market_narrative 2

In this problem, a renewable energy producer participates in a day-ahead merit-order electricity market competing against three other conventional producers. The producer submits hourly bids that specify a delivery quantity of electricity and an offer price for each hour of the next day. If a bid is accepted, the producer is obligated to deliver the specified quantity of electricity at the offer price. Otherwise, the producer earns zero profit for that hour. However, renewable generation is deeply uncertain, as it depends on weather conditions that cannot be predicted. Meanwhile, the hourly clearing price depends on the bids of all producers and is unknown in advance. The renewable energy producer's goal is to maximize its expected profit by strategically selecting the quantity of electricity b_t and offer price p_t to bid for each hour of the next day, accounting for the uncertainty in both renewable generation and clearing prices.

Please formalize this problem description as a model. Please also specify key components, such as state variables, stochastic variables, constant parameters, uncertainties and transition functions.

Excellent — this is a classic **stochastic optimization problem** under uncertainty in both **generation** and **market prices**, often modeled as a **two-stage stochastic programming** or **stochastic dynamic programming** problem.

Let's formalize it carefully and precisely.

🧩 Problem setup

We consider a **renewable energy producer** participating in a **day-ahead electricity market** over a planning horizon $t = 1, 2, \dots, T$ (hours of the next day).

For each hour t, the producer must decide:

- how much energy b_t (MWh) to **offer** to the market, and
- at what **offer price** p_t ($/MWh).

---

1. Decision Variables

Symbol	Meaning	Type
b_t	Quantity bid for delivery at hour t	Decision (continuous ≥ 0)
p_t	Offer price for hour t	Decision (continuous ≥ 0)
x_t	Indicator of whether bid is accepted (depends on market clearing)	Endogenous binary / stochastic outcome

---

2. Stochastic (Uncertain) Variables

Symbol	Meaning	Type / Source
G_t	Actual renewable generation available (MWh)	Random variable (depends on weather)
π_t	Market clearing price ($/MWh)	Random variable (depends on system supply/demand & competitors' bids)

Both G_t and π_t are **unknown when the bids** (b_t, p_t) are submitted, but their **probability distributions** (or scenario sets) are assumed known from forecasts.

---

3. Bid acceptance rule

For each hour t, the producer's bid is accepted if its offer price is **below or equal** to the market clearing price:

$$x_t = \begin{cases} 1, & \text{if } p_t \le \pi_t, \\ 0, & \text{otherwise.} \end{cases}$$

When the bid is accepted ($x_t = 1$), the producer is **obligated** to deliver b_t MWh.

---

4. Delivery constraint under renewable uncertainty

If generation is insufficient to meet the accepted bid, the producer must buy the shortfall from the market (at a penalty or replacement cost).

Let c^{pen} denote the per-unit cost of buying electricity to cover shortfall.

Then the delivered (or balanced) energy d_t is:

$$d_t = \min(G_t, b_t)$$

and the **imbalance cost** is incurred when $G_t < b_t$:

$$\text{Shortfall cost} = c^{\text{pen}}(b_t - G_t)^+,$$

where $(x)^+ = \max(x, 0)$.

---

5. Profit Function per Hour

The realized profit at hour t is:

$$\Pi_t(b_t, p_t, G_t, \pi_t) = x_t \left[\pi_t b_t - c^{\text{pen}}(b_t - G_t)^+\right].$$

If the bid is not accepted ($x_t = 0$), profit = 0.

---

6. Objective Function

The producer aims to **maximize expected total profit** over all hours:

$$\max_{\{b_t, p_t\}_{t=1}^{T}} \mathbb{E}_{G_t, \pi_t}\left[\sum_{t=1}^{T} \Pi_t(b_t, p_t, G_t, \pi_t)\right].$$

Subject to:

$$b_t \geq 0, \quad p_t \geq 0, \quad \forall t.$$

7. Uncertainty model

- **Stochastic process for renewable generation:**

$$G_t \sim f_G(G_t \mid \text{weather forecast, time } t)$$

- **Stochastic process for market price:**

$$\pi_t \sim f_\pi(\pi_t \mid \text{demand forecast, merit order, time } t)$$

Optionally, there may be **correlation** between G_t and π_t: low renewable output often corresponds to higher market prices.

8. State Variables and Transitions (for dynamic formulation)

If we extend this to a **multi-stage stochastic dynamic programming** model, we can define:

Symbol	Meaning
s_t	State at time t (e.g., remaining forecast uncertainty, previous outcomes)
$w_t = (G_t, \pi_t)$	Random disturbance (realized weather and market price)
$s_{t+1} = f(s_t, w_t)$	State transition function (e.g., update of forecast distributions or cumulative profit)

Then the **Bellman equation** is:

$$V_t(s_t) = \max_{b_t, p_t} \mathbb{E}_{w_t}\left[\Pi_t(b_t, p_t, w_t) + V_{t+1}(f(s_t, w_t))\right].$$

For a one-day market with no intertemporal constraints (no storage), the problem **decouples by hour**, and the optimal bid for each hour can be determined independently.

9. Constant Parameters

Symbol	Description
T	Number of hourly periods (e.g., 24)
c^{pen}	Penalty or replacement energy cost ($/MWh)
Forecast distributions f_G, f_π	Probabilistic models for renewable generation and market price

10. Compact Model Summary

$$\begin{aligned} \max_{\{b_t, p_t\}} \quad & \mathbb{E}\left[\sum_{t=1}^{T} x_t\left(\pi_t b_t - c^{\text{pen}}(b_t - G_t)^+\right)\right] \\ \text{s.t.} \quad & x_t = \mathbf{1}_{\{p_t \le \pi_t\}}, \quad \forall t, \\ & b_t \ge 0,\ p_t \ge 0, \quad \forall t. \end{aligned}$$

Extensions (optional)

You can enrich the model to include:

- Storage or curtailment decisions (S_t),
- Correlated stochastic processes for G_t and π_t,
- Risk aversion (maximize CVaR of profit instead of expectation),
- Market participation limits or minimum bid sizes.

Would you like me to write this in **mathematical-programming form** (e.g., a stochastic optimization formulation with explicit scenarios and probabilities)? That would make it ready to implement in software like Pyomo or GAMS.

First, the day-ahead market is cleared once per day for 24 hourly intervals of the following day, so you can assume the problem only considers a 24-hour period. Second, penalties would apply if the renewable producer under-delivers or over-delivers relative to its bid. Please refine the specification.

Excellent clarification. Since the **day-ahead market** is cleared **once per day** for the **24 hourly intervals** of the *next* day, we're dealing with a **single-day, multi-hour stochastic optimization** problem.

Also, you noted an important refinement: **penalties apply for both under- and over-delivery**, not only shortages. That changes the payoff and the model structure slightly. Let's formalize it cleanly and rigorously.

⚙️ Model: Day-Ahead Bidding of a Renewable Producer

1. Time Horizon

Let

$$t \in \mathcal{T} = \{1, 2, \dots, 24\}$$

denote each **hour of the delivery day**.

The day-ahead market is cleared once (today) for the whole 24-hour horizon, and all decisions $(b_t, p_t)_{t\in\mathcal{T}}$ must be submitted **before uncertainties are realized**.

2. Decision Variables

Symbol	Description	Type
$b_t \geq 0$	Energy quantity bid for hour t (MWh)	Continuous
$p_t \geq 0$	Offer price for hour t (\$/MWh)	Continuous

All bids are chosen *before* the realization of random variables.

3. Stochastic Variables (Uncertainties)

Symbol	Description
G_t	Realized renewable generation available in hour t (MWh)
π_t	Market clearing price for hour t (\$/MWh)

- Both G_t and π_t are **random variables** with known joint distribution $f_{G,\pi}(\cdot)$.
- The joint distribution may exhibit **correlation** (e.g., high renewable output tends to lower prices).

4. Bid Acceptance

For each hour t, the bid is accepted if the offer price is **less than or equal to** the market clearing price:

$$x_t = \begin{cases} 1, & \text{if } p_t \leq \pi_t, \\ 0, & \text{otherwise.} \end{cases}$$

When $x_t = 1$, the producer is **committed** to deliver b_t MWh.

5. Delivery and Deviations

Let the realized generation G_t be compared to the committed delivery b_t.

Define deviation:

$$\Delta_t = G_t - b_t.$$

Then:

- **Under-delivery:** $\Delta_t < 0 \rightarrow$ the producer delivers less than promised.
- **Over-delivery:** $\Delta_t > 0 \rightarrow$ the producer produces more than promised.

6. Penalty Structure

Let:

- c_t^{under} = penalty cost per MWh of under-delivery ($/MWh),
- c_t^{over} = penalty cost per MWh of over-delivery ($/MWh).

The **penalty cost** in hour t is:

$$C_t(G_t, b_t) = c_t^{\text{under}}(b_t - G_t)^+ + c_t^{\text{over}}(G_t - b_t)^+,$$

where $(x)^+ = \max(x, 0)$.

This formulation covers both sides of imbalance risk.

---

7. Profit per Hour

The **realized profit** for hour t is:

$$\Pi_t(b_t, p_t, G_t, \pi_t) = x_t \left[\pi_t b_t - C_t(G_t, b_t)\right],$$

since:

- If the bid is **accepted** ($x_t = 1$): the producer sells b_t at π_t, then pays penalties for deviation.
- If **not accepted** ($x_t = 0$): no delivery, no penalty, no revenue.

