TUTORIAL - 4

SELF LEARNING TOPICS

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Question 1

To prove: $\frac{\sin 6\theta}{\sin 2\theta} = 16\cos^4\theta - 16\cos^2\theta + 3$.

L.H.S:

$\frac{\sin 6\theta}{\sin 2\theta}$.

To simplify $\sin 6\theta$:

$(\cos\theta + i\sin\theta)^6 = \cos 6\theta + i\sin 6\theta$.

using binomial expansion

$\cos 6\theta + i\sin 6\theta = \cos^6\theta + 6i\cos^5\theta\sin\theta + 15i^2\cos^4\theta\sin^2\theta + 20i^3\cos^3\theta\sin^3\theta + 15i^4\cos^2\theta\sin^4\theta + 6i^5\cos\theta\sin^5\theta + i^6\sin^6\theta$

$\because i^2 = -1, \quad i^3 = -i, \quad i^4 = 1$.

$\therefore \sin 6\theta = 6\cos^5\theta\sin\theta - 20i\cos^3\theta\sin^3\theta + 6\cos\theta\sin^5\theta$.

Now, $\frac{\sin 6\theta}{\sin 2\theta} = 3\cos^4\theta - 10\cos^2\theta\sin^2\theta + 3\sin^4\theta$

$\because \sin 2\theta = 2\sin\theta\cos\theta$

$= 3\cos^4\theta - 10\cos^2\theta(1-\cos^2\theta) + 3(1-\cos^2\theta)^2$

$= 3\cos^4\theta - 10\cos^2\theta + 10\cos^4\theta + 3 + 3\cos^4\theta - 6\cos^2\theta$

~~= 16cos⁴θ -~~

$= \boxed{16\cos^4\theta - 16\cos^2\theta + 3.}$

Hence proved.

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Question 2

$$\left(x-\frac{1}{x}\right)^8 = x^8 - 8x^7\frac{1}{x} + 28x^6\frac{1}{x^2} - 56x^5\frac{1}{x^3}$$
$$+70x^4\frac{1}{x^4} - 56\frac{x^3}{x^5} + 28x^2\frac{1}{x^6} - 8x\frac{1}{x^7} + \frac{1}{x^8}$$

where $\boxed{x = \cos\theta + i\sin\theta}$ & $\boxed{\frac{1}{x} = \cos\theta - i\sin\theta}$

$$\therefore 2^8\sin^8\theta = \left(x^8+\frac{1}{x^8}\right) - 8\left(x^6+\frac{1}{x^6}\right) + 28\left(x^4+\frac{1}{x^4}\right)$$
$$-56\left(x^2+\frac{1}{x^2}\right) + 70$$

$$\cancel{\therefore \frac{1}{2}\left(x+\frac{1}{x}\right) = \frac{\cos\theta}{2}}$$

$$\therefore \frac{1}{2^n}\left(x+\frac{1}{x}\right)^n = \cos n\theta$$

$$\therefore \boxed{2^8\sin^8\theta = \cos 8\theta - 8\cos 6\theta + 28\cos 4\theta - 56\cos 2\theta + 70.}$$

$\therefore$ Sin expression in the form of cosine series.

Question 3

(i) $A = \begin{bmatrix} 1 & 2 & 1 \\ 2 & 1 & 1 \\ 1 & 2 & 2 \end{bmatrix}$

$$\operatorname{adj} A = \begin{bmatrix} +\begin{vmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{vmatrix} & -\begin{vmatrix} a_{12} & a_{13} \\ a_{32} & a_{33} \end{vmatrix} & +\begin{vmatrix} a_{12} & a_{13} \\ a_{22} & a_{23} \end{vmatrix} \\ -\begin{vmatrix} a_{21} & a_{23} \\ a_{31} & a_{33} \end{vmatrix} & +\begin{vmatrix} a_{11} & a_{13} \\ a_{31} & a_{33} \end{vmatrix} & -\begin{vmatrix} a_{11} & a_{13} \\ a_{21} & a_{23} \end{vmatrix} \\ +\begin{vmatrix} a_{21} & a_{22} \\ a_{31} & a_{32} \end{vmatrix} & -\begin{vmatrix} a_{11} & a_{12} \\ a_{31} & a_{32} \end{vmatrix} & +\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix} \end{bmatrix}$$

$\therefore$ ~~A(ad~~ $\text{Adj}(A) = \begin{bmatrix} 0 & -2 & 1 \\ -3 & 1 & 1 \\ 3 & 0 & -3 \end{bmatrix}$

$$A \cdot \text{adj}(A) = \begin{bmatrix} 1 & 2 & 1 \\ 2 & 1 & 1 \\ 1 & 2 & 2 \end{bmatrix} \begin{bmatrix} 0 & -2 & 1 \\ -3 & 1 & 1 \\ 3 & 0 & -3 \end{bmatrix} = \begin{bmatrix} -3 & 0 & 0 \\ 0 & -3 & 0 \\ 0 & 0 & -3 \end{bmatrix}$$

$= L.H.S$

$$|A| = \begin{vmatrix} 1 & 2 & 1 \\ 2 & 1 & 1 \\ 1 & 2 & 2 \end{vmatrix} = 1\overset{0}{\cancel{(2-2)}} - 2(4-1) + 1(4-1)$$
$$= -6 + 3$$
$$= -3$$

$$|A| \cdot I = -3 \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} -3 & 0 & 0 \\ 0 & -3 & 0 \\ 0 & 0 & -3 \end{bmatrix}$$

$= R.H.S$

Hence proved by L.H.S. = R.H.S.

(ii) $\text{adj}(\text{adj}(A)) = \begin{bmatrix} 3 & -6 & -3 \\ -6 & -3 & -3 \\ -3 & -6 & -6 \end{bmatrix} \longrightarrow 3$

— L.H.S

$$|A|A = -3 \begin{bmatrix} 1 & 2 & 1 \\ 2 & 1 & 1 \\ 1 & 2 & 2 \end{bmatrix} = \begin{bmatrix} -3 & -6 & -3 \\ -6 & -3 & -3 \\ -3 & -6 & -6 \end{bmatrix}$$

$= R.H.S.$

Hence proved by L.H.S = R.H.S

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(iii) Now $[adj(A)]^{-1} = \frac{adj(adj(A))}{|Adj(A)|}$

$$= \begin{bmatrix} -3 & -6 & -3 \\ -6 & -3 & -3 \\ -3 & -6 & -6 \end{bmatrix} \frac{1}{9}.$$

$$= \frac{1}{3} \begin{bmatrix} -1 & -2 & -1 \\ -2 & -1 & -1 \\ -1 & -2 & -2 \end{bmatrix}.$$

$$= -\frac{1}{3} \begin{bmatrix} 1 & 2 & 1 \\ 2 & 1 & 1 \\ 1 & 2 & 2 \end{bmatrix} = \boxed{-\frac{1}{3} A}$$