Market_brief_2

An energy company is interested in investing in wind-power production. To assess the viability of this investment, the company wants to evaluate the expected revenue of a wind-power producer in a competitive spot market.

Suppose there are five energy producers participating in the day-ahead spot market. Three of them use conventional sources for energy production, such as coal, while the other two producers use solar and wind power, respectively. The market is cleared once per day for 24 hourly intervals of the following day. For each dispatch interval, every producer submits a bid specifying the energy quantity (MWh) it is willing to supply and the corresponding minimum acceptable price (per MWh). Then, the market operator follows a merit-order process to determine the market-clearing price and dispatch schedule, ensuring the total market demand is met. If a bid is accepted, the producer is committed to delivering the bid quantity at the market-clearing price. Otherwise, the producer earns zero revenue.

The energy company aims to build a simulation model to evaluate the expected revenue of the wind-power producer on any future day by selecting the hourly bid quantity and price, while accounting for uncertainties. This model can then be used to explore outcomes across various short-term and long-term scenarios, thereby informing the viability of the investment.

Please formalize this problem description as a simulation model and specify the key components, including state variables, decision variables, transition functions, objective functions, stochastic variables, constant parameters and deep uncertainties.

Below is a **formal simulation-model formulation** of the wind-power producer's revenue problem in a competitive day-ahead electricity market. The goal is to make the structure explicit so it can later be implemented (e.g., Monte Carlo simulation, stochastic optimization, agent-based modeling, or robust decision-making).

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1. Model Scope and Structure

- **Type**: Stochastic simulation of a day-ahead spot market
- **Time resolution**: Hourly dispatch intervals
- **Horizon**: 24 hours for a single day; extended to multiple days/years for scenario analysis
- **Agents**:
  - 3 conventional producers (coal/gas)
  - 1 solar producer
  - 1 wind producer (decision maker)

The wind producer chooses **hourly bid quantities and prices** to maximize expected revenue under uncertainty, given the market-clearing mechanism.

2. Indices and Sets

Symbol	Description
$h \in \{1, \ldots, 24\}$	Hour of day
$i \in \mathcal{P} = \{1, \ldots, 5\}$	Producers
$i = w$	Wind producer
$i \in \mathcal{C} \subset \mathcal{P}$	Conventional producers
$\omega \in \Omega$	Scenario index

3. State Variables

State variables represent information available at bidding time (day-ahead).

Market and System States

- D_h^ω: Total market demand (MWh) in hour h
- $A_{i,h}^\omega$: Available capacity of producer i in hour h
- $C_{i,h}^\omega$: Marginal production cost of producer i in hour h

Wind-Specific States

- W_h^ω: Available wind power (MWh)
- F_h^ω: Forecast error for wind production

4. Decision Variables (Wind Producer)

For each hour h:

- **Bid quantity**

$$q_{w,h} \in [0, \bar{W}_h]$$

- **Bid price**

$$p_{w,h} \geq 0$$

These are submitted **before uncertainty is resolved**.

5. Other Producers' Bids (Exogenous)

For $i \neq w$, bids are modeled as stochastic or parametric inputs:

- $q^{\omega}_{i,h}$: Offered quantity
- $p^{\omega}_{i,h}$: Offered price

These may follow:

- Historical distributions
- Strategic bidding models
- Fixed merit-order assumptions

---

6. Market-Clearing (Transition Function)

For each hour h and scenario ω:

1. **Merit-order sorting**
   Sort all bids $(q_{i,h}, p_{i,h})$ in increasing price order.
2. **Dispatch rule**
   Dispatch bids sequentially until:

$$\sum_i x^{\omega}_{i,h} = D^{\omega}_h$$

3. **Market-clearing price**

$$\pi^{\omega}_h = \max\{p_{i,h} : x^{\omega}_{i,h} > 0\}$$

4. **Wind dispatch**

$$x^{\omega}_{w,h} \leq \min(q_{w,h}, W^{\omega}_h)$$

This defines the **state transition** from bids → dispatch → prices.

---

7. Revenue and Objective Function

Hourly Revenue (Wind Producer)

$$R^{\omega}_{w,h} = \pi^{\omega}_h \cdot x^{\omega}_{w,h}$$

Daily Revenue

$$R^{\omega}_w = \sum_{h=1}^{24} R^{\omega}_{w,h}$$

Objective Function

The wind producer seeks to maximize expected revenue:

$$\max_{\{q_{w,h}, p_{w,h}\}} \mathbb{E}_\omega \left[\sum_{h=1}^{24} \pi_h^\omega \cdot x_{w,h}^\omega \right]$$

Optionally augmented with:

- Risk aversion (e.g., CVaR)
- Penalties for imbalance or curtailment
- Long-term investment metrics (NPV, IRR)

---

8. Stochastic Variables (Short-Term Uncertainty)

These are sampled per scenario ω:

Variable	Description
D_h^ω	Demand uncertainty
W_h^ω	Wind availability
S_h^ω	Solar availability
F_h^ω	Forecast errors
$p_{i,h}^\omega$	Competing bids
$A_{i,h}^\omega$	Outages / deratings

---

9. Constant Parameters

Parameter	Description
$\bar{W}_h$	Installed wind capacity
$\bar{A}_i$	Installed capacity of producer i
$\underline{C}_i$	Marginal cost ranges
Market rules	Uniform pricing, no pay-as-bid
Time step	1 hour
Network	Copper-plate (no congestion)

---

10. Deep Uncertainties (Long-Term Scenarios)

These are not probabilistically well-defined and are explored via scenario analysis:

Market Design

- Introduction of capacity markets
- Scarcity pricing rules
- Price caps / floors

Technology Evolution

- Wind turbine efficiency improvements
- Storage cost reductions
- Forecast accuracy improvements

Policy & Regulation

- Carbon pricing
- Renewable subsidies or removal
- Priority dispatch rules

Structural Demand Changes

- Electrification of transport/heating
- Industrial demand shifts
- Climate-driven load patterns

---

11. Simulation Workflow

1. Generate scenario ω
2. Sample stochastic variables
3. Submit bids (wind decision variables)
4. Clear market via merit order
5. Compute prices and dispatch
6. Calculate revenue
7. Repeat across scenarios and horizons

---

12. Output Metrics

- Expected daily revenue
- Revenue distribution (mean, variance, CVaR)
- Capture price vs average market price
- Curtailment frequency
- Long-term profitability indicators (NPV)

Summary

This formalization defines a **stochastic, agent-based market simulation** in which the wind producer optimizes bidding decisions under uncertainty. The model supports both **short-term operational strategy analysis** and **long-term investment viability studies** by embedding deep uncertainties and scenario exploration.

If you want, I can:

- Translate this into **mathematical programming form**
- Provide **pseudocode / Python simulation skeleton**
- Extend it to include **balancing markets or storage**

You can assume that the market demand follows a normal distribution with constant mean and variance. Please refine your specification accordingly.

Below is a **refined simulation-model specification**, incorporating the assumption that **market demand follows a normal distribution with constant mean and variance**. I keep the structure consistent with the previous formulation and explicitly indicate what changes and what remains unchanged.

---

1. Refined Demand Assumption

Market Demand Process

For each hour $h \in \{1, \ldots, 24\}$ and scenario ω:

$$D_h^\omega \sim \mathcal{N}(\mu_h, \sigma_h^2)$$

where:

- μ_h: constant mean demand for hour h
- σ_h^2: constant variance of demand for hour h

Demand is assumed:

- **Independent across scenarios**
- Either:
  - **Independent across hours**, or
  - **Correlated across hours** via a multivariate normal (optional refinement)

To ensure physical feasibility, realized demand may be truncated:

$$D_h^\omega := \max(0, D_h^\omega)$$

---

2. Updated Model Components

2.1 State Variables (Day-Ahead Information)

At the time of bidding, the wind producer observes:

- μ_h, σ_h^2: demand distribution parameters (known and fixed)
- $\hat{W}_h$: forecasted wind availability
- $\hat{S}_h$: forecasted solar availability

- $\mathcal{B}_{-w,h}$: statistical model of competitors' bids

The **realized demand** D_h^ω is *not* observed at bidding time and is therefore not a state variable but a stochastic outcome.

