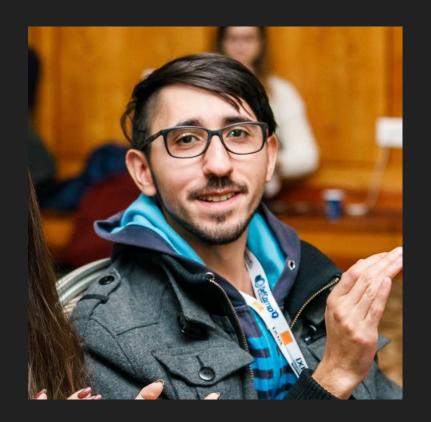
# Crypto

Bamboofox

# Who am I?

- ➤ 毛叡 (Maojui)
- ▶ 中央資工 4
- Crypto
- DoubleSigma / BFS
- > github : maojui



# Outline

- 內容
- 背景知識
- 古典密碼學
- 現代密碼學
- Hash
- 各種工具
- CTF 水題

# 內容

- 一些密碼學的小常識
- 概述 CTF Crypto 常見的題目類型
- CTF Crypto 水題解法
- 覺得我太冗我先道歉 QAQ...

# 為什麼從 CTF 講?

因為 CTF 的 Crypto 提供了不錯的情境

- 他減弱了真實情況的安全性 ... 減到讓你可以破解
- 假設存在這些漏洞,你能不能發現、找到,能不能夠破解
- 有些是真實誤用情況:參數亂放、變數打錯 ....
- 讓你的論文有應用的時機 ...
- <del>● 讓你有讀論文的理由 ...</del>

# 密碼學在幹嘛?

- 研究如何編制密碼和破譯密碼的技術
- 研究如何隱密地傳遞信息
- 研究如何破解別人的加密方法
- 研究如何玩弄數字讓其他人都無法破解
- 研究如何讓同學看不懂你在幹嘛、連隊友都不想理你
- 研究一堆跑來跑去的數字有什麼規則可以利用
- 算一堆很難的數學
- ◆ CTF 裡沒人愛的那一塊

# 密碼學可以幹嘛

- 可以讓你的聊天內容被竊聽時,也不會馬上外洩
- 可以讓你的產品有點保障
- 可以降低你們公司資料外洩的風險
- **可以侵犯他人隱私**
- 可以藏你的惡意程式
- ◆ 可以勒索別人

# 基本觀念

 真正強的加密演算法,是在提供程式原始碼,及大量明文、 密文對仍要花大量時間才能破解。通常是經過了長時間的測 試、計算才成形的。( -> 不要自己設計 ....)

Kerckhoffs's principle 不是說密碼學演算法都必須公開,而是要確保即使公開也 不會傷害安全性

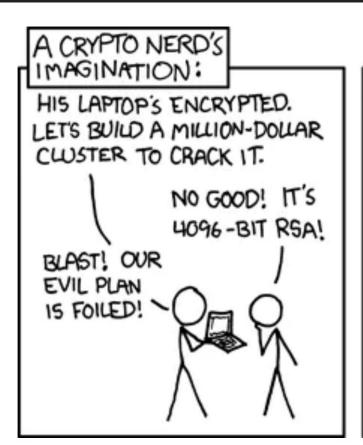
- 1. 實務上無法破解(或者要花費很大的成本)
- 2. 沒有什麼不能夠落入敵手的東西
- 3. 密鑰不能在記憶、傳遞與改變等方面太過困難
- 4. 可以數位化
- 5. 不能太笨重
- 6. 不能太難懂

#### Danger

This is a "Hazardous Materials" module. You should **ONLY** use it if you're 100% absolutely sure that you know what you're doing because this module is full of land mines, dragons, and dinosaurs with laser guns.

# 基本觀念

密碼不是靠暴力破解,而是 bypass



WHAT WOULD ACTUALLY HAPPEN: HIS LAPTOP'S ENCRYP DRUG HIM AND HIT H

HIS LAPTOP'S ENCRYPTED.

DRUG HIM AND HIT HIM WITH

THIS \$5 WRENCH UNTIL

HE TEUS US THE PASSWORD.



# 密碼學

● 古典密碼學 Classical Cryptography

● 現代密碼學 Modern Cryptography

古典密碼學,因為加密簡單 (破解也簡單) 實用性比較低

他們的存在僅供參考和教學用途

加解密方法和破解方法 大多隨便 google 到處都是

比較常出現在 Misc 或是送分的 Crypto

#### 看到他們的話

- 1. 根據規則或是提示或是各種資訊找到演算法名字
- 2. Google ...

```
<del>- 或直接丟 <u>quipquip</u> ..</del>
```

Substitution ciphers
 <a href="https://maojui.github.io/Crypto/Classical-I/">https://maojui.github.io/Crypto/Classical-I/</a>

Transposition ciphers
 <a href="https://maojui.github.io/Crypto/Classical-II/">https://maojui.github.io/Crypto/Classical-II/</a>

資料來源: <u>practical cryptography</u>

### Substitution ciphers

● 凱薩密碼 ( Caesar )

• ROT13

● 維吉尼爾加密 ( Vigenère cipher )

加密 : Ek (m) = ( m + k ) mod 26

解密 : Dk (c) = ( c - k ) mod 26

1. Message = CLASSICAL IS BORED

1. Message = CLASSICAL IS BORED

2. key = H(8)

1. Message = CLASSICAL IS BORED

2. key = H(8)

3.  $C(3) + H(8) \rightarrow K(11)$ 

```
3 12 1 19 19 .......
```

CLASSICALIS BORED

KTIAAQKITQAJWZML

11 20 9 1 1 ......

破解 ... ??

反正只有 26 種 ... 暴力解

https://planetcalc.com/1434/

# ROT13

Key = 13 的 凱薩

```
KLMNOPQRSTUVWXYZ
           KLMNOPQRSTUVWXYZ
          KLMNOPQRSTUVWXYZA
YYZABCDEFGHIIKLMNOPQRSTUVWX
ZZABCDEFGHIJKLMNOPQRSTUVWXY
```

維吉尼爾密碼 = 一堆凱薩密碼

Message = VigenereCipher

維吉尼爾密碼 = 一堆凱薩密碼

Message = VigenereCipher

Key = code

維吉尼爾密碼 = 一堆凱薩密碼

Message = VigenereCipher

Key = code

→ Extend Key = codecodeco

維吉尼爾密碼 = 一堆凱薩密碼

Message = VigenereCipher

Extend Key = codecodecodeco (Do caesar)

維吉尼爾密碼 = 一堆凱薩密碼

Message = VigenereCipher

Extend Key = codecodeco

Cipher = XwjipsuiEwslgf

破解 ... ??

南京郵電大學 Write up:

https://hackmd.io/KwQwpsDsCcoLQEYDG0AsdUqQEzqI1QAZq4RjDVskA2Q6mIA=

- Index of Coincidence (IC)
- https://en.wikipedia.org/wiki/Index\_of\_coincidence
- Frequency analysis :
  - calculate the frequency of unigram, bigram, ...
- Natural Language Processing

#### **Transposition ciphers**

• Rail Fence Cipher

● Square 類 (避免 I, J 同時出現 )

```
ex:

5x5 = 25 (26-25)

Juice -> Iuice
```

# Rail Fence Cipher (key = 3)

- Example:
- m = WE ARE DISCOVERED. FLEE AT ONCE
- c = WECRL TEERD SOEEF EAOCA IVDEN

```
W . . . E . . . C . . . R . . . L . . . T . . . E . E . R . D . S . O . E . E . F . E . A . O . C .
```

# 其他古典密碼

ADFGX Cipher	ADFGVX Cipher	Affine Cipher	Autokey Cipher
Beaufort Cipher	Enigma M3 Cipher	Foursquare Cipher	<u>Gronsfeld Cipher</u>
<u>Bifid Cipher</u>	M-209 Cipher	<u>Playfair Cipher</u>	Polybius Square  Cipher
<u>Porta Cipher</u>	Railfence Cipher	Substitution Cipher	Rot13 Cipher

# 破解方法

#### Online Break !!!

- 1. 知道大概規則
- 2. 找到演算法名字
- 3. Google ...

