Market_brief_4

An energy company is interested in investing in wind-power production. To assess the viability of this investment, the company wants to evaluate the expected revenue of a wind-power producer in a competitive spot market.

Suppose there are five energy producers participating in the day-ahead spot market. Three of them use conventional sources for energy production, such as coal, while the other two producers use solar and wind power, respectively. The market is cleared once per day for 24 hourly intervals of the following day. For each dispatch interval, every producer submits a bid specifying the energy quantity (MWh) it is willing to supply and the corresponding minimum acceptable price (per MWh). Then, the market operator follows a merit-order process to determine the market-clearing price and dispatch schedule, ensuring the total market demand is met. If a bid is accepted, the producer is committed to delivering the bid quantity at the market-clearing price. Otherwise, the producer earns zero revenue.

The energy company aims to build a simulation model to evaluate the expected revenue of the wind-power producer on any future day by selecting the hourly bid quantity and price, while accounting for uncertainties. This model can then be used to explore outcomes across various short-term and long-term scenarios, thereby informing the viability of the investment.

Please formalize this problem description as a simulation model and specify the key components, including state variables, decision variables, transition functions, objective functions, stochastic variables, constant parameters and deep uncertainties.

Below is a **formal simulation-model formulation** of the problem, structured in a way that can support Monte Carlo simulation, scenario analysis, and later optimization.

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1. Modeling Framework

The problem can be modeled as a **stochastic simulation of a competitive day-ahead electricity market** with strategic bidding by a wind-power producer under uncertainty.

- **Time structure**
  - Days indexed by $d \in \mathcal{D}$
  - Hours within a day indexed by $h \in \{1, \dots, 24\}$
- **Agents**
  - Set of producers $i \in \mathcal{I} = \{1, \dots, 5\}$
    - $\mathcal{I}^{conv}$: 3 conventional producers
    - $i = w$: wind producer (decision-maker of interest)

- $i = s$: solar producer

The simulation focuses on **price formation, dispatch, and revenues**, given bids and realized uncertainties.

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2. State Variables

State variables describe the system at the time bids are submitted or the market is cleared.

For each hour h on day d:

1. **Available generation capacity**

$$C_{i,h}^{d} \quad (\mathrm{MWh})$$

- For wind and solar: stochastic and weather-dependent
- For conventional: typically fixed or weakly stochastic

2. **Market demand**

$$D_{h}^{d} \quad (\mathrm{MWh})$$

3. **Fuel costs (conventional only)**

$$c_{i,h}^{d} \quad (€/\mathrm{MWh}), \quad i \in \mathcal{I}^{conv}$$

4. **Market information (optional, for learning models)**

- Historical prices, forecast errors, past dispatch outcomes

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3. Decision Variables (Wind Producer)

For each hour h:

1. **Bid quantity**

$$q_{w,h}^{d} \geq 0$$

2. **Bid price**

$$p_{w,h}^{d} \geq 0$$

Subject to:

$$q_{w,h}^{d} \leq \hat{C}_{w,h}^{d}$$

where $\hat{C}_{w,h}^{d}$ is the *forecasted* available wind capacity.

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4. Stochastic Variables (Short-Term Uncertainty)

OpenAI

These are sampled during simulation runs.

1. **Wind availability**

$$C_{w,h}^d \sim \mathcal{P}_w(\cdot \mid \text{weather}_h^d)$$

2. **Solar availability**

$$C_{s,h}^d \sim \mathcal{P}_s(\cdot)$$

3. **Demand**

$$D_h^d \sim \mathcal{P}_D(\cdot)$$

4. **Forecast errors**

$$\varepsilon_{w,h}^d = C_{w,h}^d - \hat{C}_{w,h}^d$$

---

5. Market-Clearing (Transition Function)

For each hour h:

1. **Bid set**

$$\mathcal{B}_h^d = \{(q_{i,h}^d, p_{i,h}^d)\}_{i \in \mathcal{I}}$$

2. **Merit-order dispatch**

- Sort bids by increasing price
- Accept bids sequentially until cumulative quantity meets demand D_h^d

3. **Market-clearing price**

$$\lambda_h^d = \max\{p_{i,h}^d \mid \text{bid accepted}\}$$

4. **Dispatch quantity**

$$x_{i,h}^d = \begin{cases} q_{i,h}^d, & \text{if accepted} \\ 0, & \text{otherwise} \end{cases}$$

For renewables:

$$x_{i,h}^d \le C_{i,h}^d$$

Curtailment or penalties may be added if actual production falls short.

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6. Revenue and Objective Function

Hourly revenue (wind producer)

$$R_{w,h}^d = \lambda_h^d \cdot x_{w,h}^d$$

Daily revenue

$$R_w^d = \sum_{h=1}^{24} R_{w,h}^d$$

Expected revenue (objective)

$$\max_{q_{w,h}^d,\, p_{w,h}^d} \quad \mathbb{E}\left[R_w^d\right]$$

Optionally include:

- Risk measures (e.g., CVaR)
- Imbalance penalties
- Maintenance or operating costs

---

7. Constant Parameters

1. **Installed capacities**

$$\bar{C}_i \quad \forall i$$

2. **Market rules**

- Uniform pricing
- Merit-order dispatch
- No strategic bidding by competitors (or parameterized strategies)

3. **Simulation horizon**

- Number of simulated days $|\mathcal{D}|$

---

8. Deep Uncertainties (Long-Term)

These are *not* well described by probability distributions and are explored via scenarios.

1. **Market structure**

- Number of competitors
- Entry/exit of producers

2. **Policy and regulation**

- Carbon prices
- Renewable subsidies

- Market design changes

3. **Technology evolution**

- Wind turbine efficiency improvements
- Storage integration

4. **Demand evolution**

- Electrification trends
- Climate-driven load changes

5. **Fuel price regimes**

- Structural shifts in coal or gas prices

These uncertainties define **scenario ensembles**, not random draws.

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9. Simulation Output Metrics

- Expected daily and annual revenue
- Revenue volatility
- Frequency of zero-dispatch events
- Sensitivity to bid strategies
- Robustness across deep-uncertainty scenarios

Summary

This formulation defines a **stochastic, agent-based market simulation** where the wind producer optimizes bidding decisions under short-term uncertainty (weather, demand) and evaluates robustness under long-term deep uncertainties (policy, technology, market design). It provides a foundation for Monte Carlo simulation, scenario analysis, and eventually stochastic or robust optimization.

You can assume that the market demand follows a normal distribution with constant mean and variance. Please refine your specification accordingly.

Below is a **refined and more concrete specification** of the simulation model, incorporating the assumption that **market demand is normally distributed with constant mean and variance**. I keep the structure compatible with simulation, Monte Carlo analysis, and later optimization.

---

1. Time and Index Sets

- Days: $d \in \mathcal{D}$
- Hours within a day: $h \in \mathcal{H} = \{1, \dots, 24\}$
- Producers:

$$\mathcal{I} = \mathcal{I}^{conv} \cup \{s, w\}, \quad |\mathcal{I}^{conv}| = 3$$

---

2. State Variables

For each day d and hour h:

1. **Available generation capacity**

$$C_{i,h}^d \quad (\text{MWh})$$

- $i = w$ (wind): stochastic, weather-driven
- $i = s$ (solar): stochastic, diurnal
- $i \in \mathcal{I}^{conv}$: constant or weakly stochastic

2. **Realized market demand**

$$D_h^d \quad (\text{MWh})$$

3. **Fuel-based marginal costs (conventional producers)**

$$c_{i,h} \quad \forall i \in \mathcal{I}^{conv}$$

(assumed constant over time for simplicity)

---

3. Demand Model (Refined)

Market demand is assumed to follow a **normal distribution**:

$$D_h^d \sim \mathcal{N}(\mu_h, \sigma_h^2)$$

where:

- μ_h: constant mean demand for hour h
- σ_h^2: constant variance for hour h

To ensure physical feasibility:

$$D_h^d := \max\{0, D_h^d\}$$

Interpretation

- Hourly demand is independent across days
- Correlation across hours may be added later but is omitted here for clarity

4. Decision Variables (Wind Producer)

For each hour h on day d:

1. **Bid quantity**

$$q_{w,h}^d \geq 0$$

2. **Bid price**

$$p_{w,h}^d \geq 0$$

Constraint:

$$q_{w,h}^d \leq \hat{C}_{w,h}^d$$

where $\hat{C}_{w,h}^d$ is the forecasted wind availability.

5. Stochastic Variables

1. **Wind availability**

$$C_{w,h}^d \sim \mathcal{P}_w(\cdot)$$

2. **Solar availability**

$$C_{s,h}^d \sim \mathcal{P}_s(\cdot)$$

3. **Demand**

$$D_h^d \sim \mathcal{N}(\mu_h, \sigma_h^2)$$

4. **Wind forecast error**

$$\varepsilon_{w,h}^d = C_{w,h}^d - \hat{C}_{w,h}^d$$

6. Bidding Behavior of Other Producers (Parameterized)

For each hour h:

- Conventional producers bid:

$$q_{i,h} = \bar{C}_i, \quad p_{i,h} = c_{i,h} \quad \forall i \in \mathcal{I}^{conv}$$

- Solar producer bids:

$$q_{s,h}^d = \hat{C}_{s,h}^d, \quad p_{s,h}^d = 0$$

This assumption isolates the strategic behavior to the wind producer.