---

8. Objective Function

The producer's goal is to **maximize expected total profit** over all 24 hours:

$$\max_{\{b_t, p_t\}_{t=1}^{24}} \mathbb{E}_{G_t, \pi_t} \left[\sum_{t=1}^{24} x_t(\pi_t b_t - C_t(G_t, b_t))\right].$$

---

9. Constraints

$$\begin{aligned} &b_t \geq 0, \quad p_t \geq 0, \quad \forall t \in \mathcal{T}, \\ &x_t = \mathbf{1}_{\{p_t \leq \pi_t\}}, \quad \forall t. \end{aligned}$$

Optionally, you could add:

- Capacity limits: $0 \leq b_t \leq \bar{G}_t^{\text{forecast}}$,
- Price bounds: $0 \leq p_t \leq \bar{p}$,
- Correlation or scenario consistency across hours.

---

10. Stochastic Structure

All uncertainty enters through random vectors:

$$\omega = (G_1, \pi_1, \ldots, G_{24}, \pi_{24})$$

with probability measure $P(\omega)$.

Expected profit:

$$\max_{b,p} \mathbb{E}_\omega[\Pi(b,p;\omega)].$$

In a **discrete-scenario form** (for implementation):

$$\max_{b,p} \sum_{s\in S} p_s \sum_{t=1}^{24} x_t^s(\pi_t^s b_t - C_t(G_t^s, b_t)),$$

where S is the set of possible scenarios, each with probability p_s.

---

11. State and Transition View (for completeness)

Although the market is cleared once and each hour is independent (no intertemporal linkage), we can define the dynamic components:

Element	Definition
State s_t	Forecast distributions (f_{G_t}, f_{π_t}) at bidding time
Decision a_t	Bid pair (b_t, p_t)
Random outcome w_t	Realized (G_t, π_t)
Transition	$s_{t+1} = f(s_t, w_t)$, e.g. update of forecasts or cumulative profit
Reward	$r_t = x_t(\pi_t b_t - C_t(G_t, b_t))$

But since bids are all submitted simultaneously, the optimization is **one-stage stochastic** with multiple hourly components.

---

12. Compact Model Summary

$$\begin{aligned} \max_{\{b_t,p_t\}_{t=1}^{24}} \quad & \mathbb{E}\left[\sum_{t=1}^{24} \mathbf{1}_{\{p_t\le\pi_t\}}\left(\pi_t b_t - c_t^{\text{under}}(b_t - G_t)^+ - c_t^{\text{over}}(G_t - b_t)^+\right)\right] \\ \text{s.t.} \quad & b_t \ge 0,\ p_t \ge 0, \quad \forall t. \end{aligned}$$

---

13. Interpretation

- The producer must **balance two tradeoffs**:
  1. Bidding high p_t increases the chance of non-acceptance (zero profit).
  2. Bidding low p_t increases acceptance but exposes the producer to imbalance penalties if G_t deviates from b_t.
- The optimal strategy depends on the **joint distribution** of (G_t, π_t), the **penalty asymmetry**, and the **risk attitude** of the producer (if risk-averse objectives are later introduced).

Would you like me to now extend this to a **discrete scenario-based formulation** (explicit scenario index and probabilities, ready for implementation in Pyomo/GAMS or as a stochastic MILP)?

There are still several areas where the model specification can be improved: 1. The renewable generation is deeply uncertain, so its probability distribution is unknown; 2. The market clearing price can be calculated based on the total energy demand and the bids of all energy producers for each hour.

Excellent — that's a crucial refinement.
You're now describing a **strategic bidding problem under deep uncertainty** in renewable output, coupled with **endogenous market prices** determined by **market clearing conditions** (not exogenous stochastic prices).

Let's now formalize this properly — including both the **merit-order price formation** and the **ambiguity (deep uncertainty)** in renewable generation.

⚙️ Refined Model: Strategic Day-Ahead Bidding under Deep Uncertainty and Market Clearing

1. Market setup

The **day-ahead electricity market** is cleared **once per day** for **24 hourly intervals**,

$$t \in \mathcal{T} = \{1, 2, \ldots, 24\}.$$

There are $N + 1$ producers:

- $i = 0$: the *renewable* producer (decision-maker);
- $i = 1, \ldots, N$: conventional producers (competitors).

Each producer submits for each hour t:

- a **bid quantity** $b_{i,t}$ (MWh),
- an **offer price** $p_{i,t}$ ($/MWh).

---

2. Decision variables (for the renewable producer)

Symbol	Meaning
$b_t \geq 0$	Energy quantity bid by renewable producer for hour t (MWh)
$p_t \geq 0$	Offer price for hour t ($/MWh)

All bids (b_t, p_t) are chosen simultaneously before market clearing.

---

3. Conventional producers (given competitors)

Each conventional producer $i = 1, \ldots, N$ submits known or forecasted bids $(b_{i,t}, p_{i,t})$.
These are treated as *parameters* from the renewable producer's perspective (though they determine

market clearing).

4. Demand

Let D_t denote total system demand in hour t (MWh).
This is generally **known or forecasted** at market clearing time and is treated as a constant parameter.

5. Market-clearing mechanism (merit-order)

For each hour t:

- All offers $(p_{i,t}, b_{i,t})$ are sorted in **ascending order of price**.
- The market **accepts** bids until cumulative supply equals demand D_t.

Formally, define:

$$\sum_{i:p_{i,t} \leq \pi_t} b_{i,t} = D_t,$$

where π_t is the **clearing price** (the highest accepted offer price).

Then for each producer i:

$$x_{i,t} = \begin{cases} 1, & \text{if } p_{i,t} \leq \pi_t, \\ 0, & \text{otherwise.} \end{cases}$$

For the renewable producer ($i = 0$):

$$x_t = \mathbf{1}_{\{p_t \leq \pi_t\}}.$$

Hence, π_t is **endogenously determined** by all submitted bids and the demand.

6. Renewable generation and deep uncertainty

Let G_t denote the **actual renewable generation** available in hour t (MWh).

- G_t is **deeply uncertain** — its probability distribution is **unknown or ambiguous**.
- Instead of a known stochastic model, assume $G_t \in \mathcal{G}_t$, where $\mathcal{G}_t$ is an **uncertainty set** (e.g., derived from forecast intervals or scenario envelopes).

We can represent the producer's robustness attitude via a **min-max** structure:

$$\max_{b_t, p_t} \min_{G_t \in \mathcal{G}_t} (\text{profit}).$$

Alternatively, if partial probabilistic information exists (e.g., moments), we could use **distributionally robust** optimization.

7. Penalties for deviation

After market clearing, if the renewable producer's bid is accepted ($x_t = 1$), it must deliver b_t MWh. Deviation penalties apply for both under- and over-delivery:

$$C_t(G_t, b_t) = c_t^{\text{under}}(b_t - G_t)^+ + c_t^{\text{over}}(G_t - b_t)^+,$$

where $c_t^{\text{under}}, c_t^{\text{over}} \geq 0$ are penalty parameters.

8. Profit per hour

Given the market outcome, the renewable producer's **realized profit** is:

$$\Pi_t(b_t, p_t, G_t, \pi_t) = x_t\big[\pi_t b_t - C_t(G_t, b_t)\big].$$

If $x_t = 0$ (bid not accepted), profit = 0.

9. Coupled equilibrium condition

The market clearing price π_t depends on all bids:

$$\pi_t = \text{ClearingPrice}(p_t, b_t; \{p_{i,t}, b_{i,t}\}_{i=1}^N, D_t),$$

where this function represents the **merit-order dispatch rule** (monotone, non-decreasing in b_t, p_t).

Thus, the renewable producer's problem is **strategic**:
its own bid influences π_t through the clearing mechanism.

10. Objective: robust or distributionally robust expected profit

(a) Robust formulation (deep uncertainty in G_t)

$$\max_{\{b_t, p_t\}_{t=1}^{24}} \min_{G_t \in \mathcal{G}_t \ \forall t} \sum_{t=1}^{24} x_t(\pi_t b_t - C_t(G_t, b_t)).$$

(b) If a partial probabilistic model is available:

$$\max_{\{b_t, p_t\}_{t=1}^{24}} \mathbb{E}_{G_t \sim \mathcal{P}_t,\ \pi_t = \text{ClearingPrice}(\cdot)} \left[\sum_{t=1}^{24} x_t(\pi_t b_t - C_t(G_t, b_t))\right],$$

where $\mathcal{P}_t$ is an **ambiguity set** of distributions consistent with known information (e.g., mean and bounds).

11. Constraints

$$\begin{aligned} & b_t \geq 0,\ p_t \geq 0, \quad \forall t \in \mathcal{T}, \\ & \pi_t \text{ satisfies market-clearing equations}, \\ & G_t \in \mathcal{G}_t, \quad \forall t. \end{aligned}$$

12. Compact representation

$$
\begin{aligned}
\max_{\{b_t,p_t\}_{t=1}^{24}} \min_{G_t \in \mathcal{G}_t} \sum_{t=1}^{24} \mathbf{1}_{\{p_t \le \pi_t(p,b)\}} \Big(\pi_t(p,b)\, b_t - c_t^{\text{under}} (b_t - G_t)^+ - c_t^{\text{over}} (G_t - b_t)^+ \Big) \\
\text{s.t.}\;\; \pi_t(p,b) \text{ satisfies } \sum_{i:p_{i,t} \le \pi_t} b_{i,t} = D_t, \quad \forall t, \\
b_t \ge 0,\; p_t \ge 0, \quad \forall t, \\
G_t \in \mathcal{G}_t, \quad \forall t.
\end{aligned}
$$

13. Key model components summary

Component	Symbol / Definition	Nature
Decision variables	b_t, p_t	Renewable producer's hourly bid quantity and offer price
Uncertain variables	G_t	Deeply uncertain renewable output
Endogenous variables	π_t, x_t	Determined by market clearing mechanism
Parameters	$D_t, (b_{i,t}, p_{i,t}), c_t^{\text{under}}, c_t^{\text{over}}$	Given inputs
State variables	(none intertemporal; decisions simultaneous for 24h)	Single-stage
Transition	None (each hour independent except through common bidding)	—
Objective	Maximize worst-case (or expected) total profit	—

14. Interpretation

- The renewable producer faces **two intertwined uncertainties**:
  1. **Deep uncertainty in generation** (modeled via an uncertainty set or ambiguity set).
  2. **Endogenous price** (determined by merit-order equilibrium, affected by own bid).
- This creates a **robust bilevel optimization problem**:
  - Upper level: renewable producer chooses b_t, p_t to maximize (robust) profit.
  - Lower level: market clears, determining π_t and x_t from all bids and demand.

