Market_brief_1

An energy company is interested in investing in wind-power production. To assess the viability of this investment, the company wants to evaluate the expected revenue of a wind-power producer in a competitive spot market.

Suppose there are five energy producers participating in the day-ahead spot market. Three of them use conventional sources for energy production, such as coal, while the other two producers use solar and wind power, respectively. The market is cleared once per day for 24 hourly intervals of the following day. For each dispatch interval, every producer submits a bid specifying the energy quantity (MWh) it is willing to supply and the corresponding minimum acceptable price (per MWh). Then, the market operator follows a merit-order process to determine the market-clearing price and dispatch schedule, ensuring the total market demand is met. If a bid is accepted, the producer is committed to delivering the bid quantity at the market-clearing price. Otherwise, the producer earns zero revenue.

The energy company aims to build a simulation model to evaluate the expected revenue of the wind-power producer on any future day by selecting the hourly bid quantity and price, while accounting for uncertainties. This model can then be used to explore outcomes across various short-term and long-term scenarios, thereby informing the viability of the investment.

Please formalize this problem description as a simulation model and specify the key components, including state variables, decision variables, transition functions, objective functions, stochastic variables, constant parameters and deep uncertainties.

Below is a clean **formalization of the wind-power producer revenue problem as a simulation model** suitable for scenario analysis, Monte Carlo simulation, and policy evaluation. The emphasis is on *decision making under uncertainty* in a competitive electricity spot market.

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1. Modeling Framework

We model the problem as a **stochastic simulation of a day-ahead electricity market**, repeated over many simulated days.
Each simulated day consists of **24 hourly market-clearing problems** with uncertain demand, renewable availability, and competitor bids.

The wind producer is a **price-taker but strategic bidder**, choosing hourly bid quantities and prices to maximize expected revenue.

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2. Time Structure

- **Decision horizon**: One day ahead

- **Time steps**:

$$t \in \{1, 2, \ldots, 24\} \quad \text{(hourly dispatch intervals)}$$

- **Simulation horizon**: Many days (Monte Carlo or scenario-based)

3. State Variables

State variables describe the system conditions at each hour t:

$$S_t = \left(D_t,\ W_t,\ S_t^{\text{solar}},\ B_{-w,t}\right)$$

Where:

- D_t: electricity demand (MWh)
- W_t: available wind generation capacity (MWh)
- S_t^{solar}: available solar generation
- $B_{-w,t}$: competitors' bid stacks (coal + solar)

These states are **partially observed** and uncertain at bidding time.

4. Decision Variables (Wind Producer)

For each hour t, the wind producer chooses:

$$x_t = (q_t^w, p_t^w)$$

Where:

- q_t^w: bid quantity (MWh)
- p_t^w: bid price (€/MWh)

Subject to:

$$0 \leq q_t^w \leq W_t$$

5. Stochastic Variables

Uncertainty is represented by random variables:

Short-Term (Operational) Uncertainty

- $D_t \sim F_D(t)$: demand uncertainty
- $W_t \sim F_W(t)$: wind availability
- $S_t^{\text{solar}} \sim F_S(t)$: solar availability

- $B_{-w,t} \sim F_B(t)$: competitor bids

Correlations

- Temporal correlation (e.g., wind persistence)
- Cross-correlation (e.g., wind and solar anticorrelation)

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6. Market-Clearing Transition Function

For each hour t, the market operator applies a **merit-order dispatch**:

$$(P_t^*, \delta_{i,t}) = \mathcal{M}\big(D_t,\ \{(q_{i,t}, p_{i,t})\}_{i=1}^{5}\big)$$

Where:

- P_t^*: market-clearing price
- $\delta_{i,t} \in \{0, 1\}$: acceptance indicator of producer i

The wind producer is accepted if:

$$p_t^w \leq P_t^*$$

---

7. Revenue Function

Hourly revenue of the wind producer:

$$R_t = \delta_{w,t} \cdot q_t^w \cdot P_t^*$$

Daily revenue:

$$R = \sum_{t=1}^{24} R_t$$

---

8. Objective Function

The wind producer seeks to maximize **expected daily revenue**:

$$\max_{\{q_t^w, p_t^w\}_{t=1}^{24}} \mathbb{E}\left[\sum_{t=1}^{24} \delta_{w,t} \cdot q_t^w \cdot P_t^*\right]$$

Expectation is taken over all stochastic variables.

---

9. Constant Parameters

Parameter	Description
$N = 5$	Number of producers
$T = 24$	Dispatch intervals
c_i	Marginal cost of conventional producers
$\underline{p}$	Market price floor
$\overline{p}$	Market price cap
Network assumptions	No congestion, single node

10. Deep Uncertainties (Structural & Long-Term)

These are **not probabilistically well-defined** and require scenario analysis:

Market Design

- Price caps and scarcity pricing rules
- Renewable subsidies or penalties
- Carbon pricing

Competitor Behavior

- Strategic vs marginal-cost bidding
- Entry/exit of generators
- Capacity expansion or retirement

Technology & Climate

- Long-term wind patterns
- Turbine efficiency improvements
- Climate-driven demand shifts

Policy & Regulation

- Renewable mandates
- Capacity markets
- Market coupling across regions

11. Simulation Outputs

The simulation model can generate:

- Expected daily and annual revenue

- Revenue distributions (risk measures)
- Sensitivity to bidding strategies
- Stress tests under extreme scenarios

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12. Use of the Model

This framework supports:

- **Investment viability analysis**
- **Bid-strategy optimization**
- **Risk and downside exposure analysis**
- **Robust decision-making under deep uncertainty**

If you'd like, I can next:

- Convert this into a **Markov Decision Process (MDP)**
- Show **pseudo-code or Python structure**
- Extend it to **storage + wind**
- Add **risk-averse objectives (CVaR)**

You can assume that the market demand follows a normal distribution with constant mean and variance. Please refine your specification accordingly.

Below is a **refined and more explicit specification** of the simulation model incorporating the assumption that **market demand follows a Normal distribution with constant mean and variance**. I keep the structure rigorous so it can be directly implemented.

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1. Time and Scope (unchanged)

- Dispatch intervals:

$$t = 1, 2, \ldots, 24$$

- Simulation evaluates one day-ahead market and is repeated over many simulated days.

---

2. State Variables (Refined)

At each hour t, the system state is:

$$S_t = \left(D_t,\ W_t,\ S_t^{\text{solar}},\ B_{-w,t}\right)$$

Where:

- D_t: system demand (MWh)
- W_t: wind availability (MWh)
- S_t^{solar}: solar availability (MWh)
- $B_{-w,t}$: competitors' bid curves

---

3. Stochastic Variables (Refined)

3.1 Demand Uncertainty (Key Refinement)

Demand is modeled as an **hourly independent normal random variable**:

$$D_t \sim \mathcal{N}(\mu_D, \sigma_D^2)$$

with:

- μ_D: constant mean demand (MWh)
- σ_D^2: constant variance

To ensure physical feasibility:

$$D_t := \max(0, D_t)$$

(or equivalently, a truncated normal distribution).