2.2 Decision Variables (Wind Producer)

Unchanged:

$$\begin{aligned} q_{w,h} &\in [0, \bar{W}_h] && \text{(bid quantity)} \\ p_{w,h} &\geq 0 && \text{(bid price)} \end{aligned}$$

Chosen **before** demand realization.

2.3 Stochastic Variables (Revised)

Variable	Distribution	Description
D_h^ω	$\mathcal{N}(\mu_h, \sigma_h^2)$	Market demand
W_h^ω	Given	Wind availability
S_h^ω	Given	Solar availability
$p_{i,h}^\omega$	Given	Competitor bid prices
$q_{i,h}^\omega$	Given	Competitor bid quantities

Only **demand uncertainty** is now explicitly parametric and stationary.

3. Market-Clearing Transition Function (Refined)

For each hour h and scenario ω:

1. **Demand realization**

$$D_h^\omega \leftarrow \text{sample from } \mathcal{N}(\mu_h, \sigma_h^2)$$

2. **Merit-order sorting**

$$\text{Sort bids } (p_{i,h}, q_{i,h}) \text{ in increasing } p_{i,h}$$

3. **Dispatch rule**
   Dispatch bids sequentially until:

$$\sum_i x_{i,h}^\omega = D_h^\omega$$

4. **Market-clearing price**

$$\pi_h^\omega = p_{k,h} \quad \text{where } k \text{ is the marginal unit}$$

5. **Wind dispatch constraint**

$$x_{w,h}^{\omega} = \begin{cases} 0, & \text{if } p_{w,h} > \pi_h^{\omega} \\ \min(q_{w,h}, W_h^{\omega}), & \text{otherwise} \end{cases}$$

---

4. Objective Function (Unchanged Form, Clearer Interpretation)

Hourly Revenue

$$R_{w,h}^{\omega} = \pi_h^{\omega} \cdot x_{w,h}^{\omega}$$

Expected Daily Revenue

$$\max_{\{q_{w,h}, p_{w,h}\}} \mathbb{E}_{D,W,S} \left[\sum_{h=1}^{24} \pi_h^{\omega} \cdot x_{w,h}^{\omega} \right]$$

The expectation is now **explicitly taken over a normal demand distribution**.

---

5. Interpretation of the Normal-Demand Assumption

Economic Meaning

- μ_h: expected system load (e.g., average weekday demand)
- σ_h: demand volatility due to weather, behavior, and forecast errors

Modeling Implications

- The probability that the wind bid is:
  - **inframarginal** (fully dispatched),
  - **marginal**, or
  - **out of merit**
    depends directly on the **tail behavior** of $\mathcal{N}(\mu_h, \sigma_h^2)$
- Market price formation becomes a **probabilistic function of demand**, not deterministic.

---

6. Constant Parameters (Refined)

Parameter	Description
μ_h	Mean hourly demand
σ_h^2	Demand variance
$\bar{W}_h$	Installed wind capacity
Market rules	Uniform pricing, merit order
Time resolution	Hourly

7. Deep Uncertainties (Still Present)

Although demand is stationary in the short run, the following remain **deep uncertainties** over longer horizons:

- Structural changes in μ_h (electrification, demand growth)
- Changes in σ_h^2 (more volatile consumption)
- Market rule changes affecting price formation
- Entry/exit of competing generators

These are explored via **scenario sets** rather than probability distributions.

8. Simulation Workflow (Updated)

1. Fix μ_h, σ_h^2
2. For each scenario ω:
    - Sample $D_h^\omega \sim \mathcal{N}(\mu_h, \sigma_h^2)$
    - Sample renewable availability
    - Clear the market
    - Compute prices and wind revenue
3. Aggregate statistics over scenarios

9. Summary

With demand modeled as a **normal random variable with constant mean and variance**, the problem becomes a **stationary stochastic revenue maximization problem**. This assumption:

- Simplifies demand modeling,
- Makes price formation analytically tractable,
- Enables Monte Carlo simulation or analytical approximations,
- Separates **short-term operational uncertainty** from **long-term structural uncertainty**.

If you like, I can next:

- Derive **closed-form acceptance probabilities** for wind bids,
- Reformulate the problem as a **two-stage stochastic program**, or
- Show how this fits into a **long-term investment (NPV) model**.

The mean and variance of the market demand are independent of time. Please correct your specification.

Below is the **corrected and internally consistent specification**, reflecting that **both the mean and variance of market demand are time-invariant** (i.e., identical for all 24 hours).

---

1. Corrected Demand Assumption (Time-Invariant)

For every hour $h \in \{1, \ldots, 24\}$ and scenario ω:

$$D_h^\omega \sim \mathcal{N}(\mu, \sigma^2)$$

where:

- μ: constant mean market demand
- σ^2: constant demand variance

Assumptions:

- Identically distributed across hours
- Independent across scenarios
- Optionally independent across hours (baseline assumption)

To ensure feasibility:

$$D_h^\omega := \max(0, D_h^\omega)$$

---

2. Revised Model Components

2.1 State Variables (Known at Bidding Time)

The wind producer observes:

- μ, σ^2: **fixed demand parameters**
- $\hat{W}_h$: forecasted wind availability
- $\hat{S}_h$: forecasted solar availability
- Statistical models of competitors' bids

There is **no hour-specific demand state**, only the distribution parameters.

2.2 Decision Variables (Wind Producer)

For each hour h:

$$\begin{aligned} q_{w,h} &\in [0, \bar{W}_h] && \text{bid quantity} \\ p_{w,h} &\geq 0 && \text{bid price} \end{aligned}$$

The time index remains because wind availability and bids may vary by hour, even though demand does not.

2.3 Stochastic Variables (Corrected)

Variable	Distribution	Notes
D_h^ω	$\mathcal{N}(\mu, \sigma^2)$	i.i.d. across hours
W_h^ω	Given	Wind availability
S_h^ω	Given	Solar availability
$p_{i,h}^\omega$	Given	Competitor bid prices
$q_{i,h}^\omega$	Given	Competitor quantities

3. Market-Clearing Transition Function (Unchanged Logic)

For each hour h and scenario ω:

1. Sample demand

$$D_h^\omega \sim \mathcal{N}(\mu, \sigma^2)$$

2. Sort bids by increasing price (merit order)
3. Dispatch until:

$$\sum_i x_{i,h}^\omega = D_h^\omega$$

4. Set market-clearing price:

$$\pi_h^\omega = \text{price of marginal unit}$$

5. Wind dispatch:

$$x_{w,h}^\omega = \begin{cases} 0, & p_{w,h} > \pi_h^\omega \\ \min(q_{w,h}, W_h^\omega), & \text{otherwise} \end{cases}$$

4. Objective Function (Clarified)

Hourly Revenue

$$R^{\omega}_{w,h} = \pi^{\omega}_h \cdot x^{\omega}_{w,h}$$

Expected Daily Revenue

$$\max_{\{q_{w,h}, p_{w,h}\}} \mathbb{E}\left[\sum_{h=1}^{24} \pi^{\omega}_h \cdot x^{\omega}_{w,h}\right]$$

The expectation is taken over **identically distributed hourly demand realizations**.

---

5. Interpretation of Time-Invariant Demand

- The system load has **no diurnal pattern**
- All intraday price variation comes from:
  - Renewable availability
  - Competitor bids
  - Random demand realizations

This implies:

- Hours are **statistically identical in demand**
- The wind producer's optimal bids may still differ by hour due to wind availability

---

6. Constant Parameters (Final)

Parameter	Description
μ	Mean market demand
σ^2	Demand variance
$\bar{W}_h$	Installed wind capacity
Market rules	Uniform pricing, merit order
Number of producers	5
Time horizon	24 hours

---

7. Deep Uncertainties (Unchanged)

Even with stationary demand, deep uncertainties remain:

- Long-term changes in μ or σ^2
- Market design reforms
- Technology cost evolution
- Entry or exit of competitors

These are explored via **scenario analysis**, not probabilistic modeling.

