

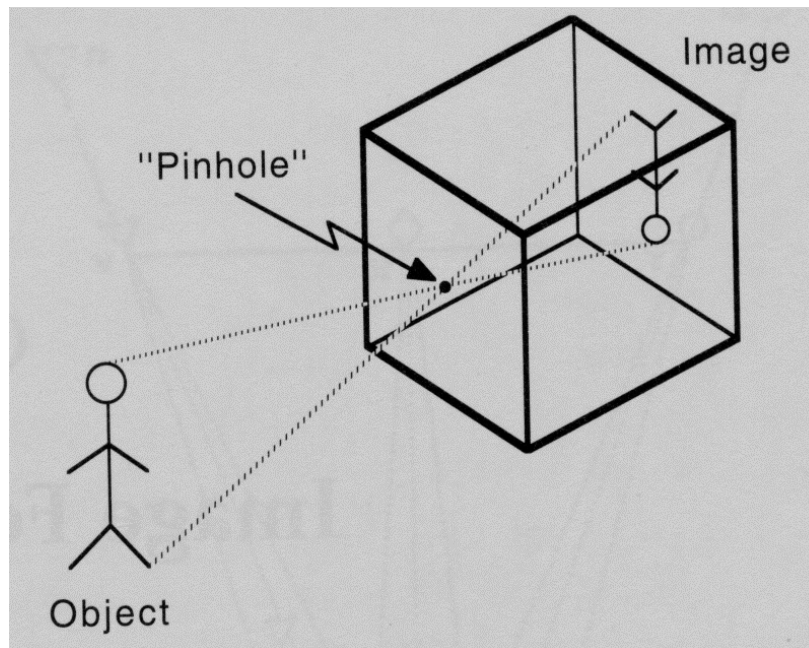
Image Processing

Image Processing

- **Image Fundamental**
- **Digital Image Processing**

Pinhole camera

- This is the simplest device to form an image of a 3D scene on a 2D surface.
- Straight rays of light pass through a “pinhole” and form an inverted image of the object on the image plane.

