nuclear size $\simeq 10^{-15}$ m | atomic size $\simeq 10^{-10}$ m

take $\Delta x \simeq 10$ nuclei $\simeq 10^{-14}$ m

$\Delta p = \frac{\hbar}{\Delta x} \simeq 1.05 \times 10^{-20}$

$\Delta x \simeq 10$ atom $\simeq 10^{-9}$ m

$\Delta p = \frac{\hbar}{\Delta x} \simeq 1.05 \times 10^{-25}$

1 nm, Δx; $n^\bullet$ 10^{-14} m, 10 nuclei; 10^{-9} m, 10 atom

$E = 20$ MeV $= 3.2 \times 10^{-12}$ J

$p = \sqrt{2 m_n E} = \sqrt{- - - -}$

$= 1.03 \times 10^{-19}$

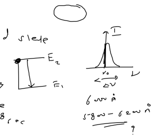
nuclear: $\frac{\Delta p}{p} \times 100\% \simeq 10\%$ | atomic: $\frac{\Delta p}{p} \times 100\% \simeq 0.0001\%$

The atomic dimensions are large for a neutron so its position is highly uncertain therefore the momentum is almost accurate.
at the nuclear dimensions, the position uncertainty is small but then it leads to larger uncertianty in momentum. Thus, more we try to confine a nucleon to determine its position accurately, its mometum becomes lesser and lesser accurate and vice versa.

typical lifetime of excited state $\simeq 10$ ns

take this as uncertainty in measurement of time

i.e. $\Delta t = 10$ ns $= 10^{-8}$ sec



use uncertainty product

$\Delta E \Delta t = \hbar$

also we have $E = h\nu$

$\Delta E = h \Delta\nu$

$h \Delta\nu \Delta t = \hbar = \frac{h}{2\pi}$

$\Delta\nu = \frac{1}{2\pi \times \Delta t} = \frac{1}{2 \times 3.14 \times 10^{-8}}$

$= 15.6 \times 10^{6}$ Hz $= 15.6$ MHz

what could be energy of the e^- when it is generated in the nucleus ?

at that instant, e^- is within the nuclear dimensions

so take uncertainty in the measurement of its position to be the nuclear size

avg nuclear size $\simeq 10^{-15}$ m

$\therefore \Delta x \simeq 10^{-15}$ m for electron

$\Rightarrow \Delta p = \frac{\hbar}{\Delta x} \simeq 1.05 \times 10^{-19}$ kg m/s

let momentum of this electron be at least equal to uncertainty in measurement of momentum

i.e. $p \simeq \Delta p \approx 1.05 \times 10^{-19}$ kg m/s

for KE, we can't use $KE = \frac{p^2}{2m}$ since this is a relativistic electron (its speed $v \to c$)

we have to use relativistic more general eqn

$KE = \sqrt{m_0^2 c^4 + p^2 c^2} - m_0 c^2$ ★

★ for $v << c$ this reduces to $\frac{p^2}{2m}$

here $m_0 c^2 << pc$ (almost 10^6 times, check)

$\therefore KE \simeq pc = 3.15 \times 10^{-11}$ J

i.e. $E \simeq 3.15 \times 10^{-11}$ J

let uncertainty in the measurement of energy be at the most equals the energy itself

$\therefore \Delta E = E = 3.15 \times 10^{-11}$ J

using uncertainty principle,

$\Delta E \Delta t = \hbar \Rightarrow \Delta t = \frac{\hbar}{\Delta E}$

$= 3.33 \times 10^{-24}$ sec

this is uncertainty in measurement of time interval

at microscopic levels, uncertainties are quite significant

let $\Delta t \simeq 10\%\ t$

$\therefore t \simeq 3.33 \times 10^{-23}$ sec

this is the time interval for which an electron stays in the nucleus as per the uncertainty principle

As we can see the time is infinitesimally small, it means an electron is emitted almost instantaneously as it is generated during β-decay

nucleon → proton, neutron

both are present in nucleus

but their exact position is unknown

it can be present anywhere within the nucleus

avg nuclear size $\simeq 10^{-15}$ m

using this as uncertainty in the position of nucleon

i.e. $\Delta x \simeq 10^{-15}$ m

use uncertainty product

$\Delta p \simeq \frac{\hbar}{\Delta x} \simeq \frac{- -}{- -}$

$\simeq 1.05 \times 10^{-19}$ kg m/s

let p of a nucleon be at least equal to Δp

$p \simeq p \simeq 1.05 \times 10^{-19}$ kg m/s

find KE

$KE = \frac{p^2}{2m}$ [why relativistic formula is NOT required]

$= \frac{- - -}{- - - -}$

$= 3.3 \times 10^{-12}$ J $\simeq 20.6$ MeV

★ here, take the size of the quantum well to which, that electron is confined as the "uncertainty" in the determination of position of that elecron

use $\Delta x \Delta p = \frac{h}{2\pi}$, $\frac{h}{2\pi} = \hbar$

$\Delta x = 1$ nm $= 10^{-9}$ m

$\Delta p = \frac{\hbar}{\Delta x} = \frac{1.05 \times 10^{-34}}{10^{-9}}$

$\Delta p = 1.05 \times 10^{-25}$ kg m/s

avg $v_e = 10^6$ m/s

$p = m_e v_e = 9.1 \times 10^{-31} \times 10^6$

$= 9.1 \times 10^{-25}$ kg m/s

% uncertainty in momentum

$\frac{\Delta p}{p} \times 100\% = \frac{- - -}{- - -} = 11.5\%$

here, Δx will be the size of the box

i.e. $\Delta x = 50$ cm $= 0.5$ m

$\Delta p = \frac{\hbar}{\Delta x} = \frac{- -}{- -} = 2.1 \times 10^{-34}$ kg m/s

