CS101 Discrete Mathematics

Maximum Flow Problem

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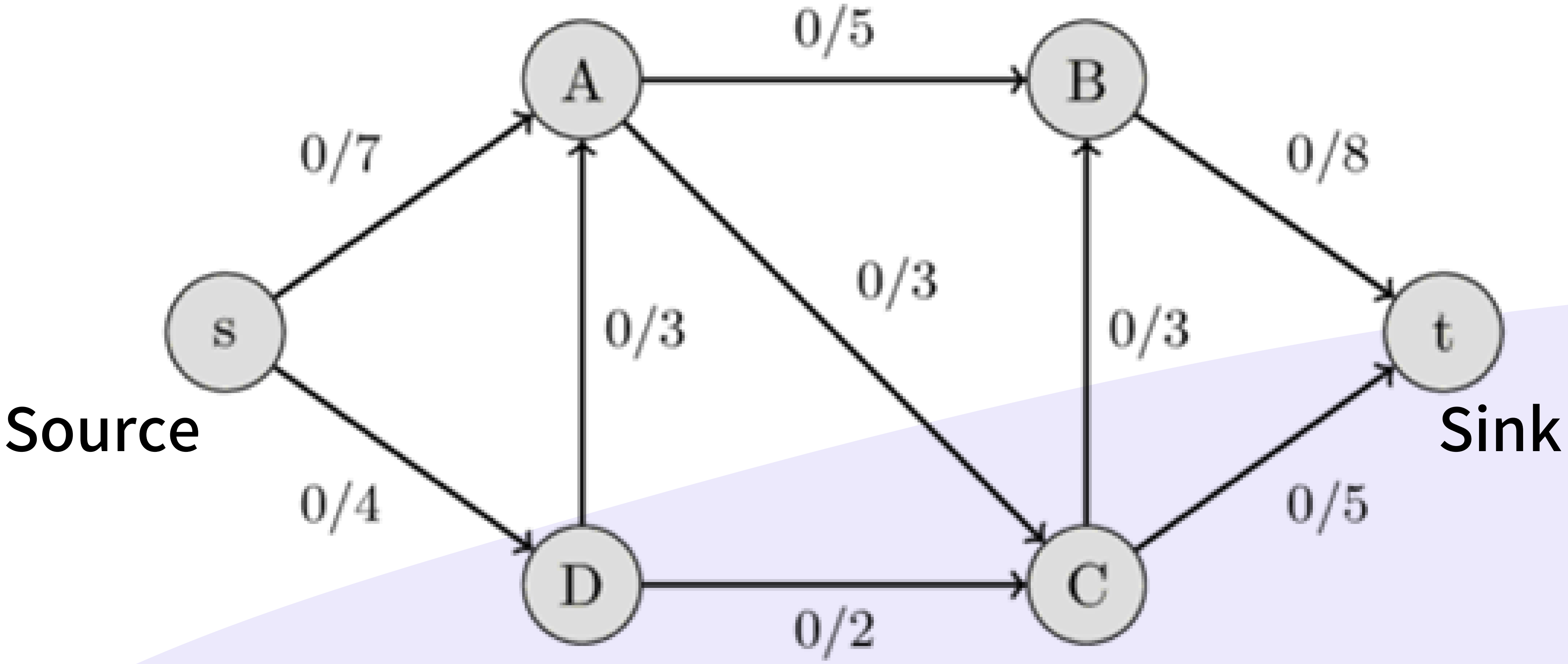
Problem Description

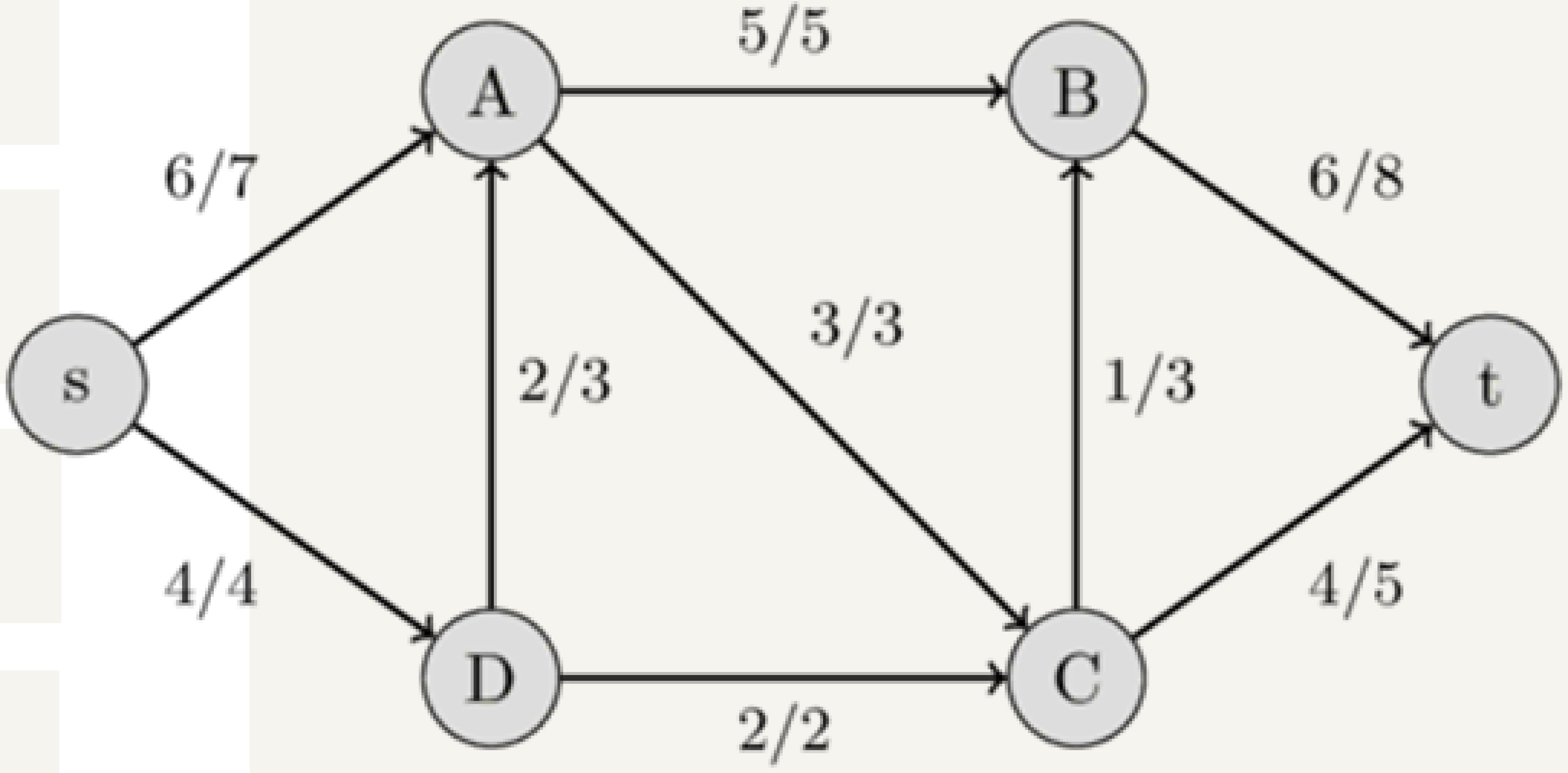
The problem involves determining the most amount of flow that can be sent via a system of pipes, channels and other paths while taking into account the capacity restrictions.



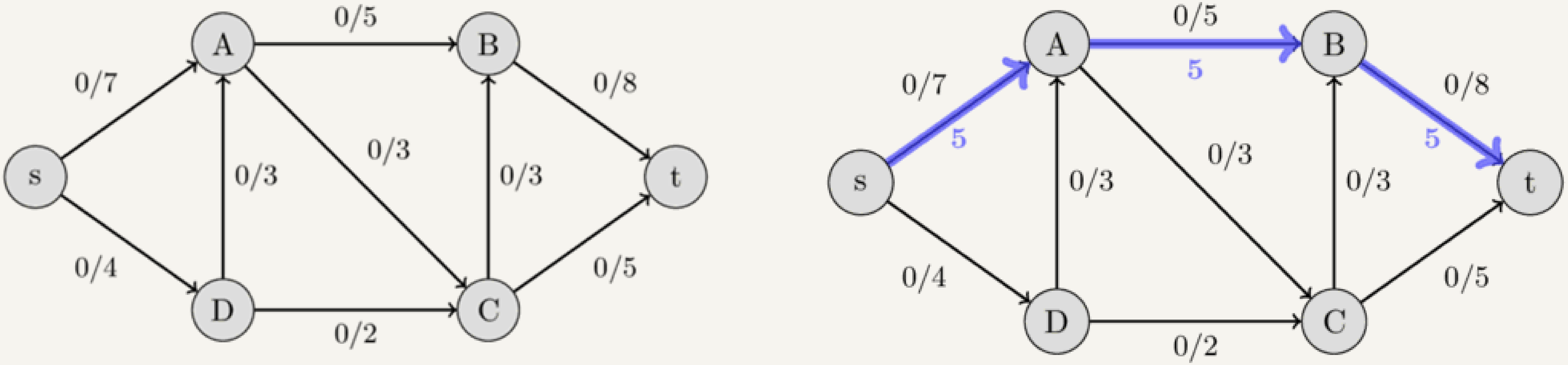
The Directed edge from S to A has maximum capacity of 7 and 0 depicts the amount of flow from the edge. There can be numerous paths to follow to reach sink from source.

How to find the Maximal Flow?

