = 2$

8 $x = -4$

$y = 4$

$z = -2$

9 $x = 3$

$y = -2$

$z = 6$

10 **a** $\begin{pmatrix} \frac{4}{5} \\ \frac{3}{5} \end{pmatrix}$ **b** $\begin{pmatrix} -\frac{3}{5} \\ \frac{4}{5} \end{pmatrix}$ **c** $\begin{pmatrix} \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \end{pmatrix}$

11 $\begin{pmatrix} \frac{3}{\sqrt{14}} \\ -\frac{1}{\sqrt{14}} \\ \sqrt{\frac{2}{7}} \end{pmatrix}$

$\begin{pmatrix} -\frac{3}{\sqrt{14}} \\ \frac{1}{\sqrt{14}} \\ -\sqrt{\frac{2}{7}} \end{pmatrix}$

12 $z = \frac{\sqrt{3}}{4}$

13 $y = -\frac{\sqrt{23}}{6}$

$y = \frac{\sqrt{23}}{6}$

14 $a = -\sqrt{14}$

$a = \sqrt{14}$

Exercise 5B

1 **a** $CB = \begin{pmatrix} 2 \\ 5 \end{pmatrix}$

$AB = \begin{pmatrix} 5 \\ -1 \end{pmatrix}$

$AC = \begin{pmatrix} 3 \\ -6 \end{pmatrix}$

b $\sqrt{29}$

$\sqrt{26}$

$3\sqrt{5}$

2 $-3\underset{\sim}{i} - 5\underset{\sim}{j}$

$-2\underset{\sim}{i} - 7\underset{\sim}{j}$

$5\underset{\sim}{i} + 12\underset{\sim}{j}$

3 $5\underset{\sim}{i} - 1\underset{\sim}{j} - 4\underset{\sim}{k}$

$-2\underset{\sim}{i} - 8\underset{\sim}{j} - 10\underset{\sim}{k}$

$-3\underset{\sim}{i} + 9\underset{\sim}{j} - 6\underset{\sim}{k}$

$\sqrt{42}$

$2\sqrt{42}$

$3\sqrt{14}$

4 $-8\underset{\sim}{i} + 2\underset{\sim}{j} - 5\underset{\sim}{k}$

$9\underset{\sim}{i} - 5\underset{\sim}{j} + 3\underset{\sim}{k}$

$-1\underset{\sim}{i} + 3\underset{\sim}{j} + 2\underset{\sim}{k}$

5 **a** (–4, 9)
b (–1, 3)
c (–4, 0, 12)
d (–1, 3, 5)
e (9, 2, 7)

6 (4, 2)

7 (–2, 4, 4)

8 Q(2, 0, 0)
R(12, 0, 0)

Exercise 5C

1 B(4, 0, 2)

2 Q(2, 3, 13)

3 (–4, –1, 0)

4 D(9, –3, 2)

5 B(1, 1, –3)

6 B(2, 2, –2)
D(0, –4, –14)

7 Q(4, 2, –1)
S(8, –4, –5)
T(10, –7, –7)

8 C(–2, 7, 10)

9 A(–2, 24, –20)

Exercise 5D

1 **a** $\vec{RP}$
b $\vec{QS}$
c 0

2 **a** $\vec{AD}$
b $\vec{AD}$
c $\vec{EC}$
d $\vec{AD}$

3 **a** $\underset{\sim}{a} + \underset{\sim}{b} + \underset{\sim}{c}$
b $\underset{\sim}{a} + \underset{\sim}{b}$
c $\underset{\sim}{b} + \underset{\sim}{c}$
d $\underset{\sim}{a} - \underset{\sim}{b} - \underset{\sim}{c}$
e $-\underset{\sim}{a} - \underset{\sim}{c}$
f $-\underset{\sim}{a} - \underset{\sim}{c}$

4 **a** $\begin{pmatrix} 5 \\ 7 \\ 7 \end{pmatrix}$ **b** $\begin{pmatrix} -1 \\ 5 \\ 7 \end{pmatrix}$ **c** $\begin{pmatrix} 3 \\ -3 \\ 1 \end{pmatrix}$

5 $\overrightarrow{EA} = \begin{pmatrix} 3 \\ -5 \\ -6 \end{pmatrix}$ $\overrightarrow{BE} = \begin{pmatrix} -5 \\ -1 \\ 4 \end{pmatrix}$

6 **a** $\vec{OQ} = \begin{pmatrix} 2 \\ 3 \\ 4 \end{pmatrix}$
Q(2, 3, 4)
b $\vec{OP} = \begin{pmatrix} 6 \\ \frac{3}{2} \\ 2 \end{pmatrix}$
$P\left(6, \frac{3}{2}, 2\right)$

7 **a** $9\underset{\sim}{i} + 9\underset{\sim}{j} + 6\underset{\sim}{k}$
b $6\underset{\sim}{i} + 8\underset{\sim}{j} + 5\underset{\sim}{k}$
c $8\underset{\sim}{i} + 8\underset{\sim}{j} + 3\underset{\sim}{k}$
d $0\underset{\sim}{i} + 6\underset{\sim}{j} + 3\underset{\sim}{k}$
e $2\underset{\sim}{i} + 0\underset{\sim}{j} - 2\underset{\sim}{k}$

8 **a** $\vec{PA} = \begin{pmatrix} 9 \\ 2 \\ 4 \end{pmatrix}$
b $\vec{PB} = \begin{pmatrix} 5 \\ -5 \\ \frac{15}{4} \end{pmatrix}$
c $\vec{QV} = \begin{pmatrix} 2 \\ -8 \\ 10 \end{pmatrix}$

d $\vec{PV} = \begin{pmatrix} 10 \\ -4 \\ 13 \end{pmatrix}$

e $\vec{AB} = \begin{pmatrix} -4 \\ -7 \\ -\frac{1}{4} \end{pmatrix}$

9 $\vec{PA} = \begin{pmatrix} -\frac{31}{4} \\ -\frac{11}{2} \\ \frac{129}{4} \end{pmatrix}$

$\vec{PB} = \begin{pmatrix} -\frac{17}{3} \\ -\frac{14}{3} \\ 33 \end{pmatrix}$

Exercise 5E

1 **a** $t = 10$

b $t = 4$

2 $h = -2$

$k = 8$

3 $c = 7$

$d = -5$

4 **a** first = 3 (second)

b second = 2 (first)

c second = 2.5 (first)

d second = $-\frac{2}{3}$ (first)

First = multiple (second) => parallel

5 D(23, –2, –7)

6 **a** collinear

b not collinear

c not collinear

d collinear

7 **a** AB = $\left(\frac{1}{3}\right)$ AC so A, B, C collinear

b ratio is 1:2

8 **a** PQ = $\left(\frac{1}{4}\right)$ PR so PQR collinear

b ratio is 1:3

9 **a** MN = $\left(\frac{2}{3}\right)$ MP so MNP collinear

b ratio is 2:1

10 **a** AB = $\left(\frac{2}{5}\right)$ AC so ABC collinear

b ratio is 2:3

11 EF is not multiple of EG so EFG not collinear

12 **a** PQ = $\left(\frac{1}{3}\right)$ PR so collinear

b S(10, 5, –1)

13 **a** PA = $\left(\frac{1}{4}\right)$ PB so PAB collinear

b 14:45

14 AB = $\left(\frac{3}{5}\right)$ AC so collinear

15 **a** C(9, –41, 32)

b not collinear so no.