

Experiments 3 - Market_narrative 3



In this problem, a renewable energy producer participates in a day-ahead merit-order electricity market competing against three other conventional producers. The producer submits hourly bids that specify a delivery quantity of electricity and an offer price for each hour of the next day. If a bid is accepted, the producer is obligated to deliver the specified quantity of electricity at the clearing price. Otherwise, the producer earns zero profit for that hour. However, renewable generation is deeply uncertain, as it depends on weather conditions that cannot be predicted. Meanwhile, the hourly clearing price depends on the bids of all producers and is unknown in advance. The renewable energy producer's goal is to maximize its expected profit by strategically selecting the quantity of electricity b_t and offer price p_t to bid for each hour of the next day, accounting for the uncertainty in both renewable generation and clearing prices.

Please formalize this problem description as a model. Please also specify key components, such as state variables, stochastic variables, constant parameters, uncertainties and transition functions.

Excellent — this is a well-posed **stochastic decision optimization** problem in an electricity market context. Below is a **formal model formulation** capturing all the structural elements you described.

1. Problem Overview

A renewable producer participates in a **day-ahead electricity market**, submitting hourly bids (b_t, p_t) for $t = 1, \dots, T$, where T is the number of hourly trading periods in a day (typically 24).

The profit depends on:

- whether the bid is accepted (determined by the market clearing price),
- the realized renewable generation,
- and the realized market price.

Both renewable generation and market price are **stochastic**.

The goal is to **maximize expected total profit** over all hours, subject to renewable availability and bid acceptance constraints.

2. Sets and Time Horizon

- $t \in \{1, 2, \dots, T\}$: hourly time index (e.g., 24 hours)
- $i \in \{1, 2, 3, 4\}$: producer index (1 = renewable, 2–4 = conventional competitors)

3. Decision Variables

For the renewable producer (indexed by 1):

- $b_t \geq 0$: quantity of electricity (MW or MWh) bid for delivery in hour t
- $p_t \geq 0$: offer price (\$/MWh) submitted for hour t

The vector of all hourly bids is $B = \{(b_t, p_t)\}_{t=1}^T$.

4. Stochastic Variables (Uncertainties)

- \tilde{G}_t : **available renewable generation** in hour t (MW); random variable depending on weather
- \tilde{P}_t^{clr} : **market clearing price** in hour t (\$/MWh); depends on all bids from all producers

Both are uncertain at the time of bidding.

Optional: If prices and generation are temporally correlated, we may represent the joint process as

$$\tilde{\omega}_t = (\tilde{G}_t, \tilde{P}_t^{\text{clr}})$$

following some stochastic process with transition law $\Pr(\tilde{\omega}_{t+1} \mid \tilde{\omega}_t)$.

5. Constant Parameters

- c^{pen} : penalty cost (\$/MWh) for shortfall (when committed delivery exceeds actual generation)
- T : number of hourly periods (e.g., 24)
- \mathcal{P}_t : joint distribution of uncertainties at time t
- p_t^{\min}, p_t^{\max} : admissible bidding price bounds
- b_t^{\max} : maximum feasible bid (technical limit or expected upper bound on renewable generation)

6. Market Acceptance Logic

Bid acceptance indicator:

$$\delta_t = \begin{cases} 1, & \text{if } p_t \leq \tilde{P}_t^{\text{clr}}, \\ 0, & \text{otherwise.} \end{cases}$$

If accepted ($\delta_t = 1$), the producer must deliver b_t at the clearing price.

7. Realized Profit Function

For a given realization of uncertainties:

$$\Pi_t(b_t, p_t; \tilde{G}_t, \tilde{P}_t^{\text{clr}}) = \begin{cases} \tilde{P}_t^{\text{clr}} \cdot b_t - c^{\text{pen}} \cdot \max(0, b_t - \tilde{G}_t), & \text{if } p_t \leq \tilde{P}_t^{\text{clr}}, \\ 0, & \text{otherwise.} \end{cases}$$

The first term is revenue, the second is a penalty for underproduction relative to the committed bid.

8. Objective Function

Maximize expected total profit:

$$\max_{\{b_t, p_t\}_{t=1}^T} \mathbb{E} \left[\sum_{t=1}^T \Pi_t(b_t, p_t; \tilde{G}_t, \tilde{P}_t^{\text{clr}}) \right]$$

subject to constraints below.

9. Constraints

$$\begin{aligned} 0 &\leq b_t \leq b_t^{\max}, \quad \forall t \\ p_t^{\min} &\leq p_t \leq p_t^{\max}, \quad \forall t \end{aligned}$$

(optional temporal constraints) : ramping or correlation limits on bids, if applicable.

10. State Variables and Transition Functions

If the producer models intertemporal dependence or learning:

- **State variable:**
 $x_t = (\text{forecast of renewable generation, forecast of price, } t)$
- **Transition function:**
 $x_{t+1} = f(x_t, \tilde{\omega}_t)$ — updates forecasts or posterior distributions after observing partial outcomes.

In a static day-ahead setting, x_t may be deterministic (no feedback between hours). In a **dynamic stochastic optimization** (e.g., sequential bidding), the transition captures updated forecasts or correlations across hours.

11. Compact Model Formulation

$$\begin{aligned} \max_{b_t, p_t} \quad & \mathbb{E}_{\tilde{G}, \tilde{P}^{\text{clr}}} \left[\sum_{t=1}^T \left(1_{\{p_t \leq \tilde{P}_t^{\text{clr}}\}} \cdot \left(\tilde{P}_t^{\text{clr}} b_t - c^{\text{pen}} \max(0, b_t - \tilde{G}_t) \right) \right) \right] \\ \text{s.t.} \quad & 0 \leq b_t \leq b_t^{\max}, \quad p_t^{\min} \leq p_t \leq p_t^{\max}, \quad \forall t. \end{aligned}$$

12. Notes on Extensions

- If forecasts are updated over time: → **stochastic dynamic programming** formulation
- If the joint distribution is known: → **stochastic programming** with scenarios
- If the market price depends on all producers' bids: → **stochastic game** formulation

- If the producer is risk-averse: replace expectation with **CVaR** or utility-based objective.

✓ Summary of Key Model Components

Component	Symbol / Example	Description
Decision Variables	b_t, p_t	Bid quantity and price
State Variables	x_t	Forecasts or market state
Stochastic Variables	$\tilde{G}_t, \tilde{P}_t^{\text{clr}}$	Generation and market price
Constants	$c^{\text{pen}}, b_t^{\text{max}}, p_t^{\text{min}}, p_t^{\text{max}}$	Technical and economic parameters
Transition Function	$x_{t+1} = f(x_t, \tilde{\omega}_t)$	Update of state or forecasts
Objective	$\max \mathbb{E}[\sum_t \Pi_t]$	Maximize expected profit
Uncertainties	Weather and market bids	Affect generation and clearing price

Would you like me to express this as a **stochastic programming formulation with discrete scenarios** (e.g., a two-stage or multi-stage model)? That would make it directly solvable with optimization software like Pyomo or GAMS.



First, the day-ahead market is cleared once per day for 24 hourly intervals of the following day, so you can assume the problem only considers a 24-hour period. Second, penalties would apply if the renewable producer under-delivers or over-delivers relative to its bid. Please refine the specification.

Excellent clarification — those two adjustments substantially refine the model:

- 1 The optimization covers **exactly 24 hours** (a single day-ahead market session).
- 2 Deviations — both **shortfalls** (under-delivery) and **surpluses** (over-delivery) — are penalized.

Below is a **fully refined formal model** reflecting those points.

⚙️ 1. Problem Context

A renewable energy producer participates in a **day-ahead merit-order electricity market**, submitting for each hour $t = 1, \dots, 24$ a bid consisting of:

- a **quantity** b_t [MWh], and
- an **offer price** p_t [\$/MWh].

All bids are submitted **simultaneously** for the next day's 24 hourly intervals.

The market operator clears the market once, determining hourly **clearing prices** \tilde{P}_t^{clr} based on all producers' bids.

If a bid is **accepted** (because $p_t \leq \tilde{P}_t^{\text{clr}}$), the renewable producer must deliver b_t MWh in hour t . The actual renewable generation \tilde{G}_t is uncertain. Deviations from b_t are penalized.

The goal: **maximize expected profit over the 24-hour period**, accounting for both price and generation uncertainty.

⚙️ 2. Time and Index Sets

$$t \in \mathcal{T} = \{1, 2, \dots, 24\}$$

⚙️ 3. Decision Variables

For each hour $t \in \mathcal{T}$:

$$\begin{aligned} b_t &\geq 0 && \text{: bid quantity (MWh)} \\ p_t &\geq 0 && \text{: offer price (\$/MWh)} \end{aligned}$$

All bids $B = \{(b_t, p_t)\}_{t=1}^{24}$ are chosen **before** any uncertainties are realized.

⚙️ 4. Stochastic Variables (Uncertainties)

- \tilde{G}_t : renewable generation available (MWh), random
- \tilde{P}_t^{clr} : market clearing price (\\$/MWh), random

Define the joint random vector

$$\tilde{\omega} = \{(\tilde{G}_t, \tilde{P}_t^{\text{clr}})\}_{t=1}^{24}$$

with known or estimated distribution $\mathbb{P}(\tilde{\omega})$.

⚙️ 5. Constant Parameters

Symbol	Description	Units
c^{under}	penalty cost for under-delivery (\\$/MWh short)	\\$/MWh
c^{over}	penalty cost for over-delivery (\\$/MWh excess)	\\$/MWh
p_t^{\min}, p_t^{\max}	bounds on offer price	\\$/MWh
b_t^{\max}	upper bound on bid quantity	MWh

⚙️ 6. Market Acceptance Logic

Bid acceptance indicator (random):

$$\delta_t = \begin{cases} 1, & \text{if } p_t \leq \tilde{P}_t^{\text{clr}}, \\ 0, & \text{otherwise.} \end{cases}$$

⚙️ 7. Realized Hourly Profit

If accepted ($\delta_t = 1$), the producer earns revenue at the clearing price but pays penalties for deviations:

$$\Pi_t(b_t, p_t; \tilde{G}_t, \tilde{P}_t^{\text{clr}}) = \delta_t \left[\tilde{P}_t^{\text{clr}} b_t - c^{\text{under}} \max(0, b_t - \tilde{G}_t) - c^{\text{over}} \max(0, \tilde{G}_t - b_t) \right]$$

If not accepted ($\delta_t = 0$), then $\Pi_t = 0$.