$v = 20$ cm/s $= 0.2$ m/s

$m = 10$ gm $= 10^{-2}$ kg

$p = mv = 2 \times 10^{-3}$ kg m/s

% uncertainty here, $\frac{\Delta p}{p} \times 100\% = 0.00000 - - - . - ..$

$\simeq 0$

for marble, unertainty in measurment of momentum is almost zero. Hence at macroscopic level, the uncertainties in measurements can be neglected while for electron, the uncertainty in measurement of momentum is significant (11.5%) hence it cannot be ignored at microscopic level

Numerical problems on QM (Set-1)
(de'Broglie hypothesis and Uncertainty principle)

te de' Broglie wavelengths of (i) cricket ball of mass 150 gm ~~thrown at a speed of~~ 150 km/hr electron orbiting in hydrogen atom at a speed of 10^6 m/s. Comment on your results.

cricket ball

$m = 150$ gm $= 0.15$ kg

$v = 150$ km/hr $= 41.67$ m/s

$\lambda = \frac{h}{mv} = \frac{6.63 \times 10^{-34}}{0.15 \times 41.67}$

$= 1.06 \times 10^{-34}$ m

e^- in H-atom

$m_e = 9.1 \times 10^{-31}$ kg

$v_e = 10^6$ m/s

$\lambda_e = \frac{h}{m_e v_e} = \frac{6.63 \times 10^{-34}}{9.1 \times 10^{-31} \times 10^6}$

$= 7.28 \times 10^{-10}$ m

The limit of experimental measurements of dimentions is ~ 10^-15 m. The wave nature of electron is experimentally verifiable since it is within the limits of measurements while the wavelength of the cricket ball is way beyond any experimental measurement.
Hence the wave nature of matter is significant at the microscopic level, whereas it can be neglected at the macroscopic level.

$\lambda = \frac{h}{mv} \Rightarrow \lambda = \frac{h}{\sqrt{2mE}}$ $m_n = 1.67 \times 10^{-27}$ kg

E = kinetic energy

For all elementary particles when we say "energy", it is mostly kinetic energy

$E = 1$ MeV $= 1.6 \times 10^{-13}$ J

$\lambda_n = \frac{h}{\sqrt{2mE}} = \frac{h}{\sqrt{2 m_n E}}$

$= \frac{- -}{\sqrt{\cdots - -}}$

$\lambda_n = 2.87 \times 10^{-14}$ m

$\lambda = \frac{h}{mv} \Rightarrow \lambda = \frac{h}{\sqrt{2 m q V}}$

where V ; accelerating p.d

$\lambda_p^2 = \frac{h^2}{2 m_p q V} \Rightarrow V = \frac{h^2}{2 m_p q \lambda_p^2}$

$\lambda_p = \lambda_n = 2.87 \times 10^{-14}$ m

$V = \frac{- - - .^2}{- - - - - - ..}$

$= 9.98 \times 10^5$ volt

$\simeq 1$ million volt

$E_{ph} = 100$ keV $= 1.6 \times 10^{-14}$ J

for $\lambda_{ph} = \frac{hc}{E_{ph}}$ $\left[E = h\nu = \frac{hc}{\lambda} \right]$

for electron, $\lambda_e = \frac{h}{\sqrt{2 m_e E_e}}$

$\lambda_e^2 = \frac{h^2}{2 m_e E_e} \Rightarrow E_e = \frac{h^2}{2 m_e \lambda_e^2}$

it is given that $\lambda_e = \lambda_{ph}$

$E_e = \frac{h^2}{2 m_e \lambda_{ph}^2} = \frac{h^2}{2 m_e \left(\frac{hc}{E_{ph}}\right)^2}$

$= \frac{E_{ph}^2}{2 m_e c^2} = \frac{- - - -}{- - - -}$

$= 1.56 \times 10^{-15}$ J

$= \frac{1.56 \times 10^{-15}}{1.6 \times 10^{-19}}$ eV $= 9768$ eV

$\lambda_e = \frac{h}{\sqrt{2 m_e E_e}}$; $\lambda_p = \frac{h}{\sqrt{2 m_p E_p}}$

$\therefore E_e = \frac{h^2}{2 m_e \lambda_e^2}$; $E_p = \frac{h^2}{2 m_p \lambda_p^2}$

given that $E_e = E_p$

$\therefore \frac{h^2}{2 m_e \lambda_e^2} = \frac{h^2}{2 m_p \lambda_p^2}$

$\therefore \frac{m_p}{m_e} = \frac{\lambda_e^2}{\lambda_p^2}$

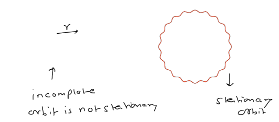
i.e. $\lambda_e : \lambda_p = \sqrt{\frac{m_p}{m_e}} : 1$

$= \sqrt{\frac{1800 m_e}{m_e}} : 1$

$\therefore \lambda_e : \lambda_p = \sqrt{1800} : 1$

$= 42.4 : 1$

$r = a_0 = 0.5$ Å $= 0.5 \times 10^{-10}$ m $= 5 \times 10^{-11}$ m



Stationary orbits are those for which $2\pi r = n\lambda$

for 1st orbit, $n = 1$

$\therefore \lambda = 2\pi r$

$= 2 \times 3.14 \times 5 \times 10^{-11}$

$= 3.14 \times 10^{-10}$ m

$\lambda = \frac{h}{mv} \Rightarrow v = \frac{h}{m\lambda} = \frac{- - -}{- - -}$

$= 2.3 \times 10^6$ m/s