Optional refinement (still consistent with your assumption):

- Hourly demand is i.i.d. across t
- No demand learning between hours

3.2 Renewable Availability

- Wind availability:

$$W_t \sim F_W(t)$$

(e.g., Weibull-derived or empirical distribution)

- Solar availability:

$$S_t^{\text{solar}} \sim F_S(t)$$

These may be independent or correlated with each other, but **independent of demand** under the simplified assumption.

3.3 Competitor Bids

Competitor bids are modeled as random bid stacks:

$$B_{-w,t} \sim F_B$$

Each bid consists of price–quantity pairs reflecting:

- Marginal-cost bidding for coal
- Near-zero bids for solar

---

4. Decision Variables (Wind Producer)

For each hour t, the wind producer selects:

$$x_t = (q_t^w, p_t^w)$$

Subject to:

$$0 \leq q_t^w \leq W_t$$

Decisions are made **before realizations of** D_t.

---

5. Market-Clearing Transition Function

The market operator clears the market via merit order:

$$(P_t^*, \delta_{i,t}) = \mathcal{M}\left(D_t,\ \{(q_{i,t}, p_{i,t})\}_{i=1}^5\right)$$

Where:

- P_t^*: clearing price
- $\delta_{i,t} \in \{0, 1\}$: acceptance indicator

The wind producer is dispatched if:

$$p_t^w \leq P_t^*$$

---

6. Revenue Function

Hourly revenue:

$$R_t = \delta_{w,t} \cdot q_t^w \cdot P_t^*$$

Daily revenue:

$$R = \sum_{t=1}^{24} R_t$$

---

7. Objective Function (Refined Expectation)

The wind producer maximizes **expected daily revenue**, where expectation is now explicit over the normal demand distribution:

$$\max_{\{q_t^w, p_t^w\}_{t=1}^{24}} \mathbb{E}_{D,W,S,B}\left[\sum_{t=1}^{24} \delta_{w,t}(D_t, B_{-w,t}) \cdot q_t^w \cdot P_t^*(D_t, B_{-w,t})\right]$$

with:

$$D_t \sim \mathcal{N}(\mu_D, \sigma_D^2)$$

---

8. Transition Dynamics (Simplified)

Because demand is i.i.d.:

$$\mathbb{P}(S_{t+1} \mid S_t, x_t) = \mathbb{P}(S_{t+1})$$

This makes the model a **stage-wise independent stochastic program**, not a fully dynamic MDP.

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9. Constant Parameters (Updated)

Parameter	Description
μ_D	Mean hourly demand
σ_D	Std. dev. of hourly demand
$T = 24$	Number of dispatch intervals
$N = 5$	Number of producers
$\underline{p}, \overline{p}$	Market price bounds
c_i	Marginal costs of coal units

10. Deep Uncertainty (Unchanged but Clearer)

Even with normally distributed demand, **deep uncertainties remain**:

- Whether μ_D and σ_D are stable over years
- Market-rule changes affecting clearing prices
- Strategic bidding by competitors
- Long-term climate effects on wind availability

These are handled via **scenario ensembles**, not probabilities.

11. Key Modeling Implication of Normal Demand

Because demand has **unbounded support**, rare high-demand events:

- Increase clearing prices
- Increase upside revenue for wind
- Create heavy right tails in revenue distribution

This justifies Monte Carlo simulation rather than closed-form evaluation.

If you want, I can next:

- Derive the **acceptance probability** of the wind bid as a function of (μ_D, σ_D)
- Show how to approximate expected revenue analytically
- Extend the objective to **risk-averse metrics** (e.g., CVaR)
- Provide **pseudo-code** for the simulation loop

You can also make the following assumptions regarding the behavioral logic of the producers: (1) the bid quantities of the conventional producers are stable, while their bid prices follow normal distributions with deeply uncertain means and variances; (2) the bid quantities of the solar-power producer can be modeled as $max(0, a+b\cos(\frac{2\pi t}{24}))$, and its bid prices also follow a normal distribution with deeply uncertain mean and variance; and (3) the actual wind-power production is stochastic and follows a normal distribution with constant mean and variance. Please refine your specification accordingly.

Below is a **fully refined, internally consistent simulation specification** that incorporates your new **behavioral assumptions for all producers**, while keeping the model suitable for Monte Carlo analysis and deep-uncertainty exploration.

---

1. Time Structure

- Dispatch intervals:

$$t \in \{1, 2, \ldots, 24\}$$

- Market cleared independently for each hour.
- Simulation repeated over many synthetic days.

---

2. Producers

$$\mathcal{I} = \{\text{Coal}_1, \text{Coal}_2, \text{Coal}_3, \text{Solar}, \text{Wind}\}$$

---

3. State Variables (Refined)

At each hour t, the system state is:

$$S_t = \left(D_t,\ \tilde{W}_t,\ Q_t^{\text{solar}},\ B_{-w,t}\right)$$

Where:

- D_t: market demand
- $\tilde{W}_t$: realized wind production
- Q_t^{solar}: realized solar quantity
- $B_{-w,t}$: competitors' bid prices and quantities

---

4. Stochastic Variables (Explicit)

4.1 Market Demand

$$D_t \sim \mathcal{N}(\mu_D, \sigma_D^2), \quad D_t \geq 0$$

Mean and variance are constant across hours and days.

4.2 Conventional (Coal) Producers

For each coal producer $i \in \{\text{Coal}_1, \text{Coal}_2, \text{Coal}_3\}$:

- **Bid quantity (deterministic):**

$$Q_{i,t} = \bar{Q}_i$$

- **Bid price (stochastic):**

$$P_{i,t} \sim \mathcal{N}(\mu_i, \sigma_i^2)$$

with:

- μ_i, σ_i^2 = **deeply uncertain parameters**, varied across scenarios
- Prices truncated below at marginal cost if desired

4.3 Solar Producer

- **Bid quantity (deterministic, diurnal pattern):**

$$Q_t^{\text{solar}} = \max\left(0, a + b\cos\left(\frac{2\pi t}{24}\right)\right)$$

Parameters:

- a: baseline solar output
- b: diurnal amplitude
- **Bid price (stochastic):**

$$P_t^{\text{solar}} \sim \mathcal{N}(\mu_{\text{solar}}, \sigma_{\text{solar}}^2)$$

with deeply uncertain $\mu_{\text{solar}}, \sigma_{\text{solar}}^2$.

4.4 Wind Producer

- **Actual wind production (stochastic):**

$$\tilde{W}_t \sim \mathcal{N}(\mu_W, \sigma_W^2), \quad \tilde{W}_t \geq 0$$

- **Bid quantity (decision variable):**

$$0 \leq q_t^w \leq \tilde{W}_t$$

- **Bid price (decision variable):**

$$p_t^w \in [\underline{p}, \overline{p}]$$

5. Decision Variables (Wind Producer)

For each hour t:

$$x_t = (q_t^w, p_t^w)$$

Decisions are made **ex ante**, before realizations of:

$$(D_t, P_{i,t}, P_t^{\text{solar}}, \tilde{W}_t)$$

6. Market-Clearing Mechanism

For each hour t, the market operator executes a merit-order dispatch:

$$(P_t^*, \delta_{i,t}) = \mathcal{M}\left(D_t,\ \{(Q_{i,t}, P_{i,t})\}_{i \in \mathcal{I}}\right)$$

Where:

- P_t^*: clearing price
- $\delta_{i,t} \in \{0, 1\}$: acceptance indicator

The wind producer is accepted if:

$$p_t^w \leq P_t^*$$

7. Feasibility and Curtailment

If:

$$q_t^w > \tilde{W}_t$$

then actual delivery is:

$$q_t^{\text{del}} = \min(q_t^w, \tilde{W}_t)$$

(Optionally include imbalance penalties; omitted here for simplicity.)

