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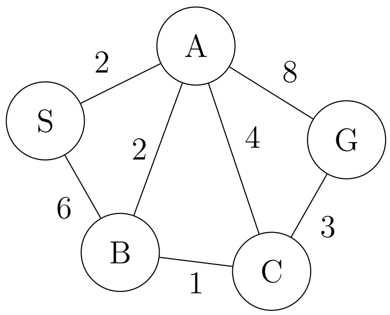
# CS460 Assignment 1

## Problem 1

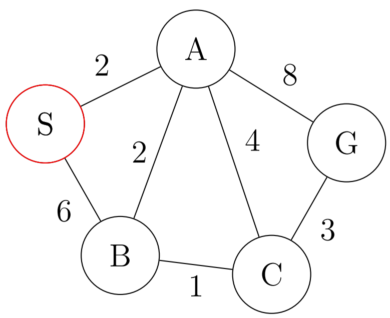
Uniform Cost Search ($f(n) = g(n)$) (Tiebreaker: Alphatecial Value)

1.



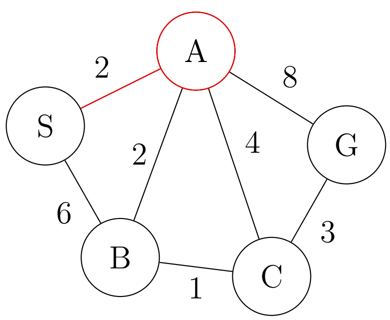

| Open List | $f(n)$ |
|---|---|
| S | 0 |
| | |
| | |
| | |

2.



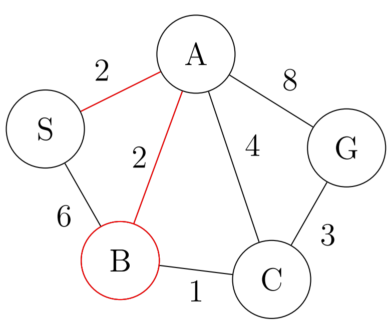

| Open List | $f(n)$ |
|---|---|
| A | 2 |
| B | 6 |
| | |
| | |

3.



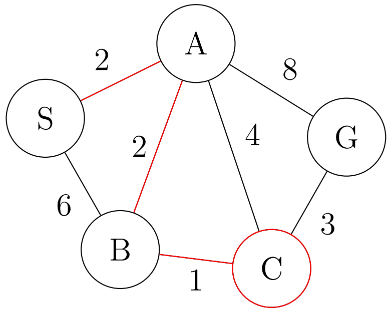

| Open List | $f(n)$ |
|---|---|
| G | 10 |
| B | 4 |
| C | 6 |
| | |

4.



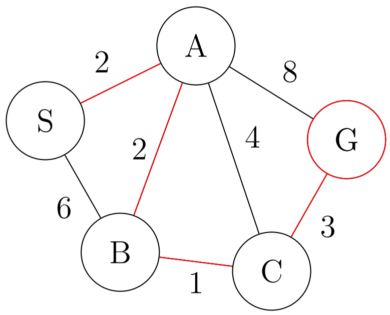

| Open List | $f(n)$ |
|---|---|
| G | 10 |
| C | 5 |
| | |
| | |

5.




| Open List | $f(n)$ |
|---|---|
| G | 8 |
| | |
| | |
| | |

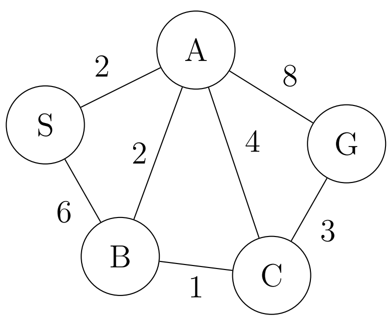
6.




Final Path: S $\rightarrow$ A $\rightarrow$ B $\rightarrow$ C $\rightarrow$ G
Cost: 8

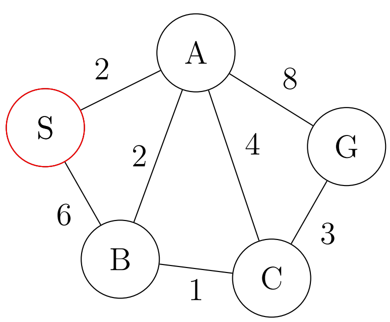
A* Search ($f(n) = g(n) + h(n)$) (Tiebreaker: Alphatecial Value)

1.



