Wind-power bidding model

An energy company is interested in investing in wind-power production. To assess the viability of this investment, the company wants to evaluate the expected revenue of a wind-power producer employing the optimal bidding strategy in a competitive spot market.

Suppose there are five energy producers participating in the day-ahead spot market. Three of them use conventional sources for energy production, such as coal, while the other two producers use solar and wind power, respectively. The market is cleared once per day for 24 hourly intervals of the following day. For each dispatch interval, every producer submits a bid specifying the energy quantity (MWh) it is willing to supply and the corresponding minimum acceptable price (per MWh). Then, the market operator follows a merit-order process to determine the market-clearing price and dispatch schedule, ensuring the total market demand is met.

The wind-power producer aims to maximize its expected revenue by strategically selecting the hourly bid quantity and price for the following day. This exercise will inform the company's investment strategy in wind-power production. However, this process involves some uncertainties. First, the actual energy production of the wind-power producer is also uncertain, as it depends on weather conditions. If the producer under-delivers or over-delivers, the shortfall or surplus would incur a cost. Secondly, the market clearing price is stochastic, determined by the bids from all producers, thus the wind-power producer needs to compete in the market.

Please formalize this problem description as a model. Please also specify key components, such as state variables, stochastic variables, constant parameters, uncertainties and transition functions.

Excellent — this is a rich modeling problem involving **stochastic optimization** under uncertainty. Let's build a **formal model** for the wind-power producer's optimal bidding strategy in the day-ahead electricity market.

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1. Problem Setting

We model the decision-making of a **wind-power producer (WP)** participating in a **day-ahead spot market** with 24 hourly intervals $t = 1, 2, \dots, 24$.

The WP chooses, for each hour, a **bid price** and a **bid quantity** to maximize expected daily revenue, accounting for:

- stochastic **market-clearing prices**, and
- stochastic **actual wind generation**.

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2. Sets and Indices

Symbol	Description
$t \in T = \{1, \dots, 24\}$	Set of hourly market intervals

Symbol	Description
$i \in I = \{1, \ldots, 5\}$	Set of producers (3 conventional, 1 solar, 1 wind)

3. Decision Variables

Symbol	Type	Description
$q_t \geq 0$	Decision	Bid quantity (MWh) offered by WP for hour t
$p_t^{\text{bid}} \geq 0$	Decision	Minimum acceptable price (€/MWh) offered by WP for hour t

The pair (p_t^{bid}, q_t) defines the **hourly bid** submitted to the market.

4. Stochastic Variables

Symbol	Description
$\tilde{P}_t^{\text{market}}$	Market-clearing price (€/MWh) — stochastic outcome of the merit-order process
$\tilde{G}_t$	Actual available wind generation (MWh) for WP
$\tilde{D}_t$	Total market demand (MWh) — affects clearing price indirectly

These variables represent **uncertainties** that are only realized *after* the bidding decision.

5. Constant Parameters

Symbol	Description
c_t^{short}	Penalty cost (€/MWh) for shortfall: if actual generation < committed delivery
c_t^{surp}	Penalty cost (€/MWh) for surplus: if actual generation > committed delivery
π_t	Forecast distribution parameters for wind availability (mean, variance, etc.)
μ_t	Forecast distribution parameters for market price
D_t^{forecast}	Forecast demand
$Q_i^{\max}$	Capacity of each producer i
C_i	Marginal production cost (€/MWh) for conventional producers

6. Market Clearing Mechanism (Merit Order)

The market operator clears the market as follows:

1. Collect all bids $(p_{i,t}^{\text{bid}}, q_{i,t})$ from all producers $i \in I$.
2. Sort bids by ascending price.
3. Determine the **market-clearing price** $\tilde{P}_t^{\text{market}}$ such that total supply = demand $\tilde{D}_t$.
4. All producers with bid price ≤ $\tilde{P}_t^{\text{market}}$ are **dispatched** and receive the market price.

For the wind producer:

$$\text{Dispatched quantity: } Q_t^{\text{disp}} = \begin{cases} q_t, & \text{if } p_t^{\text{bid}} \le \tilde{P}_t^{\text{market}} \\ 0, & \text{otherwise} \end{cases}$$

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7. Revenue and Penalty Structure

After realization of uncertainties:

1. **Revenue from market participation:**

$$R_t = Q_t^{\text{disp}} \times \tilde{P}_t^{\text{market}}$$

2. **Shortfall/surplus adjustment cost:**

$$C_t = c_t^{\text{short}} \cdot \max(0, Q_t^{\text{disp}} - \tilde{G}_t) + c_t^{\text{surp}} \cdot \max(0, \tilde{G}_t - Q_t^{\text{disp}})$$

3. **Net profit (random variable):**

$$\tilde{\Pi}_t = R_t - C_t$$

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8. Objective Function

The wind producer chooses bids to **maximize expected daily revenue (or profit)**:

$$\max_{\{q_t, p_t^{\text{bid}}\}_{t \in T}} \mathbb{E}\left[\sum_{t=1}^{24} \tilde{\Pi}_t\right]$$

Subject to:

- Capacity constraint: $0 \le q_t \le \bar{G}_t^{\text{forecast}}$
- Non-negativity: $p_t^{\text{bid}} \ge 0, q_t \ge 0$
- Risk constraints (optional): e.g. CVaR or variance limit on profit

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9. State and Transition Structure

Although each hour is a separate market interval, wind conditions and prices are often temporally correlated — so we can model it as a **stochastic dynamic programming problem** over t:

State Variables

Symbol	Description
$S_t = (\hat{G}_t, \hat{P}_t^{\text{market}}, \hat{D}_t)$	Forecasts of wind availability, price, and demand at time t
B_t	Bidding decision made at t for day-ahead market (applied to $t+1$)

Transition Function

The evolution of the state is driven by exogenous stochastic processes:

$$S_{t+1} = f(S_t, \xi_t)$$

where ξ_t represents random innovations (e.g., weather shocks, demand uncertainty).

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10. Modeling Approaches

- **Static stochastic optimization (per hour):**

$$\max_{q_t, p_t^{\text{bid}}} \mathbb{E}[\tilde{\Pi}_t]$$

  Each hour independent.

- **Dynamic stochastic programming:**
  If intertemporal correlation or ramping constraints exist, use:

$$V_t(S_t) = \max_{(q_t, p_t^{\text{bid}})} \mathbb{E}\left[\tilde{\Pi}_t + V_{t+1}(S_{t+1}) \mid S_t\right]$$

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11. Summary of Key Model Components

Component	Description
State Variables	Forecasts of wind availability, prices, and demand S_t
Decision Variables	Bid price p_t^{bid}, bid quantity q_t
Stochastic Variables	Realized wind output $\tilde{G}_t$, market price $\tilde{P}_t^{\text{market}}$, demand $\tilde{D}_t$
Constants	Penalty parameters $c_t^{\text{short}}, c_t^{\text{surp}}$, capacity limits, cost coefficients
Transition Function	$S_{t+1} = f(S_t, \xi_t)$ driven by weather and market dynamics
Objective	Maximize expected total profit $\mathbb{E}[\sum_t \tilde{\Pi}_t]$
Uncertainties	Weather-driven wind availability; market-clearing prices driven by competitor bids and demand

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Would you like me to extend this into a **mathematical program (e.g., stochastic mixed-integer formulation)** suitable for simulation or optimization (e.g., Monte Carlo or stochastic programming form)?

In fact, the behavior logic of the conventional producers and solar-power producer is known. The bid quantities of the conventional producers are stable, while their bid prices follow normal distributions with deeply uncertain means and variances. The bid quantities of the solar-power producer can be modeled as $max(0, a+b\cos(\frac{2\pi t}{24}))$, and its bid prices also follow a normal distribution with deeply uncertain mean and variance.

Excellent — this is a crucial refinement. Now the model must explicitly **account for the behaviors of the other market participants**, which drive the **market-clearing price distribution** the wind-power producer faces.

Let's integrate these new assumptions into the formal structure of the model.

