Linear FMCW radar techniques

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Indexing term: Frequency modulated continuous wave radar

Abstract: Frequency modulated continuous wave (FMCW) radar uses a very low probability of intercept waveform, which is also well suited to make good use of simple solid-state transmitters. FMCW is finding applications in such diverse fields as naval tactical navigation radars, smart ammunition sensors and automotive radars. The paper discusses some features of FMCW radar which are not dealt with in much detail in the generally available literature. In particular, it discusses the effects of noise reflected back from the transmitter to the receiver and the application of moving target indication to FMCW radars. Some of the strengths and weaknesses of FMCW radar are considered. The paper describes how the strengths are utilised in some systems and how the weaknesses can be mitigated. It also discusses a modern implementation of a reflected power canceller, which can be used to suppress the leakage between the transmitter and the receiver, a well known problem with continous wave radars.

1 Introduction

As the name suggests, frequency modulated continuous wave (FMCW) radar is a technique for obtaining range information from a radar by frequency modulating a continous signal. The technique has a very long history [1], but in the past its use has been limited to certain specialised applications, such as radio altimeters. However, there is now renewed interest in the technique for three main reasons. First, the most general advantage possessed by FMCW is that the modulation is readily compatible with a wide variety of solid-state transmitters. Secondly, the frequency measurement which must be performed to obtain range measurement from such a system can now be performed digitally, using a processor based on the fast Fourier transform [2] (FFT). A third reason for the interest in FMCW radars is that their signals are very difficult to detect with conventional intercept receivers [3].

The frequency modulation used by the radar can take many forms. Linear and sinusoidal modulations have both been used in the past. Linear frequency modulation is the most versatile, however, and is most suitable, when used with an FFT processor, for obtaining range information from targets over a wide range. For that reason the renewed interest in FMCW radar is almost all concentrated on linear FMCW, on which this article will concentrate.

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2 Basic analysis of the FMCW principle

The basic explanation of the working of a linear FMCW radar is well known and has been described in many other places, for example in Reference 4. It is not entirely obvious what one should call the beat signal, the frequency of which carries the range information. In this paper this signal will be called the intermediate frequency (IF) signal because it corresponds most appropriately with the IF of a pulse radar, although the information is not modulated onto a conventional carrier. The signal after the frequency analysis can best be referred to as the video signal. This signal is indeed an exact analogue of the video of a pulse radar. If a fast Fourier transform is used for the frequency analysis, then the FMCW video is coherent.

3 Moving target indication

The simple analysis of the FMCW radar is adequate for a single sweep if the target is not moving. In fact sweep-to-sweep processing is possible for performing moving target indication (MTI) and moving target Doppler (MTD) [5] processing with FMCW radars in the same way as for pulse radars. The Appendix contains a more detailed analysis of the FMCW signal in this case.

The analysis shows that a moving Target indicator (MTI) system operates in an almost identical manner for an FMCW radar as for a pulse radar. There are some additional second-order factors, but they have no practical effect on the MTI performance.

FMCW radars incorporating MTI are not yet common because FMCW has not often been applied to systems where MTI is needed. However, Fig. 1 shows the operation of the MTI canceller on an experimental S-band FMCW radar built at Philips Research Laboratories. The upper trace shows the video 'A-scope' picture from one sweep of the radar. The lower trace shows the signal at the output of a digital three-pulse MTI canceller. It can be seen that the static targets are indeed cancelled and that a static cancellation of better than 40 dB has been achieved.

The analysis shown in the Appendix can be extended to cover multiple cancellers and staggered sweep repetition frequencies, although these are best analysed by using the conventional equations for pulse radar MTI, and checking that the additional terms in the FMCW MTI equation are indeed negligible.

Because the MTI technique analysed in the Appendix is the same as is used in pulse radars, it can likewise be extended to encompass moving target Doppler (MTD) processing. MTD processing of FMCW signals is implemented by measuring the rate of change of phase of the output of each FFT range bin from one sweep to the next, the same as is done for pulse radars. Fig. 2 shows a simple block diagram of such a system. The Signal SQUIRE FMCW battlefield surveillance radar uses

MTD processing to measure the speed of its targets, to help the user to evaluate their importance. The coherent, CW, nature of the FMCW transmissions means that it is in fact very easy to add such a facility to an FMCW radar.

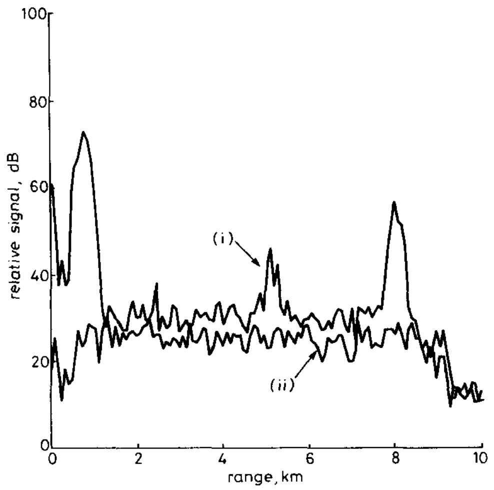

Fig. 1 *Performance of three-sweep canceller on S-band FMCW radar*
(i) Uncancelled
(ii) Cancelled

