Solⁿ: (4): We have $P-Q = \dfrac{\cos x + \sin x - e^{-y}}{2\cos x - e^{y} - e^{-y}}$

$$P-Q = \frac{1}{2}\left[1 + \frac{2\cos x + 2\sin x - 2e^{-y}}{2\cos x - e^{y} - e^{-y}} - 1\right]$$

$$= \frac{1}{2}\left[1 + \frac{2\sin x + e^{y} - e^{-y}}{2\cos x - e^{y} - e^{-y}}\right]$$

or, $$P-Q = \frac{1}{2}\left[1 + \frac{\sin x + \sinh y}{\cos x - \cosh y}\right] \longrightarrow (1)$$

Differentiating (1) w.r.t. x and y partially, we have

$$\frac{\partial P}{\partial x} - \frac{\partial Q}{\partial x} = \frac{1}{2}\left[\frac{\cos x(\cos x - \cosh y) + \sin x(\sin x + \sinh y)}{(\cos x - \cosh y)^2}\right]$$

$$= \frac{1}{2}\left[\frac{1 - \cos x\cosh y + \sin x\sinh y}{(\cos x - \cosh y)^2}\right] \longrightarrow (2)$$

and $$\frac{\partial P}{\partial y} - \frac{\partial Q}{\partial y} = \frac{1}{2}\left[\frac{\cosh y(\cos x - \cosh y) + \sinh y(\sin x + \sin y)}{(\cos x - \cosh y)^2}\right]$$

or, $$-\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial x} = \frac{1}{2}\left[\frac{\cos x\cosh y + \sin x\sinh y - 1}{(\cos x - \cosh y)^2}\right] \longrightarrow (3)$$

Solving (2) and (3), we get

$$\frac{\partial P}{\partial x} = \frac{1}{2}\,\frac{1 - \cos x\cosh y}{(\cos x - \cosh y)^2} = \phi_1(x,y)$$

and $$\frac{\partial Q}{\partial x} = -\frac{1}{2}\,\frac{\sin x\sinh y}{(\cos x - \cosh y)^2} = \phi_2(x,y).$$

$$\therefore f'(z) = \frac{\partial P}{\partial x} + i\frac{\partial Q}{\partial x} = \phi_1(z,0) + i\,\phi_2(z,0)$$

$$= \frac{1}{2}\,\frac{1-\cos z}{(\cos z - 1)^2} = \frac{1}{2(1-\cos z)}.$$

$$\therefore f(z) = \frac{1}{4}\int \operatorname{cosec}^2\frac{z}{2}\,dz + c = -\frac{1}{2}\cot\frac{z}{2} + A.$$

When $z = \pi/2$, $f(z) = 0$, $\therefore A = \frac{1}{2}$

Hence, $f(z) = \frac{1}{2}\left(1 - \cot \frac{z}{2}\right)$. Ans.