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1. Updated Context

We still have **five producers**:

- 3 conventional (coal, gas, etc.) $\rightarrow i \in I_C = \{1, 2, 3\}$
- 1 solar $\rightarrow i = s$
- 1 wind $\rightarrow i = w$ (the decision-maker)

The market operates in 24 hourly intervals $t = 1, \ldots, 24$.

The **market-clearing price** $\tilde{P}_t^{\text{market}}$ is determined by all producers' bids and total demand.

Now, the *stochasticity* of the market price is **endogenously linked** to the uncertain bids of other producers.

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2. Conventional Producers

For each conventional producer $i \in I_C$:

- **Bid quantity (MWh)**:

  $$Q_{i,t}^{\text{bid}} = \bar{Q}_i$$

  (stable and deterministic)

- **Bid price (€/MWh)**:

  $$P_{i,t}^{\text{bid}} \sim \mathcal{N}(\mu_{i,t}, \sigma_{i,t}^2)$$

  where both $\mu_{i,t}$ and $\sigma_{i,t}$ are **deeply uncertain parameters** (i.e., not known precisely, may vary by scenario).

The "deep uncertainty" means that the wind producer does not even know the probability distributions of these parameters with confidence.
In a decision-theoretic framework, this would suggest a **robust stochastic optimization** or **info-gap decision** approach rather than a pure expected-value optimization.

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3. Solar Producer

The solar producer's bidding behavior follows:

- **Bid quantity (MWh):**

$$Q_{s,t}^{\text{bid}} = \max\left(0,\ a + b\cos\left(\frac{2\pi t}{24}\right)\right)$$

  where a, b are known constants characterizing typical solar generation over the day.

- **Bid price (€/MWh):**

$$P_{s,t}^{\text{bid}} \sim \mathcal{N}(\mu_{s,t}, \sigma_{s,t}^2)$$

  with $\mu_{s,t}, \sigma_{s,t}$ also deeply uncertain.

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4. Wind Producer (Decision-Maker)

The **wind producer** decides:

Symbol	Description
q_t	Bid quantity (MWh)
p_t^{bid}	Bid price (€/MWh)

and faces stochastic actual generation $\tilde{G}_t$ and stochastic market price $\tilde{P}_t^{\text{market}}$.

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5. Market Clearing with Known Competitor Behavior

For each hour t, the **market-clearing price** $\tilde{P}_t^{\text{market}}$ is determined by the **intersection of total bid supply** and **market demand** $\tilde{D}_t$:

$$\tilde{P}_t^{\text{market}} = \text{ClearingPrice}\left(\{(P_{i,t}^{\text{bid}}, Q_{i,t}^{\text{bid}})\}_{i \in I_C \cup \{s,w\}}, \tilde{D}_t\right)$$

where the clearing function sorts bids in ascending price and finds the price where cumulative supply equals demand.

Because the other producers' bidding behaviors are random, this induces a **stochastic mapping**

$$\tilde{P}_t^{\text{market}} = f_t(p_t^{\text{bid}}, q_t; \Theta_t, \tilde{D}_t)$$

where Θ_t denotes the random vector of all other producers' bid parameters and realizations:

$$\Theta_t = \{P_{i,t}^{\text{bid}} : i \in I_C \cup \{s\}\}$$

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6. Stochastic Components Summary

Type	Symbol	Description
Uncertain competitor prices	$P_{i,t}^{\text{bid}} \sim \mathcal{N}(\mu_{i,t}, \sigma_{i,t}^2)$ for $i \in I_C \cup \{s\}$	
Uncertain demand	$\tilde{D}_t$ (possibly random or scenario-based)	

Type	Symbol	Description
Uncertain wind output	$\tilde{G}_t \sim$ forecast-based distribution	
Deeply uncertain parameters	$\mu_{i,t}, \sigma_{i,t}$: not well characterized probabilistically	
Derived random variable	$\tilde{P}_t^{\text{market}} = f_t(\Theta_t, \tilde{D}_t)$	

7. Revised Objective Function

The wind producer seeks to maximize **expected or robust expected profit** over all hours:

$$\max_{\{q_t, p_t^{\text{bid}}\}_{t\in T}} \mathbb{E}_{\tilde{G}_t, \tilde{P}_t^{\text{market}}} \left[\sum_{t=1}^{24} \left(Q_t^{\text{disp}} \tilde{P}_t^{\text{market}} - c_t^{\text{short}} \max(0, Q_t^{\text{disp}} - \tilde{G}_t) - c_t^{\text{surp}} \max(0, \tilde{G}_t - Q_t^{\text{disp}}) \right) \right]$$

where

$$Q_t^{\text{disp}} = \begin{cases} q_t, & p_t^{\text{bid}} \leq \tilde{P}_t^{\text{market}} \\ 0, & \text{otherwise} \end{cases}$$

If deep uncertainty is represented through **scenarios of the uncertain parameters** $(\mu_{i,t}, \sigma_{i,t}) \in \Omega$, a **robust counterpart** can be written as:

$$\max_{\{q_t, p_t^{\text{bid}}\}} \min_{\omega\in\Omega} \mathbb{E} \left[\sum_t \tilde{\Pi}_t(\omega) \right]$$

This represents a **max–min expected revenue** over possible deep uncertainty scenarios.

8. Revised Model Structure (Summary Table)

Model Element	Description
Decision Variables	q_t, p_t^{bid} (wind producer's bids)
State Variables	Forecasts of wind generation, demand, and competitor parameters $S_t = (\hat{G}_t, \hat{D}_t, \hat{\mu}_t)$
Stochastic Variables	$\tilde{G}_t, \tilde{D}_t, P_{i,t}^{\text{bid}}$
Deeply Uncertain Parameters	$\mu_{i,t}, \sigma_{i,t}$
Transition Function	$S_{t+1} = f(S_t, \xi_t)$ with ξ_t capturing weather/demand evolution
Merit-Order Function	$\tilde{P}_t^{\text{market}} = f_t(p_t^{\text{bid}}, q_t; \Theta_t, \tilde{D}_t)$
Objective	Maximize expected (or robust expected) profit
Uncertainties	Competitor bid prices, demand, and wind generation

9. Conceptual Interpretation

This enhanced model is a **two-level stochastic optimization** problem:

- **Level 1 (strategic decision):** The wind producer chooses (p_t^{bid}, q_t).
- **Level 2 (market realization):** Random draws from the other producers' bids and demand determine the market-clearing price and dispatch outcome.

Because conventional and solar bids are random but structured, the wind producer can simulate many scenarios of market outcomes to estimate its expected profit distribution — or use robust optimization to ensure viability under adverse combinations of competitor behavior and uncertainty.

Would you like me to extend this into a **stochastic programming formulation with explicit scenarios** (e.g., a Monte Carlo or robust counterpart form), suitable for computational solution or simulation (e.g., with scenario trees)?

You can also assume that the market demand and the actual wind-power production follows normal distributions with known means and standard deviations. Please refine your specification and explicitly specify the constant parameters.

Perfect — this allows us to **close the model formally** by specifying all distributions and constants explicitly. Below is a **refined, complete mathematical model** of the wind-power producer's optimal bidding problem in the day-ahead electricity market, incorporating all information so far.

🧭 1. Model Overview

The **wind-power producer (WP)** participates in a **day-ahead spot market** for 24 hourly intervals $t = 1, \dots, 24$.

The WP chooses hourly **bid quantities** q_t and **bid prices** p_t^{bid} to maximize expected profit, considering:

- stochastic **market-clearing price**, driven by competitor bids and market demand,
- stochastic **actual wind generation**, and
- penalties for over- or under-delivery.

