Design and Analysis of Algorithms I

Graph Primitives

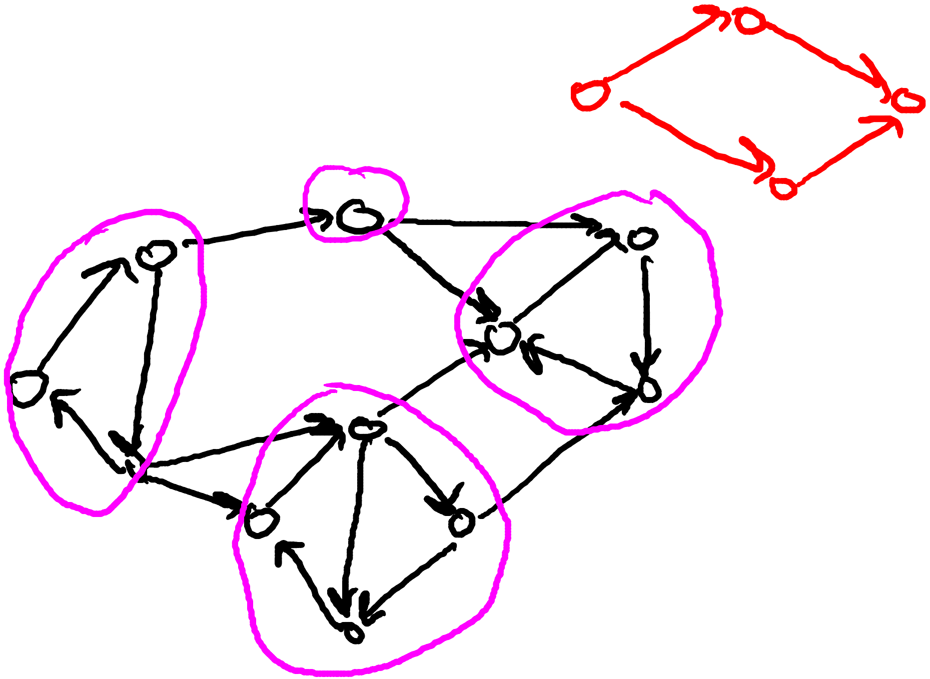
An O(m+n) Algorithm for Computing Strong Components

Strongly Connected Components

Formal Definition : the strongly connected components (SCCs) of a directed graph G are the equivalence classes of the relation

u<->v <==> there exists a path u->v
and a path v->u in G

You check : <-> is an equivalence relation



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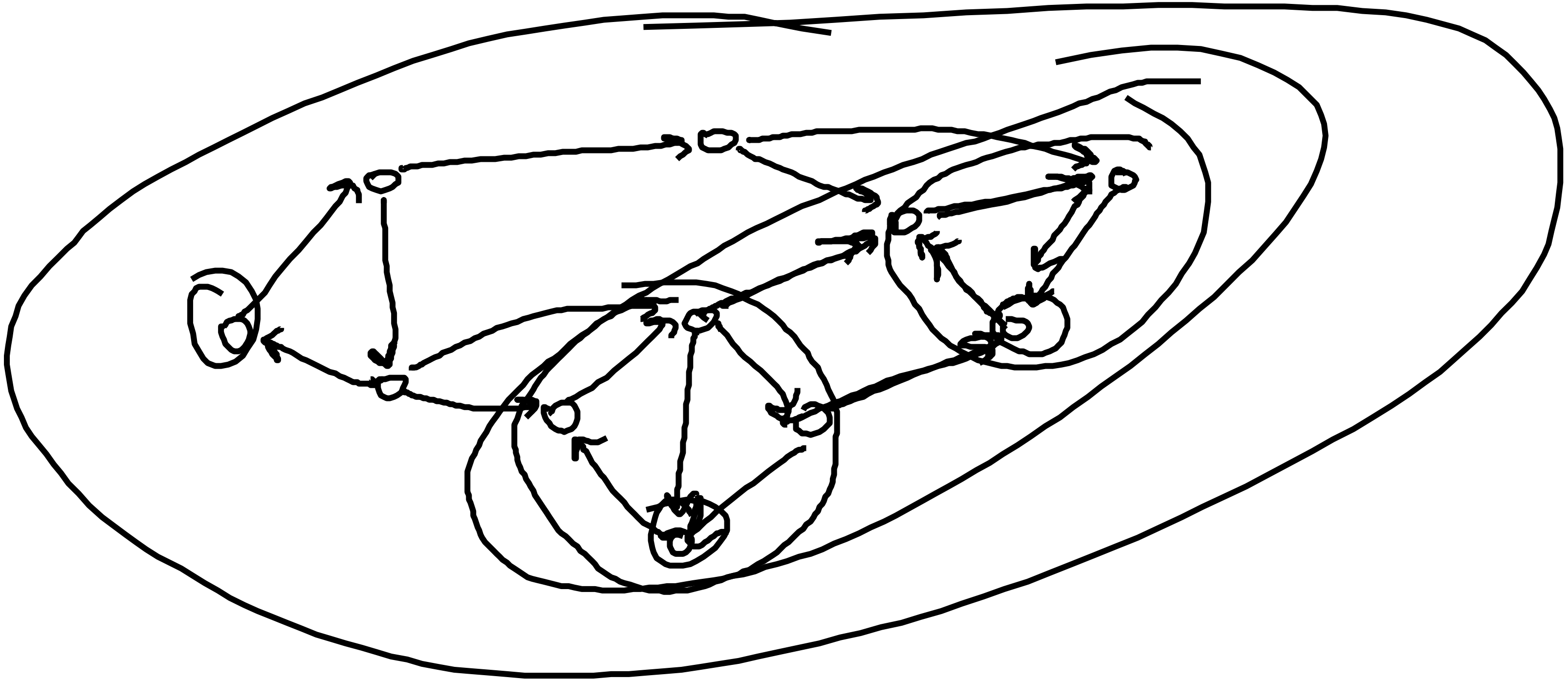
Why Depth-First Search?



