Experiments 2 - Market 4

In this problem, four energy producers participate in a day-ahead electricity market, including three conventional ones and a renewable one. These producers submit hourly bids that specify a delivery quantity of electricity and an offer price for each hour of the next day. The market operator uses these bids to determine the market-clearing price and dispatch schedule. If a bid is accepted, the producer is obligated to deliver the specified quantity of electricity at the offer price. Otherwise, the producer earns zero profit for that hour.

However, renewable generation is deeply uncertain, as it depends on weather conditions that cannot be predicted. If the producer under-delivers relative to its bid (due to overestimation of production), or over-delivers (due to underestimation), penalties or imbalance costs would apply. These costs are imposed by the system operator to maintain grid stability and to ensure supply-demand balance in real time. Meanwhile, the clearing price is unknown to the producer in advance, so they need to compete in this market.

The renewable energy producer's goal is to maximize its expected profit by strategically selecting the quantity of electricity to bid for each hour of the next day, accounting for the uncertainty in both renewable generation and clearing prices.

Market structure and timing:
1. The day-ahead market is cleared once per day (e.g., at noon) for 24 hourly intervals of the following day.
2. Each producer submits a bid (b_t, p_t) for each hour $t=1, ..., 24$, where:
2.1. b_t: the quantity of energy (in MWh) the producer is willing to supply
2.2. p_t: the minimum acceptable price per MWh
3. The market operator computes the market-clearing price c_t for each hour.
4. If $p_t \leq c_t$, the bid is accepted, and the producer is committed to delivering b_t MWh in hour t.
5. The total energy demand in the market is D_t, which follows a normal distribution with constant mean and variance.

Penalty structure:
1. If actual generation $P_t < b_t$, the shortfall $b_t - P_t$ incurs a penalty of q_u per MWh.
2. If actual generation $P_t \geq b_t$, the surplus $P_t - b_t$ incurs a curtailment penalty of q_o per MWh.

Conventional producers:
1. For each conventional producer, the delivery quantity of electricity and the offer price in bids follow normal distributions with constant means and variances.

Please formalize this problem description as a model. Please also specify key components, such as the state variables, initial state, uncertainties and transition functions.

Let's formalize the problem as a **stochastic day-ahead electricity market model** with one *renewable producer* (decision-maker) and three *conventional producers* (competitors).

1. Common Market Environment

Time horizon

- Discrete hourly intervals indexed by $t = 1, 2, \ldots, 24$.

Global parameters

Symbol	Description
D_t	Total market demand in hour t, $D_t \sim \mathcal{N}(\mu_D, \sigma_D^2)$.
c_t	Market-clearing price at hour t (determined endogenously).
q_u	Penalty per MWh for **under-delivery**.
q_o	Penalty per MWh for **over-delivery** (curtailment).
$N = 4$	Total number of producers (3 conventional + 1 renewable).

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2. Agents and Control Variables

Renewable producer (agent $i = R$)

- Control (decision) variables for each hour t:

$$b_t^R, \quad p_t^R$$

  where b_t^R is the bid quantity (MWh) and p_t^R is the offer price ($/MWh).

- Uncertain renewable generation:

$$P_t^R \sim f_P(\cdot; \theta_P)$$

  where f_P is a probability distribution (e.g., truncated normal) depending on weather uncertainty.

- Profit realization:

$$\pi_t^R = \begin{cases} c_t b_t^R - q_u \max(0, b_t^R - P_t^R) - q_o \max(0, P_t^R - b_t^R), & \text{if } p_t^R \leq c_t, \\ 0, & \text{otherwise.} \end{cases}$$

- Objective:

$$\max_{\{b_t^R, p_t^R\}_{t=1}^{24}} \mathbb{E}\left[\sum_{t=1}^{24} \pi_t^R\right]$$

Conventional producers (agents $i = 1, 2, 3$)

- Each submits bids (b_t^i, p_t^i) drawn from stochastic distributions:

$$b_t^i \sim \mathcal{N}(\mu_{b,i}, \sigma_{b,i}^2), \qquad p_t^i \sim \mathcal{N}(\mu_{p,i}, \sigma_{p,i}^2)$$

- These determine their participation in the market and contribute to price formation.

3. Market Clearing Mechanism

The market operator determines the **clearing price** c_t such that:

$$\sum_{i \in \{R,1,2,3\}} b_t^i \, \mathbf{1}_{\{p_t^i \le c_t\}} = D_t$$

That is, c_t equates total accepted supply and total demand.
This mechanism induces endogenous coupling between producers' bids and the market outcome.

4. State, Controls, and Transitions

Type	Symbol	Description	Update rule
State variables	$s_t = (D_t, P_t^R, c_t)$	Market demand, renewable generation, and clearing price at hour t.	Determined by stochastic realizations and market clearing.
Control variables	$a_t = (b_t^R, p_t^R)$	Renewable producer's decision variables.	Chosen at the start of each day (before uncertainty).
Uncertainties	$\xi_t = (D_t, P_t^R, \{b_t^i, p_t^i\}_{i=1}^3)$	Random factors for demand, renewable generation, and competitors' bids.	Drawn independently across hours (or following a known correlation).
Transition function	$s_{t+1} = f(s_t, a_t, \xi_t)$	Represents the evolution of stochastic variables to the next hour.	Usually Markovian or independent across hours for the day-ahead setting.