⚙️ 2. Sets and Indices

Symbol	Description
$T = \{1, \dots, 24\}$	Set of hourly dispatch intervals
$I_C = \{1, 2, 3\}$	Set of conventional producers
s	Solar-power producer
w	Wind-power producer (decision-maker)

🔢 3. Decision Variables (Chosen by Wind Producer)

Symbol	Units	Description
$q_t \geq 0$	MWh	Bid quantity offered by the wind producer at hour t
$p_t^{\text{bid}} \geq 0$	€/MWh	Minimum acceptable price offered by the wind producer at hour t

🎲 4. Stochastic Variables

Symbol	Units	Distribution	Description
$\tilde{G}_t$	MWh	$\mathcal{N}(\mu_t^G, (\sigma_t^G)^2)$	Actual wind generation available
$\tilde{D}_t$	MWh	$\mathcal{N}(\mu_t^D, (\sigma_t^D)^2)$	Market demand
$P_{i,t}^{\text{bid}}$	€/MWh	$\mathcal{N}(\mu_{i,t}, \sigma_{i,t}^2)$	Bid price of conventional producer $i \in I_C$
$P_{s,t}^{\text{bid}}$	€/MWh	$\mathcal{N}(\mu_{s,t}, \sigma_{s,t}^2)$	Bid price of solar producer
$\tilde{P}_t^{\text{market}}$	€/MWh	Endogenous (depends on bids and demand)	Market-clearing price

📉 5. Deterministic Competitor Quantities

Symbol	Units	Description
$Q_{i,t}^{\text{bid}} = \bar{Q}_i$	MWh	Constant bid quantity from each conventional producer $i \in I_C$
$Q_{s,t}^{\text{bid}} = \max(0, a + b\cos(\frac{2\pi t}{24}))$	MWh	Solar producer's bid quantity as a function of time of day

📘 6. Constant Parameters

Symbol	Units	Description
a, b	MWh	Solar generation profile coefficients
$\bar{Q}_i$	MWh	Bid quantity of conventional producer i
$\mu_{i,t}, \sigma_{i,t}$	€/MWh	Mean and standard deviation of conventional producer i's bid price distribution
$\mu_{s,t}, \sigma_{s,t}$	€/MWh	Mean and standard deviation of solar producer's bid price distribution
μ_t^G, σ_t^G	MWh	Mean and standard deviation of WP's available generation
μ_t^D, σ_t^D	MWh	Mean and standard deviation of market demand
c_t^{short}	€/MWh	Penalty for shortfall (if delivered < committed)
c_t^{surp}	€/MWh	Penalty for surplus (if delivered > committed)
$\bar{Q}_w$	MWh	Maximum bidding capacity of the wind producer
r	—	Risk aversion or robustness parameter (if used in robust optimization)

⚖️ 7. Market Clearing Mechanism

For each hour t:

1. All producers submit their bids:

$$\{(P_{i,t}^{\text{bid}}, Q_{i,t}^{\text{bid}})\}_{i \in I_C \cup \{s,w\}}$$

2. The market operator sorts bids by ascending price.
3. The **market-clearing price** $\tilde{P}_t^{\text{market}}$ is determined such that total offered quantity equals market demand:

$$\sum_{i \in I_C \cup \{s,w\}} Q_{i,t}^{\text{disp}} = \tilde{D}_t$$

4. The dispatch decision for each producer is:

$$Q_{i,t}^{\text{disp}} = \begin{cases} Q_{i,t}^{\text{bid}}, & \text{if } P_{i,t}^{\text{bid}} \leq \tilde{P}_t^{\text{market}} \\ 0, & \text{otherwise.} \end{cases}$$

For the wind producer:

$$Q_{w,t}^{\text{disp}} = \begin{cases} q_t, & p_t^{\text{bid}} \leq \tilde{P}_t^{\text{market}} \\ 0, & \text{otherwise.} \end{cases}$$

💰 8. Profit Function

Given realizations of stochastic variables, the wind producer's realized profit in hour t is:

$$\tilde{\Pi}_t = Q_{w,t}^{\text{disp}} \tilde{P}_t^{\text{market}} - c_t^{\text{short}} \max(0, Q_{w,t}^{\text{disp}} - \tilde{G}_t) - c_t^{\text{surp}} \max(0, \tilde{G}_t - Q_{w,t}^{\text{disp}})$$

🧮 9. Objective Function

The wind producer aims to **maximize the expected total daily profit**:

$$\max_{\{q_t, p_t^{\text{bid}}\}_{t \in T}} \mathbb{E}\left[\sum_{t=1}^{24} \tilde{\Pi}_t\right]$$

subject to:

$$0 \leq q_t \leq \bar{Q}_w, \quad p_t^{\text{bid}} \geq 0$$

If deep uncertainty is important (uncertain means/variances of competitor bids), a **robust counterpart** can be formulated as:

$$\max_{\{q_t, p_t^{\text{bid}}\}} \min_{\omega \in \Omega} \mathbb{E}_\omega \left[\sum_{t=1}^{24} \tilde{\Pi}_t(\omega)\right]$$

where Ω is the set of plausible scenarios for $(\mu_{i,t}, \sigma_{i,t}, \mu_{s,t}, \sigma_{s,t})$.

🔄 10. Transition Function and State Evolution

Wind generation and market demand may exhibit temporal correlation (e.g., weather patterns):

$$S_t = (\hat{G}_t, \hat{D}_t, \hat{\mu}_t)$$

$$S_{t+1} = f(S_t, \xi_t)$$

where ξ_t represents stochastic weather and demand innovations.

However, if each hour is considered independent (no ramping or carryover effects), the problem decomposes into **24 independent hourly optimizations**.

🧩 11. Summary of Model Components

Category	Symbol / Description
Decision Variables	q_t, p_t^{bid}
Stochastic Variables	$\tilde{G}_t, \tilde{D}_t, P_{i,t}^{\text{bid}}, P_{s,t}^{\text{bid}}$
State Variables	Forecasts $S_t = (\hat{G}_t, \hat{D}_t, \hat{\mu}_t)$
Constants	$a, b, \bar{Q}_i, \mu_{i,t}, \sigma_{i,t}, \mu_t^G, \sigma_t^G, \mu_t^D, \sigma_t^D, c_t^{\text{short}}, c_t^{\text{surp}}, \bar{Q}_w$
Transition Function	$S_{t+1} = f(S_t, \xi_t)$ (optional for dynamic case)
Objective	Maximize expected total profit or robust expected profit
Uncertainties	Market demand, competitor bids, actual wind production

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🔍 12. Interpretation

- The **known competitor behavior** defines a *stochastic but structured market environment* for the wind producer.
- The **normal distributions** of demand and wind generation capture standard forecast uncertainty.
- The **deeply uncertain** bid price parameters for competitors suggest either:
  - Scenario-based **robust optimization**, or
  - Bayesian **hierarchical stochastic modeling**.

Would you like me to now express this as a **stochastic optimization program with explicit scenario indices** (e.g., $\omega \in \Omega$) and expectation approximated by scenario probabilities — ready for numerical simulation or solver implementation?

Various stakeholders approach this problem from different perspectives. Please refine the model specification from the perspectives of: (1) the renewable producer; and (2) the system regulator. These two perspectives should share a common environment, with each stakeholder independently controlling its own control variables. In the specification, specify the common environment, including global parameters and variables. For each perspective, also specify the corresponding model and its key components, such as control variables, transition functions, and objective functions. If there are multiple objectives, provide multiple objective functions instead of a weighted objective function.

Excellent — this request moves from a **single-agent stochastic optimization** to a **multi-agent (multi-stakeholder) system model**, where different agents share a **common environment** but act with distinct goals and control variables.

We'll develop a **two-level, multi-agent model** with:

1. a **shared stochastic market environment**, and
2. two stakeholders:
    - **(1) the renewable (wind) producer**, and
    - **(2) the system regulator** (e.g., market operator or policy planner).

🌍 1. Common Environment (Shared System)

Both the renewable producer and the regulator operate in the same electricity market, with the same uncertainties and system structure.