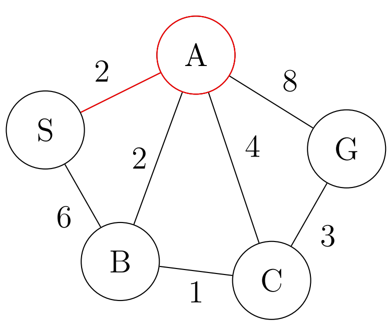

| Open List | $f(n)$ |
|---|---|
| S | 7 |
| | |
| | |
| | |

2.



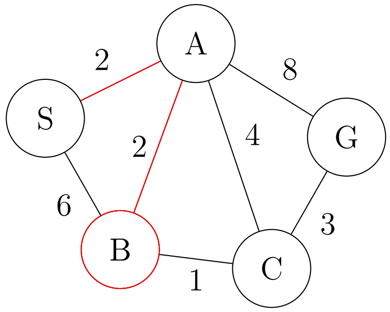

| Open List | $f(n)$ |
|---|---|
| A | $2 + 6 = 8$ |
| B | $6 + 2 = 8$ |
| | |
| | |

3.



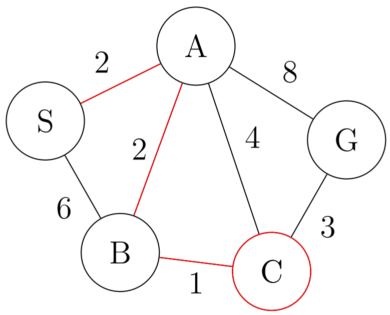

| Open List | $f(n)$ |
|---|---|
| G | 10 |
| B | $4 + 2 = 6$ |
| C | $6 + 1 = 7$ |
| | |

4.



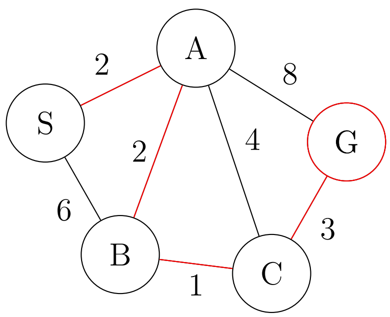

| Open List | $f(n)$ |
|---|---|
| G | 10 |
| C | $5 + 1 = 6$ |
| | |
| | |

5.




| Open List | $f(n)$ |
|---|---|
| G | 8 |
| | |
| | |
| | |

6.

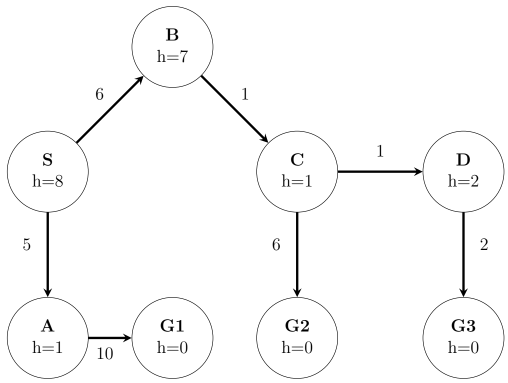



Final Path: S $\rightarrow$ A $\rightarrow$ B $\rightarrow$ C $\rightarrow$ G
Cost: 8

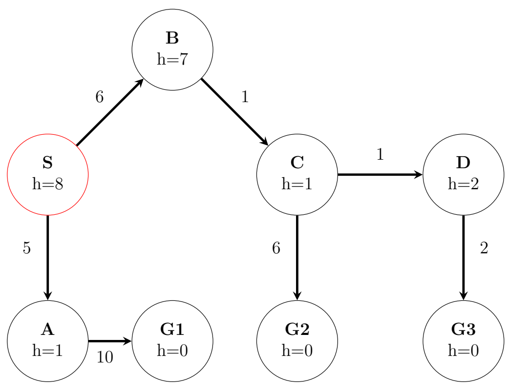
Both the algorithms utilized on this graph returned the optimal path from $S$ to $G$. The tiebreaker was utilized in A* but not in Uniform Cost Search. Both algorithms expanded to the same number of nodes. The heuristic of A* seems to be admissible and consistent, which explains to its optimality.