1. Ford-Fulkerson's Algorithm
2. Edmonds-Karp's Algorithm
3. Dinic's Algorithm
4. Capacity Scaling Heuristic

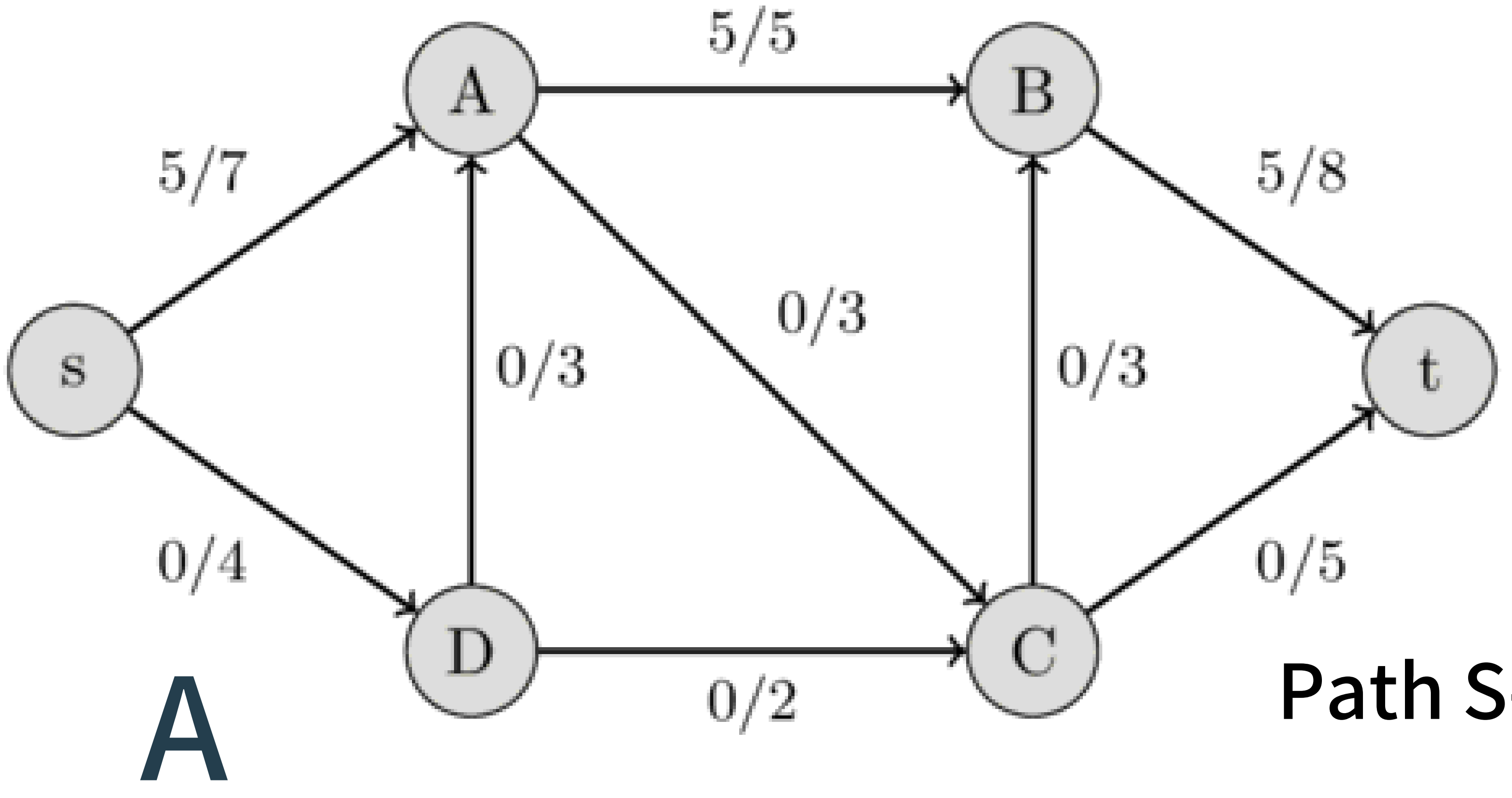


Ford–Fulkerson Algorithm

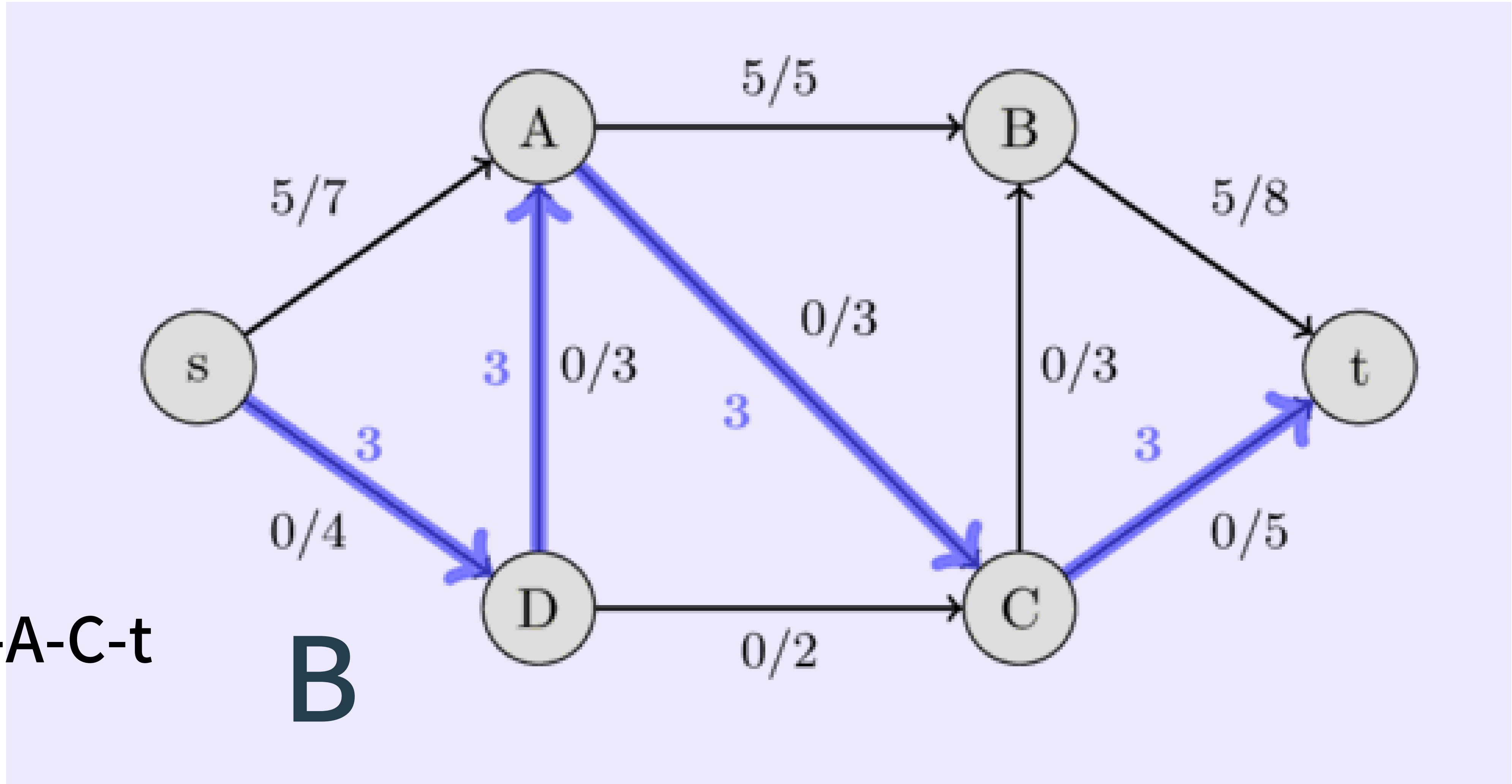


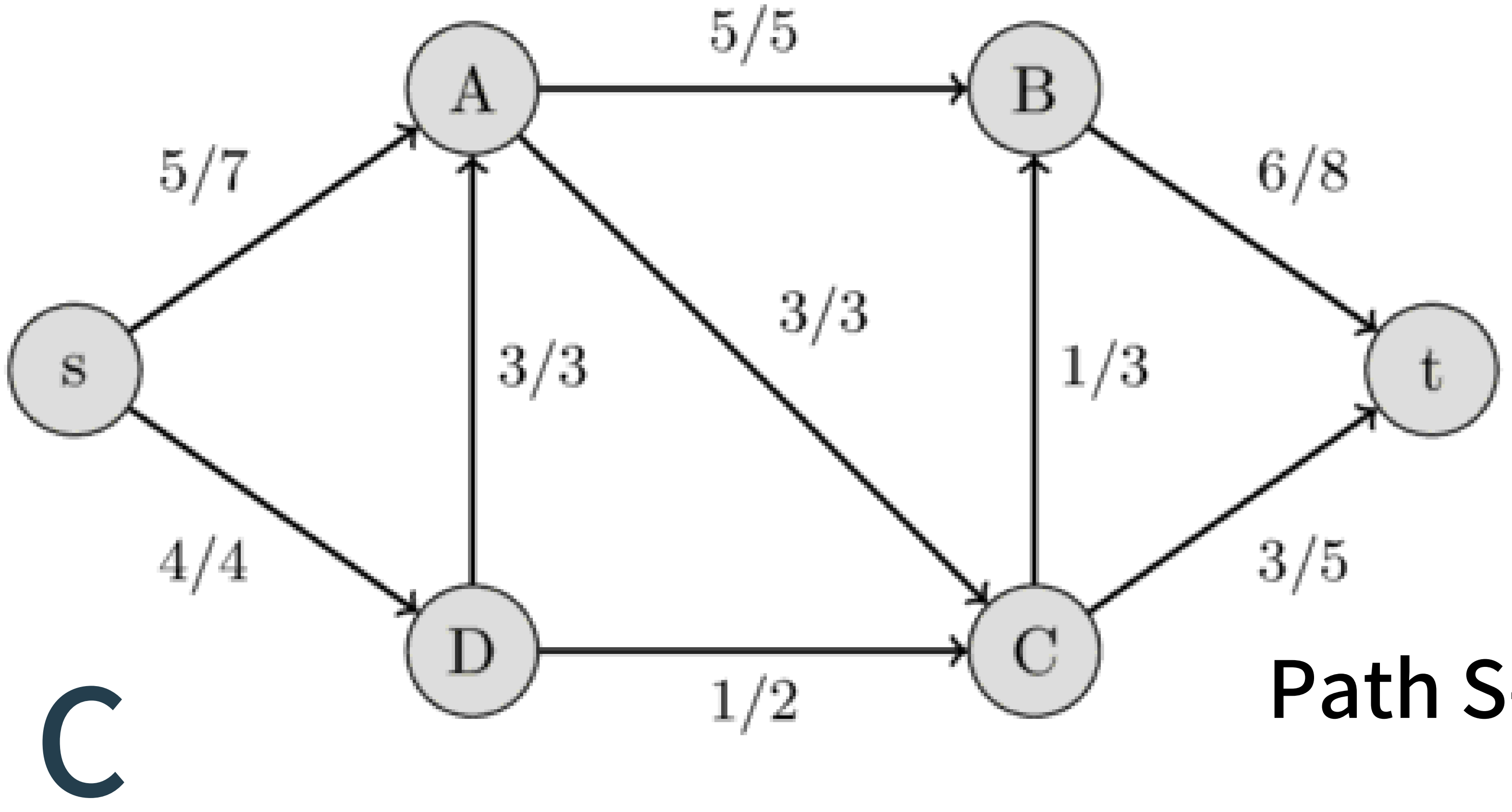
Path S-A-B-t

Uses DFS to find augmenting paths