1.1 Sets and Indices

Symbol	Description
$T = \{1, \ldots, 24\}$	Set of hourly market intervals
$I_C = \{1, 2, 3\}$	Set of conventional producers
s	Solar producer
w	Wind producer (renewable decision-maker)

1.2 Global (Shared) Parameters

Symbol	Units	Description
a, b	MWh	Coefficients in the solar generation profile
$\bar{Q}_i$	MWh	Bid quantity of conventional producer i
$c_t^{\text{short}}, c_t^{\text{surp}}$	€/MWh	Penalty costs for under/over-delivery (apply to renewables)
$\bar{Q}_w$	MWh	Wind producer's capacity limit
$\mu_{i,t}, \sigma_{i,t}$	€/MWh	Mean and std. deviation of bid prices for conventional producers
$\mu_{s,t}, \sigma_{s,t}$	€/MWh	Mean and std. deviation of solar producer's bid prices
μ_t^G, σ_t^G	MWh	Mean and std. deviation of actual wind production
μ_t^D, σ_t^D	MWh	Mean and std. deviation of market demand
ρ_t	—	Correlation between demand and renewable generation (optional)

1.3 Global Stochastic Variables

Symbol	Units	Distribution	Description
$\tilde{D}_t$	MWh	$\mathcal{N}(\mu_t^D, (\sigma_t^D)^2)$	Market demand
$\tilde{G}_t$	MWh	$\mathcal{N}(\mu_t^G, (\sigma_t^G)^2)$	Available wind generation
$P_{i,t}^{\text{bid}}$	€/MWh	$\mathcal{N}(\mu_{i,t}, \sigma_{i,t}^2)$	Conventional bid price
$P_{s,t}^{\text{bid}}$	€/MWh	$\mathcal{N}(\mu_{s,t}, \sigma_{s,t}^2)$	Solar bid price
$Q_{s,t}^{\text{bid}}$	MWh	$\max(0, a + b\cos(\frac{2\pi t}{24}))$	Solar bid quantity
$\tilde{P}_t^{\text{market}}$	€/MWh	Endogenous	Market-clearing price

1.4 Common Transition Function (Environment Dynamics)

The environment evolves through **exogenous stochastic processes** (e.g., weather and demand):

$$S_t = (\tilde{G}_t, \tilde{D}_t, \{P_{i,t}^{\text{bid}}\}, P_{s,t}^{\text{bid}})$$

$$S_{t+1} = f(S_t, \xi_t)$$

where ξ_t captures random shocks (e.g., weather, demand, competitor bids).

This transition function is **common to both stakeholders**.

1.5 Market-Clearing Mechanism (Shared Function)

For each hour t:

$$\tilde{P}_t^{\text{market}} = \text{ClearingPrice}\Big(\{(P_{i,t}^{\text{bid}}, Q_{i,t}^{\text{bid}})\}_{i \in I_C \cup \{s,w\}}, \tilde{D}_t\Big)$$

with the dispatch rule:

$$Q_{i,t}^{\text{disp}} = \begin{cases} Q_{i,t}^{\text{bid}}, & \text{if } P_{i,t}^{\text{bid}} \le \tilde{P}_t^{\text{market}} \\ 0, & \text{otherwise.} \end{cases}$$

The **market price** and **dispatch outcomes** are shared results that both stakeholders observe.

🌬️ 2. Renewable Producer Perspective (Wind Producer)

The wind producer acts as an **individual profit-maximizing agent** operating within the shared stochastic environment.

2.1 Control Variables (Wind Producer's Decisions)

Symbol	Description
q_t	Bid quantity (MWh) offered at hour t
p_t^{bid}	Bid price (€/MWh) offered at hour t

Control set:

$$U_w = \{(q_t, p_t^{\text{bid}}) \mid 0 \leq q_t \leq \bar{Q}_w,\ p_t^{\text{bid}} \geq 0\}$$

2.2 State Variables (Wind Producer's Knowledge)

Symbol	Description
$S_t = (\hat{G}_t, \hat{D}_t, \hat{\mu}_t)$	Forecasts of wind, demand, and competitor prices
$\tilde{G}_t$	Realized generation
$\tilde{P}_t^{\text{market}}$	Realized market price (after clearing)

2.3 Transition Function

$$S_{t+1} = f(S_t, \xi_t)$$

where ξ_t are random shocks (weather, market dynamics).

2.4 Objective Functions (Multiple Possible Perspectives)

The renewable producer may consider **multiple objectives** beyond pure expected profit:

1. **Expected profit maximization:**

$$\max_{U_w} \mathbb{E}\left[\sum_{t=1}^{24}\left(Q_{w,t}^{\text{disp}}\tilde{P}_t^{\text{market}} - c_t^{\text{short}}\max(0, Q_{w,t}^{\text{disp}} - \tilde{G}_t) - c_t^{\text{surp}}\max(0, \tilde{G}_t - Q_{w,t}^{\text{disp}})\right)\right]$$

2. **Risk minimization (optional second objective):**

$$\min_{U_w} \operatorname{Var}\left[\sum_{t=1}^{24}\tilde{\Pi}_t\right]$$

3. **Reliability maximization (optional):**

$$\max_{U_w} \Pr\left(\tilde{G}_t \geq Q_{w,t}^{\text{disp}},\ \forall t\right)$$

Thus, the renewable producer's model is potentially **multi-objective**: maximize expected profit, minimize risk, and ensure delivery reliability.

🏛️ 3. System Regulator Perspective

The system regulator (e.g., market operator, planner, or policymaker) acts in the same environment but with **different control levers** and **societal objectives**.

3.1 Regulator's Control Variables

Symbol	Description
τ_t^{renew}	Renewable subsidy or feed-in tariff (€/MWh)
c_t^{pen}	Penalty adjustment factor for imbalance costs
η_t^{cap}	Capacity target or renewable penetration requirement
r_t^{CO2}	Carbon price or tax (€/ton CO_2) applied to conventional producers

Control set:

$$U_R = \{(\tau_t^{\text{renew}}, c_t^{\text{pen}}, \eta_t^{\text{cap}}, r_t^{\text{CO2}})\}$$

These controls influence:

- Market-clearing prices,
- Producer incentives, and
- System-level renewable integration outcomes.

3.2 State Variables

Symbol	Description
S_t	Same as global state (market environment)
R_t	Renewable share in generation mix at time t
E_t	System-wide emissions (e.g., from conventional units)
P_t^{avg}	Average consumer price at hour t

3.3 Transition Function

The regulator's interventions affect system evolution:

$$S_{t+1} = g(S_t, U_R, \xi_t)$$

where, for instance, increasing r_t^{CO2} raises conventional producers' bid prices, shifting the merit order and altering $\tilde{P}_t^{\text{market}}$.

3.4 Objective Functions (Multiple Policy Objectives)

1. **Market Efficiency Objective (Welfare Maximization):**

$$\max_{U_R} \mathbb{E}\left[\sum_{t=1}^{24}(\text{Consumer Surplus}_t + \text{Producer Surplus}_t - \text{External Costs}_t)\right]$$

2. **Reliability Objective (Supply Security):**

$$\max_{U_R} \Pr\left(\sum_i Q_{i,t}^{\text{disp}} \geq \tilde{D}_t, \forall t\right)$$

3. **Sustainability Objective (Emission Minimization):**

$$\min_{U_R} \mathbb{E}\left[\sum_{t=1}^{24} E_t(r_t^{\text{CO2}})\right]$$

4. **Price Stability Objective:**

$$\min_{U_R} \text{Var}\left[\tilde{P}_t^{\text{market}}\right]$$

These objectives reflect **societal trade-offs** between economic efficiency, reliability, and environmental goals.

⚖️ 4. Summary: Shared Environment with Independent Agents

Component	Common	Wind Producer	Regulator
State Variables	S_t (market demand, renewable availability, competitor bids)	Observed forecasts and outcomes	Observed system-wide indicators (price, emissions, renewables share)
Control Variables	—	q_t, p_t^{bid}	$\tau_t^{\text{renew}}, c_t^{\text{pen}}, \eta_t^{\text{cap}}, r_t^{\text{CO2}}$
Transition Function	$S_{t+1} = f(S_t, \xi_t)$	Same environment	Modified by regulator's policies $S_{t+1} = g(S_t, U_R, \xi_t)$
Objectives	—	Profit, risk, reliability	Welfare, reliability, sustainability, price stability
Interaction	Market-clearing price $\tilde{P}_t^{\text{market}}$ and dispatch outcomes couple their behavior		

🧠 5. Conceptual Interpretation

- The **renewable producer** optimizes its private expected return subject to uncertain market and production conditions.
- The **regulator** designs **market rules and policy instruments** (tariffs, penalties, carbon prices) to achieve system-level objectives.
- They share the same **stochastic market environment**, linked via the **market-clearing mechanism** and **environmental transition functions**.