15. Optional reformulations

- The market clearing (lower level) can be replaced by its **KKT conditions** or **complementarity formulation**, making the full model a **Mathematical Program with Equilibrium Constraints (MPEC)**.
- If the renewable producer assumes its own bid does *not* affect the clearing price (price-taker), the model simplifies to a **robust optimization** problem rather than a bilevel one.

Would you like me to write out the **explicit bilevel (MPEC) formulation** next — showing the market-clearing subproblem and how it links to the renewable producer's upper-level optimization (robust version)? That would yield a complete equilibrium-consistent optimization model.

> You can assume the total energy demand D_t follows a normal distribution with constant mean and variance. You can also assume that for each conventional producer, its delivery quantity of energy and the offer price in bids follow normal distributions with constant means and variances. Please refine the model specification.

Perfect — now we can formalize a **stochastic equilibrium model** for the renewable producer's **day-ahead bidding problem**, in which:

- Market prices are *endogenous* (from supply-demand balance),
- Conventional producers' bids and total demand are *random* (with known normal distributions), and
- The renewable producer's generation is *deeply uncertain* (distribution unknown but bounded).

Let's carefully write the complete refined model, step by step.

⚙️ Final Refined Model:

Renewable Producer's Day-Ahead Bidding under Deep Generation Uncertainty and Stochastic Market Clearing

---

1. Market horizon

The day-ahead market covers

$$t \in \mathcal{T} = \{1, 2, \ldots, 24\},$$

each t corresponding to one delivery hour of the following day.

All bids (b_t, p_t) are submitted simultaneously **before** market clearing.

---

2. Players

There are $N + 1$ producers:

- **Renewable producer (index 0)** — decision-maker.
- **Conventional producers** $i = 1, \ldots, N$ — competitors.

---

3. Decision variables (renewable producer only)

Symbol	Meaning	Domain
b_t	Bid quantity for hour t (MWh)	$b_t \geq 0$
p_t	Offer price for hour t (\$/MWh)	$p_t \geq 0$

All 24 pairs (b_t, p_t) form the producer's **bidding strategy**.

4. Random variables

Symbol	Meaning	Distribution / Source
D_t	Total system energy demand (MWh)	$D_t \sim \mathcal{N}(\mu_D, \sigma_D^2)$
$b_{i,t}$	Bid quantity of conventional producer i	$b_{i,t} \sim \mathcal{N}(\mu_{b_i}, \sigma_{b_i}^2)$
$p_{i,t}$	Offer price of conventional producer i	$p_{i,t} \sim \mathcal{N}(\mu_{p_i}, \sigma_{p_i}^2)$

All these distributions are **known** and **stationary across hours** (constant mean and variance).

These random variables jointly determine the **market clearing price**.

5. Deeply uncertain renewable generation

Symbol	Meaning	Model
G_t	Realized renewable generation available (MWh)	Unknown distribution, but known bounds $G_t \in [\underline{G}_t, \overline{G}_t]$

Hence, generation uncertainty is modeled **robustly**, not probabilistically.

6. Market-clearing mechanism (merit order)

For each hour t, the market-clearing price π_t satisfies:

$$\sum_{i:p_{i,t} \leq \pi_t} b_{i,t} + x_t b_t = D_t,$$

where:

$$x_t = \begin{cases} 1, & \text{if } p_t \leq \pi_t, \\ 0, & \text{otherwise.} \end{cases}$$

Equivalently, π_t is the **lowest price** such that total supply equals demand:

$$\pi_t = \inf \left\{ \pi : \sum_{i=1}^{N} b_{i,t} \mathbf{1}_{\{p_{i,t} \leq \pi\}} + b_t \mathbf{1}_{\{p_t \leq \pi\}} \geq D_t \right\}.$$

Because D_t, $b_{i,t}$, and $p_{i,t}$ are random, the market clearing price π_t is also **random**.

7. Penalties for deviation

After market clearing, if the bid is accepted ($x_t = 1$) and actual generation G_t differs from the committed b_t:

$$C_t(G_t, b_t) = c_t^{\text{under}}(b_t - G_t)^+ + c_t^{\text{over}}(G_t - b_t)^+,$$

with constants $c_t^{\text{under}}, c_t^{\text{over}} \geq 0$.

8. Profit function (per hour)

$$\Pi_t(b_t, p_t, G_t, \pi_t) = x_t\left[\pi_t b_t - C_t(G_t, b_t)\right].$$

If the bid is not accepted ($x_t = 0$), the profit is zero.

9. Uncertainty structure

We distinguish two types of uncertainty:

Type	Variables	Modeling approach
Aleatory (stochastic)	$D_t, b_{i,t}, p_{i,t}$	Known joint distribution (normal)
Epistemic (deep)	G_t	Ambiguity set (bounded interval or worst case)

Thus, the renewable producer faces a **hybrid stochastic–robust** optimization problem.

10. Objective function

The renewable producer seeks to **maximize expected profit** over the 24 hours, under the **worst-case** renewable generation scenario within its uncertainty set:

$$\max_{\{b_t, p_t\}_{t=1}^{24}} \mathbb{E}_{D_t, b_{i,t}, p_{i,t}}\left[\min_{G_t \in [\underline{G}_t, \overline{G}_t]} \sum_{t=1}^{24} x_t(\pi_t b_t - C_t(G_t, b_t))\right].$$

Here:

- The **expectation** is taken over the stochastic variables (market demand and conventional bids);
- The **inner minimization** represents worst-case renewable output within its feasible range.

11. Constraints

$$\begin{aligned}
&b_t \geq 0,\ p_t \geq 0, \quad \forall t, \\
&G_t \in [\underline{G}_t, \overline{G}_t], \quad \forall t, \\
&\pi_t \text{ satisfies the merit-order clearing condition}, \\
&x_t = \mathbf{1}_{\{p_t \leq \pi_t\}}, \quad \forall t.
\end{aligned}$$

12. Compact stochastic–robust formulation

$$
\max_{\{b_t,p_t\}_{t=1}^{24}} \mathbb{E}_{D_t,b_{i,t},p_{i,t}}\left[\min_{G_t\in[\underline{G}_t,\overline{G}_t]} \sum_{t=1}^{24} \mathbf{1}_{\{p_t\le \pi_t(D_t,b,p)\}} \left(\pi_t(D_t,b,p)\, b_t - c_t^{\text{under}}(b_t-G_t)^+ - c_t^{\text{over}}(G_t-b_t)^+\right)\right]
$$
$$
\text{s.t. } b_t, p_t \ge 0, \quad \forall t.
$$

where $\pi_t(D_t, b, p)$ denotes the market clearing price determined endogenously by the random demand and all bids.

13. Model classification

Aspect	Type
Problem type	**Hybrid stochastic–robust bilevel optimization**
Upper level	Renewable producer chooses bids (b_t, p_t)
Lower level	Market clearing (determines π_t, x_t) given all bids
Uncertainty	Stochastic (demand, competitor bids) + Deep (generation)
Objective	Maximize expected worst-case profit
Solution concept	Expected-value robust Stackelberg equilibrium

14. Interpretation

- **Demand** and **conventional bids** create randomness in the clearing price π_t, with tractable (normal) uncertainty.
  The expectation captures the *average profit opportunity* across possible market outcomes.
- **Renewable generation** is not statistically known; the inner minimization ensures the bid strategy is **robust** to all feasible realizations.
- The renewable producer must **balance**:
  - Offering low prices (to increase acceptance probability),
  - Bidding quantities aligned with uncertain generation (to avoid penalties),
  - And anticipating stochastic price outcomes driven by random market clearing.

15. Summary of key components

Category	Symbol	Description
Decision variables	b_t, p_t	Bid quantity and offer price for hour t
Endogenous variables	π_t, x_t	Market clearing price, bid acceptance
Random variables (known distributions)	$D_t, b_{i,t}, p_{i,t}$	Demand and competitors' bids, all normally distributed
Deep uncertainty variables	G_t	Renewable generation, bounded but distribution unknown
Parameters	$c_t^{\text{under}}, c_t^{\text{over}}$	Penalty coefficients
Objective	$\max_{b,p} \mathbb{E}\left[\min_{G_t} \sum_t \Pi_t\right]$	Expected profit under worst-case generation

Category	Symbol	Description
Equilibrium rule	$\sum_i b_{i,t}\mathbf{1}_{\{p_{i,t}\le\pi_t\}} + b_t\mathbf{1}_{\{p_t\le\pi_t\}} = D_t$	Market clearing per hour

Would you like me to next **express this explicitly as a bilevel stochastic–robust program**, including the **market-clearing subproblem as an optimization (social welfare maximization)** that determines π_t?
That would make the model ready for computational solution or equilibrium analysis.

Various stakeholders approach this problem from different perspectives. Please identify four different perspectives related to this problem. These perspectives should share a common environment, along with global parameters, state variables, and an initial state. For each perspective, also provide the corresponding model with the key components, such as control variables, transition functions, and objective functions. If there are multiple objectives, provide multiple objective functions instead of a weighted objective function.

Excellent — now we're moving beyond a single decision-maker and looking at the **multi-perspective system** around the day-ahead electricity market with uncertain renewable generation.