7. Market-Clearing Mechanism (Transition Function)

For each (d, h):

1. **Bid set**

$$\mathcal{B}_h^d = \{(q_{i,h}^d, p_{i,h}^d)\}_{i \in \mathcal{I}}$$

2. **Merit-order dispatch**

- Sort bids by ascending $p_{i,h}^d$
- Accept bids until cumulative quantity meets D_h^d

3. **Market-clearing price**

$$\lambda_h^d = \max\{p_{i,h}^d \mid \text{accepted bids}\}$$

4. **Dispatch**

$$x_{i,h}^d = \begin{cases} \min(q_{i,h}^d, C_{i,h}^d), & \text{if accepted} \\ 0, & \text{otherwise} \end{cases}$$

8. Revenue Model

Hourly revenue (wind)

$$R_{w,h}^d = \lambda_h^d \cdot x_{w,h}^d$$

Daily revenue

$$R_w^d = \sum_{h=1}^{24} R_{w,h}^d$$

9. Objective Function

The wind producer seeks to maximize **expected daily revenue**:

$$\max_{\{q^d_{w,h}, p^d_{w,h}\}} \quad \mathbb{E}_{D, C_w, C_s} \left[\sum_{h=1}^{24} \lambda^d_h \, x^d_{w,h} \right]$$

Expectation is taken with respect to:

- $D^d_h \sim \mathcal{N}(\mu_h, \sigma^2_h)$
- Renewable availability distributions

---

10. Constant Parameters

- μ_h, σ^2_h: demand distribution parameters
- $\bar{C}_i$: installed capacities
- $c_{i,h}$: marginal costs
- Market rules (uniform pricing, merit order)
- Number of producers

---

11. Deep Uncertainties (Unchanged, Long-Term)

Handled via scenarios rather than probability distributions:

- Policy and carbon pricing
- Market design changes
- Technology evolution
- Structural demand shifts (changes in μ_h, σ^2_h)
- Entry or exit of competitors

---

12. Simulation Outputs

- Expected revenue $\mathbb{E}[R^d_w]$
- Revenue variance
- Probability of zero dispatch
- Sensitivity to μ_h and σ_h
- Robustness across deep-uncertainty scenarios

Key Refinement

The explicit assumption

$$D_h^d \sim \mathcal{N}(\mu_h, \sigma_h^2)$$

allows:

- Closed-form intuition about scarcity pricing
- Efficient Monte Carlo sampling
- Sensitivity analysis on demand volatility

If you want, I can next:

- Derive the **distribution of clearing prices** given this demand model, or
- Reformulate this as a **stochastic optimization problem**, or
- Show how to implement the simulation algorithmically (e.g., pseudocode or Python).

The mean and variance of the market demand are independent of time. Please correct your specification.

Good point — below is the **corrected and internally consistent specification**, assuming that **market demand has a time-invariant mean and variance**, i.e. no hourly heterogeneity in demand uncertainty.

---

1. Time and Index Sets

- Days: $d \in \mathcal{D}$
- Hours: $h \in \mathcal{H} = \{1, \ldots, 24\}$
- Producers:

$$\mathcal{I} = \mathcal{I}^{conv} \cup \{s, w\}, \quad |\mathcal{I}^{conv}| = 3$$

---

2. State Variables

For each day d and hour h:

1. **Available generation capacity**

$$C_{i,h}^d \quad (\text{MWh})$$

2. **Realized market demand**

$$D_h^d \quad (\text{MWh})$$

3. **Marginal costs of conventional producers**

$$c_i \quad \forall i \in \mathcal{I}^{conv}$$

(constant across time)

---

3. Demand Model (Corrected)

Market demand is **identically distributed across all hours and days**:

$$D_h^d \sim \mathcal{N}(\mu, \sigma^2) \quad \forall h, d$$

Assumptions:

- $\mu > 0$, $\sigma > 0$
- D_h^d are i.i.d. across h and d

- Physical feasibility:

$$D_h^d := \max\{0, D_h^d\}$$

4. Decision Variables (Wind Producer)

For each hour h on day d:

1. **Bid quantity**

$$q_{w,h}^d \geq 0$$

2. **Bid price**

$$p_{w,h}^d \geq 0$$

Constraint:

$$q_{w,h}^d \leq \hat{C}_{w,h}^d$$

5. Stochastic Variables

1. **Wind availability**

$$C_{w,h}^d \sim \mathcal{P}_w(\cdot)$$

2. **Solar availability**

$$C_{s,h}^d \sim \mathcal{P}_s(\cdot)$$

3. **Market demand**

$$D_h^d \sim \mathcal{N}(\mu, \sigma^2)$$

4. **Wind forecast error**

$$\varepsilon_{w,h}^d = C_{w,h}^d - \hat{C}_{w,h}^d$$

6. Bidding Behavior of Other Producers

For all h, d:

- **Conventional producers**

$$q_{i,h} = \bar{C}_i, \quad p_{i,h} = c_i \quad \forall i \in \mathcal{I}^{conv}$$

- **Solar producer**

$$q_{s,h}^d = \hat{C}_{s,h}^d, \quad p_{s,h}^d = 0$$

7. Market-Clearing Mechanism (Transition Function)

For each (d, h):

1. Collect bids:

$$\mathcal{B}_h^d = \{(q_{i,h}^d, p_{i,h}^d)\}_{i \in \mathcal{I}}$$

2. Merit-order dispatch:

- Sort bids by increasing $p_{i,h}^d$
- Accept bids until:

$$\sum_i x_{i,h}^d \geq D_h^d$$

3. Market-clearing price:

$$\lambda_h^d = \max\{p_{i,h}^d \mid x_{i,h}^d > 0\}$$

4. Dispatch:

$$x_{i,h}^d = \begin{cases} \min(q_{i,h}^d, C_{i,h}^d), & \text{if accepted} \\ 0, & \text{otherwise} \end{cases}$$

---

8. Revenue Model

Hourly revenue (wind)

$$R_{w,h}^d = \lambda_h^d \cdot x_{w,h}^d$$

Daily revenue

$$R_w^d = \sum_{h=1}^{24} R_{w,h}^d$$

---

9. Objective Function

The wind producer chooses bids to maximize **expected daily revenue**:

$$\max_{\{q_{w,h}^d, p_{w,h}^d\}} \quad \mathbb{E}\left[\sum_{h=1}^{24} \lambda_h^d \, x_{w,h}^d\right]$$

Expectation taken over:

$$D_h^d \sim \mathcal{N}(\mu, \sigma^2), \quad C_{w,h}^d, \quad C_{s,h}^d$$

---

10. Constant Parameters

- μ, σ^2: demand distribution parameters (time-invariant)
- $\bar{C}_i$: installed capacities
- c_i: marginal costs
- Market rules (uniform pricing, merit order)
- Number of producers

---

11. Deep Uncertainties (Long-Term)

Handled via scenario analysis:

- Structural changes in μ and σ^2
- Policy and regulatory regimes
- Market design changes
- Technology evolution
- Competitor entry/exit

---

12. Simulation Outputs

- Expected daily and annual revenue
- Revenue variance
- Probability of zero dispatch
- Sensitivity to μ and σ
- Robustness across deep-uncertainty scenarios

Final Note

With this correction, **all inter-temporal variation in outcomes arises from renewable availability and bidding decisions**, not from systematic changes in demand. This significantly simplifies the stochastic structure while preserving realistic price uncertainty driven by demand fluctuations.

You can also make the following assumptions regarding the behavioral logic of the producers: (1) the bid quantities of the conventional producers are stable, while their bid prices follow normal distributions with deeply uncertain means and variances; (2) the bid quantities of the solar-power producer can be modeled as $max(0, a+b\cos(\frac{2\pi t}{24}))$, and its bid prices also follow a normal distribution with deeply uncertain mean and variance; and (3) the actual wind-power production is stochastic and follows a normal distribution with constant mean and variance. Please refine your specification accordingly.

Below is a **fully refined and internally consistent simulation-model specification**, incorporating **all behavioral assumptions** you listed while keeping the model suitable for Monte Carlo simulation and deep-uncertainty scenario analysis.