8. Revenue Function

Hourly revenue of the wind producer:

$$R_t = \delta_{w,t} \cdot q_t^{\text{del}} \cdot P_t^*$$

Daily revenue:

$$R = \sum_{t=1}^{24} R_t$$

9. Objective Function

The wind producer maximizes expected daily revenue:

$$\max_{\{q_t^w, p_t^w\}_{t=1}^{24}} \mathbb{E}\left[\sum_{t=1}^{24} \delta_{w,t}(D_t, P_{-w,t}) \cdot \min(q_t^w, \tilde{W}_t) \cdot P_t^*\right]$$

Expectation taken over:

$$D_t,\ P_{i,t},\ P_t^{\text{solar}},\ \tilde{W}_t$$

10. Constant Parameters (Expanded)

Parameter	Meaning
μ_D, σ_D	Demand mean and std. dev.
$\bar{Q}_i$	Coal bid quantities
a, b	Solar diurnal parameters
μ_W, σ_W	Wind production parameters
$\underline{p}, \overline{p}$	Price bounds

11. Deep Uncertainty Layer (Explicit)

The following parameters are **deeply uncertain** and explored via scenario ensembles:

Behavioral & Cost Uncertainty

- μ_i, σ_i^2 (coal bid prices)
- $\mu_{\text{solar}}, \sigma^2_{\text{solar}}$

Structural & Policy Uncertainty

- Price caps
- Renewable support mechanisms
- Entry/exit of coal units

Climate & Technology

- Long-term changes in μ_W, σ_W

- Solar intermittency patterns

---

12. Model Class

- **Mathematical type**:
  Two-stage stochastic program (per hour), repeated 24 times
- **Solution method**:
  Monte Carlo simulation with parametric scenario sampling
- **Decision use**:
  Evaluate revenue distributions and robustness of wind investment

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Next steps (optional):

- Derive **closed-form acceptance probabilities**
- Add **imbalance penalties or reserve markets**
- Introduce **risk-averse objectives (CVaR)**
- Translate this into **Python or Julia simulation code**

If you want, tell me which direction you'd like to go.

Given that the actual wind-power production is stochastic, it may differ from the quantity committed to dispatch one day in advance. If the wind-power producer under-delivers relative to its dispatched quantity, the shortfall incurs a constant penalty cost per MWh. Please also implement this penalty mechanism. Then, please provide a comprehensive model specification that specifies the key components, including state variables, decision variables, transition functions, objective functions, stochastic variables, constant parameters and deep uncertainties.

Below is a **comprehensive, self-contained simulation model specification** that incorporates **stochastic wind production with an explicit under-delivery penalty**, while fully reflecting all assumptions introduced so far.

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1. Modeling Framework

- **Type**: Stochastic simulation of a competitive day-ahead electricity spot market
- **Structure**: 24 independent hourly clearing problems per day
- **Purpose**: Evaluate expected revenue and risk of a wind-power producer under operational and deep uncertainty
- **Solution approach**: Monte Carlo simulation + scenario discovery

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2. Time Structure

- Dispatch intervals:

$$t \in \{1, 2, \dots, 24\}$$

- One simulated day consists of 24 hourly markets.
- Simulation is repeated across many synthetic days.

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3. Market Participants

$$\mathcal{I} = \{\text{Coal}_1, \text{Coal}_2, \text{Coal}_3, \text{Solar}, \text{Wind}\}$$

---

4. State Variables

The system state at hour t is:

$$S_t = \left(D_t,\ \tilde{W}_t,\ Q_t^{\text{solar}},\ P_{-w,t}\right)$$

Where:

- D_t: realized system demand (MWh)
- $\tilde{W}_t$: realized wind production (MWh)
- Q_t^{solar}: realized solar quantity (MWh)
- $P_{-w,t}$: realized bid prices of coal and solar producers

The state is **revealed after bids are submitted**.

---

5. Stochastic Variables

5.1 Demand

$$D_t \sim \mathcal{N}(\mu_D, \sigma_D^2), \qquad D_t \geq 0$$

- i.i.d. across hours
- Mean and variance constant across days

5.2 Conventional (Coal) Producers

For each coal producer i:

- **Bid quantity (deterministic):**

$$Q_{i,t} = \bar{Q}_i$$

- **Bid price (stochastic):**

$$P_{i,t} \sim \mathcal{N}(\mu_i, \sigma_i^2)$$

with:

- μ_i, σ_i^2 = **deeply uncertain parameters**

5.3 Solar Producer

- **Bid quantity (deterministic diurnal pattern):**

$$Q_t^{\text{solar}} = \max\left(0, a + b\cos\left(\frac{2\pi t}{24}\right)\right)$$

- **Bid price (stochastic):**

$$P_t^{\text{solar}} \sim \mathcal{N}(\mu_{\text{solar}}, \sigma_{\text{solar}}^2)$$

with deeply uncertain $\mu_{\text{solar}}, \sigma_{\text{solar}}^2$.

5.4 Wind Production

- **Actual production (stochastic):**

$$\tilde{W}_t \sim \mathcal{N}(\mu_W, \sigma_W^2), \qquad \tilde{W}_t \geq 0$$

Mean and variance are constant across hours and days.

---

6. Decision Variables (Wind Producer)

For each hour t, the wind producer selects **day-ahead bids**:

$$x_t = (q_t^w, p_t^w)$$

Where:

- q_t^w: committed dispatch quantity (MWh)
- p_t^w: bid price (€/MWh)

Constraints:

$$0 \leq q_t^w \leq \bar{W}$$

where $\bar{W}$ is installed wind capacity.

Decisions are made **before realizations** of all stochastic variables.

---

7. Market-Clearing Transition Function

For each hour t, the market operator clears the market via merit order:

$$(P_t^*, \delta_{i,t}) = \mathcal{M}\left(D_t,\ \{(Q_{i,t}, P_{i,t})\}_{i \in \mathcal{I}}\right)$$

Where:

- P_t^*: market-clearing price
- $\delta_{i,t} \in \{0, 1\}$: indicator of whether producer i is dispatched

The wind producer is dispatched if:

$$\delta_{w,t} = \mathbb{I}(p_t^w \leq P_t^*)$$

---

8. Delivery, Imbalance, and Penalty Mechanism

8.1 Actual Delivery

If dispatched, actual delivered energy is:

$$q_t^{\text{del}} = \delta_{w,t} \cdot \min(q_t^w, \tilde{W}_t)$$

8.2 Under-Delivery (Shortfall)

The under-delivery quantity is:

$$s_t = \delta_{w,t} \cdot \max(0, q_t^w - \tilde{W}_t)$$

8.3 Penalty Cost

A **constant imbalance penalty** applies per MWh of shortfall:

$$C_t^{\text{pen}} = \pi \cdot s_t$$

Where:

- $\pi > 0$: penalty cost per MWh (€/MWh)

9. Revenue Function (with Penalty)

9.1 Gross Revenue

$$R_t^{\text{gross}} = \delta_{w,t} \cdot q_t^w \cdot P_t^*$$

(Note: payment is based on committed quantity, as typical in day-ahead markets.)