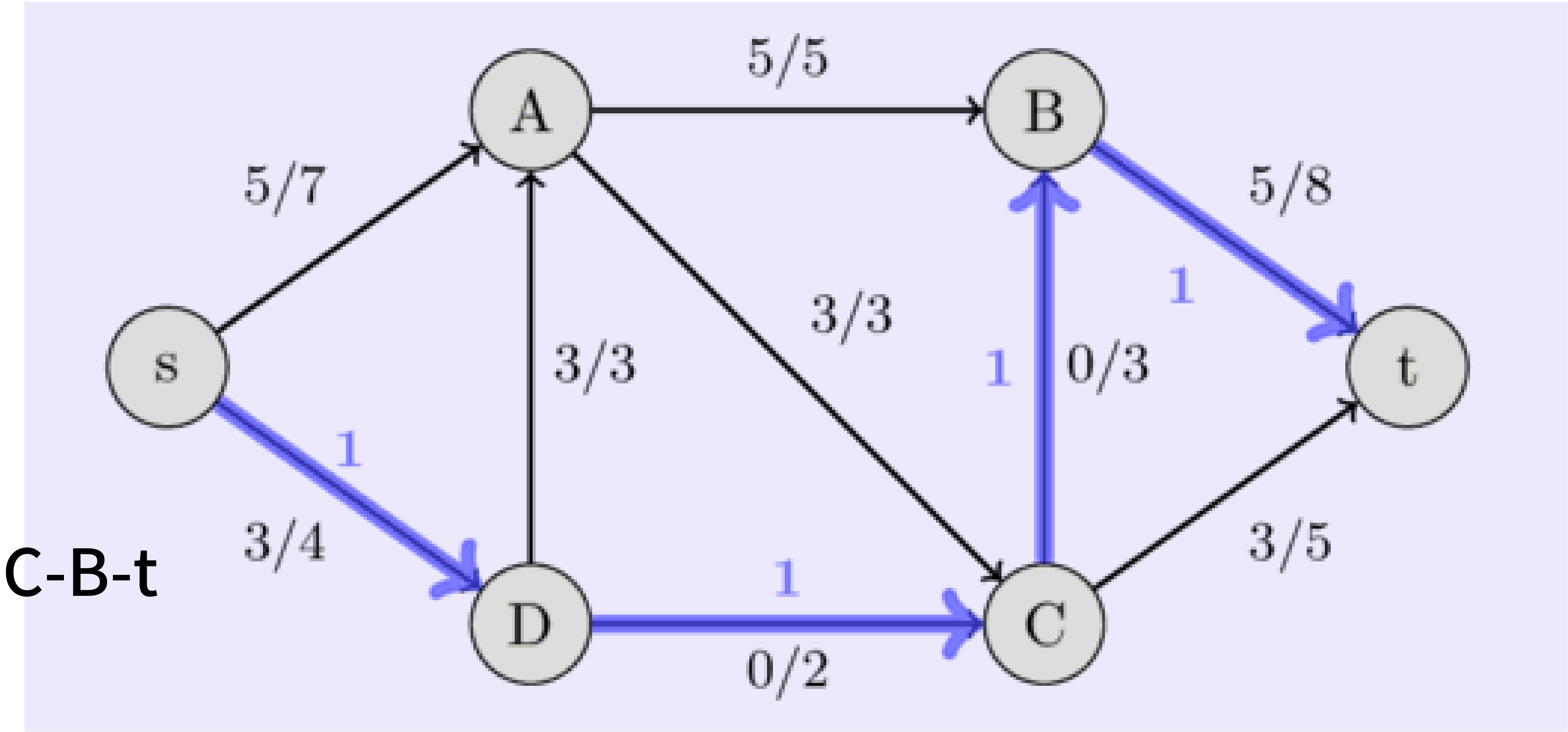


Path S-D-A-C-t

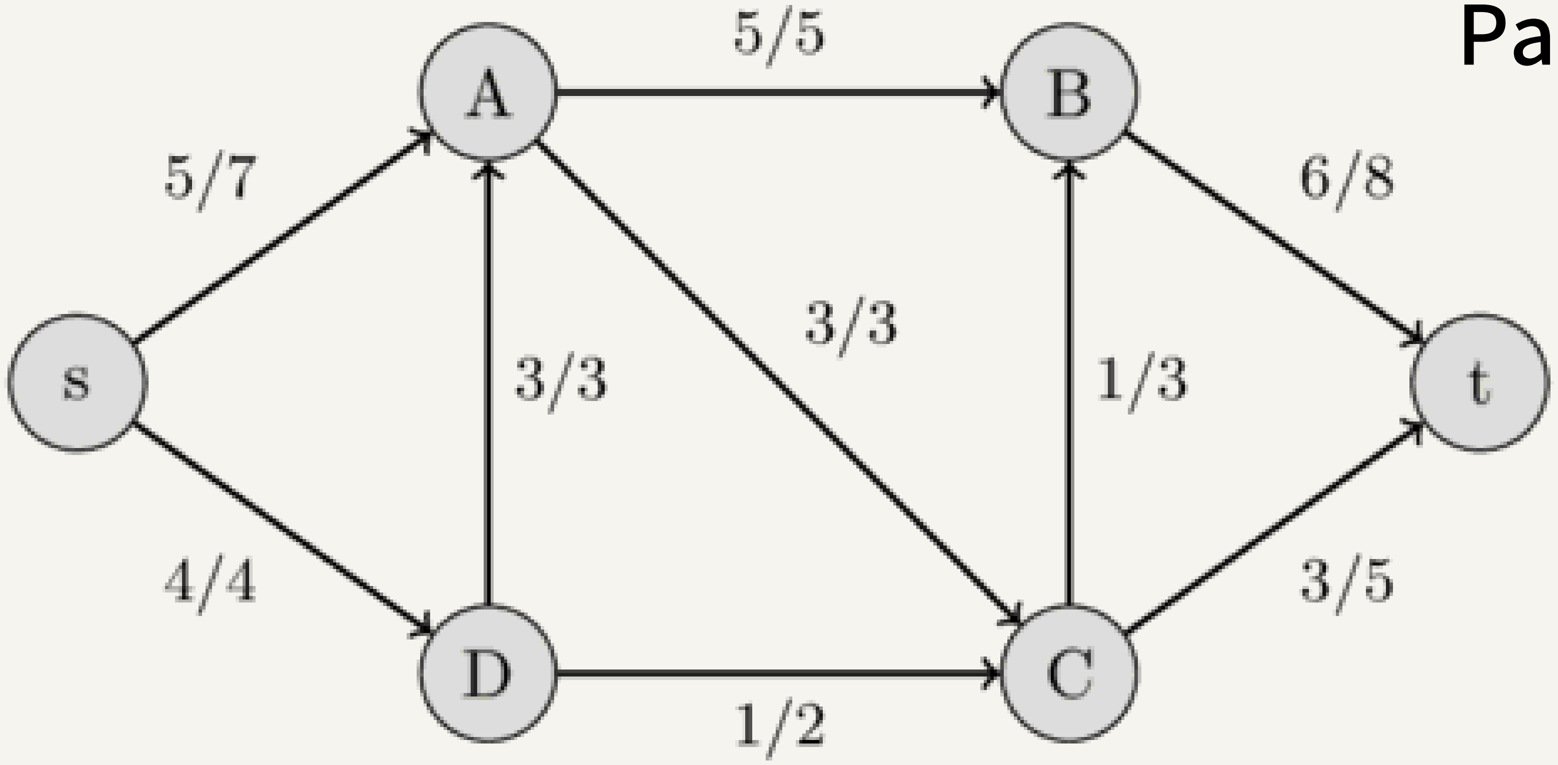




Path S-D-C-B-t



Path S-A-D-C-t



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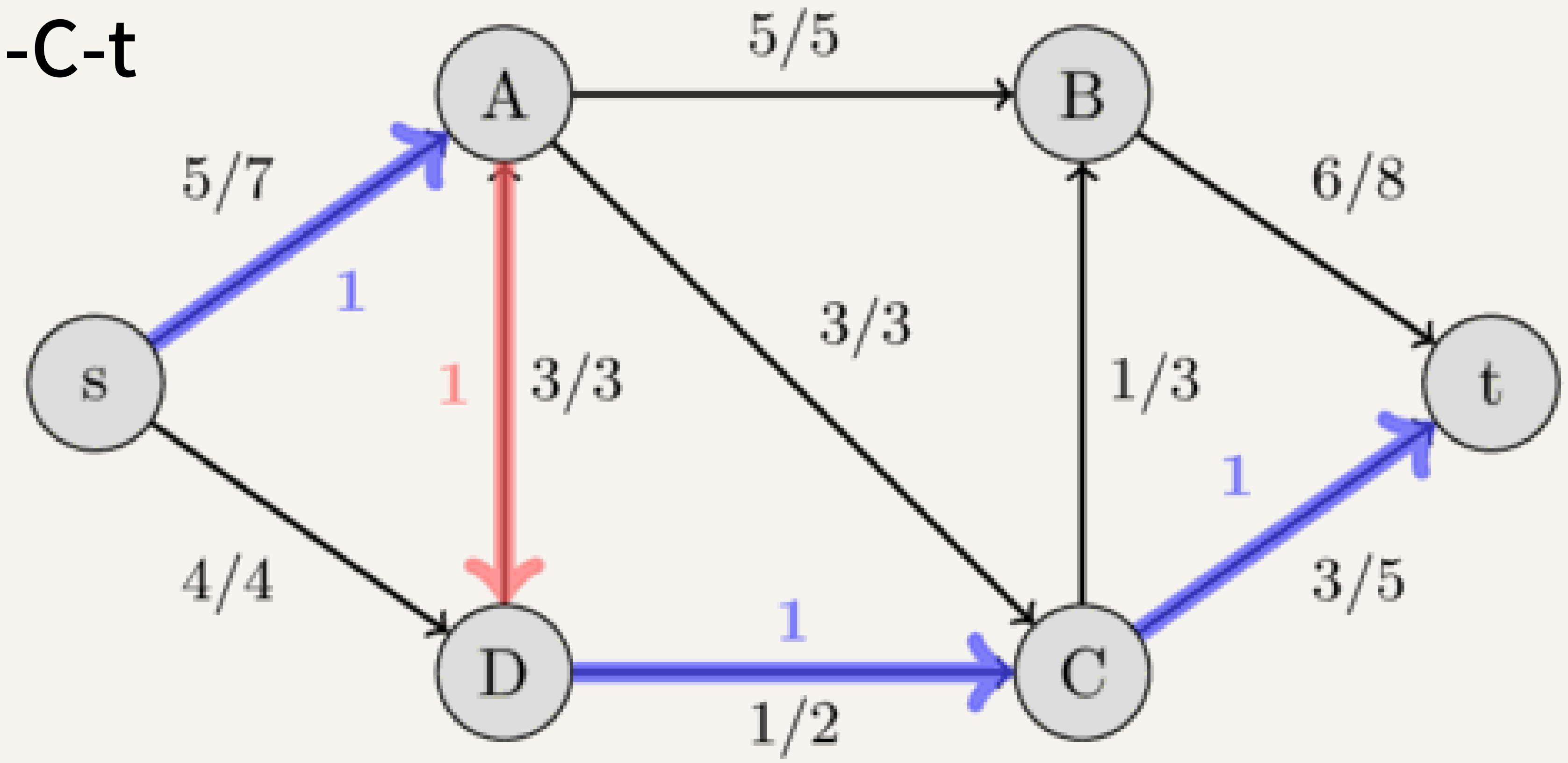

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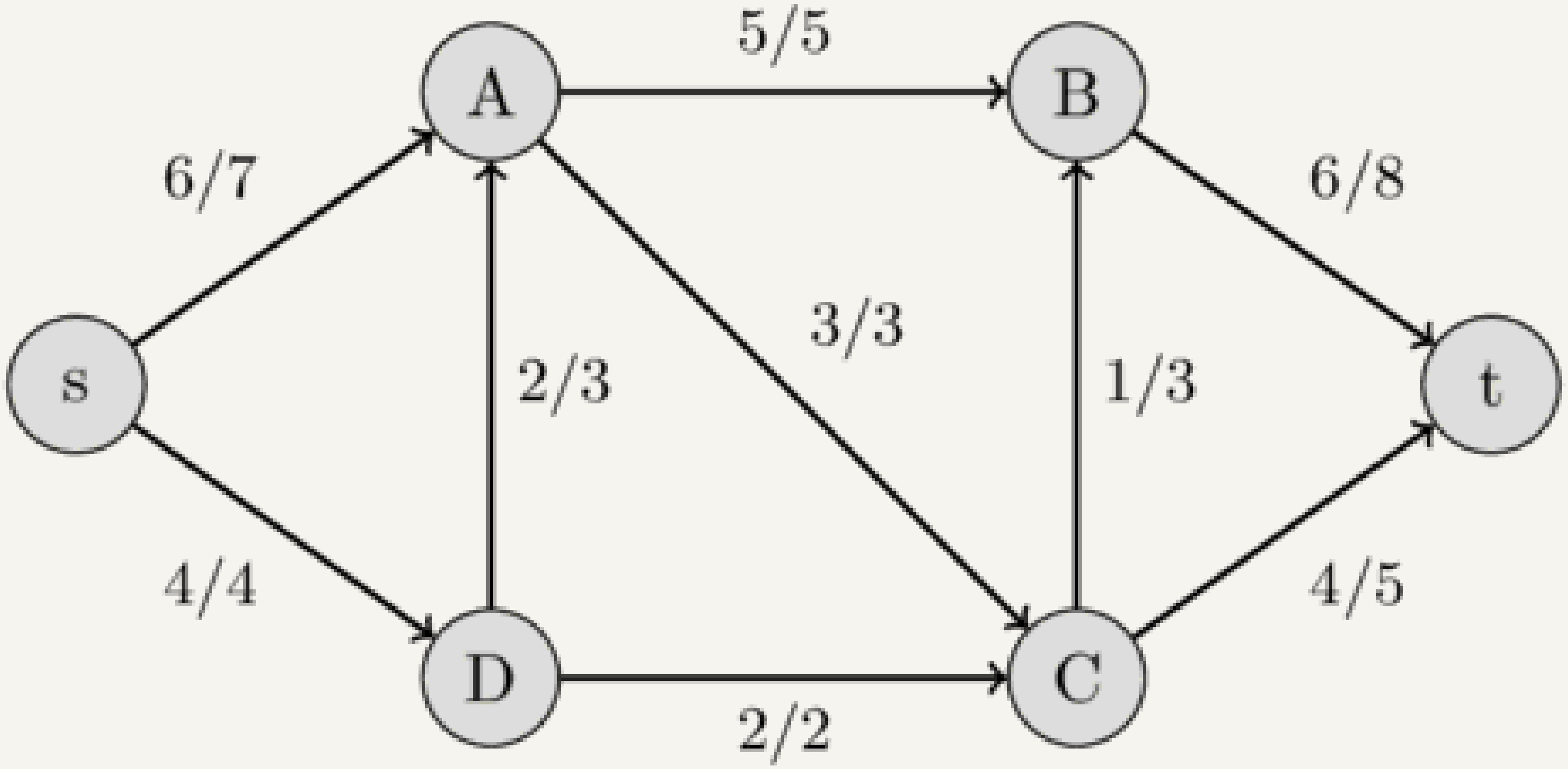