或直接:

quipquip

# XOR

Flipping bit, Inverting bit ....

Message = 'a', Key = 'k', Cipher = '\n'

Message ^ key = Cipher

A	B	XOR
0	0	0
0	1	1
1	0	1
1	1	0

10 (00001010) ('\n')

```
=> Message ( * key * key ) = Message * 0 = Message
=> xor eax, eax = 0
```

$$=> a ^b = c, c^b = a$$

#### 最簡單且省成本的加密

```
def xor(data, key):
    repeat = len(data)//len(key)+1
    return ''.join(chr(ord(a)^ord(b)) for a,b in zip(data,key* repeat ))
```

```
>> data = 'CaptureTheFlag'
>> key = 'password'

>> encrypted = xor(data,key)
'3\x00\x03\x07\x02\x1d\x170\x18\x045\x1f\x16\x08'

>> decrypted = xor(encrypted,key)
'CaptureTheFlag'
```

xortool: 真D好用的工具, 解不開的 xor, 懶得解的 Vigenère cipher, 直接丟進去即可。

## One-Time-Pad @@

#### One-Time-Pad

用嚴密的演算法得到 truly random data 來加密 (XOR 或者 啥的)

又因為用完即丟,就算被知道了上一段的 Key

下一段仍然無法預測。

#### One-Time-Pad

這種加密方法理論上絕對安全, 但是實際上會有些問題

- 1. 安全的共享一個和明文一樣長的 key(幹嘛不直接共享明文)或亂數演算法
- 2. truly random 不易實現
  - a. DVD加密
  - b. RC4
  - c. 802.11b協定
- 3. 一次性密碼本成本高,常常因多次使用造成問題 QQ
  - a. 蘇聯士兵通訊
  - b. Windows NT 的 PPTP 協定

Many time pad -> <a href="Crib Drag">Crib Drag</a>

EX:

m1 = "Hello World" m2 = "the program" key = "supersecret"

You get:

c1: "3b101c091d53320c000910"

c2: "071d154502010a04000419"

c1: "3b101c091d53320c000910"

c2: "071d154502010a04000419"

guess: 'thethetheth' 'hethethethe' 'ethethethet' (XOR cipher2)

cipher-text1: "3b101c091d53320c000910"

cipher-text2: "071d154502010a04000419"

guess: 'thethetheth' 'hethethethe' 'ethethethet' (XOR cipher2)

out : 'sup1jd~lepq' 'oxa-gubatl|' 'bi} viopham' (再 XOR cipher1)

cipher-text1: "3b101c091d53320c000910"

cipher-text2: "071d154502010a04000419"

guess: 'thethetheth' 'hethethethe' 'ethethethet' (XOR cipher2)

out : 'sup1jd~lepq' 'oxa-gubatl|' 'bi} viopham' (再 XOR cipher1)

get : 'Hel8w7L`eya'

cipher-text1: "3b101c091d53320c000910"

cipher-text2: "071d154502010a04000419"

guess: 'thethetheth' 'hethethethe' 'ethethethet' (XOR cipher2)

out : 'sup1jd~lepq' 'oxa-gubatl|' 'bi} viopham' (再 XOR cipher1)

get : 'Hel8w7L`eya'

Next ...

guess : message1 = 'HELLO ...' do again ... then again ...

Modern Cryptography

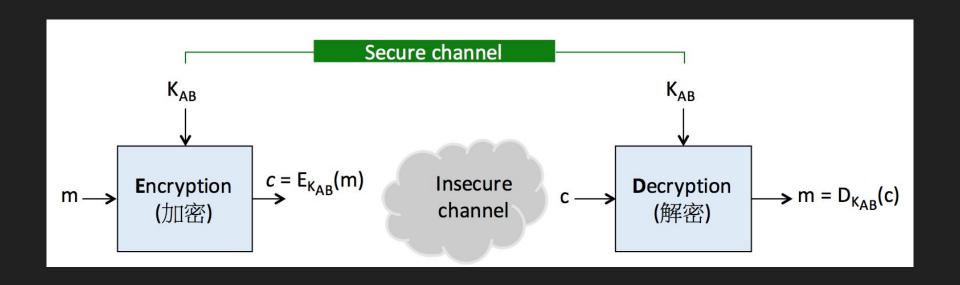
## Modern Cryptography

- 對稱式加密 Symmetric
  - 區塊(Block Ciphers): AES、DES
  - 串流 ( Stream Ciphers) : RC4

- 非對稱式加密 Asymmetric
  - Diffie-Hellman key agreement
  - Public-Private key Encryption: RSA, DSA, ECC
  - 數位簽章 Digital Signature

# Symmetric

## 對稱式加密 Symmetric



## 對稱式加密 Symmetric

#### 優點:

- 加解密速度較快
- 提供私密性( Confidential ) => 由密文推不出明文

#### 缺點:

● 不具有不可否認性 (non-repudiation) => 不能確定加密的人是誰

## 對稱式加密 Symmetric

Block Cipher

Stream Cipher

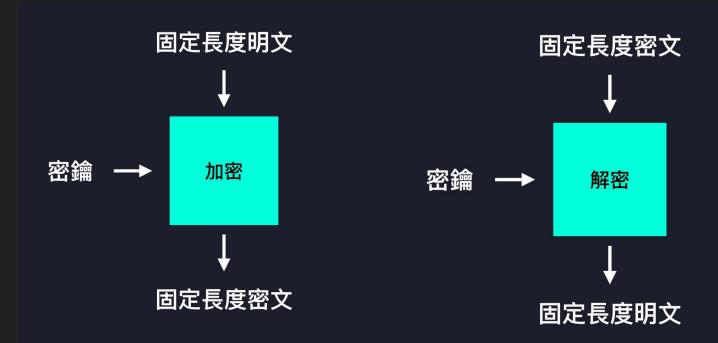
## **Block Cipher**

Data Encryption Standard (DES)

• Tripple DES (3DES)

Advenced Encryption Standard (AES)

## **Block Cipher**



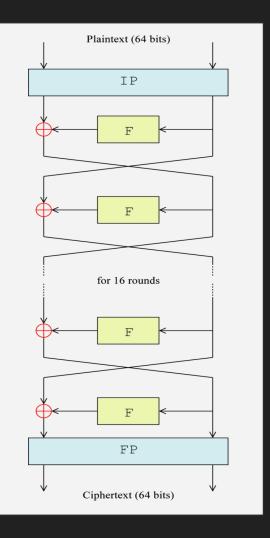
## DES (Data Encryption Standard)

#### Weakness:

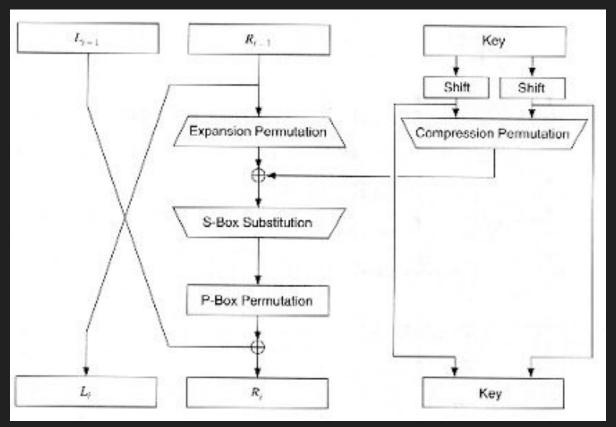
- n=64 ( Too small)
- k=56 ( <u>Weak keys</u> )