This structure can be represented as a **stochastic bi-level game**:

$$\begin{cases} \text{Wind Producer:} & \max_{U_w} \ \mathbb{E}[\Pi_w(U_w, U_R, S)] \\ \text{Regulator:} & \max_{U_R} \ \{\text{Welfare}(U_w^*(U_R), S), \text{Reliability}(S), \text{Sustainability}(S)\} \end{cases}$$

Would you like me to extend this into a **bi-level stochastic programming formulation**, where the regulator's decision anticipates the renewable producer's optimal response (e.g., a Stackelberg equilibrium formulation)?

> Your specification should only include implementable and quantifiable components. For example, your specification did not specify how capacity target or carbon price would affect the conventional producers' bids. Please consider refining these conceptual components or remove them.

Excellent — you're asking for a **fully implementable, quantitatively explicit model**, free from abstract policy levers or conceptual placeholders. Let's refine the previous structure to include only **measurable, directly parameterized, and model-implementable components**, while retaining the dual perspectives:

1. **Renewable Producer (wind power)**
2. **System Regulator (market operator or policymaker)**

Both operate in a **shared stochastic market environment** with explicitly defined relationships.

🌍 1. Common Environment (Shared Framework)

Both stakeholders share the same stochastic electricity market, where supply and demand determine a **market-clearing price** through a merit-order mechanism.

1.1 Sets and Indices

Symbol	Description
$T = \{1, \ldots, 24\}$	Set of hourly dispatch intervals
$I_C = \{1, 2, 3\}$	Set of conventional producers
s	Solar producer
w	Wind producer (renewable decision-maker)

1.2 Global Parameters (Known and Quantifiable)

Symbol	Units	Description
a, b	MWh	Solar generation coefficients
$\bar{Q}_i$	MWh	Bid quantity of conventional producer i

Symbol	Units	Description
$\bar{Q}_w$	MWh	Capacity of wind producer
$c_t^{\text{short}}, c_t^{\text{surp}}$	€/MWh	Penalty for wind under- or over-delivery
α_i	€/MWh	Marginal fuel cost component for conventional producer i
β_i	€/MWh	Bid markup sensitivity to regulatory penalty or cost adjustment
μ_t^D, σ_t^D	MWh	Mean and std. deviation of market demand
μ_t^G, σ_t^G	MWh	Mean and std. deviation of available wind generation
$\mu_{s,t}, \sigma_{s,t}$	€/MWh	Mean and std. deviation of solar producer's bid price

1.3 Global Stochastic Variables

Symbol	Distribution	Description
$\tilde{D}_t \sim \mathcal{N}(\mu_t^D, (\sigma_t^D)^2)$	Market demand	
$\tilde{G}_t \sim \mathcal{N}(\mu_t^G, (\sigma_t^G)^2)$	Actual wind generation	
$P_{s,t}^{\text{bid}} \sim \mathcal{N}(\mu_{s,t}, (\sigma_{s,t})^2)$	Solar bid price	
$P_{i,t}^{\text{bid}}$	Conventional bid price, defined below	
$\tilde{P}_t^{\text{market}}$	Market-clearing price (endogenous)	

1.4 Common Market Mechanism

For each hour t:

1. Each producer submits a bid $(P_{i,t}^{\text{bid}}, Q_{i,t}^{\text{bid}})$.
2. The market operator sorts bids in ascending price order.
3. The market-clearing price $\tilde{P}_t^{\text{market}}$ is the marginal price where cumulative supply meets demand.

Formally:

$$\tilde{P}_t^{\text{market}} = \text{ClearingPrice}\Big(\{(P_{i,t}^{\text{bid}}, Q_{i,t}^{\text{bid}})\}_{i \in I_C \cup \{s,w\}}, \tilde{D}_t\Big)$$

and

$$Q_{i,t}^{\text{disp}} = \begin{cases} Q_{i,t}^{\text{bid}}, & \text{if } P_{i,t}^{\text{bid}} \leq \tilde{P}_t^{\text{market}} \\ 0, & \text{otherwise.} \end{cases}$$

🌬️ 2. Renewable Producer (Wind Producer) Model

2.1 Control Variables

Symbol	Units	Description
q_t	MWh	Bid quantity offered for hour t
p_t^{bid}	€/MWh	Bid price offered for hour t

Constraints:

$$0 \leq q_t \leq \bar{Q}_w, \quad p_t^{\text{bid}} \geq 0$$

2.2 Stochastic Inputs (from environment)

Symbol	Description
$\tilde{P}_t^{\text{market}}$	Realized market price after clearing
$\tilde{G}_t$	Realized available wind generation

2.3 Revenue and Cost Components

- **Revenue:**

$$R_t = Q_{w,t}^{\text{disp}} \, \tilde{P}_t^{\text{market}}$$

- **Imbalance penalty cost:**

$$C_t = c_t^{\text{short}} \max(0, Q_{w,t}^{\text{disp}} - \tilde{G}_t) + c_t^{\text{surp}} \max(0, \tilde{G}_t - Q_{w,t}^{\text{disp}})$$

- **Dispatch decision:**

$$Q_{w,t}^{\text{disp}} = \begin{cases} q_t, & \text{if } p_t^{\text{bid}} \leq \tilde{P}_t^{\text{market}} \\ 0, & \text{otherwise.} \end{cases}$$

2.4 Objective Function(s)

1. **Expected Profit Maximization**

$$\max_{q_t, \, p_t^{\text{bid}}} \mathbb{E}\left[\sum_{t=1}^{24} \left(Q_{w,t}^{\text{disp}} \tilde{P}_t^{\text{market}} - c_t^{\text{short}} \max(0, Q_{w,t}^{\text{disp}} - \tilde{G}_t) - c_t^{\text{surp}} \max(0, \tilde{G}_t - Q_{w,t}^{\text{disp}})\right)\right]$$

2. **Reliability Objective (optional secondary)**

$$\max_{q_t, \, p_t^{\text{bid}}} \Pr\left(\tilde{G}_t \geq Q_{w,t}^{\text{disp}} \; \forall t\right)$$

🏛️ 3. System Regulator (Market Operator) Model

The regulator directly controls **operational parameters that influence bidding behavior and dispatch**, specifically:

- **balancing penalties** (affecting renewable bidding incentives)
- **reserve requirements** (affecting reliability and market price volatility)
- **reference cost adjustments** (affecting conventional producer bid functions)

All controls are measurable and quantifiable.

3.1 Control Variables

Symbol	Units	Description
λ_t	€/MWh	System-wide imbalance penalty factor applied to renewables (scales $c_t^{\text{short}}, c_t^{\text{surp}}$)
ϕ_t	€/MWh	Adjustment to conventional bid price (e.g., due to system cost, fuel surcharge)

Thus, the regulator's decisions affect the market environment through:

$$c_t^{\text{short,eff}} = \lambda_t\, c_t^{\text{short}}, \qquad c_t^{\text{surp,eff}} = \lambda_t\, c_t^{\text{surp}}$$

and

$$P_{i,t}^{\text{bid}} = \alpha_i + \beta_i\, \phi_t + \epsilon_{i,t}, \quad \epsilon_{i,t} \sim \mathcal{N}(0, \sigma_{i,t}^2)$$

where α_i and β_i are empirically calibrated parameters.

3.2 State Variables

Symbol	Description
$S_t = (\tilde{D}_t, \{P_{i,t}^{\text{bid}}\}, P_{s,t}^{\text{bid}}, \tilde{G}_t)$	Market environment
$\tilde{P}_t^{\text{market}}$	Clearing price (determined endogenously)
Loss_t	Total penalty payments due to imbalance

3.3 Transition Function

The regulator's control choices affect future states through expected market behavior:

$$S_{t+1} = f(S_t, \lambda_t, \phi_t, \xi_t)$$

where ξ_t represents stochastic demand and renewable fluctuations.

3.4 Objective Functions

The regulator balances **efficiency**, **reliability**, and **price stability**, all measurable from market outcomes.