## Problem 2

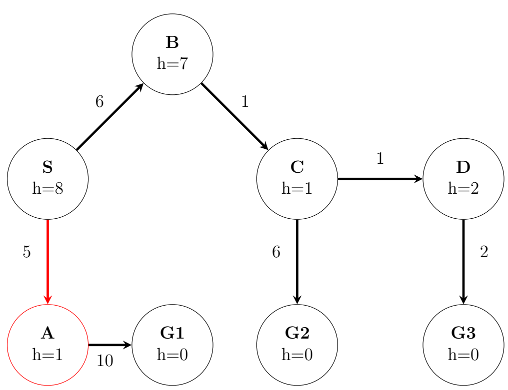
1.



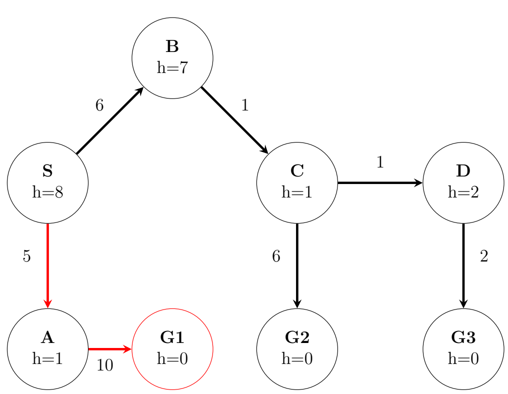

| Open List | $g(n) + \epsilon \cdot h(n) = f(n)$ |
|---|---|
| S | $0 + 2 \cdot 8 = 16$ |
| | |
| | |
| | |




| Open List | $g(n) + \epsilon \cdot h(n) = f(n)$ |
|---|---|
| B | $6 + 2 \cdot 7 = 20$ |
| A | $5 + 2 \cdot 1 = 7$ |
| | |
| | |




| Open List | $g(n) + \epsilon \cdot h(n) = f(n)$ |
|---|---|
| B | $6 + 2 \cdot 7 = 20$ |
| G1 | $15 + 2 \cdot 0 = 15$ |
| | |
| | |




Final Path: S $\rightarrow$ A $\rightarrow$ G
Cost: 15

2. The strongest bound that can be proven is $g(G) \leq \epsilon \cdot C^*$.

   Suppose there is a node $n$ on an optimal path to goal node $G^*$.
   Since $h(n)$ is admissible, $h(n) \leq c(n, G^*)$ where $c(n, G^*)$ is the optimal cost to reach $G^*$ from $n$, thusly having $g(n) + h(n) \leq g(n) + c^*(n, G) = C^*$.
   It would then follow $\epsilon \cdot (g(n) + h(n)) \leq \epsilon \cdot C^*$.
   Since a goal node $G$ was already found with a cost of $g(G)$, it must be $\leq g(n) + \epsilon \cdot h(n)$.

   Therefore, because $g(n) + \epsilon \cdot h(n) \leq \epsilon \cdot (g(n) + h(n))$, $g(G) \leq \epsilon \cdot C^*$.

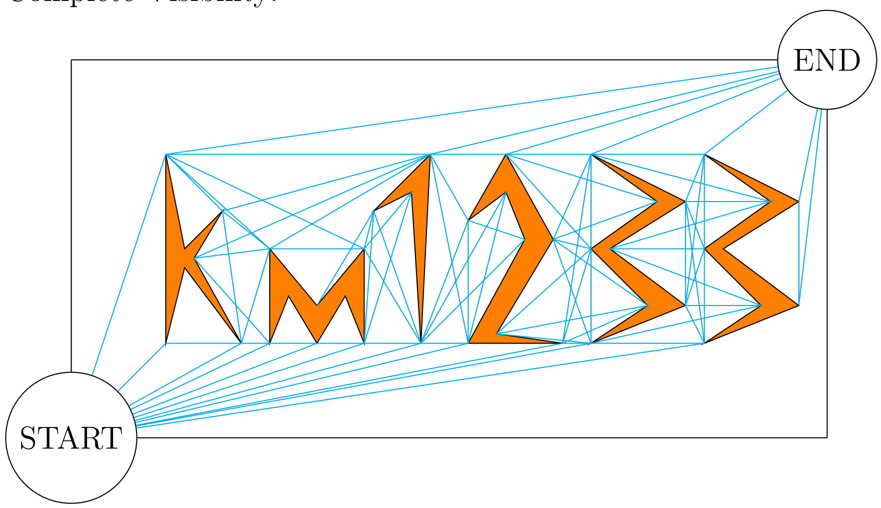
3. i Here, since $\epsilon = 1$, $f(n) = g(n) + h(n)$. Thusly, its equivalent to A*. Guaranteed optimality for path but not necessarily fast in terms of search speed.

   ii Here, the heuristic is ignored since $\epsilon = 0$, so its equivalent to Uniform Cost Search. Similar case to A*.