⚙️ 8. Objective Function

Maximize **expected total profit** over the day:

$$\max_{\{b_t, p_t\}_{t=1}^{24}} \mathbb{E}_{\tilde{\omega}} \left[\sum_{t=1}^{24} \Pi_t(b_t, p_t; \tilde{G}_t, \tilde{P}_t^{\text{clr}}) \right]$$

⚙️ 9. Constraints

$$\begin{aligned} 0 &\leq b_t \leq b_t^{\text{max}}, & \forall t \in \mathcal{T} \\ p_t^{\text{min}} &\leq p_t \leq p_t^{\text{max}}, & \forall t \in \mathcal{T} \end{aligned}$$

Optionally, if physical or operational limits couple hours (e.g., ramping, storage, or curtailment limits), those constraints can be added:

$$b_{t+1} - b_t \leq R^{\text{max}}, \quad \forall t$$

but this is not required in the simplest setting.

⚙️ 10. Compact Model Summary

$$\begin{aligned} \max_{\{b_t, p_t\}_{t=1}^{24}} \mathbb{E} &\left[\sum_{t=1}^{24} 1_{\{p_t \leq \tilde{P}_t^{\text{clr}}\}} \left(\tilde{P}_t^{\text{clr}} b_t - c^{\text{under}} \max(0, b_t - \tilde{G}_t) - c^{\text{over}} \max(0, \tilde{G}_t - b_t) \right) \right] \\ \text{s.t.} \quad &0 \leq b_t \leq b_t^{\text{max}}, \quad p_t^{\text{min}} \leq p_t \leq p_t^{\text{max}}, \quad \forall t \in \mathcal{T}. \end{aligned}$$

⚙️ 11. Key Model Components (Summary Table)

Category	Symbol	Description
Decision variables	b_t, p_t	Bid quantity and offer price per hour
Stochastic variables	$\tilde{G}_t, \tilde{P}_t^{\text{clr}}$	Random renewable generation and market clearing price
State variables	x_t (optional)	Forecasts or other market state info (if dynamic)
Constants / parameters	$c^{\text{under}}, c^{\text{over}}, b_t^{\text{max}}, p_t^{\text{min}}, p_t^{\text{max}}$	Market and technical parameters

Category	Symbol	Description
Transition function	$x_{t+1} = f(x_t, \tilde{G}_t, \tilde{P}_t^{\text{clr}})$	Forecast update rule (if dynamic version)
Objective	$\max \mathbb{E}[\sum_t \Pi_t]$	Maximize expected total profit
Uncertainties	Weather $\rightarrow \tilde{G}_t$; Bids of others $\rightarrow \tilde{P}_t^{\text{clr}}$	Affect revenue and penalties

💡 Optional: Stochastic Programming Interpretation

If we discretize the uncertainty into scenarios $s \in S$ with probabilities π_s :

$$\max_{b_t, p_t} \sum_{s \in S} \pi_s \sum_{t=1}^{24} 1_{\{p_t \leq P_{t,s}^{\text{clr}}\}} (P_{t,s}^{\text{clr}} b_t - c^{\text{under}} \max(0, b_t - G_{t,s}) - c^{\text{over}} \max(0, G_{t,s} - b_t))$$

Would you like me to **extend this into a scenario-based stochastic program** (e.g., suitable for implementation in Pyomo/GAMS), or would you prefer the **dynamic programming version** where the producer sequentially observes forecasts and updates its decisions?



There are still several areas where the model specification can be improved: (1) the renewable energy generation is deeply uncertain, so its probability distribution cannot be determined; and (2) the market-clearing price is not random but endogenous, determined by merit-order of bids. Please refine your specification.

Excellent — those two clarifications change the nature of the model **fundamentally**.

We now have:

1 Deep uncertainty in renewable generation — the probability distribution of generation is *not known*, so the producer cannot rely on expected-value optimization; instead, it must use a **robust** or **distributionally robust** approach.

2 Endogenous market clearing price — \tilde{P}_t^{clr} is *not stochastic* but is determined *endogenously* from the supply bids of all producers via the **merit-order market clearing mechanism**.

Below is a refined, internally consistent formal specification that reflects those realities.

⚙️ 1. Overview

The renewable producer participates in a **day-ahead market** for 24 hourly intervals ($t = 1, \dots, 24$). Each producer $i \in \mathcal{I} = \{1, 2, 3, 4\}$ submits for each hour t :

- an offer quantity $b_{i,t}$,
- and an offer price $p_{i,t}$.

The market operator sorts all offers by increasing price (merit order) and determines the **clearing price** and **dispatch quantities** that meet total demand.

The renewable producer (say $i = R$) must choose its bid strategy $(b_{R,t}, p_{R,t})$ before observing actual renewable generation $g_{R,t}$.

Its available generation is **deeply uncertain** — no probability distribution is known, but some feasible range or uncertainty set can be specified (e.g. derived from forecasts or weather scenarios).

⚙️ 2. Sets and Indices

$$\begin{aligned}\mathcal{T} &= \{1, 2, \dots, 24\} && \text{(hours of the day)} \\ \mathcal{I} &= \{R, 1, 2, 3\} && \text{(renewable producer and 3 conventional producers)}\end{aligned}$$

⚙️ 3. Decision Variables (for the renewable producer R)

$$\begin{aligned}b_{R,t} &\geq 0 && \text{: bid quantity (MWh)} \\ p_{R,t} &\geq 0 && \text{: offer price (\$/MWh)}\end{aligned}$$

All bids $\{(b_{R,t}, p_{R,t})\}_{t=1}^{24}$ are chosen before market clearing.

⚙️ 4. Exogenous Parameters (known before bidding)

Symbol	Meaning	Units
D_t	system demand in hour t	MWh
$(b_{i,t}, p_{i,t}), i \neq R$	bids from conventional producers	MWh, \\$/MWh
c^{under}	penalty for under-delivery	\\$/MWh
c^{over}	penalty for over-delivery	\\$/MWh
$b_{R,t}^{\text{max}}$	upper bound on bid quantity	MWh
\mathcal{G}_t	uncertainty set for actual renewable generation	subset of \mathbb{R}_+ (e.g. $[\underline{G}_t, \overline{G}_t]$)

No probability distribution is assumed for $g_{R,t} \in \mathcal{G}_t$.

⚙️ 5. Market-Clearing Mechanism (Endogenous Price Formation)

For each hour t :

1. The market operator collects all bids $\{(b_{i,t}, p_{i,t})\}_{i \in \mathcal{I}}$.
2. Orders bids by increasing $p_{i,t}$.
3. Determines **clearing price** P_t^{clr} and **dispatched quantities** $q_{i,t}$ such that:

$$\sum_{i \in \mathcal{I}} q_{i,t} = D_t$$

and

$$0 \leq q_{i,t} \leq b_{i,t}, \quad \text{for all } i.$$

The clearing price is the **marginal offer price** that meets demand:

$$P_t^{\text{clr}} = p_{i^*,t} \text{ where } i^* \text{ is the marginal accepted producer.}$$

The renewable producer is **dispatched** if its bid lies within the accepted range, i.e.

$$q_{R,t} = b_{R,t} \quad \text{if } p_{R,t} \leq P_t^{\text{clr}}, \quad \text{else } q_{R,t} = 0.$$

6. Deep Uncertainty in Renewable Generation

The actual generation $g_{R,t}$ is not random but uncertain within a **set**:

$$g_{R,t} \in \mathcal{G}_t = [\underline{G}_t, \overline{G}_t]$$

No probability distribution is assumed; the producer plans for the **worst case** (robust optimization) or a **min-max regret** criterion.

7. Penalty for Delivery Deviations

After dispatch (if accepted), the renewable producer's realized delivery deviation is penalized:

$$\text{Penalty}_t(g_{R,t}, b_{R,t}) = c^{\text{under}} \max(0, b_{R,t} - g_{R,t}) + c^{\text{over}} \max(0, g_{R,t} - b_{R,t})$$

8. Profit Expression

If the renewable bid is accepted ($p_{R,t} \leq P_t^{\text{clr}}$):

$$\Pi_t(b_{R,t}, p_{R,t}; g_{R,t}, P_t^{\text{clr}}) = P_t^{\text{clr}} b_{R,t} - \text{Penalty}_t(g_{R,t}, b_{R,t})$$

Otherwise, $\Pi_t = 0$.

9. Robust (or Distributionally Ambiguous) Objective

Because:

- $g_{R,t}$ has **no known distribution**, and
- the market-clearing price P_t^{clr} is **endogenous**,

the renewable producer faces a **bilevel robust optimization** problem:

$$\begin{aligned} \max_{\{b_{R,t}, p_{R,t}\}} \quad & \min_{g_{R,t} \in \mathcal{G}_t} \sum_{t=1}^{24} \Pi_t(b_{R,t}, p_{R,t}; g_{R,t}, P_t^{\text{clr}}(b_{R,t}, p_{R,t}, \{b_{i,t}, p_{i,t}\}_{i \neq R})) \\ \text{s.t.} \quad & 0 \leq b_{R,t} \leq b_{R,t}^{\text{max}}, \\ & p_{R,t} \geq 0, \quad \forall t. \end{aligned}$$

Here, $P_t^{\text{clr}}(\cdot)$ is *endogenous*, determined by the merit-order clearing mechanism described in Section 5.

10. Alternative Formulations

Depending on analytical needs, the model can be specified in one of three equivalent forms:

1. Bilevel robust optimization (strategic producer model)

- Upper level: renewable producer chooses $(b_{R,t}, p_{R,t})$ to maximize worst-case profit
- Lower level: market operator clears market (linear program) to determine P_t^{clr} and dispatch.

$$\begin{aligned} \max_{b_{R,t}, p_{R,t}} \min_{g_{R,t} \in \mathcal{G}_t} \sum_t [P_t^{\text{clr}} b_{R,t} - \text{Penalty}_t(g_{R,t}, b_{R,t})] \\ \text{s.t. } (P_t^{\text{clr}}, q_{i,t}) \text{ solve market clearing problem.} \end{aligned}$$

2. Single-level reformulation (using complementarity or KKT conditions for the market operator's problem)

Common in electricity market equilibrium modeling.

3. Distributionally robust extension

If partial statistical information about $g_{R,t}$ is available (e.g., bounds on mean or variance), replace

$\min_{g_{R,t}}$ with

$\min_{\mathbb{P} \in \mathcal{P}} \mathbb{E}_{\mathbb{P}}[\cdot],$

where \mathcal{P} is an ambiguity set of plausible distributions.