1. **Market Efficiency (Total Surplus):**

$$\max_{\lambda_t, \phi_t} \mathbb{E}\left[\sum_{t=1}^{24} (\text{Consumer Surplus}_t + \text{Producer Surplus}_t)\right]$$

where

$\text{Consumer Surplus}_t = \int_0^{\tilde{D}_t} P(q)\, dq - \tilde{D}_t \tilde{P}_t^{\text{market}}$,

and producer surplus can be computed from bids and clearing prices.

2. **Reliability Objective:**

$$\max_{\lambda_t, \phi_t} \Pr\left(\sum_i Q_{i,t}^{\text{disp}} \geq \tilde{D}_t, \forall t\right)$$

3. **Price Stability Objective:**

$$\min_{\lambda_t, \phi_t} \text{Var}\left[\tilde{P}_t^{\text{market}}\right]$$

Each can be evaluated directly from simulated or observed market outcomes.

🔗 4. Summary of the Implementable Shared System

Component	Common Environment	Renewable Producer	System Regulator
State Variables	$S_t = (\tilde{D}_t, \tilde{G}_t, P_{i,t}^{\text{bid}}, P_{s,t}^{\text{bid}})$	Observed forecasts and prices	Observed market outcomes
Control Variables	—	q_t, p_t^{bid}	λ_t, ϕ_t
Transition Function	$S_{t+1} = f(S_t, \xi_t)$	Passive	Affects future prices and reliability via λ_t, ϕ_t
Objective Functions	—	(1) Maximize expected profit; (2) Maximize reliability	(1) Maximize surplus; (2) Maximize reliability; (3) Minimize price variance
Coupling Mechanism	Market-clearing price $\tilde{P}_t^{\text{market}}$ and imbalance penalties link their actions		

⚙️ 5. Implementation Notes

- All quantities $(a, b, \alpha_i, \beta_i, c_t^{\text{short}}, \lambda_t)$ are **empirically calibratable** or **policy-controlled**.
- All stochastic variables follow explicit **normal distributions**.
- Market outcomes (price, surplus, reliability) can be **simulated or measured** from the bid stack and demand realization.
- The framework supports numerical optimization or Monte Carlo simulation for both agents.

✅ **Result:**

This refined specification contains only **implementable, quantifiable, and computationally operational** components, shared within a clearly defined **stochastic market environment**, with

separate, measurable control variables and objectives for the renewable producer and the system regulator.

To ensure internal consistency across these models, please compose them into a single, unified model that shares a common environment and states, while embedding perspective-specific controls and objectives. The composed model should: 1. embed each perspective's control variables and objectives; 2. enable modular evaluation of each perspective; 3. for each perspective, treat other perspectives' controls as exogenous inputs; and 4. incorporate all other components from every perspective.

Excellent — you're now asking for a **unified, internally consistent model** that:

- defines a **single shared stochastic environment**,
- embeds the **wind producer's and the regulator's control and objective structures**,
- allows each perspective to be **evaluated modularly**,
- and ensures that the **interactions** (via the market mechanism) are explicit but **treat the other's decisions as exogenous** during any single-agent optimization.

Below is a clean, mathematically rigorous **composite model specification** that satisfies all four requirements.

⚙️ Unified Market System Model with Multi-Perspective Controls

---

1. Global Environment and Shared Structure

All agents (the wind producer and the regulator) operate in the same **stochastic electricity market**. A common **state vector** evolves through time, driven by exogenous uncertainty and endogenous interactions (e.g., market prices and dispatch results).

1.1 Sets and Indices

Symbol	Description
$T = \{1, \dots, 24\}$	Set of hourly dispatch intervals
$I_C = \{1, 2, 3\}$	Conventional producers
s	Solar producer
w	Wind producer (renewable agent)
R	Regulator (system operator / policymaker)

1.2 Global State Variables (Shared by All Perspectives)

At each hour t:

$$S_t = (\tilde{D}_t, \tilde{G}_t, P^{\text{bid}}_{i,t}, P^{\text{bid}}_{s,t}, \tilde{P}^{\text{market}}_t)$$

Symbol	Units	Description
$\tilde{D}_t$	MWh	Realized market demand
$\tilde{G}_t$	MWh	Realized available wind generation
$P^{\text{bid}}_{i,t}$	€/MWh	Conventional producers' bid prices
$P^{\text{bid}}_{s,t}$	€/MWh	Solar producer's bid price
$\tilde{P}^{\text{market}}_t$	€/MWh	Market-clearing price

1.3 Exogenous Stochastic Dynamics

$$S_{t+1} = f(S_t, \xi_t)$$

where ξ_t are stochastic innovations (weather, demand shocks, etc.) driving:

$$\tilde{D}_t \sim \mathcal{N}(\mu^D_t, (\sigma^D_t)^2), \quad \tilde{G}_t \sim \mathcal{N}(\mu^G_t, (\sigma^G_t)^2)$$

and

$$P^{\text{bid}}_{s,t} \sim \mathcal{N}(\mu_{s,t}, (\sigma_{s,t})^2), \quad \epsilon_{i,t} \sim \mathcal{N}(0, \sigma^2_{i,t})$$

1.4 Global Parameters (Constants)

Symbol	Description
a, b – solar profile coefficients	
$\bar{Q}_i$ – bid quantity for conventional producer i	
$\bar{Q}_w$ – wind producer's capacity	
α_i, β_i – conventional cost and sensitivity parameters	
$c^{\text{short}}_t, c^{\text{surp}}_t$ – imbalance penalties (€/MWh)	
$\mu^D_t, \sigma^D_t, \mu^G_t, \sigma^G_t$ – demand and wind forecast parameters	
$\mu_{s,t}, \sigma_{s,t}$ – solar bid distribution parameters	

All are measurable or calibratable.

1.5 Market Clearing Function (Shared Mechanism)

For each hour t, the **market-clearing price** is determined by:

$$\tilde{P}^{\text{market}}_t = \text{ClearingPrice}\Big(\{(P^{\text{bid}}_{i,t}, Q^{\text{bid}}_{i,t})\}_{i \in I_C \cup \{s,w\}}, \tilde{D}_t\Big)$$

and the **dispatch rule**:

$$Q_{i,t}^{\text{disp}} = \begin{cases} Q_{i,t}^{\text{bid}}, & P_{i,t}^{\text{bid}} \le \tilde{P}_t^{\text{market}} \\ 0, & \text{otherwise.} \end{cases}$$

This function links both perspectives and defines the **interaction** between the wind producer's bids and the regulator's controls.

2. Perspective 1: Renewable Producer (Wind)

2.1 Control Variables

Symbol	Description
q_t	Bid quantity (MWh) for hour t
p_t^{bid}	Bid price (€/MWh) for hour t

Constraints:

$$0 \le q_t \le \bar{Q}_w, \quad p_t^{\text{bid}} \ge 0$$

The wind producer **takes regulator controls** (λ_t, ϕ_t) as **exogenous** inputs.

2.2 Relevant Exogenous Components

- Market-clearing price $\tilde{P}_t^{\text{market}}$ (depends on environment and regulator inputs)
- Actual available generation $\tilde{G}_t$
- Effective penalty coefficients adjusted by the regulator:

$$c_t^{\text{short,eff}} = \lambda_t c_t^{\text{short}}, \quad c_t^{\text{surp,eff}} = \lambda_t c_t^{\text{surp}}$$

2.3 Objective Function(s)

(1) Expected Profit

$$\max_{q_t, p_t^{\text{bid}}} \mathbb{E}\left[\sum_{t=1}^{24} \left(Q_{w,t}^{\text{disp}} \tilde{P}_t^{\text{market}} - c_t^{\text{short,eff}} \max(0, Q_{w,t}^{\text{disp}} - \tilde{G}_t) - c_t^{\text{surp,eff}} \max(0, \tilde{G}_t - Q_{w,t}^{\text{disp}})\right)\right]$$

(2) Reliability (optional secondary)

$$\max_{q_t, p_t^{\text{bid}}} \Pr(\tilde{G}_t \ge Q_{w,t}^{\text{disp}}, \ \forall t)$$

2.4 Modular Evaluation

When evaluating the **renewable producer module**, treat:

- (λ_t, ϕ_t) (regulator controls),

- stochastic state S_t, and
- market-clearing function $\text{ClearingPrice}(\cdot)$
  as **fixed exogenous inputs**.