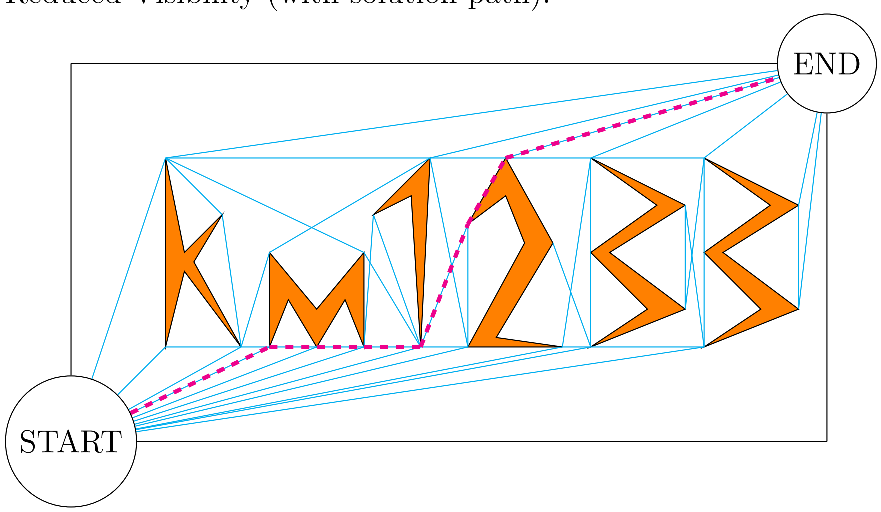
   iii Here, the least amount of nodes are expanded to since $\epsilon \to \infty$, so it will have an equivalent or better search speed than the previous cases. However, optimality is not guaranteed.

## Problem 3

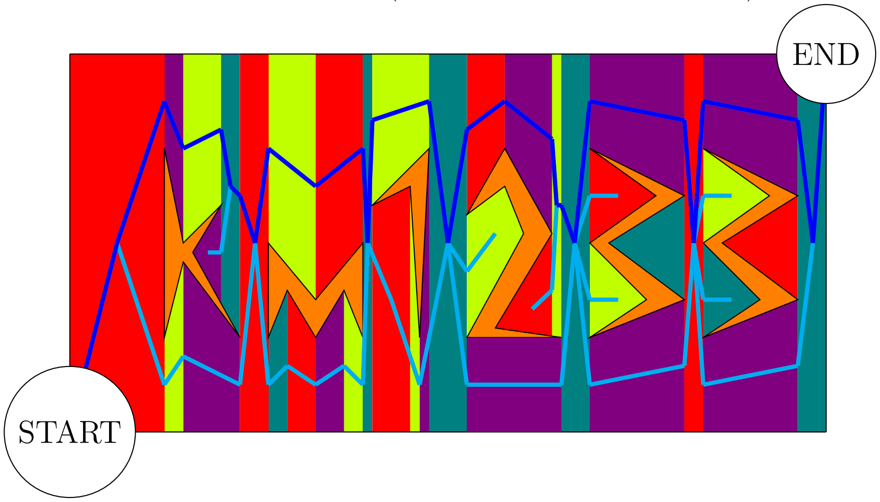
1. Complete Visibility:



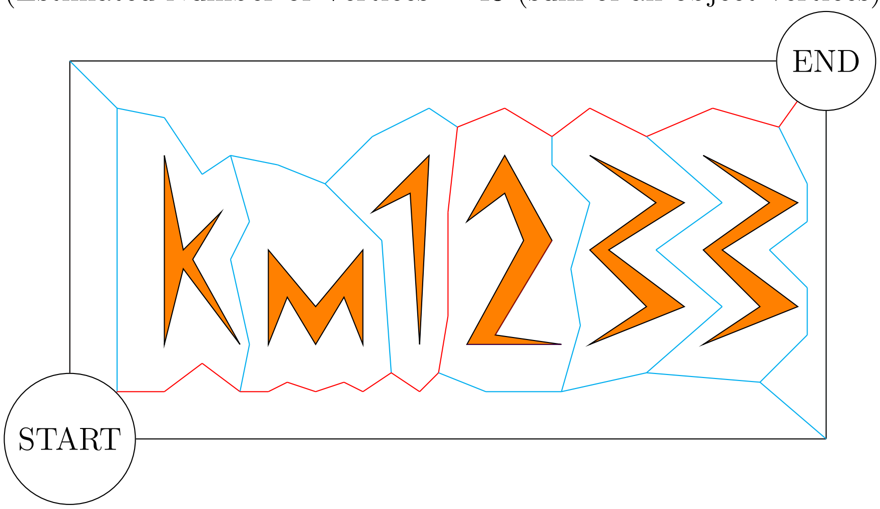

Reduced Visibility (with solution path):