11. Key Components Summary

Category	Symbol	Description
Decision variables	$b_{R,t}, p_{R,t}$	Bid quantity and offer price per hour
Endogenous variables	$P_t^{\text{clr}}, q_{i,t}$	Clearing price and accepted quantities determined by market equilibrium
Uncertain parameters (deep uncertainty)	$g_{R,t} \in \mathcal{G}_t$	Renewable generation, with no known probability law
Constants	$c^{\text{under}}, c^{\text{over}}, D_t, b_{R,t}^{\text{max}}$	Market and technical parameters
State variables	none (single-day static problem)	
Objective	$\max_{b,p} \min_{g \in \mathcal{G}} \sum_t \Pi_t(\cdot)$	Robust profit maximization
Transition function	Not applicable (no sequential coupling)	
Equilibrium constraint	Merit-order market clearing	Determines P_t^{clr} and dispatch quantities

Final Compact Model

$$\begin{aligned}
 & \max_{b_{R,t}, p_{R,t}} \min_{g_{R,t} \in \mathcal{G}_t} \sum_{t=1}^{24} \left[1_{\{p_{R,t} \leq P_t^{\text{clr}}(b,p)\}} (P_t^{\text{clr}}(b,p) b_{R,t} - c^{\text{under}} \max(0, b_{R,t} - g_{R,t}) - c^{\text{over}} \max(0, g_{R,t} - b_{R,t}) \right. \\
 & \text{s.t. } (P_t^{\text{clr}}, \{q_{i,t}\}_{i \in \mathcal{I}}) \text{ satisfy market clearing for each } t, \\
 & \quad 0 \leq b_{R,t} \leq b_{R,t}^{\max}, p_{R,t} \geq 0, \quad \forall t \in \mathcal{T}.
 \end{aligned}$$

Would you like me to **explicitly write out the lower-level market clearing optimization problem** (the operator's LP) and its **KKT-based single-level reformulation** next? That would give you a complete **mathematical program with equilibrium constraints (MPEC)** version suitable for implementation.



You can assume the total energy demand D_t follows a normal distribution with constant mean and variance. You can also assume that for each conventional producer i , its bid (b_{it}, p_{it}) for each hour t follows normal distributions with constant means and variances. Please refine the model specification.

Perfect — this now becomes a **hybrid stochastic-robust market bidding model**:

- **Renewable generation** remains *deeply uncertain* (no known distribution → handled via a robust or set-based approach).
- **Market-clearing prices** are **endogenous**, determined by the **merit-order clearing** that depends on:
 - **stochastic bids** of conventional producers, and
 - **stochastic demand** D_t .

This gives a *nested uncertainty structure*: endogenous randomness (via market equilibrium) and deep uncertainty (via renewable generation).

Below is the **refined, comprehensive model specification**.

1. Problem Overview

The renewable producer (denoted R) participates in a **24-hour day-ahead electricity market** alongside three conventional producers $i \in \mathcal{I}_C = \{1, 2, 3\}$.

All participants submit **hourly bids** (b_{it}, p_{it}) for each hour $t = 1, \dots, 24$.

The market operator clears once for the next day, determining hourly clearing prices P_t^{clr} and dispatched quantities q_{it} .

The renewable producer must choose its bids **before knowing**:

- the actual renewable generation $g_{R,t}$ (deep uncertainty),
- the total demand D_t (stochastic), and
- the competing bids of conventional producers (b_{it}, p_{it}) (stochastic).

⚙️ 2. Sets and Indices

$$\begin{aligned}\mathcal{T} &= \{1, 2, \dots, 24\} && \text{(hourly intervals)} \\ \mathcal{I} &= \{R, 1, 2, 3\} && \text{(renewable + 3 conventional producers)} \\ \mathcal{I}_C &= \mathcal{I} \setminus \{R\} && \text{(set of conventional producers)}\end{aligned}$$

⚙️ 3. Decision Variables (for the renewable producer)

$$\begin{aligned}b_{R,t} &\geq 0 && \text{: bid quantity (MWh)} \\ p_{R,t} &\geq 0 && \text{: offer price (\$/MWh)}\end{aligned}$$

Bids are made simultaneously for all hours.

⚙️ 4. Uncertain Quantities

(a) Deeply uncertain renewable generation

$$g_{R,t} \in \mathcal{G}_t = [\underline{G}_t, \overline{G}_t]$$

No known probability distribution — handled with robust optimization (worst case over \mathcal{G}_t).

(b) Stochastic system demand

$$D_t \sim \mathcal{N}(\mu_D, \sigma_D^2)$$

Constant mean μ_D and variance σ_D^2 for all t .

(c) Stochastic bids from conventional producers

$$\begin{aligned}b_{it} &\sim \mathcal{N}(\mu_{b_i}, \sigma_{b_i}^2), \\ p_{it} &\sim \mathcal{N}(\mu_{p_i}, \sigma_{p_i}^2), \quad \forall i \in \mathcal{I}_C.\end{aligned}$$

Each producer's bid distribution is independent across hours and across producers.

⚙️ 5. Market Clearing (Endogenous Prices)

For each hour t :

1. The market operator collects all offers $(b_{it}, p_{it})_{i \in \mathcal{I}}$.
2. Sorts them in **ascending order of price**.
3. Determines **clearing price** P_t^{clr} and dispatch quantities q_{it} that meet demand D_t :

$$\sum_{i \in \mathcal{I}} q_{it} = D_t, \quad 0 \leq q_{it} \leq b_{it}, \quad \forall i.$$

4. The clearing price is the **marginal accepted offer price**:

$$P_t^{\text{clr}} = p_{i^*,t}, \quad \text{where } i^* \text{ is the last accepted producer.}$$

5. For the renewable producer:

$$q_{R,t} = \begin{cases} b_{R,t}, & p_{R,t} \leq P_t^{\text{clr}}, \\ 0, & p_{R,t} > P_t^{\text{clr}}. \end{cases}$$

The market-clearing price is thus an **endogenous stochastic function**:

$$P_t^{\text{clr}} = \Phi_t(D_t, \{b_{it}, p_{it}\}_{i \in \mathcal{I}})$$

6. Penalty for Delivery Deviations

If accepted, the renewable producer must deliver $b_{R,t}$.

Actual generation may differ → penalties apply for both under- and over-delivery:

$$\text{Penalty}_t(g_{R,t}, b_{R,t}) = c^{\text{under}} \max(0, b_{R,t} - g_{R,t}) + c^{\text{over}} \max(0, g_{R,t} - b_{R,t})$$

with known penalty parameters $c^{\text{under}}, c^{\text{over}} > 0$.

7. Realized Hourly Profit

$$\Pi_t(b_{R,t}, p_{R,t}; g_{R,t}, D_t, \{b_{it}, p_{it}\}) = \begin{cases} P_t^{\text{clr}}(D_t, \{b_{it}, p_{it}\}) b_{R,t} - \text{Penalty}_t(g_{R,t}, b_{R,t}), & \text{if } p_{R,t} \leq P_t^{\text{clr}}, \\ 0, & \text{otherwise.} \end{cases}$$

8. Objective Function

Because:

- $D_t, (b_{it}, p_{it})$ are stochastic (with known distributions),
- $g_{R,t}$ is uncertain but bounded within \mathcal{G}_t ,

the renewable producer aims to **maximize its expected profit** with respect to stochastic variables, while being **robust** to the worst-case renewable realization.

$$\max_{\{b_{R,t}, p_{R,t}\}} \mathbb{E}_{D_t, \{b_{it}, p_{it}\}} \left[\min_{g_{R,t} \in \mathcal{G}_t} \sum_{t=1}^{24} \Pi_t(b_{R,t}, p_{R,t}; g_{R,t}, D_t, \{b_{it}, p_{it}\}) \right]$$

9. Constraints

$$\begin{aligned} 0 &\leq b_{R,t} \leq b_{R,t}^{\max}, & \forall t, \\ p_{R,t}^{\min} &\leq p_{R,t} \leq p_{R,t}^{\max}, & \forall t. \end{aligned}$$

10. Compact Model Summary

$$\begin{aligned} \max_{\{b_{R,t}, p_{R,t}\}} \mathbb{E}_{D_t, \{b_{it}, p_{it}\}} & \left[\min_{g_{R,t} \in \mathcal{G}_t} \sum_{t=1}^{24} 1_{\{p_{R,t} \leq P_t^{\text{clr}}(D_t, \{b_{it}, p_{it}\})\}} \left(P_t^{\text{clr}}(D_t, \{b_{it}, p_{it}\}) b_{R,t} \right. \right. \\ & \left. \left. - c^{\text{under}} \max(0, b_{R,t} - g_{R,t}) - c^{\text{over}} \max(0, g_{R,t} - b_{R,t}) \right) \right] \\ \text{s.t. } & 0 \leq b_{R,t} \leq b_{R,t}^{\max}, \quad p_{R,t}^{\min} \leq p_{R,t} \leq p_{R,t}^{\max}, \quad \forall t. \end{aligned}$$

11. Interpretation

Component	Type	Description
$b_{R,t}, p_{R,t}$	Decision	Renewable bid quantity and offer price
D_t	Random (Normal)	System demand, affects market price
b_{it}, p_{it}	Random (Normal)	Bids from conventional producers, affect price formation
$g_{R,t}$	Deeply uncertain	Renewable generation, bounded but distribution-free
P_t^{clr}	Endogenous	Clearing price determined by merit-order equilibrium
$c^{\text{under}}, c^{\text{over}}$	Parameter	Penalties for deviation
Objective	Hybrid stochastic-robust	Expectation over market variables, worst case over renewable generation

12. Notes on Solution Structure

- The inner **market-clearing problem** can be formulated as a **linear program** for each hour t :

$$\begin{aligned} \min_{P_t^{\text{clr}}, q_{it}} & P_t^{\text{clr}} \\ \text{s.t. } & \sum_i q_{it} = D_t, \quad 0 \leq q_{it} \leq b_{it}, \quad q_{it} > 0 \Rightarrow p_{it} \leq P_t^{\text{clr}}. \end{aligned}$$

This ensures endogenous clearing.

- Because the renewable producer's decision affects the clearing price through merit order, the overall problem is a **stochastic-robust bilevel optimization**:
 - upper level: renewable's bid strategy,
 - lower level: stochastic market clearing.
- If desired, the expectation over normal variables can be approximated by **Monte Carlo sampling** or a **finite-scenario set**.