The renewable submodel outputs:

- Optimal bids $(q_t^*, p_t^{\text{bid}*})$,
- Expected profit and reliability measures,
- Resulting dispatch $Q_{w,t}^{\text{disp}}$.

---

3. Perspective 2: System Regulator (Market Operator)

3.1 Control Variables

Symbol	Units	Description
λ_t	—	Imbalance penalty scaling factor (affects renewables' imbalance cost)
ϕ_t	€/MWh	Cost adjustment factor applied to conventional producers' bids

---

3.2 Effects on Market Participants

- **Renewable penalties:**

$$c_t^{\text{short,eff}} = \lambda_t c_t^{\text{short}}, \quad c_t^{\text{surp,eff}} = \lambda_t c_t^{\text{surp}}$$

- **Conventional producers' bid prices:**

$$P_{i,t}^{\text{bid}} = \alpha_i + \beta_i \phi_t + \epsilon_{i,t}, \quad \epsilon_{i,t} \sim \mathcal{N}(0, \sigma_{i,t}^2)$$

The regulator takes the **wind producer's bid strategy** (q_t, p_t^{bid}) as exogenous input.

3.3 Derived Market Variables

Given all bids and demand, the market-clearing price and dispatch are determined endogenously:

$$(\tilde{P}_t^{\text{market}}, \{Q_{i,t}^{\text{disp}}\}) = \text{ClearingPrice}\big(\lambda_t, \phi_t, q_t, p_t^{\text{bid}}, S_t\big)$$

3.4 Objective Functions

(1) Market Efficiency (Total Surplus)

$$\max_{\lambda_t, \phi_t} \mathbb{E}\left[\sum_{t=1}^{24} \Big(\text{Consumer Surplus}_t + \text{Producer Surplus}_t\Big)\right]$$

where:

$$\text{Consumer Surplus}_t = \int_0^{\tilde{D}_t} P(q)\,dq - \tilde{D}_t \tilde{P}_t^{\text{market}}$$

$$\text{Producer Surplus}_t = \sum_{i \in I_C \cup \{s,w\}} (\tilde{P}_t^{\text{market}} - P_{i,t}^{\text{bid}}) Q_{i,t}^{\text{disp}}$$

(2) Reliability

$$\max_{\lambda_t, \phi_t} \Pr\left(\sum_i Q_{i,t}^{\text{disp}} \geq \tilde{D}_t, \forall t \right)$$

(3) Price Stability

$$\min_{\lambda_t, \phi_t} \text{Var}[\tilde{P}_t^{\text{market}}]$$

3.5 Modular Evaluation

When evaluating the **regulator module**, treat:

- Wind bids (q_t, p_t^{bid}),
- Solar and conventional bidding parameters, and
- Exogenous stochastic processes S_t
  as **given inputs**.

The regulator submodel outputs:

- Optimal (λ_t^*, ϕ_t^*),
- Market-clearing prices and dispatch,
- System metrics: total surplus, reliability, price variance.

---

4. Unified System Composition

4.1 System Coupling and Execution Flow

1. **Common environment** provides S_t (demand, generation, uncertainty).
2. **Regulator** sets (λ_t, ϕ_t) — penalty and bid-cost adjustments.
3. **Wind producer** optimizes bids (q_t, p_t^{bid}) given regulator settings.
4. **Market-clearing mechanism** computes prices $\tilde{P}_t^{\text{market}}$ and dispatch outcomes.
5. **Feedback**: outcomes update the state S_{t+1} for both agents.

4.2 Unified State and Transition

$$S_{t+1} = f(S_t, q_t, p_t^{\text{bid}}, \lambda_t, \phi_t, \xi_t)$$

where $f(\cdot)$ embodies the stochastic environment and endogenous market response.

4.3 Summary Table of Unified Model Components

Category	Symbol(s)	Description	Agent
State Variables	$S_t = (\tilde{D}_t, \tilde{G}_t, P_{i,t}^{\text{bid}}, P_{s,t}^{\text{bid}}, \tilde{P}_t^{\text{market}})$	Common environment state	Shared
Renewable Controls	q_t, p_t^{bid}	Wind producer's bid decisions	Wind producer
Regulator Controls	λ_t, ϕ_t	Penalty and cost adjustment factors	Regulator
Transition Function	$S_{t+1} = f(S_t, q_t, p_t^{\text{bid}}, \lambda_t, \phi_t, \xi_t)$	State evolution	Shared
Market Mechanism	$\tilde{P}_t^{\text{market}} = \text{ClearingPrice}(\cdot)$	Determines price and dispatch	Shared
Wind Objectives	Expected profit, reliability	Renewable submodel	Wind producer
Regulator Objectives	Efficiency, reliability, price stability	Policy submodel	Regulator

4.4 Modularity

- Each agent's submodel is **modularly evaluable**:
  - The **renewable module** can be solved for optimal (q_t, p_t^{bid}) under fixed (λ_t, ϕ_t).
  - The **regulator module** can be solved for optimal (λ_t, ϕ_t) under fixed (q_t, p_t^{bid}).
- Both modules share the same **state vector**, stochastic processes, and market-clearing mechanism, ensuring internal consistency.

✅ **Final Result:**
A **unified stochastic market model** with a **shared environment**, **modular control structure**, and **agent-specific objectives**, suitable for joint or sequential evaluation (e.g., Stackelberg equilibrium, simulation-based coupling, or policy evaluation).
All quantities, controls, and relationships are explicitly quantifiable and implementable.

Now, provide a modular Python implementation of the unified model, using classes to represent different perspectives. At the end of the program, define a function as the interface. This function should accept a dictionary of control variables and a dictionary of uncertain parameter values as input, simulate the model dynamics, and return the value of each objective function as output. This function does not need to be executed.

Below is a **modular, object-oriented Python implementation** of the **unified market model**, built to:

- represent each **perspective** (renewable producer and regulator) as a class,
- share a **common environment and state evolution**,
- and provide a **single simulation interface function** that takes:
  - dictionaries of **control variables** (e.g., bids, penalties), and
  - dictionaries of **uncertain parameter values** (e.g., wind/demand realizations),
    and returns the values of all **objective functions**.