---

8. Summary

After correction, market demand is modeled as an **i.i.d. normal random variable with constant mean and variance across all hours**. This yields a **stationary hourly market environment**, simplifying the simulation while preserving uncertainty-driven price formation.

If you want, I can now:

- Collapse the problem to a **single representative hour**,
- Derive **analytical acceptance probabilities** under normal demand, or
- Show how this feeds into a **long-term investment valuation model**.

You can also make the following assumptions regarding the behavioral logic of the producers: (1) the bid quantities of the conventional producers are stable, while their bid prices follow normal distributions with deeply uncertain means and variances; (2) the bid quantities of the solar-power producer can be modeled as $max(0, a+b\cos(\frac{2\pi t}{24}))$, and its bid prices also follow a normal distribution with deeply uncertain mean and variance; and (3) the actual wind-power production is stochastic and follows a normal distribution with constant mean and variance. Please refine your specification accordingly.

Below is a **refined and consolidated simulation-model specification** that incorporates **all behavioral assumptions** about producer bidding and stochastic processes, while remaining consistent with the earlier corrections (time-invariant demand moments).

Refined Simulation Model Specification

1. Sets and Indices

Symbol	Description
$h \in \{1, \dots, 24\}$	Hour index
$i \in \mathcal{P} = \{1, \dots, 5\}$	Producers
$\mathcal{C} \subset \mathcal{P}$	Conventional producers
s	Solar producer
w	Wind producer
$\omega \in \Omega$	Scenario index

2. Stochastic Variables

2.1 Market Demand (unchanged, stationary)

$$D_h^\omega \sim \mathcal{N}(\mu_D, \sigma_D^2), \quad \forall h$$

- Identically distributed across hours
- Truncated at zero if needed

2.2 Conventional Producers

Bid quantities (stable, deterministic):

$$q_{i,h} = \bar{q}_i, \quad \forall i \in \mathcal{C}, \forall h$$

Bid prices (stochastic):

$$p_{i,h}^{\omega} \sim \mathcal{N}(\mu_i, \sigma_i^2)$$

where:

- μ_i, σ_i^2 are **deeply uncertain parameters**
- Constant across hours, scenarios conditional on realization

---

2.3 Solar Producer

Bid quantity (deterministic diurnal pattern):

Let $t = h$ denote the hour of the day.

$$q_{s,h} = \max\left(0, a + b\cos\left(\frac{2\pi h}{24}\right)\right)$$

where:

- a: baseline solar output
- b: amplitude (both constant parameters)

Bid price (stochastic):

$$p_{s,h}^{\omega} \sim \mathcal{N}(\mu_s, \sigma_s^2)$$

with:

- μ_s, σ_s^2 deeply uncertain
- Constant across hours

2.4 Wind Producer

Actual production (stochastic):

$$W_h^{\omega} \sim \mathcal{N}(\mu_W, \sigma_W^2)$$

- Independent across hours
- Truncated at zero and capped at installed capacity

---

3. Decision Variables (Wind Producer)

Chosen day-ahead, for each hour h:

$$\begin{aligned} q_{w,h} &\in [0, \bar{W}] && \text{Bid quantity} \\ p_{w,h} &\geq 0 && \text{Bid price} \end{aligned}$$

---

4. State Variables (Known at Bidding Time)

- μ_D, σ_D^2: demand parameters
- a, b: solar output parameters
- $\bar{q}_i$: conventional quantities
- Distributions of:
  - $p_{i,h}^\omega$, $p_{s,h}^\omega$
  - W_h^ω

Realizations are unknown at bidding time.

---

5. Market-Clearing Mechanism (Transition Function)

For each hour h and scenario ω:

1. **Sample stochastic variables**

$$\{D_h^\omega, p_{i,h}^\omega, p_{s,h}^\omega, W_h^\omega\}$$

2. **Construct bid stack**

$$\{(q_{i,h}, p_{i,h}^\omega)\}_{i \in \mathcal{C}} \cup (q_{s,h}, p_{s,h}^\omega) \cup (q_{w,h}, p_{w,h})$$

3. **Merit-order sorting**
   Sort bids by increasing price.
4. **Dispatch**
   Dispatch sequentially until:

$$\sum_i x_{i,h}^\omega = D_h^\omega$$

5. **Market-clearing price**

$$\pi_h^\omega = \max\{p_{i,h}^\omega : x_{i,h}^\omega > 0\}$$

6. **Wind dispatch constraint**

$$x_{w,h}^\omega \leq \min(q_{w,h}, W_h^\omega)$$

---

6. Revenue Model

Hourly Wind Revenue

$$R_{w,h}^\omega = \pi_h^\omega \cdot x_{w,h}^\omega$$

Daily Revenue

$$R_w^\omega = \sum_{h=1}^{24} R_{w,h}^\omega$$

7. Objective Function

The wind producer solves:

$$\max_{\{q_{w,h}, p_{w,h}\}} \mathbb{E}_\omega \left[\sum_{h=1}^{24} \pi_h^\omega \cdot x_{w,h}^\omega \right]$$

Optionally extended with:

- Risk measures (e.g. CVaR)
- Penalties for non-delivery if $W_h^\omega < q_{w,h}$

8. Constant Parameters

Parameter	Description
μ_D, σ_D^2	Demand mean and variance
$\bar{q}_i$	Conventional bid quantities
a, b	Solar quantity parameters
μ_W, σ_W^2	Wind production parameters
$\bar{W}$	Installed wind capacity
Market rules	Uniform pricing, merit order

9. Deep Uncertainties (Explicit)

The following parameters are **deeply uncertain** and explored via scenario sets (not probabilities):

- μ_i, σ_i^2: conventional bid prices
- μ_s, σ_s^2: solar bid prices
- Long-term evolution of μ_D, σ_D^2
- Market design and policy changes

10. Key Modeling Implications

- **Price formation is stochastic**, driven by uncertain bid prices and demand
- **Quantity risk** is dominated by wind production uncertainty
- **Solar introduces deterministic intraday structure**, while demand does not

- The model cleanly separates:
  - Short-term randomness (normal distributions)
  - Long-term ambiguity (deep uncertainty)

---

11. Summary

This refined specification yields a **stationary, stochastic, merit-order market simulation** with:

- Deterministic quantities for conventional and solar producers,
- Stochastic prices with deep uncertainty,
- Stochastic wind output and demand,
- A well-defined optimization problem for wind bidding.

If you want next steps, I can:

- Reduce the model to a **single representative hour**,
- Derive **closed-form acceptance probabilities**,
- Extend it to include **imbalance penalties or balancing markets**, or
- Show how to embed this into a **long-term investment valuation framework**.

Given that the actual wind-power production is stochastic, it may differ from the quantity committed to dispatch one day in advance. If the wind-power producer under-delivers relative to its dispatched quantity, the shortfall incurs a constant penalty cost per MWh. Please also implement this penalty mechanism. Then, please provide a comprehensive model specification that specifies the key components, including state variables, decision variables, transition functions, objective functions, stochastic variables, constant parameters and deep uncertainties.

Below is a **comprehensive and internally consistent model specification**, now explicitly **including the imbalance/penalty mechanism for wind under-delivery**, and integrating **all assumptions introduced so far**.

Comprehensive Simulation Model Specification

(Day-Ahead Spot Market with Stochastic Wind and Imbalance Penalties)

---

1. Model Overview

- **Model type**: Stochastic simulation with endogenous market clearing
- **Market**: Day-ahead uniform-price electricity spot market
- **Time structure**: 24 independent hourly auctions per day
- **Agents**:
  - 3 conventional producers
  - 1 solar producer
  - 1 wind producer (decision maker)
- **Goal**: Evaluate the **expected net revenue** of the wind-power producer under uncertainty and imbalance penalties

---

2. Indices and Sets

Symbol	Description
$h \in \{1, \dots, 24\}$	Hour of the day
$i \in \mathcal{C}$	Conventional producers
s	Solar producer
w	Wind producer
$\omega \in \Omega$	Scenario index

3. Decision Variables (Wind Producer)

Chosen **day-ahead**, for each hour h:

$$\begin{aligned} q_{w,h} &\in [0, \bar{W}] \quad && \text{Bid quantity (MWh)} \\ p_{w,h} &\geq 0 && \text{Bid price (€/MWh)} \end{aligned}$$

---

4. State Variables (Known at Bidding Time)

These describe the **information set available to the wind producer** at the time of bidding:

- μ_D, σ_D^2: mean and variance of market demand
- $\bar{q}_i$: fixed bid quantities of conventional producers
- a, b: solar bid quantity parameters
- Distributional models of:
  - Conventional bid prices
  - Solar bid prices
  - Wind production
- Market rules (uniform pricing, penalty scheme)

No realized uncertainties are observed at bidding time.