13. Summary Table of Model Components

Category	Symbol	Description	Type
Decision variables	$b_{R,t}, p_{R,t}$	Bid quantity and price	Continuous
Stochastic variables	D_t, b_{it}, p_{it}	Demand and conventional bids (Normal)	Random
Deep uncertainty	$g_{R,t} \in [\underline{G}_t, \overline{G}_t]$	Renewable generation	Uncertain
Endogenous variables	P_t^{clr}, q_{it}	Market clearing price and accepted quantities	Determined by equilibrium
Constants	$c^{\text{under}}, c^{\text{over}}, b_{R,t}^{\text{max}}$	Parameters	Fixed
Objective		Expected value over market stochasticity, robust to worst renewable realization	Hybrid stochastic-robust

✓ Interpretation Summary

This refined model represents:

- a **stochastic market** (uncertain demand and competitors' bids),
- an **endogenous clearing mechanism** (merit-order equilibrium),
- and **deep uncertainty** in renewable generation (robust component).

Would you like me to take this one step further and **write the bilevel structure explicitly** (upper-level renewable optimization, lower-level market clearing LP), with all variables and constraints formalized? That would yield a complete **Mathematical Program with Equilibrium Constraints (MPEC)** form ready for numerical solution.



Various stakeholders approach this problem from different perspectives. Please refine the model specification from the perspectives of: (1) the renewable producer; and (2) the system regulator. These two perspectives should share a common environment, with each stakeholder independently controlling its own control variables and indirectly influencing each other through the shared environment. In the specification, specify the common environment, including global parameters and variables. For each perspective, also specify the corresponding model and its key components, such as control variables, transition functions, and objective functions. If there are multiple objectives, provide multiple objective functions instead of a weighted objective function.

Excellent — this question now moves from a single-actor optimization problem to a **multi-agent system** formulation.

We'll construct a **shared-environment framework** where:

- both the **renewable producer (RP)** and the **system regulator (SR)** operate in the same market environment,
- each controls different variables (their **control variables**) and pursues different **objectives**,

- their actions and outcomes interact *indirectly* through the **shared environment**, which includes market clearing, price formation, and system balance.

Below is a formal, structured specification.

Common Environment (Shared System)

1. Time and Participants

$$t \in \mathcal{T} = \{1, 2, \dots, 24\} \quad (\text{hourly periods of the day})$$

Producers:

$$\mathcal{I} = \{R, 1, 2, 3\}$$

where R = renewable producer, and $i = 1, 2, 3$ are conventional producers.

2. Global Parameters (Known to Both Stakeholders)

Symbol	Description	Type
$D_t \sim \mathcal{N}(\mu_D, \sigma_D^2)$	Total system demand	Stochastic
b_{it}, p_{it} for $i \in \{1, 2, 3\}$	Conventional producers' bids	Stochastic (Normal)
$\mathcal{G}_t = [\underline{G}_t, \overline{G}_t]$	Feasible set of renewable generation	Deep uncertainty
$c^{\text{under}}, c^{\text{over}}$	Penalties for under/over-delivery	Constants
$b_{R,t}^{\text{max}}, p_{R,t}^{\text{min}}, p_{R,t}^{\text{max}}$	Renewable bid bounds	Constants
$E_i^{\text{CO}_2}$	Emission factor for producer i (tCO ₂ /MWh)	Constant
E^{cap}	Total emission cap (if applicable)	Constant

3. Global Variables (Endogenous Market Outcomes)

Symbol	Description	Determined by
$q_{i,t}$	Accepted (dispatched) quantity of producer i	Market clearing
P_t^{clr}	Market clearing price (\$/MWh)	Merit-order equilibrium
$E_t = \sum_{i \in \mathcal{I}_C} E_i^{\text{CO}_2} q_{i,t}$	Total system emissions	Function of dispatch
$\text{Balance}_t = D_t - \sum_i q_{i,t}$	Energy balance residual	Should be zero at equilibrium

System interaction:

The renewable producer's bid affects P_t^{clr} and dispatch \rightarrow affects both the producer's profit and the regulator's welfare and emissions outcomes.

(1) Renewable Producer's Model (RP)

A. Perspective

The renewable producer is a **strategic market participant** aiming to maximize its own profit while facing:

- deep uncertainty in its generation capability,
- stochastic demand and competitors' bids,
- and possible regulatory constraints (e.g., penalties, emission markets).

B. Control Variables

$b_{R,t} \geq 0$: bid quantity (MWh)

$p_{R,t} \geq 0$: offer price (\$/MWh)

C. Exogenous Inputs

- $D_t, (b_{it}, p_{it})$ for $i \in \mathcal{I}_C$: random from normal distributions.
- $g_{R,t} \in \mathcal{G}_t$: actual generation (deeply uncertain).
- Market rules and penalties set by regulator (fixed for the producer).

D. Transition Function (Information / System Coupling)

The market-clearing operator determines:

$$(P_t^{\text{clr}}, q_{i,t}) = \Phi_t(D_t, \{b_{it}, p_{it}\}_{i \in \mathcal{I}})$$

This function represents the **shared environment** — mapping all participants' bids and demand to prices and dispatch.

If accepted ($p_{R,t} \leq P_t^{\text{clr}}$), the renewable producer's commitment is $q_{R,t} = b_{R,t}$.

E. Objective Function(s)

(1) Profit Maximization (economic objective)

$$J_{\text{RP}}^{(1)} = \mathbb{E}_{D_t, \{b_{it}, p_{it}\}} \left[\min_{g_{R,t} \in \mathcal{G}_t} \sum_{t=1}^{24} 1_{\{p_{R,t} \leq P_t^{\text{clr}}\}} (P_t^{\text{clr}} b_{R,t} - c^{\text{under}} \max(0, b_{R,t} - g_{R,t}) - c^{\text{over}} \max(0, g_{R,t} - b_{R,t})) \right]$$

(2) Reliability Objective (optional technical goal)

Minimize expected magnitude of delivery deviation:

$$J_{\text{RP}}^{(2)} = \mathbb{E} \left[\sum_t |b_{R,t} - g_{R,t}| \right]$$

This objective can reflect a desire for contractual reliability or reputation.

F. Constraints

$$\begin{aligned}
 0 &\leq b_{R,t} \leq b_{R,t}^{\max}, \\
 p_{R,t}^{\min} &\leq p_{R,t} \leq p_{R,t}^{\max}, \\
 \text{Market clearing: } (P_t^{\text{clr}}, q_{i,t}) &= \Phi_t(D_t, \{b_{it}, p_{it}\}).
 \end{aligned}$$

G. Summary of RP Components

Type	Symbol	Description
Control variables	$b_{R,t}, p_{R,t}$	Renewable bid decision
State / exogenous	$g_{R,t}, D_t, (b_{it}, p_{it})$	Uncertain quantities
Transition	$\Phi_t(\cdot)$	Market clearing map (shared environment)
Objective 1	$J_{\text{RP}}^{(1)}$	Maximize expected profit (robust to generation)
Objective 2	$J_{\text{RP}}^{(2)}$	Minimize delivery deviation (optional)
Coupling	via $P_t^{\text{clr}}, q_{i,t}$	Interacts with regulator and other producers

■ (2) System Regulator's Model (SR)

A. Perspective

The system regulator (market operator / energy authority) designs or oversees mechanisms to ensure:

- efficient, reliable, and environmentally sustainable operation;
- fair pricing and supply-demand balance;
- potentially, emission limits and renewable integration goals.

The regulator **does not directly control** producer bids, but can influence:

- penalty parameters $c^{\text{under}}, c^{\text{over}}$,
- emission cap E^{cap} ,
- or market-clearing rules $\Phi_t(\cdot)$.

B. Control Variables

$$\begin{aligned}
 c^{\text{under}}, c^{\text{over}} &\geq 0 && : \text{penalty rates (policy levers)} \\
 E^{\text{cap}} &\geq 0 && : \text{emission cap (policy constraint)} \\
 \rho^{\text{CO}_2} &\geq 0 && : \text{emission permit price (if applicable)}
 \end{aligned}$$

Optionally, SR may also adjust renewable incentive or subsidy rates s_t per MWh.

C. Exogenous Inputs

- $D_t, (b_{it}, p_{it})$: random variables (Normal)
- $g_{R,t}$: deep uncertainty
- Producers' strategic bids, including $b_{R,t}, p_{R,t}$: treated as external from SR's perspective

D. Transition / System Dynamics

At each hour t :

$$\begin{aligned} (P_t^{\text{clr}}, q_{i,t}) &= \Phi_t(D_t, \{b_{it}, p_{it}\}), \\ E_t &= \sum_{i \in \mathcal{I}_C} E_i^{\text{CO}_2} q_{i,t}, \\ \text{Balance}_t &= D_t - \sum_i q_{i,t}. \end{aligned}$$

These shared variables depend on all bids and on the policy parameters set by SR.

E. Objective Function(s)

The regulator typically faces **multiple distinct objectives**, not aggregated into a single weighted utility.

(1) Economic Efficiency

Minimize expected total generation cost (proxied by accepted offers):

$$J_{\text{SR}}^{(1)} = \mathbb{E}_{D_t, \{b_{it}, p_{it}\}} \left[\sum_{t=1}^{24} \sum_{i \in \mathcal{I}} p_{it} q_{it} \right]$$

(2) Environmental Objective

Minimize expected emissions:

$$J_{\text{SR}}^{(2)} = \mathbb{E} \left[\sum_{t=1}^{24} E_t \right]$$

subject to $E_t \leq E^{\text{cap}}$ if a cap is enforced.

(3) Reliability Objective

Minimize supply-demand imbalance:

$$J_{\text{SR}}^{(3)} = \mathbb{E} \left[\sum_{t=1}^{24} |\text{Balance}_t| \right]$$

F. Constraints

$$\begin{aligned} E_t &\leq E^{\text{cap}}, \quad \forall t \text{ (if binding cap)} \\ \text{Balance}_t &= D_t - \sum_i q_{i,t} = 0 \text{ (in ideal clearing)} \\ c^{\text{under}}, c^{\text{over}}, \rho^{\text{CO2}} &\geq 0. \end{aligned}$$

G. Summary of SR Components

Type	Symbol	Description
Control variables	$c^{\text{under}}, c^{\text{over}}, E^{\text{cap}}, \rho^{\text{CO2}}$	Regulatory levers
Exogenous variables	$D_t, (b_{it}, p_{it}), g_{R,t}$	System inputs
Transition / coupling	$\Phi_t(\cdot), E_t, \text{Balance}_t$	Shared market & physical system equations
Objective 1	$J_{\text{SR}}^{(1)}$	Economic efficiency
Objective 2	$J_{\text{SR}}^{(2)}$	Environmental performance
Objective 3	$J_{\text{SR}}^{(3)}$	Reliability / balance
Coupling	via $P_t^{\text{clr}}, q_{i,t}$	Shared with producer and market

Interdependence Through the Shared Environment

Coupling Channel	From	To	Mechanism
Market price P_t^{clr}	RP's bid, conventional bids, demand	Both RP & SR	Determines profit and welfare
Dispatch quantities $q_{i,t}$	All producers' bids	Both RP & SR	Affect revenues, balance, and emissions
Penalty parameters $c^{\text{under}}, c^{\text{over}}$	SR \rightarrow RP	Change producer's risk & incentives	
Emission cap / permits	SR \rightarrow market	Influence bidding behavior and cost structure	

The result is a **coupled stochastic-robust dynamic game**:

- RP optimizes profit under deep uncertainty and policy parameters.
- SR optimizes system-level welfare under uncertainty and strategic producer behavior.
- Both interact via the **shared market-clearing environment**.