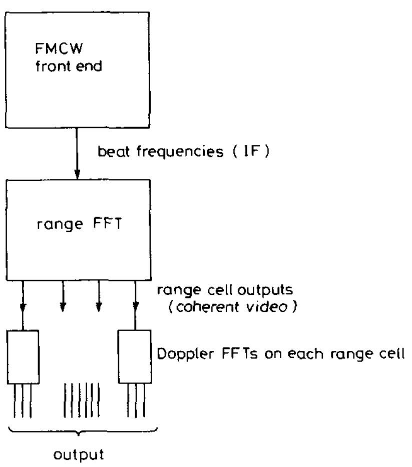

Fig. 2 *MTD processing applied to FMCW radar*
Output is a set of range Doppler cells

There is an interesting relationship between the range-Doppler cross coupling and the MTI blind speeds. The range-Doppler cross coupling is the well known effect in FMCW radars whereby the Doppler shift due to a moving target changes the apparent range of that target. The first blind speed of the radar is at $f_d = 1/T$, which is the Doppler shift which causes a range error of exactly one range cell. This is obvious because the first blind speed is the speed at which the Doppler goes through a complete cycle from one sweep to the next, i.e. the beat frequency has been increased by one cycle per sweep. It follows from this that if an FMCW radar is unambiguous in Doppler, then the range-Doppler coupling gives range errors of less than one range cell.

4 Advantages of FMCW radar

4.1 Simple solid-state transmitters

Unlike vacuum tubes, solid-state devices, because of their small physical size, are inherently unable to raise their peak power handling much above their continuous ratings so they are best used to handle continuous transmissions. Because it is a continuous wave transmission, FMCW is thus much more readily compatible with simple solid-state transmitters than are pulse waveforms. For low-power systems, FMCW undoubtedly offers the simplest transceiver designs of any radar which can give range information, although this is to some extent offset by the need to perform some sort of frequency analysis on the IF spectrum.

It is indeed possible to make a complete FMCW radar using only one active device. The oscillator can in principle be bias modulated to control its frequency and can simultaneously be used as a self-oscillating receiver mixer. For most applications this idea is only a theoretical curiosity, because the sensitivity will not be particularly good and it is difficult to separate the IF signal coming out of the device from the sweep signal going in. The concept, nonetheless, illustrates the simplicity which can be obtained from the principle. Its use has, however, been suggested for some fusing applications, where sensitivity is less important than low cost. Fig. 3 illustrates the possible circuit of such a fuse.

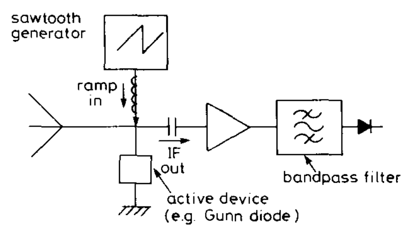

Fig. 3 *Simplest possible FMCW radar*
Bandpass filter gives single range bin

One deficiency of current solid-state transmitters is their poor DC to RF power conversion, but this is not a great problem for low-power systems, and the development of more efficient microwave amplifiers using heterojunction bipolar transistors or pseudomorphic HEMTs [6] promises to ease this problem.

4.2 Resistance to interception

FMCW is a simple way of giving a radar an extremely high time bandwidth product, which makes such systems very resistant to interception by electronic support measure (ESM) systems, because it is impractical to match the ESM receiver to the radar's sweep pattern. It will be shown below in the discussion of the PILOT radar that the resulting mismatch between the ESM receiver and the FMCW transmissions is often sufficient to give the FMCW radar a range advantage over the intercept receiver, even against intercepts of its main beam.

4.3 Good range resolution

The fact that the excursion of the linear FM ramp determines the range resolution of the system makes it easy for FMCW radars to have very good range resolution. It also makes it very easy to change the range resolution at will, so that, for example, on a navigation radar it is possible to use a different range cell width on each range scale, so as to optimise the resolution of the display on each range scale.

4.4 Advantages over other modulated CW waveforms

Other waveforms, such as phase shift keying, have been proposed for continuous wave radars. Such waveforms share many of the advantages of FMCW, such as compatibility with solid-state transmitters and difficulty of interception, but FMCW has some very important practical advantages due to the deterministic nature of its waveform.

4.4.1 FFT as correlator: The FFT processor which is used to recover the range information from the IF signals is much simpler than the comparable correlators which would be required if other forms of pulse compression were used. The fact that the target range is proportional to the beat frequencies means that the number of range cells which need to be processed by the FFT can be limited by simply filtering the signals to reduce the bandwidth into the FFT to cover only those ranges which are of interest. This can lead to a dramatic reduction in the complexity of the digital processor.

4.4.2 Resistance to interference: The fact that the FMCW waveform is highly deterministic is actually an advantage in rejecting interference because it allows definite predictions to be made about the form which any genuine signals should have. This allows interference which is uncorrelated with the expected returns to be very effectively suppressed. This applies in particular to interference from pulse radars and to narrow-band interference. Because FMCW radars are so hard to detect, a potential jammer would find it virtually impossible to measure the parameters of the FMCW signal with sufficient accuracy for it to be able to match the jamming waveform to the radar waveform.