#### Break

- Brute-force
- <u>linear cryptanalysis</u> (sbox:5th bit) 2^43 plaintxt
- DES-cracking hardware



## DES (Data Encryption Standard)

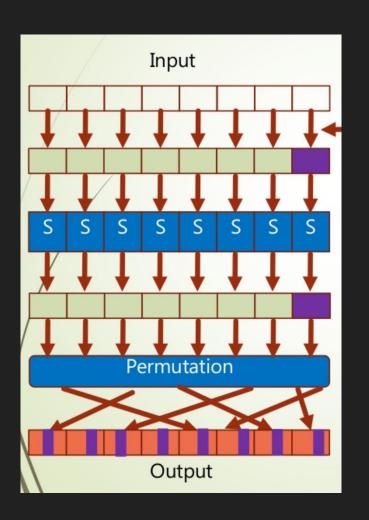


### SPN (Substition Permutation Networks)

```
• Sbox : [14, 4, 13, 1, 2, 15, 11, 8, 3, 10, 6, 12, 5, 9, 0, 7]
output = sbox[ input ]
```

```
Pbox: [ 0, 4, 8, 12,
1, 5, 9, 13,
2, 6, 10, 14,
3, 7, 11, 15 ]
```

```
output = [input[pbox[i]] for i in range(64)]
```



## Tripple DES (3DES) → 修正 DES weak keys

- n = 64
- k = 56, 112, 168
- Encryption of P: EK3(DK2(EK1(P)))
- Decryption of C: DK1(EK2(DK3(C)))
- 3 keys, each is 56-bit long
  - Option 1: K1, K2, K3 are independent
  - Option 2: K1, K2 are independent, K3 = K1
  - Option 3: K1 = K2 = K3 => reduced to DES

## Linear cryptanalysis

- Known-plaintext 攻擊法
- 利用大量蒐集到的明文/密文對的相關性, 對加密法進行攻擊
- 明文/密文對的相關性由線性軌跡 (Linear trails) 所組成, 由於線性軌跡的相關係數與Round keys的值有密切關係, 透過相關係數的正負號, 線性攻擊 法就可以找出金鑰值。

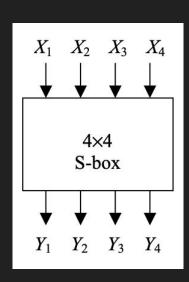
#### 優質論文:

https://www.engr.mun.ca/~howard/PAPERS/ldc\_tutorial.pdf

## Linear cryptanalysis

一個好的 Sbox 從哪裡進來跟從哪裡出去 0,1 的機率應該要一樣為 ½

如果不同, 就存在 bias



$X_1$	$X_2$	$X_3$	<i>X</i> <sub>4</sub>	<i>Y</i> <sub>1</sub>	$Y_2$	$Y_3$	$Y_4$	$X_2$ $\oplus X_3$	$Y_1$ $\oplus Y_3$	$X_1$ $\oplus X_4$	<i>Y</i> <sub>7</sub>	$X_3$ $\oplus X_4$	$Y_1$ $\oplus Y_4$
									$\oplus Y_4$				
0	0	0	0	1	1	1	0	0	0	0	1	0	1
0	0	0	1	0	1	0	0	0	0	1	1	1	0
0	0	1	0	1	1	0	1	1	0	0	1	1	0
0	0	1	1	0	0	0	1	1	1	1	0	0	1
0	1	0	0	0	0	1	0	1	1	0	0	0	0
0	1	0	1	1	1	1	1	1	1	1	1	1	0
0	1	1	0	1	0	1	1	0	1	0	0	1	0
0	1	1	1	1	0	0	0	0	1	1	0	0	1
1	0	0	0	0	0	1	1	0	0	1	0	0	1
1	0	0	1	1	0	1	0	0	0	0	0	1	1
1	0	1	0	0	1	1	0	1	1	1	1	1	0
1	0	1	1	1	1	0	0	1	1	0	1	0	1
1	1	0	0	0	1	0	1	1	1	1	1	0	1
1	1	0	1	1	0	0	1	1	0	0	0	1	0
1	1	1	0	0	0	0	0	0	0	1	0	1	0
1	1	1	1	0	1	1	1	0	0	0	1	0	1

bias = -¾ ( ½ - ½ )

		Output Sum															
		0	1	2	3	4	5	6	7	8	9	Α	В	C	D	E	F
	0	+8	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	1	0	0	-2	-2	0	0	-2	+6	+2	+2	0	0	+2	+2	0	0
	2	0	0	-2	-2	0	0	-2	-2	0	0	+2	+2	0	0	-6	+2
I	3	0	0	0	0	0	0	0	0	+2	-6	-2	-2	+2	+2	-2	-2
n	4	0	+2	0	-2	-2	-4	-2	0	0	-2	0	+2	+2	-4	+2	0
p u	5	0	-2	-2	0	-2	0	+4	+2	-2	0	-4	+2	0	-2	-2	0
t	6	0	+2	-2	+4	+2	0	0	+2	0	-2	+2	+4	-2	0	0	-2
	7	0	-2	0	+2	+2	-4	+2	0	-2	0	+2	0	+4	+2	0	+2
S	8	0	0	0	0	0	0	0	0	-2	+2	+2	-2	+2	-2	-2	-6
u	9	0	0	-2	-2	0	0	-2	-2	-4	0	-2	+2	0	+4	+2	-2
m	Α	0	+4	-2	+2	<b>-4</b>	0	+2	-2	+2	+2	0	0	+2	+2	0	0
	В	0	+4	0	-4	+4	0	+4	0	0	0	0	0	0	0	0	0
	C	0	-2	+4	-2	-2	0	+2	0	+2	0	+2	+4	0	+2	0	-2
	D	0	+2	+2	0	-2	+4	0	+2	-4	-2	+2	0	+2	0	0	+2
	E	0	+2	+2	0	-2	-4	0	+2	-2	0	0	-2	-4	+2	-2	0
·	F	0	-2	-4	-2	-2	0	+2	0	0	-2	+4	-2	-2	0	+2	0

## Linear cryptanalysis

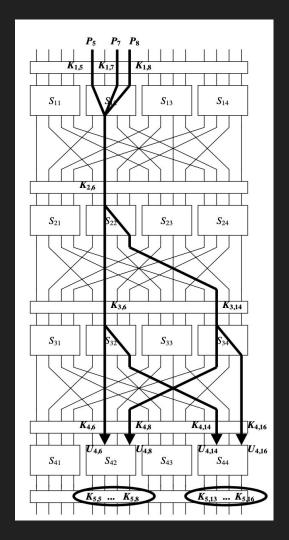
一層的 bias 可能很大,但是也可以在幾 Round 之後減小

右圖的 bias 相當於

 $\frac{1}{2} + \frac{2^3}{3} \left( \frac{3}{4} - \frac{1}{2} \right) \left( \frac{1}{4} - \frac{1}{2} \right)^3 = \frac{15}{32} = -\frac{1}{32}$ 

 $U_{4,6} \oplus U_{4,8} \oplus U_{4,14} \oplus U_{4,16} \oplus P_5 \oplus P_7 \oplus P_8 \oplus \Sigma_K = 0$ 