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1. Time and Index Sets

- Days: $d \in \mathcal{D}$
- Hours: $h \in \mathcal{H} = \{1, \dots, 24\}$
- Producers:

$$\mathcal{I} = \mathcal{I}^{conv} \cup \{s, w\}, \quad |\mathcal{I}^{conv}| = 3$$

---

2. State Variables

For each day d and hour h:

1. **Market demand**

$$D_h^d \in \mathbb{R}_+$$

2. **Bid prices of competitors**

$$p_{i,h}^d \quad \forall i \in \mathcal{I}^{conv} \cup \{s\}$$

3. **Available renewable production**

$$C_{s,h}^d, \ C_{w,h}^d$$

4. **Market-clearing price**

$$\lambda_h^d$$

---

3. Demand Model (Unchanged, Time-Invariant)

Market demand is i.i.d. across hours and days:

$$D_h^d \sim \mathcal{N}(\mu_D, \sigma_D^2), \quad D_h^d := \max\{0, D_h^d\}$$

4. Behavioral Models of Producers

4.1 Conventional Producers (Deep Uncertainty in Prices)

For each conventional producer $i \in \mathcal{I}^{conv}$:

- **Bid quantity (stable)**

$$q_{i,h} = \bar{C}_i$$

- **Bid price (stochastic)**

$$p_{i,h}^d \sim \mathcal{N}(\mu_i, \sigma_i^2)$$

where:

- μ_i, σ_i^2 are **deeply uncertain parameters**
- Sampled once per scenario (not per realization)

4.2 Solar Producer

Bid Quantity (Deterministic Diurnal Pattern)

Let $t = h$ be the hour index.

$$q_{s,h} = \max\left(0,\ a + b\cos\left(\frac{2\pi t}{24}\right)\right)$$

Parameters:

- a: average solar output level
- b: diurnal amplitude
- a, b are scenario parameters

Bid Price (Deep Uncertainty)

$$p_{s,h}^d \sim \mathcal{N}(\mu_s, \sigma_s^2)$$

where μ_s, σ_s^2 are **deeply uncertain** and fixed per scenario.

4.3 Wind Producer (Decision Maker)

Actual Production (Stochastic)

$$C_{w,h}^d \sim \mathcal{N}(\mu_w, \sigma_w^2), \quad C_{w,h}^d := \max\{0, C_{w,h}^d\}$$

Mean and variance are **time-invariant**.

Decision Variables

For each hour h:

- **Bid quantity**

$$q_{w,h}^d \geq 0$$

- **Bid price**

$$p_{w,h}^d \geq 0$$

Constraint:

$$q_{w,h}^d \leq \hat{C}_{w,h}^d$$

where $\hat{C}_{w,h}^d$ is a forecast of $C_{w,h}^d$.

5. Stochastic Variables (Summary)

Variable	Distribution	Nature
D_h^d	$\mathcal{N}(\mu_D, \sigma_D^2)$	Aleatory
$p_{i,h}^d$ (conv.)	$\mathcal{N}(\mu_i, \sigma_i^2)$	Aleatory, parameters deeply uncertain
$p_{s,h}^d$	$\mathcal{N}(\mu_s, \sigma_s^2)$	Aleatory, parameters deeply uncertain
$C_{w,h}^d$	$\mathcal{N}(\mu_w, \sigma_w^2)$	Aleatory

6. Deep Uncertainty Parameters (Scenario-Level)

Sampled **once per scenario**, not per Monte Carlo draw:

- μ_i, σ_i^2: conventional bid price distributions
- μ_s, σ_s^2: solar bid price distribution
- a, b: solar production parameters
- μ_w, σ_w^2: wind production parameters (if long-term uncertainty considered)
- Structural demand parameters μ_D, σ_D^2

7. Market-Clearing Mechanism (Transition Function)

For each (d, h):

1. **Bid set**

$$\mathcal{B}_h^d = \{(q_{i,h}, p_{i,h}^d),\ (q_{s,h}, p_{s,h}^d),\ (q_{w,h}^d, p_{w,h}^d)\}$$

2. **Merit-order dispatch**

- Sort bids by increasing price
- Accept bids until cumulative quantity meets D_h^d

3. **Dispatch**

$$x_{i,h}^d = \begin{cases} q_{i,h}, & \text{if accepted (conv., solar)} \\ \min(q_{w,h}^d, C_{w,h}^d), & \text{if accepted (wind)} \\ 0, & \text{otherwise} \end{cases}$$

4. **Market-clearing price**

$$\lambda_h^d = \max\{p_{i,h}^d \mid x_{i,h}^d > 0\}$$

---

8. Revenue Model (Wind Producer)

Hourly Revenue

$$R_{w,h}^d = \lambda_h^d \cdot x_{w,h}^d$$

Daily Revenue

$$R_w^d = \sum_{h=1}^{24} R_{w,h}^d$$

---

9. Objective Function

The wind producer aims to maximize **expected daily revenue**:

$$\max_{\{q_{w,h}^d, p_{w,h}^d\}} \quad \mathbb{E}\left[\sum_{h=1}^{24} \lambda_h^d \, x_{w,h}^d\right]$$

Expectation taken over:

- D_h^d
- $p_{i,h}^d, p_{s,h}^d$
- $C_{w,h}^d$

---

10. Simulation Logic (Two-Level Uncertainty)

1. **Outer loop (scenarios)**
   Sample deeply uncertain parameters:

$$(\mu_i, \sigma_i^2), (\mu_s, \sigma_s^2), (a, b), (\mu_w, \sigma_w^2)$$

2. **Inner loop (Monte Carlo runs)**
   Sample realizations of:

$$D_h^d,\ p_{i,h}^d,\ p_{s,h}^d,\ C_{w,h}^d$$

---

11. Key Outputs

- Expected wind revenue
- Revenue variance and downside risk
- Dispatch probability
- Sensitivity to deep uncertainties
- Robustness of bidding strategies

Conceptual Takeaway

This refined formulation cleanly separates:

- **Aleatory uncertainty** (weather, demand, bidding noise),
- **Deep uncertainty** (cost structures, technology, market behavior),
- **Strategic control** (wind bidding decisions),

making it well suited for **robust decision-making (RDM)**, **exploratory modeling**, or **stochastic optimization under deep uncertainty**.

If you'd like, I can next:

- Translate this into **pseudocode or Python**, or
- Show how to embed **risk aversion** (e.g., CVaR), or
- Derive analytical insights about price formation under these assumptions.

Given that the actual wind-power production is stochastic, it may differ from the quantity committed to dispatch one day in advance. If the wind-power producer under-delivers relative to its dispatched quantity, the shortfall incurs a constant penalty cost per MWh. Please also implement this penalty mechanism.

Then, please provide a comprehensive model specification that specifies the key components, including state variables, decision variables, transition functions, objective functions, stochastic variables, constant parameters and deep uncertainties.

Below is a **complete and internally consistent simulation-model specification**, now **explicitly incorporating imbalance risk and penalty costs for wind under-delivery**, and clearly organizing **all key components** as requested.

Comprehensive Simulation Model Specification

Day-Ahead Electricity Market with Wind Imbalance Penalties

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1. Modeling Framework

- **Market type**: Competitive day-ahead spot market
- **Clearing rule**: Merit-order dispatch with uniform pricing
- **Time structure**:
  - Days $d \in \mathcal{D}$
  - Hours $h \in \mathcal{H} = \{1, \ldots, 24\}$
- **Agents**:
  - 3 conventional producers $\mathcal{I}^{conv}$
  - 1 solar producer s
  - 1 wind producer w (decision-maker)

The model is designed for **Monte Carlo simulation under deep uncertainty**, with bidding decisions fixed ex ante and realizations occurring ex post.

---

2. State Variables

For each day d and hour h:

1. **Realized market demand**

$$D_h^d \in \mathbb{R}_+$$

2. **Realized bid prices of competitors**

$$p_{i,h}^d \quad \forall i \in \mathcal{I}^{conv} \cup \{s\}$$

3. **Actual renewable production**

$$C_{s,h}^d, \quad C_{w,h}^d$$

4. **Dispatch quantities**

$$x_{i,h}^d \quad \forall i \in \mathcal{I}$$

5. **Market-clearing price**

$$\lambda_h^d$$

6. **Wind imbalance**

$$\Delta_{w,h}^d = x_{w,h}^d - C_{w,h}^d$$

---

3. Decision Variables (Wind Producer)

Chosen **day-ahead**, before uncertainties are realized.

For each hour h:

1. **Bid quantity**

$$q_{w,h} \geq 0$$

2. **Bid price**

$$p_{w,h} \geq 0$$

Constraint (based on forecasted availability):

$$q_{w,h} \leq \hat{C}_{w,h}$$

---

4. Stochastic Variables (Aleatory Uncertainty)

Sampled independently across hours and days.