4.4.3 Implementation of sensitivity time control: Sensitivity time control (STC) is an essential technique in many pulse radars and is used to control the dynamic range of the returns, to counteract the r^4 fall off in signal strength with increasing range in order to limit the dynamic range of the IF signals. An analogue of this technique may be implemented in the frequency domain in an FMCW radar by recognising that the connection between the frequency of the returns and the range of the target allows such processing to be done early in the processing chain; in a way analogous to the way that the relationship between time and target range is used in the STC of pulse radars. The ability to control the dynamic range is important for a CW radar as any intermodulation due to overload of the IF will generate spurious signals. FMCW allows a range gain control to be used to control the dynamic range of the signals, whereas this is not possible with other CW-type waveforms.

4.4.4 Compatibility with reflected power canceller: Section 6.2 will discuss the use of a reflected power canceller (RPC) to adaptively cancel the transmit-receive feedthrough which can threaten to limit the dynamic range of single-antenna CW radars. The linear ramp of an FMCW system means that a simple RPC is able to adapt during the sweep to cope with a wide total RF bandwidth, because the instantaneous bandwidth is small. If other modulated CW waveforms are used, the RPC must operate over a much wider instantaneous RF bandwidth and it will be correspondingly more complex, or else it will be less effective.

5 Applications of FMCW radar

This Section looks at two applications which exploit the advantages of the FMCW technique. The first application is a current product, the PILOT X-band naval navigation radar, and the other, a radar headway monitor for cars, is in development.

5.1 PILOT indetectable navigation radar

The PILOT radar [7] operates in the conventional radar X-band, around 9.375 GHz, for the conventional reasons of forming a compromise between antenna size, bad weather performance and component cost. It is a tactical navigation radar, and is designed to be used by warships for navigation where the ability to perform accurate navigation in poor weather is essential for the accomplishment of the ships's tactical mission. As such, the reason for using FMCW is to make the radar tactically indetectable, that is, PILOT cannot be detected by a tactical ESM system at sufficient range for the detections to be tactically significant. This is achieved by using an FMCW waveform to give it a frequency sweep of typically 10 MHz or more and a sweep time of about 1 ms, which gives it a time-bandwidth product of 10000. With this time bandwidth product a sensitivity and resolution equivalent to that of a conventional pulse navigation radar can be obtained with an output power of 1 W CW.

The following discusssion will illustrate the basis on which PILOT can be called 'tactically indetectable'. Table 1 lists the salient parameters of PILOT and of a

Table 1: Parameters of FMCW radar and ESM system

PILOT:	antenna gain	30 dB
	output power	1 W
	noise figure	3 dB
	sweep period	1 ms
ESM:	antenna gain	0 dB
	minimum detectable signal	−60 dBm

representative high performance naval tactical ESM system.

PILOT can detect a target of 100 m^2 radar cross-section, equivalent to a small ship, at a free space range of 17 km. The ESM can only detect PILOT at a range of 2.5 km. Hence the ESM cannot detect PILOT until well after the platform which is carrying it has been detected by PILOT. This is the basis for calling PILOT 'tactically indetectable'.

The low output power of PILOT is compensated by the narrow receiver bandwidth of 1 kHz, whereas the tactical ESM system must be sensitive to emitters on all frequencies in all directions. Nonetheless, the ESM could detect a typical pulse navigation radar with 10 kW peak power at a free space range of 250 km. The pulse radar's signals would be matched to an IF bandwidth of perhaps 10 MHz whilst PILOT's are matched to 1 kHz. This further mismatch reduces the range at which PILOT's main bean can be detected to 2.5 km, which is so low as to be of no tactical use.

Other configurations of ESM system can be made more sensitive and can thus detect FMCW radars such as PILOT at longer ranges, but they generally achieve this greater sensitivity at the expense of reduced probability of intercept. Extensive investigations have shown that the resulting times between intercepts for more sensitive receivers means that any information they can gather is still tactically useless, so the radar is still tactically indetectable again any affordable ESM system.

This tactical indetectability was the principal reason for developing PILOT as an FMCW radar. Once the choice to use FMCW has been made, however, the advantages of a solid-state transmitter and of improved range resolution come for free. They cannot themselves currently justify the development of an FMCW navigation radar, but they are real additional advantages. PILOT actually has a minimum range cell width of just under 3 m, which is three times as good as a conventional navigation radar, and it can display a full 512 range cells on each range scale from 0.75 nm upwards, which is the limit of the number of range bins which a plan position indicator (PPI) can distinguish. This feature has proved very popular with operators of the system.

The minimum range resolution of 3 m, corresponding to a 50 MHz sweep, would normally require a receiver bandwidth of at least 30 MHz and a video bandwidth of typically 20 MHz. However, as the maximum range of PILOT with this resolution is limited to 0.75 nm, the IF bandwidth can be limited to 512 kHz, reducing the sampling rate for the analogue-to-digital converter (ADC) to just over 1 MHz, and keeping the video bandwidth correspondingly low.

5.2 Automotive headway monitoring

The second area of application for FMCW which is of growing interest is in radar systems for cars. The most promising short-term application in this area is headway monitoring sensor which can be used as an input to a so-called autonomous intelligent cruise control system (AICC). The AICC adapts the speed of a car to keep it a safe distance away from any vehicle in front of it. To do this it needs a sensor to measure its distance from that vehicle (the headway). The particular automotive applications of radar are discussed further in Reference 8.

The principal design requirements for automotive radars are low cost and small size. An operating frequency of around 80 GHz is preferred, as being the highest which is practical and for which a frequency allocation is potentially available. The system range is limited to the order of a hundred metres, so atmospheric effects are not particularly important. The highest possible frequency is used to obtain the smallest possible antenna size compatible with the required resolution. An antenna width of about 15 cm, corresponding to about 1.8 degrees beamwidth, is desired. The required transmitter power is only of the order of a milliwatt.