In this shared environment, different stakeholders have **distinct objectives, control levers, and information**, but they operate over the same physical and market system.
Below, I'll define the **common environment**, and then give **four coherent modeling perspectives**, each with its own model structure.

🌍 Common Environment

All perspectives share a single underlying system defined by **global parameters**, **state variables**, and an **initial state**.

Global Parameters

Symbol	Meaning
$t \in \mathcal{T} = \{1, \dots, 24\}$	Hours of the delivery day
$N+1$	Total producers: 1 renewable + N conventional
D_t	Total energy demand (random, normally distributed)
$c_t^{\text{under}}, c_t^{\text{over}}$	Penalty coefficients for delivery deviations
$\mu_{b_i}, \sigma_{b_i}, \mu_{p_i}, \sigma_{p_i}$	Mean and variance of conventional bids
$[\underline{G}_t, \overline{G}_t]$	Bounds for renewable generation (deep uncertainty)
μ_D, σ_D	Mean and variance of hourly demand distribution

Shared State Variables

Symbol	Meaning
G_t	Realized renewable generation (uncertain)
D_t	Realized demand (stochastic)
$\{p_{i,t}, b_{i,t}\}_{i=1}^N$	Realized bids of conventional producers
π_t	Market clearing price determined endogenously
$x_{i,t}$	Acceptance indicator for each producer's bid

Initial State

At the start of the day-ahead market (time 0):

$$s_0 = \left\{\text{forecasts of } D_t, [\underline{G}_t, \overline{G}_t], \mu_{b_i}, \mu_{p_i}, \text{and variances for all producers}\right\}.$$

This represents the **information state** all agents share when submitting bids.

⚖️ Perspective 1 — Renewable Producer (Strategic Bidder)

Control Variables

Symbol	Description
b_t	Bid quantity (MWh)
p_t	Offer price ($/MWh)

Transition Function

No intertemporal physical dynamics (bids are one-shot).
State evolves only through realization of market outcomes:

$$s_{t+1} = f(s_t, G_t, D_t, \pi_t).$$

Objective Function(s)

Maximize profit under stochastic prices and deep uncertainty in generation:

$$\max_{b_t, p_t} \mathbb{E}_{D_t, b_{i,t}, p_{i,t}} \left[\min_{G_t \in [\underline{G}_t, \overline{G}_t]} \sum_{t=1}^{24} x_t \left(\pi_t b_t - c_t^{\text{under}} (b_t - G_t)^+ - c_t^{\text{over}} (G_t - b_t)^+\right) \right].$$

If the producer is **risk-averse**, an alternative second objective could be:

$$\max_{b_t, p_t} \ \mathrm{CVaR}_\alpha(\text{profit}),$$

capturing risk exposure.

⚙️ Perspective 2 — Conventional Producers (Price-Takers)

Each conventional producer $i = 1, \ldots, N$ operates under known cost structure and aims to maximize expected profit.

Control Variables

Symbol	Description
$b_{i,t}$	Bid quantity (MWh)
$p_{i,t}$	Offer price ($/MWh)

Transition Function

No intertemporal link; market outcomes depend on all bids:

$$s_{t+1} = f(s_t, \pi_t) \text{ where } \pi_t = \text{ClearingPrice}(D_t, \{p_{j,t}, b_{j,t}\}_{j=0}^{N}).$$

Objective Function

For each producer i:

$$\max_{b_{i,t}, p_{i,t}} \mathbb{E}_{D_t, p_{-i,t}, b_{-i,t}} \left[x_{i,t}(\pi_t b_{i,t} - C_i(b_{i,t}))\right],$$

where $C_i(b_{i,t})$ is the production cost function, e.g. $C_i(b_{i,t}) = a_i b_{i,t}^2 + c_i b_{i,t}$.

Alternative objectives (multi-criteria for a regulated producer):

- **Profit** maximization (as above),
- **Reliability** or **availability** target:

$$\max \ \mathbb{E}[b_{i,t}/b_{i,t}^{\max}]$$

  to ensure capacity utilization.

🧮 Perspective 3 — Market Operator (System Operator / ISO)

The Market Operator (Independent System Operator) clears the market once per day for 24 hours.

Control Variables

Symbol	Description
π_t	Market clearing price ($/MWh)
$x_{i,t}$	Acceptance indicators for bids (binary)

These are determined endogenously from the **market-clearing problem**.

Transition Function

Given bids and demand, the operator enforces the supply-demand balance:

$$\sum_{i=0}^{N} x_{i,t} b_{i,t} = D_t.$$

Objective Function(s)

The operator may pursue multiple goals, typically:

1. **Social welfare maximization:**

$$\max_{x_{i,t},\pi_t} \mathbb{E}_{D_t,b_{i,t},p_{i,t}} \left[U_t(D_t) - \sum_{i=0}^{N} x_{i,t} p_{i,t} b_{i,t} \right],$$

   where $U_t(D_t)$ is the aggregate consumer utility.

2. **Market efficiency or fairness:**

$$\min_{x_{i,t},\pi_t} \operatorname{Var}(\pi_t - p_{i,t}),$$

   encouraging prices close to marginal bids to reduce market power.

3. **Reliability objective:**

$$\min_{x_{i,t}} \sum_t (|D_t - \sum_i x_{i,t} b_{i,t}|)$$

   ensuring load–generation balance.

🌱 Perspective 4 — Regulator / Policy Maker

The Regulator oversees the market and renewable integration, seeking policy objectives over time, not per hour.

Control Variables

Symbol	Description
s_t	Subsidy or penalty scheme (e.g. renewable feed-in tariff, carbon tax, balancing penalty scaling)
γ_t	Market rule parameters (e.g. cap on offer price, penalty coefficients)

Transition Function

Regulatory choices influence market equilibrium outcomes:

$$s_{t+1} = g(s_t, \text{market outcomes } (\pi_t, G_t, D_t)).$$

For instance, penalties may adapt to renewable deviation statistics.

Objective Functions

The regulator has **multiple societal objectives**, not reducible to a single weighted sum:

1. **Affordability (Consumer Welfare):**

$$\max \mathbb{E}[U_t(D_t) - \pi_t D_t].$$

2. **Reliability (Adequacy of Supply):**

$$\min \mathbb{E}[|D_t - \sum_i x_{i,t} b_{i,t}|].$$

3. **Sustainability (Renewable share):**

$$\max \mathbb{E}\left[\frac{x_0 b_t}{\sum_i x_{i,t} b_{i,t}}\right],$$

proportion of renewable generation in total cleared supply.

4. **Market competitiveness (Fairness):**

$$\min \mathrm{Var}(\pi_t)$$

to avoid volatility and market manipulation.

🔄 Shared Dynamics Across Perspectives

Element	Shared Meaning
State evolution	$s_{t+1} = f(s_t, D_t, G_t, \pi_t)$: system transitions through stochastic realizations of demand and generation.
Uncertainties	Demand D_t, conventional bids $(b_{i,t}, p_{i,t})$, renewable generation G_t.
Common constraints	Market clearing: $\sum_i x_{i,t} b_{i,t} = D_t$.
Common time scale	24-hour day-ahead horizon, all decisions made at $t = 0$.

🧩 Summary Table of Perspectives

Perspective	Decision-maker	Control variables	Key uncertainties	Transition	Objective(s)
Renewable producer	Renewable firm	b_t, p_t	G_t (deep), $D_t, p_{i,t}, b_{i,t}$ (stochastic)	Market outcome update	Max expected worst-case profit; (optionally) max CVaR of profit
Conventional producers	Firms $i = 1..N$	$b_{i,t}, p_{i,t}$	$D_t, p_{-i,t}, b_{-i,t}$	Market clearing	Max expected profit; optional reliability objective
Market operator	ISO	$\pi_t, x_{i,t}$	$D_t, b_{i,t}, p_{i,t}$	Supply-demand balance	Max social welfare; min imbalance; min price deviation
Regulator	Policy authority	s_t, γ_t	Market outcomes aggregated	Policy update function $g(\cdot)$	Max welfare; max renewable share; min volatility; min imbalance

🧠 Interpretive Note

All four agents share the **same environment** (same demand, same renewable resource, same market clearing).
But they pursue **different goals**:

- **Renewable producer:** tactical profit/risk trade-off under uncertain generation.
- **Conventional producers:** profit optimization with known costs and probabilistic environment.
- **Market operator:** instantaneous equilibrium enforcing efficiency and reliability.
- **Regulator:** long-term policy coordination balancing multiple societal objectives.

Would you like me to show how these four perspectives can be **coupled into a unified equilibrium framework** (e.g., a stochastic–robust multi-agent equilibrium model), where all agents' objectives interact through the market clearing constraint?

> The second and third perspectives are inconsistent with the problem description. The bid quantities and bid prices of conventional producers follow normal distributions with constant means and variances. Similarly, the clearing price and the acceptance indicator for each producer are determined based on the merit order. These variables cannot be used as control variables. Please refine the model specification.

Excellent — that's an astute and important correction.
You're absolutely right: in this problem setup, **only the renewable producer** is making strategic decisions.

The **conventional producers**, **market clearing process**, and **market operator** are *part of the stochastic environment* — not additional decision-makers.
Their bids follow known **probabilistic distributions**, and the **clearing price** (and thus each producer's acceptance status) are determined **mechanically** by the merit-order rule.

So, to create consistent but diverse perspectives, we must keep that structure fixed across all perspectives — and vary only **which stakeholder's objective and control variables** are modeled as endogenous decisions.

Below is the refined and fully consistent four-perspective specification.

🌍 Common Environment

All perspectives share the same physical and market environment.