U<sub>4,6</sub> ⊕ U<sub>4,8</sub> ⊕ U<sub>4,14</sub> ⊕ U<sub>4,16</sub> ⊕ P<sub>5</sub> ⊕ P<sub>7</sub> ⊕ P<sub>8</sub> = 0 的機率為 1/32

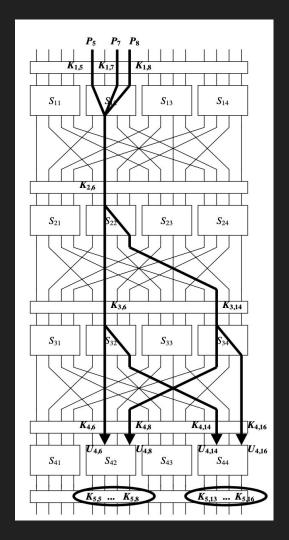


## Linear cryptanalysis

亂猜 key 的中間 2bytes

把該 key 加密過的 plaintext 撒過去計算

有誰的 bias 為 1/32,就猜中這倆 bytes 了

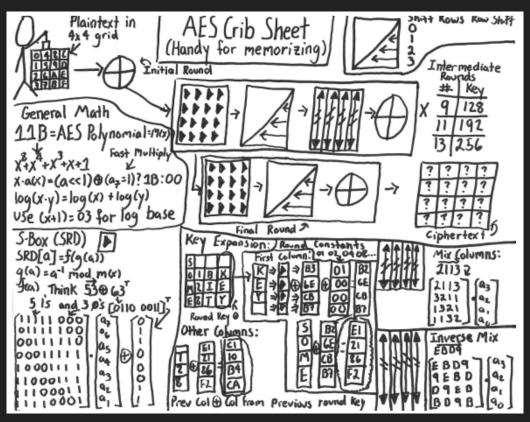


partial subkey $[K_{5,5}K_{5,8}, K_{5,13}K_{5,16}]$	bias	partial subkey $[K_{5,5}K_{5,8}, K_{5,13}K_{5,16}]$	bias
1 C	0.0031	2 A	0.0044
1 D	0.0078	2 B	0.0186
1 E	0.0071	2 C	0.0094
1 F	0.0170	2 D	0.0053
2 0	0.0025	2 E	0.0062
2 1	0.0220	2 F	0.0133
2 2	0.0211	3 0	0.0027
2 3	0.0064	3 1	0.0050
2 4	0.0336	3 2	0.0075
2 5	0.0106	3 3	0.0162
2 6	0.0096	3 4	0.0218
2 7	0.0074	3 5	0.0052
2 8	0.0224	3 6	0.0056
2 9	0.0054	3 7	0.0048

## Advenced Encryption Standard (AES)

- n = 128
- k = 128, 192, 256
- Encrypt & Decrypt :
  - AddRoundKey
  - SubBytes
  - SiftRows
  - MixColumns

## Advenced Encryption Standard (AES)

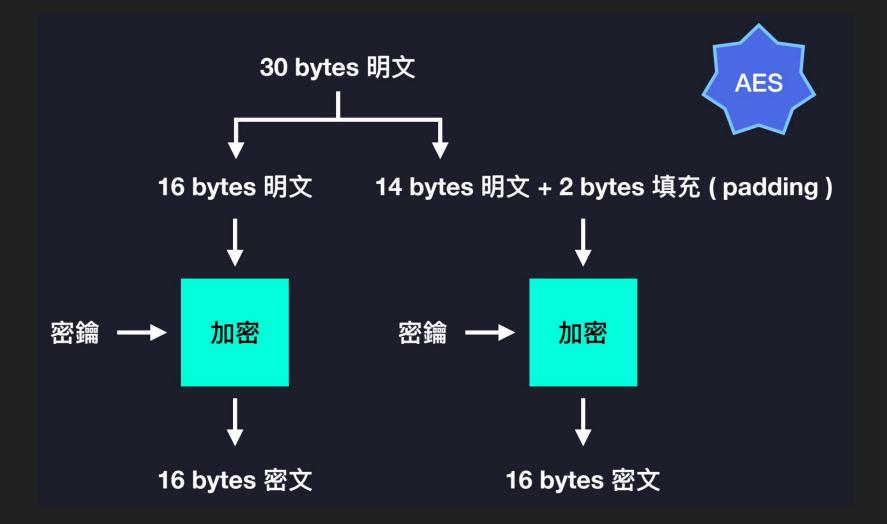


## Block Cipher

一次加密一個 Block

那如果明文很長呢 QAQ ?? → 切成好幾個 Block

切完有剩呢 OAQ?? → 加 Padding 補齊

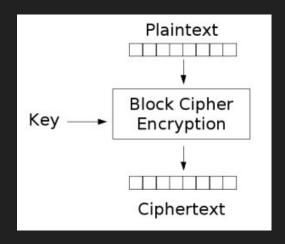


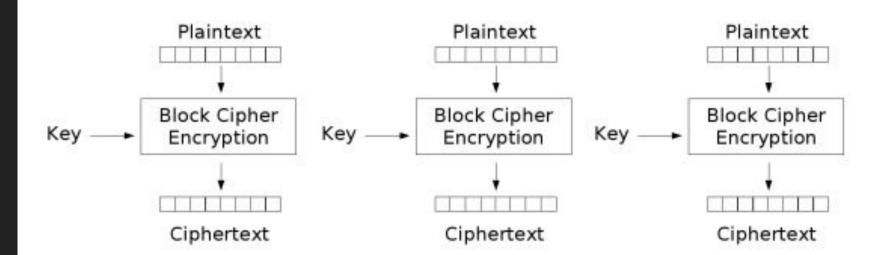
## Mode of Operation

● 區塊加密:

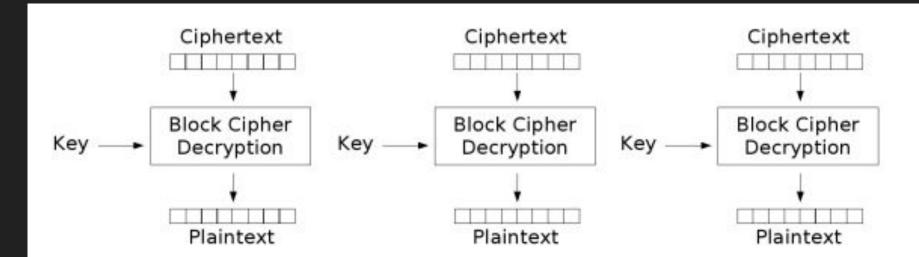
```
ECB (Electronic codebook)
```

- CBC (Cipher-block chaining)
- CTR (Counter)
- PCBC (Propagating cipher-block chaining)
- 0 ...

