# Traço de Tensores:

$$Tr : T^n_m(V) \longrightarrow T^{n-1}_{m-1}(V)$$

• Ex: $T^1_1 \simeq End(V)$

$T \in T^1_1(V) := T_j^i (\phi_j \otimes \phi^i)$

$Tr : T^1_1(V) \to \underbrace{T^0_0(V)}_{\mathbb{R}}$

## Definição:

O traço é a única operação linear em $T^1_1(V) \to \mathbb{R}$ tal que:

Se $T = v \otimes \beta$ onde $v \in V$ e $\beta \in V^*$

$$Tr(v \otimes \beta) = \beta(v)$$

• Linear: $T = T_j^i \, \phi_i \otimes \phi^i$

$$\mathrm{Tr}(T) = \mathrm{Tr}(T^i_j\, \phi_i \otimes \phi^j)$$
$$= T^i_j\, \mathrm{Tr}(\phi_i \otimes \phi^j)$$
$$= T^i_j\, \phi^j(\phi_i) = T^i_i$$

Generalização

$$\mathrm{Tr}^i_j : T^n_m(V) \longrightarrow T^{n-1}_{m-1}(V)$$

$$\mathrm{Tr}(v_1 \otimes \cdots \otimes v_i \otimes \cdots \otimes v_m \otimes \beta^1 \otimes \cdots \otimes \beta^j \otimes \cdots \otimes \beta^n)$$

Omitidos

$$= (v_1 \otimes \cdots \otimes \hat{v}_i \otimes v_{i+1} \otimes \cdots \otimes v_n \otimes \beta^1 \otimes \cdots \otimes \hat{\beta}^j \otimes \cdots \otimes \beta^n)$$
$$\beta^j(v_i)$$

Exemplo:

$$T = T^{ij}_{klm}\, \phi_i \otimes \phi_j \otimes \phi^k \otimes \phi^l \otimes \phi^m$$
$$\mathrm{Tr}^1_1(T) = T^{ij}_{ilm}\, \phi_j \otimes \phi^l \otimes \phi^m$$