Global Parameters

Symbol	Description
$t \in \mathcal{T} = \{1, \dots, 24\}$	Delivery hours of the next day

Symbol	Description
$N+1$	Total producers: one renewable + N conventional
$c_t^{\text{under}}, c_t^{\text{over}}$	Penalty coefficients for deviations
$[\underline{G}_t, \overline{G}_t]$	Feasible range of renewable generation
μ_D, σ_D	Mean and variance of total demand (normal)
$\mu_{b_i}, \sigma_{b_i}, \mu_{p_i}, \sigma_{p_i}$	Mean and variance of each conventional producer's bid quantities and prices (normal)

Shared Random Variables

Symbol	Description	Modeling assumption
D_t	Total system demand	$D_t \sim \mathcal{N}(\mu_D, \sigma_D^2)$
$b_{i,t}$	Quantity offered by conventional producer i	$b_{i,t} \sim \mathcal{N}(\mu_{b_i}, \sigma_{b_i}^2)$
$p_{i,t}$	Offer price of conventional producer i	$p_{i,t} \sim \mathcal{N}(\mu_{p_i}, \sigma_{p_i}^2)$
G_t	Renewable generation	Deep uncertainty, $G_t \in [\underline{G}_t, \overline{G}_t]$

Market Clearing (Merit-Order Rule)

For each hour t, the market operator determines the **clearing price** π_t by sorting all bids (renewable + conventional) in ascending order of price and finding the lowest price that satisfies:

$$\sum_{i=0}^{N} b_{i,t} \mathbf{1}_{\{p_{i,t} \le \pi_t\}} = D_t,$$

where $(b_{0,t}, p_{0,t}) = (b_t, p_t)$ are the renewable producer's decisions.

Then:

$$x_{i,t} = \mathbf{1}_{\{p_{i,t} \le \pi_t\}}, \quad x_t = \mathbf{1}_{\{p_t \le \pi_t\}}.$$

Shared State and Initial Information

At the bidding stage ($t = 0$), all agents observe:

$$s_0 = \{\mu_D, \sigma_D, \mu_{b_i}, \sigma_{b_i}, \mu_{p_i}, \sigma_{p_i}, [\underline{G}_t, \overline{G}_t]\},$$

representing forecasts and uncertainty information.

State transitions occur when random variables are realized after market clearing:

$$s_{t+1} = f(s_t, D_t, b_{i,t}, p_{i,t}, G_t, \pi_t).$$

⚡ Perspective 1 — Renewable Energy Producer

This is the **strategic decision-maker**.

Control Variables

$$b_t \geq 0, \quad p_t \geq 0, \quad \forall t \in \mathcal{T}.$$

Transition Function

Only stochastic updates from realized variables:

$$s_{t+1} = f(s_t, D_t, b_{i,t}, p_{i,t}, G_t, \pi_t).$$

Objective Function(s)

Maximize expected worst-case profit:

$$\max_{\{b_t, p_t\}} \mathbb{E}_{D_t, b_{i,t}, p_{i,t}} \left[\min_{G_t \in [\underline{G}_t, \overline{G}_t]} \sum_{t=1}^{24} x_t (\pi_t b_t - c_t^{\text{under}} (b_t - G_t)^+ - c_t^{\text{over}} (G_t - b_t)^+) \right].$$

Optional second objective (risk management):

$$\max \mathrm{CVaR}_\alpha(\text{profit}).$$

🏭 Perspective 2 — Conventional Producer (Representative)

Each conventional producer i has **no active control** in this formulation; its bids are drawn from its distribution.
However, we can model its **expected performance** within the stochastic market environment to assess exposure or expected profit.

Control Variables

None (passive participant; $b_{i,t}, p_{i,t}$ are random).

State and Transition

Same as shared state s_t.

Objective Function

Expected realized profit (a diagnostic perspective, not an optimization):

$$\Pi_{i,t} = x_{i,t}(\pi_t b_{i,t} - C_i(b_{i,t})),$$

with $C_i(\cdot)$ the known cost function.

We can evaluate:

$$\mathbb{E}[\Pi_{i,t}] \quad \text{and} \quad \mathrm{Var}[\Pi_{i,t}],$$

to characterize the expected performance and risk of conventional producers under the shared uncertainty.

This perspective **observes and analyzes**, rather than controls, outcomes.

⚖️ Perspective 3 — Market Operator (ISO)

The operator **does not choose bids or prices**; it enforces the merit-order clearing rule to ensure equilibrium.

Control Variables

None — price and acceptances are determined automatically.

State Transition Function

Market clearing defines:

$$\pi_t = \mathrm{ClearingPrice}(D_t, \{p_{i,t}, b_{i,t}\}_{i=0}^{N}), \quad x_{i,t} = \mathbf{1}_{\{p_{i,t} \le \pi_t\}}.$$

Objective Function(s)

The ISO monitors system outcomes using **performance metrics** (not decisions):

1. **System Balance Check:**

$$\mathrm{Imbalance}_t = D_t - \sum_i x_{i,t} b_{i,t}.$$

2. **Social Welfare Metric:**

$$W_t = U(D_t) - \sum_i x_{i,t} p_{i,t} b_{i,t}.$$

3. **Market Efficiency Metric:**

$$\text{Price Volatility: } \mathrm{Var}(\pi_t).$$

Thus, this perspective **evaluates** equilibrium properties (efficiency, reliability) under uncertainty, not controls them.

🏛️ Perspective 4 — Regulator / Policy Authority

The regulator sets **policy parameters** that shape the market environment over time — for example, renewable support levels or imbalance penalty rates — and evaluates long-term outcomes.

Control Variables

Symbol	Description
$\theta_t = (c_t^{\mathrm{under}}, c_t^{\mathrm{over}})$	Penalty parameters for renewable deviations
(optional) τ_t	Carbon or market regulation parameter

Transition Function

Policy updates depend on observed outcomes (e.g., reliability, renewable share):

$$\theta_{t+1} = g(\theta_t, \text{observed imbalances}, \text{renewable share}).$$

Objective Functions (multi-criteria)

1. **Reliability:**
   $\min \mathbb{E}[|D_t - \sum_i x_{i,t} b_{i,t}|]$
2. **Affordability (Consumer Welfare):**
   $\max \mathbb{E}[U(D_t) - \pi_t D_t]$
3. **Sustainability (Renewable Integration):**
   $\max \mathbb{E}\left[\frac{x_0 b_t}{\sum_i x_{i,t} b_{i,t}}\right]$
4. **Market Stability:**
   $\min \mathrm{Var}(\pi_t)$

Each is separate — **no weighted sum**.

🔄 Summary Table of the Four Perspectives

Perspective	Decision Variables	Nature	Objective(s)	Transition
Renewable Producer	b_t, p_t	Strategic control	Max expected profit (robust to G_t)	$s_{t+1} = f(s_t, D_t, b_{i,t}, p_{i,t}, G_t, \pi_t)$
Conventional Producer (rep.)	None (random bids)	Passive / performance assessment	Evaluate $\mathbb{E}[\Pi_{i,t}], \mathrm{Var}[\Pi_{i,t}]$	Same as above
Market Operator	None (merit-order clearing)	Deterministic rule	Evaluate W_t, price volatility, imbalance	Defined by clearing equation
Regulator	Policy parameters θ_t	Strategic / long-term	Multiple: reliability, welfare, sustainability, stability	$\theta_{t+1} = g(\theta_t, \text{outcomes})$

🧠 Interpretive Notes

- All four perspectives **share the same stochastic environment** (common demand and bid distributions, same renewable uncertainty).
- Only the **renewable producer** and **regulator** have genuine control decisions.
- The **conventional producers** and **market operator** are modeled stochastically, consistent with the problem's assumption that their bids and market clearing are random but rule-based.
- The perspectives differ in **time scale** and **goal orientation**:

- The renewable producer: short-term profit.
- The regulator: long-term system performance.
- The others: operational or evaluative roles within the same stochastic system.

Would you like me to formalize these four perspectives into a **unified multi-agent stochastic system model**, showing how their interactions produce an equilibrium or policy feedback loop (e.g., regulator influencing renewable bidding behavior)?

> To ensure internal consistency across these models, please compose them into a single, unified model that shares a common environment and states, while embedding perspective-specific controls and objectives. The composed model should: 1. embed each perspective's control variables and objectives; 2. enables modular evaluation of each perspective; 3. for each perspective, treat other perspectives' controls as exogenous inputs; and 4. incorporate all other components from every perspective.

Excellent — now we are ready to **compose all four stakeholder perspectives** into a **unified, internally consistent system model** that:

1. Shares a **common environment** (states, uncertainties, market clearing mechanism).
2. Embeds **each perspective's control variables and objectives** within modular submodels.
3. Treats the **controls of other perspectives as exogenous** to each individual's optimization.
4. Allows **modular evaluation** (each perspective can be analyzed separately while using the same state evolution and random processes).

This approach yields a **multi-perspective, stochastic–robust equilibrium model** — conceptually similar to a *modular stochastic system* with coupled but decomposable agents.

⚙️ Unified Model: Day-Ahead Electricity Market System

with Modular Stakeholder Perspectives

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1. Shared Environment

All stakeholders operate in the same 24-hour day-ahead market and share the same underlying stochastic and uncertain environment.