Summary of the Full Two-Perspective Specification

Layer	Element	Description
Common environment	Demand D_t , conventional bids, generation uncertainty $g_{R,t}$, market clearing Φ_t , penalties, emission factors	Shared stochastic/uncertain system dynamics
Renewable Producer (RP)	Controls $b_{R,t}, p_{R,t}$; maximizes expected profit and reliability; robust to $g_{R,t}$; interacts via prices and dispatch	Firm-level, hybrid stochastic-robust optimization

Layer	Element	Description
System Regulator (SR)	Controls penalties, emission caps, and incentive parameters; minimizes total cost, emissions, and imbalance	Policy-level, multi-objective optimization
Interaction	Coupled through equilibrium prices, penalties, and emissions	Strategic environment shared by both

Would you like me to **explicitly express the two models as coupled mathematical programs** (e.g., a *bi-level game formulation* where the regulator sets policy first and the producer responds), or keep them conceptually separate as above?



Your specification should only include implementable and quantifiable components. For those conceptual components, please refine them or remove them.

Excellent — this is the right next step.

You're asking for a **clean, implementable, and quantifiable specification** of the two interdependent models (renewable producer and system regulator) within a **shared, stochastic electricity market environment**.

We'll now eliminate any conceptual or abstract elements (like "reputation," "fairness," or "policy intent") and keep only quantities that can be *numerically evaluated or simulated*.

1. Common Environment (Shared System)

This section defines **the system within which both the renewable producer and the system regulator act**.

All random variables, parameters, and endogenous outcomes are explicitly defined and quantifiable.

1.1 Time and Participants

$$t \in \mathcal{T} = \{1, 2, \dots, 24\}$$

Producers:

$$\mathcal{I} = \{R, 1, 2, 3\}$$

where R = renewable producer, and $i = 1, 2, 3$ = conventional producers.

1.2 Global Stochastic and Deterministic Inputs

Symbol	Description	Type
$D_t \sim \mathcal{N}(\mu_D, \sigma_D^2)$	Total system electricity demand (MWh)	Random
$b_{it} \sim \mathcal{N}(\mu_{b_i}, \sigma_{b_i}^2)$	Conventional producer i 's bid quantity	Random

Symbol	Description	Type
$p_{it} \sim \mathcal{N}(\mu_{p_i}, \sigma_{p_i}^2)$	Conventional producer i 's offer price	Random
$g_{R,t} \in [\underline{G}_t, \overline{G}_t]$	Renewable producer's available generation	Deeply uncertain (bounded set)
$E_i^{\text{CO}_2}$	Emission intensity of producer i (tCO ₂ /MWh)	Constant
E^{cap}	Systemwide emission cap (tCO ₂ /hour)	Constant
$c^{\text{under}}, c^{\text{over}}$	Penalty rates for under/over-delivery (\$/MWh)	Constant
$b_{R,t}^{\text{max}}, p_{R,t}^{\text{min}}, p_{R,t}^{\text{max}}$	Bid bounds for renewable producer	Constants

1.3 Market-Clearing Mechanism (Endogenous Variables)

Given bids and demand in hour t :

$$\begin{aligned}
 &\text{Minimize total cost:} && \min_{q_{i,t}, P_t^{\text{clr}}} \sum_{i \in \mathcal{I}} p_{i,t} q_{i,t} \\
 &\text{s.t.} && \sum_{i \in \mathcal{I}} q_{i,t} = D_t, \\
 & && 0 \leq q_{i,t} \leq b_{i,t}, \quad \forall i, \\
 & && q_{i,t} > 0 \Rightarrow p_{i,t} \leq P_t^{\text{clr}}, \\
 & && q_{i,t} = 0 \Rightarrow p_{i,t} \geq P_t^{\text{clr}}.
 \end{aligned}$$

The solution yields:

- **Market clearing price** P_t^{clr}
- **Dispatched quantities** $q_{i,t}$

1.4 Derived Quantities (Shared by Both Models)

Symbol	Definition	Meaning
$q_{R,t}$	Dispatch of renewable producer (equals $b_{R,t}$ if accepted, 0 otherwise)	Accepted quantity
$E_t = \sum_{i \in \mathcal{I}_C} E_i^{\text{CO}_2} q_{i,t}$	Total CO ₂ emissions in hour t	System emission outcome

2. Renewable Producer's Model (RP)

2.1 Control Variables

$$b_{R,t} \geq 0, \quad p_{R,t} \geq 0, \quad \forall t$$

Decision variables: bid quantity and offer price for each hour.

2.2 Inputs

- Random variables: $D_t, (b_{it}, p_{it})_{i \in \mathcal{I}_C}$
- Uncertain renewable availability: $g_{R,t} \in [\underline{G}_t, \overline{G}_t]$
- Constants: $c^{\text{under}}, c^{\text{over}}$

2.3 Profit Function per Hour

If renewable bid is accepted ($p_{R,t} \leq P_t^{\text{clr}}$):

$$\Pi_{R,t}(b_{R,t}, p_{R,t}; g_{R,t}, D_t, \{b_{it}, p_{it}\}) = P_t^{\text{clr}}(D_t, \{b_{it}, p_{it}\}) b_{R,t} - c^{\text{under}} \max(0, b_{R,t} - g_{R,t}) - c^{\text{over}} \max(0, g_{R,t} - b_{R,t})$$

Otherwise, $\Pi_{R,t} = 0$.

2.4 Objective Function

Maximize expected daily profit under stochastic market conditions and worst-case renewable availability:

$$\max_{\{b_{R,t}, p_{R,t}\}} \mathbb{E}_{D_t, \{b_{it}, p_{it}\}} \left[\min_{g_{R,t} \in [\underline{G}_t, \overline{G}_t]} \sum_{t=1}^{24} \Pi_{R,t}(b_{R,t}, p_{R,t}; g_{R,t}, D_t, \{b_{it}, p_{it}\}) \right]$$

2.5 Constraints

$$\begin{aligned} 0 &\leq b_{R,t} \leq b_{R,t}^{\max}, \\ p_{R,t}^{\min} &\leq p_{R,t} \leq p_{R,t}^{\max}, \\ (P_t^{\text{clr}}, q_{i,t}) &\text{ satisfy market-clearing equations.} \end{aligned}$$

2.6 Outputs

- Optimal bids $(b_{R,t}^*, p_{R,t}^*)$
- Implied renewable dispatch $q_{R,t}^*$
- Realized profits $\Pi_{R,t}^*$

3. System Regulator's Model (SR)

3.1 Control Variables

Symbol	Description	Units
$c^{\text{under}}, c^{\text{over}}$	Penalty coefficients	\$/MWh
E^{cap}	Hourly CO ₂ emission cap	tCO ₂ /h

All are continuous and quantifiable parameters the regulator can adjust.

3.2 Inputs

- Producer bids: $(b_{R,t}, p_{R,t})$ from RP, and (b_{it}, p_{it}) from conventional producers.
- Random demand $D_t \sim \mathcal{N}(\mu_D, \sigma_D^2)$.
- Market clearing mechanism Φ_t (same as above).

3.3 Transition Function (Market Clearing)

$$(P_t^{\text{clr}}, q_{i,t}) = \Phi_t(D_t, \{b_{it}, p_{it}\})$$

identical to Section 1.3.

3.4 Quantifiable Objective Functions

The regulator has **three measurable system-level objectives** evaluated in terms of expected values:

(1) Economic Efficiency (Minimize Expected Total Cost)

$$J_{\text{SR}}^{(1)} = \mathbb{E}_{D_t, \{b_{it}, p_{it}\}} \left[\sum_{t=1}^{24} \sum_{i \in \mathcal{I}} p_{i,t} q_{i,t} \right]$$

(2) Environmental Compliance (Minimize Emissions)

$$J_{\text{SR}}^{(2)} = \mathbb{E} \left[\sum_{t=1}^{24} \max(0, E_t - E^{\text{cap}}) \right]$$

The term $\max(0, E_t - E^{\text{cap}})$ quantifies emission violations.

(3) Market Reliability (Minimize Supply-Demand Imbalance)

$$J_{\text{SR}}^{(3)} = \mathbb{E} \left[\sum_{t=1}^{24} |D_t - \sum_i q_{i,t}| \right]$$

All objectives are expressed in observable, implementable terms.

3.5 Constraints

$$E_t = \sum_{i \in \mathcal{I}_C} E_i^{\text{CO}_2} q_{i,t},$$

$$(P_t^{\text{clr}}, q_{i,t}) = \Phi_t(D_t, \{b_{it}, p_{it}\}),$$

$$c^{\text{under}}, c^{\text{over}}, E^{\text{cap}} \geq 0.$$

3.6 Outputs

- Optimal penalty rates $(c^{\text{under}*}, c^{\text{over}*})$,
- Optimal emission cap $E^{\text{cap}*}$,
- Expected system cost, emission, and imbalance statistics.

4. Interaction Between the Two Models

Interaction Variable	Determined by	Used in
$P_t^{\text{clr}}, q_{i,t}$	Market clearing mechanism Φ_t	Both RP and SR objectives
$c^{\text{under}}, c^{\text{over}}$	Regulator	Producer's penalty function
E^{cap}	Regulator	Market clearing feasibility (emission constraint)
$b_{R,t}, p_{R,t}$	Producer	Market clearing input (affects price & emissions)

The **renewable producer** and **regulator** interact indirectly via these shared variables.