2. Trapezoidal Decomposition (optimal path is dark blue):




3. Generalized Voronoi:
(Estimated Number of Vertices = 43 (sum of all object vertices))



## Problem 4

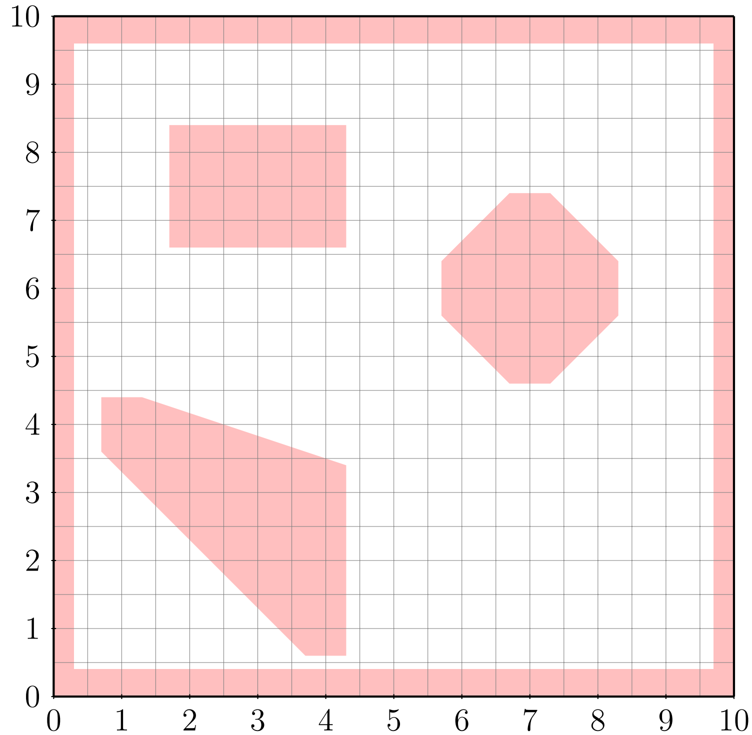
i. The C-space for the car is $S^2 \times S^1$ since the car moving around the 2D surface of the sphere ($S^2$) and can also rotate itself along the surface as it does on the ground ($S^1$).

ii. The arms-plus-steering wheel system has $2 \cdot (n - 3)$ degrees of freedom since the elbows cannot bend due to the torso and the steering wheel being stationary relative to the car ($-1$ degree per arm) and the wrists cannot twist around or through the steering wheel ($-2$ degrees per arm).

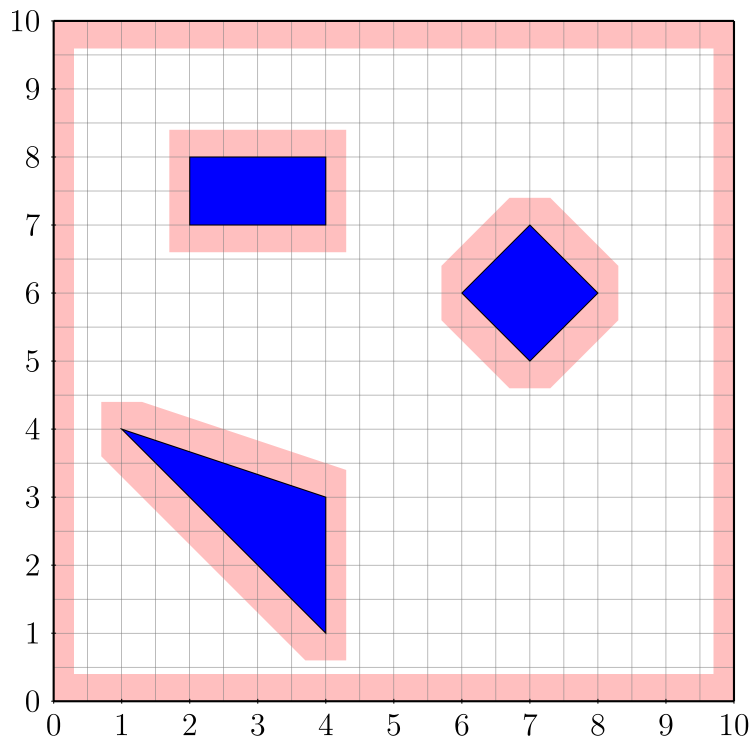
iii.
- The C-space of the mobile manipulator is $(\mathbb{R}^2 \times T^2) \times T^6$.
- The mechanism formed by the arm and the open door has 5 degrees of freedom, with 6 degrees of freedom from the arm, 1 from the door, and 2 rotational degrees lost due to the hand being attached to the door.
- The mechanism formed by the two arms and the open door has 9 degrees of freedom, with 12 degrees of freedom from the two arms, 1 from the door, and the same losses in rotational degrees for the arms as the previous part stated.

