Subspace State Space Identification

Hardik Patel (202103032)

June 2024

Introduction: Subspace Identification

- Subspace Identification means Decoding System Behavior from Data
- System Identification: This approach creates a mathematical model of a system based solely on its input (u) and output (y) data.
- System identification offers an alternative approach, where the model is built directly from data collected from the actual system's operation. This eliminates the need for complete knowledge of the system's internal workings and allows us to model complex systems where deriving physical equations might be challenging.

Motivation and A New Approach

$$x(t+1) = Ax(t) + Bu(t) + w(t) \tag{1}$$

$$y(t) = Cx(t) + Du(t) + v(t) \tag{2}$$

- where x(t) is the state vector at time t, u(t) is the input vector, and y(t) is the output vector. A, B, C, and D are matrices that define the relationships between these variables
- State vector (x): Captures the system's internal variables that influence its behavior (e.g., car's speed, engine temperature).
- State space equations: Describe how the system's state evolves over time based on inputs and outputs. (Equation 1)
- Subspace Identification at Work: Analyzes input-output data to extract the hidden states and build the state-space model.

Orthogonal/Oblique Projections:

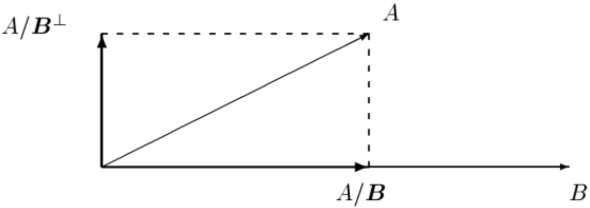
$$\frac{A}{B} = AB^{\dagger}B = AB^{T}(BB^{T})^{-1}B \tag{3}$$

$$\frac{A}{B^{\perp}} = A - \frac{A}{B} \tag{4}$$

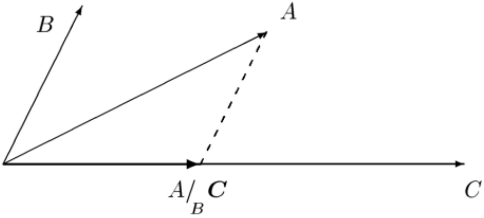
$$\frac{B}{B^{\perp}} = 0 \tag{5}$$

- Orthogonal Projections: Imagine a shadow cast by an object (the system) onto a flat surface. Orthogonal projection isolates the part of the shadow that falls directly on the surface, effectively removing irrelevant information (noise) and highlighting the core structure of the object.
- Orthogonal projection helps us to Filtering the Noise to Reveal the Signal while oblique projections can distort the system's dynamics, making them less suitable for building accurate models.

Orthogonal/Oblique Projections:



(a) Orthogonal Projection



(b) The oblique projection of rowspace of A along the rowspace of B on the row space of C

Figure 1: Projection of vectors