5. Summary of Implementable Two-Model Specification

Role	Control Variables	Stochastic Inputs	Objective Functions	Constraints	Outputs
Renewable Producer (RP)	$b_{R,t}, p_{R,t}$	$D_t, (b_{it}, p_{it}), g_{R,t}$	Maximize expected profit (robust to $g_{R,t}$)	Bid bounds; market clearing	Optimal bids, profits
System Regulator (SR)	$c^{\text{under}}, c^{\text{over}}, E^{\text{cap}}$	$D_t, (b_{it}, p_{it}), (b_{R,t}, p_{R,t})$	(1) Min total cost; (2) Min emission violations; (3) Min imbalance	Market clearing; nonnegativity	Optimal penalty rates and caps
Shared Environment	$P_t^{\text{clr}}, q_{i,t}, E_t$	Determined by merit-order equilibrium	Links both agents		

This specification is now **fully implementable**:

- All parameters and variables are numeric and quantifiable.
- Each objective is expressed as a measurable expectation over well-defined random variables.
- All coupling is through explicit, solvable equilibrium equations.

Would you like me to now express this shared-environment system as a **computationally solvable bilevel (or multi-agent) stochastic optimization problem**—for example, suitable for GAMS, Pyomo, or AMPL implementation?



To ensure internal consistency across these models, please compose them into a single, unified model that shares a common environment and states, while embedding perspective-specific controls and objectives. The composed model should: 1. embed each perspective's control variables and objectives; 2. enables modular evaluation of each perspective; 3. for each perspective, treat other perspectives' controls as exogenous inputs; and 4. incorporate all other components from every perspective.

Excellent — you're now asking for a **unified, internally consistent, multi-perspective model** that:

- 1 lives in a single **shared environment** with consistent states and dynamics,
- 2 embeds both the **renewable producer's** and the **system regulator's** submodels (each with its own control variables and objectives),
- 3 allows each perspective to be analyzed **modularly**, treating the other's controls as exogenous, and
- 4 retains all previously defined, **quantifiable** and **implementable** components.

Below is a fully composed, implementable, and internally consistent specification.

Unified Model Specification

(Common environment + perspective-specific control and objectives)

1. Global Environment and System Structure

This environment is *shared* by all stakeholders and contains all stochastic, uncertain, and endogenous components that define how the electricity market functions.

1.1 Time and Participants

$$t \in \mathcal{T} = \{1, 2, \dots, 24\}, \quad \mathcal{I} = \{R, 1, 2, 3\}$$

- R : renewable producer
- $i = 1, 2, 3$: conventional producers

1.2 Random and Uncertain Inputs

Symbol	Description	Type
$D_t \sim \mathcal{N}(\mu_D, \sigma_D^2)$	System electricity demand (MWh)	Random
$b_{it} \sim \mathcal{N}(\mu_{b_i}, \sigma_{b_i}^2)$	Conventional producer i 's bid quantity	Random
$p_{it} \sim \mathcal{N}(\mu_{p_i}, \sigma_{p_i}^2)$	Conventional producer i 's offer price	Random
$g_{R,t} \in [\underline{G}_t, \overline{G}_t]$	Renewable producer's actual generation	Deep uncertainty (bounded)

1.3 Deterministic Parameters

Symbol	Description
$b_{R,t}^{\max}, p_{R,t}^{\min}, p_{R,t}^{\max}$	Renewable producer's bid bounds
$E_i^{\text{CO}_2}$	Emission factor for conventional producer i (tCO ₂ /MWh)
E^{cap}	System-level emission cap (tCO ₂ /hour)
$c^{\text{under}}, c^{\text{over}}$	Penalty coefficients (\$/MWh)

These parameters can be controlled by the regulator or treated as exogenous constants depending on the perspective.

1.4 Endogenous Market Variables (Shared State Variables)

Symbol	Definition	Description
$q_{i,t}$	Dispatch of producer i	Accepted generation (MWh)
P_t^{clr}	Market clearing price (\$/MWh)	Equilibrium price
$E_t = \sum_{i \in \mathcal{I}_C} E_i^{\text{CO}_2} q_{i,t}$	System emissions (tCO ₂)	Environmental outcome
$\text{Bal}_t = D_t - \sum_{i \in \mathcal{I}} q_{i,t}$	Power balance residual (MWh)	System reliability indicator

1.5 Market-Clearing Mechanism (Common Transition Function)

At each hour t , the market operator determines price and dispatch given all bids and demand:

$$\begin{aligned}
 \Phi_t(D_t, \{b_{it}, p_{it}\}_{i \in \mathcal{I}}) : \quad & \text{Minimize } \sum_{i \in \mathcal{I}} p_{i,t} q_{i,t} \\
 \text{s.t. } \quad & \sum_{i \in \mathcal{I}} q_{i,t} = D_t, \\
 & 0 \leq q_{i,t} \leq b_{i,t}, \quad \forall i, \\
 & q_{i,t} > 0 \Rightarrow p_{i,t} \leq P_t^{\text{clr}}, \\
 & q_{i,t} = 0 \Rightarrow p_{i,t} \geq P_t^{\text{clr}}.
 \end{aligned}$$

The outcome of this function is:

$$(P_t^{\text{clr}}, q_{i,t}) = \Phi_t(D_t, \{b_{it}, p_{it}\})$$

This transition function defines how all actors' decisions and stochastic factors combine to determine prices and dispatch.

It serves as the **shared state evolution mechanism** in the unified model.

2. Renewable Producer Submodel (RP)

2.1 Control Variables

$$b_{R,t} \geq 0, \quad p_{R,t} \geq 0, \quad \forall t \in \mathcal{T}$$

2.2 Inputs

- Random: $D_t, (b_{it}, p_{it})_{i \in \mathcal{I}_C}$
- Deep uncertainty: $g_{R,t} \in [\underline{G}_t, \overline{G}_t]$
- Exogenous regulatory parameters: $c^{\text{under}}, c^{\text{over}}, E^{\text{cap}}$

2.3 Transition Coupling

Market clearing maps producer bids into accepted dispatch and price:

$$(P_t^{\text{clr}}, q_{i,t}) = \Phi_t(D_t, \{b_{it}, p_{it}\})$$

Renewable dispatch:

$$q_{R,t} = \begin{cases} b_{R,t}, & \text{if } p_{R,t} \leq P_t^{\text{clr}}, \\ 0, & \text{otherwise.} \end{cases}$$

2.4 Profit Function

$$\Pi_{R,t}(b_{R,t}, p_{R,t}; g_{R,t}, D_t, \{b_{it}, p_{it}\}) = 1_{\{p_{R,t} \leq P_t^{\text{clr}}\}} [P_t^{\text{clr}} b_{R,t} - c^{\text{under}} \max(0, b_{R,t} - g_{R,t}) - c^{\text{over}} \max(0, g_{R,t} - b_{R,t})]$$

2.5 Renewable Producer Objective

$$J_{\text{RP}}(b_R, p_R; c^{\text{under}}, c^{\text{over}}, E^{\text{cap}}) = \mathbb{E}_{D_t, \{b_{it}, p_{it}\}} \left[\min_{g_{R,t} \in [\underline{G}_t, \overline{G}_t]} \sum_{t=1}^{24} \Pi_{R,t}(\cdot) \right]$$

The RP's optimization problem:

$$\begin{aligned} \max_{\{b_{R,t}, p_{R,t}\}} \quad & J_{\text{RP}}(b_R, p_R; c^{\text{under}}, c^{\text{over}}, E^{\text{cap}}) \\ \text{s.t.} \quad & 0 \leq b_{R,t} \leq b_{R,t}^{\text{max}}, \\ & p_{R,t}^{\text{min}} \leq p_{R,t} \leq p_{R,t}^{\text{max}}, \\ & (P_t^{\text{clr}}, q_{i,t}) = \Phi_t(D_t, \{b_{it}, p_{it}\}). \end{aligned}$$

3. System Regulator Submodel (SR)

3.1 Control Variables

$$c^{\text{under}} \geq 0, \quad c^{\text{over}} \geq 0, \quad E^{\text{cap}} \geq 0$$

3.2 Inputs

- Producer bids $(b_{R,t}, p_{R,t})$ and $(b_{it}, p_{it})_{i \in \mathcal{I}_C}$
- Random demand D_t
- Market clearing outcome from Φ_t

3.3 Derived Quantities

$$E_t = \sum_{i \in \mathcal{I}_C} E_i^{\text{CO}_2} q_{i,t}, \quad \text{Bal}_t = D_t - \sum_{i \in \mathcal{I}} q_{i,t}$$

3.4 Regulator Objectives

The regulator's goals are **measurable** and **quantitative**:

(1) Economic Efficiency

$$J_{\text{SR}}^{(1)}(c^{\text{under}}, c^{\text{over}}, E^{\text{cap}}) = \mathbb{E} \left[\sum_{t=1}^{24} \sum_{i \in \mathcal{I}} p_{i,t} q_{i,t} \right]$$

(2) Environmental Compliance

$$J_{\text{SR}}^{(2)}(c^{\text{under}}, c^{\text{over}}, E^{\text{cap}}) = \mathbb{E} \left[\sum_{t=1}^{24} \max(0, E_t - E^{\text{cap}}) \right]$$

(3) System Reliability

$$J_{\text{SR}}^{(3)}(c^{\text{under}}, c^{\text{over}}, E^{\text{cap}}) = \mathbb{E} \left[\sum_{t=1}^{24} |\text{Bal}_t| \right]$$

The regulator treats the producers' bids as exogenous inputs when evaluating these objectives.

3.5 Regulator Problem

$$\begin{aligned} \min_{c^{\text{under}}, c^{\text{over}}, E^{\text{cap}}} \quad & \{J_{\text{SR}}^{(1)}, J_{\text{SR}}^{(2)}, J_{\text{SR}}^{(3)}\} \\ \text{s.t.} \quad & E_t = \sum_{i \in \mathcal{I}_C} E_i^{\text{CO}_2} q_{i,t}, \\ & (P_t^{\text{clr}}, q_{i,t}) = \Phi_t(D_t, \{b_{it}, p_{it}\}). \end{aligned}$$

This problem may be analyzed as a multi-objective optimization or in a scenario-based simulation.

4. Unified Model Composition

The two perspectives are **modularly embedded** in a shared framework:

$$\mathcal{M} = (\mathcal{E}, \text{RP}, \text{SR})$$

where:

- **Environment** \mathcal{E} : common stochastic and uncertain processes, parameters, and market-clearing function Φ_t .

- **Renewable Producer submodel (RP):** controls $(b_{R,t}, p_{R,t})$, optimizes J_{RP} .
- **System Regulator submodel (SR):** controls $(c^{\text{under}}, c^{\text{over}}, E^{\text{cap}})$, optimizes $\{J_{SR}^{(1)}, J_{SR}^{(2)}, J_{SR}^{(3)}\}$.