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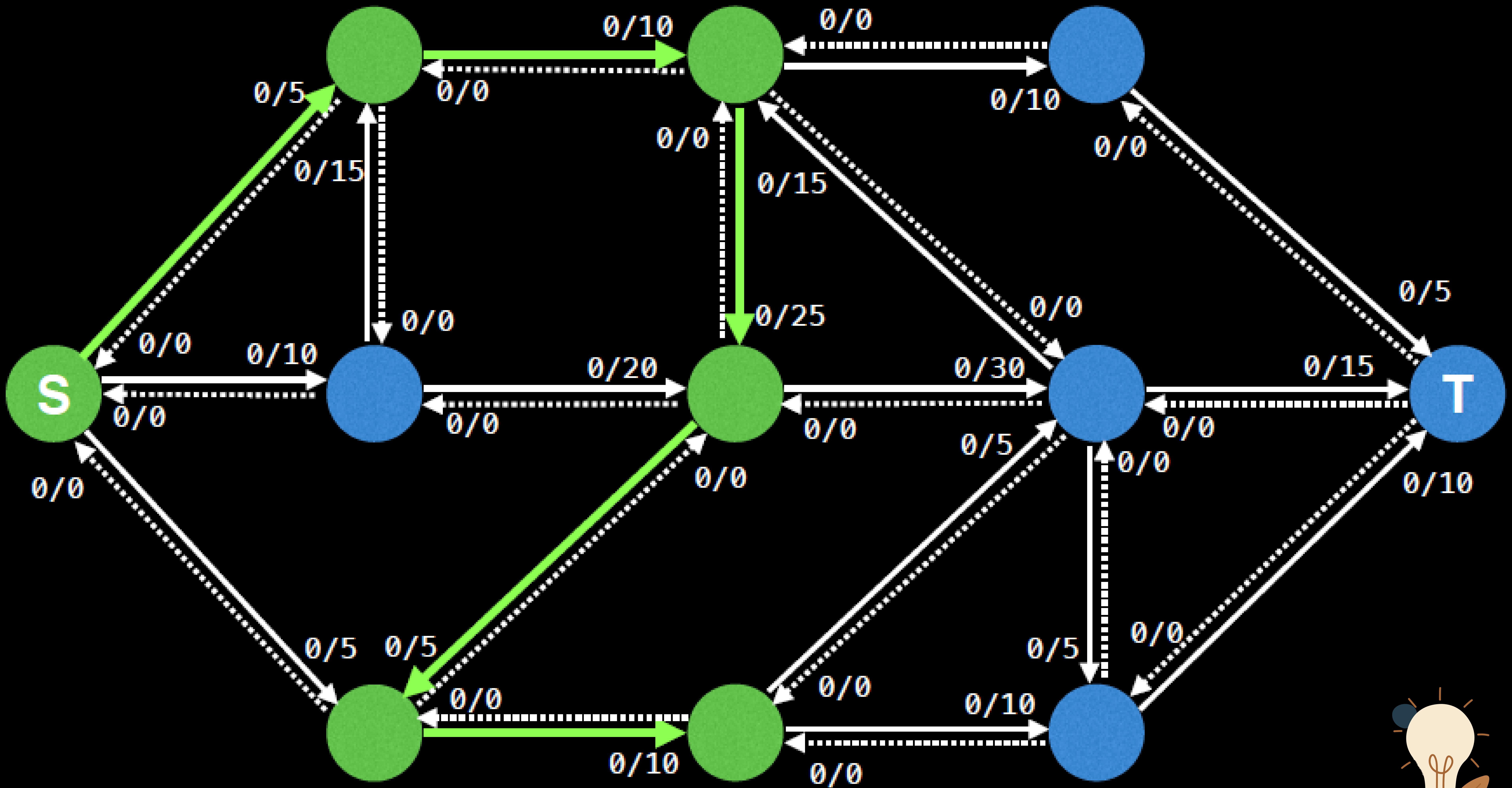
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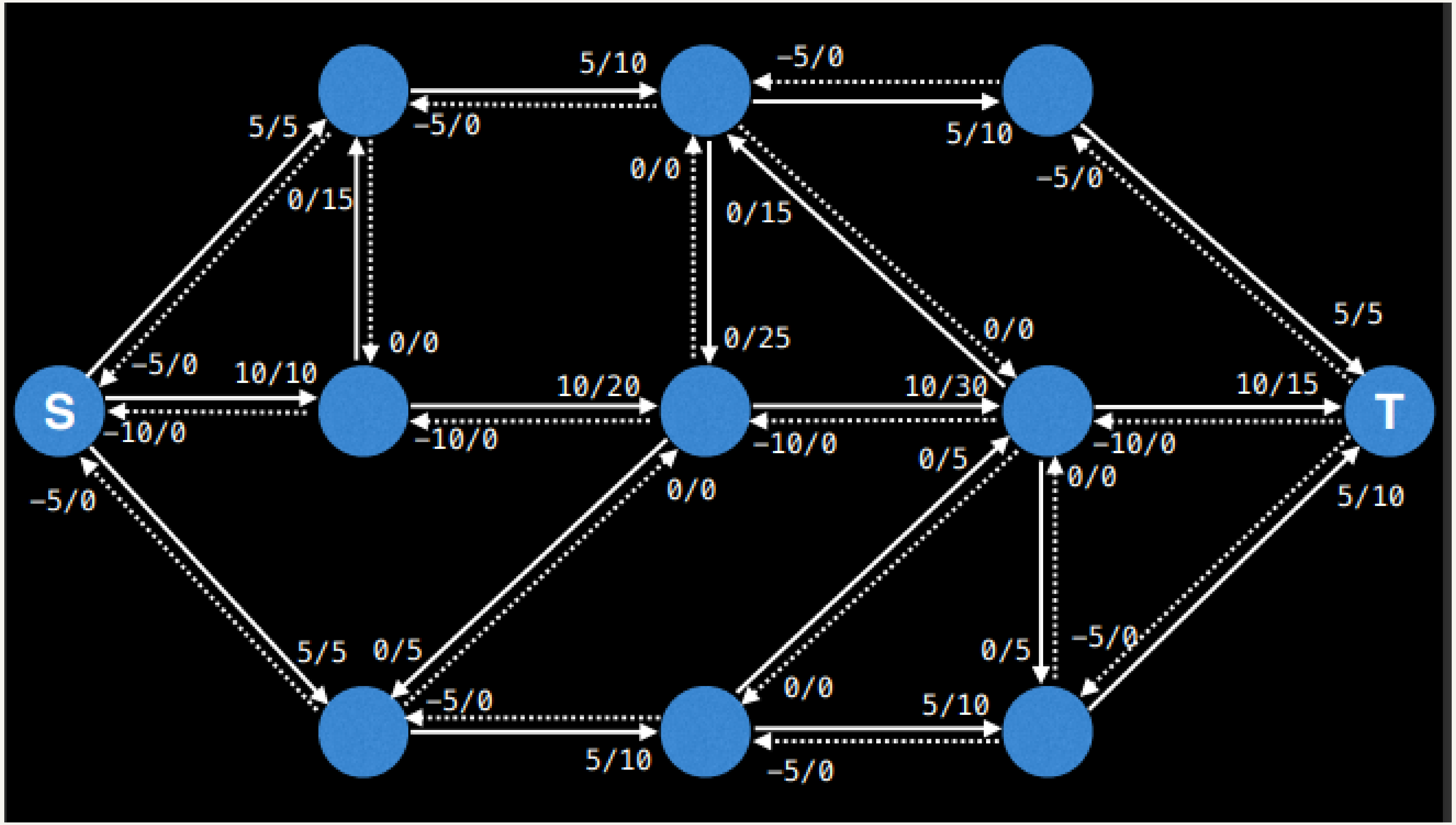
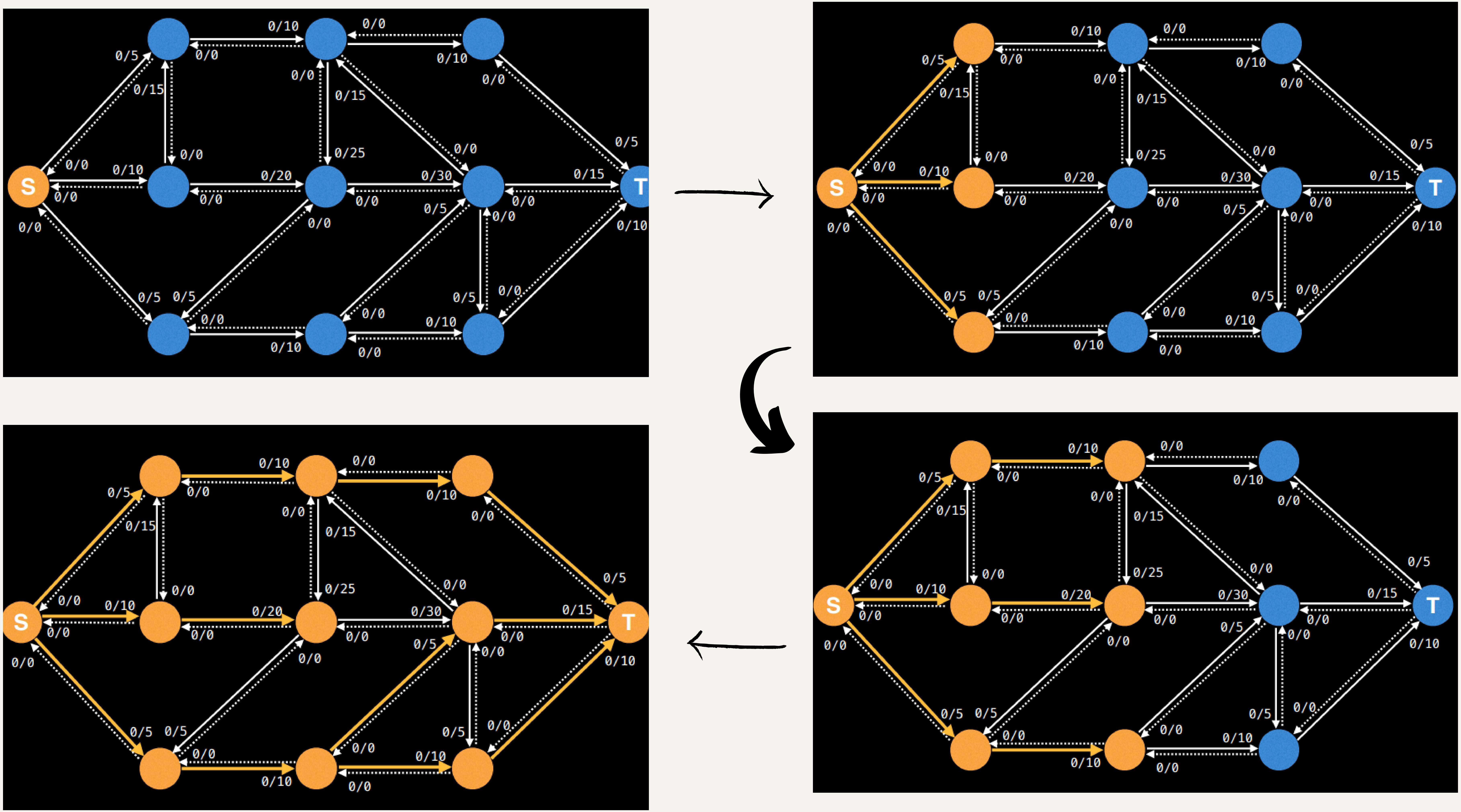
Edmonds Karp Algorithm

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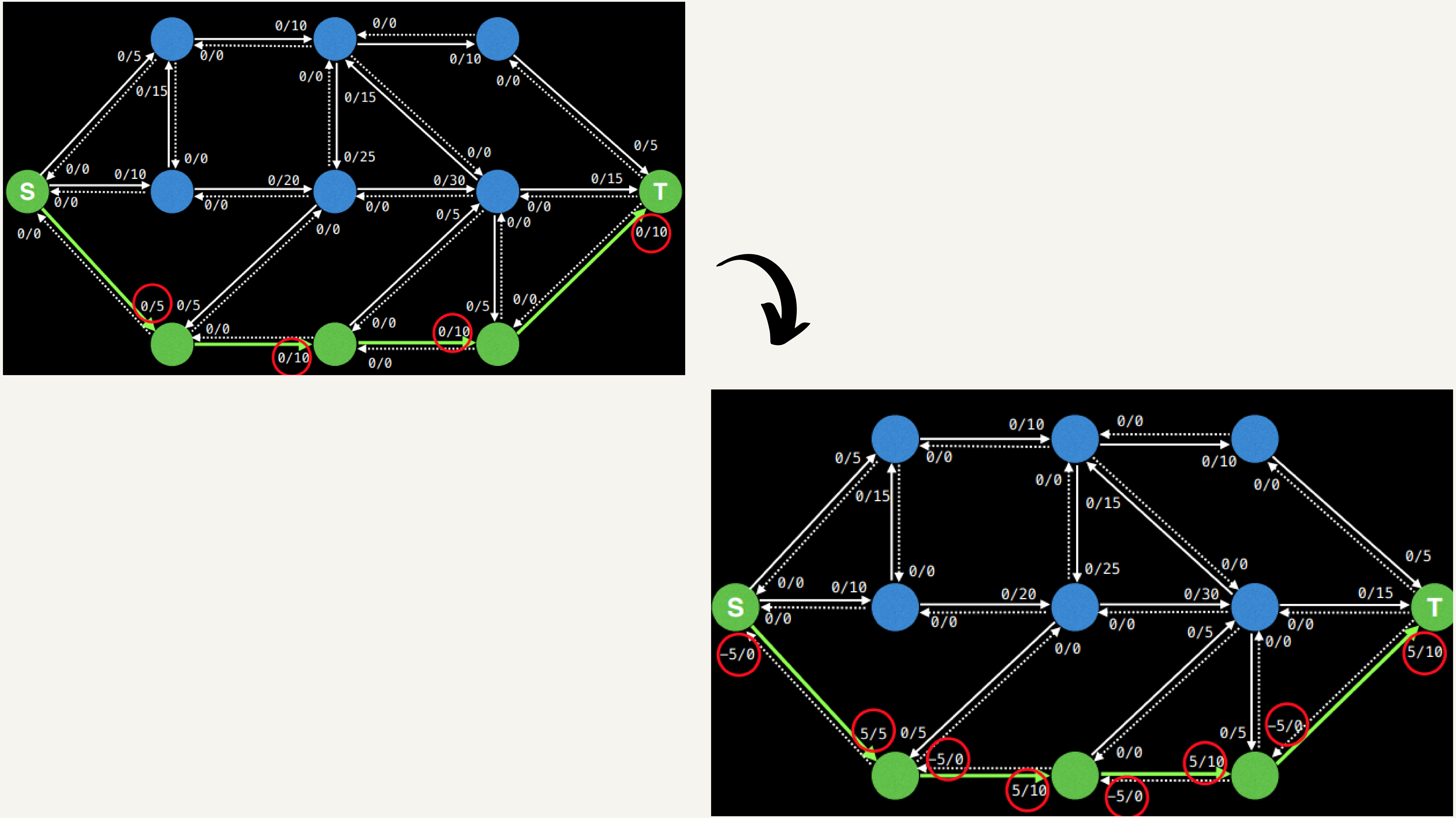