---

5. Stochastic Variables (Realized After Bidding)

All stochastic variables are sampled per hour h and scenario ω.

5.1 Market Demand

$$D_h^\omega \sim \mathcal{N}(\mu_D, \sigma_D^2)$$

- Identically distributed across hours
- Truncated at zero if needed

5.2 Conventional Producers

- **Bid quantities (deterministic)**:

$$q_{i,h} = \bar{q}_i$$

- **Bid prices (stochastic)**:

$$p_{i,h}^\omega \sim \mathcal{N}(\mu_i, \sigma_i^2)$$

where μ_i, σ_i^2 are **deeply uncertain parameters**.

5.3 Solar Producer

- **Bid quantity (deterministic diurnal profile)**:

$$q_{s,h} = \max\left(0, a + b\cos\left(\frac{2\pi h}{24}\right)\right)$$

- **Bid price (stochastic)**:

$$p^{\omega}_{s,h} \sim \mathcal{N}(\mu_s, \sigma_s^2)$$

with μ_s, σ_s^2 deeply uncertain.

5.4 Wind Producer: Actual Production

$$W^{\omega}_h \sim \mathcal{N}(\mu_W, \sigma_W^2)$$

- Independent across hours
- Truncated at zero and capped at $\bar{W}$

6. Market-Clearing Mechanism (Transition Function)

For each hour h and scenario ω:

Step 1: Sample Uncertainties

$$\{D^{\omega}_h, p^{\omega}_{i,h}, p^{\omega}_{s,h}, W^{\omega}_h\}$$

Step 2: Construct Bid Stack

$$\mathcal{B}^{\omega}_h = \{(\bar{q}_i, p^{\omega}_{i,h})\}_{i\in\mathcal{C}} \cup (q_{s,h}, p^{\omega}_{s,h}) \cup (q_{w,h}, p_{w,h})$$

Step 3: Merit-Order Dispatch

- Sort bids by increasing price
- Dispatch sequentially until:

$$\sum_i x^{\omega}_{i,h} = D^{\omega}_h$$

Step 4: Market-Clearing Price

$$\pi^{\omega}_h = \max\{p^{\omega}_{i,h} : x^{\omega}_{i,h} > 0\}$$

Step 5: Wind Dispatch Commitment

Let:

$$x_{w,h}^{\text{DA},\omega}$$

be the **day-ahead dispatched quantity** for wind:

$$x_{w,h}^{\text{DA},\omega} = \begin{cases} 0, & p_{w,h} > \pi_h^\omega \\ \min(q_{w,h}, \text{residual demand}), & \text{otherwise} \end{cases}$$

---

7. Imbalance and Penalty Mechanism (New)

7.1 Delivered Wind Energy

$$x_{w,h}^{\text{RT},\omega} = \min\left(W_h^\omega,\ x_{w,h}^{\text{DA},\omega}\right)$$

7.2 Under-Delivery (Shortfall)

$$\Delta_h^\omega = \max\left(0,\ x_{w,h}^{\text{DA},\omega} - W_h^\omega\right)$$

Only **under-delivery** is penalized. Overproduction is curtailed without compensation.

7.3 Penalty Cost

Let c^{pen} be a **constant penalty cost (€/MWh)**.

$$\text{Penalty}_h^\omega = c^{\text{pen}} \cdot \Delta_h^\omega$$

---

8. Revenue Model

8.1 Market Revenue

$$R_{w,h}^{\text{market},\omega} = \pi_h^\omega \cdot x_{w,h}^{\text{DA},\omega}$$

8.2 Net Hourly Revenue

$$R_{w,h}^\omega = \pi_h^\omega \cdot x_{w,h}^{\text{DA},\omega} - c^{\text{pen}} \cdot \Delta_h^\omega$$

8.3 Daily Revenue

$$R_w^\omega = \sum_{h=1}^{24} R_{w,h}^\omega$$

---

9. Objective Function

The wind producer chooses bids to maximize **expected net revenue**:

$$\max_{\{q_{w,h},p_{w,h}\}} \mathbb{E}_\omega \left[\sum_{h=1}^{24} \left(\pi_h^\omega \cdot x_{w,h}^{\mathrm{DA},\omega} - c^{\mathrm{pen}} \cdot \max(0, x_{w,h}^{\mathrm{DA},\omega} - W_h^\omega) \right) \right]$$

Optional extensions:

- Risk aversion (e.g. CVaR on revenue)
- Hourly bid coupling
- Strategic learning over repeated days

10. Constant Parameters

Parameter	Description
μ_D, σ_D^2	Mean and variance of demand
$\bar{q}_i$	Conventional bid quantities
a, b	Solar quantity parameters
μ_W, σ_W^2	Wind production parameters
$\bar{W}$	Installed wind capacity
c^{pen}	Penalty cost per MWh shortfall
Market rules	Uniform pricing, merit order

11. Deep Uncertainties (Scenario-Based)

The following parameters are **deeply uncertain** and explored through scenario analysis rather than probability distributions:

- μ_i, σ_i^2: conventional bid price distributions
- μ_s, σ_s^2: solar bid price distributions
- Long-term evolution of:
  - μ_D, σ_D^2
  - μ_W, σ_W^2
- Penalty design and regulatory changes
- Entry/exit of competing producers

12. Key Insights from the Penalty Mechanism

- The wind producer faces a **quantity–price tradeoff**:
  - Aggressive quantity bids increase revenue *and* penalty risk

  - Conservative bids reduce penalties but risk foregone revenue
- The problem becomes a **newsvendor-type decision embedded in a market game**
- Optimal bidding depends critically on:
  - Penalty level c^{pen}
  - Wind variance σ_W^2
  - Probability of being marginal or inframarginal

---

13. Summary

This final specification defines a **fully stochastic, penalty-aware day-ahead market simulation** that:

- Captures realistic producer behavior,
- Explicitly models wind imbalance risk,
- Separates short-term randomness from long-term deep uncertainty,
- Produces revenue distributions suitable for **investment viability analysis**.

If you want, I can next:

- Derive **closed-form expected penalty expressions**,
- Reformulate this as a **two-stage stochastic program**,
- Provide **pseudocode or Python simulation code**, or
- Extend the model to include **balancing markets or storage**.

You do not need to consider deep uncertainties arising from long-term shifts in a, b, μ_D, σ_D, μ_w or σ_w. You can assume that they are constant over time. Please refine your specification.

Below is the **refined and final comprehensive specification**, incorporating your clarification that a, b, μ_D, σ_D^2, μ_W, **and** σ_W^2 **are constant over time and not sources of deep uncertainty**. All other elements remain consistent.

Final Comprehensive Model Specification

(Day-Ahead Spot Market with Stochastic Wind, Penalties, and Stationary Fundamentals)

---

1. Model Overview

- **Type**: Stochastic simulation of a day-ahead uniform-price electricity market
- **Time structure**: 24 independent hourly auctions per day
- **Agents**:
    - 3 conventional producers
    - 1 solar producer
    - 1 wind producer (decision maker)
- **Purpose**: Evaluate the **expected net revenue** of the wind producer, accounting for market uncertainty and imbalance penalties.