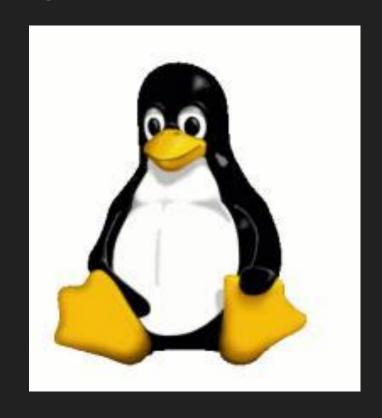


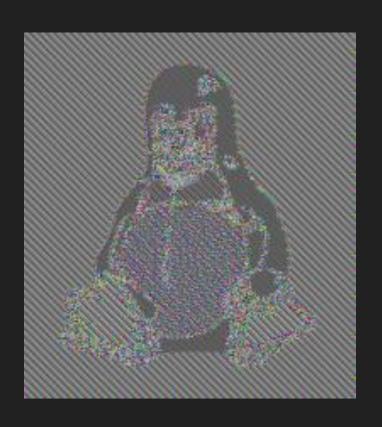


Electronic Codebook (ECB) mode encryption



Electronic Codebook (ECB) mode decryption





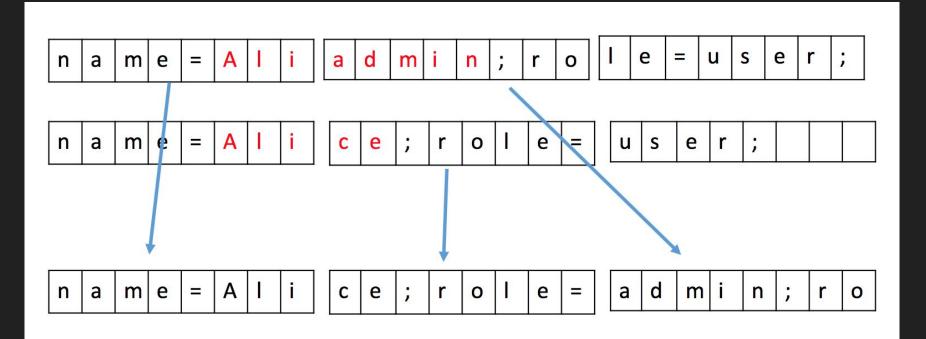
Encryption Oracle + Cut-and-paste

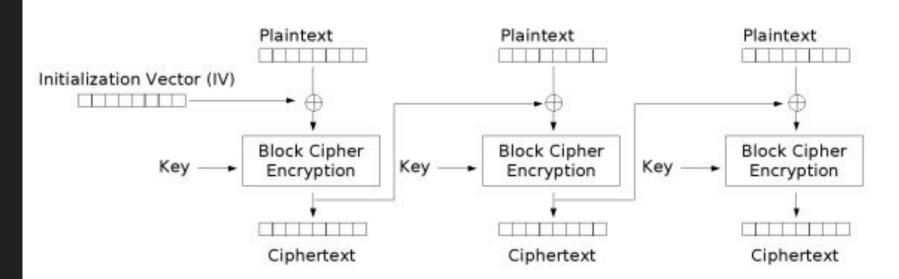
#### Encryption Oracle Attack:

- E: for ECB mode,  $E_k(P | A | S)$
- 你不知道 key, Padding, secret 但是可以控制 A
- Oracle : return EK(P | A | S)

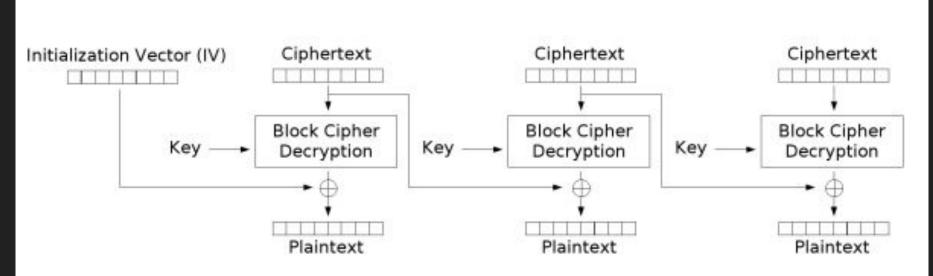
#### Example:

- A cookie is EK(name = Alice | | role = user), username is alice
   Ek(P | name = Alice | | role = user | S)
- user-controllable but not the role
- You want to obtain EK(name = Alice | | role = admin)



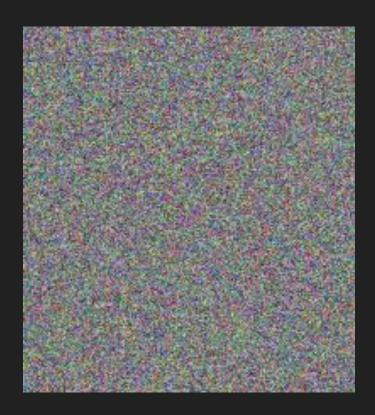


Cipher Block Chaining (CBC) mode encryption

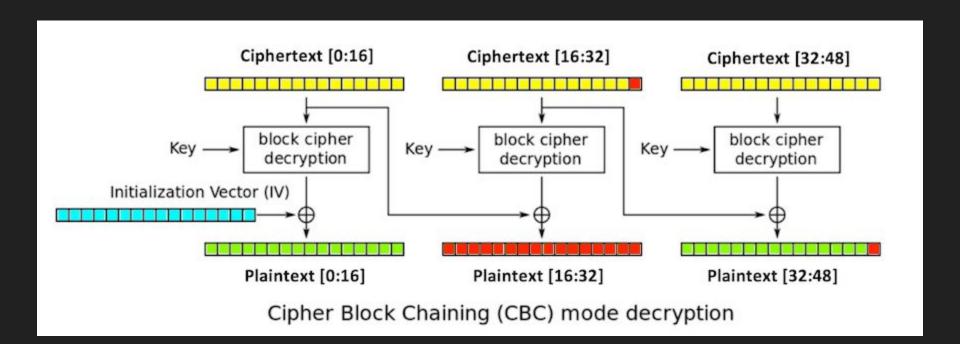


Cipher Block Chaining (CBC) mode decryption





# Bit flipping attacks

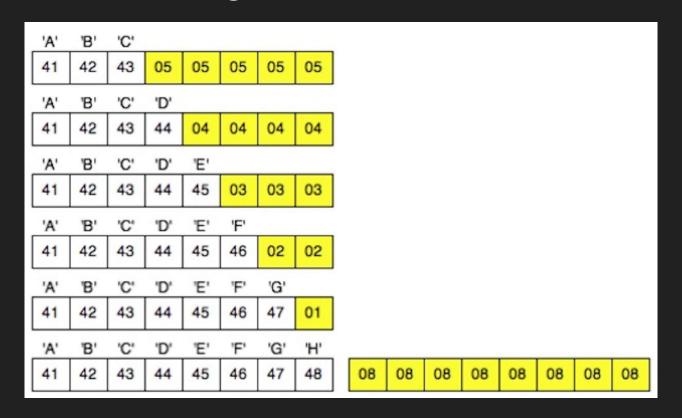


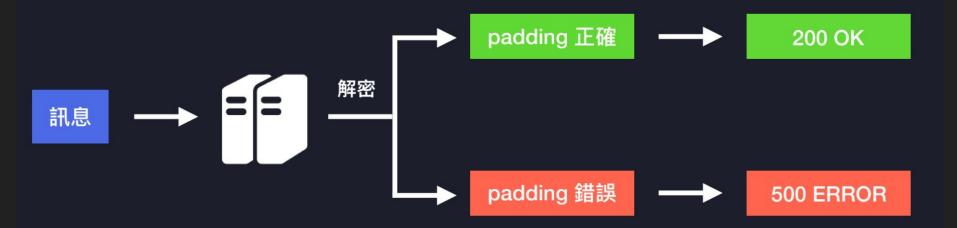
Padding Oracle Attack => 利用 PKCS #7 Padding 挖出 明文