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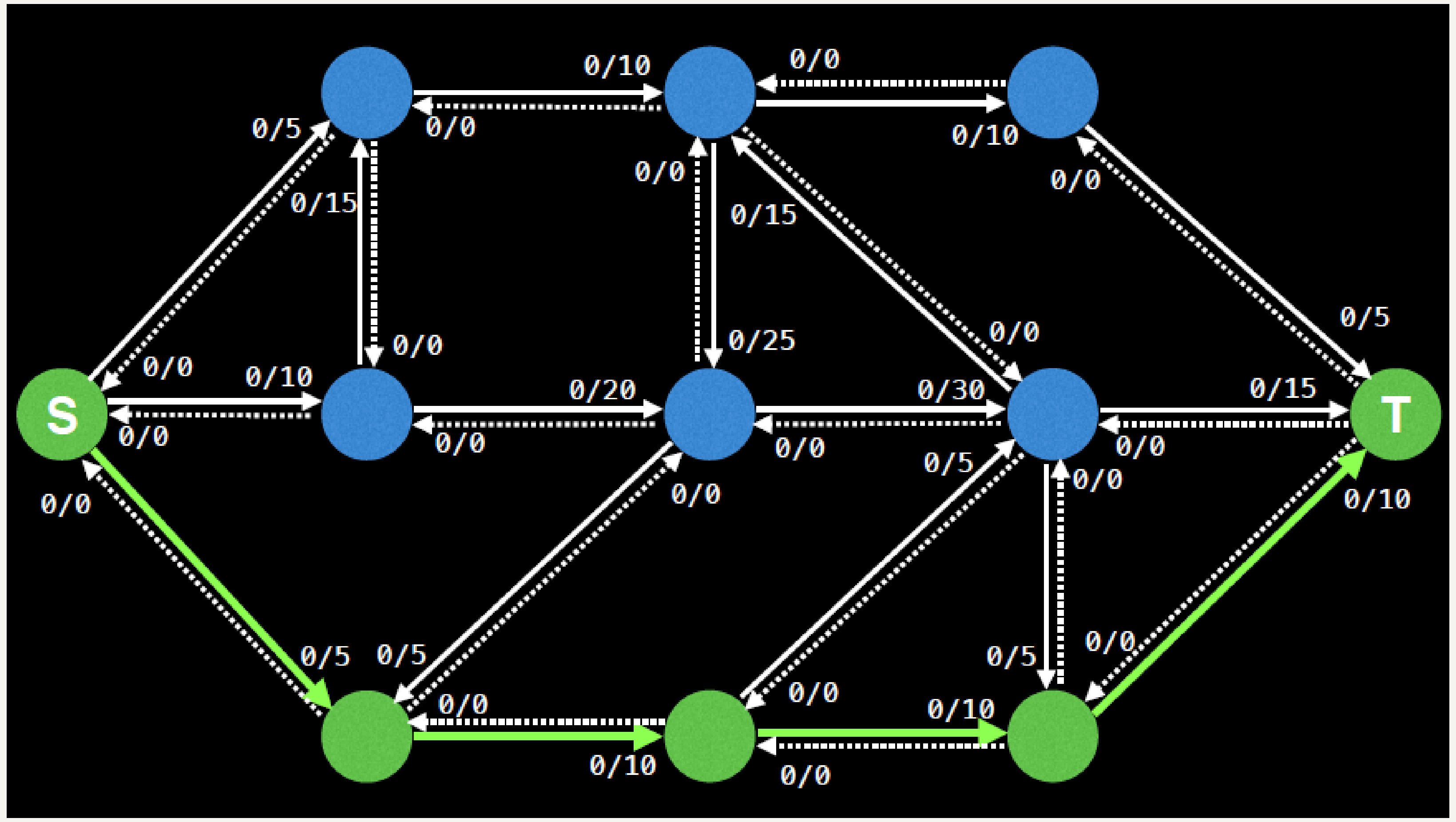
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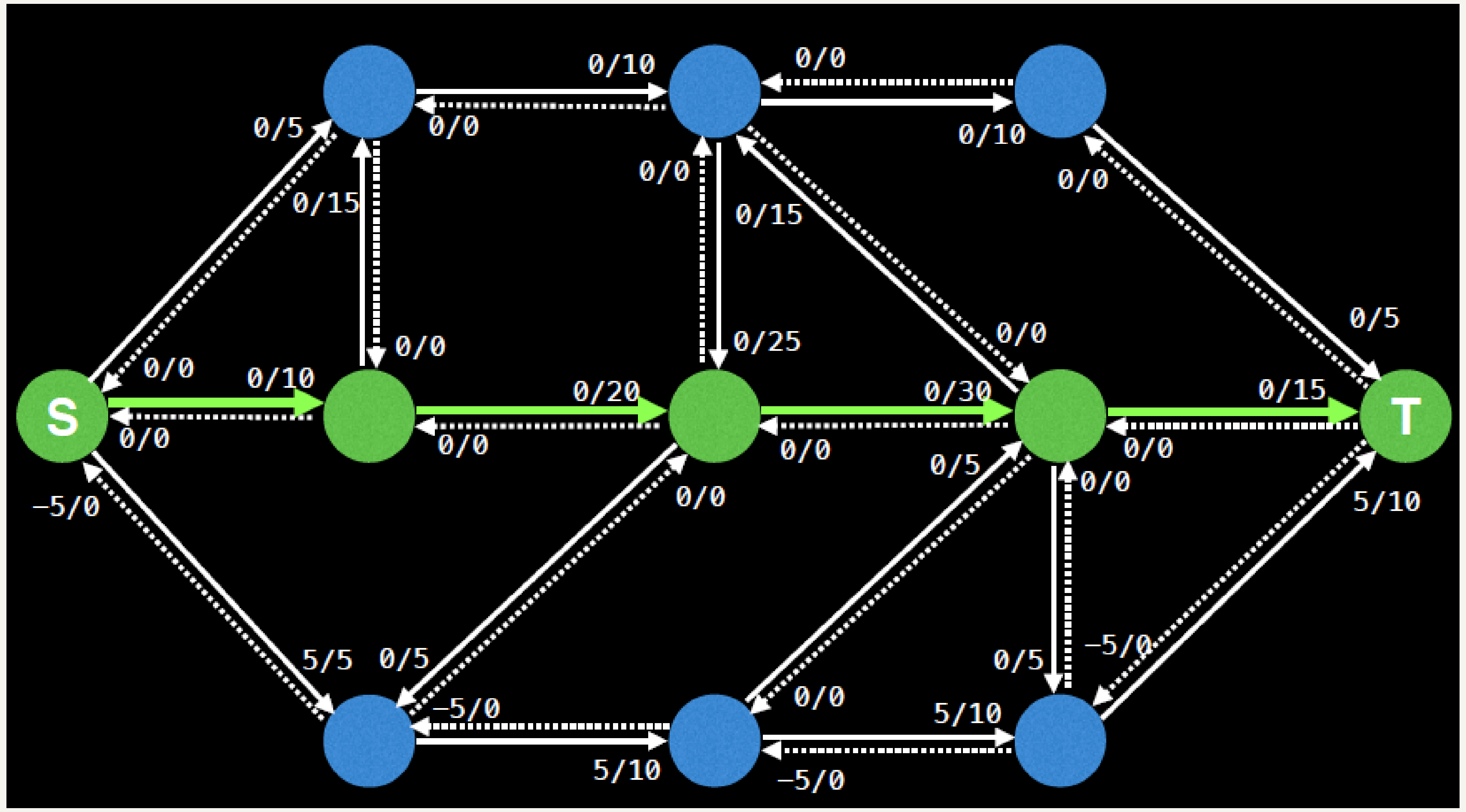
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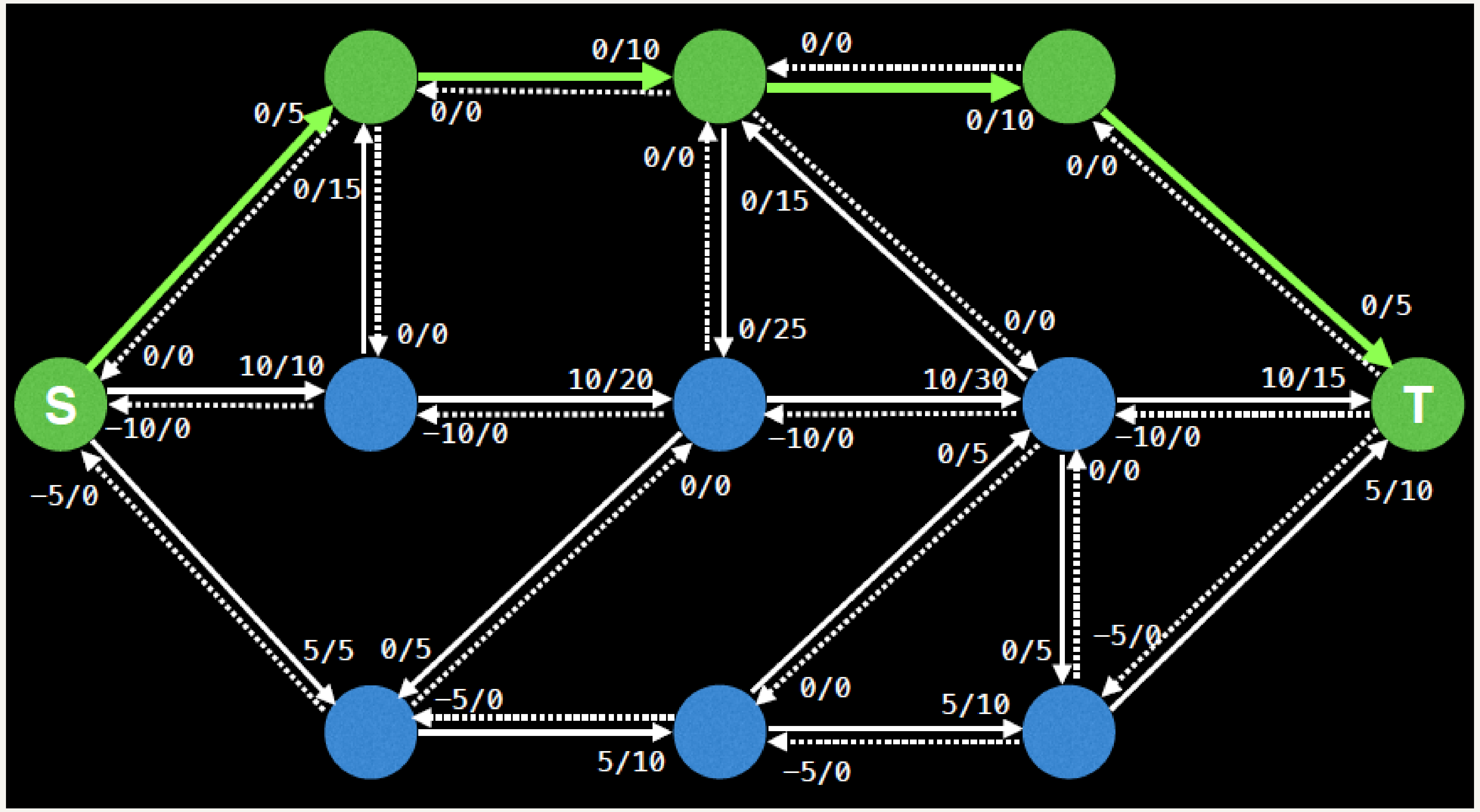
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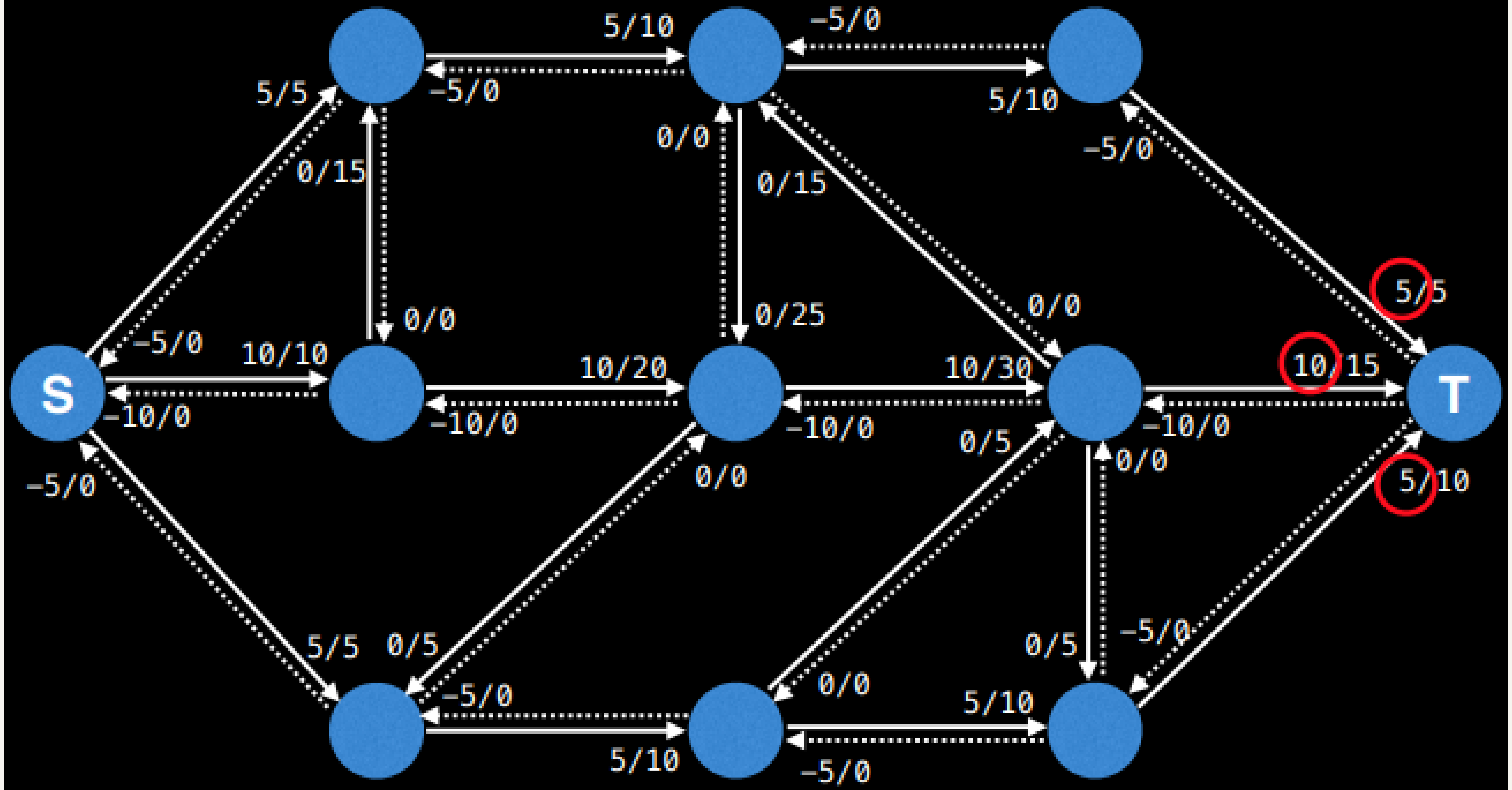
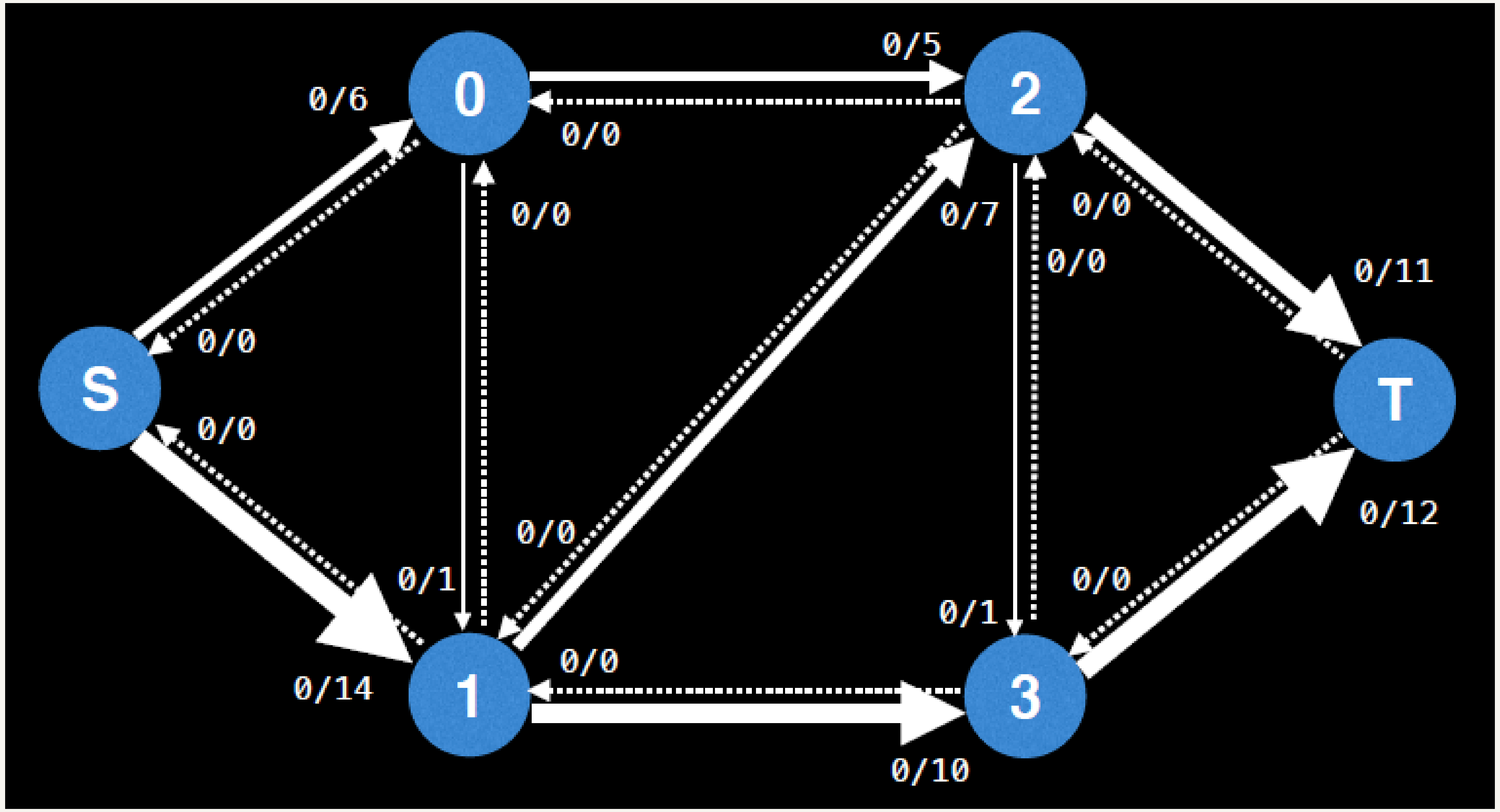
Max flow = 5 + 10 + 5 = 20