Since each hour is typically cleared independently in the day-ahead market, there is **no dynamic coupling** between hours except through shared distributions. Thus:

$$s_{t+1} \text{ independent of } s_t \text{ given parameters.}$$

Initial state:

$$s_0 = (\mu_D, \mathbb{E}[P^R], \text{initial forecast})$$

5. Summary of Model Components

Component	Description
Decision variables	(b_t^R, p_t^R) for $t = 1, \ldots, 24$.
State variables	(D_t, P_t^R, c_t).
Uncertainties	D_t, P_t^R, b_t^i, p_t^i for $i = 1, 2, 3$.
Transition	Hourly independent market clearing process: $c_t = g(D_t, \{b_t^i, p_t^i\}_{i=R,1,2,3})$.
Objective	Maximize $\mathbb{E}[\sum_t \pi_t^R]$ with penalties for imbalance.
Constraints	$b_t^R \ge 0, \;\; p_t^R \ge 0$.

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6. Model Interpretation

- The model is a **stochastic optimization problem under competition**.
- The renewable producer's optimal bidding strategy must balance:
  - The trade-off between a higher probability of acceptance (low p_t^R) and expected profit (dependent on c_t);
  - The risks from forecast uncertainty P_t^R and imbalance penalties q_u, q_o;
  - The uncertainty in competitors' bids that influences c_t.

Would you like me to extend this formal model into a **computational structure** (e.g., a stochastic programming or simulation-based optimization framework) next?

Various stakeholders approach this problem from different perspectives. Please identify four different perspectives related to this problem. These perspectives should share a common environment, along with global parameters, state variables, and an initial state. For each perspective, also provide the corresponding model with the key components, such as control variables, transition functions, and objective functions. If there are multiple objectives, provide multiple objective functions instead of a weighted objective function.

Excellent — we can now develop a **multi-perspective formalization** of this day-ahead electricity market problem.
Each perspective operates within the *same market environment* (shared parameters, state variables, and uncertainty) but has distinct **objectives** and **controls** that reflect its role in the system.

🔹 Common Market Environment

Global Parameters

Symbol	Description
$T = 24$	Number of hourly periods in the day-ahead market
D_t	Total demand in hour t, $D_t \sim \mathcal{N}(\mu_D, \sigma_D^2)$
q_u	Penalty per MWh for under-delivery (shortfall)
q_o	Penalty per MWh for over-delivery (surplus)
$N = 4$	Number of producers (3 conventional, 1 renewable)

Shared State Variables

Symbol	Description
$s_t = (D_t, P_t^R, c_t)$	Market state at hour t: demand, renewable production, and clearing price

Initial State

$$s_0 = (\mu_D, \mathbb{E}[P^R], c_0)$$

where c_0 may represent the forecast or previous clearing price.

Uncertainties

$$\xi_t = (D_t, P_t^R, \{b_t^i, p_t^i\}_{i=1}^3)$$

drawn from known distributions (demand and renewable variability, conventional bids).

Market-Clearing Condition

$$\sum_{i \in \{R,1,2,3\}} b_t^i \mathbf{1}_{\{p_t^i \le c_t\}} = D_t$$

Transition Function

The system evolves hour-by-hour according to:

$$s_{t+1} = f(s_t, a_t, \xi_t)$$

but since day-ahead hours are typically independent:

$$f(s_t, a_t, \xi_t) = (D_{t+1}, P_{t+1}^R, g(D_{t+1}, \{b_{t+1}^i, p_{t+1}^i\}))$$

🔹 Four Perspectives

Perspective 1: Renewable Producer

Controls

$$a_t^R = (b_t^R, p_t^R)$$

State Dynamics

Exogenous: s_t evolves via $f(\cdot)$ determined by market clearing and uncertainty.

Objective Functions

1. **Expected profit maximization:**

$$\max_{a_t^R} \ \mathbb{E}\left[\sum_{t=1}^{T} \left(c_t b_t^R - q_u \max(0, b_t^R - P_t^R) - q_o \max(0, P_t^R - b_t^R)\right) \mathbf{1}_{\{p_t^R \le c_t\}}\right]$$

2. **Risk management (variance minimization):**

$$\min_{a_t^R} \ \mathrm{Var}\left[\sum_{t=1}^{T} \pi_t^R\right]$$

Perspective 2: Conventional Producer (Representative of Producers 1–3)

Controls

$$a_t^C = (b_t^C, p_t^C)$$

Each conventional producer sets offer quantity and price.

State Dynamics

Driven by market clearing, identical to the renewable case.

Objective Functions

1. **Profit maximization:**

$$\max_{a_t^C} \mathbb{E}\left[\sum_{t=1}^{T} (c_t - p_t^C) b_t^C \mathbf{1}_{\{p_t^C \le c_t\}}\right]$$

2. **Operational reliability minimization (optional secondary):**

$$\min_{a_t^C} \mathbb{E}\left[\sum_{t=1}^{T} (b_t^C - \bar{b}_t^C)^2\right]$$

   where $\bar{b}_t^C$ is nominal capacity, penalizing excessive ramping.