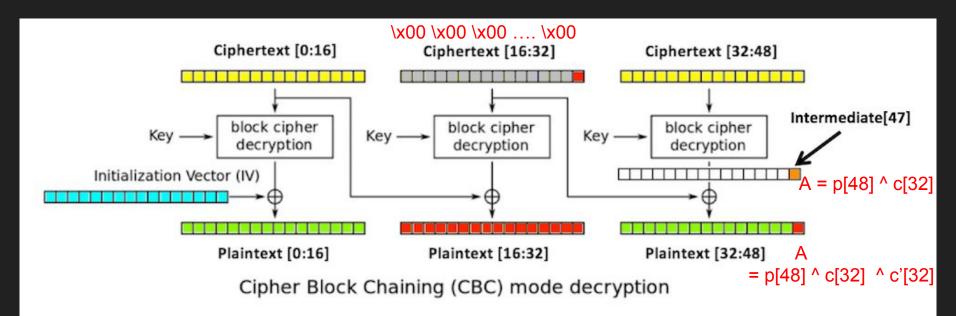
加解密 圖

我的小筆記

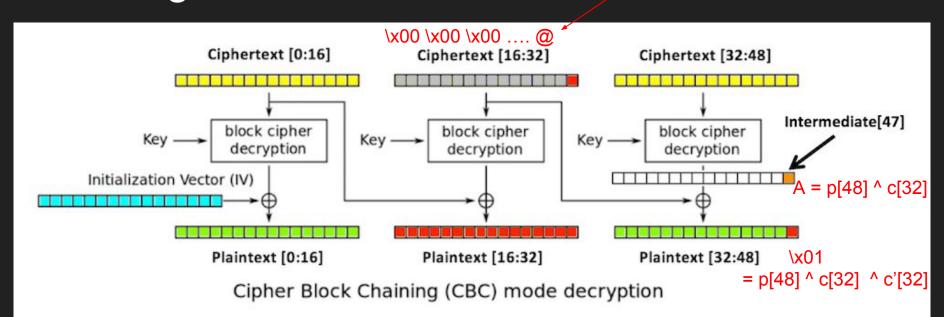
# PKCS #7 Padding

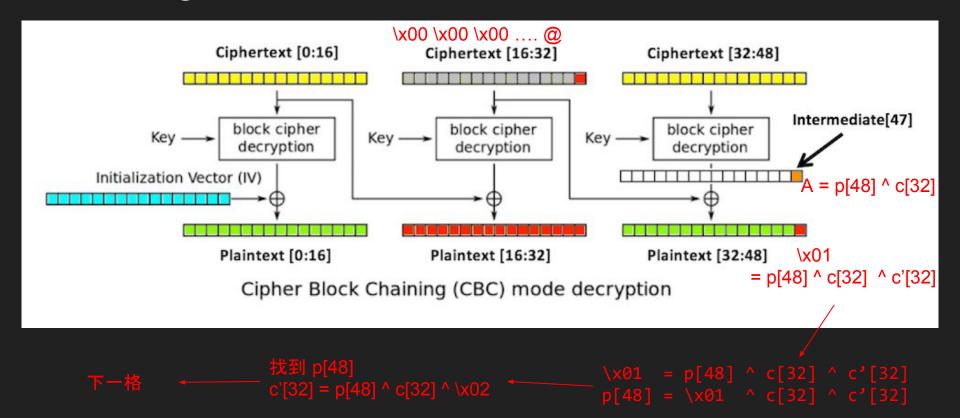






嘗試 256 次,找 Padding Ok 的





#### 三層迴圈:

猜 0 - 255 直到猜出一個 byte

- 一次解出一個 byte 直到解完一個 block
- 一次解出一個 block 直到解完所有 blocks

#### ❖ 第一個區塊:

我們需要前一個區塊來解目前的區塊

所以我們需要知道原始 Ⅳ 和能夠操控 Ⅳ 才能解出第一個區塊

#### 嘗試次數:

解出一個 byte 最多需要 256 次嘗試

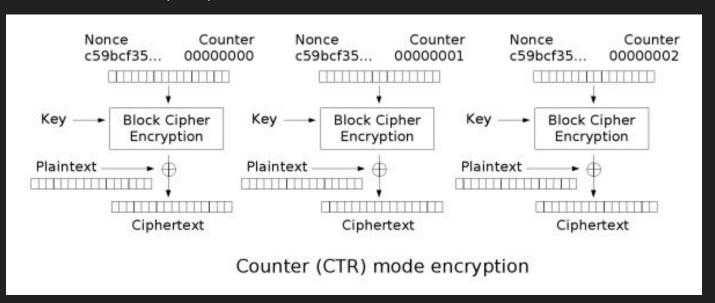
解出一個 block (16 bytes) 最多需要 4096 次嘗試

看他有幾個 Block ... 一個一個解回去

# CTR

## CTR

- counter 從 0 開始遞加
- nonce 為隨機生成(IV)



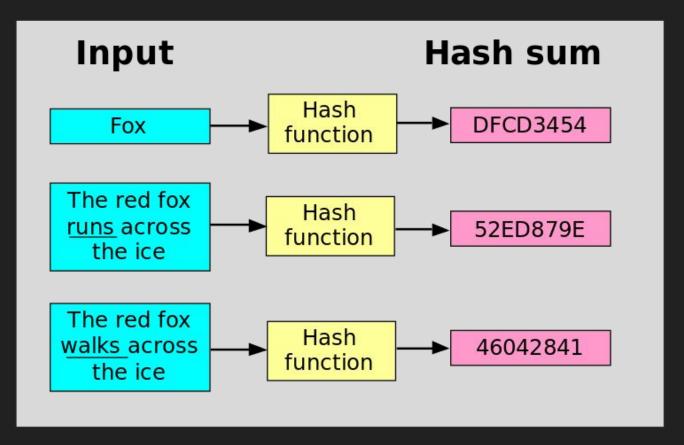
# 對稱式加密 Symmetric

Stream Cipher: (ex: RC4)

A (synchronous) stream cipher is an algorithm which maps some fixed-length key to an arbitrary-length key-stream

=> 很像 One-Time-Pad ...

# Hash 亂入

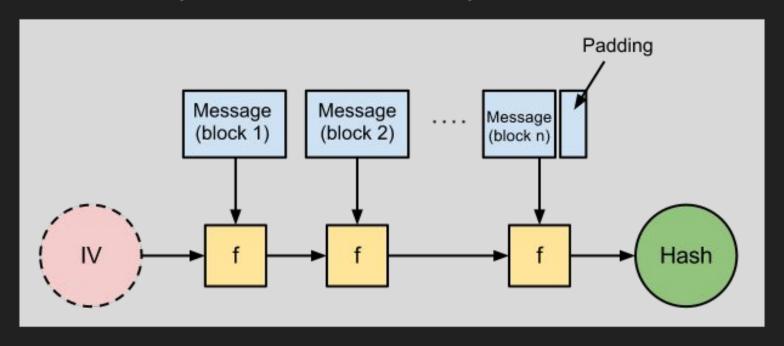


Hash function 就像 Block cipher 一段一段 hash 起來

他們各自也有自己的 IV (起始向量) 叫做 magic number

ex: md5 0x67452301, 0xefcdab89, 0x98badcfe, 0x10325476

剩下不夠的 + Padding 補齊,最後補上該 message 的長度



Hash(message)

- => Hash(message + padding + length)
- => Hash(message + (\x80 + \x00 ... \x00) + \x02\x28)

 $Hash(m1) \rightarrow digest (d1)$ 

把 d1 當 magic number 再繼續 Hash(m2) 相當於

Hash( m1 + padding + length + m2 )

