University of Leicester

CO2114
All Candidates

SEMESTER 2 2022/23 CO2114 FINAL EXAMINATION

DO NOT OPEN THE QUESTION PAPER UNTIL INSTRUCTED TO DO SO BY THE CHIEF INVIGILATOR

School of Computing and Mathematical Sciences	
Module Code	CO2114
Module Title	Foundations of Artificial Intelligence
Exam Duration	Two hours

CHECK YOU HAVE THE CORRECT QUESTION PAPER

Number of Pages	7
Number of Questions	6
Instructions to Candidates	Answer all questions.

For this exam, you are allowed to use the following:	
Calculators	PERMITTED
Books/Statutes	NOT PERMITTED
Additional Stationary	NOT REQUIRED

Question 1 (True/False Questions) [8 marks]

For each of the statements below, say whether it is true or false, and then provide your **justification**.

- If f_1, f_2 and f_3 are all admissible functions, then $\frac{f_1}{6} + \frac{f_2}{3} + \frac{f_3}{2}$ is admissible. (4 marks)

- If breadth-first search can find a solution to a search problem, then A* is guaranteed to find a solution. (4 marks)

Question 2 (Heuristics) [4 marks)

A sliding-tile puzzle consists of three black tiles, three white tiles and an empty space, thus:

B1	B2	B3		W1	W2	W3

There are three legal ways of moving a tile, each with an associated cost:

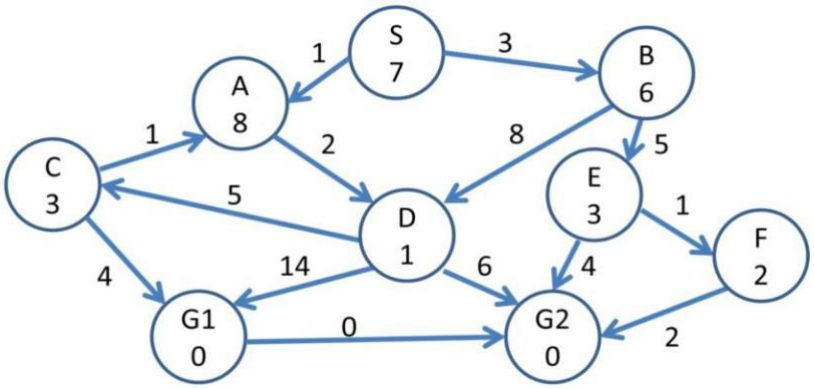
- slide into the adjacent empty location — cost 1
- jump over one tile into the empty location — cost 1
- jump over two tiles into the empty location — cost 2

The goal is to have all the white tiles to the left of all the black tiles and to achieve this at minimum cost. The final position of the empty space is not important.

State **two** possible **admissible heuristics** to help solve this problem and explain. (4 marks)

Question 3 (Search) [18 marks]

Consider the following search space, where S is the initial state and G1 and G2 are goal states, where the cost of the operator that takes you from one state to the next is given along the edge between the states, and where the value of the heuristic evaluation function h applied to the state is the number written inside the state. In any cases of a tie pick the state that comes earlier in the alphabet.



- Give the sequence of states that breadth-first graph search visits. (4 marks)
- Give the sequence of states that depth-first graph search visits. (4 marks)
- Give the sequence of states that greedy best first graph search visits (4 marks)
- Give the sequence of states that A* graph visits if it uses f(s) = g(s) + h(s). (4 marks)
- Is h(s) admissible? (2 marks)

Question 4. (Uncertainty) [16 marks]

(a) For each expression, state whether it is equal to $P(A, B, C)$ given no independence assumptions. [8 marks – 2 marks for each part]

i. $P(C \mid A, B) \,.\, P(A, B)$ (2 marks)

ii. $P(C \mid A, B) \,.\, P(A) \,.\, P(B)$ (2 marks)

iii. $P(C \mid A, B) \,.\, P(A, B)$ (2 marks)

iv. $P(A \mid B, C) \,.\, P(B \mid C) \,.\, P(C)$ (2 marks)

(b) Suppose A and B are independent boolean variables. Determine the missing entries x and y in the joint distribution of $P(A, B)$ shown below. You must show the detailed workings. [8 marks – 4 marks for correct value of x with correct workings and 4 marks for correct value of y with correct workings]

$$P(A = T, B = T) = 0.15$$
$$P(A = T, B = F) = 0.45$$
$$P(A = F, B = T) = x$$
$$P(A = F, B = F) = y$$

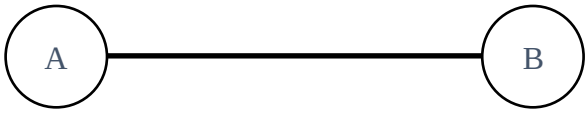
$x =$ _____ (4 marks) $y =$ _____ (4 marks)

Question 5. (Bayesian Networks and CSP) [10 marks]

Let us consider how to solve CSPs by converting them to Bayesian networks.

We can solve for values of variables that satisfy CSPs by running inference for the query on the constraints, where the values of the variables with nonzero probability are those that satisfy the constraints.

Let us analyse a CSP of only two integer variables A and B each with domain {+1, −1}. **The two variables are subject to the constraint that** $A + B \neq 0$. This is shown below with the edge between the two nodes representing the constraint.



Enforce the above constraint by representing the variables in a Bayesian Network. This formation must represent each variable as a node with a **directed** edge showing influence between the two variables. **Draw the Bayes' net** (*2 marks*) and **define all necessary conditional probability tables** (*8 marks*) with values such that consistent settings of the variables correspond to nonzero probabilities.

$\boldsymbol{P(A)}$ **has been provided** to get you started.

A	$P(A)$
$+1$	0.5
-1	0.5

Question 6. (Machine Learning) [14 marks]

Cluster the following eight points (*with (x, y) representing locations*) into **three clusters**:

$A1$ (2, 10), $A2$ (2, 5), $A3$ (8, 4), $A4$ (5, 8), $A5$ (7, 5), $A6$ (6, 4), $A7$ (1, 2), $A8$ (4, 9)

The initial cluster centers are given as $Cluster - 01$ (2, 10), $Cluster - 02$ (5, 8) and $Cluster - 03$ (1, 2).

The distance function, $d(a, b)$, between two points a = ($x1$, $y1$) and b = ($x2$, $y2$) is defined as:

$$\boldsymbol{d(a, b) = |x2 - x1| + |y2 - y1|}$$

For example, if a = (2, 10) and b = (5, 8), then
$d(a, b)$
$= |5 - 2| + |8 - 10|$
$= 3 + 2$
$= 5$

Using the K-Means Algorithm, find the three cluster centers after the <u>first</u> iteration.

Complete the below table to show the following (8 marks):

- distance between every point to every cluster center
- the cluster that each point belongs to

No marks will be awarded if the calculations are wrong.

Given Points	Distance from center (2, 10) of $Cluster - 01$	Distance from center (5, 8) of $Cluster - 02$	Distance from center (1, 2) of $Cluster - 03$	Point belongs to Cluster
A1(2, 10)				
A2(2, 5)				
A3(8, 4)				
A4(5, 8)				
A5(7, 5)				
A6(6, 4)				
A7(1, 2)				
A8(4, 9)				

Now, compute the new cluster centers below (6 marks):

$Cluster - 01$: _____ $Cluster - 02$: _____ $Cluster - 03$: _____