## Problem 5

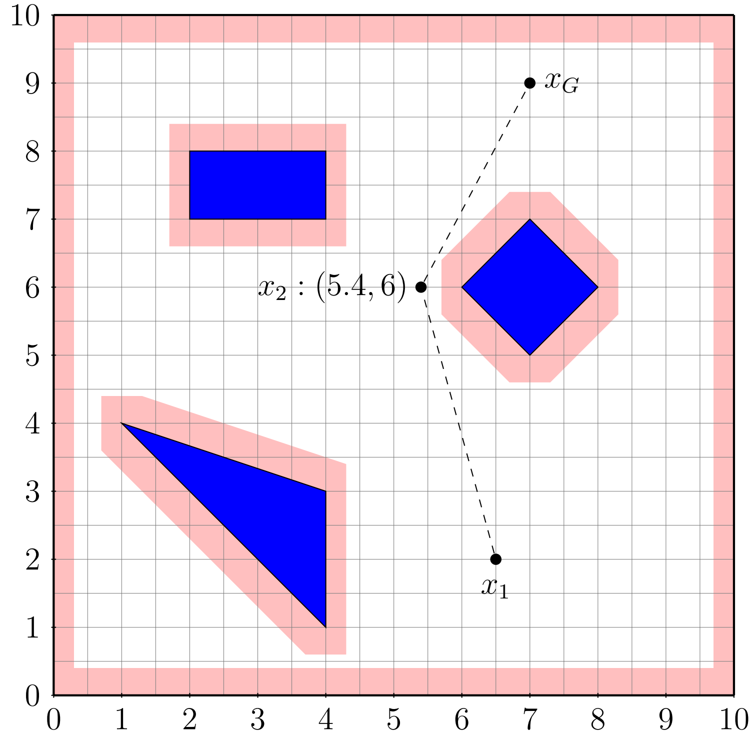
i $C_{free}$ is respresented by the white area in the grid. The $CO$ robot is




ii $C_{free}$ is connected.



iii Collision Free Path from $x_1$ to $x_G$:




Path: $x_1 \rightarrow x_2 \rightarrow x_G$

## Problem 6

i $J = \frac{A+B+C+D+E}{5}$ (approximate coordinates of polygon centroid)

$$T = \begin{bmatrix} 1 & 0 & J_x \\ 0 & 1 & J_y \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos(40^\circ) & -\sin(40^\circ) & 3 \\ \sin(40^\circ) & \cos(40^\circ) & 4 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & -J_x \\ 0 & 1 & -J_y \\ 0 & 0 & 1 \end{bmatrix}$$

ii Approximate Results:
$A' = (11.49, 6.76)$, $B' = (11.74, 9.58)$, $C' = (13.91, 10.1)$, $D' = (14.43, 7.93)$, $E' = (13.54, 5.87)$.

iii Yes, edge $FG$ and $GH$ intersect $AB$ on the transformed rigid body.

iv The transformation involves tranlation of (5,4) and a 90° rotation about $A$.

$$T' = \begin{bmatrix} 1 & 0 & 8 \\ 0 & 1 & 4 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos(90^\circ) & -\sin(90^\circ) & 5 \\ \sin(90^\circ) & \cos(90^\circ) & 4 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & -8 \\ 0 & 1 & -4 \\ 0 & 0 & 1 \end{bmatrix}$$