Perspective 3: Market Operator (System Operator / ISO)

Controls

$$a_t^M = c_t$$

— implicitly, through the market-clearing algorithm or ancillary market mechanisms (e.g., reserve capacity).

Objective Functions

1. **Market-clearing feasibility:**

$$\text{s.t. } \sum_i b_t^i \mathbf{1}_{\{p_t^i \le c_t\}} = D_t$$

2. **Minimize total system imbalance (real-time correction cost):**

$$\min_{a_t^M} \mathbb{E}\left[\sum_{t=1}^{T} q_u \max(0, D_t - \sum_i P_t^i) + q_o \max(0, \sum_i P_t^i - D_t)\right]$$

3. **Ensure price stability (secondary):**

$$\min_{a_t^M} \ \mathrm{Var}(c_t)$$

Perspective 4: System Planner / Regulator

Controls

$$a_t^G = (\text{policy parameters, e.g. } q_u, q_o, \text{subsidies, capacity limits})$$

Objective Functions

1. **Social welfare maximization:**

$$\max_{a_t^G} \ \mathbb{E}\left[\sum_{t=1}^{T}(U(D_t) - \sum_i C_i(b_t^i))\right]$$

   where $U(D_t)$ is consumer utility and $C_i(b_t^i)$ producers' costs.

2. **Reliability target:**

$$\min_{a_t^G} \ \Pr\left(\left|\sum_i P_t^i - D_t\right| > \epsilon\right)$$

3. **Environmental performance:**

$$\min_{a_t^G} \ \sum_{t=1}^{T} \mathrm{Emissions}_t = \sum_{t=1}^{T}\sum_{i=1}^{3} \alpha_i b_t^i$$

   where α_i is emission intensity of conventional producer i.

🔹 Summary Table

Perspective	Controls	Objective(s)	Key Interaction
Renewable producer	(b_t^R, p_t^R)	Max expected profit; Min risk	Competes via price–quantity bid; affected by uncertainty
Conventional producer	(b_t^C, p_t^C)	Max profit; Min ramp cost	Competes with others; influences price
Market operator	c_t (implicit)	Min imbalance cost; Min price volatility	Ensures market clearing
System planner	Policy parameters ($q_u, q_o,$ incentives)	Max social welfare; Min emissions; Ensure reliability	Designs market environment

🔹 Conceptual Integration

All four perspectives coexist within the same environment:

$$s_{t+1} = f(s_t, a_t^R, a_t^C, a_t^M, a_t^G, \xi_t)$$

Each perspective optimizes its own objective given others' controls as **exogenous** inputs.
This modular structure enables analysis of equilibrium behavior, policy interventions, and stochastic simulation under consistent state dynamics.

Would you like me next to **compose these four perspectives into a unified modular model** (with interlinked Python class structure and objective evaluation interface) as in your previous market examples?

> The second perspective is not consistent with the initial description, since I have specified that the bid quantity and bid price of conventional producers follow normal distributions with constant means and variances. These variables cannot be used as control variables. Please refine the model specification.

Excellent — you are absolutely right.
Since the **conventional producers' bids are exogenous stochastic inputs** (with constant mean and variance), they cannot act as *control variables* in their perspective.
We therefore reinterpret their "perspective" as one of **stochastic participation** rather than *strategic optimization*.

Below is the refined and fully consistent **four-perspective model**, with a common environment and modularly defined perspectives.

🔹 Common Environment

Global Parameters

Symbol	Description
$T = 24$	Number of hourly intervals in the day-ahead market
$N = 4$	Number of producers (3 conventional + 1 renewable)
q_u	Penalty per MWh for under-delivery
q_o	Penalty per MWh for over-delivery
μ_D, σ_D	Mean and standard deviation of demand
$\mu_{b,i}, \sigma_{b,i}, \mu_{p,i}, \sigma_{p,i}$	Mean and variance parameters for conventional producers' bids
θ_P	Parameters describing renewable generation uncertainty (e.g., scale, shape, correlation)

State Variables

$$s_t = (D_t, P_t^R, c_t)$$

where

- D_t = stochastic demand at hour t;

- P_t^R = renewable generation realization;
- c_t = market-clearing price determined endogenously.

Initial state:

$$s_0 = (\mu_D, \mathbb{E}[P^R], c_0)$$

Uncertainties

$$\xi_t = \left(D_t, P_t^R, \{b_t^i, p_t^i\}_{i=1}^3\right)$$

- $D_t \sim \mathcal{N}(\mu_D, \sigma_D^2)$
- $P_t^R \sim f_P(\cdot; \theta_P)$
- $b_t^i \sim \mathcal{N}(\mu_{b,i}, \sigma_{b,i}^2)$, $p_t^i \sim \mathcal{N}(\mu_{p,i}, \sigma_{p,i}^2)$

Market-Clearing Mechanism

The clearing price c_t satisfies:

$$\sum_{i \in \{R,1,2,3\}} b_t^i \mathbf{1}_{\{p_t^i \leq c_t\}} = D_t$$

That is, the total accepted supply equals total demand.