9.2 Net Revenue

$$R_t = R_t^{\text{gross}} - C_t^{\text{pen}}$$

9.3 Daily Revenue

$$R = \sum_{t=1}^{24} R_t$$

10. Objective Function

The wind producer maximizes **expected daily net revenue**:

$$\max_{\{q_t^w, p_t^w\}_{t=1}^{24}} \mathbb{E}\left[\sum_{t=1}^{24} \left(\delta_{w,t} \cdot q_t^w \cdot P_t^* - \pi \cdot \delta_{w,t} \cdot \max(0, q_t^w - \tilde{W}_t)\right)\right]$$

Expectation is taken over:

$$D_t,\ P_{i,t},\ P_t^{\text{solar}},\ \tilde{W}_t$$

11. Transition Dynamics

Because all stochastic variables are i.i.d. across hours:

$$\mathbb{P}(S_{t+1} \mid S_t, x_t) = \mathbb{P}(S_{t+1})$$

Thus:

- No intertemporal coupling
- Model is a **stage-wise independent stochastic program**

---

12. Constant Parameters

Parameter	Description
μ_D, σ_D	Demand distribution
$\bar{Q}_i$	Coal bid quantities
a, b	Solar diurnal parameters
μ_W, σ_W	Wind production distribution
π	Under-delivery penalty
$\underline{p}, \overline{p}$	Market price bounds
$\bar{W}$	Installed wind capacity

---

13. Deep Uncertainties

The following parameters are **not probabilistically known** and are explored via scenario ensembles:

Market Behavior

- μ_i, σ_i^2: coal bid prices
- $\mu_{\text{solar}}, \sigma_{\text{solar}}^2$: solar bid prices

Regulation & Market Design

- Penalty magnitude π
- Price caps and imbalance settlement rules

Technology & Climate

- Long-term shifts in μ_W, σ_W
- Structural changes in demand (μ_D, σ_D)

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14. Key Modeling Implications

- Higher q_t^w increases expected revenue **and** penalty exposure
- Penalty introduces an **implicit risk-aversion mechanism**
- Optimal bids trade off:
  - Acceptance probability
  - Market price
  - Wind shortfall risk

Summary

This specification yields a **realistic, policy-relevant simulation model** that:

- Captures day-ahead market mechanics
- Represents renewable intermittency
- Explicitly prices imbalance risk
- Supports robust investment and bidding analysis under deep uncertainty

If you'd like, I can now:

- Derive the **optimal bidding logic** under penalties
- Add **risk-averse objectives (CVaR)**
- Translate this into **executable simulation code**
- Extend it to include **storage or balancing markets**

You do not need to consider deep uncertainties arising from long-term shifts in μ_D, σ_D, μ_W or σ_W. You can assume that they are constant over time. Please refine your specification.

Below is a **refined, consolidated model specification** that **removes deep uncertainty about demand and wind statistics** ($\mu_D, \sigma_D, \mu_W, \sigma_W$ are now fixed constants), while **retaining deep uncertainty only where you explicitly allow it** (producer bidding behavior, market rules). The specification is complete and internally consistent.

Comprehensive Simulation Model Specification

(Day-Ahead Electricity Market with Wind Imbalance Penalties)

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1. Modeling Framework

- **Model class**: Stochastic simulation / two-stage stochastic program
- **Purpose**: Evaluate expected revenue and risk exposure of a wind-power producer bidding into a competitive day-ahead spot market
- **Time resolution**: Hourly
- **Horizon**: One day (24 hours), repeated across simulated days
- **Solution approach**: Monte Carlo simulation with scenario sampling

---

2. Time Structure

$$t \in \{1, 2, \ldots, 24\}$$

Each hour is cleared independently by the market operator.

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3. Market Participants

$$\mathcal{I} = \{\mathrm{Coal}_1, \mathrm{Coal}_2, \mathrm{Coal}_3, \mathrm{Solar}, \mathrm{Wind}\}$$

---

4. State Variables

The realized system state at hour t is:

$$S_t = \left(D_t,\ \tilde{W}_t,\ Q_t^{\mathrm{solar}},\ P_{-w,t}\right)$$

Where:

- D_t: realized electricity demand (MWh)
- $\tilde{W}_t$: realized wind production (MWh)
- Q_t^{solar}: realized solar bid quantity (MWh)
- $P_{-w,t}$: realized bid prices of coal and solar producers

State variables are revealed **after day-ahead bids are submitted**.

---

5. Stochastic Variables

5.1 Market Demand (Fixed Distribution)

$$D_t \sim \mathcal{N}(\mu_D, \sigma_D^2), \qquad D_t \geq 0$$

- i.i.d. across hours and days
- μ_D, σ_D are **known constants**

5.2 Coal Producers

For each coal producer i:

- **Bid quantity (deterministic)**:

$$Q_{i,t} = \bar{Q}_i$$

- **Bid price (stochastic)**:

$$P_{i,t} \sim \mathcal{N}(\mu_i, \sigma_i^2)$$

with μ_i, σ_i^2 treated as **behavioral parameters**.

5.3 Solar Producer

- **Bid quantity (deterministic diurnal profile)**:

$$Q_t^{\text{solar}} = \max\left(0, a + b\cos\left(\frac{2\pi t}{24}\right)\right)$$

- **Bid price (stochastic)**:

$$P_t^{\text{solar}} \sim \mathcal{N}(\mu_{\text{solar}}, \sigma_{\text{solar}}^2)$$

5.4 Wind Production (Fixed Distribution)

- **Actual wind generation**:

$$\tilde{W}_t \sim \mathcal{N}(\mu_W, \sigma_W^2), \qquad \tilde{W}_t \geq 0$$

- i.i.d. across hours and days
- μ_W, σ_W are **known constants**

6. Decision Variables (Wind Producer)

For each hour t, the wind producer submits:

$$x_t = (q_t^w, p_t^w)$$

Where:

- q_t^w: committed dispatch quantity (MWh)
- p_t^w: bid price (€/MWh)

Constraints:

$$0 \leq q_t^w \leq \bar{W}, \qquad \underline{p} \leq p_t^w \leq \bar{p}$$

Decisions are made **day-ahead**, before uncertainty is realized.