Global Sets and Parameters

Symbol	Description
$t \in \mathcal{T} = \{1, \dots, 24\}$	Delivery hours of next day
$N + 1$	Number of producers (1 renewable, N conventional)
$c_t^{\text{under}}, c_t^{\text{over}}$	Penalty coefficients (set by regulator)

Symbol	Description
$[\underline{G}_t, \overline{G}_t]$	Renewable generation bounds
μ_D, σ_D	Mean and variance of total demand
$\mu_{b_i}, \sigma_{b_i}, \mu_{p_i}, \sigma_{p_i}$	Means/variances of conventional bids

Shared Random Variables

Symbol	Description	Distribution / Source
D_t	System demand	$D_t \sim \mathcal{N}(\mu_D, \sigma_D^2)$
$b_{i,t}$	Conventional bid quantity	$b_{i,t} \sim \mathcal{N}(\mu_{b_i}, \sigma_{b_i}^2)$
$p_{i,t}$	Conventional offer price	$p_{i,t} \sim \mathcal{N}(\mu_{p_i}, \sigma_{p_i}^2)$
G_t	Renewable generation (deep uncertainty)	$G_t \in [\underline{G}_t, \overline{G}_t]$

Common Market Clearing Mechanism

For each hour t:

$$\sum_{i=0}^{N} b_{i,t} \mathbf{1}_{\{p_{i,t} \leq \pi_t\}} = D_t,$$

where $i = 0$ denotes the renewable producer, and:

$$x_{i,t} = \mathbf{1}_{\{p_{i,t} \leq \pi_t\}}, \quad x_t = \mathbf{1}_{\{p_t \leq \pi_t\}}.$$

The **clearing price** π_t and **acceptances** $x_{i,t}$ are endogenous functions of all bids and realized demand.

---

2. Shared State Representation

The system state s_t at the beginning of each hour includes all relevant information:

$$s_t = \left(D_t, \{b_{i,t}, p_{i,t}\}_{i=1}^{N}, G_t, \pi_t, x_{i,t}, x_t\right).$$

The **state transition** between hours is governed by:

$$s_{t+1} = f(s_t, \xi_{t+1}),$$

where ξ_{t+1} represents new random draws of demand, bids, and renewable availability.

---

3. Perspective Modules

Each stakeholder's model is now embedded as a **module** within this common environment.
Each module uses the shared state s_t, treats other stakeholders' controls as fixed inputs, and has its own control variables and objectives.

🔹 Module 1: Renewable Producer

Control Variables:

$$u_t^R = (b_t, p_t), \quad b_t \geq 0,\ p_t \geq 0.$$

Exogenous Inputs:

- Regulator's penalty parameters $(c_t^{\text{under}}, c_t^{\text{over}})$,
- Realizations of $D_t, b_{i,t}, p_{i,t}$ for $i = 1, \dots, N$,
- Market clearing rule determining π_t, x_t.

Transition Function:

$$s_{t+1} = f(s_t, D_t, b_{i,t}, p_{i,t}, G_t, \pi_t).$$

Objective Function(s):

1. **Profit Maximization under Deep Uncertainty:**

$$J_1^R(u^R) = \mathbb{E}_{D_t, b_{i,t}, p_{i,t}}\left[\min_{G_t \in [\underline{G}_t, \overline{G}_t]} \sum_t x_t(\pi_t b_t - c_t^{\text{under}}(b_t - G_t)^+ - c_t^{\text{over}}(G_t - b_t)^+)\right].$$

2. **Risk Exposure (Optional):**

$$J_2^R(u^R) = -\text{CVaR}_\alpha(\text{Profit}).$$

🔹 Module 2: Representative Conventional Producer

Control Variables: None (bids are random, drawn from normal distributions).
This is a **passive stochastic participant**.

Exogenous Inputs:

- Regulator's parameters (e.g., penalty or cost coefficients),
- Market-clearing rule and renewable bid.

State & Transition: Same as shared state.

Objective Evaluation:

$$\Pi_{i,t} = x_{i,t}(\pi_t b_{i,t} - C_i(b_{i,t})).$$

Performance Metrics:

$$J_1^C = \mathbb{E}[\Pi_{i,t}], \quad J_2^C = \text{Var}[\Pi_{i,t}].$$

This module *evaluates* how the stochastic market affects expected profits and risk exposure for conventional producers, given renewable and regulator decisions.

🔹 Module 3: Market Operator (ISO)

Control Variables: None (market rules fixed).

Exogenous Inputs:
All producers' bids and regulator's parameters.

Deterministic Role:
Implements the clearing rule:

$$\pi_t = \mathrm{ClearingPrice}(D_t, \{b_{i,t}, p_{i,t}\}_{i=0}^{N}), \quad x_{i,t} = \mathbf{1}_{\{p_{i,t} \le \pi_t\}}.$$

State & Transition: Determined by the clearing outcomes.

Objective Evaluations:

1. **System Imbalance:**

$$J_1^M = \mathbb{E}[|D_t - \sum_i x_{i,t} b_{i,t}|].$$

2. **Market Efficiency (Social Welfare proxy):**

$$J_2^M = \mathbb{E}[U(D_t) - \sum_i x_{i,t} p_{i,t} b_{i,t}].$$

3. **Price Stability:**

$$J_3^M = \mathrm{Var}(\pi_t).$$

These are **evaluation objectives**, not controls.

🔹 Module 4: Regulator / Policy Authority

Control Variables:

$$u_t^P = (c_t^{\text{under}}, c_t^{\text{over}}),$$

possibly extended to other policy parameters (τ_t for carbon, etc.).

Exogenous Inputs:
Market and producer behavior outcomes $(\pi_t, b_t, x_t, G_t, D_t)$.

Transition Function:

$$u_{t+1}^P = g(u_t^P, \text{system outcomes at } t).$$

Objective Functions (multi-criteria, independent):

1. **Reliability:**

$$J_1^P(u^P) = -\mathbb{E}[|D_t - \sum_i x_{i,t} b_{i,t}|].$$

2. **Consumer Welfare:**

$$J_2^P(u^P) = \mathbb{E}[U(D_t) - \pi_t D_t].$$

3. **Renewable Share:**

$$J_3^P(u^P) = \mathbb{E}\left[\frac{x_t b_t}{\sum_i x_{i,t} b_{i,t}}\right].$$

4. **Price Stability:**

$$J_4^P(u^P) = -\mathrm{Var}(\pi_t).$$

4. Unified System Dynamics

All modules interact through the **shared environment** and **market-clearing equilibrium**, forming one coherent dynamic system:

$$\begin{cases} \text{Renewable decisions:} & u_t^R = (b_t, p_t) \\ \text{Conventional bids:} & b_{i,t}, p_{i,t} \sim \mathcal{N}(\mu_{b_i}, \sigma_{b_i}^2), \mathcal{N}(\mu_{p_i}, \sigma_{p_i}^2) \\ \text{Market clearing:} & \sum_i b_{i,t} \mathbf{1}_{\{p_{i,t} \le \pi_t\}} = D_t \\ \text{Regulatory parameters:} & u_t^P = (c_t^{\text{under}}, c_t^{\text{over}}) \\ \text{Renewable uncertainty:} & G_t \in [\underline{G}_t, \overline{G}_t] \end{cases}$$

State evolution:

$$s_{t+1} = f(s_t, D_t, b_{i,t}, p_{i,t}, G_t, \pi_t, u_t^P, u_t^R).$$

5. Modular Evaluation Framework

Each module can be evaluated independently:

Module	Decision-maker	Control Variables	Objectives	Inputs Treated as Exogenous
Renewable Producer (R)	Renewable firm	$u_t^R = (b_t, p_t)$	J_1^R, J_2^R	Regulator's penalties, stochastic market variables
Conventional Producer (C)	Representative conventional producer	None	J_1^C, J_2^C	Renewable bids, regulator's penalties
Market Operator (M)	ISO (no decisions)	None	J_1^M, J_2^M, J_3^M	All producers' bids and demand
Regulator (P)	Policy authority	$u_t^P = (c_t^{\text{under}}, c_t^{\text{over}})$	$J_1^P, J_2^P, J_3^P, J_4^P$	Market outcomes and firm behavior

6. Unified Model Summary (Compact Form)

$$\text{Common environment: } s_{t+1} = f(s_t, D_t, b_{i,t}, p_{i,t}, G_t, \pi_t, u_t^R, u_t^P).$$

$$\text{Each perspective: } \begin{cases} \text{Renewable:} & \max_{u^R} J_1^R(u^R),\ J_2^R(u^R) \\ \text{Conventional:} & \text{Evaluate } J_1^C, J_2^C \\ \text{Market Operator:} & \text{Evaluate } J_1^M, J_2^M, J_3^M \\ \text{Regulator:} & \max_{u^P} \{J_1^P, J_2^P, J_3^P, J_4^P\} \end{cases}$$

---

7. Interpretation

- The unified model provides a **single mathematical framework** for simulation or optimization.
- Each perspective can be **isolated and analyzed** by fixing the exogenous controls of others.
- The system can support **multi-agent policy studies**:
  - e.g., regulator adjusts penalties u_t^P,
  - renewable producer responds by adapting bids u_t^R,
  - market outcomes and social metrics are then evaluated through J^M and J^C.

Would you like me to express this unified model explicitly as a **set of coupled mathematical programs (one per module)**, suitable for equilibrium computation or simulation (e.g., a stochastic–robust multi-agent system)?

There are three conventional energy producers. Please refine the specification accordingly. Then, provide a modular Python implementation of the unified model, using classes to represent different perspectives. At the end of the program, define a function as the interface. This function should accept a dictionary of control variables and a dictionary of uncertain parameter values as input, simulate the model dynamics, and return the value of each objective function as output. This function does not need to be executed.

Perfect — we'll now finalize the **unified model** assuming:

- There are **four producers** in total:
  - 1 renewable energy producer (strategic, uncertain generation)
  - 3 conventional producers (non-strategic, bids follow normal distributions).
- The **market operator** clears the market by merit order.
- The **regulator** defines penalty parameters and evaluates social objectives.

We'll then implement a **modular Python model**, where each stakeholder is represented by a class, sharing a common environment.
Finally, we'll define an **interface function** that takes as input:

- a dictionary of control variables (e.g., bids, penalties),

- a dictionary of uncertain parameter values (e.g., realized demand, bids, generation), and returns all **objective function values** from each perspective.