The code is written for simulation and extension — **not for execution now**.

```python
import numpy as np
from dataclasses import dataclass, field
from typing import Dict, List, Tuple

# ===================================================================
# 1. COMMON ENVIRONMENT
# ===================================================================

@dataclass
class MarketEnvironment:
    """Shared stochastic electricity market environment."""
    T: int = 24

    # Exogenous stochastic parameters
    mu_D: np.ndarray = field(default_factory=lambda: np.zeros(24))      # Mean demand
    sigma_D: np.ndarray = field(default_factory=lambda: np.ones(24))    # Std dev demand
    mu_G: np.ndarray = field(default_factory=lambda: np.zeros(24))      # Mean wind generation
    sigma_G: np.ndarray = field(default_factory=lambda: np.ones(24))    # Std dev wind
generation

    # Solar parameters
    a: float = 50.0
    b: float = 40.0
    mu_s: np.ndarray = field(default_factory=lambda: np.full(24, 30.0))
    sigma_s: np.ndarray = field(default_factory=lambda: np.full(24, 5.0))

    # Conventional producer parameters
    alpha: np.ndarray = field(default_factory=lambda: np.array([25.0, 27.0, 30.0]))
    beta: np.ndarray = field(default_factory=lambda: np.array([0.5, 0.6, 0.4]))
    sigma_conv: np.ndarray = field(default_factory=lambda: np.array([2.0, 2.5, 3.0]))
    Q_conv: np.ndarray = field(default_factory=lambda: np.array([100.0, 120.0, 150.0]))

    # Global penalties (nominal)
    c_short: np.ndarray = field(default_factory=lambda: np.full(24, 80.0))
    c_surp: np.ndarray = field(default_factory=lambda: np.full(24, 10.0))

    # Wind capacity
    Q_w_max: float = 100.0

    def solar_quantity(self, t: int) -> float:
        """Solar producer's deterministic bid quantity."""
        return max(0.0, self.a + self.b * np.cos(2 * np.pi * t / 24))

    def sample_state(self, uncertainties: Dict[str, np.ndarray]) -> Tuple[np.ndarray,
np.ndarray, np.ndarray]:
        """Generate stochastic realizations for demand, wind, and solar bids."""
        D = np.random.normal(self.mu_D, self.sigma_D)
        G = np.random.normal(self.mu_G, self.sigma_G)
        P_s = np.random.normal(self.mu_s, self.sigma_s)
        return D, G, P_s


# ===================================================================
# 2. RENEWABLE PRODUCER MODULE
# ===================================================================
```

```
@dataclass
class WindProducer:
    """Wind-power producer's model."""
    env: MarketEnvironment
    q: np.ndarray = field(default_factory=lambda: np.full(24, 50.0))   # bid quantity (control)
    p_bid: np.ndarray = field(default_factory=lambda: np.full(24, 30.0))  # bid price (control)

    # Effective penalties (scaled by regulator)
    lambda_penalty: float = 1.0

    def dispatch_quantity(self, p_market: float, t: int) -> float:
        """Determine dispatched quantity based on bid acceptance."""
        return self.q[t] if self.p_bid[t] <= p_market else 0.0

    def profit_objective(self, p_market: np.ndarray, G: np.ndarray) -> float:
        """Compute expected profit over all hours."""
        profit = 0.0
        for t in range(self.env.T):
            Q_disp = self.dispatch_quantity(p_market[t], t)
            revenue = Q_disp * p_market[t]
            c_short_eff = self.lambda_penalty * self.env.c_short[t]
            c_surp_eff = self.lambda_penalty * self.env.c_surp[t]
            penalty = c_short_eff * max(0, Q_disp - G[t]) + c_surp_eff * max(0, G[t] - Q_disp)
            profit += revenue - penalty
        return profit

    def reliability_objective(self, p_market: np.ndarray, G: np.ndarray) -> float:
        """Probability (fraction of hours) of meeting or exceeding commitment."""
        reliability = 0.0
        for t in range(self.env.T):
            Q_disp = self.dispatch_quantity(p_market[t], t)
            reliability += 1.0 if G[t] >= Q_disp else 0.0
        return reliability / self.env.T


# ================================================================
# 3. REGULATOR MODULE
# ================================================================

@dataclass
class Regulator:
    """System regulator or market operator model."""
    env: MarketEnvironment
    lambda_t: np.ndarray = field(default_factory=lambda: np.ones(24))    # imbalance penalty
scale
    phi_t: np.ndarray = field(default_factory=lambda: np.zeros(24))      # conventional price
adjustment

    def conv_bid_prices(self, t: int) -> np.ndarray:
        """Compute conventional producers' bid prices."""
        prices = []
        for i in range(len(self.env.alpha)):
            base = self.env.alpha[i] + self.env.beta[i] * self.phi_t[t]
            prices.append(np.random.normal(base, self.env.sigma_conv[i]))
        return np.array(prices)

    def market_clearing(self, t: int, q_w: float, p_w: float, P_s: float) -> Tuple[float,
Dict[str, float]]:
        """Compute market-clearing price using simplified merit order."""
        # Construct supply stack
        bids = []
        # Conventional producers
        for i, Q_i in enumerate(self.env.Q_conv):
            bids.append((self.conv_bid_prices(t)[i], Q_i))
        # Solar
        bids.append((P_s, self.env.solar_quantity(t)))
        # Wind
        bids.append((p_w, q_w))
        # Sort by price
        bids = sorted(bids, key=lambda x: x[0])
        # Demand draw
        D_t = np.random.normal(self.env.mu_D[t], self.env.sigma_D[t])
        supply_cum, price = 0.0, 0.0
        for (p, q) in bids:
```

```
            supply_cum += q
            if supply_cum >= D_t:
                price = p
                break
        return price, {"demand": D_t, "supply": supply_cum}

    def evaluate_market(
        self,
        q_w: np.ndarray,
        p_w: np.ndarray,
        P_s: np.ndarray
    ) -> Tuple[np.ndarray, float, float, float]:
        """Simulate market over T and compute objectives."""
        prices, reliability_flags = [], []
        total_consumer_surplus, total_producer_surplus = 0.0, 0.0
        for t in range(self.env.T):
            p_market, stats = self.market_clearing(t, q_w[t], p_w[t], P_s[t])
            prices.append(p_market)
            # Approximate surpluses (simplified for demonstration)
            consumer_surplus = 0.5 * stats["demand"] * max(0, 100 - p_market)
            producer_surplus = stats["supply"] * max(0, p_market - 20)
            total_consumer_surplus += consumer_surplus
            total_producer_surplus += producer_surplus
            reliability_flags.append(stats["supply"] >= stats["demand"])
        prices = np.array(prices)
        reliability = np.mean(reliability_flags)
        efficiency = total_consumer_surplus + total_producer_surplus
        price_variance = np.var(prices)
        return prices, efficiency, reliability, price_variance


# ================================================================
# 4. UNIFIED SIMULATION INTERFACE
# ================================================================

def simulate_unified_model(
    controls: Dict[str, np.ndarray],
    uncertainties: Dict[str, np.ndarray],
    env_params: Dict[str, float] = {}
) -> Dict[str, float]:
    """
    Unified simulation interface.
    Inputs:
        controls: dict with control variables:
            {
              "q_w": np.ndarray(24),
              "p_w": np.ndarray(24),
              "lambda_t": np.ndarray(24),
              "phi_t": np.ndarray(24)
            }
        uncertainties: dict with stochastic parameter overrides (optional)
        env_params: optional overrides for MarketEnvironment parameters
    Output:
        dict with all objective function values:
            {
              "wind_profit": float,
              "wind_reliability": float,
              "reg_efficiency": float,
              "reg_reliability": float,
              "price_variance": float
            }
    """
    # Initialize environment
    env = MarketEnvironment(**env_params)

    # Initialize agents
    wind = WindProducer(env=env, q=controls["q_w"], p_bid=controls["p_w"],
lambda_penalty=np.mean(controls["lambda_t"]))
    regulator = Regulator(env=env, lambda_t=controls["lambda_t"], phi_t=controls["phi_t"])

    # Sample stochastic components
    D, G, P_s = env.sample_state(uncertainties)

    # Run market simulation from regulator’s perspective
    p_market, eff, rel_sys, var_p = regulator.evaluate_market(
```

```
        q_w=controls["q_w"], p_w=controls["p_w"], P_s=P_s
    )

    # Evaluate wind producer objectives given market outcome
    profit = wind.profit_objective(p_market, G)
    rel_wind = wind.reliability_objective(p_market, G)

    return {
        "wind_profit": profit,
        "wind_reliability": rel_wind,
        "reg_efficiency": eff,
        "reg_reliability": rel_sys,
        "price_variance": var_p
    }
```

🧩 Explanation

Structure and Modularity

- **`MarketEnvironment`** → defines shared global state, parameters, and stochastic sampling.
- **`WindProducer`** → encapsulates the renewable producer's bidding controls and objectives.
- **`Regulator`** → controls market-level policy levers (**`λ_t`**, **`φ_t`**), computes clearing prices, and evaluates system-level objectives.
- **`simulate_unified_model()`** → unified callable interface that:
  1. Initializes all components with user-specified controls and uncertainties,
  2. Simulates market outcomes,
  3. Returns all perspective-specific objective values in one dictionary.

Usage Example (conceptual)

```python
controls = {
    "q_w": np.full(24, 60.0),
    "p_w": np.full(24, 35.0),
    "lambda_t": np.full(24, 1.0),
    "phi_t": np.full(24, 0.0)
}
uncertainties = {}  # use defaults
results = simulate_unified_model(controls, uncertainties)
print(results)
```

✅ **Result:**
A **unified, modular, and quantitative Python implementation** of the multi-perspective stochastic market model — ready for simulation, optimization, or scenario analysis.