7. Market-Clearing Transition Function

For each hour t, the market operator applies merit-order dispatch:

$$(P_t^*, \delta_{i,t}) = \mathcal{M}\left(D_t, \ \{(Q_{i,t}, P_{i,t})\}_{i \in \mathcal{I}}\right)$$

Where:

- P_t^*: market-clearing price
- $\delta_{i,t} \in \{0, 1\}$: acceptance indicator

Wind producer is dispatched if:

$$\delta_{w,t} = \mathbb{I}(p_t^w \leq P_t^*)$$

8. Delivery, Imbalance, and Penalty Mechanism

8.1 Actual Delivered Energy

$$q_t^{\text{del}} = \delta_{w,t} \cdot \min(q_t^w, \tilde{W}_t)$$

8.2 Under-Delivery (Shortfall)

$$s_t = \delta_{w,t} \cdot \max(0, q_t^w - \tilde{W}_t)$$

8.3 Penalty Cost

$$C_t^{\text{pen}} = \pi \cdot s_t$$

Where:

- π: constant penalty per MWh of shortfall (€/MWh)

---

9. Revenue Function

9.1 Gross Revenue (Day-Ahead Settlement)

$$R_t^{\text{gross}} = \delta_{w,t} \cdot q_t^w \cdot P_t^*$$

9.2 Net Revenue

$$R_t = R_t^{\text{gross}} - C_t^{\text{pen}}$$

9.3 Daily Revenue

$$R = \sum_{t=1}^{24} R_t$$

---

10. Objective Function

The wind producer maximizes expected daily net revenue:

$$\max_{\{q_t^w, p_t^w\}_{t=1}^{24}} \mathbb{E}\left[\sum_{t=1}^{24} \left(\delta_{w,t} \cdot q_t^w \cdot P_t^* - \pi \cdot \delta_{w,t} \cdot \max(0, q_t^w - \tilde{W}_t)\right)\right]$$

Expectation is taken over:

$$D_t,\ P_{i,t},\ P_t^{\text{solar}},\ \tilde{W}_t$$

---

11. Transition Dynamics

Because all stochastic processes are i.i.d.:

$$\mathbb{P}(S_{t+1} \mid S_t, x_t) = \mathbb{P}(S_{t+1})$$

- No intertemporal coupling
- Model is **stage-wise independent**

---

12. Constant Parameters (Final)

Parameter	Description
μ_D, σ_D	Demand distribution (fixed)
μ_W, σ_W	Wind production distribution (fixed)
$\bar{Q}_i$	Coal bid quantities
a, b	Solar diurnal parameters
π	Imbalance penalty
$\bar{W}$	Installed wind capacity
$\underline{p}, \overline{p}$	Price bounds

13. Deep Uncertainties (Remaining)

Deep uncertainty is **restricted to behavioral and institutional dimensions**:

Producer Bidding Behavior

- μ_i, σ_i^2: coal bid prices
- $\mu_{\text{solar}}, \sigma_{\text{solar}}^2$: solar bid prices

Market Design

- Penalty level π
- Price caps or scarcity pricing rules

These are explored through **scenario ensembles**, not probability distributions.

14. Key Implications

- Wind bidding faces a **risk–reward trade-off** due to penalties
- Over-commitment increases expected revenue but raises imbalance costs
- Model supports:
  - Expected revenue estimation
  - Downside-risk analysis
  - Investment viability assessment

If you want, I can now:

- Derive **analytical acceptance probabilities**
- Add **risk-averse objectives (e.g., CVaR)**
- Convert this into **executable simulation code**

- Extend the framework to **storage or balancing markets**

Just tell me how you'd like to proceed.

Various stakeholders approach this problem from different perspectives. Please refine the model specification from the perspectives of (1) the wind-power producer; and (2) the system regulator. These two perspectives should share a common environment, with each stakeholder independently controlling its own decision variables. In the specification, specify the common environment, including the global parameters and variables. For each perspective, also specify the corresponding model and its key components, including the decision variables, transition functions, and objective functions. If there are multiple objectives, provide multiple objective functions instead of a weighted objective function.

Below is a **refined, multi-stakeholder model specification** that separates **objectives and controls** while embedding both stakeholders in a **shared market environment**. The wind-power producer and the system regulator act **independently**, each controlling its own decision variables, but interacting through the same stochastic electricity market.

I. Common Environment (Shared by All Stakeholders)

This layer defines the **physical market, stochastic processes, and clearing rules**. It is exogenous to individual stakeholders' objectives.

I.1 Time Structure

- Dispatch intervals:

$$t \in \{1, \ldots, 24\}$$

- Each hour is cleared independently.
- Simulation repeated over many synthetic days.

I.2 Market Participants

$$\mathcal{I} = \{\mathrm{Coal}_1, \mathrm{Coal}_2, \mathrm{Coal}_3, \mathrm{Solar}, \mathrm{Wind}\}$$

I.3 Global State Variables

At hour t, the realized environment state is:

$$S_t = \left(D_t,\ \tilde{W}_t,\ Q_t^{\mathrm{solar}},\ P_{-w,t}\right)$$

Where:

- D_t: realized electricity demand (MWh)

- $\tilde{W}_t$: realized wind production (MWh)
- Q_t^{solar}: solar bid quantity (MWh)
- $P_{-w,t}$: realized bid prices of coal and solar producers

These variables are **not controlled** by either stakeholder.

I.4 Stochastic Processes (Fixed Distributions)

Demand

$$D_t \sim \mathcal{N}(\mu_D, \sigma_D^2), \quad D_t \geq 0$$

Wind production

$$\tilde{W}_t \sim \mathcal{N}(\mu_W, \sigma_W^2), \quad \tilde{W}_t \geq 0$$

Coal producers (for each i)

$$Q_{i,t} = \bar{Q}_i, \qquad P_{i,t} \sim \mathcal{N}(\mu_i, \sigma_i^2)$$

Solar producer

$$Q_t^{\text{solar}} = \max\left(0, a + b\cos\frac{2\pi t}{24}\right), \qquad P_t^{\text{solar}} \sim \mathcal{N}(\mu_{\text{solar}}, \sigma_{\text{solar}}^2)$$

I.5 Market-Clearing Transition Function

For each hour t:

$$(P_t^*, \delta_{i,t}) = \mathcal{M}\big(D_t, \{(Q_{i,t}, P_{i,t})\}_{i \in \mathcal{I}}\big)$$

Where:

- P_t^*: market-clearing price
- $\delta_{i,t} \in \{0, 1\}$: dispatch indicator

This transition function is **common and immutable**.

I.6 Global Parameters

Parameter	Description
μ_D, σ_D	Demand distribution
μ_W, σ_W	Wind production distribution
$\bar{Q}_i$	Coal quantities
a, b	Solar profile

Parameter	Description
$\underline{p}, \overline{p}$	Price bounds
$\bar{W}$	Wind capacity

II. Perspective 1: Wind-Power Producer

The wind producer is a **profit-maximizing agent** exposed to imbalance penalties.

II.1 Decision Variables

For each hour t:

$$x_t^w = (q_t^w, p_t^w)$$

Subject to:

$$0 \le q_t^w \le \bar{W}, \qquad \underline{p} \le p_t^w \le \overline{p}$$

Decisions are made **day-ahead**, before uncertainty is realized.

II.2 Producer-Specific Transition Components

Dispatch indicator

$$\delta_{w,t} = \mathbb{I}(p_t^w \le P_t^*)$$

Delivered energy

$$q_t^{\text{del}} = \delta_{w,t} \min(q_t^w, \tilde{W}_t)$$

Shortfall

$$s_t = \delta_{w,t} \max(0, q_t^w - \tilde{W}_t)$$

II.3 Revenue and Cost Functions

Gross revenue

$$R_t^{\text{gross}} = \delta_{w,t}\, q_t^w\, P_t^*$$

Imbalance penalty

$$C_t^{\text{pen}} = \pi\, s_t$$

Net revenue

$$R_t^w = R_t^{\text{gross}} - C_t^{\text{pen}}$$

II.4 Objective Function (Wind Producer)

Objective W1: Expected profit maximization

$$\max_{\{q_t^w, p_t^w\}} \mathbb{E}\left[\sum_{t=1}^{24} R_t^w\right]$$

(Optional extensions such as risk constraints can be added, but are not imposed here.)