Length Extend Attack (LEA) (for: MD5, SHA-1, and SHA-2)

tool: <u>hashpumpy</u>

Sha-1 Google 已經破解了, 隨便 google 都有

https://github.com/maojui/cryptools/blob/master/cryptools/hash.py

偶爾會出現要 sha1-collision 的東西, 可以用上面的 code 生

# Asymmetric

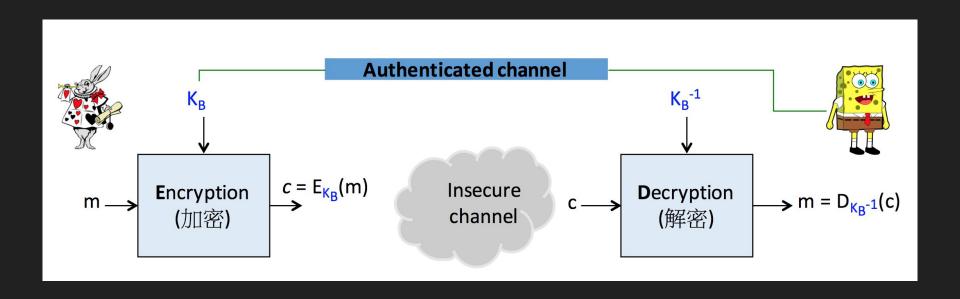
# 非對稱式加密 Asymmetric

1. Diffie-Hellman key exchange (D-H)

2. Public-Private key Encryption: RSA, DSA, ECC

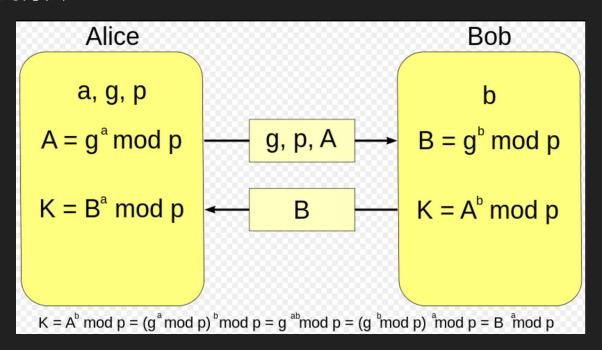
3. 數位簽章 Digital Signature

# 非對稱式加密 Asymmetric



## Diffie-Hellman Key Exchange

#### 一種交換密鑰的方法:



## Diffie-Hellman Key Exchange

最簡單,最早提出的這個協定使用一個質數p的整數模n乘法群以及其原根g。下面展示這個演算法,綠色表示非秘密資訊,紅色粗體表示秘密資訊:

愛麗絲		
秘密	非秘密	計算
	p, g	
а		
		g <sup>a</sup> mod p
	(g <sup>b</sup> mod p) <sup>a</sup> mod p	

鮑伯			
計算	非秘密	秘密	
	p, g		
		b	
	•••		
<b>g</b> <sup>b</sup> mod p			
	(g <sup>a</sup> mod p) <sup>b</sup> mod p		

- 1. 愛麗絲與鮑伯協定使用 p=23以及base g=5.
- 2. 愛麗絲選擇一個秘密整數a=6, 計算 $A=g^2 \mod p$ 並行送給鮑伯。
  - $A = 5^6 \mod 23 = 8$ .
- 3. 鮑伯選擇一個秘密整數b=15, 計算 $B=g^b \mod p$ 並行送給愛麗絲。
  - B =  $5^{15}$  mod 23 = 19.
- 4. 愛麗絲計算**s** = B a mod p
  - $19^6 \mod 23 = 2$ .
- 5. 鮑伯計算**s** = A b mod p
  - $8^{15} \mod 23 = 2$ .

Key =  $(g^a \mod p)^b \mod p = (g^b \mod p)^a \mod p = g^a \pmod p$ 

## Diffie-Hellman Key Exchange

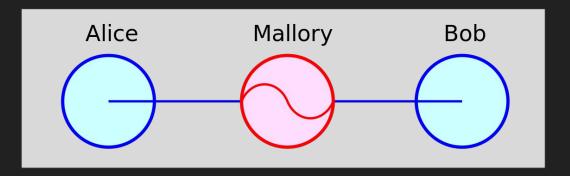
中間就算被監聽:對方得到

- 1.  $A = g^a \mod p$
- 2.  $B = g^b \mod p$
- 3. g
- 4. p

他還是算不出 Key: g^ab mod p

## Man In the Middle (MITM) Attack

可是如果他從一開始就偽裝成你交換的對象 ...



## Public-Private key Encryption

想要有自己的 Key,又不想因為通訊方外洩造成問題 ....

Public Key 加密過的,只有擁有 Private Key 的人可以解開 (訊息一定傳給你)

Private Key 加密的東西, 只有 Public Key 可以解(訊息一定是你傳的)

安全在哪?

安全在 N 雖然是合數, 但是兩個質數的因數分解超爆幹難。

懂?

143 ???

11

X

13

 $11706419524121703981169462650571415580634814515508830394031236629231239096879060679307091710465880099593241\\ 56890476924399115987295002514512363284816380910931716305629337272132394183012044490174099598958998619361237\\ 85912877632115236868068240062899322948890286991592977121183245275916261915916506559089600633642940791215633\\ 94307257727223287994176475287190040789626096715069741585535460171254504405800971810903555742002284616584915\\ 98791887490113266745870667017599133806709959025364960543973145607357341171413726935927742274278393455749736\\ 6968757476854657854735736618019200213059434669079878135257529127645832291986911087 = ?????$ 

Few months after ....

95474338312218210998112918490280041453507072579441458411551887347487123857090489541288757069990797973918374 29786307635088968627339677006807580469109278505753693936850995759226191485095758660975794393134379833667359 2489160105072883119840130956240371877556630164166624480189383412782557267438170932307849980613

X

12261325641073952688685275396191914994258545375588073456236900615006505663195519880227884945880510407976963 27016863514305311556471075442985629111024971314985814162100965835181718613769104530486967377098003874351604 91530124834970538550266360946103032277268693327915643706609369966325158751073626167458888180899

 $601846749745964573884689344393031538631057582811191869541915558824076177011971683775962946042104337099172166050034918457515501282712139131582146244\\802551546700307177344515745952316959346932989668467495121800603076587652703080403049796121689939912056204836720587829590069719895803319422909994515\\638886095614240740594775559633049332402703302748958179031461121157073573759505116987801699919705129270775324940265135420025420407156783554219195237\\250668724641362125212036841431009377802117265918740238995812157525934442346229868133997025137574642992279543190983721010958717947985798020524328710\\934902753423113404927814838640252115638907578689077878457597754193085812860663776989596762265311838316238941419905689340574109938643388504773773963\\697189728446764345247460023523274332615073262570897510523286883158759286129302345900558251045005559677266291388484866720216484502576314539378259590\\721648349990558369145393568044116742275044633712109798635821040276403059928912878952365629613511337323584494216958756687815466594074021411709373498\\502201860191153889132716324348494578774061426942683446483003541229197122080234492199140225902386039793389606076596318409207673901687921429204685040\\790980114718682234258003783137556777190554991549198508847$ 

#### ????? Few years after ....

 $248685272994095731184120554458548026849803985832714117509961834280732727160551011978108875615998546145840786606595765695120700465843941672322380397\\677348490453455057214789850026480190876262870678907729356987849054464101584566830226528460145802007641403623300755703901948369961309917126821112669\\833127400659588429676765185684706392299465166482627110990892217113206644991658802687248198769385413117789414986879441962817767431569794386477266570\\024858636833651266305579632483340367483666794242827028588406038870569316106315364555575437919169111424418935853666595690909142855669448403303587428\\45142040589111873515992365211$ 

Х

 $242011415674081182294700056598040417190377803073960118316453103929706140488561066708447921867530272991391869936959580669697067939916612396600787407\\889214415942282077912501974815975321942482435974281248950288329423132383054802144773111934864889009243291168817560432375669121147541732947285719147\\957261214479713948555093250023121513195103119018342785262160833200556459907010235194119451414769916838252345629760112099112934532173648871952913165\\085441339450919242027658000137182269936830070148809354637970244036226628245826072435517826708442806808114904302922992354687660352585172617776886588\\94610305771271862432176551677$ 