FMCW modulation is attractive for automotive radars in order to minimise the overall system cost. The generation of a few milliwatts of CW power at 80 GHz is not now a problem, but getting even a milliwatt of mean power from an equivalent solid-state pulse system is still a difficult task. FMCW allows good use to be made of the CW power. It also gives the simplest radar configuration, and hence the best change of minimising the cost. The cost of the FFT processor for this application is not a major consideration.

The wide time-bandwidth product available from an FMCW system also helps to minimise the chances of interference from other radars operating in the same frequency band.

6 Transmit/receive leakage

Despite their advantages, FMCW radars have not been common in the past. Some of the reasons for this have been based on limitations in the technology which have now been overcome. One such limitation was in the availability of solid-state transmitters, which make the principle much more attractive. Another former limitation was the availability of a suitable means for analysing the IF spectrum, which is now accomplished with FFT processors using digital signal processing chips.

However, there are still a number of considerations which must be addressed in the design of an FMCW radar and which do not occur with pulse radars. Ignorance of these factors can cause problems which can appear insuperable, and it is some of these factors which led to FMCW not being used more widely.

One of the most important factors which must be addressed is leakage of the transmitter signal into the receiver. The problems associated with this leakage are not insuperable, but they must be treated with respect if an FMCW system is to be successful. Any CW radar must be able to receive at the same time as it is transmitting. This means that the inevitable direct breakthrough of the transmitter signal into the receiver must be controlled so as to stop it degrading the receiver sensitivity. This has two aspects. The most severe problem occurs when the leakage signal is so powerful that it threatens to damage the receiver mixer. Even at lower power levels, the noise sidebands on the transmitted signal may still degrade the receiver sensitivity if care is not taken.

For low-power radars, actual damage of the receiver is not normally the problem. If a common transmit/receive antenna is used then the dominant source of leakage will be reflections from the antenna and leakage around the circulator. The combination will typically give an isolation between transmitter and receiver of about 15–20 dB, which can be improved by careful design to about 30–35 dB over a narrow frequency band.

6.1 Separate transmit and receive antennas

The simplest way of achieving greater isolation is by using separate antennas for transmission and reception. This can be inconvenient, but will give an isolation of up to about 60 dB. Even in this case the effects of noise leakage cannot necessarily be ignored, although with careful design, noise need not be a problem.

6.2 Reflected power canceller

When the use of two antennas is not desirable, the level of the transmit/receive leakage can be reduced by 'bucking off' the leakage signal using a reflected power canceller (RPC). Fig. 4 shows the basic principle. The amplitude and phase of the leakage power are estimated by measuring the DC levels of the inphase (I) and quadrature (Q) outputs of the receiver mixer. These I and Q signals are then used to control the amplitude and phase of the buck-off signal. The RPC forms a closed-loop controller.

The control loops for the I and Q channels are essentially independent, which means that leakage of signals from one channel to the other, due to lack of ideality in the components, merely acts as a disturbance in the other channel. With a reasonable level of loop gain this disturbance has only a small effect on the overall performance of the loop, so the RPC is quite robust to phase errors of the order of 45 degrees around the loop, which means that the specifications of the vector modulator and of the quadrature mixer do not need to be particularly stringent.

The principle of the RPC has been known for a long time and its practical use goes back to about 1960 [9].

The idea has been revived for the PILOT navigation radar, using modern microwave component techniques and a simplified phase compensation scheme [10]. In the case of PILOT, the RPC improves the transmit/receive isolation from about 20 dB (without the RPC) to about 50 dB, which is comparable with the isolation obtained from a dual antenna system.

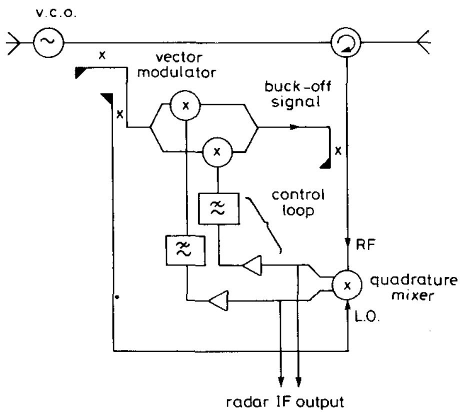

Fig. 4 *Basic reflected power canceller*

6.3 Transmitter noise leakage

The problem of transmit-receive noise leakage is known as one of the most severe problems facing the designer of an FMCW radar. In fact one well-known radar textbook [11] even goes as far as to suggest that FM radars can never be made to meet their nominal power budget. However, that suggestion is false and it is quite possible to design an FMCW radar, such as PILOT, in which the transmitter noise makes at most a second-order contribution to the overall noise figure of the system.

The problem of noise leakage has been addressed before [11], but the full derivation of the appropriate formulas is given in the Appendix for completeness. Note that the FM sidebands at a deviation rate ω_m, like AM noise sidebands at the same frequency, give rise to noise only at an IF of ω_m. There is thus a simple relationship between the transmitter's noise spectra and the spectrum of the detected noise in the receiver's IF.