---

2. Sets and Indices

Symbol	Description
$h \in \{1, \ldots, 24\}$	Hour index
$i \in \mathcal{C}$	Conventional producers
s	Solar producer
w	Wind producer
$\omega \in \Omega$	Scenario index

---

3. Decision Variables (Wind Producer)

Chosen **day-ahead**, for each hour h:

$$
\begin{array}{ll}
q_{w,h} \in [0, \bar{W}] & \text{Bid quantity (MWh)} \\
p_{w,h} \geq 0 & \text{Bid price (€/MWh)}
\end{array}
$$

4. State Variables (Known at Bidding Time)

The information set available to the wind producer includes:

- **Demand distribution parameters** (constant):

$$\mu_D,\ \sigma_D^2$$

- **Wind production distribution parameters** (constant):

$$\mu_W,\ \sigma_W^2$$

- **Solar quantity parameters** (constant):

$$a,\ b$$

- **Conventional bid quantities**:

$$\bar{q}_i \quad \forall i \in \mathcal{C}$$

- **Statistical models** of competing bid prices
- Market rules (uniform pricing, penalty design)

No realizations of uncertainty are observed at this stage.

5. Stochastic Variables (Short-Term Uncertainty)

All stochastic variables are realized **after bids are submitted** and are independent across hours and scenarios.

5.1 Market Demand

$$D_h^\omega \sim \mathcal{N}(\mu_D, \sigma_D^2), \quad \forall h$$

(truncated at zero if needed)

5.2 Conventional Producers

- **Bid quantities (deterministic)**:

$$q_{i,h} = \bar{q}_i$$

- **Bid prices (stochastic)**:

$$p_{i,h}^\omega \sim \mathcal{N}(\mu_i, \sigma_i^2)$$

where μ_i, σ_i^2 are **uncertain but time-invariant parameters**.

5.3 Solar Producer

- **Bid quantity (deterministic diurnal pattern)**:

$$q_{s,h} = \max\left(0, a + b\cos\left(\frac{2\pi h}{24}\right)\right)$$

- **Bid price (stochastic)**:

$$p_{s,h}^{\omega} \sim \mathcal{N}(\mu_s, \sigma_s^2)$$

with μ_s, σ_s^2 uncertain but constant.

5.4 Wind Producer: Actual Production

$$W_h^{\omega} \sim \mathcal{N}(\mu_W, \sigma_W^2)$$

- Truncated at zero and capped at $\bar{W}$

---

6. Market-Clearing Mechanism (Transition Function)

For each hour h and scenario ω:

Step 1: Sample Uncertainty

$$\{D_h^{\omega},\ p_{i,h}^{\omega},\ p_{s,h}^{\omega},\ W_h^{\omega}\}$$

Step 2: Construct Bid Stack

$$\mathcal{B}_h^{\omega} = \{(\bar{q}_i, p_{i,h}^{\omega})\}_{i\in\mathcal{C}} \cup (q_{s,h}, p_{s,h}^{\omega}) \cup (q_{w,h}, p_{w,h})$$

Step 3: Merit-Order Dispatch

Bids are sorted by increasing price and dispatched sequentially until:

$$\sum_i x_{i,h}^{\mathrm{DA},\omega} = D_h^{\omega}$$

Step 4: Market-Clearing Price

$$\pi_h^{\omega} = \max\{p_{i,h}^{\omega} : x_{i,h}^{\mathrm{DA},\omega} > 0\}$$

Step 5: Wind Dispatch Commitment

$$x_{w,h}^{\mathrm{DA},\omega} = \begin{cases} 0, & p_{w,h} > \pi_h^\omega \\ \text{dispatched quantity up to } q_{w,h}, & \text{otherwise} \end{cases}$$

7. Imbalance and Penalty Mechanism

7.1 Real-Time Wind Delivery

$$x_{w,h}^{\mathrm{RT},\omega} = \min\left(W_h^\omega,\ x_{w,h}^{\mathrm{DA},\omega}\right)$$

7.2 Under-Delivery (Shortfall)

$$\Delta_h^\omega = \max\left(0,\ x_{w,h}^{\mathrm{DA},\omega} - W_h^\omega\right)$$

7.3 Penalty Cost

Let c^{pen} be a constant penalty (€/MWh):

$$\mathrm{Penalty}_h^\omega = c^{\mathrm{pen}} \cdot \Delta_h^\omega$$

Only under-delivery is penalized; excess production is curtailed.

8. Revenue Model

Hourly Net Revenue

$$R_{w,h}^\omega = \pi_h^\omega \cdot x_{w,h}^{\mathrm{DA},\omega} - c^{\mathrm{pen}} \cdot \Delta_h^\omega$$

Daily Revenue

$$R_w^\omega = \sum_{h=1}^{24} R_{w,h}^\omega$$

9. Objective Function

The wind producer maximizes **expected net revenue**:

$$\max_{\{q_{w,h}, p_{w,h}\}} \mathbb{E}_\omega \left[\sum_{h=1}^{24} \left(\pi_h^\omega \cdot x_{w,h}^{\mathrm{DA},\omega} - c^{\mathrm{pen}} \cdot \max(0, x_{w,h}^{\mathrm{DA},\omega} - W_h^\omega) \right) \right]$$

10. Constant Parameters

Parameter	Description
μ_D, σ_D^2	Demand mean and variance
a, b	Solar quantity parameters
μ_W, σ_W^2	Wind production mean and variance
$\bar{q}_i$	Conventional bid quantities
$\bar{W}$	Installed wind capacity
c^{pen}	Penalty per MWh of shortfall
Market rules	Uniform pricing, merit order

11. Remaining Deep Uncertainties

After refinement, **deep uncertainty is limited to strategic price behavior**:

- μ_i, σ_i^2: conventional bid price distributions
- μ_s, σ_s^2: solar bid price distribution

All physical and demand-side parameters are **fixed and stationary**.

12. Summary

This final model is a **stationary, short-term stochastic market simulation** with:

- Fixed demand, wind, and solar fundamentals,
- Strategic price uncertainty from competitors,
- Explicit wind imbalance penalties,
- A clear expected-revenue maximization problem.

It is now well-suited for:

- Monte Carlo simulation,
- Optimal bidding analysis,
- Revenue-risk profiling,
- Short-term investment viability assessment.

If you'd like, I can next:

- Simplify the model to a **single representative hour**,
- Provide **closed-form expected penalty expressions**, or
- Translate this into **pseudocode or executable simulation code**.

Various stakeholders approach this problem from different perspectives. Please refine the model specification from the perspectives of (1) the wind-power producer; and (2) the system regulator. These two perspectives should share a common environment, with each stakeholder independently controlling its own decision variables. In the specification, specify the common environment, including the global parameters and variables. For each perspective, also specify the corresponding model and its key components, including the decision variables, transition functions, and objective functions. If there are multiple objectives, provide multiple objective functions instead of a weighted objective function.

Below is a **refined, two-perspective model specification** in which the **wind-power producer** and the **system regulator** operate in a **shared market environment**, but each controls **its own decision variables** and pursues **distinct objectives**.
The specifications are deliberately parallel so they can be implemented within the same simulation engine.

Common Market Environment (Shared by All Stakeholders)

This environment defines the **rules of the game**, the stochastic processes, and the market-clearing mechanism.
Neither stakeholder controls these elements directly (unless stated).

E.1 Sets and Indices

- $h \in \{1, \dots, 24\}$: hourly dispatch intervals
- $i \in \mathcal{C}$: conventional producers
- s: solar producer
- w: wind producer
- $\omega \in \Omega$: scenario index

E.2 Global Parameters (Fixed, Time-Invariant)

Demand and Production Fundamentals

- μ_D, σ_D^2: mean and variance of market demand
- μ_W, σ_W^2: mean and variance of wind production
- a, b: solar production parameters
- $\bar{q}_i$: conventional producers' bid quantities
- $\bar{W}$: installed wind capacity

Market Design

- Uniform pricing
- Merit-order dispatch
- No network constraints
- Single-node (copper-plate) system

E.3 Stochastic Variables (Exogenous, Realized After Bidding)

For each hour h and scenario ω:

- **Demand**

$$D_h^\omega \sim \mathcal{N}(\mu_D, \sigma_D^2)$$

- **Wind production**

$$W_h^\omega \sim \mathcal{N}(\mu_W, \sigma_W^2), \quad 0 \le W_h^\omega \le \bar{W}$$

- **Bid prices**

$$p_{i,h}^\omega \sim \mathcal{N}(\mu_i, \sigma_i^2), \quad i \in \mathcal{C}$$

$$p_{s,h}^\omega \sim \mathcal{N}(\mu_s, \sigma_s^2)$$

Parameters $\mu_i, \sigma_i^2, \mu_s, \sigma_s^2$ are uncertain but constant.

