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AM - 1

Tutorials on Matrices

Question 1

$$A = \begin{bmatrix} 1 & 2 & -1 & 3 \\ 3 & 4 & 0 & -1 \\ -1 & 0 & -2 & 7 \end{bmatrix}$$

Elementary row transformation :

$$\begin{array}{l} R_2 \rightarrow R_2 - 3R_1 \\ R_3 \rightarrow R_3 + R_1 \end{array} = \begin{bmatrix} 1 & 2 & -1 & 3 \\ 0 & -2 & 3 & -10 \\ 0 & 2 & -3 & 10 \end{bmatrix}$$

$$R_3 \rightarrow R_3 + R_2 = \begin{bmatrix} 1 & 2 & -1 & 3 \\ 0 & -2 & 3 & 10 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

No. of non-zero rows = 2.

Hence, Rank of matrix A is 2.

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Question 2

$x + y + 4z = 1$
$x + 2y - 2z = 1$
$\lambda x + y + z = 1$

$$\Rightarrow \begin{bmatrix} 1 & 1 & 4 \\ 1 & 2 & -2 \\ \lambda & 1 & 1 \end{bmatrix}$$

(i) unique solution : $D \neq 0$.

$\Rightarrow 1(2+2) - 1(1+2\lambda) + 4(1-2\lambda) \neq 0.$

$\Rightarrow 4 - 1 - 2\lambda + 4 - 8\lambda \neq 0.$

$\Rightarrow \lambda \neq \frac{7}{10}$

$\therefore$ we get a unique solution at $\lambda \neq 7/10$

(ii) No solution $\Rightarrow D = 0$

$\therefore$ At $\lambda = 7/10$. we get ~~a un~~ no solution.

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Question 3

$$x_1 + 2x_2 + 3x_3 + x_4 = 0$$
$$x_1 + x_2 - x_3 - x_4 = 0.$$
$$3x_1 - x_2 + 2x_3 + 3x_4 = 0$$

Given set of equations = non-homogenous.

$\therefore A = \begin{bmatrix} 1 & 2 & 3 & 1 \\ 1 & 1 & -1 & -1 \\ 3 & -1 & 2 & 3 \end{bmatrix}$ since $AX = 0$.

Argument matrix = [A], since $B = 0$ for a homogenous matrix.

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Continuing in Echelon form using row transformations:

$R_2 \rightarrow R_2 - R_1$; $R_3 \rightarrow R_3 - 3R_1$

$A \Rightarrow \begin{bmatrix} 1 & 2 & 3 & 1 \\ 0 & -1 & -4 & 0 \\ 0 & -7 & -7 & 0 \end{bmatrix}$

$R_3 \rightarrow R_3 - 7R_2 \Rightarrow A = \begin{bmatrix} 1 & 2 & 3 & 1 \\ 0 & -1 & -4 & 0 \\ 0 & 0 & 21 & 0 \end{bmatrix}$

$R_2 \rightarrow (-1) R_2$

$A = \begin{bmatrix} 1 & 2 & 3 & 1 \\ 0 & 1 & 4 & 0 \\ 0 & 0 & 21 & 1 \end{bmatrix}$

$R_3 \rightarrow \left(\frac{1}{21}\right) R_3 \Rightarrow \begin{bmatrix} 1 & 2 & 3 & 1 \\ 0 & 1 & 4 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

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$P(A) = 3$

$P(A) <$ no. of unknown variables.

i.e. $P(A) < 4$.

$\therefore$ The system of eq^n. is consistent but has infinite solutions; the solutions can be found out as below:

$$x_1 + 2x_2 + 3x_3 + x_4 = 0 \quad \text{--- (1)}$$

$$x_2 + 4x_3 = 0 \quad \text{--- (2)}$$

$$x_3 = 0 \quad \text{--- (3)}$$

We can choose, $(n-r)$ variable i.e. $(4-3) = 1$ variable of our choice:

Let $\Rightarrow x_1 = k$; $k \in R$.

$\boxed{x_3 = 0}$

substitute in (2)

$x_2 + 4(0) = 0$, $\boxed{x_2 = 0}$

Subs. x_3 and x_2, x_1 in (1).

$k + 2(0) + 3(0) + x_4 = 0$.

$k + x_4 = 0$

$\boxed{x_4 = -k}$

$\boxed{\text{Hence } x_1 = k,\ x_2 = 0,\ x_3 = 0\ \&\ x_4 = -k}$