III. Perspective 2: System Regulator

The regulator does **not bid energy**. It influences outcomes through **market design parameters** and evaluates system-level performance.

III.1 Decision Variables (Regulatory Controls)

The regulator selects parameters that define market incentives:

$$x^{\text{reg}} = (\pi, \overline{p})$$

Where:

- π: imbalance penalty per MWh
- $\overline{p}$: price cap (optional)

These are **set ex ante** and apply uniformly.

III.2 Regulator-Relevant System Metrics

Total system cost

$$C_t^{\text{sys}} = \sum_{i \in \mathcal{I}} \delta_{i,t} \, Q_{i,t} \, P_t^*$$

Wind shortfall

$$s_t$$

Renewable utilization

$$U_t^{\text{wind}} = \frac{q_t^{\text{del}}}{\tilde{W}_t}$$

OpenAI

III.3 Objective Functions (Multiple, Non-Weighted)

The regulator evaluates **multiple objectives simultaneously**:

Objective R1: Minimize expected imbalance

$$\min_{\pi,\overline{p}} \ \mathbb{E}\left[\sum_{t=1}^{24} s_t\right]$$

Objective R2: Minimize expected system cost

$$\min_{\pi,\overline{p}} \ \mathbb{E}\left[\sum_{t=1}^{24} C_t^{\text{sys}}\right]$$

Objective R3: Maximize renewable utilization

$$\max_{\pi,\overline{p}} \ \mathbb{E}\left[\sum_{t=1}^{24} U_t^{\text{wind}}\right]$$

(No scalarization is imposed; trade-offs are analyzed via Pareto frontiers.)

IV. Interaction Structure

- The **environment** maps:

$$(x_t^w, x^{\text{reg}}, S_t) \ \longrightarrow \ (P_t^*, \delta_{i,t}, R_t^w, C_t^{\text{sys}})$$

- The wind producer responds optimally to regulatory parameters.
- The regulator evaluates system outcomes induced by producer behavior.

This defines a **hierarchical but non-cooperative simulation**:

- Regulator: market designer / evaluator
- Wind producer: strategic bidder within given rules

V. Summary

Aspect	Wind Producer	Regulator
Control	q_t^w, p_t^w	$\pi, \overline{p}$
Goal	Maximize expected profit	Cost, reliability, renewables
Risk	Price + imbalance	System-level performance
Interaction	Via market clearing	Via incentive design

If you want, I can next:

- Formulate this as a **bi-level optimization problem**
- Show **policy–response maps** for π
- Implement a **multi-objective simulation experiment**
- Extend the regulator's view to **emissions or reliability metrics**

To ensure internal consistency across these models, please compose them into a single, unified model that shares a common environment and states, while embedding perspective-specific decisions and objectives. The composed model should: 1. embed each perspective's decision variables and objectives; 2. enable modular evaluation of each perspective; 3. for each perspective, treat other perspectives' decisions as exogenous inputs; and 4. incorporate all other components from every perspective.

Below is a **single, unified, internally consistent model** that **composes the wind-producer and regulator perspectives into one shared environment**, while preserving **modularity, independence of objectives, and exogeneity of other actors' decisions** for each perspective.

The model is written as a **multi-agent stochastic environment** with **perspective-specific controls and objectives** layered on top of a common state, dynamics, and market-clearing mechanism.

Unified Multi-Perspective Simulation Model

(Day-Ahead Electricity Market with Wind Imbalance Penalties)

---

1. Common Environment (Authoritative Core)

This layer is **shared**, immutable, and used identically by all perspectives.

1.1 Time Structure

$$t \in \{1, \ldots, 24\}$$

- Each hour is cleared independently.
- The day is the fundamental simulation episode.
- Episodes are repeated across Monte Carlo samples.

1.2 Agents

$$\mathcal{A} = \{\text{Wind Producer},\ \text{System Regulator}\}$$

Other generators (coal, solar) are modeled as **environmental processes**, not decision-making agents.

1.3 Global State Variables

At each hour t, the environment state is:

$$S_t = \left(D_t,\ \tilde{W}_t,\ Q_t^{\text{solar}},\ P_{-w,t}\right)$$

Where:

- D_t: realized electricity demand (MWh)
- $\tilde{W}_t$: realized wind production (MWh)
- Q_t^{solar}: solar bid quantity (MWh)
- $P_{-w,t}$: realized bid prices of coal and solar producers

These states are **not controlled by any perspective**.

1.4 Stochastic Processes (Fixed)

All stochastic variables are i.i.d. across hours and days.

Demand

$$D_t \sim \mathcal{N}(\mu_D, \sigma_D^2), \quad D_t \geq 0$$

Wind production

$$\tilde{W}_t \sim \mathcal{N}(\mu_W, \sigma_W^2), \quad \tilde{W}_t \geq 0$$

Coal producers (for each $i = 1, 2, 3$)

$$Q_{i,t} = \bar{Q}_i, \qquad P_{i,t} \sim \mathcal{N}(\mu_i, \sigma_i^2)$$

Solar producer

$$Q_t^{\text{solar}} = \max\left(0, a + b\cos\frac{2\pi t}{24}\right), \qquad P_t^{\text{solar}} \sim \mathcal{N}(\mu_{\text{solar}}, \sigma_{\text{solar}}^2)$$

1.5 Market-Clearing Transition Function

For each hour t:

$$(P_t^*, \delta_{i,t}) = \mathcal{M}\Big(D_t, \{(Q_{i,t}, P_{i,t})\}_{i \in \mathcal{I}}\Big)$$

Where:

- P_t^*: market-clearing price
- $\delta_{i,t} \in \{0, 1\}$: dispatch indicator

This transition function is **global and fixed**.

1.6 Global Parameters

$$\Theta = (\mu_D, \sigma_D, \mu_W, \sigma_W, \bar{Q}_i, a, b, \bar{W}, \underline{p})$$

2. Perspective-Specific Embedding

Each perspective is defined by:

1. Its **decision variables**
2. Its **induced state augmentations**
3. Its **objective functions**
4. **Exogenous treatment** of other perspectives' decisions

2A. Perspective 1 — Wind-Power Producer

2A.1 Decision Variables

For each hour t:

$$x_t^w = (q_t^w, p_t^w)$$

Subject to:

$$0 \le q_t^w \le \bar{W}, \qquad \underline{p} \le p_t^w \le \overline{p}$$

Exogenous inputs:

- Regulator decisions x^{reg}
- Market prices P_t^*

2A.2 Producer-Specific State Augmentation

From the common environment:

$$\delta_{w,t} = \mathbb{I}(p_t^w \le P_t^*)$$

$$s_t = \delta_{w,t} \max(0, q_t^w - \tilde{W}_t)$$

2A.3 Producer Revenue Function

$$R_t^w = \delta_{w,t}\, q_t^w\, P_t^* - \pi\, s_t$$

2A.4 Objective (Wind Producer)

W1: Expected profit maximization

$$\max_{\{q_t^w, p_t^w\}} \mathbb{E}\left[\sum_{t=1}^{24} R_t^w\right]$$

Other perspectives' decisions are treated as **fixed parameters**.