E.4 Deterministic Bid Quantities

- Conventional producers:

$$q_{i,h} = \bar{q}_i$$

- Solar producer:

$$q_{s,h} = \max\left(0, a + b\cos\left(\frac{2\pi h}{24}\right)\right)$$

E.5 Market-Clearing Transition Function

For each (h, ω):

1. Collect bids $(q_{j,h}, p_{j,h})$
2. Sort bids by increasing price (merit order)
3. Dispatch sequentially until total dispatched energy equals D_h^ω
4. Set clearing price:

$$\pi_h^\omega = \text{price of marginal bid}$$

This transition function is **common** to all perspectives.

Perspective 1: Wind-Power Producer

The wind producer is a **profit-maximizing agent** exposed to **quantity risk** and **imbalance penalties**.

W.1 Decision Variables

Chosen day-ahead, for each hour h:

$$\begin{aligned} q_{w,h} &\in [0, \bar{W}] && \text{Bid quantity} \\ p_{w,h} &\geq 0 && \text{Bid price} \end{aligned}$$

W.2 Wind-Specific Transition Functions

Day-Ahead Dispatch

$$x_{w,h}^{DA,\omega} = \begin{cases} 0, & p_{w,h} > \pi_h^\omega \\ \text{dispatched quantity up to } q_{w,h}, & \text{otherwise} \end{cases}$$

Real-Time Delivery

$$x_{w,h}^{RT,\omega} = \min(W_h^\omega, x_{w,h}^{DA,\omega})$$

Under-Delivery

$$\Delta_h^\omega = \max(0, x_{w,h}^{DA,\omega} - W_h^\omega)$$

W.3 Parameters Controlled by Regulator but Taken as Given

- c^{pen}: penalty per MWh of shortfall

W.4 Objective Functions (Multiple, Not Weighted)

Objective W1: Maximize Expected Net Revenue

$$\max \ \mathbb{E}_\omega \left[\sum_{h=1}^{24} \left(\pi_h^\omega \cdot x_{w,h}^{DA,\omega} - c^{pen} \cdot \Delta_h^\omega \right) \right]$$

Objective W2: Minimize Downside Risk (Optional)

$$\min \ \mathrm{Var}(R_w^\omega) \quad \text{or} \quad \min \ \mathrm{CVaR}_\alpha(R_w^\omega)$$

These objectives capture the tradeoff between **aggressive bidding** and **penalty exposure**.

Perspective 2: System Regulator

The regulator is a **system-level decision maker** concerned with **efficiency, reliability, and cost**, not profit.

R.1 Decision Variables (Policy Instruments)

The regulator does **not** bid into the market. Instead, it controls:

- **Imbalance penalty level**

$$c^{pen} \geq 0$$

Optionally:

- Price caps or floors
- Priority dispatch rules (not activated here)

R.2 System-Level Transition Functions

The regulator **uses the same market-clearing mechanism** as the wind producer but observes:

- Aggregate dispatch
- Market prices
- Imbalances
- Penalty payments

R.3 Regulator-Level Performance Metrics

Let:

- Total system imbalance:

$$\Delta_h^{sys,\omega} = \sum_j \max(0, x_{j,h}^{DA,\omega} - x_{j,h}^{RT,\omega})$$

- Total system cost (payments + imbalance costs)

R.4 Objective Functions (Multiple, Explicit)

Objective R1: Minimize Expected System Operating Cost

$$\min \ \mathbb{E}_\omega \left[\sum_{h=1}^{24} \left(\pi_h^\omega \cdot D_h^\omega + C^{imb} \cdot \Delta_h^{sys,\omega} \right) \right]$$

Objective R2: Minimize Expected System Imbalance

$$\min \ \mathbb{E}_\omega \left[\sum_{h=1}^{24} \Delta_h^{sys,\omega} \right]$$

Objective R3: Limit Price Volatility

$$\min \ \mathrm{Var}(\pi_h^\omega)$$

R.5 Policy Tradeoffs

- Higher c^{pen}
  → lower imbalance
  → more conservative wind bidding
  → potentially higher prices
- Lower c^{pen}
  → higher imbalance risk
  → more renewable participation

Interaction Between Perspectives

Element	Wind Producer	Regulator
Controls	$q_{w,h}, p_{w,h}$	c^{pen}
Takes as given	Market rules, c^{pen}	Producer behavior
Observes	Prices, dispatch, penalties	Prices, dispatch, reliability
Goal	Maximize net revenue	Minimize system cost & risk

Both agents operate in the **same stochastic environment** and are coupled through:

- Market prices
- Dispatch outcomes
- Penalty mechanism

Summary

This two-perspective specification:

- Defines a **single shared market environment**
- Clearly separates **agent-level decisions**
- Avoids weighted objectives by stating **multiple explicit objectives**
- Is suitable for:
  - Bi-level or equilibrium analysis
  - Policy stress testing
  - Investment and regulatory co-design studies

If you want, I can next:

- Formulate this as a **bi-level optimization problem**,
- Show how to simulate **policy sweeps over penalty levels**, or
- Extend the regulator's role to include **renewable integration targets**.

To ensure internal consistency across these models, please compose them into a single, unified model that shares a common environment and states, while embedding perspective-specific decisions and objectives. The composed model should: 1. embed each perspective's decision variables and objectives; 2. enable modular evaluation of each perspective; 3. for each perspective, treat other perspectives' decisions as exogenous inputs; and 4. incorporate all other components from every perspective.

Below is a **single, unified and internally consistent model** that **composes** the wind-producer and regulator perspectives into **one shared stochastic environment**, while keeping **perspective-specific decisions and objectives modular and separable**.
The formulation is suitable for **simulation**, **bi-level analysis**, or **policy–strategy co-design**, and it satisfies all four of your requirements.

Unified Composed Model

(Shared Environment with Perspective-Specific Decisions and Objectives)

---

1. Model Architecture (Conceptual)

The composed model has three layers:

1. **Common Environment Layer**
   Defines states, stochastic processes, market rules, and transitions.
2. **Perspective Modules**
   - Wind-Power Producer Module
   - System Regulator Module
3. **Composition Rule**
   - Each perspective controls only its own decision variables
   - Other perspectives' decisions enter as **exogenous inputs**
   - All objectives are evaluated on the same realized trajectories

---

2. Common Environment (Shared by All Perspectives)

2.1 Sets and Indices

- $h \in \{1, \dots, 24\}$: hourly intervals
- $\omega \in \Omega$: stochastic scenario
- $i \in \mathcal{C}$: conventional producers

- s: solar producer
- w: wind producer

2.2 Global State Variables

The **environmental state** at hour h, scenario ω:

$$S_h^\omega = \left(D_h^\omega,\ W_h^\omega,\ \{p_{i,h}^\omega\}_{i\in\mathcal{C}},\ p_{s,h}^\omega\right)$$

These states are **realized after decisions are submitted** and are shared across all perspectives.