Capacity Scaling Heuristic

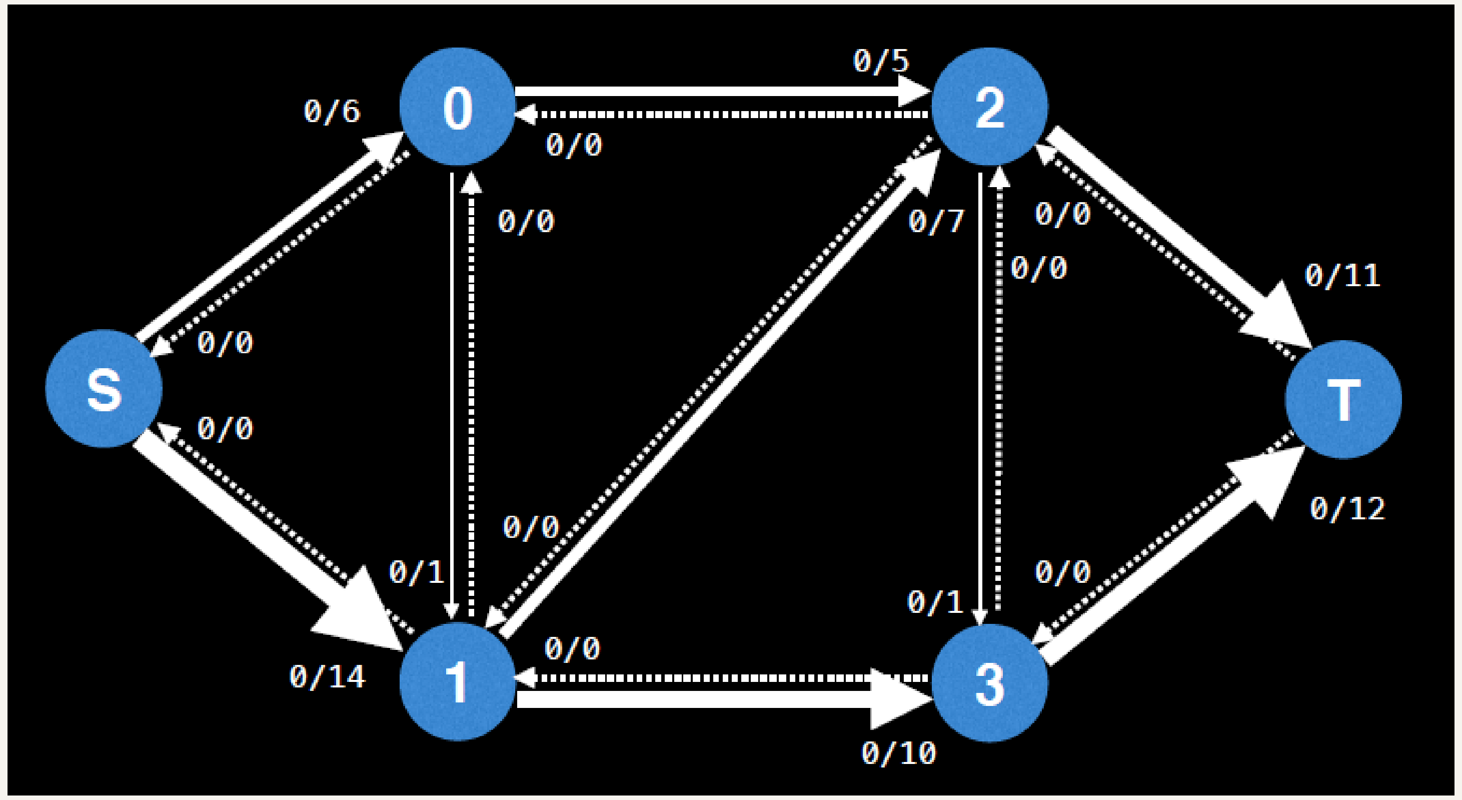


Adjust the size of each edge based on the capacity value highlighting the edges that should be given priority while finding maximal flow