It is interesting to note that in the case where $\Delta\omega$ is independent of frequency, then for small $\omega_m \delta\tau$, the detected noise is also the same at all IF frequencies, which is the same as the case for white AM noise. In the FM case this corresponds to a sideband level, $(\Delta\omega/\omega_m)^2/4$, which falls at 20 dB per decade as m increases. This then counteracts the fact that, for small $\omega_m \delta\tau$, the degree of cancellation is reduced by 20 dB/decade as the modulation frequency rises. Any $1/f$ noise in the oscillator's FM noise spectrum will likewise show itself as $1/f^3$ noise on the oscillator's power spectrum, which will appear as $1/f$ noise in the receiver IF.

In practice the phase term will add a fast ripple to the fall-off in cancellation as the path length error $\delta\tau$ increases. If we compare the expressions for detected AM and FM noise (eqns. 29 and 35):

$$\mathrm{AM} = 4L \cos 2\phi$$

and

$$\mathrm{FM} = 4L \sin 2\phi \cdot 4 \sin 2(\omega_m \delta\tau/2)$$

we can identify

$$C = 4 \sin 2(\omega_m \delta\tau/2) \tag{1}$$

as the formula for the FM noise cancellation. An alternative form replaces $f = \omega_m/2\pi$ and $\delta\tau = l/v$ where l is the mismatched path length and v is the transmission medium

$$C = 4 \sin 2(\pi f l/v) \tag{2}$$

The contribution of the FM noise can be minimised by identifying the major sources of the leakage and arranging that the path length from the transmitter to the mixer via the leakage is the same as the local oscilator path length from the transmitter to the mixer. The limit to the degree of cancellation which can be achieved will either be due to incidental FM to AM conversion (caused by imperfections in the frequency responses of the various components in the RF path) or to the presence of multiple leakage sources. The FM noise cancellation effect counteracts the fact that, for a typical oscillator, the FM noise power is much higher than the AM noise power, and in many systems the contribution of the FM noise to the overall noise figure of the receiver is less important than that of the AM noise.

6.3.1 Example FM and AM noise calculation: As an example of how this may work in practice, we may take the case of a system with a transmitter power of 1 W, an AM noise level of −160 dBc/Hz and an FM noise level of −120 dBc/Hz at 100 kHz offset. The maximum IF frequency will be assumed to be 100 kHz and the system will be assumed to have a receiver noise figure of 3 dB.

If the variation of transmitter noise with distance from carrier is assumed to be reasonably well behaved, then the critical case will be at 100 kHz IF, corresponding to maximum range, where the receiver sensitivity is most needed.

If the AM and FM noise levels are to be allowed to make equal contributions to the overall noise figure, then the FM noise at 100 kHz must be cancelled by 40 dB. From eqn. 2, this means that $\delta\tau$ must be less than 16 ns. This corresponds to a path length error of nearly 5 m in air, so the requirement for FM noise correlation is not particularly strict in this case.

If the AM and FM noise levels are equally significant, then from eqns. 29 and 35 we can see that the effective noise level will be −154 dBc/Hz, referred to the receiver input. The loss factor, L, can either be included here or, more conveniently, it can be included when the added noise is compared with the receiver noise. If the transmit/receive isolation is 50 dB, achieved by either using two antennas or an RPC, then the leakage level will be −20 dBm, so the total leakage power seen in the receiver will be −174 dBm/Hz. This is equal to the thermal noise floor at room temperature.

For the purpose of the calculation the receiver noise figure will be included by assuming that the receiver has no loss, but that excess noise, corresponding to its noise figure, is added in at its input. If the receiver noise figure is 3 dB, the equivalent added noise power will be −171 dBm/Hz. The noise due to the leakage of the transmitter noise sidebands will be 3 dB below the noise due to the receiver noise figure. If these two noise powers at the receiver input are added, the sum will be −169 dBm/Hz.

In this simple, but not atypical, example, it will be seen that the noise sidebands affect the sensitivity of the FMCW radar, but the effect is not too serious. Note also that the -20 dBm leakage signal is near the limit which a low noise amplifier/mixer combination can handle linearly. It is higher than the signals which a 1 W radar will see back from almost any targets, so FM noise reflected back from close-in targets will also not be a problem.

6.3.2 Rules for defining transmitter noise specifications: A simple rule can now be derived from eqns. 29 and 35 for calculating the allowable transmitter leakage power. If the permitted degradation of the system sensitivity by AM noise leakage is, for example, 0.5 dB, then it can contribute about 12% to the total noise figure, i.e. the detected AM noise should be 9 dB below the basic receiver noise level. Let transmitter power = P_t dBm, oscillator AM noise level = n_{AM} dBc/Hz s.s.b., and transmit-receive leakage = R dB. Receiver noise figure = N_r dB at the input to the receiver and thermal noise = -174 dBm/Hz.

The average power level over all phases of the detected double sideband AM will be twice that detected from one sideband. If the overall degradation is to be less than 0.5 dB then the following inequality must be satisfied.

$$P_t + n_{AM} + R \leqslant -174 + N_r - 12 \tag{3}$$

Given P_t and N_r, the AM noise level and the isolation must then be arranged to satisfy the inequality if the desired performance for the whole system is to be achieved.

