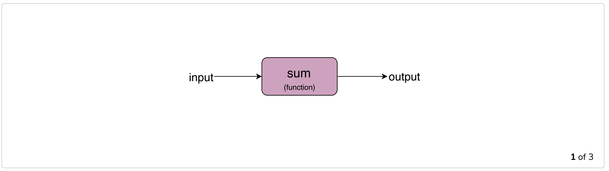
What does the sum of integers from 1 to n mean?

Natural numbers are positive numbers starting from 1. These can be written as:

$1, 2, 3, 4, 5, 6, 7, 8, 9, 10$.....

We want to write a program that takes a particular number and sums up all the numbers from 1 up until that number.

The illustration below explains the concept to help you understand.



The sum of all numbers up to a particular number is equal to the sum of that number and the sum of all the numbers before it. It can be seen as follows:

$\sum_{i=1}^{5} i$

$= 5 + \sum_{i=1}^{4} i$

$= 5 + 4 + \sum_{i=1}^{3}$

$= 5 + 4 + 3 + \sum_{i=1}^{2}$

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$= 5 + 4 + 3 + 2 + 1$

Generic Mathematical Notation

$\sum_{i=1}^{n} i$

$= n + \sum_{i=1}^{n-1} i$

$= n + (n-1) + \sum_{i=1}^{n-2}$

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$= n + (n-1) + (n-2) \ldots + 2 + 1$

Implementing the Code

```
class SummationClass {

    public static int sumAll(int num) {
        if (num == 1) {
            return num;
        }
        else {
            return num + sumAll(num-1);
        }
    }

    public static void main( String args[] ) {
        int input = 5;
        int sum = sumAll(input);
        System.out.println("The sum of integers from 1 to " + input + " is: " + sum);
    }
}
```

Run Save Reset

Understanding the Code

In the code above, the method `sumAll` is a recursive method, since it calls itself in the function body. Below is an explanation of the above code:

Driver Method

- Inside the `main` method, we have defined the integer variable `input` on **line 13**. The code calculates the sum of all the numbers up to the value stored in the `input`.
- The method `sumAll` is called on **line 14** and its return value is stored in an `int` variable, which is defined as `sum`.
- The `System.out.println` command on **line 15** prints the answer when the `sumAll` method is called.

Recursive Method

- The return type of this method is `int` since the sum of all the integers will be an integer.
- This method takes an integer, `num`, as the input argument.

Base Case

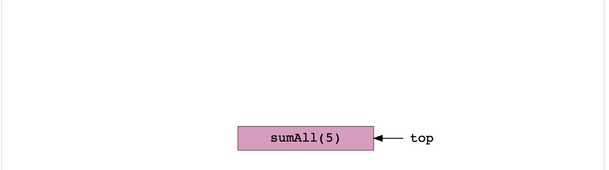
- The base case of the method is defined on **line 4** where the method will terminate and return `num` if the `num<=1` condition is met. This due to the fact that, if the `num` is 1, there are no integers less than 1; hence the sum is 1. All the numbers including 0 and below are returned because they are considered an invalid positive integer input.

Recursive Case

- If the base case condition does not compute as true, the method enters the else block where it makes a recursive call.
- This recursive call takes an input argument of `num-1`. The value returned from `sumAll(num-1)` is added to the input argument, `num`, and returned. This is because, as discussed above, the sum of the integers up to that number is equal to the sum of that number and all the numbers below it.

  $n + \sum_{i=1}^{n-1} i$

Understanding through a Stack



Now that you have learned about computing the sum of integers from 1 to n using recursion, we will solve another interesting mathematical problem using recursion, computing modulo.