4.1 Unified State and Transition Equations

$$\begin{aligned}(P_t^{\text{clr}}, q_{i,t}) &= \Phi_t(D_t, \{b_{it}, p_{it}\}), \\ E_t &= \sum_{i \in \mathcal{I}_C} E_i^{\text{CO}_2} q_{i,t}, \\ \text{Bal}_t &= D_t - \sum_i q_{i,t}.\end{aligned}$$

These jointly define the system's state evolution and link both perspectives.

4.2 Modular Evaluation Property

- **RP evaluation:** Fix $(c^{\text{under}}, c^{\text{over}}, E^{\text{cap}})$, optimize over $(b_{R,t}, p_{R,t})$.
- **SR evaluation:** Fix $(b_{R,t}, p_{R,t})$, optimize over $(c^{\text{under}}, c^{\text{over}}, E^{\text{cap}})$.
- Both use the **same environment and state equations**, ensuring internal consistency and comparable outcomes.

✓ 5. Summary of Unified, Implementable Multi-Perspective Model

Component	Description	Shared / Specific
Random variables	$D_t, (b_{it}, p_{it})_{i \in \mathcal{I}_C}$	Shared
Deep uncertainty	$g_{R,t} \in [\underline{G}_t, \overline{G}_t]$	Shared
Transition function	Market clearing $\Phi_t(D_t, \{b_{it}, p_{it}\})$	Shared
State variables	$P_t^{\text{clr}}, q_{i,t}, E_t, \text{Bal}_t$	Shared
RP controls	$b_{R,t}, p_{R,t}$	Renewable producer specific
SR controls	$c^{\text{under}}, c^{\text{over}}, E^{\text{cap}}$	Regulator specific
RP objective	$J_{RP} = \mathbb{E}[\min_{g_R} \sum_t \Pi_{R,t}]$	Producer
SR objectives	$J_{SR}^{(1)}, J_{SR}^{(2)}, J_{SR}^{(3)}$	Regulator
Coupling variables	$P_t^{\text{clr}}, q_{i,t}$	Shared environment link

* Mathematical Structure

The unified model can be viewed as a **multi-agent system** (or bilevel optimization framework):

$$\begin{aligned} \text{Environment: } & (P_t^{\text{clr}}, q_{i,t}, E_t, \text{Bal}_t) = \Phi_t(D_t, \{b_{it}, p_{it}\}), \\ \text{RP: } & \max_{b_R, p_R} J_{\text{RP}}(\cdot), \\ \text{SR: } & \min_{c^{\text{under}}, c^{\text{over}}, E^{\text{cap}}} \{J_{\text{SR}}^{(1)}, J_{\text{SR}}^{(2)}, J_{\text{SR}}^{(3)}\}. \end{aligned}$$

Both modules can be simulated or solved sequentially, each treating the other's controls as given, within the same quantifiable system state and transition structure.

Would you like me to go one step further and express this **unified model** explicitly as a **computational framework** (e.g., modular Python/Pyomo structure where each perspective is a subproblem sharing environment state variables)?



Now, provide a modular Python implementation of the unified model, using classes to represent different perspectives. At the end of the program, define a function as the interface. This function should accept a dictionary of control variables and a dictionary of uncertain parameter values as input, simulate the model dynamics, and return the value of each objective function as output. This function does not need to be executed.

Below is a **modular, implementable Python specification** of the unified model using **object-oriented structure**, reflecting the shared environment and two stakeholder perspectives.

The implementation is **fully self-contained, quantitative**, and designed for **simulation and evaluation** (not optimization).

It assumes a user will call a single interface function at the end that accepts:

- a dictionary of **control variables** (both renewable and regulator), and
- a dictionary of **uncertain/stochastic parameters**,

and returns the computed **objective values** for each perspective.

```
python

import numpy as np

# =====
# 1. Common Environment Class
# =====

class MarketEnvironment:
    """
    Shared environment representing the 24-hour day-ahead electricity market.
    Includes stochastic demand, stochastic conventional producer bids,
    deep uncertainty in renewable generation, and endogenous market clearing.
    """

    def __init__(self, params):
        """
        Initialize environment with known constants and random distributions.
        params: dict containing deterministic parameters and configuration.
        """
        self.T = 24
```



```

self.mu_D = params.get("mu_D", 1000)
self.sigma_D = params.get("sigma_D", 100)
self.mu_b = params.get("mu_b", [400, 400, 400])
self.sigma_b = params.get("sigma_b", [50, 50, 50])
self.mu_p = params.get("mu_p", [40, 50, 60])
self.sigma_p = params.get("sigma_p", [5, 5, 5])
self.G_lower = params.get("G_lower", [0]*24)
self.G_upper = params.get("G_upper", [200]*24)
self.E_CO2 = params.get("E_CO2", [0.5, 0.6, 0.7]) # tCO2/MWh for conventional producers

def sample_demand(self):
    return np.random.normal(self.mu_D, self.sigma_D, self.T)

def sample_conventional_bids(self):
    b = np.zeros((3, self.T))
    p = np.zeros((3, self.T))
    for i in range(3):
        b[i, :] = np.random.normal(self.mu_b[i], self.sigma_b[i], self.T)
        p[i, :] = np.random.normal(self.mu_p[i], self.sigma_p[i], self.T)
    return b, p

def sample_renewable_generation(self):
    """
    Renewable generation is deeply uncertain, but for simulation we can pick midpoint or
    worst case.
    Here we return midpoint of uncertainty interval.
    """
    g = np.array([(low + high)/2 for low, high in zip(self.G_lower, self.G_upper)])
    return g

def market_clearing(self, D_t, bids):
    """
    Perform simple merit-order market clearing given demand and offers.
    bids: list of tuples (b_i, p_i) including renewable.
    Returns clearing price P_clr and accepted quantities q_i.
    """
    offers = sorted(enumerate(bids), key=lambda x: x[1][1]) # sort by price
    q = np.zeros(len(bids))
    remaining = D_t
    P_clr = 0.0
    for idx, (b_i, p_i) in offers:
        accept = min(b_i, remaining)
        q[idx] = accept
        remaining -= accept
        P_clr = p_i
        if remaining <= 0:
            break
    return P_clr, q

# =====
# 2. Renewable Producer Perspective
# =====

class RenewableProducer:
    """
    Renewable producer selects bid quantity and offer price.
    Computes profit given environment and market-clearing results.
    """

    def __init__(self, env, controls):
        self.env = env
        self.b_R = np.array(controls.get("b_R", [100]*env.T))
        self.p_R = np.array(controls.get("p_R", [45]*env.T))
        # Penalty coefficients set by regulator
        self.c_under = controls.get("c_under", 10)
        self.c_over = controls.get("c_over", 5)

    def profit(self, g_R, P_clr, accepted, b_R_t):
        """
        Compute profit for one hour given clearing price and generation.
        """
        if accepted:
            penalty = self.c_under * max(0, b_R_t - g_R) + self.c_over * max(0, g_R - b_R_t)
            return P_clr * b_R_t - penalty

```



```

else:
    return 0.0

def evaluate(self, demand, conv_bids, g_R):
    """
    Evaluate expected (simulated) profit over 24 hours.
    """
    T = self.env.T
    total_profit = 0.0
    for t in range(T):
        D_t = demand[t]
        bids = [(self.b_R[t], self.p_R[t])] + [(conv_bids[0][i, t], conv_bids[1][i, t]) for
i in range(3)]
        P_clr, q = self.env.market_clearing(D_t, bids)
        accepted = self.p_R[t] <= P_clr
        profit_t = self.profit(g_R[t], P_clr, accepted, self.b_R[t])
        total_profit += profit_t
    return total_profit

# =====
# 3. System Regulator Perspective
# =====

class SystemRegulator:
    """
    Regulator controls penalty parameters and emission cap.
    Evaluates system-wide objectives: cost, emissions, and balance.
    """

    def __init__(self, env, controls):
        self.env = env
        self.c_under = controls.get("c_under", 10)
        self.c_over = controls.get("c_over", 5)
        self.E_cap = controls.get("E_cap", 300)

    def evaluate(self, demand, conv_bids, ren_bids):
        """
        Compute expected system cost, emission, and imbalance metrics.
        """
        T = self.env.T
        total_cost = 0.0
        total_violation = 0.0
        total_imbalance = 0.0

        for t in range(T):
            D_t = demand[t]
            bids = [ren_bids[t]] + [(conv_bids[0][i, t], conv_bids[1][i, t]) for i in range(3)]
            P_clr, q = self.env.market_clearing(D_t, bids)

            # Cost
            total_cost += sum([p * q_i for (_, p), q_i in zip(bids, q)])

            # Emissions (only conventional producers)
            E_t = sum([self.env.E_CO2[i] * q[i + 1] for i in range(3)])
            total_violation += max(0, E_t - self.E_cap)

            # Supply-demand balance
            imbalance = abs(D_t - sum(q))
            total_imbalance += imbalance

        return {
            "system_cost": total_cost,
            "emission_violation": total_violation,
            "imbalance": total_imbalance
        }

# =====
# 4. Unified Model Interface Function
# =====

def simulate_market(controls, uncertainties):
    """
    Unified interface function.

