Excellent — that's an important and realistic refinement.
Let's now build a **complete and rigorous model** that accounts for both points:

1. **Deep uncertainty** in renewable generation (unknown probability distribution); and
2. **Endogenous market clearing price**, determined by the **merit-order mechanism** given total demand and all producers' bids.

We'll produce a well-structured, research-grade model that identifies all key elements and relationships precisely.

Formal Model of Renewable Producer's Day-Ahead Bidding Problem

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1. Setting and Time Horizon

The day-ahead market is cleared once for 24 hourly intervals $t \in \mathcal{T} = \{1, 2, \ldots, 24\}$ of the following day.
All producers submit bids before uncertainties are realized.

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2. Market Participants

Let the set of producers be:

$$\mathcal{I} = \{0, 1, 2, 3\},$$

where:

- $i = 0$: **renewable producer** (our decision maker);
- $i = 1, 2, 3$: **conventional producers** (competitors).

Each producer i submits for each hour t:

$$(b_{i,t}, p_{i,t}) \quad \text{where} \begin{cases} b_{i,t} = \text{bid quantity (MWh)} \\ p_{i,t} = \text{offer price (\$/MWh)} \end{cases}$$

All bids are submitted simultaneously.

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3. Market Clearing Mechanism (Merit-Order Rule)

For each hour t:

- Let total system **demand** D_t (MWh) be known and inelastic.
- The **market clearing price** P_t is determined as:

$$P_t = \min\left\{ p : \sum_{i\in\mathcal{I}} b_{i,t}\,\mathbf{1}_{\{p_{i,t}\le p\}} \ge D_t \right\}.$$

In words: the lowest price such that total accepted supply meets or exceeds demand.

Each producer i's **accepted bid quantity** is:

$$q_{i,t}(P_t) = \begin{cases} b_{i,t}, & \text{if } p_{i,t} < P_t, \\ \tilde{b}_{i,t}, & \text{if } p_{i,t} = P_t \text{ and partial acceptance occurs}, \\ 0, & \text{if } p_{i,t} > P_t, \end{cases}$$

where $\tilde{b}_{i,t}$ ensures that $\sum_i q_{i,t}(P_t) = D_t$.

Thus, the clearing price P_t is **endogenous**, determined jointly by all producers' bids and total demand.

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4. Renewable Producer's Physical Uncertainty

Let G_t denote the renewable producer's **available generation** for hour t.

- G_t is *deeply uncertain*: its probability distribution is **unknown or only partially characterized**.
- Instead of a known distribution, we assume that G_t lies in a known **uncertainty set**:

  $$G_t \in \mathcal{G}_t = [\underline{G}_t, \overline{G}_t],$$

  representing plausible lower and upper bounds from forecasts or historical data.

We can model this using:

- a **robust optimization** approach (maximize the worst-case expected profit across all plausible realizations), or
- a **distributionally robust** approach (maximize expected profit under the most adverse probability distribution in a given ambiguity set).

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5. Delivery and Penalty Mechanism

After market clearing, if the renewable producer's bid is **fully or partially accepted**, the producer must deliver $q_{0,t}$ MWh.

If actual generation G_t differs from $q_{0,t}$, imbalance penalties apply:

$$\text{Deviation: } \Delta_t = G_t - q_{0,t}.$$

Then, the **penalized payoff** in hour t is:

$$\pi_t(G_t, q_{0,t}, P_t) = P_t q_{0,t} - c_t^{\text{under}}(\max\{0, -\Delta_t\}) - c_t^{\text{over}}(\max\{0, \Delta_t\}),$$

where:

- c_t^{under}: penalty ($/MWh) for shortfall (under-delivery),

- c_t^{over}: penalty ($/MWh) for surplus (over-delivery).

6. Renewable Producer's Objective

The renewable producer (index $i = 0$) seeks to determine bids $(b_{0,t}, p_{0,t})$ to **maximize expected profit** under uncertainty in both:

- own generation G_t (deep uncertainty),
- market price P_t (endogenous and dependent on others' bids).