#### 要怎麼做?

- 1. 首先 找出兩個超大質數 p,q (2048 bit)
- 2. N = p\*q
- 3. e = 任意質數
- 4. 找到 phi = (p-1) \* (q-1) 必須跟e 互質
- 5. d = invmod(e,phi) ed mod phi = 1
- 6. 把 p,q 丟掉

public key : (N,e)
private key : (N,d)

93 ÷ 11 = 8 ... 5 : 93 % 11 = 5 寫成 : 93 ≡ 5 (mod 11)

關於 mod 的特性太多了,有興趣請 Google 數論

他好像就把前面的數字套入一個循環中,永遠不會超出來

費馬小定理: (當p為質數)

$$a^{p-1} \equiv 1 \pmod{p}$$

ex: mod 7

```
def fm(p) :
    for _ in range(1,p) :
        print( pow( _,p-1,p) )
```

歐拉 (Euler) 定理:

$$a^{arphi(n)} \equiv 1 \pmod n$$

n 為質數: Φ(n) = n-1 即為 費馬小定理

#### 歐拉函數:

```
ex: n = 7, Φ(n) = 6
n 為合數 (n = p^a * q^b * r^c * ...): Φ(n) = [(p-1) * p^(a-1)] * [(q-1) * q^(b-1)] * ....
ex: n = 1125 (3*3*5*5*5), Φ(n) = (2*3) * (4* 5*5) = 600
```

$$n = p * q$$
  $phi = (p-1)*(q-1)$ 

#### 歐拉定理:

$$a^{arphi(n)} \equiv 1 \pmod{n}$$

#### 重要特性:

```
ex: n = 7, \Phi(n) = 6, a<n
a^{19} \pmod{7} \equiv a^{19} \pmod{\Phi(7)} \pmod{7}
a^{19} \pmod{6} \pmod{7} \equiv a^{1} \pmod{7}
pow(5,19,7) >> 5
```

歐拉定理:

$$a^{arphi(n)} \equiv 1 \pmod n$$

可以幹嘛? 反函數!!

如果可以找到一個 d

使得 ed mod Φ(n) = 1

那麼 當我們遇到 a^e, 將他乘上 d 次方, 變成:a^ed (mod n) 相當於 a^1 (mod n)

就可以"把 a^e 還原成 a "

 $c^d = (m^e)^d = m^ed = m^(ed mod phi) = m^1 = m$ 

如何找到一個 d,使得 ed mod Φ(n) = 1 ? Extended Euclidean algorithm

好工具:

pip2 install git+https://github.com/hellman/libnum pip3 install git+https://github.com/maojui/libnum

>> d = invmod( e, phi ) # e 跟 phi 要互質

## Public-Private key Encryption

RSA 還有一堆怪招, CTF 最愛出的

DSA ... (特色:給 p,q,g 三個質數, 一個亂數 r)

Rabin Cryptosystem ... (m^2 % p)

Paillier cryptosystem ... (g^m \* r^n mod n^2)

## Public-Private key Encryption

ECC ... 橢圓曲線密碼學

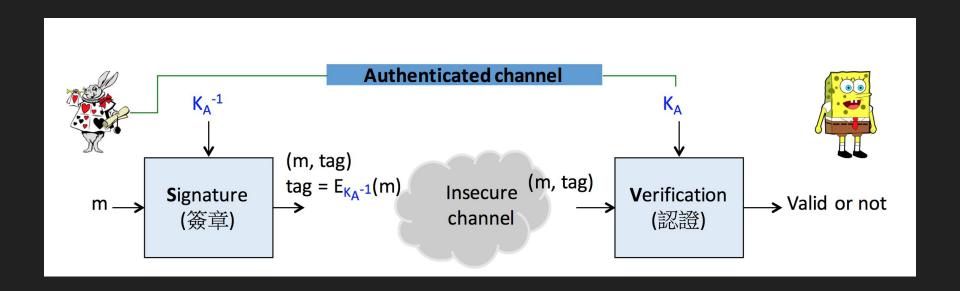
http://andrea.corbellini.name/2015/05/17/elliptic-curve-cryptography-a-gentle-introduction/

強度:

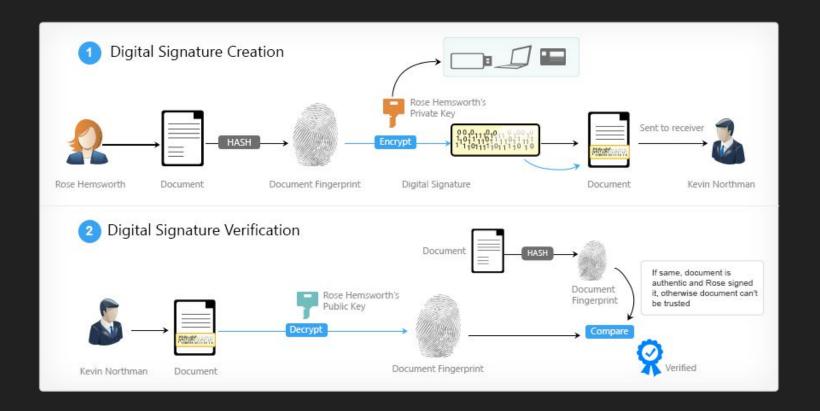
RSA: 1024 bit | 15360 bit

ECC: 160 bit | 512 bit

## Digital Signature



## Digital Signature



## Digital Signature

數位簽章就是 ...

把非對稱式加密 和 Hash 喇來喇去在一起 @@

## 工具

SageMath: 數學運算軟體

factordb:因式分解資料庫

● Yafu:幫助你因式分解

## 工具

- pycipher:古典密碼學加密演算法
- pycrypto:現代密碼學加解密演算法
- cryptography:同上
- <u>Sympy:符號運算</u>

```
Classical
picoCTF 2018, Caesar Cipher 1 (150)
picoCTF 2018, Caesar Cipher 2 (150)
Seccon CTF 2016, Vigenere (100)
Seccon CTF 2017, Vigenere3d (100)
Qiwi-Infosec CTF-2016, Crypto 100 1 (100)
```

One Time Pad - crib drag

<u>EasyCTF 2018 - Not OTP</u> (100)

Pragyan CTF 2018, Improper encryption (100)

AlexCTF 2017 - CR2 Many time secrets (100)

Linear Cryptanalysis

**TUMCTF Teaser 2015** (350)

<u>OCTF Qual 2018 - zerOSPN</u> (550)

ECB Cut & paste

CSAW Quals 2017, BabyCrypt (350)

PicoCTF 2018, SpyFi (300)

Nuit du Hack CTF Quals 2016, Toil33t (400)

CBC Padding Oracle

BambooFox CTF 2018, mini-padding (200)

<u>Hitcon CTF 2017, Secret Server</u> (221)

<u>CSAW Qual 2016, Neo (200)</u>

Hack.lu CTF 2016, Cryptolocker (200)

ASIS Qual 2018, Yunny It (500)

picoCTF 2018, Magic Padding Oracle (450)

Length Extend Attack (LEA)

TW.edu CTF 2015, LEA (200)

RCTF 2018, cpushop (176)

<u>Viettel Mates CTF 2018, Viettel Store</u> (100)

BambooFox CTF 2018, baby-lea (100)

<u>Hackover CTF 2015, securelogin</u> (250)

Sha1 collision

Defcon CTF 2018, Easy Pisy (104)

Seccon CTF 2017, SHA-1 is dead (100)

RSA

**NTUSTISC** 

