

Levi-Civita

$$\epsilon_{ijk} = \begin{cases} 1, & (i,j,k) \text{ cíclicos} \\ -1, & (i,j,k) \text{ anticíclicos} \\ 0, & \text{índice repetido} \end{cases}$$

$$\epsilon^{ijk} \epsilon_{klm} = \delta^i_l \delta^j_m - \delta^i_m \delta^j_l$$

Propriedade

• Considerando \mathbb{R}^3 , a base $\{\phi_1, \phi_2, \phi_3\}$

$$\det(\phi_i, \phi_j, \phi_k) = \begin{vmatrix} \delta_{i1} & \delta_{j1} & \delta_{k1} \\ \delta_{i2} & \delta_{j2} & \delta_{k2} \\ \delta_{i3} & \delta_{j3} & \delta_{k3} \end{vmatrix} = \epsilon_{ijk}$$
$$\left[\phi_1 = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, \phi_2 = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}, \phi_3 = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \right]$$

Produto Vetorial entre dois vetores:

$$(\vec{v} \times \vec{w})^k = \epsilon_{ij}^k v^i w^j \quad \leftarrow \begin{matrix} \text{componente} \\ \text{do prod. vetorial} \end{matrix}$$

$$\langle \vec{v} \times \vec{w}, \vec{z} \rangle = \epsilon_{ijk} v^i w^j z^k$$

$$\begin{aligned}
 \cdot \quad \vec{A} \times \vec{B} \times \vec{C} &= \epsilon_{j k l}^i \epsilon_{l m n}^k A^j B^l C^m \\
 &= (\delta_{i m}^j \delta_{j n}^k - \delta_{i n}^j \delta_{j m}^k) A^j B^l C^m \\
 &= B^i (\vec{A} \cdot \vec{C}) - C^i (\vec{A} \cdot \vec{B}) \quad \text{✓}
 \end{aligned}$$