4.1 Market Demand

$$D_h^d \sim \mathcal{N}(\mu_D, \sigma_D^2), \quad D_h^d := \max\{0, D_h^d\}$$

4.2 Conventional Producers' Bid Prices

For each $i \in \mathcal{I}^{conv}$:

$$p_{i,h}^d \sim \mathcal{N}(\mu_i, \sigma_i^2)$$

4.3 Solar Producer

- **Bid quantity (deterministic diurnal pattern)**
  Let $t = h$:

$$q_{s,h} = \max\left(0,\ a + b\cos\left(\frac{2\pi t}{24}\right)\right)$$

- **Bid price**

$$p^d_{s,h} \sim \mathcal{N}(\mu_s, \sigma_s^2)$$

4.4 Wind Production

$$C^d_{w,h} \sim \mathcal{N}(\mu_w, \sigma_w^2), \quad C^d_{w,h} := \max\{0, C^d_{w,h}\}$$

5. Constant Parameters

- μ_D, σ_D^2: demand distribution parameters
- $\bar{C}_i$: installed capacities of conventional producers
- μ_w, σ_w^2: wind production parameters (short-term)
- a, b: solar production parameters
- π^{imb}: **penalty cost per MWh of wind under-delivery**
- Market rules: uniform pricing, merit order, one-sided penalty
- Simulation horizon $|\mathcal{D}|$

6. Deep Uncertainties (Scenario-Level)

Sampled **once per scenario**, fixed across Monte Carlo runs.

- μ_i, σ_i^2: conventional bid-price distributions
- μ_s, σ_s^2: solar bid-price distribution
- a, b: solar generation profile
- Long-term shifts in μ_D, σ_D^2
- Long-term changes in μ_w, σ_w^2
- Market design or regulatory regimes

These are **not assigned probabilities** but explored via scenario ensembles.

7. Transition Function: Market Clearing

For each (d, h):

Step 1: Bid Collection

$$\mathcal{B}_h^d = \left\{ (\bar{C}_i, p_{i,h}^d)_{i \in \mathcal{I}^{conv}}, (q_{s,h}, p_{s,h}^d), (q_{w,h}, p_{w,h}) \right\}$$

Step 2: Merit-Order Dispatch

- Sort bids by ascending price
- Accept bids until cumulative quantity meets demand D_h^d

Dispatch quantity:

$$x_{i,h}^d = \begin{cases} q_{i,h}, & \text{if accepted} \\ 0, & \text{otherwise} \end{cases}$$

Step 3: Market-Clearing Price

$$\lambda_h^d = \max\{p_{i,h}^d \mid x_{i,h}^d > 0\}$$

---

8. Wind Imbalance and Penalty Mechanism

Under-Delivery

If actual wind production is insufficient:

$$\Delta_{w,h}^d = x_{w,h}^d - C_{w,h}^d > 0$$

Penalty Cost

$$\text{Penalty}_{w,h}^d = \pi^{imb} \cdot \max\{0, \Delta_{w,h}^d\}$$

(No reward or penalty for over-delivery.)

---

9. Revenue Model

Hourly Revenue (Wind)

$$R_{w,h}^d = \lambda_h^d \cdot x_{w,h}^d \; - \; \pi^{imb} \cdot \max\{0, x_{w,h}^d - C_{w,h}^d\}$$

Daily Revenue

$$R_w^d = \sum_{h=1}^{24} R_{w,h}^d$$

---

10. Objective Function

The wind producer selects bids to maximize **expected daily profit**:

$$\max_{\{q_{w,h},p_{w,h}\}} \quad \mathbb{E}\left[\sum_{h=1}^{24}\left(\lambda_h^d\, x_{w,h}^d - \pi^{imb}\max\{0, x_{w,h}^d - C_{w,h}^d\}\right)\right]$$

Expectation taken over:

$$D_h^d,\ p_{i,h}^d,\ p_{s,h}^d,\ C_{w,h}^d$$

11. Simulation Structure (Two-Level Uncertainty)

1. **Outer loop (deep uncertainty scenarios)**
   Sample:

$$\mu_i, \sigma_i^2,\ \mu_s, \sigma_s^2,\ a, b,\ \mu_D, \sigma_D^2$$

2. **Inner loop (Monte Carlo realizations)**
   For each scenario, simulate many days:

$$D_h^d,\ p_{i,h}^d,\ p_{s,h}^d,\ C_{w,h}^d$$

12. Key Outputs

- Expected revenue and profit
- Revenue variance and downside risk
- Expected imbalance penalties
- Probability of under-delivery
- Sensitivity to penalty level π^{imb}
- Robustness of bidding strategies across scenarios

Final Interpretation

This model now captures the **central economic trade-off for wind investment**:

> **Higher bid quantities increase dispatch probability and price exposure but also raise expected imbalance penalties due to stochastic wind production.**

The formulation is suitable for:

- Monte Carlo simulation
- Robust decision-making under deep uncertainty
- Risk-aware bidding strategy analysis

- Investment viability assessment

If you want, I can next:

- Reformulate this as a **stochastic program with recourse**, or
- Show **pseudocode / Python implementation**, or
- Add **risk aversion (e.g., CVaR)** to the objective.

You do not need to consider deep uncertainties arising from long-term shifts in a, b, μ_D, σ_D, μ_w or σ_w. You can assume that they are constant over time. Please refine your specification.

Below is the **refined and simplified comprehensive model specification**, updated to reflect your instruction that

$$a,\ b,\ \mu_D,\ \sigma_D,\ \mu_w,\ \sigma_w$$

are **constant over time and not sources of deep uncertainty**.
Deep uncertainty now applies **only to competitors' bid-price distributions**.

Refined Comprehensive Simulation Model Specification

Day-Ahead Electricity Market with Wind Imbalance Penalties

---

1. Modeling Framework

- **Market**: Day-ahead spot market with uniform pricing
- **Clearing rule**: Merit-order dispatch
- **Time structure**:
  - Days $d \in \mathcal{D}$
  - Hours $h \in \{1, \ldots, 24\}$
- **Producers**:
  - $\mathcal{I}^{conv}$: 3 conventional producers
  - s: solar producer
  - w: wind producer (decision-maker)

The model supports **Monte Carlo simulation** with **scenario-based deep uncertainty** in competitor behavior only.

---

2. State Variables

For each day d and hour h:

1. **Market demand**

$$D_h^d \in \mathbb{R}_+$$

2. **Competitors' realized bid prices**

$$p_{i,h}^d \quad \forall i \in \mathcal{I}^{conv} \cup \{s\}$$

3. **Actual wind production**

$$C_{w,h}^d$$

4. **Dispatch quantities**

$$x_{i,h}^d \quad \forall i \in \mathcal{I}$$

5. **Market-clearing price**

$$\lambda_h^d$$

6. **Wind shortfall**

$$\Delta_{w,h}^d = x_{w,h}^d - C_{w,h}^d$$

---

3. Decision Variables (Wind Producer)

Chosen day-ahead, before uncertainty realization.

For each hour h:

1. **Bid quantity**

$$q_{w,h} \geq 0$$

2. **Bid price**

$$p_{w,h} \geq 0$$

Constraint:

$$q_{w,h} \leq \hat{C}_{w,h}$$

---

4. Stochastic Variables (Aleatory Uncertainty)

All stochastic variables are **time-invariant in distribution**.

4.1 Market Demand

$$D_h^d \sim \mathcal{N}(\mu_D, \sigma_D^2), \quad D_h^d := \max\{0, D_h^d\}$$

4.2 Conventional Producers' Bid Prices

For each $i \in \mathcal{I}^{conv}$:

$$p_{i,h}^d \sim \mathcal{N}(\mu_i, \sigma_i^2)$$

4.3 Solar Producer

Bid quantity (deterministic diurnal pattern)
Let $t = h$:

$$q_{s,h} = \max\left(0,\ a + b\cos\left(\frac{2\pi t}{24}\right)\right)$$

Bid price

$$p^d_{s,h} \sim \mathcal{N}(\mu_s, \sigma_s^2)$$

4.4 Wind Production

$$C^d_{w,h} \sim \mathcal{N}(\mu_w, \sigma_w^2), \quad C^d_{w,h} := \max\{0, C^d_{w,h}\}$$

5. Constant Parameters

These parameters are **fixed across time and scenarios**:

- μ_D, σ_D^2: demand distribution
- μ_w, σ_w^2: wind production distribution
- a, b: solar diurnal production parameters
- $\bar{C}_i$: conventional producers' bid quantities
- π^{imb}: penalty cost per MWh of wind under-delivery
- Market rules (uniform pricing, merit order)

6. Deep Uncertainties (Scenario-Level)

Deep uncertainty is limited to **competitor pricing behavior**.

Sampled once per scenario:

- μ_i, σ_i^2: conventional producers' bid-price distributions
- μ_s, σ_s^2: solar producer's bid-price distribution

No probabilities are assigned to these parameters; they define **alternative plausible market environments**.

7. Transition Function: Market Clearing

For each (d, h):

Step 1: Bid Assembly

$$\mathcal{B}_h^d = \left\{ (\bar{C}_i, p_{i,h}^d)_{i \in \mathcal{I}^{conv}}, (q_{s,h}, p_{s,h}^d), (q_{w,h}, p_{w,h}) \right\}$$

Step 2: Merit-Order Dispatch

- Sort bids by ascending price
- Accept bids until cumulative supply meets D_h^d

Dispatch:

$$x_{i,h}^d = \begin{cases} q_{i,h}, & \text{if accepted} \\ 0, & \text{otherwise} \end{cases}$$

Step 3: Market-Clearing Price

$$\lambda_h^d = \max\{p_{i,h}^d \mid x_{i,h}^d > 0\}$$

8. Wind Imbalance and Penalty Mechanism

If wind under-delivers:

$$\Delta_{w,h}^d = x_{w,h}^d - C_{w,h}^d > 0$$

Penalty:

$$\text{Penalty}_{w,h}^d = \pi^{imb} \cdot \max\{0, \Delta_{w,h}^d\}$$

No compensation for over-delivery.

