Solutions of Introduction to Stochastic Process

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Preliminaries

0.1

Find all functions x(t), y(t) satisfying

$$x'(t) = y(t) - x(t)$$

$$y'(t) = 3x(t) - 3y(t)$$

Find the particular pair of functions satisfying $x(0) = y(0) = 1/2$. **Solution:**
Let $\mathbf{Y} = \begin{bmatrix} x \\ y \end{bmatrix}$, and $A = \begin{bmatrix} -1 & 1 \\ 3 & -3 \end{bmatrix}$, so rewrite original equations to $\mathbf{Y}' = A\mathbf{Y}$
$\left|A - \lambda I\right| = \begin{vmatrix} -1-\lambda & 1 \\ 3 & -3-\lambda \end{vmatrix}$, or $\lambda^2 + 4\lambda + 3 - 3 = 0 \Rightarrow \lambda^2 + 4\lambda = 0$, eigenvalues are $\lambda_1 = 0$, and $\lambda_2 = -4$
for λ_1, $(A - \lambda_1 I)\mathbf{Y} = \begin{bmatrix} -1 & 1 \\ 3 & -3 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \mathbf{0} \Rightarrow \begin{cases} -x + y = 0 \\ 3x - 3y = 0 \end{cases}$
we have $\mathbf{x_1} = (1,1)^T$ is a eigenvector for λ_1
for λ_2, $(A - \lambda_1 I)\mathbf{Y} = \begin{bmatrix} 3 & 1 \\ 3 & -7 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \mathbf{0}$ yield $\mathbf{x_2} = (0,0)^T$ is eigenvector for λ_2.
so, $\mathbf{Y} = c_1 e^{\lambda_1 t}\mathbf{x_1} + c_2 e^{\lambda_2 t}\mathbf{x_2} = c_1 e^{0 \cdot t}\mathbf{x_1} = c_1\mathbf{x_1} = \begin{bmatrix} c_1 \\ c_1 \end{bmatrix}$

Part I

Limit

Chapter 1

2

1.1 a

1.1.1 b

Appendix A

First Appendix

Last note