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Kosaraju's Two-Pass Algorithm

Theorem : can compute SCCs in O(m+n) time.

Algorithm : (given directed graph G)

1. Let Grev = G with all arcs reversed
2. Run DFS-Loop on Grev

   Goal : compute "magical ordering" of nodes

   Let f(v) = "finishing time" of each v in V

1. Run DFS-Loop on G

   Goal : discover the SCCs one-by-one

   processing nodes in decreasing order of finishing times

[SCCs = nodes with the same "leader"]

DFS-Loop

<u>DFS-Loop (graph G)</u>
Global variable t = 0
[# of nodes processed so far]
Global variable s = NULL
[current source vertex]
Assume nodes labeled 1 to n
For i = n down to 1
    if i not yet explored
        s := i
        DFS(G,i)

For finishing times in 1st pass

For leaders in 2nd pass

DFS (graph G, node i)
-- mark i as explored
-- set leader(i) := node s
-- for each arc (i,j) in G :
    -- if j not yet explored
        -- DFS(G,j)
-- t++
-- set f(i) := t

For rest of DFS-Loop

if no nodes left then label it with the biggest number

i's finishing time

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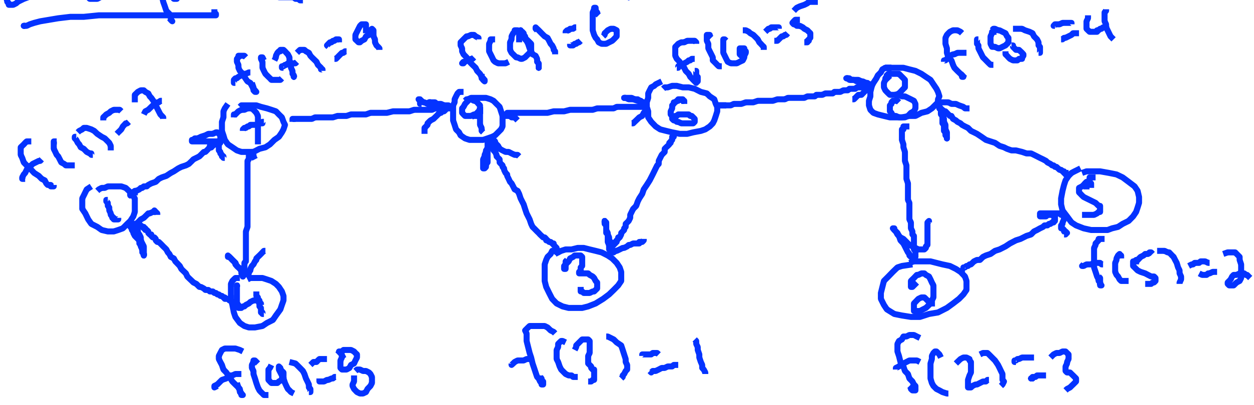
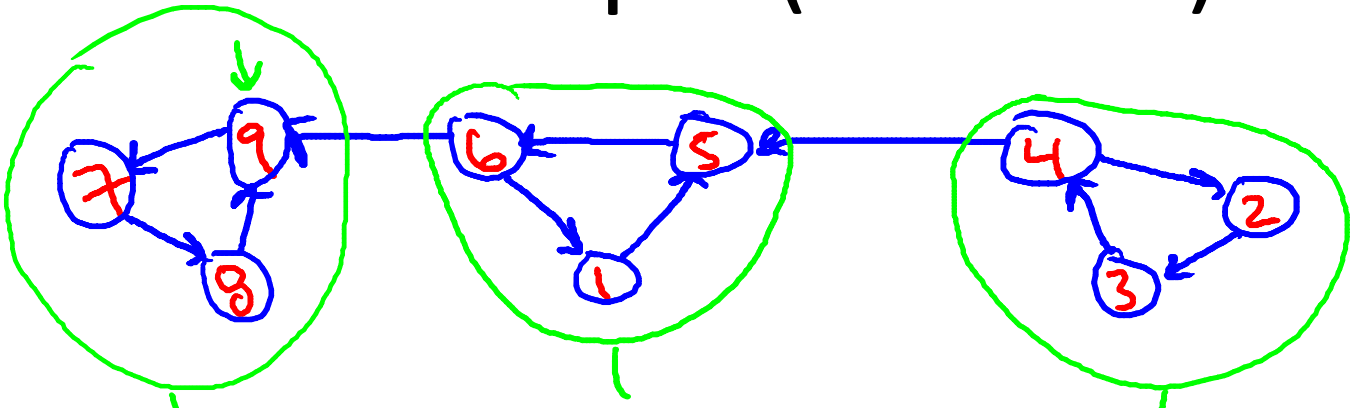
Only one of the following is a possible set of finishing times for the nodes 1,2,3,…,9, respectively, when the DFS-Loop subroutine is executed on the graph below. Which is it?

Example: [1st DFS-Loop on Grev]

- 9,8,7,6,5,4,3,2,1
- 1,7,4,9,6,3,8,2,5
- 1,7,9,6,8,2,5,3,4
- 7,3,1,8,2,5,9,4,6



Example (2nd Pass)



Leader = 9

Leader = 6

Leader = 4

Running Time : 2*DFS = O(m+n)