9. Revenue Model (Wind Producer)

Hourly Profit

$$R_{w,h}^d = \lambda_h^d \cdot x_{w,h}^d - \pi^{imb} \cdot \max\{0, x_{w,h}^d - C_{w,h}^d\}$$

Daily Profit

$$R_w^d = \sum_{h=1}^{24} R_{w,h}^d$$

10. Objective Function

The wind producer maximizes **expected daily profit**:

$$\max_{\{q_{w,h},p_{w,h}\}} \quad \mathbb{E}\left[\sum_{h=1}^{24} \left(\lambda_h^d x_{w,h}^d - \pi^{imb} \max\{0, x_{w,h}^d - C_{w,h}^d\}\right)\right]$$

Expectation is taken over:

$$D_h^d,\ p_{i,h}^d,\ p_{s,h}^d,\ C_{w,h}^d$$

---

11. Simulation Structure

1. **Scenario loop**
   Fix:

$$(\mu_i, \sigma_i^2),\ (\mu_s, \sigma_s^2)$$

2. **Monte Carlo loop**
   Simulate many days by sampling:

$$D_h^d,\ p_{i,h}^d,\ p_{s,h}^d,\ C_{w,h}^d$$

---

12. Key Outputs

- Expected wind profit
- Profit variance and downside risk
- Expected imbalance penalties
- Dispatch probability
- Sensitivity to π^{imb}
- Robustness of bidding strategies across competitor-price scenarios

---

Final Takeaway

With this refinement, the model cleanly distinguishes:

- **Fixed physical and demand fundamentals**
- **Stochastic short-term operational uncertainty**
- **Deep uncertainty limited to competitors' pricing behavior**

This yields a **tractable yet realistic simulation framework** for evaluating wind-power investment viability and bidding strategies in competitive electricity markets.

Various stakeholders approach this problem from different perspectives. Please refine the model specification from the perspectives of (1) the wind-power producer; and (2) the system regulator. These two perspectives should share a common environment, with each stakeholder independently controlling its own decision variables. In the specification, specify the common environment, including the global parameters and variables. For each perspective, also specify the corresponding model and its key components, including the decision variables, transition functions, and objective functions. If there are multiple objectives, provide multiple objective functions instead of a weighted objective function.

Below is a **stakeholder-specific refinement** of the model, structured around a **shared common environment** with **two independent decision-makers**:

1. **Wind-power producer** (profit-maximizing market participant)
2. **System regulator** (market designer / system overseer)

The two perspectives are **embedded in the same stochastic market environment**, but each stakeholder:

- controls **its own decision variables only**;
- treats the other stakeholder's decisions as **exogenous inputs**;
- has **distinct objectives**.

---

1. Common Environment (Shared by All Stakeholders)

The common environment defines the **physical system, market rules, uncertainties, and state evolution**.

1.1 Time and Index Sets

- Days: $d \in \mathcal{D}$
- Hours: $h \in \mathcal{H} = \{1, \dots, 24\}$
- Producers:
  - $\mathcal{I}^{conv}$: 3 conventional producers
  - s: solar producer
  - w: wind producer

1.2 Global State Variables

For each (d, h):

1. **Market demand**

$$D_h^d$$

2. **Actual wind production**

$$C_{w,h}^d$$

3. **Competitors' bid prices**

$$p_{i,h}^d \quad (i \in \mathcal{I}^{conv} \cup \{s\})$$

4. **Dispatch quantities**

$$x_{i,h}^d \quad (i \in \mathcal{I}^{conv} \cup \{s, w\})$$

5. **Market-clearing price**

$$\lambda_h^d$$

6. **Wind shortfall**

$$\Delta_{w,h}^d = x_{w,h}^d - C_{w,h}^d$$

1.3 Global Stochastic Processes

All distributions are **time-invariant**.

Demand

$$D_h^d \sim \mathcal{N}(\mu_D, \sigma_D^2), \quad D_h^d := \max(0, D_h^d)$$

Wind production

$$C_{w,h}^d \sim \mathcal{N}(\mu_w, \sigma_w^2), \quad C_{w,h}^d := \max(0, C_{w,h}^d)$$

Conventional bid prices

$$p_{i,h}^d \sim \mathcal{N}(\mu_i, \sigma_i^2), \quad i \in \mathcal{I}^{conv}$$

Solar bid price

$$p_{s,h}^d \sim \mathcal{N}(\mu_s, \sigma_s^2)$$

1.4 Deterministic Producer Behavior (Non-Strategic)

Conventional producers

$$q_{i,h} = \bar{C}_i$$

Solar producer (diurnal quantity)

$$q_{s,h} = \max\left(0, a + b\cos\left(\frac{2\pi h}{24}\right)\right)$$

1.5 Market-Clearing Transition Function

Given bids $(q_{i,h}, p_{i,h})$:

1. **Merit-order dispatch**
   Sort bids by price and accept until

$$\sum_i x^d_{i,h} \geq D^d_h$$

2. **Market-clearing price**

$$\lambda^d_h = \max\{p^d_{i,h} \mid x^d_{i,h} > 0\}$$

3. **Dispatch**

$$x^d_{i,h} = \begin{cases} q_{i,h}, & \text{if accepted} \\ 0, & \text{otherwise} \end{cases}$$

1.6 Global Parameters

- μ_D, σ^2_D: demand distribution
- μ_w, σ^2_w: wind production distribution
- a, b: solar profile parameters
- $\bar{C}_i$: conventional capacities
- π^{imb}: imbalance penalty per MWh
- Market rules (uniform pricing, merit order)

---

2. Perspective 1: Wind-Power Producer

The wind producer is a **strategic market participant**.

2.1 Decision Variables (Controlled by Wind Producer)

For each hour h:

- **Bid quantity**

$$q_{w,h} \geq 0$$

- **Bid price**

$$p_{w,h} \geq 0$$

Constraint:

$$q_{w,h} \leq \hat{C}_{w,h}$$

2.2 Transition Function (From Wind Perspective)

Given:

- regulator-defined rules and penalties;
- competitor bids and prices (exogenous);
- stochastic realizations;

the environment produces:

$$(\lambda_h^d, x_{w,h}^d, C_{w,h}^d)$$

Wind imbalance:

$$\Delta_{w,h}^d = x_{w,h}^d - C_{w,h}^d$$

2.3 Objective Functions (Wind Producer)

Objective 1: Maximize expected profit

$$\max_{q_{w,h}, p_{w,h}} \mathbb{E}\left[\sum_{h=1}^{24} \left(\lambda_h^d x_{w,h}^d - \pi^{imb} \max(0, x_{w,h}^d - C_{w,h}^d)\right)\right]$$

(Optional additional objectives, if desired):

- minimize expected imbalance volume;
- minimize revenue volatility.

Each objective remains **separate**, not weighted.

---

3. Perspective 2: System Regulator

The regulator **does not bid into the market**. Instead, it designs or tunes **market parameters**.

3.1 Decision Variables (Controlled by Regulator)

Examples (subset may be active):

1. **Imbalance penalty**

$$\pi^{imb} \geq 0$$

2. **Price-cap or floor**

$$\lambda_h^d \leq \bar{\lambda}$$

3. **Renewable priority rules** (binary or categorical)

$$\rho \in \{\text{neutral}, \text{priority}\}$$

These variables shape the environment but are **not controlled by producers**.

3.2 Transition Function (From Regulator Perspective)

Given:

- producers' bids (exogenous);
- stochastic realizations;

the market rules selected by the regulator determine:

$$\lambda_h^d,\ x_{i,h}^d,\ \Delta_{w,h}^d$$

3.3 Objective Functions (System Regulator)

The regulator may have **multiple, non-commensurate objectives**.

Objective 1: Minimize expected imbalance

$$\min_{\pi^{imb},\dots} \ \mathbb{E}\left[\sum_{h=1}^{24} \max(0, x_{w,h}^d - C_{w,h}^d)\right]$$

Objective 2: Minimize price volatility

$$\min_{\pi^{imb},\dots} \ \mathrm{Var}\left(\lambda_h^d\right)$$

Objective 3: Maintain revenue adequacy for renewables

$$\max_{\pi^{imb},\dots} \ \mathbb{E}\left[R_w^d\right]$$

Objective 4: Minimize expected system cost

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$$\min_{\pi^{imb},...} \mathbb{E}\left[\sum_{h,i} \lambda_h^d x_{i,h}^d\right]$$

These objectives are **not aggregated** and can be evaluated independently.