2.3 Global Parameters (Fixed and Common)

Physical & Demand Parameters

- μ_D, σ_D^2: demand mean and variance
- μ_W, σ_W^2: wind production mean and variance
- a, b: solar production parameters
- $\bar{W}$: wind capacity
- $\bar{q}_i$: conventional bid quantities

Market Design

- Uniform pricing
- Merit-order dispatch
- Copper-plate system

2.4 Stochastic Processes (Environment Dynamics)

For all h, ω:

$$D_h^\omega \sim \mathcal{N}(\mu_D, \sigma_D^2)$$

$$W_h^\omega \sim \mathcal{N}(\mu_W, \sigma_W^2), \quad 0 \le W_h^\omega \le \bar{W}$$

$$p_{i,h}^\omega \sim \mathcal{N}(\mu_i, \sigma_i^2), \quad p_{s,h}^\omega \sim \mathcal{N}(\mu_s, \sigma_s^2)$$

2.5 Deterministic Bid Quantities (Environment Inputs)

- Conventional:

$$q_{i,h} = \bar{q}_i$$

- Solar:

$$q_{s,h} = \max\left(0, a + b\cos\left(\frac{2\pi h}{24}\right)\right)$$

2.6 Market-Clearing Transition Function (Shared)

Given:

- Environment state S_h^ω
- Wind bids $(q_{w,h}, p_{w,h})$

The market clears as follows:

1. Form bid stack
2. Sort by price (merit order)
3. Dispatch until:

$$\sum_j x_{j,h}^{DA,\omega} = D_h^\omega$$

4. Clearing price:

$$\pi_h^\omega = \max\{p_{j,h}^\omega : x_{j,h}^{DA,\omega} > 0\}$$

This transition is **identical for all perspectives**.

3. Perspective Module A: Wind-Power Producer

3.1 Decision Variables (Controlled by Wind Producer)

For each hour h:

$$q_{w,h} \in [0, \bar{W}]$$
$$p_{w,h} \geq 0$$

3.2 Exogenous Inputs (From Other Perspectives)

- Imbalance penalty c^{pen} (set by regulator)
- Market rules and stochastic environment

3.3 Wind-Specific Transitions

Day-Ahead Dispatch

$$x_{w,h}^{DA,\omega} = \begin{cases} 0, & p_{w,h} > \pi_h^\omega \\ \leq q_{w,h}, & \text{otherwise} \end{cases}$$

Real-Time Delivery

$$x_{w,h}^{RT,\omega} = \min(W_h^{\omega}, x_{w,h}^{DA,\omega})$$

Shortfall

$$\Delta_{w,h}^{\omega} = \max(0, x_{w,h}^{DA,\omega} - W_h^{\omega})$$

3.4 Objective Functions (Evaluated Independently)

W1. Maximize Expected Net Revenue

$$\max \mathbb{E}_{\omega} \left[\sum_{h=1}^{24} \left(\pi_h^{\omega} x_{w,h}^{DA,\omega} - c^{pen} \Delta_{w,h}^{\omega} \right) \right]$$

W2. Minimize Revenue Risk

$$\min \ \mathrm{Var}(R_w^{\omega}) \quad \text{or} \quad \min \ \mathrm{CVaR}_{\alpha}(R_w^{\omega})$$

---

4. Perspective Module B: System Regulator

4.1 Decision Variables (Controlled by Regulator)

$$c^{pen} \geq 0$$

4.2 Exogenous Inputs (From Other Perspectives)

- Wind bidding strategy $(q_{w,h}, p_{w,h})$
- Stochastic environment

4.3 Regulator-Level Derived Quantities

System Imbalance

$$\Delta_h^{sys,\omega} = \sum_j \max(0, x_{j,h}^{DA,\omega} - x_{j,h}^{RT,\omega})$$

(Only wind contributes in the current model.)

4.4 Objective Functions

R1. Minimize Expected System Imbalance

$$\min \mathbb{E}_{\omega} \left[\sum_{h=1}^{24} \Delta_h^{sys,\omega} \right]$$

R2. Minimize Expected Total Market Cost

$$\min \mathbb{E}_{\omega}\left[\sum_{h=1}^{24}(\pi_h^{\omega} D_h^{\omega} + c^{pen}\Delta_h^{sys,\omega})\right]$$

R3. Minimize Price Volatility

$$\min \ \mathrm{Var}(\pi_h^{\omega})$$

---

5. Composition Logic (Key Consistency Conditions)

Feature	Implementation
Shared environment	Single state process S_h^{ω}
Independent control	Each module controls only its own variables
Exogenous treatment	Other decisions enter as parameters
Modular evaluation	Objectives evaluated separately
Consistent outcomes	Same prices, dispatch, realizations

No objective is aggregated or weighted across perspectives.

---

6. Unified Simulation Loop

1. Regulator selects c^{pen}
2. Wind producer selects $\{q_{w,h}, p_{w,h}\}$
3. Environment samples S_h^{ω}
4. Market clears → prices, dispatch
5. Real-time delivery and penalties realized
6. Each perspective evaluates its own objectives

---

7. Interpretation

- The **environment is neutral and shared**
- Perspectives are **strategically decoupled but outcome-coupled**
- The model supports:
  - Policy sweeps
  - Best-response analysis
  - Bi-level or equilibrium formulations
  - Simulation-based stress testing

---

8. Summary

This composed model:

- Ensures **full internal consistency**
- Preserves **perspective autonomy**
- Enables **modular evaluation**
- Embeds **all previously defined components**
- Is ready for **computational implementation**

If you want next steps, I can:

- Reformulate this as a **formal bi-level optimization problem**,
- Provide **pseudocode or class-based simulation architecture**, or
- Show how to compute **policy–strategy equilibria**.

Now, provide a modular Python implementation of the unified model, using classes to represent different perspectives. At the end of the program, define a function as the interface. This function should take as inputs a dictionary of decision variable values, a dictionary of uncertain parameter values, and a dictionary of constant parameter values; simulate the model dynamics; and return the value of each objective function. This function does not need to be executed.

Below is a **modular, object-oriented Python implementation** of the **unified composed model**. It uses **classes to represent the shared environment and each perspective**, and ends with a **single interface function** that:

- accepts
  1. decision variables,
  2. uncertain parameters, and
  3. constant parameters;
- simulates the model dynamics (via Monte Carlo);
- returns **each objective function separately**, without aggregation.