Transition Function

Each hour is independently determined:

$$s_{t+1} = f(s_t, a_t^R, \xi_t) = (D_{t+1}, P_{t+1}^R, g(D_{t+1}, \{b_{t+1}^i, p_{t+1}^i\}))$$

so there is no intertemporal coupling within the day.

🔹 Four Perspectives (Refined)

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1. Renewable Producer Perspective

Control variables

$$a_t^R = (b_t^R, p_t^R)$$

State transition
Driven by exogenous market outcomes $s_t \to s_{t+1}$.

Profit function

$$\pi_t^R = \begin{cases} c_t b_t^R - q_u \max(0, b_t^R - P_t^R) - q_o \max(0, P_t^R - b_t^R), & \text{if } p_t^R \le c_t, \\ 0, & \text{otherwise.} \end{cases}$$

Objective functions

1. **Expected profit maximization:**

$$J_1^R = \max_{a_t^R} \mathbb{E}\left[\sum_{t=1}^{T} \pi_t^R\right]$$

2. **Profit risk minimization:**

$$J_2^R = \min_{a_t^R} \mathrm{Var}\left[\sum_{t=1}^{T} \pi_t^R\right]$$

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2. Conventional Producers' Perspective (Aggregate Stochastic Participation)

Nature of this perspective
These producers do **not control** their bids strategically; their bids are *stochastic realizations* governed by fixed parameters. Their perspective represents **aggregate market behavior** under uncertainty, not decision-making.

Control variables
None (bids are exogenous).

Stochastic bid generation

$$b_t^i \sim \mathcal{N}(\mu_{b,i}, \sigma_{b,i}^2), \qquad p_t^i \sim \mathcal{N}(\mu_{p,i}, \sigma_{p,i}^2)$$

Derived quantities
Expected supply:

$$\mathbb{E}[S_t^{(C)}] = \sum_{i=1}^{3} \mathbb{E}[b_t^i \mathbf{1}_{\{p_t^i \le c_t\}}]$$

Implicit objective
Maintain mean-variance stability of generation; modeled descriptively, not as an optimization.

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3. Market Operator Perspective

Control variables

$$a_t^M = c_t$$

implicitly determined to ensure market clearing.

Constraints

$$\sum_i b_t^i \mathbf{1}_{\{p_t^i \le c_t\}} = D_t$$

Objective functions

1. **Imbalance cost minimization:**

$$J_1^M = \min_{a_t^M} \mathbb{E}\left[\sum_{t=1}^{T} \left(q_u \max(0, D_t - \sum_i P_t^i) + q_o \max(0, \sum_i P_t^i - D_t)\right)\right]$$

2. **Price stability:**

$$J_2^M = \min_{a_t^M} \operatorname{Var}(c_t)$$

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4. System Planner / Regulator Perspective

Control variables

$$a_t^G = (\text{policy parameters such as } q_u, q_o, \text{subsidies, or caps})$$

Objective functions

1. **Social welfare maximization:**

$$J_1^G = \max_{a_t^G} \mathbb{E}\left[\sum_{t=1}^{T} (U(D_t) - \sum_i C_i(b_t^i))\right]$$

2. **System reliability:**

$$J_2^G = \min_{a_t^G} \Pr\left(\left|\sum_i P_t^i - D_t\right| > \epsilon\right)$$

3. **Environmental performance:**

$$J_3^G = \min_{a_t^G} \sum_{t=1}^{T} \sum_{i=1}^{3} \alpha_i b_t^i$$

where α_i is the emission intensity of producer i.

🔹 Structural Summary

Perspective	Controls	Objectives	Treatment of Other Perspectives
Renewable Producer	(b_t^R, p_t^R)	Maximize expected profit; minimize risk	Faces stochastic market and clearing price
Conventional Producers (aggregate)	None (stochastic behavior)	Represent supply uncertainty	Inputs to market clearing and price formation
Market Operator	c_t (implicit)	Minimize imbalance cost, price variance	Responds to all bids and demand

Perspective	Controls	Objectives	Treatment of Other Perspectives
System Planner	Policy parameters $(q_u, q_o, \text{subsidies})$	Maximize welfare, minimize emissions, ensure reliability	Designs and regulates the entire environment

Common Transition Structure

$$s_{t+1} = f(s_t, a_t^R, \xi_t; a_t^G)$$

where a_t^G influences penalty rates and policy incentives that shape payoffs and system dynamics.

This refined formulation ensures:

1. **Conventional producers are modeled stochastically (not as controllers).**
2. **All perspectives share a consistent environment** with common states and parameters.
3. Each perspective has **distinct objectives** appropriate to its role.

Would you like me to now **compose these four refined perspectives into a unified modular Python model**, where each perspective is implemented as a class sharing the same environment (and where the simulation interface accepts the renewable producer's controls and uncertain renewable generation as inputs)?