For the FM noise a similar inequality must be satisfied, but with an additional term to take account of the FM noise cancellation.

$$P_t + n_{FM} + R + C \leqslant -174 + Nr - 12 \tag{4}$$

Expr. 4 assumes a mean level of leakage, averaged over all phases of the local oscillator. The worst case will be 3 dB poorer, but the phase which gives the worst case detection of AM noise will give no detection of FM noise and *vice versa*, so that if both make similar contributions to the overall noise level of the system, then the average value for each should be taken. If one type of noise is known to be dominant, then a worst case calculation can be made.

In the principle the local oscillator phase can be adjusted to minimise the detected noise, but because of the difficulty of keeping this adjustment over time and temperature this is normally impractical.

The most significant practical difficulty in applying these formulas is the accurate determination of the AM noise level. Whereas the FM noise of many oscillators can these days be measured directly on a spectrum analyser, the much lower levels of AM noise can be difficult to measure with any accuracy. It is correspondingly also frequently difficult to get manufacturers to accept a specification for a worst-case AM noise level of an oscillator which is representative of what the oscillator is actually capable of achieving, because of this difficulty in measuring these very low levels of AM noise.

7 Conclusions

As was mentioned in the preceeding Section, the practical difficulties in implementing FMCW radars have in the past led one well-known radar textbook to say that [12] 'The badge of a novice in the FM-radar field is a carefully worked out performance appraisal based only on the methods of Chap. 2', where chapter 2 of the work described the power budget calculations for a pulse radar. The same source goes on to say that 'In general, it may be said that, with two antennas with good decoupling between them, a well-designed FM homodyne radar may come within 20 to 30 dB of the performance indicated by Chap. 2'.

The principal conclusion of this paper must be that even if the first of those statements is true, the second is definitely wrong. Many FMCW radar systems have been designed and built which do have the sensitivity one would expect of them. Although care must be taken in the design, provided that care is taken, the radar will perform as one would expect from its transmitter power and its receiver noise figure. Its range discrimination is as would be expected from the RF bandwidth used. Once it has been shown that FMCW is a workable technique then one can begin to consider its advantages, in particular its 'indetectability', its simplicity, its compatibility with solid-state transmitters and the ease with which it can be used to give very fine range resolution.

This paper has discussed some applications of FMCW radar which make use of these advantages. It has also discussed the problems associated with the leakage of transmitter signals into the receiver in an FMCW radar and it has shown how these problems can be controlled. So the old FMCW technique can be combined with modern digital and microwave technology to produce attractive, low cost, high performance radar systems for a variety of applications.

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9 Appendix

9.1 Analysis of FMCW signal

The following is a more detailed analysis of the behaviour of linear FMCW signals taking account of Doppler and second-order terms. The analysis will be performed for a single target. The effect of multiple targets can easily be extrapolated because the system is linear with respect to the received signals. End effects due to the flyback will be neglected. The following symbols will be used:

t = time since start of sweep
f_t = instantaneous transmitter frequency
f_0 = transmitter frequency at time $t = 0$
α = chirp rate

τ = time of flight of the signal from the transmitter to the target and back
r_1 = range of target at time $t = 0$
r_0 = mean range of target during sweep
v = radial velocity of the target
T = sweep repetition interval (SRI) of the radar
c = velocity of propagation.

The radial velocity of the target is assumed to be constant. The analysis could relatively easily be extended to include accelerating targets.

Now, $f_t = f_0 + \alpha t$. The standing phase of the transmitted signal is

$$\phi = 2\pi \int_0^t f_t \, dt = 2\pi[f_0 t + (1/2)\alpha t^2] \quad (5)$$

assuming $\phi = 0$ at time $t = 0$.

In the following analysis, a_0, b_0 and c_0 are constants which determine the amplitudes of the signals, but have no effect on the rest of the calculations. The instantaneous amplitude of the transmitted signal is

$$a(t) = a_0 \sin 2\pi[f_0 t + (1/2)\alpha t^2] \quad (6)$$

The received signal from the target is delayed and attenuated

$$b(t) = b_0 \sin 2\pi[f_0(t - \tau) + (1/2)\alpha(t - \tau)^2] \quad (7)$$

The IF signal is

$$c(t) = c_0 \cos 2\pi[f_0 \tau + \alpha t\tau - (1/2)\alpha\tau^2] \quad (8)$$

If the target is moving

$$r(t) = r_1 + vt \quad (9)$$

and

$$\tau = 2r(t)/c$$

Then, after some manipulation

$$c(t) = c_0 \cos 2\pi[2r_1 t(1 - 2v/c)/c + 2f_0 vt/c + 2\alpha vt^2(1 - v/c)/c + 2(f_0 - \alpha r_1/c)r_1/c] \quad (10)$$

The expression contains 'frequency' terms, which are time varying, and 'phase' terms, which are not. The first frequency term, $2r_1(1 - 2v/c)/c$ is the range beat, which is proportional to the range of the target. It is normally assumed to be equal to $2\alpha r_0/c$. The second frequency term, $2f_0 vt/c$, is the Doppler shift. This is in itself a well known result.

The third frequency term in eqn. 10, $2\alpha vt^2(1 - v/c)/c$, is a cross-term which may either be interpreted as chirp on the range beat, due to the changing range, or as chirp on the Doppler, due to the changing transmitter frequency. The final term is a constant phase term, $4\pi(f_0 - \alpha r_1/c)r_1/c$.