```



```

Accepts:
    controls: dict with keys:
        - "b_R", "p_R": renewable producer bids
        - "c_under", "c_over", "E_cap": regulator controls
    uncertainties: dict with keys:
        - "D_t", "b_conv", "p_conv", "g_R"
Returns:
    dict with values of each objective:
        - Renewable producer profit
        - Regulator: system cost, emission violation, imbalance
"""
# Initialize environment
env = MarketEnvironment({
    "mu_D": uncertainties.get("mu_D", 1000),
    "sigma_D": uncertainties.get("sigma_D", 100),
    "mu_b": uncertainties.get("mu_b", [400, 400, 400]),
    "sigma_b": uncertainties.get("sigma_b", [50, 50, 50]),
    "mu_p": uncertainties.get("mu_p", [40, 50, 60]),
    "sigma_p": uncertainties.get("sigma_p", [5, 5, 5]),
    "G_lower": uncertainties.get("G_lower", [0]*24),
    "G_upper": uncertainties.get("G_upper", [200]*24),
    "E_CO2": uncertainties.get("E_CO2", [0.5, 0.6, 0.7])
})

# Sample stochastic inputs if not given explicitly
D_t = uncertainties.get("D_t", env.sample_demand())
b_conv = uncertainties.get("b_conv", None)
p_conv = uncertainties.get("p_conv", None)
if b_conv is None or p_conv is None:
    b_conv, p_conv = env.sample_conventional_bids()
g_R = uncertainties.get("g_R", env.sample_renewable_generation())

# Instantiate perspectives
rp = RenewableProducer(env, controls)
sr = SystemRegulator(env, controls)

# Evaluate each perspective
rp_profit = rp.evaluate(D_t, (b_conv, p_conv), g_R)
sr_results = sr.evaluate(D_t, (b_conv, p_conv), list(zip(rp.b_R, rp.p_R)))

return {
    "renewable_profit": rp_profit,
    **sr_results
}

# =====
# Example Usage (not executed)
# =====

# Example control and uncertainty dictionaries:
"""
controls = {
    "b_R": [100]*24,
    "p_R": [45]*24,
    "c_under": 10,
    "c_over": 5,
    "E_cap": 300
}

uncertainties = {
    "mu_D": 1000,
    "sigma_D": 100,
    "mu_b": [400, 400, 400],
    "sigma_b": [50, 50, 50],
    "mu_p": [40, 50, 60],
    "sigma_p": [5, 5, 5],
    "G_lower": [0]*24,
    "G_upper": [200]*24
}

results = simulate_market(controls, uncertainties)
print(results)
"""

```


✓ Model Highlights

- **Shared environment** (`MarketEnvironment`) holds stochastic processes and clearing rules.
- **RenewableProducer** and **SystemRegulator** each:
 - have independent control variables,
 - share the same environment and state transitions,
 - evaluate separate, quantifiable objective functions.
- `simulate_market()` acts as the **interface**, performing:
 1. environment initialization,
 2. sampling or accepting uncertain inputs,
 3. evaluating both perspectives, and
 4. returning a dictionary of all objective values.

This modular structure makes it easy to embed in an optimization or simulation framework (e.g., Monte Carlo analysis, bilevel optimization, or reinforcement learning).



`mu_D`, `sigma_D`, `mu_b`, `sigma_b`, `mu_p`, `sigma_p` are constants instead of uncertain parameters, so the implementation should not take them as inputs. Please refine the implementation.

Excellent refinement — we'll now cleanly separate **constants** (fixed model parameters) from **uncertain variables** (realizations that change between simulations).

Here's the **refined, modular Python implementation**:

- Constants such as `mu_D`, `sigma_D`, `mu_b`, `sigma_b`, `mu_p`, `sigma_p` are **embedded inside the environment** as fixed configuration values.
- The interface accepts only **control variables** and **uncertain realizations** (e.g., realized demand, renewable generation, etc.).

python

```
import numpy as np

# =====
# 1. Common Environment Class
# =====

class MarketEnvironment:
    """
    Shared 24-hour day-ahead electricity market environment.
    Defines constants, stochastic structure, and market clearing logic.
    """

    def __init__(self):
        # Fixed constants (global market configuration)
        self.T = 24
        # Demand distribution parameters
        self.mu_D = 1000
```



```

self.sigma_D = 100
# Conventional producers: mean bid quantities and prices
self.mu_b = [400, 400, 400]
self.sigma_b = [50, 50, 50]
self.mu_p = [40, 50, 60]
self.sigma_p = [5, 5, 5]
# Renewable generation uncertainty range
self.G_lower = [0] * 24
self.G_upper = [200] * 24
# Emission factors for conventional producers (tCO2/MWh)
self.E_CO2 = [0.5, 0.6, 0.7]

# -----
# Sampling methods
# -----

def sample_demand(self):
    """Sample hourly demand from Normal( $\mu$ ,  $\sigma$ )."""
    return np.random.normal(self.mu_D, self.sigma_D, self.T)

def sample_conventional_bids(self):
    """Sample (quantity, price) for conventional producers."""
    b = np.zeros((3, self.T))
    p = np.zeros((3, self.T))
    for i in range(3):
        b[i, :] = np.random.normal(self.mu_b[i], self.sigma_b[i], self.T)
        p[i, :] = np.random.normal(self.mu_p[i], self.sigma_p[i], self.T)
    return b, p

def sample_renewable_generation(self):
    """Return midpoints of uncertainty intervals for renewable generation."""
    return np.array([(low + high) / 2 for low, high in zip(self.G_lower, self.G_upper)])

# -----
# Market clearing
# -----

def market_clearing(self, D_t, bids):
    """
    Merit-order market clearing given demand and offers.
    bids: list of tuples (b_i, p_i) including renewable first.
    Returns clearing price P_clr and accepted quantities q_i.
    """
    offers = sorted(enumerate(bids), key=lambda x: x[1][1]) # sort by price
    q = np.zeros(len(bids))
    remaining = D_t
    P_clr = 0.0
    for idx, (b_i, p_i) in offers:
        accept = min(b_i, remaining)
        q[idx] = accept
        remaining -= accept
        P_clr = p_i
        if remaining <= 0:
            break
    return P_clr, q

# =====
# 2. Renewable Producer Perspective
# =====

class RenewableProducer:
    """
    Renewable producer: selects bid quantity and offer price.
    Computes profit given market clearing and realized generation.
    """

    def __init__(self, env, controls):
        self.env = env
        self.b_R = np.array(controls.get("b_R", [100] * env.T))
        self.p_R = np.array(controls.get("p_R", [45] * env.T))
        # Penalty coefficients set by regulator
        self.c_under = controls.get("c_under", 10)
        self.c_over = controls.get("c_over", 5)

```



```
def profit(self, g_R, P_clr, accepted, b_R_t):
    """Compute profit for one hour."""
    if accepted:
        penalty = self.c_under * max(0, b_R_t - g_R) + self.c_over * max(0, g_R - b_R_t)
        return P_clr * b_R_t - penalty
    else:
        return 0.0

def evaluate(self, demand, conv_bids, g_R):
    """Evaluate expected (simulated) profit over 24 hours."""
    total_profit = 0.0
    for t in range(self.env.T):
        D_t = demand[t]
        # Combine renewable + conventional offers
        bids = [(self.b_R[t], self.p_R[t])] + [
            (conv_bids[0][i, t], conv_bids[1][i, t]) for i in range(3)
        ]
        P_clr, q = self.env.market_clearing(D_t, bids)
        accepted = self.p_R[t] <= P_clr
        profit_t = self.profit(g_R[t], P_clr, accepted, self.b_R[t])
        total_profit += profit_t
    return total_profit
```

```
# =====
# 3. System Regulator Perspective
# =====
```

```
class SystemRegulator:
    """
    Regulator: controls penalty parameters and emission cap.
    Evaluates system-wide cost, emission, and imbalance metrics.
    """

    def __init__(self, env, controls):
        self.env = env
        self.c_under = controls.get("c_under", 10)
        self.c_over = controls.get("c_over", 5)
        self.E_cap = controls.get("E_cap", 300)

    def evaluate(self, demand, conv_bids, ren_bids):
        """
        Compute system cost, emission violation, and imbalance metrics.
        """
        total_cost = 0.0
        total_violation = 0.0
        total_imbalance = 0.0

        for t in range(self.env.T):
            D_t = demand[t]
            bids = [ren_bids[t]] + [
                (conv_bids[0][i, t], conv_bids[1][i, t]) for i in range(3)
            ]
            P_clr, q = self.env.market_clearing(D_t, bids)

            # System cost (accepted offers)
            total_cost += sum([p * q_i for (_, p), q_i in zip(bids, q)])

            # Emissions (only conventional producers)
            E_t = sum([self.env.E_CO2[i] * q[i + 1] for i in range(3)])
            total_violation += max(0, E_t - self.E_cap)

            # Imbalance (supply-demand mismatch)
            total_imbalance += abs(D_t - sum(q))

        return {
            "system_cost": total_cost,
            "emission_violation": total_violation,
            "imbalance": total_imbalance,
        }
```

```
# =====
# 4. Unified Model Interface Function
# =====
```



```
def simulate_market(controls, uncertainties):
    """
    Unified interface to simulate the shared market environment.
    Inputs:
        controls: dict containing
            - "b_R", "p_R" : renewable bids
            - "c_under", "c_over", "E_cap" : regulator controls
        uncertainties: dict containing
            - "D_t" : realized demand array (optional)
            - "b_conv", "p_conv" : realized conventional bids (optional)
            - "g_R" : realized renewable generation array (optional)
    Returns:
        dict with:
            - 'renewable_profit'
            - 'system_cost'
            - 'emission_violation'
            - 'imbalance'
    """
    # Initialize environment (constants embedded)
    env = MarketEnvironment()

    # Sample or use provided uncertain realizations
    D_t = uncertainties.get("D_t", env.sample_demand())
    b_conv = uncertainties.get("b_conv", None)
    p_conv = uncertainties.get("p_conv", None)
    if b_conv is None or p_conv is None:
        b_conv, p_conv = env.sample_conventional_bids()
    g_R = uncertainties.get("g_R", env.sample_renewable_generation())

    # Instantiate both perspectives
    rp = RenewableProducer(env, controls)
    sr = SystemRegulator(env, controls)

    # Evaluate each perspective
    rp_profit = rp.evaluate(D_t, (b_conv, p_conv), g_R)
    sr_results = sr.evaluate(D_t, (b_conv, p_conv), list(zip(rp.b_R, rp.p_R)))

    # Combine results
    return {
        "renewable_profit": rp_profit,
        **sr_results
    }

# =====
# Example (not executed)
# =====

"""
controls = {
    "b_R": [100]*24,
    "p_R": [45]*24,
    "c_under": 10,
    "c_over": 5,
    "E_cap": 300
}

uncertainties = {
    # Optionally provide specific realizations
    # "D_t": np.ones(24)*1000,
    # "b_conv": ...,
    # "p_conv": ...,
    # "g_R": ...
}

results = simulate_market(controls, uncertainties)
print(results)
"""
```

✓ Key Refinements and Features

Component	Description
Constants (μ_D , σ_D , μ_b , etc.)	Embedded in MarketEnvironment , not user inputs.
Uncertainties	Only realized variables (D_t , b_{conv} , p_{conv} , g_R) are optional inputs.
Control dictionaries	Contain renewable and regulator controls, fully modular.
Market dynamics	Merit-order clearing ensures shared, endogenous prices and dispatch.
Output	Returns numerical values of all objectives: profit, cost, emissions, imbalance.

This design allows the unified model to be **simulated, optimized, or embedded** in higher-level analyses (e.g., bilevel optimization, Monte Carlo sampling) while maintaining a **clear separation of constants, controls, and uncertainties**.