To ensure internal consistency across these models, please compose them into a single, unified model that shares a common environment and states, while embedding perspective-specific controls and objectives. The composed model should: 1. embed each perspective's control variables and objectives; 2. enables modular evaluation of each perspective; 3. for each perspective, treat other perspectives' controls as exogenous inputs; and 4. incorporate all other components from every perspective.

Below is a **unified formal model** that composes the four perspectives — renewable producer, conventional producers, market operator, and system planner — into a single modular framework.

The unified model integrates a **common environment and shared states**, embeds **perspective-specific controls and objectives**, and ensures **mutual consistency** where each perspective treats the others' controls as *exogenous*.

🔹 1. Common Environment

Let the day-ahead market operate over $T = 24$ hourly intervals, $t = 1, \dots, T$.

Global Parameters

Symbol	Description
$N = 4$	Number of producers (3 conventional + 1 renewable)
μ_D, σ_D	Mean and standard deviation of demand D_t
q_u, q_o	Penalties for under- and over-delivery
$\{\mu_{b,i}, \sigma_{b,i}, \mu_{p,i}, \sigma_{p,i}\}_{i=1}^3$	Means and variances of conventional producers' bids
θ_P	Parameters describing renewable generation uncertainty
α_i	Emission factors for conventional producers
$U(\cdot)$	Consumer utility function
$C_i(\cdot)$	Cost function of producer i
ϵ	Reliability threshold

State Variables

$$s_t = (D_t, P_t^R, c_t)$$

- $D_t \sim \mathcal{N}(\mu_D, \sigma_D^2)$: total demand
- $P_t^R \sim f_P(\cdot; \theta_P)$: stochastic renewable generation
- c_t: market-clearing price

Exogenous Uncertainties

$$\xi_t = \left(D_t, P_t^R, \{b_t^i, p_t^i\}_{i=1}^3\right)$$

where conventional producers' bids follow:

$$b_t^i \sim \mathcal{N}(\mu_{b,i}, \sigma_{b,i}^2), \quad p_t^i \sim \mathcal{N}(\mu_{p,i}, \sigma_{p,i}^2)$$

Market-Clearing Condition

At each hour t, the clearing price c_t is determined by:

$$\sum_{i \in \{R,1,2,3\}} b_t^i \mathbf{1}_{\{p_t^i \le c_t\}} = D_t$$

ensuring total accepted supply equals demand.

Transition Function

Each hour's state evolves independently:

$$s_{t+1} = f(s_t, a_t^R, \xi_t; a_t^G) = \left(D_{t+1}, P_{t+1}^R, g(D_{t+1}, \{b_{t+1}^i, p_{t+1}^i\})\right)$$

where a_t^G (policy parameters) can affect penalties q_u, q_o and cost structure.

2. Embedded Perspectives

Each perspective operates within this shared environment, observing the same states but with distinct controls and objectives.

Perspective 1 — Renewable Producer

Controls

$$a_t^R = (b_t^R, p_t^R)$$

Profit per hour

$$\pi_t^R = \begin{cases} c_t b_t^R - q_u \max(0, b_t^R - P_t^R) - q_o \max(0, P_t^R - b_t^R), & \text{if } p_t^R \leq c_t, \\ 0, & \text{otherwise.} \end{cases}$$

Objective functions

1. **Expected profit maximization**

$$J_1^R = \mathbb{E}\left[\sum_{t=1}^{T} \pi_t^R\right]$$

2. **Profit variance minimization**

$$J_2^R = \mathrm{Var}\left[\sum_{t=1}^{T} \pi_t^R\right]$$

Exogenous inputs

- Market-clearing price c_t
- Penalties q_u, q_o
- Conventional producers' stochastic bids

Perspective 2 — Conventional Producers (Aggregate Stochastic Participation)

No controls — bids are exogenous.

Bid distributions

$$b_t^i \sim \mathcal{N}(\mu_{b,i}, \sigma_{b,i}^2), \quad p_t^i \sim \mathcal{N}(\mu_{p,i}, \sigma_{p,i}^2)$$

Expected profit per producer

$$\pi_t^i = \mathbb{E}\left[(c_t - p_t^i) b_t^i \mathbf{1}_{\{p_t^i \leq c_t\}}\right]$$

Descriptive metrics (objectives)

1. **Expected profit**

$$J_1^{C,i} = \mathbb{E}\left[\sum_t \pi_t^i\right]$$

2. **Supply stability (variance of bid quantities)**

$$J_2^{C,i} = \mathrm{Var}(b_t^i)$$

Exogenous inputs

- Clearing price c_t
- Demand D_t
- Policy parameters q_u, q_o

Perspective 3 — Market Operator

Control variable

$$a_t^M = c_t$$

implicitly solved by the market-clearing condition.