4. Relationship Between Perspectives

- Both stakeholders:
  - operate in the **same stochastic environment**;
  - observe the same prices, dispatch, and realizations.
- Each stakeholder:
  - controls **only its own decision variables**;
  - treats the other's decisions as **fixed inputs**.
- The model supports:
  - policy stress-testing;
  - equilibrium or bilevel extensions;
  - robust or exploratory analysis.

Final Conceptual Summary

Element	Wind Producer	System Regulator
Control variables	Bids (q, p)	Penalties, rules
Main concern	Profit & risk	Reliability & efficiency
Time of action	Day-ahead	Structural / policy
Environment	Shared stochastic market	Shared stochastic market

This specification provides a **clean, modular foundation** for simulation, comparative evaluation, and future extensions such as bilevel optimization or equilibrium analysis.

According to the specification, imbalance penalty is a decision variable controlled by the system regulator and therefore should not be treated as a global parameter. Please refine your specification.

You are absolutely right. Below is the **refined and internally consistent specification**, where the **imbalance penalty is no longer a global parameter** but a **decision variable controlled exclusively by the system regulator**.
All other components remain unchanged unless logically required by this correction.

Refined Two-Perspective Model Specification

(With Regulator-Controlled Imbalance Penalty)

---

1. Common Environment (Shared by All Stakeholders)

The common environment defines **states, uncertainties, physical constraints, and market-clearing logic**, but **does not include any stakeholder decision variables**.

1.1 Time and Index Sets

- Days: $d \in \mathcal{D}$
- Hours: $h \in \mathcal{H} = \{1, \ldots, 24\}$
- Producers:
  - $\mathcal{I}^{conv}$: 3 conventional producers
  - s: solar producer
  - w: wind producer

1.2 Global State Variables

For each (d, h):

1. **Market demand**

$$D_h^d$$

2. **Actual wind production**

$$C_{w,h}^d$$

3. **Competitors' realized bid prices**

$$p^d_{i,h} \quad i \in \mathcal{I}^{conv} \cup \{s\}$$

4. **Dispatch quantities**

$$x^d_{i,h} \quad i \in \mathcal{I}^{conv} \cup \{s, w\}$$

5. **Market-clearing price**

$$\lambda^d_h$$

6. **Wind shortfall**

$$\Delta^d_{w,h} = x^d_{w,h} - C^d_{w,h}$$

1.3 Global Stochastic Variables

All distributions are **time-invariant**.

Market demand

$$D^d_h \sim \mathcal{N}(\mu_D, \sigma^2_D), \quad D^d_h := \max(0, D^d_h)$$

Wind production

$$C^d_{w,h} \sim \mathcal{N}(\mu_w, \sigma^2_w), \quad C^d_{w,h} := \max(0, C^d_{w,h})$$

Conventional producers' bid prices

$$p^d_{i,h} \sim \mathcal{N}(\mu_i, \sigma^2_i), \quad i \in \mathcal{I}^{conv}$$

Solar bid price

$$p^d_{s,h} \sim \mathcal{N}(\mu_s, \sigma^2_s)$$

1.4 Deterministic Non-Strategic Behavior

Conventional producers

$$q_{i,h} = \bar{C}_i$$

Solar producer (diurnal quantity)

$$q_{s,h} = \max\left(0,\ a + b\cos\left(\frac{2\pi h}{24}\right)\right)$$

1.5 Market-Clearing Transition Function

Given bids $(q_{i,h}, p_{i,h})$ and realizations:

1. **Merit-order dispatch**

$$\sum_i x_{i,h}^d \geq D_h^d$$

2. **Market-clearing price**

$$\lambda_h^d = \max\{p_{i,h}^d \mid x_{i,h}^d > 0\}$$

3. **Dispatch**

$$x_{i,h}^d = \begin{cases} q_{i,h}, & \text{if accepted} \\ 0, & \text{otherwise} \end{cases}$$

1.6 Global Parameters (Corrected)

These are **true parameters**, fixed and not controlled by any stakeholder:

- μ_D, σ_D^2: demand distribution
- μ_w, σ_w^2: wind production distribution
- a, b: solar diurnal parameters
- $\bar{C}_i$: conventional bid quantities
- Market design primitives:
  - uniform pricing
  - merit-order dispatch

Note:
⚠️ *Imbalance penalty is intentionally excluded from this list.*

---

2. Perspective 1: Wind-Power Producer

2.1 Decision Variables (Wind Producer)

For each hour h:

- **Bid quantity**

$$q_{w,h} \geq 0$$

- **Bid price**

$$p_{w,h} \geq 0$$

Constraint:

$$q_{w,h} \le \hat{C}_{w,h}$$

2.2 Transition Function (Wind Perspective)

Given:

- regulator-selected imbalance penalty π^{imb};
- competitor bids (exogenous);
- stochastic realizations;

the environment produces:

$$(\lambda_h^d, x_{w,h}^d, C_{w,h}^d)$$

Wind imbalance:

$$\Delta_{w,h}^d = x_{w,h}^d - C_{w,h}^d$$

2.3 Objective Functions (Wind Producer)

Objective 1: Maximize expected profit

$$\max_{q_{w,h}, p_{w,h}} \mathbb{E}\left[\sum_{h=1}^{24} \left(\lambda_h^d x_{w,h}^d - \pi^{imb} \max(0, \Delta_{w,h}^d)\right)\right]$$

Additional (optional) objectives, evaluated separately:

- minimize expected imbalance volume;
- minimize profit volatility.

3. Perspective 2: System Regulator

The regulator **does not participate in bidding**, but **sets market rules**.

3.1 Decision Variables (System Regulator)

These variables **shape the common environment**.

1. **Imbalance penalty**

$$\pi^{imb} \ge 0$$

2. *(Optional extensions)*
    - price caps or floors
    - renewable priority rules

3.2 Transition Function (Regulator Perspective)

Given:

- producers' bids (exogenous);
- stochastic realizations;

the regulator's decision π^{imb} determines:

- realized imbalance costs;
- producer incentives;
- dispatch outcomes via induced bidding behavior.

3.3 Objective Functions (System Regulator)

Objectives are **not aggregated**.

Objective 1: Minimize expected wind imbalance

$$\min_{\pi^{imb}} \ \mathbb{E}\left[\sum_{h=1}^{24} \max(0, \Delta^d_{w,h})\right]$$

Objective 2: Minimize price volatility

$$\min_{\pi^{imb}} \ \mathrm{Var}(\lambda^d_h)$$

Objective 3: Maintain renewable revenue adequacy

$$\max_{\pi^{imb}} \ \mathbb{E}[R^d_w]$$

Objective 4: Minimize expected system cost

$$\min_{\pi^{imb}} \ \mathbb{E}\left[\sum_{h,i} \lambda^d_h x^d_{i,h}\right]$$

4. Corrected Consistency Summary

Element	Status
Imbalance penalty π^{imb}	**Regulator decision variable**
Wind profit function	Depends on π^{imb} exogenously

Element	Status
Global parameters	Physical + statistical only
Environment	Shared, policy-agnostic
Stakeholders	Independent control

Final Note

This correction restores **full conceptual consistency**:

- **No decision variable appears as a global parameter**
- **Each stakeholder controls only its own levers**
- **Interactions occur solely through the shared environment**

The model is now ready for:

- modular simulation,
- bilevel or leader–follower analysis,
- or robust policy stress-testing.

If you want, I can next **compose these perspectives into a single unified mathematical or computational model**, exactly as required for implementation.

To ensure internal consistency across these models, please compose them into a single, unified model that shares a common environment and states, while embedding perspective-specific decisions and objectives. The composed model should: 1. embed each perspective's decision variables and objectives; 2. enable modular evaluation of each perspective; 3. for each perspective, treat other perspectives' decisions as exogenous inputs; and 4. incorporate all other components from every perspective.

Below is a **single, unified model specification** that **composes the wind-producer and system-regulator perspectives into one internally consistent framework**, while satisfying all four requirements you listed.

The key idea is to define **one common stochastic environment and state space**, and then **embed perspective-specific decision variables and objectives as modular layers** that interact only through that environment.

Unified Composed Model

Day-Ahead Electricity Market with Wind Imbalance Penalties

---

1. Common Environment (Shared State and Dynamics)

The common environment is **policy- and strategy-agnostic**.
It contains **no stakeholder decision variables**, only states, uncertainties, and transition rules.

1.1 Time and Index Sets

- Days: $d \in \mathcal{D}$
- Hours: $h \in \mathcal{H} = \{1, \dots, 24\}$
- Producers:

$$\mathcal{I} = \mathcal{I}^{conv} \cup \{s, w\}, \quad |\mathcal{I}^{conv}| = 3$$

1.2 Global State Variables

For each (d, h):

1. **Market demand**

$$D_h^d$$

2. **Actual wind production**

$$C^d_{w,h}$$

3. **Competitors' realized bid prices**

$$p^d_{i,h} \quad i \in \mathcal{I}^{conv} \cup \{s\}$$

4. **Dispatch quantities**

$$x^d_{i,h} \quad i \in \mathcal{I}$$

5. **Market-clearing price**

$$\lambda^d_h$$

6. **Wind imbalance**

$$\Delta^d_{w,h} = x^d_{w,h} - C^d_{w,h}$$

1.3 Global Stochastic Variables

All distributions are **time-invariant**.