6.1 Deterministic equivalent representation (if others' bids fixed)

If other producers' bids $\{(b_{i,t}, p_{i,t})\}_{i=1}^{3}$ are fixed and known, P_t becomes a deterministic function of our bid:

$$P_t = \Phi_t(p_{0,t}, b_{0,t}),$$

as determined by the market-clearing rule given D_t.

Then the renewable producer's *realized* hourly profit is:

$$\Pi_t(b_{0,t}, p_{0,t}, G_t) = \pi_t(G_t, q_{0,t}(P_t), P_t),$$

where $q_{0,t}(P_t)$ is the cleared quantity for the renewable producer.

7. Optimization Formulation

(a) Distributionally-Robust Formulation

Because the probability distribution of G_t is unknown, we consider a worst-case (robust) expectation over all distributions $\mathbb{P} \in \mathcal{P}_t$ consistent with known moments or bounds:

$$\max_{b_{0,t}, p_{0,t}} \min_{\mathbb{P} \in \mathcal{P}} \mathbb{E}_{\mathbb{P}} \left[\sum_{t=1}^{24} \Pi_t(b_{0,t}, p_{0,t}, G_t) \right]$$

subject to market-clearing equilibrium constraints and capacity limits.

Here:

$$0 \leq b_{0,t} \leq \overline{G}_t, \quad p_{0,t} \geq 0.$$

(b) Robust Counterpart (worst-case realization)

Alternatively, a purely robust (min-max) form is:

$$\max_{\{b_{0,t}, p_{0,t}\}} \min_{G_t \in [\underline{G}_t, \overline{G}_t]} \sum_{t=1}^{24} \Pi_t(b_{0,t}, p_{0,t}, G_t)$$

subject to the market-clearing rule defining P_t.

This captures a *hedging strategy* against worst-case renewable generation within credible bounds.

8. Constraints

1. **Market-Clearing Equilibrium (per hour t):**

$$P_t = \min\left\{p : \sum_{i \in \mathcal{I}} b_{i,t}\,\mathbf{1}_{\{p_{i,t} \le p\}} \ge D_t\right\}.$$

2. **Feasibility and Capacity:**

$$0 \le b_{0,t} \le \overline{G}_t, \quad p_{0,t} \ge 0.$$

3. **Delivery Balance:**

$$q_{0,t} = \begin{cases} b_{0,t}, & \text{if } p_{0,t} \le P_t, \\ 0, & \text{otherwise.} \end{cases}$$

9. Model Components Summary

Type	Symbol	Description
Decision variables	$b_{0,t}, p_{0,t}$	Renewable producer's bid quantity and offer price per hour
Uncertain variables	G_t (deep uncertainty), P_t (endogenous)	Available renewable generation and market clearing price
Parameters	$D_t, c_t^{\text{under}}, c_t^{\text{over}}, \overline{G}_t$	Demand, penalties, and generation bounds
Competitor bids	$(b_{i,t}, p_{i,t}), i = 1, 2, 3$	Other producers' offers (given or estimated)
State variables	None (single-stage, day-ahead)	All bids submitted simultaneously
Transition function	N/A (no time evolution; one market clearing event)	
Objective	$\max_{b,p} \min_{\mathbb{P} \in \mathcal{P}} \mathbb{E}_{\mathbb{P}}[\sum_t \Pi_t]$	Distributionally robust expected profit
Clearing constraint	$P_t = \Phi_t(p_{0,t}, b_{0,t})$	Market price determined endogenously by bids and demand

10. Interpretation

- The **renewable generation** uncertainty is handled via a **robust or distributionally robust** formulation, not a classical expectation—since its distribution is unknown.
- The **market clearing price** is **endogenous**, computed by the **merit-order rule** given all bids and total demand.
- The model captures the renewable producer's **strategic interaction** with the market and its **risk-hedging behavior** under deep uncertainty.

Would you like me to expand this further into a **distributionally robust equilibrium model** (i.e., where all four producers are strategic and the clearing price arises as an equilibrium outcome)? That's the natural next formal refinement.