Besides showing the range Doppler cross coupling explicitly, the above analysis can be developed a little further to show that moving target indication is possible with FMCW radars without having to resolve the range Doppler cross coupling. Reverting to eqn. 8, the IF signal for two successive sweeps can be written as

$$c_1(t) = c_0 \cos 2\pi[f_0 \tau_1 + \alpha\tau_1 t - (1/2)\alpha\tau_1^2] \quad (11)$$

and

$$c_2(t) = c_0 \cos 2\pi[f_0 \tau_2 + \alpha\tau_2 t - (1/2)\alpha\tau_2^2] \quad (12)$$

The simplest form of MTI processing is a simple canceller which substracts the returns from the two sweeps.

$$d(t) = c_2(t) - c_1(t) \quad (13)$$

Eqns. 11 and 12 may substituted into eqn. 13, using standard trigonometrical identities and defining $\tau_0 = (1/2)(\tau_1 + \tau_2)$, the mean time of flight of the signals for the two sweeps and $\delta\tau = (1/2)(\tau_1 - \tau_2)$, half the change in time of flight between the two sweeps. Then

$$D(t) = 2C_0 \cos 2\pi[f_0 \tau_0 + \alpha\tau_0 t - (1/2)\alpha\tau_0^2 - (1/2)\alpha\delta\tau^2] \times \sin 2\pi[f_0 \delta\tau + \alpha\delta\tau t - (1/2)\alpha\tau_0 \delta\tau] \quad (14)$$

Now

$$\tau_1 = 2(r_1 + vt)/c, \tau_2 = (r_1 + vt + vT)/c$$

and

$$\delta\tau = Tv/c$$

The velocity is assumed to be a constant, so $\delta\tau$ is not a function of time and $-(1/2)\alpha\delta\tau^2$ can be written as a constant phase, ϕ, so

$$D(t) = C_0 \sin 2\pi[f_0 \tau_0 + \alpha\tau_0 t - (1/2)\alpha\tau_0^2 - \phi] \times 2 \sin 2\pi[f_0 \delta\tau + \alpha\delta\tau t - (1/2)\alpha\tau_0 \delta\tau] \quad (15)$$

Except for a phase term, the first half of the expression on the right-hand side of eqn. 15 is the same as the right-hand side of eqn. 8, which was the expression for the signal received from a single sweep. The second term is the effect of the MTI filter.

In a conventional pulse radar, the MTI term is $2 \sin 2\pi f_0 \delta\tau$. The FMCW case contains two additional terms. One, $+\alpha\delta\tau t$, is time varying and represents the change in transmitter frequency during the sweep. The other, $-(1/2)\alpha\tau_0 \delta\tau$, represents the fact that the range beat frequency is slightly different between one sweep to the next because the range has changed. As both of these extra terms are functions of $\delta\tau$, they have no effect on the static cancellation, for which $\delta\tau = 0$.

The terms $\alpha\delta\tau t$ and $(1/2)\alpha t_0 \delta\tau$ are negligible for most practical purposes. It can therefore be seen that a simple MTI canceller behaves the same for an FMCW radar as for a pulse radar.

9.2 Effect of transmitter noise sidebands

The amplitude modulation (AM) noise on a carrier in a narrow frequency band about $\pm\omega_m$ from a carrier at frequency ω_0 is quasisinusoidal and so can be expressed by the well-known expression for amplitude modulation:

$$E = E_0[(1 - \alpha) + \alpha \cos \omega_m t] \cos \omega_0 t \quad (16)$$

where $\alpha/(1 - \alpha)$ is the modulation index, and the peak signal level is E_0. This can be reformulated to separate the carrier and the sidebands:

$$E = E_0(1 - \alpha) \sin \omega_0 t + E_0 \alpha/2 \cos (\omega_0 - \omega_m)t + E_0 \alpha/2 \cos (\omega_0 + \omega_m)t \quad (17)$$

It is well known that a similar frequency-modulated signal can be expressed as

$$E = E_0 \sin (\omega_0 + \Delta\omega \sin \omega_m t)t \quad (18)$$

which can be expanded as a series of Bessel functions of the deviation ratio, $\Delta\omega/\omega_m$. If $\Delta\omega$ is the noise in 1 Hz bandwidth it is typically of the order of 1–10 Hz and ω_m is typically many kilohertz or megahertz. The FM noise thus typically gives rise to narrowband modulation, i.e. $\Delta\omega \ll \omega_m$, in which case the Bessel expansion can be reduced to

$$E = E_0 \sin \omega_0 t + E_0/2 \cdot \Delta\omega/\omega_m \sin (\omega_0 - \omega_m)t - E_0/2 \cdot \Delta\omega/\omega_m \sin (\omega_0 + \omega_m)t \quad (19)$$

It will be noted that in the expression for AM noise, the two sidebands have the same sign whereas in the expression for FM noise they have opposite signs. Whereas both modulations individually give equal sideband levels on either side of the carrier, any arbitrary ratio of sidebands can be generated by adding frequency and amplitude modulations of the appropriate level and phase. Eqns. 17 and 19 can therefore be used to completely characterise the noise sidebands of the oscillator at a given frequency from the carrier, even if the noise has no obvious correlation with the oscillations themselves.