- **Demand**

$$D^d_h \sim \mathcal{N}(\mu_D, \sigma^2_D), \quad D^d_h := \max(0, D^d_h)$$

- **Wind production**

$$C^d_{w,h} \sim \mathcal{N}(\mu_w, \sigma^2_w), \quad C^d_{w,h} := \max(0, C^d_{w,h})$$

- **Conventional bid prices**

$$p^d_{i,h} \sim \mathcal{N}(\mu_i, \sigma^2_i), \quad i \in \mathcal{I}^{conv}$$

- **Solar bid price**

$$p^d_{s,h} \sim \mathcal{N}(\mu_s, \sigma^2_s)$$

1.4 Deterministic Non-Strategic Behavior

- **Conventional quantities**

$$q_{i,h} = \bar{C}_i$$

- **Solar quantity (diurnal)**

$$q_{s,h} = \max\left(0,\ a + b\cos\left(\frac{2\pi h}{24}\right)\right)$$

1.5 Market-Clearing Transition Function

Given bids $(q_{i,h}, p_{i,h})$:

1. **Merit-order dispatch**

$$\sum_i x_{i,h}^d \geq D_h^d$$

2. **Dispatch**

$$x_{i,h}^d = \begin{cases} q_{i,h}, & \text{if accepted} \\ 0, & \text{otherwise} \end{cases}$$

3. **Market-clearing price**

$$\lambda_h^d = \max\{p_{i,h}^d \mid x_{i,h}^d > 0\}$$

This transition function is **shared by all perspectives**.

1.6 Global Parameters

- μ_D, σ_D^2: demand distribution
- μ_w, σ_w^2: wind production distribution
- a, b: solar profile parameters
- $\bar{C}_i$: conventional capacities
- Market design primitives (uniform pricing, merit order)

---

2. Embedded Perspective A: Wind-Power Producer

This perspective is **fully embedded** in the common environment.

2.1 Wind Decision Variables

For each hour h:

- **Bid quantity**

$$q_{w,h} \geq 0$$

- **Bid price**

$$p_{w,h} \geq 0$$

Constraint:

$$q_{w,h} \leq \hat{C}_{w,h}$$

2.2 Wind Perspective Transition Mapping

Given:

- regulator decision π^{imb} (exogenous);
- competitor bids (exogenous);
- stochastic realizations;

the environment induces:

$$(q_{w,h}, p_{w,h}) \longrightarrow (\lambda_h^d, x_{w,h}^d, C_{w,h}^d)$$

Wind imbalance:

$$\Delta_{w,h}^d = x_{w,h}^d - C_{w,h}^d$$

2.3 Wind Objective Functions

Objective W1: Maximize expected profit

$$\max_{q_{w,h}, p_{w,h}} \mathbb{E}\left[\sum_{h=1}^{24} \left(\lambda_h^d x_{w,h}^d - \pi^{imb} \max(0, \Delta_{w,h}^d)\right)\right]$$

Optional secondary objectives (evaluated separately):

- minimize expected imbalance volume;
- minimize profit volatility.

---

3. Embedded Perspective B: System Regulator

The regulator **modifies the environment**, but does not participate in dispatch.

3.1 Regulator Decision Variables

- **Imbalance penalty**

$$\pi^{imb} \geq 0$$

(Other market rules could be added, but are omitted for clarity.)

3.2 Regulator Perspective Transition Mapping

Given:

- wind bids $(q_{w,h}, p_{w,h})$ (exogenous);
- stochastic realizations;

the regulator's decision π^{imb} affects:

- imbalance costs;

- wind incentives;
- resulting dispatch and prices.

3.3 Regulator Objective Functions

Objectives are **not aggregated**.

Objective R1: Minimize expected wind imbalance

$$\min_{\pi^{imb}} \mathbb{E}\left[\sum_{h=1}^{24} \max(0, \Delta^d_{w,h})\right]$$

Objective R2: Minimize price volatility

$$\min_{\pi^{imb}} \mathrm{Var}(\lambda^d_h)$$

Objective R3: Maintain renewable revenue adequacy

$$\max_{\pi^{imb}} \mathbb{E}[R^d_w]$$

Objective R4: Minimize expected system cost

$$\min_{\pi^{imb}} \mathbb{E}\left[\sum_{h,i} \lambda^d_h x^d_{i,h}\right]$$

4. Composition Properties (Explicitly Verified)

Requirement	Satisfied How
1. Embed each perspective's decisions & objectives	Sections 2 and 3
2. Modular evaluation	Each perspective has its own decision set and objectives
3. Treat other decisions as exogenous	Wind treats π^{imb} as given; regulator treats bids as given
4. Incorporate all components	All states, uncertainties, penalties, and market rules unified

5. Conceptual Structure

$$\boxed{\text{Regulator decisions} \longrightarrow \text{Market rules} \longrightarrow \text{Wind bidding} \longrightarrow \text{Stochastic market outcomes}}$$

Each stakeholder **optimizes within the same realized world**, but **from its own viewpoint**.

Final Remark

This unified model is now:

- internally consistent,
- modular,
- extensible to bilevel or equilibrium analysis,
- directly implementable in simulation or optimization code.

If you want, the next natural step is to **translate this unified model into a modular Python implementation**, with each perspective represented as a class sharing a common environment.

Now, provide a modular Python implementation of the unified model, using classes to represent different perspectives. At the end of the program, define a function as the interface. This function should take as inputs a dictionary of decision variable values, a dictionary of uncertain parameter values, and a dictionary of constant parameter values; simulate the model dynamics; and return the value of each objective function. This function does not need to be executed.

Below is a **modular Python implementation** of the **unified composed model**, written to closely mirror the mathematical specification.

Key design principles satisfied:

- ✅ **Single shared environment and state**
- ✅ **Separate classes for perspectives (wind producer, regulator)**
- ✅ **Each perspective controls only its own decision variables**
- ✅ **Other perspectives' decisions treated as exogenous**
- ✅ **Objectives evaluated modularly**
- ✅ **Interface function with required inputs and outputs**

The code is **not executed** and is intended as a **reference implementation** suitable for simulation, Monte Carlo analysis, or later optimization.

High-Level Structure

- `MarketEnvironment`: shared stochastic environment and market-clearing logic
- `WindProducer`: wind producer decisions and objectives
- `SystemRegulator`: regulator decisions and objectives
- `simulate_market`: interface function

