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Tutorial 7

Question 1

$$A = \begin{bmatrix} 221025 & 0 & 0 \\ 0 & 221025 & 0 \\ 0 & 0 & 221025 \end{bmatrix}$$

1) Char eq^n. of A = $|A - \lambda I| = 0$.

$$A - \lambda I = \begin{bmatrix} 221025-\lambda & 0 & 0 \\ 0 & 221025-\lambda & 0 \\ 0 & 0 & 221025-\lambda \end{bmatrix}$$

$$A - \lambda I \Rightarrow (221025-\lambda)(221025-\lambda)^2 = 0.$$

$$\Rightarrow (221025-\lambda)^3 = 0.$$

Taking cube root on both sides:

$$\Rightarrow \boxed{\lambda = 221025}$$

This is the only Eigen value of A.

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$$\Rightarrow \begin{bmatrix} 221025-221025 & 0 & 0 \\ 0 & 221025-221025 & 0 \\ 0 & 0 & 221025-221025 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$\therefore$ x_1, x_2, x_3 can take any arbitrary value.

$$\therefore \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} p \\ q \\ r \end{bmatrix} = p \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} + q \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} + r \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

If $p \cdot q \cdot r = 1$.

$$x_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \quad x_2 = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}, \quad x_3 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

are the Eigen ~~value~~ vectors of A.

3.) Let k_1, k_2, k_3 be scalars such that
$k_1 x_1 + k_2 x_2 + k_3 x_3 = 0$

$$\therefore k_1 \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} + k_2 \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} + k_3 \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} = 0.$$

$$\Rightarrow \begin{bmatrix} k_1 \\ k_2 \\ k_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$\therefore$ $k_1 = k_2 = k_3 = 0$

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$\therefore$ The vectors x_1, x_2, x_3 are linearly independent.

Question 2

$$A = \begin{bmatrix} a & b & c \\ b & c & a \\ c & a & b \end{bmatrix}$$

$a = 24$
$b = a+1 = 25$
$c = a+2 = 27$

1.) $$P = \begin{bmatrix} 24 & 25 & 26 \\ 25 & 26 & 24 \\ 26 & 24 & 25 \end{bmatrix}$$

char eqn. of $P \Rightarrow |P - \lambda I| = 0$.

$$\begin{bmatrix} 24-\lambda & 25 & 26 \\ 25 & 26-\lambda & 24 \\ 26 & 24 & 25-\lambda \end{bmatrix} = 0.$$

Solving the determinant :

$$(24-\lambda)\left[(26-\lambda)(25-\lambda) - (24)^2\right]$$
$$-25\left[25(25-\lambda) - (24\times 26)\right] = 0$$
$$+26\left[(25\times 24) - 26(26-\lambda)\right]$$

$$(24-\lambda)\left[650 - 51\lambda + \lambda^2 - 576\right]$$
$$-25\left[625 - 25\lambda - 624\right] = 0.$$
$$+26\left[600 - 676 + 26\lambda\right]$$

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$\Rightarrow 24\lambda^2 - 1224\lambda + 1776 - \lambda^3 + 51\lambda^2 - 74\lambda - 25$
$+ 625\lambda + 676\lambda - 1976 = 0.$

$\Rightarrow -\lambda^3 + 75\lambda^2 + 3\lambda - 225 = 0.$
$\Rightarrow \lambda^3 - 75\lambda^2 - 3\lambda + 225 = 0.$

$\Rightarrow \lambda^2(\lambda - 75) - 3(\lambda - 75) = 0.$
$\Rightarrow (\lambda^2 - 3)(\lambda - 75) = 0.$

$\Rightarrow (\lambda^2 - 3)(\lambda - 75) = 0.$
$\Rightarrow (\lambda + \sqrt{3})(\lambda - \sqrt{3})(\lambda - 75) = 0.$
$\Rightarrow \lambda = 75, \sqrt{3}, -\sqrt{3}$
Now, $a+b+c = 24+25+26 = 75.$
$\therefore$ Proved $\Rightarrow \lambda = (a+b+c, \sqrt{3}, -\sqrt{3})$.

3.) Char eq^n. of P is
$\lambda^3 - 75\lambda^2 - 3\lambda + 225 = 0.$

To verify Cayley - Hamilton's theorem:

$P^3 - 75P^2 - 3P + 225 = 0.$

$$P^2 = \begin{bmatrix} 24 & 25 & 26 \\ 25 & 26 & 24 \\ 26 & 24 & 25 \end{bmatrix} \begin{bmatrix} 24 & 25 & 26 \\ 25 & 26 & 24 \\ 26 & 24 & 25 \end{bmatrix}$$

$$P^2 = \begin{bmatrix} 1877 & 1874 & 1874 \\ 1874 & 1877 & 1874 \\ 1874 & 1874 & 1877 \end{bmatrix} \begin{bmatrix} 24 & 25 & 26 \\ 25 & 26 & 24 \\ 26 & 24 & 25 \end{bmatrix}$$

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$$P^3 = \begin{bmatrix} 140622 & 140625 & 140628 \\ 140625 & 140628 & 140622 \\ 140628 & 140622 & 140625 \end{bmatrix}$$

$$75P^2 = 75\begin{bmatrix} 1877 & 1874 & 1874 \\ 1874 & 1877 & 1874 \\ 1874 & 1874 & 1877 \end{bmatrix}$$

$$3P = 3\begin{bmatrix} 24 & 25 & 26 \\ 25 & 26 & 24 \\ 26 & 24 & 25 \end{bmatrix} = \begin{bmatrix} 72 & 75 & 78 \\ 75 & 78 & 72 \\ 78 & 72 & 75 \end{bmatrix}$$

$$225\,I = \begin{bmatrix} 225 & 0 & 0 \\ 0 & 225 & 0 \\ 0 & 0 & 225 \end{bmatrix}$$

$$= \begin{bmatrix} 140622 & 140625 & 140628 \\ 140625 & 140628 & 140622 \\ 140628 & 140622 & 140625 \end{bmatrix}$$

$$= \begin{bmatrix} 140775 & 140550 & 140550 \\ 140550 & 140775 & 140550 \\ 140550 & 140550 & 140775 \end{bmatrix}$$

$$= \begin{bmatrix} 72 & 75 & 78 \\ 75 & 78 & 72 \\ 78 & 72 & 75 \end{bmatrix} + \begin{bmatrix} 225 & 0 & 0 \\ 0 & 225 & 0 \\ 0 & 0 & 225 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} = \underline{R.H.S}$$

$\therefore$ Cayley Hamiton theorem is verified.