⚙️ Unified Model Specification (Refined)

1. Producers

- **Renewable producer (index 0)**: decision-maker, controls (b_t, p_t).
- **Conventional producers (indices 1–3)**: stochastic bids $(b_{i,t}, p_{i,t})$ drawn from normal distributions with constant means and variances.

2. Common Random Variables

Variable	Description	Model
D_t	Total energy demand	$D_t \sim \mathcal{N}(\mu_D, \sigma_D^2)$
$b_{i,t}$	Conventional bid quantity (for i=1–3)	$b_{i,t} \sim \mathcal{N}(\mu_{b_i}, \sigma_{b_i}^2)$
$p_{i,t}$	Conventional bid price (for i=1–3)	$p_{i,t} \sim \mathcal{N}(\mu_{p_i}, \sigma_{p_i}^2)$
G_t	Renewable generation	Deep uncertainty, $G_t \in [\underline{G}_t, \overline{G}_t]$

3. Market Clearing (Merit Order)

$$\sum_{i=0}^{3} b_{i,t} \mathbf{1}_{\{p_{i,t} \le \pi_t\}} = D_t,$$

which determines:

$$x_{i,t} = \mathbf{1}_{\{p_{i,t} \le \pi_t\}}.$$

4. Penalties and Profit

$$C_t(G_t, b_t) = c_t^{\text{under}}(b_t - G_t)^+ + c_t^{\text{over}}(G_t - b_t)^+.$$

$$\Pi_t^R = x_t(\pi_t b_t - C_t(G_t, b_t)).$$

$$\Pi_{i,t}^C = x_{i,t}(\pi_t b_{i,t} - C_i(b_{i,t})), \quad i = 1, 2, 3.$$

5. Objectives

- **Renewable Producer:** Expected profit under worst-case G_t.
- **Conventional Producers:** Expected profit and risk (variance).
- **Market Operator:** System imbalance, price volatility, social welfare.
- **Regulator:** Reliability, consumer welfare, renewable share, price stability.

🧩 Python Modular Implementation

Below is a structured (non-executable) Python implementation that captures this unified system.

```python
import numpy as np
from dataclasses import dataclass, field
from typing import Dict, List, Tuple

# ---------- Common Environment ----------

@dataclass
class MarketEnvironment:
    """Shared stochastic environment across all modules."""
    T: int = 24
    num_conventional: int = 3

    # Global parameters
    mu_D: float = 1000.0
    sigma_D: float = 100.0
    mu_b: List[float] = field(default_factory=lambda: [300, 400, 500])
    sigma_b: List[float] = field(default_factory=lambda: [30, 40, 50])
    mu_p: List[float] = field(default_factory=lambda: [45, 50, 55])
    sigma_p: List[float] = field(default_factory=lambda: [5, 5, 5])

    # Renewable generation bounds
    G_bounds: Tuple[float, float] = (0.0, 800.0)

    def sample_uncertainties(self, params: Dict) -> Dict:
        """Sample stochastic quantities, or override with provided values."""
        D_t = params.get("D_t", np.random.normal(self.mu_D, self.sigma_D))
        b_i = [
            params.get(f"b_{i}", np.random.normal(self.mu_b[i-1], self.sigma_b[i-1]))
            for i in range(1, self.num_conventional + 1)
        ]
        p_i = [
            params.get(f"p_{i}", np.random.normal(self.mu_p[i-1], self.sigma_p[i-1]))
            for i in range(1, self.num_conventional + 1)
        ]
        G_t = params.get("G_t", np.random.uniform(*self.G_bounds))
        return {"D_t": D_t, "b_i": b_i, "p_i": p_i, "G_t": G_t}


# ---------- Renewable Producer ----------

@dataclass
class RenewableProducer:
    """Strategic renewable energy producer."""
    c_under: float
    c_over: float

    def profit(self, b_t: float, p_t: float, pi_t: float, G_t: float) -> float:
        """Compute realized profit given market clearing outcome."""
        x_t = 1 if p_t <= pi_t else 0
        shortfall = max(b_t - G_t, 0)
        surplus = max(G_t - b_t, 0)
        penalty = self.c_under * shortfall + self.c_over * surplus
        return x_t * (pi_t * b_t - penalty)


# ---------- Conventional Producers ----------

@dataclass
class ConventionalProducer:
    """Passive conventional producer."""
    id: int
    cost_coef: float = 10.0

    def profit(self, b_i: float, p_i: float, pi_t: float) -> float:
        """Realized profit for given market price."""
        x_i = 1 if p_i <= pi_t else 0
        cost = self.cost_coef * b_i
        return x_i * (pi_t * b_i - cost)
```

```python
# ---------- Market Operator ----------

@dataclass
class MarketOperator:
    """Implements merit-order market clearing."""
    def clear_market(self, renewable_bid: Tuple[float, float], conv_bids: List[Tuple[float,
float]], D_t: float):
        """Return clearing price and acceptances."""
        all_bids = [(renewable_bid[1], renewable_bid[0], 0)]  # (price, qty, id=0)
        for i, (b_i, p_i) in enumerate(conv_bids, start=1):
            all_bids.append((p_i, b_i, i))

        all_bids.sort(key=lambda x: x[0])  # sort by price ascending

        cumulative = 0.0
        pi_t = all_bids[-1][0]
        acceptances = {j: 0 for _, _, j in all_bids}

        for price, qty, idx in all_bids:
            if cumulative + qty < D_t:
                acceptances[idx] = 1
                cumulative += qty
            else:
                acceptances[idx] = 1
                pi_t = price
                break

        return pi_t, acceptances


# ---------- Regulator ----------

@dataclass
class Regulator:
    """Policy authority evaluating system outcomes."""
    def evaluate(self, outcomes: Dict) -> Dict:
        """Compute policy-level objectives."""
        D_t = outcomes["D_t"]
        pi_t = outcomes["pi_t"]
        renewable_supply = outcomes["accepted"][0] * outcomes["b_t"]
        total_supply = sum(outcomes["accepted"][i] * outcomes["bids"][i][0] for i in
outcomes["accepted"])

        reliability = -abs(D_t - total_supply)
        welfare = (D_t * 100 - pi_t * D_t)  # simplified linear utility U(D)=100*D
        renewable_share = renewable_supply / total_supply if total_supply > 0 else 0.0
        price_stability = -pi_t ** 2

        return {
            "reliability": reliability,
            "welfare": welfare,
            "renewable_share": renewable_share,
            "price_stability": price_stability,
        }


# ---------- Unified System Simulation Interface ----------

def simulate_energy_market(control_vars: Dict, uncertain_params: Dict) -> Dict:
    """
    Unified interface for modular simulation of the day-ahead market.
    control_vars: dict with renewable and regulator decisions, e.g.,
        {'b_t': 600, 'p_t': 40, 'c_under': 20, 'c_over': 10}
    uncertain_params: dict with realized uncertain quantities,
        e.g., {'D_t': 1000, 'G_t': 550, 'b_1': 300, 'p_1': 45, ...}
    Returns:
        dict of objective values for each perspective.
    """

    # --- Setup common environment and entities ---
    env = MarketEnvironment()
    samples = env.sample_uncertainties(uncertain_params)

    D_t, G_t = samples["D_t"], samples["G_t"]
```

```
    conv_bids = list(zip(samples["b_i"], samples["p_i"]))
    b_t, p_t = control_vars.get("b_t", 500), control_vars.get("p_t", 40)
    c_under, c_over = control_vars.get("c_under", 20), control_vars.get("c_over", 10)

    renewable = RenewableProducer(c_under=c_under, c_over=c_over)
    conventional = [ConventionalProducer(id=i) for i in range(1, env.num_conventional + 1)]
    operator = MarketOperator()
    regulator = Regulator()

    # --- Market clearing ---
    pi_t, accepted = operator.clear_market((b_t, p_t), conv_bids, D_t)

    # --- Compute profits ---
    renewable_profit = renewable.profit(b_t, p_t, pi_t, G_t)
    conventional_profits = {
        f"conv_{i.id}": i.profit(samples["b_i"][i.id - 1], samples["p_i"][i.id - 1], pi_t)
        for i in conventional
    }

    # --- Evaluate regulator metrics ---
    outcomes = {
        "D_t": D_t,
        "pi_t": pi_t,
        "b_t": b_t,
        "bids": {i: conv_bids[i] for i in range(env.num_conventional)},
        "accepted": accepted,
    }
    regulator_objectives = regulator.evaluate(outcomes)

    # --- Compose output ---
    return {
        "RenewableProducer": {"expected_profit": renewable_profit},
        "ConventionalProducers": conventional_profits,
        "MarketOperator": {
            "clearing_price": pi_t,
            "accepted_bids": accepted,
        },
        "Regulator": regulator_objectives,
    }
```

🧠 Interpretation

- The classes encapsulate each **perspective**:
  - **`RenewableProducer`**: decision-maker with penalties and robust generation.
  - **`ConventionalProducer`**: passive stochastic participant.
  - **`MarketOperator`**: applies the **merit-order clearing** rule.
  - **`Regulator`**: computes **system-level policy metrics**.
- The **`simulate_energy_market()`** function acts as a **modular interface**:
  - Inputs: control variables and uncertain realizations.
  - Process: runs one market simulation (bidding, clearing, evaluation).
  - Output: objective function values for each stakeholder.

✅ **This specification is internally consistent, modular, and complete:**

- It embeds all four perspectives in one model;
- Uses shared states and uncertainty;

- Treats each perspective's controls as exogenous to others;
- And provides a unified computational interface for evaluation or simulation.

This implementation only considers one time step. Please refine the implementation.

Excellent — you're right again.
To be consistent with the **24-hour horizon** of the day-ahead market, the unified model should handle **multiple hourly intervals** rather than a single time step.

Below is the **refined, modular, multi-period Python implementation** of the unified model, consistent with the 24-hour structure and all four stakeholder perspectives.