Python Implementation

```python
import numpy as np
from typing import Dict, Any


# ============================================================
# Common Environment
# ============================================================

class MarketEnvironment:
    """
    Shared stochastic environment and market-clearing mechanism.
    Contains no stakeholder decision variables.
    """
```

```python
    def __init__(self, constants: Dict[str, Any], uncertainties: Dict[str, Any]):
        # Constant parameters
        self.mu_D = constants["mu_D"]
        self.sigma_D = constants["sigma_D"]
        self.mu_w = constants["mu_w"]
        self.sigma_w = constants["sigma_w"]
        self.a = constants["solar_a"]
        self.b = constants["solar_b"]
        self.conventional_capacities = constants["conventional_capacities"]
        self.hours = constants.get("hours", 24)

        # Uncertain parameters (scenario-level)
        self.conv_price_params = uncertainties["conventional_price_params"]
        self.solar_price_params = uncertainties["solar_price_params"]

    # ----------------------------
    # Stochastic processes
    # ----------------------------

    def sample_demand(self):
        return max(0.0, np.random.normal(self.mu_D, self.sigma_D))

    def sample_wind_production(self):
        return max(0.0, np.random.normal(self.mu_w, self.sigma_w))

    def sample_conventional_prices(self):
        prices = []
        for params in self.conv_price_params:
            mu, sigma = params["mu"], params["sigma"]
            prices.append(np.random.normal(mu, sigma))
        return prices

    def sample_solar_price(self):
        mu, sigma = self.solar_price_params["mu"], self.solar_price_params["sigma"]
        return np.random.normal(mu, sigma)

    def solar_quantity(self, hour: int):
        return max(0.0, self.a + self.b * np.cos(2 * np.pi * hour / 24))

    # ----------------------------
    # Market clearing
    # ----------------------------

    def clear_market(self, bids, demand):
        """
        bids: list of tuples (quantity, price, producer_id)
        """
        bids_sorted = sorted(bids, key=lambda x: x[1])

        dispatched = {}
        supplied = 0.0
        clearing_price = 0.0

        for q, p, pid in bids_sorted:
            if supplied >= demand:
                break
            dispatched_qty = q
            supplied += dispatched_qty
            dispatched[pid] = dispatched_qty
            clearing_price = p

        return dispatched, clearing_price


# ============================================================
# Wind Producer Perspective
# ============================================================

class WindProducer:
    """
    Wind producer decisions and objectives.
    """

    def __init__(self, decisions: Dict[str, Any]):
```

```
        self.bid_quantities = decisions["bid_quantities"]  # list length 24
        self.bid_prices = decisions["bid_prices"]          # list length 24

    def profit(self, prices, dispatch, wind_output, imbalance_penalty):
        """
        Compute total profit over all hours.
        """
        profit = 0.0
        imbalance_volume = 0.0

        for h in range(len(prices)):
            dispatched = dispatch[h]
            produced = wind_output[h]
            shortfall = max(0.0, dispatched - produced)

            revenue = prices[h] * dispatched
            penalty = imbalance_penalty * shortfall

            profit += revenue - penalty
            imbalance_volume += shortfall

        return profit, imbalance_volume


# ============================================================
# System Regulator Perspective
# ============================================================

class SystemRegulator:
    """
    System regulator decisions and objectives.
    """

    def __init__(self, decisions: Dict[str, Any]):
        self.imbalance_penalty = decisions["imbalance_penalty"]

    def objectives(self, prices, imbalance_volume, wind_profit):
        return {
            "expected_imbalance": imbalance_volume,
            "price_variance": np.var(prices),
            "wind_revenue": wind_profit,
            "total_system_payment": np.sum(prices)
        }


# ============================================================
# Unified Simulation Interface
# ============================================================

def simulate_market(
    decision_variables: Dict[str, Any],
    uncertain_parameters: Dict[str, Any],
    constant_parameters: Dict[str, Any]
) -> Dict[str, Any]:
    """
    Interface function for the unified model.

    Inputs
    ------
    decision_variables:
        {
            "wind": {
                "bid_quantities": [...],
                "bid_prices": [...]
            },
            "regulator": {
                "imbalance_penalty": float
            }
        }

    uncertain_parameters:
        {
            "conventional_price_params": [
                {"mu": float, "sigma": float}, ...
            ],
```

```python
        "solar_price_params": {
            "mu": float,
            "sigma": float
        }
    }

constant_parameters:
    {
        "mu_D": float,
        "sigma_D": float,
        "mu_w": float,
        "sigma_w": float,
        "solar_a": float,
        "solar_b": float,
        "conventional_capacities": [float, float, float],
        "hours": int
    }

Returns
-------
Dictionary of objective values for each perspective.
"""

# Instantiate components
env = MarketEnvironment(constant_parameters, uncertain_parameters)
wind = WindProducer(decision_variables["wind"])
regulator = SystemRegulator(decision_variables["regulator"])

prices = []
wind_dispatch = []
wind_output = []

# ----------------------------
# Simulate one day
# ----------------------------

for h in range(env.hours):
    demand = env.sample_demand()
    wind_prod = env.sample_wind_production()

    conv_prices = env.sample_conventional_prices()
    solar_price = env.sample_solar_price()

    bids = []

    # Conventional bids
    for i, cap in enumerate(env.conventional_capacities):
        bids.append((cap, conv_prices[i], f"conv_{i}"))

    # Solar bid
    solar_q = env.solar_quantity(h)
    bids.append((solar_q, solar_price, "solar"))

    # Wind bid
    bids.append((
        wind.bid_quantities[h],
        wind.bid_prices[h],
        "wind"
    ))

    dispatch, clearing_price = env.clear_market(bids, demand)

    prices.append(clearing_price)
    wind_dispatch.append(dispatch.get("wind", 0.0))
    wind_output.append(wind_prod)

# ----------------------------
# Evaluate objectives
# ----------------------------

wind_profit, imbalance_volume = wind.profit(
    prices,
    wind_dispatch,
    wind_output,
    regulator.imbalance_penalty
```

```
    )

    regulator_objectives = regulator.objectives(
        prices,
        imbalance_volume,
        wind_profit
    )

    return {
        "wind_producer": {
            "expected_profit": wind_profit,
            "expected_imbalance": imbalance_volume
        },
        "system_regulator": regulator_objectives
    }
```

Interpretation

- The **environment** is shared and agnostic to stakeholder decisions.
- The **wind producer** controls bids and maximizes expected profit.
- The **regulator** controls the imbalance penalty and evaluates system-level outcomes.
- Each perspective can be **evaluated independently** using the same simulated states.
- The interface function can be embedded in:
  - Monte Carlo loops
  - Robust decision-making workflows
  - Bilevel or equilibrium solvers

If you want, next steps could include:

- adding **Monte Carlo averaging** inside **`simulate_market`**,
- extending to **multi-day horizons**,
- or reformulating this as a **bilevel optimization problem**.

Please refine your implementation so that any bid with a price less than or equal to the clearing price is fully accepted, even if the total market demand has already been met.

Good catch — this is an important consistency detail.
Below is a **refined Python implementation** that **correctly enforces the acceptance rule**:

> **Any bid with a price less than or equal to the clearing price is fully accepted, even if total demand has already been met.**

This requires a **two-step market-clearing procedure**:

1. Determine the **clearing price** (first bid where cumulative supply meets or exceeds demand).
2. **Accept all bids** with prices **≤ clearing price**, regardless of whether demand is already satisfied.

Everything else in the unified model remains unchanged.

Key Change (Conceptual)

Instead of stopping dispatch once demand is met, the market now:

- determines the marginal (clearing) price using cumulative supply,
- then **fully accepts all bids priced at or below that clearing price**.

This can result in **total dispatched quantity exceeding demand**, which is consistent with your specification.

Refined Python Implementation (Corrected Market Clearing)

Only the **`clear_market`** method in **`MarketEnvironment`** is modified.
The rest of the program is unchanged and remains valid.

```python
class MarketEnvironment:
    """
    Shared stochastic environment and market-clearing mechanism.
    Contains no stakeholder decision variables.
    """

    def __init__(self, constants, uncertainties):
        self.mu_D = constants["mu_D"]
        self.sigma_D = constants["sigma_D"]
        self.mu_w = constants["mu_w"]
        self.sigma_w = constants["sigma_w"]
        self.a = constants["solar_a"]
        self.b = constants["solar_b"]
        self.conventional_capacities = constants["conventional_capacities"]
        self.hours = constants.get("hours", 24)
```

```
        self.conv_price_params = uncertainties["conventional_price_params"]
        self.solar_price_params = uncertainties["solar_price_params"]

    # ---------------------------
    # Stochastic processes
    # ---------------------------

    def sample_demand(self):
        return max(0.0, np.random.normal(self.mu_D, self.sigma_D))

    def sample_wind_production(self):
        return max(0.0, np.random.normal(self.mu_w, self.sigma_w))

    def sample_conventional_prices(self):
        prices = []
        for params in self.conv_price_params:
            prices.append(np.random.normal(params["mu"], params["sigma"]))
        return prices

    def sample_solar_price(self):
        return np.random.normal(
            self.solar_price_params["mu"],
            self.solar_price_params["sigma"]
        )

    def solar_quantity(self, hour):
        return max(0.0, self.a + self.b * np.cos(2 * np.pi * hour / 24))

    # ---------------------------
    # Corrected market clearing
    # ---------------------------

    def clear_market(self, bids, demand):
        """
        bids: list of tuples (quantity, price, producer_id)

        Rule:
        1. Determine clearing price as the marginal price where
           cumulative supply first meets or exceeds demand.
        2. Fully accept ALL bids with price <= clearing price,
           even if total supply exceeds demand.
        """

        # Sort bids by price (merit order)
        bids_sorted = sorted(bids, key=lambda x: x[1])

        cumulative_supply = 0.0
        clearing_price = None

        # Step 1: find clearing price
        for q, p, _ in bids_sorted:
            cumulative_supply += q
            clearing_price = p
            if cumulative_supply >= demand:
                break

        # Step 2: accept all bids priced <= clearing price
        dispatch = {}
        for q, p, pid in bids_sorted:
            if p <= clearing_price:
                dispatch[pid] = q
            else:
                dispatch[pid] = 0.0

        return dispatch, clearing_price
```

Why This Is Correct

- ✅ Clearing price is still set by the **marginal accepted bid**

- ✅ All infra-marginal and marginal bids are **fully accepted**
- ✅ Dispatch can exceed demand, as explicitly required
- ✅ Wind imbalance penalties still apply correctly to dispatched quantity
- ✅ Regulator and wind producer perspectives remain modular and exogenous

---

What This Changes Economically

- Wind may be **fully dispatched even when demand is already met**, increasing:
  - revenue exposure,
  - imbalance risk.
- This strengthens the **trade-off** the wind producer faces between:
  - aggressive bidding for dispatch,
  - expected imbalance penalties.
- It also gives the **regulator's imbalance penalty** real leverage over outcomes.

If you'd like, I can next:

- update the **objective interpretations** under this rule,
- add **unit tests** for the clearing logic,
- or extend the model to include **curtailment or pro-rata acceptance** variants for comparison.