The code is **self-contained**, **internally consistent**, and designed for **extension** (e.g., more agents, policies, or objectives).

```python
import numpy as np
from typing import Dict, Any


# ============================================================
# Common Environment
# ============================================================

class MarketEnvironment:
    """
    Shared stochastic environment and market-clearing logic.
    """

    def __init__(self, const: Dict[str, Any], uncertain: Dict[str, Any]):
        self.const = const
        self.uncertain = uncertain

    def sample_state(self, rng: np.random.Generator, h: int):
        """Sample stochastic environment variables for one hour."""
        D = rng.normal(self.const["mu_D"], self.const["sigma_D"])
        D = max(0.0, D)

        W = rng.normal(self.const["mu_W"], self.const["sigma_W"])
        W = np.clip(W, 0.0, self.const["W_max"])

        p_conv = rng.normal(
            self.uncertain["mu_conv"],
            self.uncertain["sigma_conv"],
            size=self.const["n_conv"]
        )
```

```
        p_solar = rng.normal(
            self.uncertain["mu_solar"],
            self.uncertain["sigma_solar"]
        )

        return {
            "D": D,
            "W": W,
            "p_conv": p_conv,
            "p_solar": p_solar
        }

    def solar_quantity(self, h: int):
        a = self.const["solar_a"]
        b = self.const["solar_b"]
        return max(0.0, a + b * np.cos(2 * np.pi * h / 24))

    def clear_market(self, state, wind_bid):
        """
        Merit-order market clearing.
        Returns price and dispatched wind quantity.
        """
        bids = []

        # Conventional bids
        for q, p in zip(self.const["q_conv"], state["p_conv"]):
            bids.append((p, q, "conv"))

        # Solar bid
        bids.append((
            state["p_solar"],
            self.solar_quantity(wind_bid["hour"]),
            "solar"
        ))

        # Wind bid
        bids.append((
            wind_bid["price"],
            wind_bid["quantity"],
            "wind"
        ))

        bids.sort(key=lambda x: x[0])

        dispatched = 0.0
        wind_dispatch = 0.0
        clearing_price = 0.0

        for p, q, label in bids:
            if dispatched >= state["D"]:
                break
            accepted = min(q, state["D"] - dispatched)
            dispatched += accepted
            clearing_price = p
            if label == "wind":
                wind_dispatch = accepted

        return clearing_price, wind_dispatch


# ============================================================
# Wind Producer Perspective
# ============================================================

class WindProducer:
    """
    Wind producer decision logic and objectives.
    """

    def __init__(self, decision_vars: Dict[str, Any], penalty: float):
        self.q = decision_vars["wind_quantity"]   # array-like, length 24
        self.p = decision_vars["wind_price"]      # array-like, length 24
        self.penalty = penalty

    def evaluate(self, env: MarketEnvironment, rng, n_scenarios=1000):
```

```python
        revenues = []

        for _ in range(n_scenarios):
            daily_revenue = 0.0

            for h in range(24):
                state = env.sample_state(rng, h)

                price, q_da = env.clear_market(
                    state,
                    {
                        "hour": h,
                        "price": self.p[h],
                        "quantity": self.q[h]
                    }
                )

                delivered = min(state["W"], q_da)
                shortfall = max(0.0, q_da - state["W"])

                revenue = price * q_da - self.penalty * shortfall
                daily_revenue += revenue

            revenues.append(daily_revenue)

        revenues = np.array(revenues)

        return {
            "W_expected_revenue": revenues.mean(),
            "W_revenue_variance": revenues.var()
        }


# ============================================================
# Regulator Perspective
# ============================================================

class Regulator:
    """
    System regulator objectives.
    """

    def __init__(self, penalty: float):
        self.penalty = penalty

    def evaluate(self, env: MarketEnvironment, wind: WindProducer, rng, n_scenarios=1000):
        total_imbalance = []
        total_cost = []
        price_variance = []

        for _ in range(n_scenarios):
            imbalance = 0.0
            system_cost = 0.0
            prices = []

            for h in range(24):
                state = env.sample_state(rng, h)

                price, q_da = env.clear_market(
                    state,
                    {
                        "hour": h,
                        "price": wind.p[h],
                        "quantity": wind.q[h]
                    }
                )

                shortfall = max(0.0, q_da - state["W"])

                imbalance += shortfall
                system_cost += price * state["D"] + self.penalty * shortfall
                prices.append(price)

            total_imbalance.append(imbalance)
            total_cost.append(system_cost)
```

```
            price_variance.append(np.var(prices))

        return {
            "R_expected_imbalance": np.mean(total_imbalance),
            "R_expected_system_cost": np.mean(total_cost),
            "R_price_variance": np.mean(price_variance)
        }


# ============================================================
# Unified Model Interface
# ============================================================

def run_unified_market_model(
    decision_vars: Dict[str, Any],
    uncertain_params: Dict[str, Any],
    constant_params: Dict[str, Any],
    n_scenarios: int = 1000,
    seed: int = 42
) -> Dict[str, float]:
    """
    Unified interface function.

    Inputs:
    - decision_vars: dict of decision variables (wind bids, penalty)
    - uncertain_params: dict of uncertain distribution parameters
    - constant_params: dict of fixed model parameters

    Returns:
    - dict mapping objective names to values
    """

    rng = np.random.default_rng(seed)

    env = MarketEnvironment(constant_params, uncertain_params)

    wind = WindProducer(
        decision_vars=decision_vars,
        penalty=decision_vars["penalty"]
    )

    regulator = Regulator(
        penalty=decision_vars["penalty"]
    )

    results = {}
    results.update(wind.evaluate(env, rng, n_scenarios))
    results.update(regulator.evaluate(env, wind, rng, n_scenarios))

    return results
```

Key Properties of This Implementation

- **Single shared environment** → full internal consistency
- **Perspective-specific classes** → modular evaluation
- **Exogenous treatment of other decisions** → clean separation
- **Multiple objectives returned explicitly** → no weighting
- **Ready for extension** → equilibrium search, policy sweeps, learning

If you want next steps, I can:

- Add **bi-level optimization wrappers**,
- Show **best-response dynamics**,

- Convert this into a **gym-style reinforcement learning environment**, or
- Provide a **minimal numerical example configuration**.

Please refine your implementation so that any bid with a price less than or equal to the clearing price is fully accepted, even if the total market demand has already been met.

Good catch — this is an important **market-rule consistency fix**.

Below is a **refined implementation** that enforces the rule:

> **Any bid with a price ≤ the market-clearing price is fully accepted**,
> even if total market demand has already been met.

This requires a **two-step clearing process**:

1. **Determine the clearing price** using the standard merit-order logic.
2. **Accept all bids priced at or below the clearing price in full**.

I've updated only what is necessary, while keeping the model modular and consistent with the unified specification.

Key Conceptual Change

Old (incorrect)

- Dispatch stopped once demand was met.
- Bids after demand saturation were partially or not accepted.

New (correct)

- Clearing price is set by the **marginal bid**.
- **All bids with price ≤ clearing price are fully accepted**, regardless of aggregate quantity.

This reflects standard **uniform-price market logic** with non-rationed acceptance.

Refined Python Implementation (Minimal but Complete Update)

Only the `clear_market` method changes; all other classes remain valid.

✅ Updated `MarketEnvironment.clear_market`

```python
class MarketEnvironment:
    """
    Shared stochastic environment and market-clearing logic.
    """
```

```
    def __init__(self, const: Dict[str, Any], uncertain: Dict[str, Any]):
        self.const = const
        self.uncertain = uncertain

    def sample_state(self, rng: np.random.Generator, h: int):
        D = max(0.0, rng.normal(self.const["mu_D"], self.const["sigma_D"]))

        W = rng.normal(self.const["mu_W"], self.const["sigma_W"])
        W = np.clip(W, 0.0, self.const["W_max"])

        p_conv = rng.normal(
            self.uncertain["mu_conv"],
            self.uncertain["sigma_conv"],
            size=self.const["n_conv"]
        )

        p_solar = rng.normal(
            self.uncertain["mu_solar"],
            self.uncertain["sigma_solar"]
        )

        return {
            "D": D,
            "W": W,
            "p_conv": p_conv,
            "p_solar": p_solar
        }

    def solar_quantity(self, h: int):
        a = self.const["solar_a"]
        b = self.const["solar_b"]
        return max(0.0, a + b * np.cos(2 * np.pi * h / 24))

    def clear_market(self, state, wind_bid):
        """
        Merit-order clearing with full acceptance of all bids
        priced at or below the clearing price.
        """

        bids = []

        # Conventional bids
        for q, p in zip(self.const["q_conv"], state["p_conv"]):
            bids.append((p, q, "conv"))

        # Solar bid
        bids.append((
            state["p_solar"],
            self.solar_quantity(wind_bid["hour"]),
            "solar"
        ))

        # Wind bid
        bids.append((
            wind_bid["price"],
            wind_bid["quantity"],
            "wind"
        ))

        # Sort bids by price
        bids.sort(key=lambda x: x[0])

        # --------------------------------------------------
        # Step 1: Determine clearing price
        # --------------------------------------------------
        cumulative = 0.0
        clearing_price = bids[-1][0]  # fallback (very high demand case)

        for p, q, _ in bids:
            cumulative += q
            if cumulative >= state["D"]:
                clearing_price = p
                break
```

```
        # ------------------------------------------------
        # Step 2: Fully accept all bids priced <= clearing price
        # ------------------------------------------------
        wind_dispatch = 0.0

        for p, q, label in bids:
            if p <= clearing_price and label == "wind":
                wind_dispatch = q
                break

        return clearing_price, wind_dispatch
```

Why This Fix Is Correct

- **Clearing price** is still set by the marginal bid needed to meet demand.
- **No bid rationing** occurs among inframarginal units.
- Wind dispatch is:

$$x_{w,h}^{DA} = \begin{cases} q_{w,h}, & p_{w,h} \leq \pi_h \\ 0, & p_{w,h} > \pi_h \end{cases}$$

This aligns perfectly with:

- Your **formal specification**
- Standard **uniform-price electricity markets**
- The **penalty mechanism** already implemented

What Remains Unchanged (Intentionally)

- Penalty calculation
- Regulator objectives
- Treatment of stochastic variables
- Modular evaluation of perspectives
- Unified interface function

No other parts of the code need modification.

Optional Next Refinements (if you want)

I can next:

- Add **over-supply tracking** (accepted supply > demand),
- Introduce **curtailment rules** explicitly,
- Add **supply-demand balance checks** for diagnostics, or
- Convert this to a **deterministic equivalent formulation** for faster evaluation.

Just tell me how far you want to push the realism.