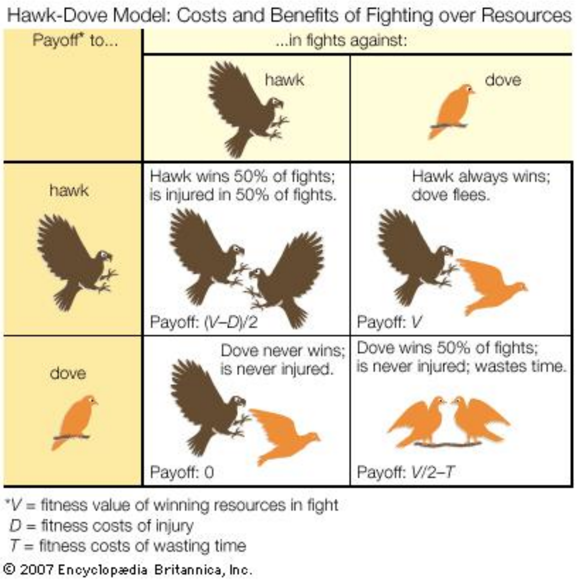
Assignment 1: Game theory

General remarks:
_ Deadline 3-11-2016
_ Mail your results to Elias Fernandez <eliasfernandez.d@gmail.com>.
_ Provide a single (self-contained) *.PDF file.
_ Put your name and your affiliation (VUB/ULB) both on the document and in the file name.

The Hawk-Dove game



The hawk-Dove game is a coordination game formulated by John Maynard Smith and Georg Price. The aim of the game was to understand the resolution of conflicts by fighting in the animal kingdom. The game consists of two players, which each have the choice between two possible actions; either they take time to display (dove) before fighting or they can escalate immediately and fight (hawk). When both players escalate (hawk), they have a 50% risk of being injured (-D/2) and 50% of wining (V/2). When a dove fights another dove she also wins 50% of the time (V/2) but only after a period of mutual displays to show of strength (-T). Hawks always win against doves, resulting in a benefit for one (V) and not for the other (0).

1. Find all the (mixed strategy) Nash equilibria of this game. How do the results change when the order of the parameters V, D and T is changed (V>D, D>T, etc.)?
2. Under which conditions does displaying become more beneficial than escalating?

Which social dilemma?

Player A knows he's confronted with one of three social dilemma's; a prisoner's dilemma, a snowdrift game or stag-hunt game (see above). In each game he needs to decide whether to cooperate (C) or defect (D), yet he is not sure in which he actually is. He's sure that each game is equally likely. The other player,

player B, knows in which game he's playing. Determine the pure Nash equilibria using the Bayesian game analysis discussed in the course.

Prisonners dilemma

	C	D
C	2,2	0,5
D	5,0	1,1

Stag-Hunt game

	C	D
C	5,5	0,2
D	2,0	1,1

Snowdrift game

	C	D
C	2,2	1,5
D	5,1	0,0

Sequential truel

Each of persons A, B, and C has a gun containing a single bullet. Each person, as long as she is alive, may shoot at any surviving person. First A can shoot, then B (if still alive), then C (if still alive).

Denote by p_i the probability that player i hits her intended target; assume that $0 < p_i < 1$. Assume that each player wish to maximize her probability of survival; among outcomes in which her survival probability is the same, she wants the danger posed by any other survivors to be as small as possible.

Model this situation as an extensive game with perfect information and chance moves. (Draw the diagram. Note that the sub-games following histories in which A misses her intended target are the same).
Find the subgame perfect equilibria of the game. (Consider only cases in which p_A, p_B, and p_C are all different.) Explain the logic behind A's equilibrium action.
Show that "weakness is strength" for C: she is better off if $p_C < p_B$ than if $p_C > p_B$.

Success!