2B. Perspective 2 — System Regulator

2B.1 Decision Variables

$$x^{\text{reg}} = (\pi, \overline{p})$$

Where:

- π: imbalance penalty (€/MWh)
- $\overline{p}$: price cap

Exogenous inputs:

- Wind producer bids x_t^w

2B.2 Regulator-Relevant Metrics

Derived from the common environment:

System cost

$$C_t^{\text{sys}} = \sum_{i \in \mathcal{I}} \delta_{i,t}\, Q_{i,t}\, P_t^*$$

Wind shortfall

$$s_t$$

Wind utilization

$$U_t^{\text{wind}} = \frac{\delta_{w,t} \min(q_t^w, \tilde{W}_t)}{\tilde{W}_t}$$

2B.3 Objectives (Regulator)

The regulator evaluates **multiple non-aggregated objectives**:

R1: Minimize expected imbalance

$$\min_{\pi, \overline{p}} \; \mathbb{E}\left[\sum_{t=1}^{24} s_t\right]$$

R2: Minimize expected system cost

$$\min_{\pi, \overline{p}} \; \mathbb{E}\left[\sum_{t=1}^{24} C_t^{\text{sys}}\right]$$

R3: Maximize renewable utilization

$$\max_{\pi,\bar{p}} \mathbb{E}\left[\sum_{t=1}^{24} U_t^{\text{wind}}\right]$$

Wind-producer decisions are treated as **exogenous**.

---

3. Composition Structure

The unified model is a **tuple**:

$$\mathcal{M} = \left(\mathcal{S},\ \mathcal{X}^w,\ \mathcal{X}^{\text{reg}},\ \mathcal{T},\ \mathcal{O}^w,\ \mathcal{O}^{\text{reg}}\right)$$

Where:

- $\mathcal{S}$: shared state space
- $\mathcal{X}^w$: wind-producer decision space
- $\mathcal{X}^{\text{reg}}$: regulator decision space
- $\mathcal{T}$: common stochastic transition + market clearing
- $\mathcal{O}^w$: wind-producer objectives
- $\mathcal{O}^{\text{reg}}$: regulator objectives

---

4. Key Properties of the Composed Model

✔️ **Single shared environment and state**
✔️ **Perspective-specific decision control**
✔️ **Other perspectives treated as exogenous inputs**
✔️ **Multiple objectives preserved without scalarization**
✔️ **Modular evaluation possible for each stakeholder**

---

5. Interpretation

- The wind producer solves a **stochastic profit-maximization problem** inside a fixed market design.
- The regulator evaluates **policy–outcome mappings** across multiple system objectives.
- Together, they form a **multi-agent stochastic environment**, suitable for:
  - bilevel analysis,
  - policy stress testing,
  - robustness and trade-off exploration.

If you'd like, I can next:

- Reformulate this as a **bilevel optimization problem**

- Provide **pseudo-code for a modular simulator**
- Show how to compute **Pareto fronts for regulator objectives**
- Extend the model to **storage, reserves, or emissions**

Now, provide a modular Python implementation of the unified model, using classes to represent different perspectives. At the end of the program, define a function as the interface. This function should take as inputs a dictionary of decision variable values, a dictionary of uncertain parameter values, and a dictionary of constant parameter values; simulate the model dynamics; and return the value of each objective function. This function does not need to be executed.

Below is a **modular Python implementation** of the **unified multi-perspective model**.
It is structured so that:

- The **environment** is shared and authoritative
- Each **perspective** (wind producer, regulator) is encapsulated in its own class
- Each perspective treats the other's decisions as **exogenous inputs**
- Objectives are evaluated **modularly**
- A single **interface function** runs the simulation and returns all objectives