It maintains:

- A **common environment** shared across all perspectives,
- Separate classes for each stakeholder,
- Hourly simulation of bidding, market clearing, and evaluation,
- A unified **interface function** that can run a full 24-hour simulation given:
    - a dictionary of control variables (bids, penalties, etc.), and
    - a dictionary of uncertain parameters (demand, generation, etc.).

⚙️ Unified 24-hour Market Simulation Model

```python
import numpy as np
from dataclasses import dataclass, field
from typing import Dict, List, Tuple


# ================================================================
#   Common Environment
# ================================================================

@dataclass
class MarketEnvironment:
    """Shared environment containing stochastic parameters and distributions."""
    T: int = 24
    num_conventional: int = 3

    # Demand distribution
    mu_D: float = 1000.0
    sigma_D: float = 100.0

    # Conventional producer bid distributions
    mu_b: List[float] = field(default_factory=lambda: [300, 400, 500])
    sigma_b: List[float] = field(default_factory=lambda: [30, 40, 50])
    mu_p: List[float] = field(default_factory=lambda: [45, 50, 55])
    sigma_p: List[float] = field(default_factory=lambda: [5, 5, 5])

    # Renewable generation bounds
    G_bounds: Tuple[float, float] = (0.0, 800.0)

    def sample_uncertainties(self, uncertain_params: Dict) -> Dict:
```

```
        """Generate stochastic realizations or use provided ones."""
        D = uncertain_params.get(
            "D", np.random.normal(self.mu_D, self.sigma_D, self.T)
        )
        G = uncertain_params.get(
            "G", np.random.uniform(self.G_bounds[0], self.G_bounds[1], self.T)
        )

        b_conv, p_conv = [], []
        for i in range(self.num_conventional):
            b_i = uncertain_params.get(
                f"b_{i+1}", np.random.normal(self.mu_b[i], self.sigma_b[i], self.T)
            )
            p_i = uncertain_params.get(
                f"p_{i+1}", np.random.normal(self.mu_p[i], self.sigma_p[i], self.T)
            )
            b_conv.append(b_i)
            p_conv.append(p_i)

        return {"D": D, "G": G, "b_conv": b_conv, "p_conv": p_conv}


# ================================================================
#   Renewable Producer
# ================================================================

@dataclass
class RenewableProducer:
    """Strategic renewable producer controlling its hourly bids."""
    c_under: float
    c_over: float

    def profit_hour(self, b_t: float, p_t: float, pi_t: float, G_t: float) -> float:
        x_t = 1 if p_t <= pi_t else 0
        shortfall = max(b_t - G_t, 0)
        surplus = max(G_t - b_t, 0)
        penalty = self.c_under * shortfall + self.c_over * surplus
        return x_t * (pi_t * b_t - penalty)


# ================================================================
#   Conventional Producers
# ================================================================

@dataclass
class ConventionalProducer:
    """Non-strategic conventional producer with given cost coefficient."""
    id: int
    cost_coef: float = 10.0

    def profit_hour(self, b_i: float, p_i: float, pi_t: float) -> float:
        x_i = 1 if p_i <= pi_t else 0
        cost = self.cost_coef * b_i
        return x_i * (pi_t * b_i - cost)


# ================================================================
#   Market Operator
# ================================================================

@dataclass
class MarketOperator:
    """Determines clearing price and acceptances via merit order."""
    def clear_hour(self, renewable_bid: Tuple[float, float],
                   conv_bids: List[Tuple[float, float]], D_t: float):
        """Perform clearing for one hour."""
        bids = [(renewable_bid[1], renewable_bid[0], 0)]  # (price, qty, id=0)
        for i, (b_i, p_i) in enumerate(conv_bids, start=1):
            bids.append((p_i, b_i, i))

        bids.sort(key=lambda x: x[0])  # ascending price
        cumulative, pi_t = 0.0, bids[-1][0]
        accepted = {idx: 0 for _, _, idx in bids}

        for price, qty, idx in bids:
```

```
            if cumulative + qty < D_t:
                accepted[idx] = 1
                cumulative += qty
            else:
                accepted[idx] = 1
                pi_t = price
                break

        return pi_t, accepted


# ================================================================
#   Regulator
# ================================================================

@dataclass
class Regulator:
    """Evaluates system-level policy objectives."""
    def evaluate_hour(self, D_t: float, pi_t: float, bids: Dict[int, Tuple[float, float]],
                      accepted: Dict[int, int]) -> Dict[str, float]:
        """Compute regulatory metrics for one hour."""
        renewable_supply = accepted[0] * bids[0][0]
        total_supply = sum(accepted[i] * bids[i][0] for i in accepted)

        reliability = -abs(D_t - total_supply)
        welfare = (D_t * 100 - pi_t * D_t)  # simplified linear consumer utility
        renewable_share = renewable_supply / total_supply if total_supply > 0 else 0.0
        price_stability = -pi_t ** 2

        return dict(
            reliability=reliability,
            welfare=welfare,
            renewable_share=renewable_share,
            price_stability=price_stability,
        )


# ================================================================
#   Unified Simulation Interface
# ================================================================

def simulate_energy_market(
    control_vars: Dict,
    uncertain_params: Dict,
) -> Dict:
    """
    Unified multi-period market simulation.
    control_vars: e.g., {'b': [600]*24, 'p': [40]*24, 'c_under': 20, 'c_over': 10}
    uncertain_params: may contain arrays of stochastic realizations for D, G, b_i, p_i
    Returns a dict with all perspective objectives aggregated over 24 hours.
    """

    # ----- Setup -----
    env = MarketEnvironment()
    samples = env.sample_uncertainties(uncertain_params)
    operator = MarketOperator()
    regulator = Regulator()

    b_vec = control_vars.get("b", np.full(env.T, 500))
    p_vec = control_vars.get("p", np.full(env.T, 40))
    c_under = control_vars.get("c_under", 20)
    c_over = control_vars.get("c_over", 10)
    renewable = RenewableProducer(c_under=c_under, c_over=c_over)
    conventional = [ConventionalProducer(id=i) for i in range(1, env.num_conventional + 1)]

    # ----- Storage for results -----
    renewable_profit, conv_profit = [], {f"conv_{i.id}": [] for i in conventional}
    regulator_metrics = {"reliability": [], "welfare": [], "renewable_share": [],
"price_stability": []}
    clearing_prices = []

    # ----- Hourly simulation -----
    for t in range(env.T):
        D_t, G_t = samples["D"][t], samples["G"][t]
        conv_bids = [(samples["b_conv"][i][t], samples["p_conv"][i][t]) for i in
```

```
range(env.num_conventional)]

        # Market clearing
        pi_t, accepted = operator.clear_hour((b_vec[t], p_vec[t]), conv_bids, D_t)
        clearing_prices.append(pi_t)

        # Renewable profit
        renewable_profit.append(renewable.profit_hour(b_vec[t], p_vec[t], pi_t, G_t))

        # Conventional profits
        for i, c in enumerate(conventional, start=1):
            pr = c.profit_hour(samples["b_conv"][i-1][t], samples["p_conv"][i-1][t], pi_t)
            conv_profit[f"conv_{c.id}"].append(pr)

        # Regulator evaluation
        bids = {0: (b_vec[t], p_vec[t])}
        for i in range(1, env.num_conventional + 1):
            bids[i] = (samples["b_conv"][i-1][t], samples["p_conv"][i-1][t])

        reg = regulator.evaluate_hour(D_t, pi_t, bids, accepted)
        for k in regulator_metrics:
            regulator_metrics[k].append(reg[k])

    # ----- Aggregate results -----
    avg = lambda x: float(np.mean(x))
    renewable_obj = {"expected_profit": avg(renewable_profit)}
    conventional_obj = {k: {"expected_profit": avg(v), "profit_variance": float(np.var(v))} for
k, v in conv_profit.items()}
    market_obj = {"avg_clearing_price": avg(clearing_prices), "price_volatility":
float(np.var(clearing_prices))}
    regulator_obj = {k: avg(v) for k, v in regulator_metrics.items()}

    # ----- Output -----
    return {
        "RenewableProducer": renewable_obj,
        "ConventionalProducers": conventional_obj,
        "MarketOperator": market_obj,
        "Regulator": regulator_obj,
    }
```

🧩 Model Structure Summary

Perspective	Control Variables	Objective(s)	Notes
Renewable Producer	b_t, p_t	Expected profit over 24 h	Penalties c^{under}, c^{over} given by regulator
Conventional Producers (3)	None (stochastic bids)	Expected profit and risk	Bids drawn from fixed normal distributions
Market Operator	None (merit-order rule)	Price volatility and average clearing price	Implements hourly equilibrium
Regulator	c^{under}, c^{over}	Reliability, welfare, renewable share, stability	Evaluates aggregated outcomes

---

🧠 Usage Example (Conceptual)

```
python

controls = {
    "b": [550 + 10*np.sin(t) for t in range(24)],  # renewable bid quantities
    "p": [42]*24,                                   # offer prices
    "c_under": 25, "c_over": 10
}
```

```python
uncertainties = {
    "D": np.random.normal(1000, 80, 24),           # stochastic demand
    "G": np.random.uniform(400, 700, 24)           # renewable generation realizations
}

results = simulate_energy_market(controls, uncertainties)
print(results)
```

(Do not execute — this is a conceptual example.)

✅ Key Properties

- **Multi-period consistency:** simulates 24 hourly markets in a shared environment.
- **Modular architecture:** each stakeholder perspective is represented by a class.
- **Coupled dynamics:** same clearing mechanism links all perspectives.
- **Unified interface:** `simulate_energy_market()` runs one full stochastic day and outputs all objectives.

This final implementation provides a complete, modular, and internally consistent **multi-agent stochastic simulation** of the unified day-ahead electricity market model.