Workflow

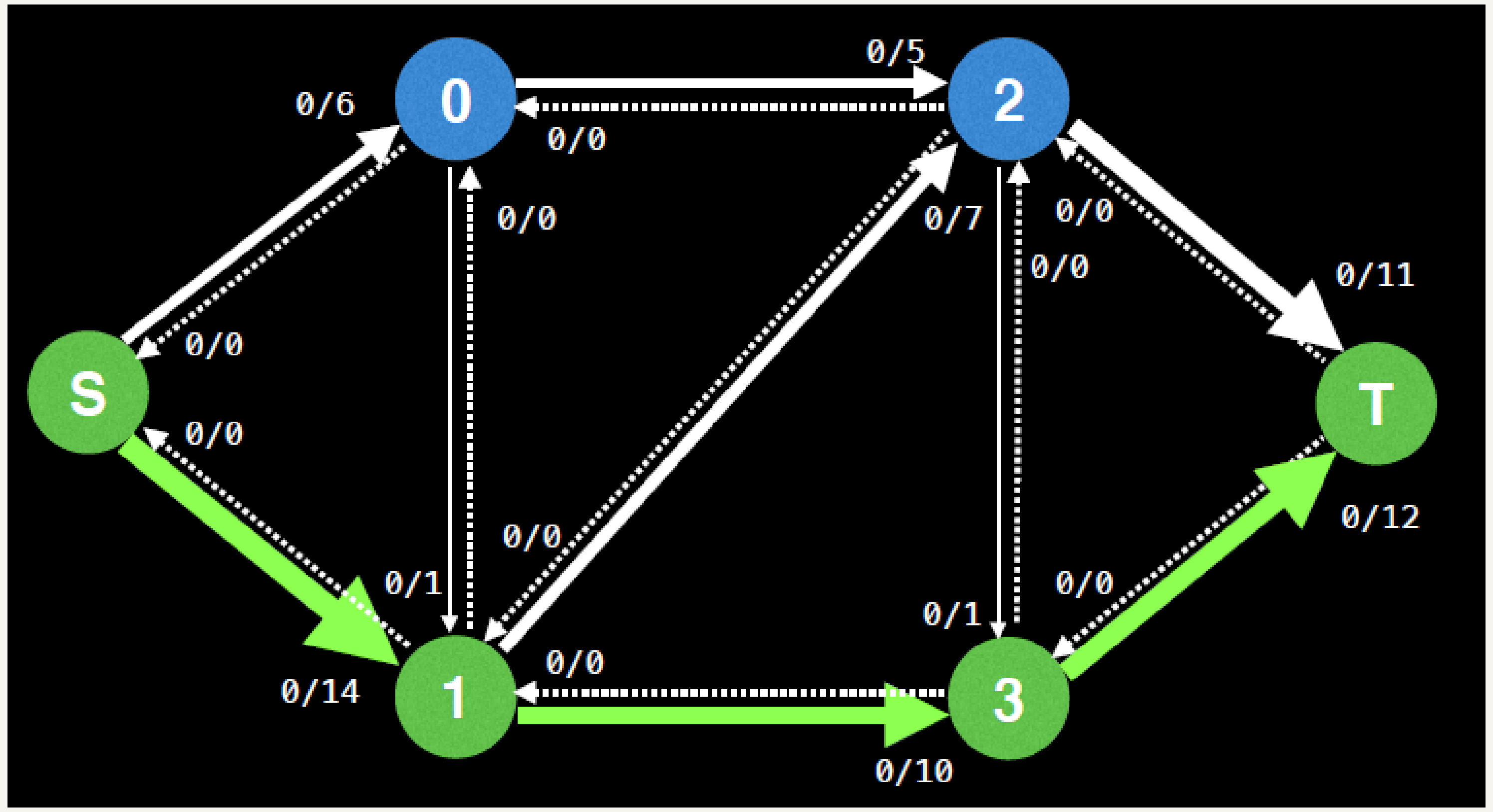
U = largets edge capacity

Δ = largest power of 2 less than or equal to U

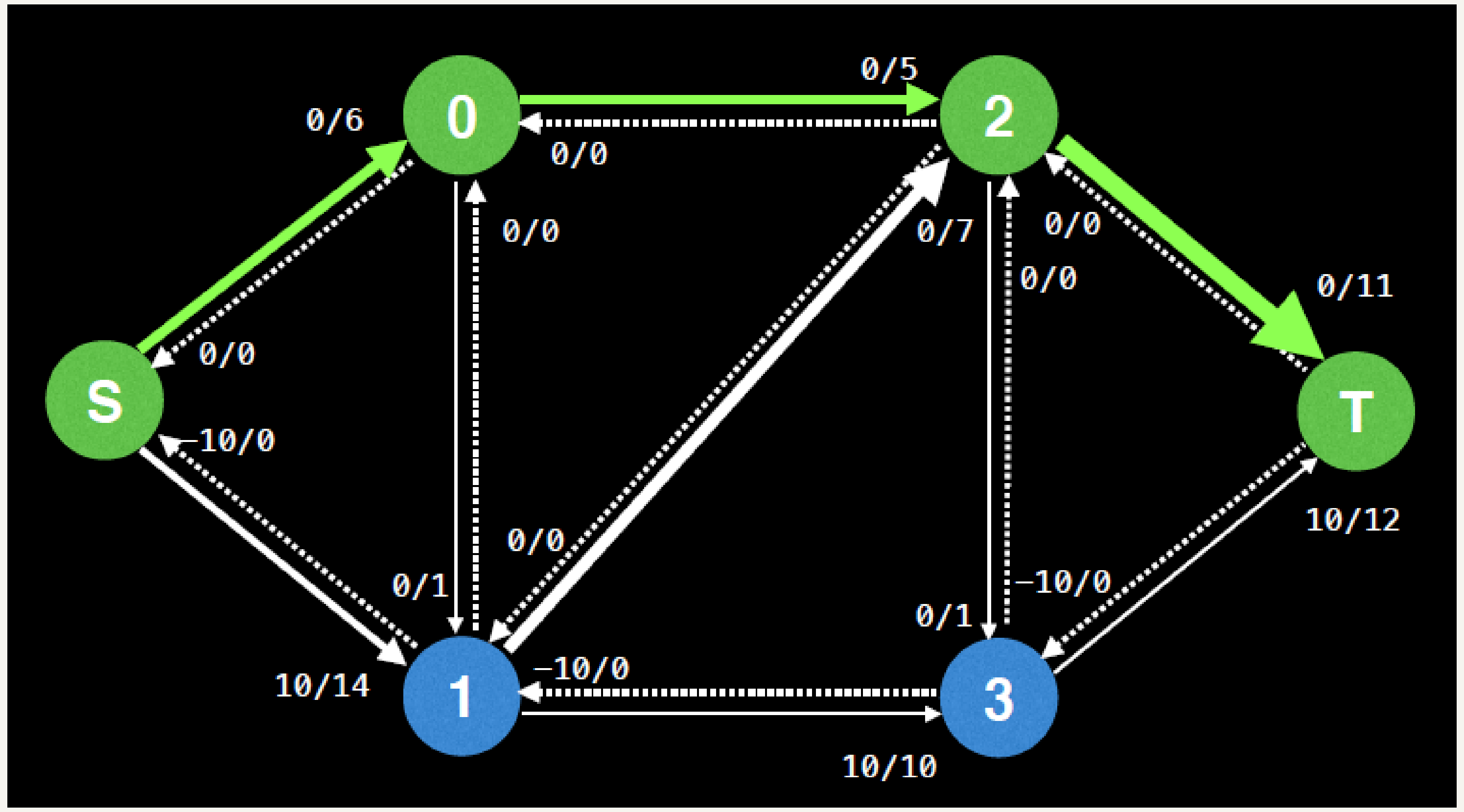


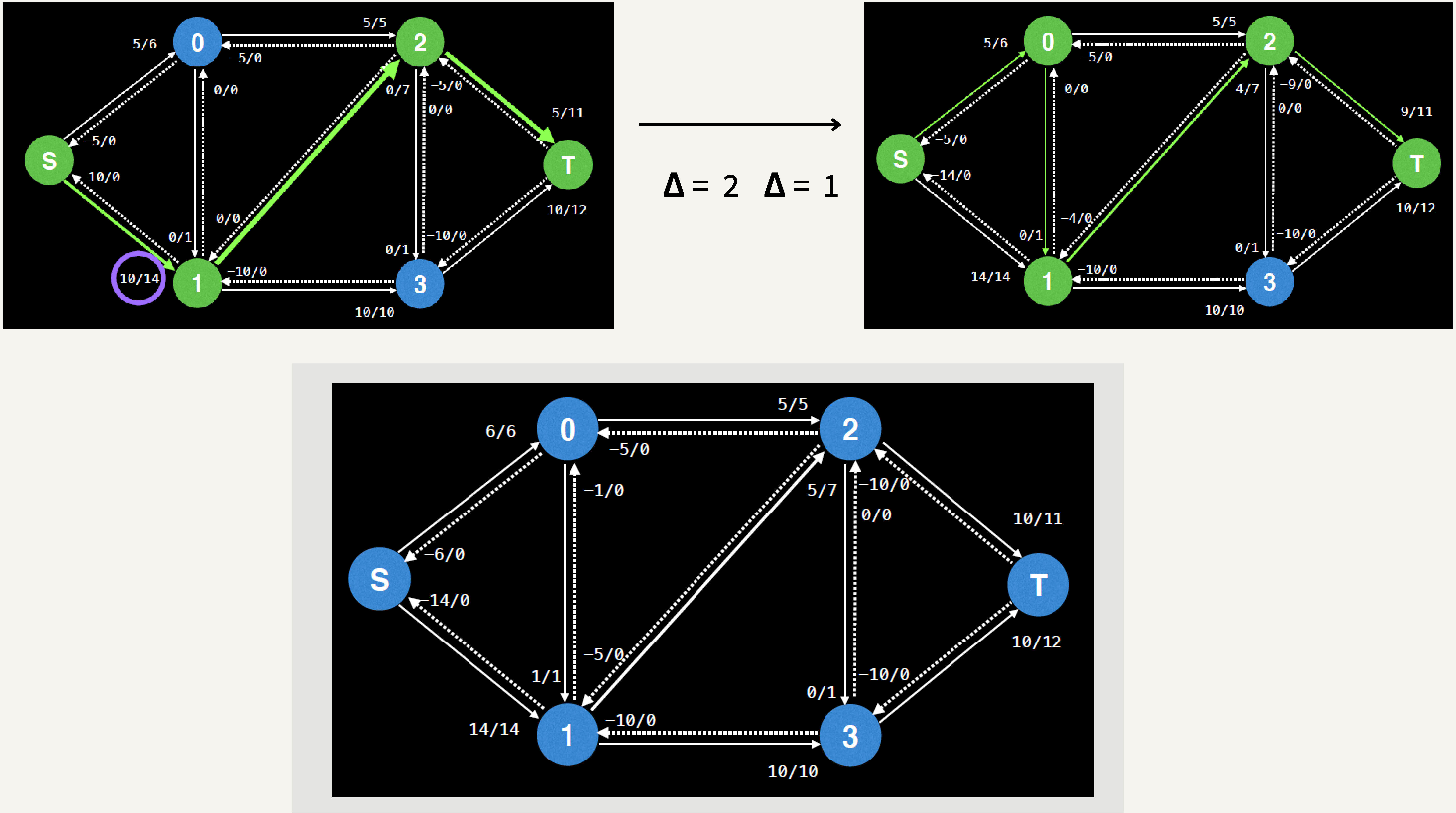
U = 14

Δ = 8



Δ = 4





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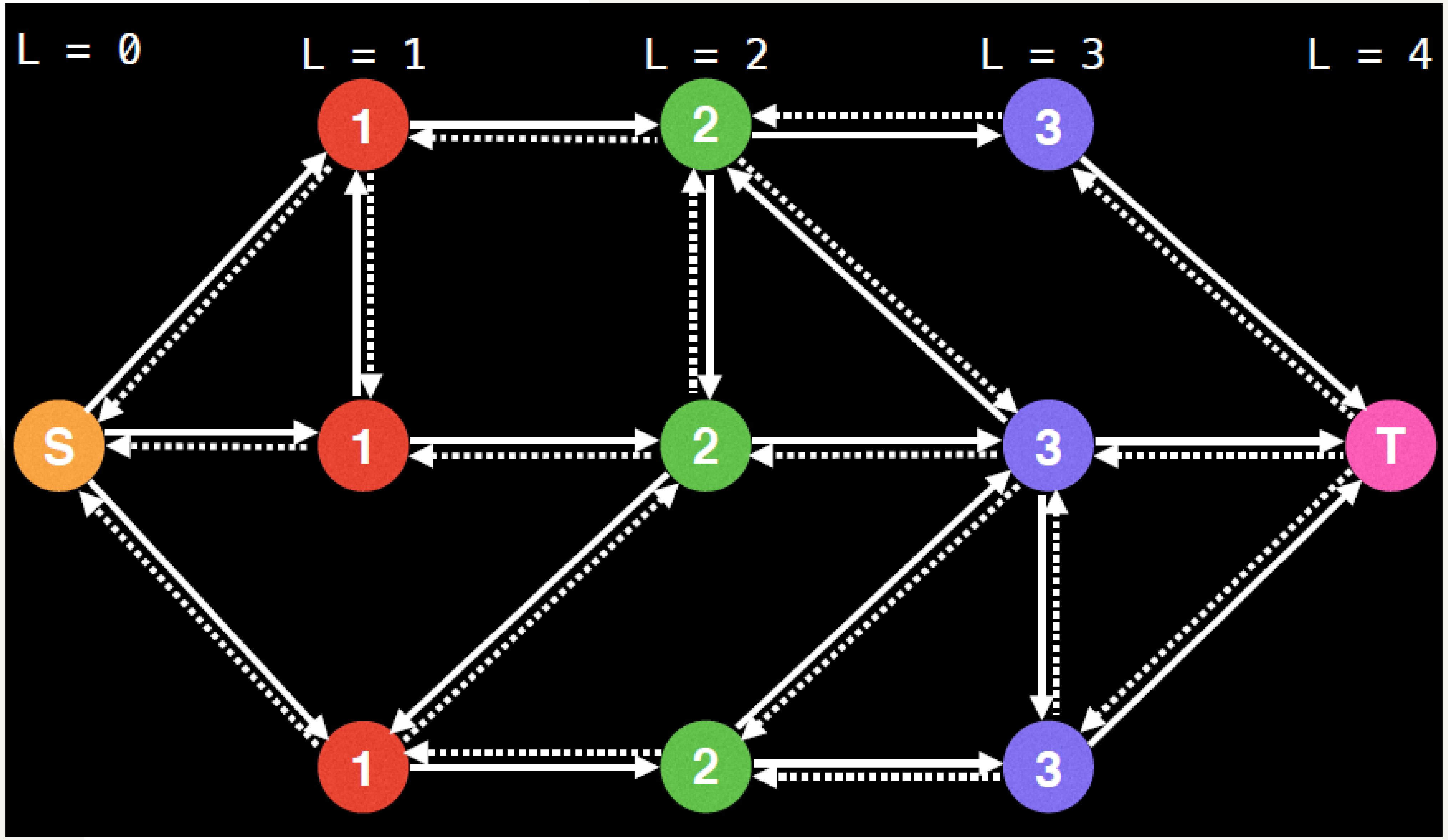
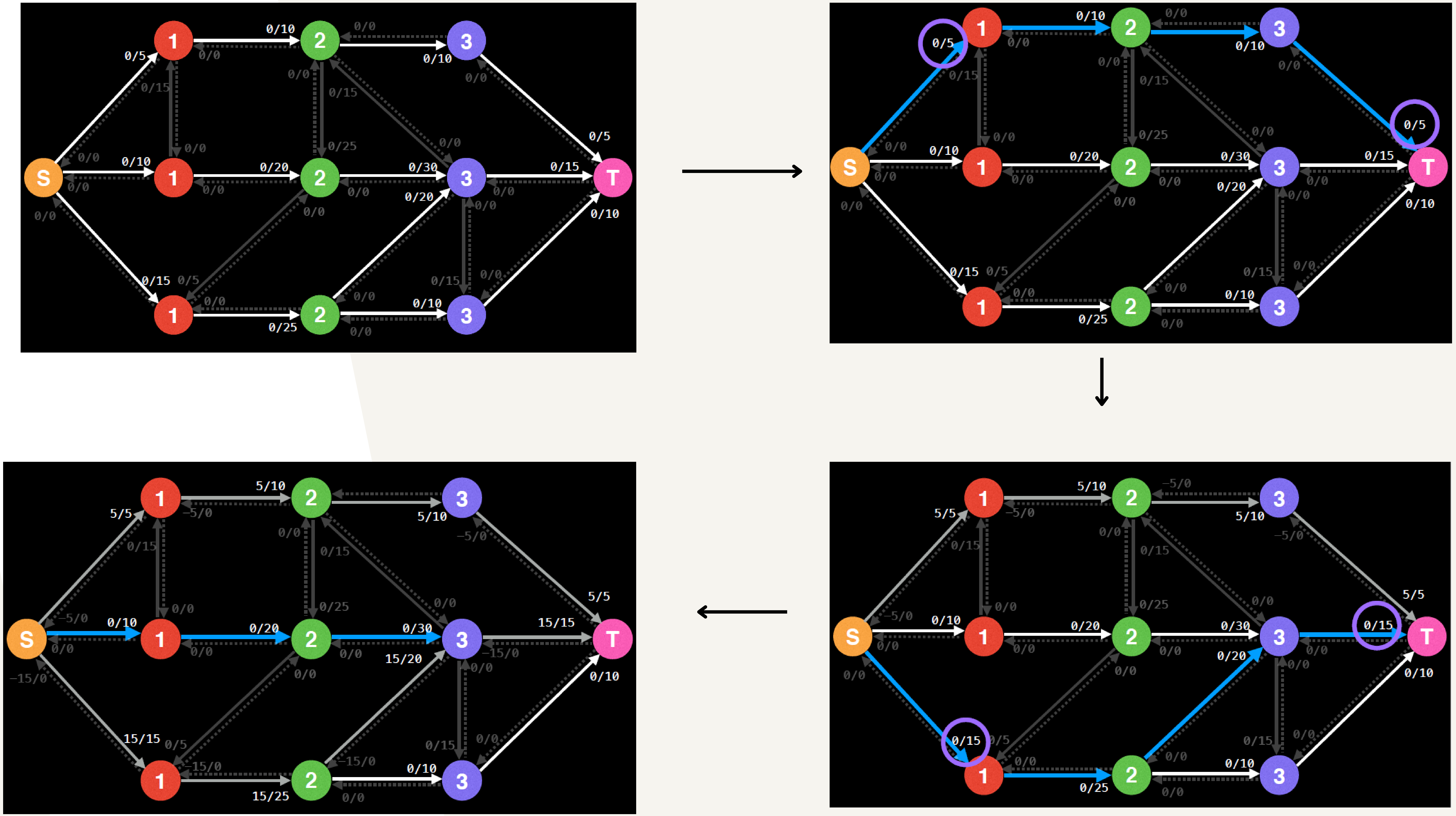
Dinic's Algorithm



Why take a detour? The main idea behind this algorithm is to guide augmenting paths from source to sink using a level graph and thus reducing the runtime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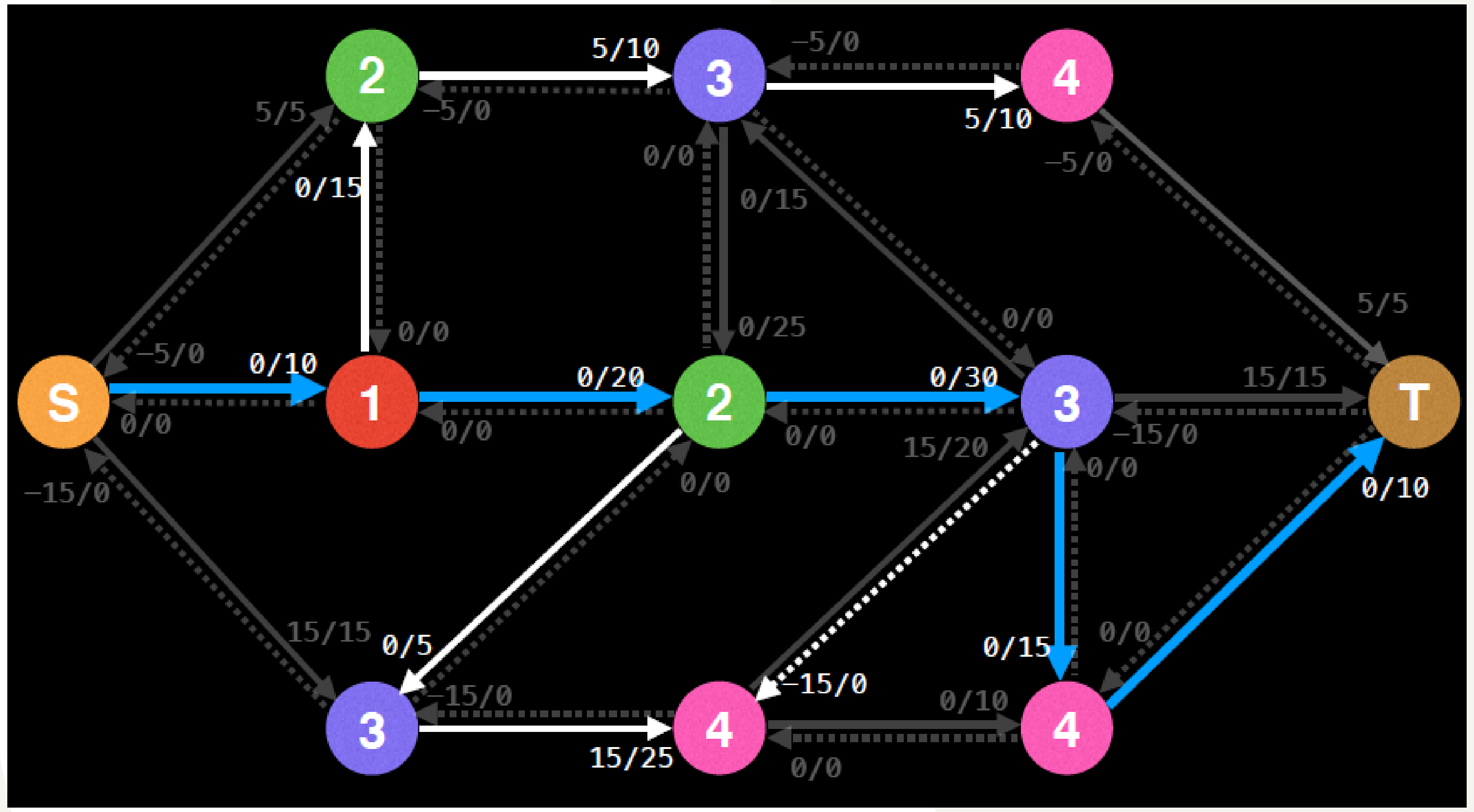

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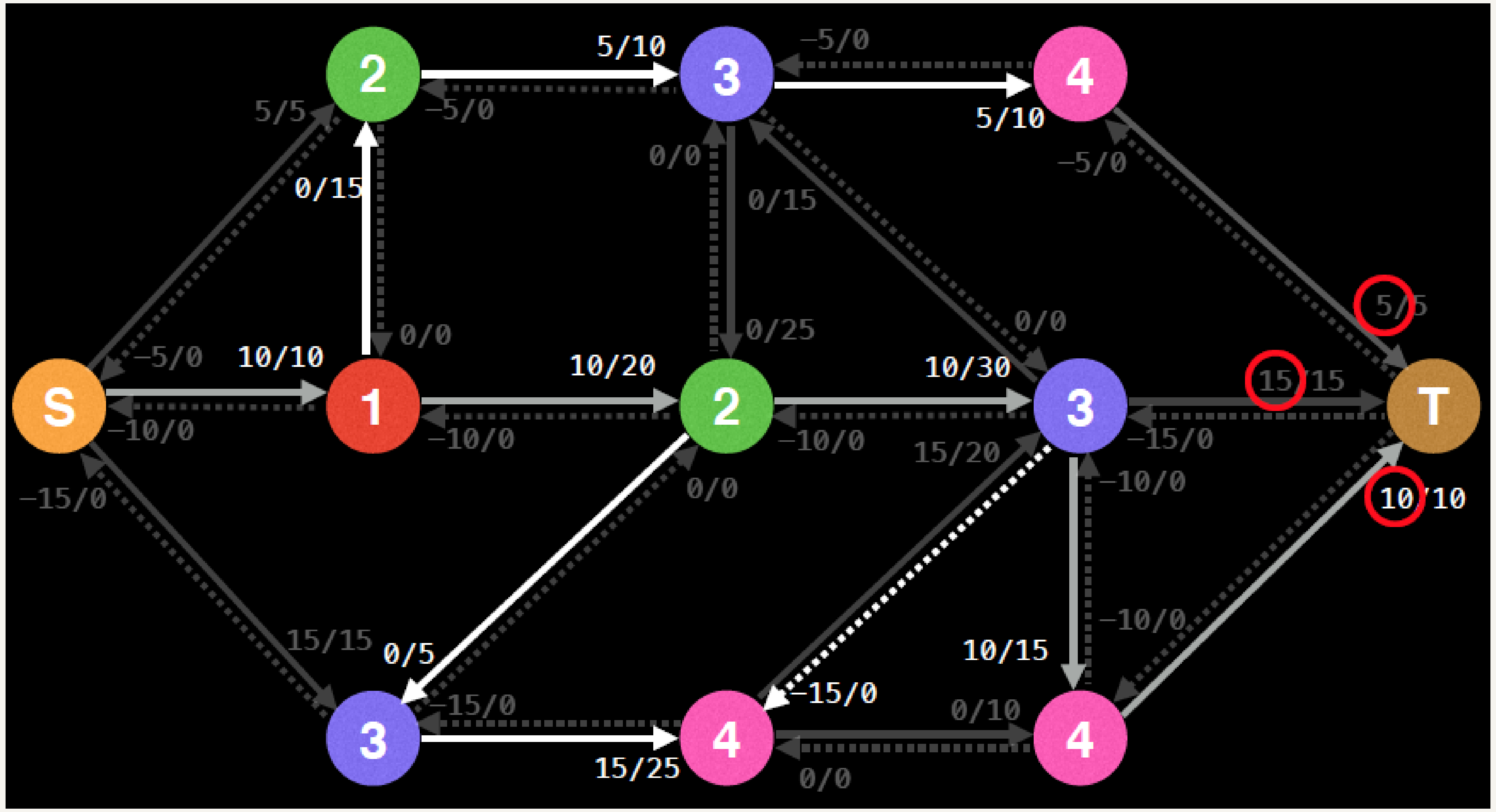

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Conclusion

Algorithm	Time Complexity
Ford Fulkerson (with DFS)	$O(EF)$, where F is the maximum flow
Edmonds-Karp	$O(VE^2)$
Dinic's algorithm	$O(V^2E)$
Capacity Scaling	$O(VE \log U)$, where U is the maximum capacity of the network

Applications

Transportation Networks - Optimizing the flow of traffic, passengers or goods through the network minimizing congestion and maximizing efficiency.

Disaster Mangement - It can be used to determine evacuation routes, resource allocation and logistics planning during emergencies

Image Segmentation - Treating image as a graph, the algorithm finds the optimal boundary between different regions based on flow of pixels

Water Management- Determining the optimal distribution of water resources in irrigation networks, hydroelectric power generation, or water supply networks

Thank You