The code is written for clarity and internal consistency rather than computational efficiency.

```python
import numpy as np
from typing import Dict, Any


# ============================================================
# Common Environment
# ============================================================

class ElectricityMarketEnvironment:
    """
    Shared stochastic environment and market-clearing mechanism.
    """

    def __init__(self, constants: Dict[str, Any], uncertainties: Dict[str, Any]):
        self.constants = constants
        self.uncertainties = uncertainties

        self.T = constants["T"]
        self.rng = np.random.default_rng(uncertainties.get("seed", None))

    # ------------------------
    # Stochastic processes
    # ------------------------

    def sample_demand(self):
        mu, sigma = self.constants["mu_D"], self.constants["sigma_D"]
        return max(0.0, self.rng.normal(mu, sigma))

    def sample_wind(self):
        mu, sigma = self.constants["mu_W"], self.constants["sigma_W"]
        return max(0.0, self.rng.normal(mu, sigma))

    def sample_coal_prices(self):
        prices = []
        for mu, sigma in self.uncertainties["coal_price_params"]:
            prices.append(self.rng.normal(mu, sigma))
        return prices
```

```
    def sample_solar_price(self):
        mu, sigma = self.uncertainties["solar_price_params"]
        return self.rng.normal(mu, sigma)

    def solar_quantity(self, t):
        a, b = self.constants["solar_a"], self.constants["solar_b"]
        return max(0.0, a + b * np.cos(2 * np.pi * t / 24))

    # ------------------------
    # Market clearing
    # ------------------------

    def clear_market(self, demand, bids):
        """
        bids: list of dicts with keys {agent, quantity, price}
        Returns clearing_price, dispatch_dict
        """
        bids_sorted = sorted(bids, key=lambda x: x["price"])
        dispatched = {}
        remaining = demand
        clearing_price = 0.0

        for bid in bids_sorted:
            if remaining <= 0:
                dispatched[bid["agent"]] = 0.0
                continue

            accepted = min(bid["quantity"], remaining)
            dispatched[bid["agent"]] = accepted
            remaining -= accepted
            clearing_price = bid["price"]

        return clearing_price, dispatched


# ============================================================
# Wind Producer Perspective
# ============================================================

class WindProducer:
    """
    Wind producer decision logic and objective evaluation.
    """

    def __init__(self, decisions: Dict[str, Any], constants: Dict[str, Any]):
        self.q = decisions["wind_quantity"]      # list of length T
        self.p = decisions["wind_price"]         # list of length T
        self.capacity = constants["wind_capacity"]

    def bid(self, t):
        return {
            "agent": "wind",
            "quantity": min(self.q[t], self.capacity),
            "price": self.p[t],
        }

    def evaluate_objective(self, prices, dispatch, wind_realizations, penalty):
        revenue = 0.0
        for t in range(len(prices)):
            dispatched = dispatch[t].get("wind", 0.0)
            shortfall = max(0.0, dispatched - wind_realizations[t])
            revenue += dispatched * prices[t] - penalty * shortfall
        return revenue


# ============================================================
# Regulator Perspective
# ============================================================

class SystemRegulator:
    """
    Regulator policy parameters and multi-objective evaluation.
    """
```

```
    def __init__(self, decisions: Dict[str, Any]):
        self.penalty = decisions["penalty"]
        self.price_cap = decisions.get("price_cap", np.inf)

    def apply_price_cap(self, price):
        return min(price, self.price_cap)

    def evaluate_objectives(self, dispatch, prices, wind_realizations):
        imbalance = 0.0
        system_cost = 0.0
        wind_utilization = 0.0

        for t in range(len(prices)):
            for agent, qty in dispatch[t].items():
                system_cost += qty * prices[t]

            wind_dispatch = dispatch[t].get("wind", 0.0)
            shortfall = max(0.0, wind_dispatch - wind_realizations[t])
            imbalance += shortfall

            if wind_realizations[t] > 0:
                wind_utilization += min(wind_dispatch, wind_realizations[t]) /
wind_realizations[t]

        return {
            "expected_imbalance": imbalance,
            "expected_system_cost": system_cost,
            "expected_wind_utilization": wind_utilization,
        }


# ============================================================
# Unified Simulation Interface
# ============================================================

def simulate_unified_model(
    decision_vars: Dict[str, Dict[str, Any]],
    uncertain_params: Dict[str, Any],
    constant_params: Dict[str, Any],
):
    """
    Interface function.

    Inputs
    ------
    decision_vars:
        {
            "wind": {
                "wind_quantity": [...],
                "wind_price": [...]
            },
            "regulator": {
                "penalty": float,
                "price_cap": float (optional)
            }
        }

    uncertain_params:
        {
            "coal_price_params": [(mu1, sigma1), (mu2, sigma2), (mu3, sigma3)],
            "solar_price_params": (mu, sigma),
            "seed": int (optional)
        }

    constant_params:
        {
            "T": 24,
            "mu_D": float,
            "sigma_D": float,
            "mu_W": float,
            "sigma_W": float,
            "coal_quantities": [Q1, Q2, Q3],
            "solar_a": float,
            "solar_b": float,
            "wind_capacity": float
```

```
        }

    Returns
    -------
    Dictionary with objective values for each perspective.
    """

    # Initialize components
    env = ElectricityMarketEnvironment(constant_params, uncertain_params)
    wind = WindProducer(decision_vars["wind"], constant_params)
    regulator = SystemRegulator(decision_vars["regulator"])

    prices = []
    dispatches = []
    wind_realizations = []

    # Simulation loop
    for t in range(constant_params["T"]):
        demand = env.sample_demand()
        wind_real = env.sample_wind()
        wind_realizations.append(wind_real)

        coal_prices = env.sample_coal_prices()
        solar_price = env.sample_solar_price()
        solar_qty = env.solar_quantity(t)

        bids = []

        # Coal bids
        for i, (q, p) in enumerate(zip(constant_params["coal_quantities"], coal_prices)):
            bids.append({"agent": f"coal_{i}", "quantity": q, "price": p})

        # Solar bid
        bids.append({"agent": "solar", "quantity": solar_qty, "price": solar_price})

        # Wind bid
        bids.append(wind.bid(t))

        price, dispatch = env.clear_market(demand, bids)
        price = regulator.apply_price_cap(price)

        prices.append(price)
        dispatches.append(dispatch)

    # Objective evaluation
    wind_profit = wind.evaluate_objective(
        prices, dispatches, wind_realizations, regulator.penalty
    )

    regulator_objectives = regulator.evaluate_objectives(
        dispatches, prices, wind_realizations
    )

    return {
        "wind_expected_profit": wind_profit,
        **regulator_objectives,
    }
```

Notes on Design

- **Single shared environment** (**`ElectricityMarketEnvironment`**)
- **Perspective-specific classes** with independent objectives
- Each perspective treats the other's decisions as **exogenous**
- The interface function:
  - accepts decisions, uncertainties, and constants

  - simulates the shared dynamics
  - returns **all objective values separately**

This structure is directly extensible to:

- bilevel optimization,
- multi-objective policy analysis,
- scenario ensembles,
- additional agents (storage, balancing market, emissions).

If you'd like, I can now:

- add **CVaR objectives**
- convert this into a **bilevel solver skeleton**
- provide **unit-test stubs**
- or refactor it into a **reinforcement-learning environment**

Please refine your implementation so that any bid with a price less than or equal to the clearing price is accepted, even if the total market demand has already been met.

Good catch — this is an important **market-clearing consistency fix**.

Below is a **refined implementation** that ensures:

> **Any bid with a price ≤ the clearing price is accepted**,
> **even if total demand has already been met**.

This corresponds to a **uniform-price market with full acceptance at the marginal price**, rather than stopping acceptance once demand is satisfied.

Key Conceptual Change

We split market clearing into **two phases**:

1. **Price discovery**
   - Sort bids by price
   - Accumulate quantities until demand is met
   - Identify the **clearing price** P^*
2. **Dispatch determination**
   - **Accept all bids with price ≤** P^*
   - Dispatch their **full bid quantities**
   - (Over-supply is allowed; no rationing is applied)

This exactly implements your requirement.

Refined `clear_market` Implementation

🔁 Replace the existing `clear_market` method with the following:

```python
def clear_market(self, demand, bids):
    """
    Merit-order clearing with full acceptance at clearing price.

    bids: list of dicts with keys {agent, quantity, price}

    Returns
    -------
    clearing_price : float
    dispatch : dict {agent: dispatched_quantity}
    """
```

```
    # Sort bids by ascending price
    bids_sorted = sorted(bids, key=lambda x: x["price"])

    # ------------------------------------------------
    # Phase 1: Determine clearing price
    # ------------------------------------------------
    cumulative_quantity = 0.0
    clearing_price = None

    for bid in bids_sorted:
        cumulative_quantity += bid["quantity"]
        if cumulative_quantity >= demand:
            clearing_price = bid["price"]
            break

    # If demand is never met, clearing price is highest bid
    if clearing_price is None:
        clearing_price = bids_sorted[-1]["price"]

    # ------------------------------------------------
    # Phase 2: Dispatch all bids priced ≤ clearing price
    # ------------------------------------------------
    dispatch = {}

    for bid in bids_sorted:
        if bid["price"] <= clearing_price:
            dispatch[bid["agent"]] = bid["quantity"]
        else:
            dispatch[bid["agent"]] = 0.0

    return clearing_price, dispatch
```

What This Fix Achieves

✔️ All bids with $p_i \leq P^*$ are accepted
✔️ Dispatch is **independent of remaining demand**
✔️ Uniform-price settlement is respected
✔️ Wind, solar, and coal are treated consistently
✔️ Compatible with imbalance penalties and oversupply

Implications for the Wind Producer

- If the wind bid price is ≤ clearing price:
  - **Entire bid quantity is dispatched**
  - **Penalty applies** if actual production is lower
- Over-commitment now carries **real risk**, as intended
- This strengthens the realism of the bidding trade-off

Model Consistency Check

This refinement preserves:

- ✅ single shared environment

- ✅ modular stakeholder perspectives
- ✅ exogenous treatment of other decisions
- ✅ unchanged objective functions

Only the **transition function** is refined — exactly where it should be.

If you'd like, the next natural steps would be:

- adding **pro-rata rationing** as an alternative market rule,
- introducing **balancing-market buy-back** instead of penalties,
- or validating the clearing logic with **edge-case tests**.