System imbalance

$$\Delta_t = \sum_i P_t^i - D_t$$

Objective functions

1. **Minimize imbalance penalties**

$$J_1^M = \mathbb{E}\left[\sum_{t=1}^{T} \left(q_u \max(0, -\Delta_t) + q_o \max(0, \Delta_t)\right)\right]$$

2. **Minimize price volatility**

$$J_2^M = \mathrm{Var}(c_t)$$

Exogenous inputs

- Bids from all producers
- Demand realizations
- Policy parameters q_u, q_o

Perspective 4 — System Planner / Regulator

Control variables

$$a_t^G = (q_u, q_o, \text{policy incentives})$$

Objective functions

1. **Social welfare maximization**

$$J_1^G = \mathbb{E}\left[\sum_{t=1}^{T} \left(U(D_t) - \sum_i C_i(b_t^i)\right)\right]$$

2. **Reliability assurance**

$$J_2^G = \min_{a_t^G} \Pr(|\Delta_t| > \epsilon)$$

3. **Environmental performance**

$$J_3^G = \min_{a_t^G} \sum_{t=1}^{T} \sum_{i=1}^{3} \alpha_i b_t^i$$

Exogenous inputs

- Market-clearing outcomes c_t, D_t, P_t^R
- Producers' bids and costs

🔹 3. Unified Model Structure

The unified model is defined as the tuple:

$$\mathcal{M} = \left(\mathcal{E}, \mathcal{S}, \{\mathcal{P}^R, \mathcal{P}^C, \mathcal{P}^M, \mathcal{P}^G\}\right)$$

where:

- $\mathcal{E}$: shared environment with parameters and stochastic processes
- $\mathcal{S} = \{s_t\}$: state variables evolving via $f(\cdot)$
- $\mathcal{P}^k = (\text{controls}, \text{objectives})$: perspective $k \in \{R, C, M, G\}$

and the joint state transition:

$$s_{t+1} = f\left(s_t, a_t^R, a_t^M, a_t^G, \xi_t\right)$$

where:

- a_t^R: renewable producer's bid decisions
- a_t^M: market operator's implicit clearing
- a_t^G: regulator's policy parameters
- ξ_t: exogenous uncertainties (demand, renewable generation, conventional bids)

Each perspective's optimization problem is **modular**, evaluated by substituting exogenous controls from other perspectives into $f(\cdot)$ and its objective J^k.

4. Modular Evaluation Framework

Perspective	Control Inputs	Exogenous Inputs	Outputs / Objectives
Renewable producer	$a_t^R = (b_t^R, p_t^R)$	$c_t, q_u, q_o, D_t, P_t^R$	J_1^R, J_2^R
Conventional producers	None	c_t, D_t	$J_1^{C,i}, J_2^{C,i}$
Market operator	Implicit c_t	Bids, D_t, P_t^R, q_u, q_o	J_1^M, J_2^M
System planner	$a_t^G = (q_u, q_o, \text{policy})$	All market outcomes	J_1^G, J_2^G, J_3^G

This structure supports **independent evaluation** of each perspective under the same simulated environment, enabling analysis of equilibrium behavior, policy sensitivity, and uncertainty propagation.

Would you like me to now **translate this unified model into a modular Python implementation** — using classes for each perspective (Renewable, Conventional, MarketOperator, SystemPlanner), all sharing a common Environment class, and an interface function for simulation and objective evaluation?

Now, please provide a modular Python implementation of the unified model, using classes to represent the different perspectives. At the end of the program, define a function that serves as the interface. This function should accept the control variables and the values of uncertain parameters as inputs, simulate the model dynamics, and return the values of each objective function as outputs. There is no need to execute the function.

Here is a **modular Python implementation** of the unified model.
Each stakeholder's perspective is encapsulated in a dedicated class, all operating within a shared **`MarketEnvironment`**.
At the end, a unified interface function **`simulate_market()`** accepts:

- control variables (e.g., renewable bids, policy parameters),
- realizations of uncertain parameters (demand, renewable generation, conventional bids), and returns **all objective values** for every perspective.

```python
import numpy as np

# ============================================================
# 1. COMMON ENVIRONMENT
# ============================================================

class MarketEnvironment:
    """
    Shared environment for all perspectives.
    Holds global parameters, stochastic generation functions, and market-clearing logic.
    """
```

```python
    def __init__(self, T=24, mu_D=100, sigma_D=10, q_u=50, q_o=30,
                 mu_b=None, sigma_b=None, mu_p=None, sigma_p=None, alpha=None):
        self.T = T
        self.mu_D = mu_D
        self.sigma_D = sigma_D
        self.q_u = q_u
        self.q_o = q_o
        self.mu_b = mu_b if mu_b is not None else [50, 60, 55]
        self.sigma_b = sigma_b if sigma_b is not None else [5, 6, 5]
        self.mu_p = mu_p if mu_p is not None else [70, 65, 68]
        self.sigma_p = sigma_p if sigma_p is not None else [5, 4, 6]
        self.alpha = alpha if alpha is not None else [0.6, 0.55, 0.58]

    def sample_demand(self):
        return np.random.normal(self.mu_D, self.sigma_D, self.T)

    def sample_conventional_bids(self):
        """Generate stochastic bids for the 3 conventional producers."""
        b = [np.random.normal(self.mu_b[i], self.sigma_b[i], self.T) for i in range(3)]
        p = [np.random.normal(self.mu_p[i], self.sigma_p[i], self.T) for i in range(3)]
        return np.array(b), np.array(p)

    def clearing_price(self, D_t, b_conv, p_conv, b_R, p_R):
        """
        Determine market clearing price c_t such that accepted bids meet demand.
        Approximate discrete clearing mechanism by sorting offers.
        """
        offers = []
        # Add renewable producer's offer
        offers.append((p_R, b_R, "R"))
        # Add conventional producers' offers
        for i in range(3):
            offers.append((p_conv[i], b_conv[i], f"C{i+1}"))

        # Sort all offers by price
        offers.sort(key=lambda x: x[0])
        total_supply = 0
        for offer_price, offer_qty, _ in offers:
            total_supply += offer_qty
            if total_supply >= D_t:
                return offer_price
        return offers[-1][0]  # if supply < demand, highest price wins


# ============================================================
# 2. PERSPECTIVES
# ============================================================

# ------------------------------
# Renewable Producer Perspective
# ------------------------------
class RenewableProducer:
    def __init__(self, env: MarketEnvironment):
        self.env = env

    def profit(self, c_t, b_t, p_t, P_t):
        """Compute realized profit for the renewable producer."""
        if p_t > c_t:  # bid rejected
            return 0.0
        q_u, q_o = self.env.q_u, self.env.q_o
        under = max(0, b_t - P_t)
        over = max(0, P_t - b_t)
        return c_t * b_t - q_u * under - q_o * over

    def evaluate_objectives(self, b_R, p_R, c, P_R):
        profits = np.array([self.profit(c[t], b_R[t], p_R[t], P_R[t]) for t in
range(self.env.T)])
        J1 = np.mean(profits.sum())  # expected profit (approx.)
        J2 = np.var(profits.sum())   # profit variance
        return J1, J2


# ------------------------------
# Conventional Producers Perspective (Aggregate Stochastic)
# ------------------------------
```

```python
class ConventionalProducers:
    def __init__(self, env: MarketEnvironment):
        self.env = env

    def expected_profit(self, c_t, b_t, p_t):
        """Expected profit for conventional producers under price c_t."""
        accepted = p_t <= c_t
        return np.mean((c_t - p_t[accepted]) * b_t[accepted])

    def evaluate_objectives(self, c, b_conv, p_conv):
        T = self.env.T
        J1_list, J2_list = [], []
        for i in range(3):
            profits = np.array([
                (c[t] - p_conv[i, t]) * b_conv[i, t] * (p_conv[i, t] <= c[t])
                for t in range(T)
            ])
            J1_list.append(np.mean(profits.sum()))
            J2_list.append(np.var(b_conv[i]))
        return J1_list, J2_list


# ------------------------------
# Market Operator Perspective
# ------------------------------
class MarketOperator:
    def __init__(self, env: MarketEnvironment):
        self.env = env

    def evaluate_objectives(self, D, P_total, c):
        """Compute imbalance and price volatility."""
        q_u, q_o = self.env.q_u, self.env.q_o
        imbalance = P_total - D
        imbalance_costs = q_u * np.maximum(0, -imbalance) + q_o * np.maximum(0, imbalance)
        J1 = np.mean(imbalance_costs.sum())  # expected imbalance cost
        J2 = np.var(c)                       # price volatility
        return J1, J2


# ------------------------------
# System Planner Perspective
# ------------------------------
class SystemPlanner:
    def __init__(self, env: MarketEnvironment):
        self.env = env

    def evaluate_objectives(self, D, b_conv, P_total):
        """Evaluate welfare, reliability, and emissions."""
        alpha = self.env.alpha
        epsilon = 5  # reliability tolerance

        # (1) Simplified welfare measure
        U = lambda d: 100 * np.log(1 + d)
        C = lambda b: 0.5 * b ** 2
        welfare = np.sum([U(D[t]) - np.sum([C(b_conv[i, t]) for i in range(3)]) for t in
range(self.env.T)])

        # (2) Reliability (probability of large imbalance)
        imbalance = np.abs(P_total - D)
        reliability = np.mean(imbalance > epsilon)

        # (3) Environmental impact
        emissions = np.sum([alpha[i] * np.sum(b_conv[i, :]) for i in range(3)])

        J1 = welfare
        J2 = reliability
        J3 = emissions
        return J1, J2, J3


# ============================================================
# 3. UNIFIED SIMULATION INTERFACE
# ============================================================

def simulate_market(b_R, p_R, P_R, policy_params, seed=None):
```

```python
    """
    Unified simulation interface.
    Inputs:
        b_R, p_R : arrays of renewable bid quantities and prices
        P_R      : array of renewable generation realizations
        policy_params : dict with possible keys {'q_u', 'q_o'}
    Output:
        dict of objective values for all perspectives
    """

    if seed is not None:
        np.random.seed(seed)

    # --- Initialize environment ---
    env = MarketEnvironment(q_u=policy_params.get('q_u', 50),
                            q_o=policy_params.get('q_o', 30))

    # --- Sample exogenous uncertainties ---
    D = env.sample_demand()
    b_conv, p_conv = env.sample_conventional_bids()

    # --- Determine market-clearing price for each hour ---
    c = np.zeros(env.T)
    for t in range(env.T):
        c[t] = env.clearing_price(D[t], b_conv[:, t], p_conv[:, t], b_R[t], p_R[t])

    # --- Compute total production ---
    P_total = P_R + np.sum(b_conv, axis=0)

    # --- Instantiate perspectives ---
    renewable = RenewableProducer(env)
    conventional = ConventionalProducers(env)
    operator = MarketOperator(env)
    planner = SystemPlanner(env)

    # --- Evaluate objectives ---
    J_R = renewable.evaluate_objectives(b_R, p_R, c, P_R)
    J_C = conventional.evaluate_objectives(c, b_conv, p_conv)
    J_M = operator.evaluate_objectives(D, P_total, c)
    J_G = planner.evaluate_objectives(D, b_conv, P_total)

    # --- Return all objectives ---
    results = {
        "RenewableProducer": {"ExpectedProfit": J_R[0], "ProfitVariance": J_R[1]},
        "ConventionalProducers": {"ExpectedProfits": J_C[0], "BidVariances": J_C[1]},
        "MarketOperator": {"ImbalanceCost": J_M[0], "PriceVariance": J_M[1]},
        "SystemPlanner": {"Welfare": J_G[0], "Reliability": J_G[1], "Emissions": J_G[2]},
    }

    return results
```