9.2.1 Power ratios: The analysis is most conveniently performed in terms of voltages. However as the final results will be in terms of power, we can define a characteristic impedance Z_0 of the system, so that the voltages can be converted into powers, using the general formula

$$P = E^2/2Z_0 \quad (20)$$

where P is the power and E is the peak voltage of the sinusoidal signal. The single sideband AM and FM noise to carrier power ratios are thus, respectively,

$$\eta_{AM} = [E_0^2 \alpha^2/8Z_0]/[E_0^2(1 - \alpha)^2/2Z_0] \quad (21)$$

and

$$\eta_{FM} = [E_0^2(\Delta\omega/\omega_m)^2/8Z_0]/[E_0^2/2Z_0] \quad (22)$$

If α is assumed to be small then

$$\eta_{AM} = \alpha^2/4 \quad (23)$$

$$\eta_{FM} = (\Delta\omega/\omega_m)^2/4 \quad (24)$$

The noise on the leakage signal will generally be detected by the receiver mixer and will degrade the receiver noise figure. The IF from an input $E = E_s \sin \omega t$ is

$$E' = kE_s \cos [(\omega - \omega_0)t + \phi] \quad (25)$$

where k is the mixer conversion loss and ϕ is the phase of the signal relative to the local oscillator. The power in the signal is

$$P_s = E_s^2/2Z_0$$

The power in the IF signal is

$$P_I = E_s^2 k^2/2Z_0$$

and the mixer power conversion loss is

$$L = P_I/P_s = k^2 \quad (26)$$

9.2.2 Detection of AM noise: If the AM noise on the local oscillator (LO) signal is assumed to be substantially suppressed by the use of a balanced mixer, then the local oscillator can be considered to be

$$E_L = E_{LO} \sin (\omega_0 t - \phi) \quad (27)$$

so the low frequency component of the AM noise at the mixer output (the IF noise) will be

$$E_I = \alpha E_0 k/2[\cos (\omega_m t + \phi) + \cos (-\omega_m t + \phi)] \quad (28)$$

The ratio of detected IF AM noise power to single sideband AM noise power is thus

$$\eta_{AM} = 4k^2 \cos^2 \phi = 4L \cos^2 \phi \quad (29)$$

This means that the detected IF noise power varies between the equivalent of four times the single sideband RF noise power and zero, as the relative phase of RF and local oscillator signals are varied and the sidebands either add up or cancel out. The mean power level is of course $2L$, that is the RF double sideband power level multiplied by the conversion loss.

9.2.3 Detection of FM noise: In the case of the FM noise, the noise on the local oscillator cannot be neglected, and the local oscillator must be written thus:

$$E_L = E_{LO}[\sin \omega_0 t + \Delta\omega/2\omega_m \sin (\omega_0 - \omega_m)t - \Delta\omega/2\omega_m \sin (\omega_0 + \omega_m)t] \quad (30)$$

The RF signal, in general, suffers a time delay $\delta\tau$ relative to the local oscillator. For the discussion of AM noise this delay was represented by the phase shift ϕ, but for the discussion of FM noise it is easier to represent it as a time delay.

$$E = E_0\{\sin \omega_0(t - \delta\tau) + \Delta\omega/2\omega_m \sin [(\omega_0 - \omega_m)(t - \delta\tau)] - \Delta\omega/2\omega_m \sin [(\omega_0 + \omega_m)(t - \delta\tau)]\} \quad (31)$$

When the RF and LO signals are mixed, the following IF components are generated:

$$\begin{aligned} E_I = \Delta\omega E_0 k/2\omega_m\{&-\cos [(\omega + \omega_m)(t - \delta\tau) - \omega_0 t] \\ &+ \cos [(\omega - \omega_m)(t - \delta\tau) - \omega_0 t] \\ &+ \cos [(\omega - \omega_m)t - \omega_0(t - \delta\tau)] \\ &+ \cos[(\omega - \omega_m)t - \omega_0(t - \delta\tau)]\} \end{aligned} \quad (32)$$

After manipulation these yield

$$E_I = 2k(\Delta\omega/\omega_m)E_0 \times \sin \omega_0 \delta\tau \cos (\omega_m \delta\tau/2) \sin [\omega_m(t - \delta\tau/2)] \quad (33)$$

The IF power level is then

$$P_I = 2k^2(\Delta\omega/\omega_m)^2 E_0^2/Z_0 \sin^2 \omega_0 \delta\tau \sin (\omega_m \delta\tau/2) \quad (34)$$

The detected power relative to the single sideband RF level is then

$$\eta_{FM} = 16L \sin^2 \omega_0 \delta\tau \sin^2(\omega_m \delta\tau/2)$$

If we now write $\phi = \omega_0 \delta\tau$, which is the equivalent of the phase shift term in eqn. 27, then

$$\eta_{FM} = 4L \sin^2 \phi 4 \sin^2(\omega_m \delta\tau/2) \quad (35)$$

The first term on the right-hand side of eqn. 35 is the phase-sensitive term equivalent to the expression in eqn. 29 except that the FM noise is detected with the local oscillator in quadrature with the RF leakage signal, whereas the AM noise is detected when the two are in phase. This is a consequence of the fact that the upper and lower AM sidebands are in phase whereas the FM sidebands are in antiphase.

The second term in the eqn. 35 is the so-called FM noise cancellation. If the time delay $\delta\tau$ is much less than the modulation rate $(\omega_m/2\pi)$ then, because the variations are the same on both the leakage signal and on the local oscillator, they are strongly correlated and no noise output is observed at the IF. As the time delay increases, the degree of correlation for a given modulation rate decreases, and more noise is seen at IF.