✅ Key Features of the Implementation

1. **Shared Environment (`MarketEnvironment`)**
    - Defines all global parameters and stochastic processes.
    - Generates demand and conventional producers' bids.
    - Implements a clearing mechanism for price formation.
2. **Perspective Classes**
    - **`RenewableProducer`** — controls: (b_t^R, p_t^R); objectives: expected profit, risk.
    - **`ConventionalProducers`** — stochastic participants; objectives: expected profit, bid stability.
    - **`MarketOperator`** — implicit control via clearing; objectives: imbalance cost, price volatility.
    - **`SystemPlanner`** — controls policy parameters; objectives: welfare, reliability, emissions.

3. **Unified Simulation Interface**
   - Function **`simulate_market()`** takes in:
     - control inputs (**`b_R`**, **`p_R`**, **`policy_params`**),
     - stochastic realizations (**`P_R`**),
   - Simulates market dynamics and returns all perspective-specific objective values.

Would you like me to include **an example call** (without execution) showing how to use **`simulate_market()`** with sample arrays and policy parameters?

> Please provide an example call to this program.

Here's an example call that demonstrates **how to use the unified simulation interface** (**`simulate_market`**) to evaluate all perspectives in the composed market model.
This example is illustrative — it shows *how the inputs are structured*, but it does **not execute** any code.

```python
# ============================================================
# EXAMPLE CALL TO THE UNIFIED MARKET SIMULATION
# ============================================================

# Import the unified model (assuming the above code is saved as unified_market.py)
# from unified_market import simulate_market

# --- Example control variables for the renewable producer ---
T = 24  # number of hourly intervals

# Renewable producer's bid quantities (in MWh)
b_R = np.array([45 + 5*np.sin(t/3) for t in range(T)])

# Renewable producer's bid prices ($/MWh)
p_R = np.array([65 + 2*np.cos(t/4) for t in range(T)])

# Realized renewable generation (deeply uncertain)
P_R = np.array([50 + 10*np.random.lognormal(mean=0, sigma=0.2) for _ in range(T)])

# --- Example policy parameters (system planner's controls) ---
policy_params = {
    'q_u': 55,   # penalty for under-delivery
    'q_o': 25    # penalty for over-delivery
}

# --- Simulate the unified model ---
results = simulate_market(
    b_R=b_R,
    p_R=p_R,
    P_R=P_R,
    policy_params=policy_params,
    seed=123  # random seed for reproducibility
)

# --- Display results ---
print("Unified Market Simulation Results:")
for perspective, metrics in results.items():
    print(f"\n{perspective}:")
    for k, v in metrics.items():
        print(f"  {k}: {v}")
```

🔍 Explanation of Inputs

Variable	Description
b_R	Renewable producer's bid quantities (control variables)
p_R	Renewable producer's bid prices (control variables)
P_R	Realized renewable generation (uncertain parameter)
policy_params	System planner's control parameters — penalties for imbalance
seed	Random seed for reproducibility of stochastic sampling

📦 Expected Output (Structure)

The call to **`simulate_market()`** returns a dictionary:

```python
{
  "RenewableProducer": {
      "ExpectedProfit": ...,
      "ProfitVariance": ...
  },
  "ConventionalProducers": {
      "ExpectedProfits": [..., ..., ...],
      "BidVariances": [..., ..., ...]
  },
  "MarketOperator": {
      "ImbalanceCost": ...,
      "PriceVariance": ...
  },
  "SystemPlanner": {
      "Welfare": ...,
      "Reliability": ...,
      "Emissions": ...
  }
}
```

This structure enables modular analysis — you can focus on one perspective (e.g., the renewable producer's profit or the planner's welfare) while holding others' behaviors constant.

Would you like me to show how to **extend this interface** for repeated simulations (e.g., Monte Carlo evaluation across many uncertainty realizations)?