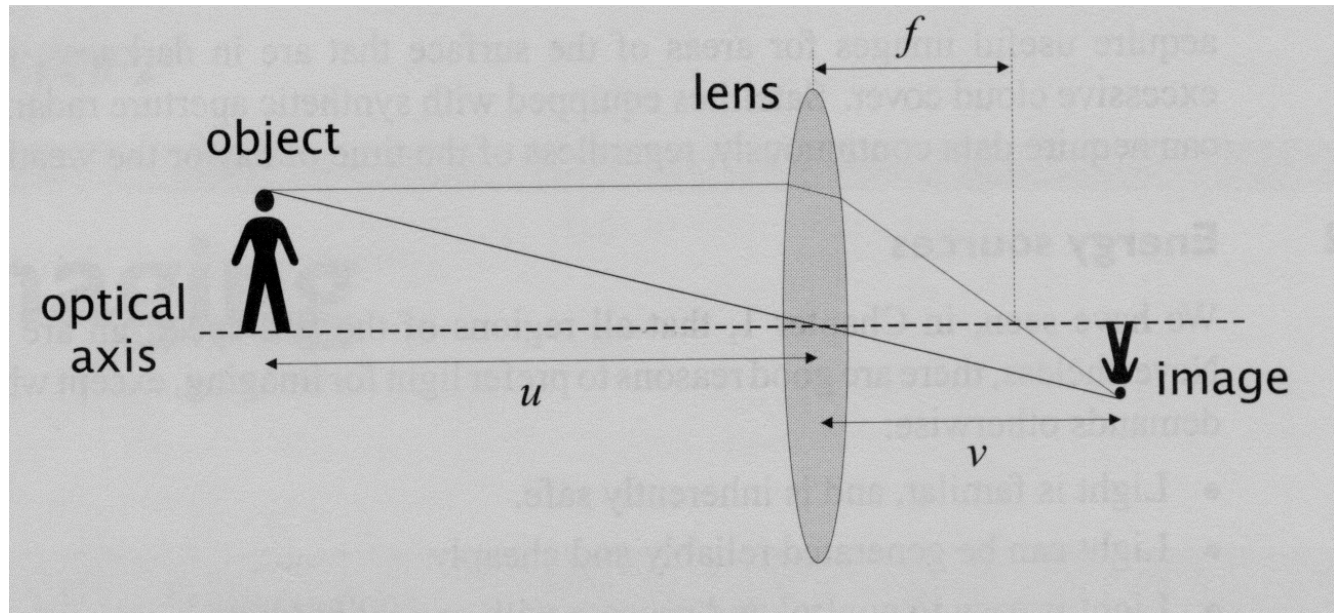


$$x = \frac{fX}{Z}$$

$$y = \frac{fY}{Z}$$

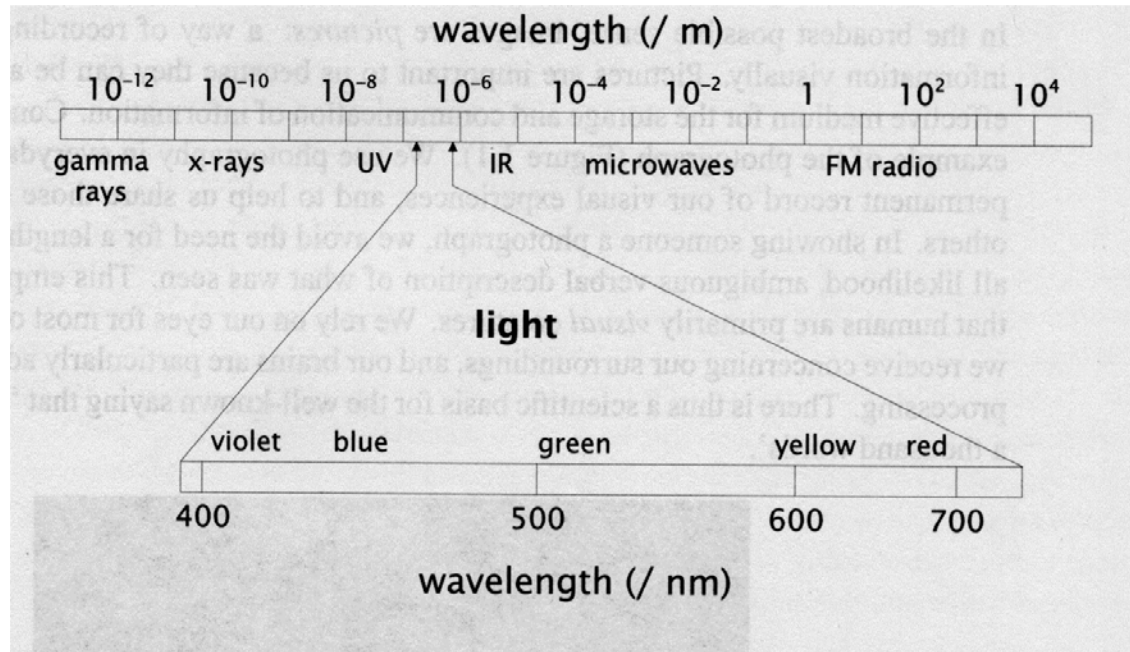
Camera optics

- In practice, the aperture must be larger to admit more light.
- Lenses are placed to in the aperture to **focus** the bundle of rays from each scene point onto the corresponding point in the image plane



What is light?

- The visible portion of the electromagnetic (EM) spectrum.
- It occurs between wavelengths of approximately 400 and 700 nanometers.



Short wavelengths

- Different wavelengths of radiation have different properties.
- The x-ray region of the spectrum, it carries sufficient energy to penetrate a significant volume or material.



Long wavelengths

- Copious quantities of infrared (IR) radiation are emitted from warm objects (e.g., locate people in total darkness).



Sonic images

- Produced by the reflection of sound waves off an object.
- High sound frequencies are used to improve resolution.



CCD (Charged-Coupled Device) cameras

- Tiny solid state cells convert light energy into electrical charge.
- The image plane acts as a digital memory that can be read row by row by a computer.

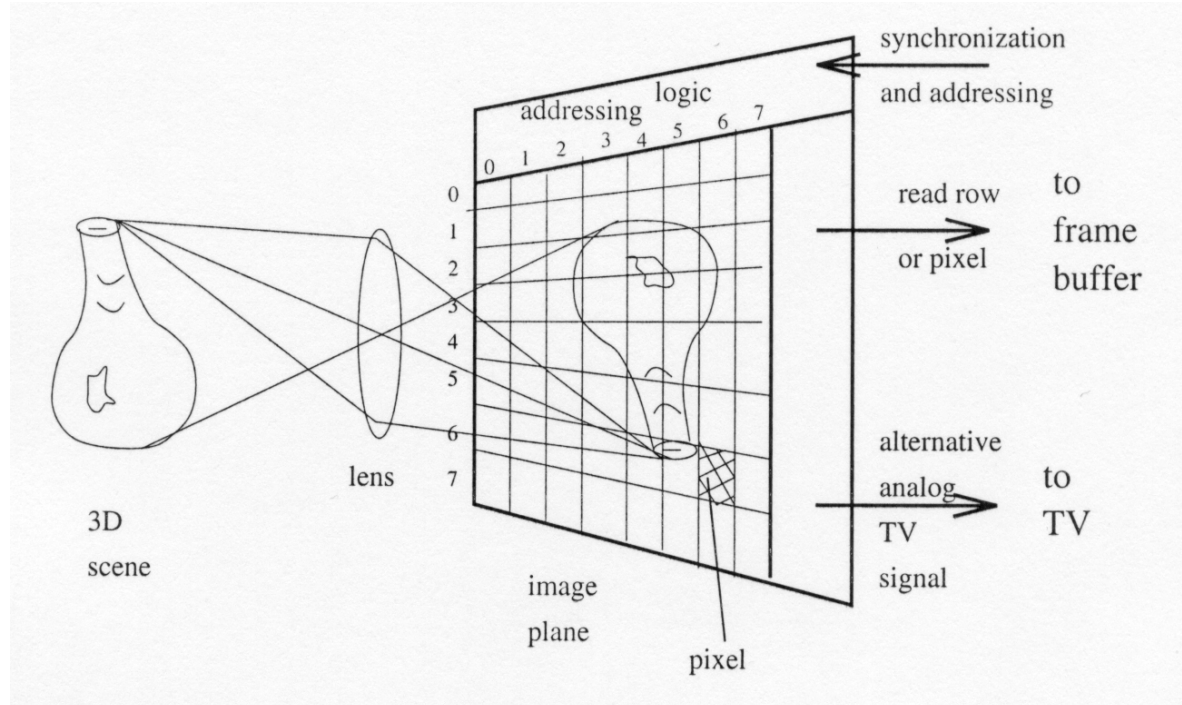
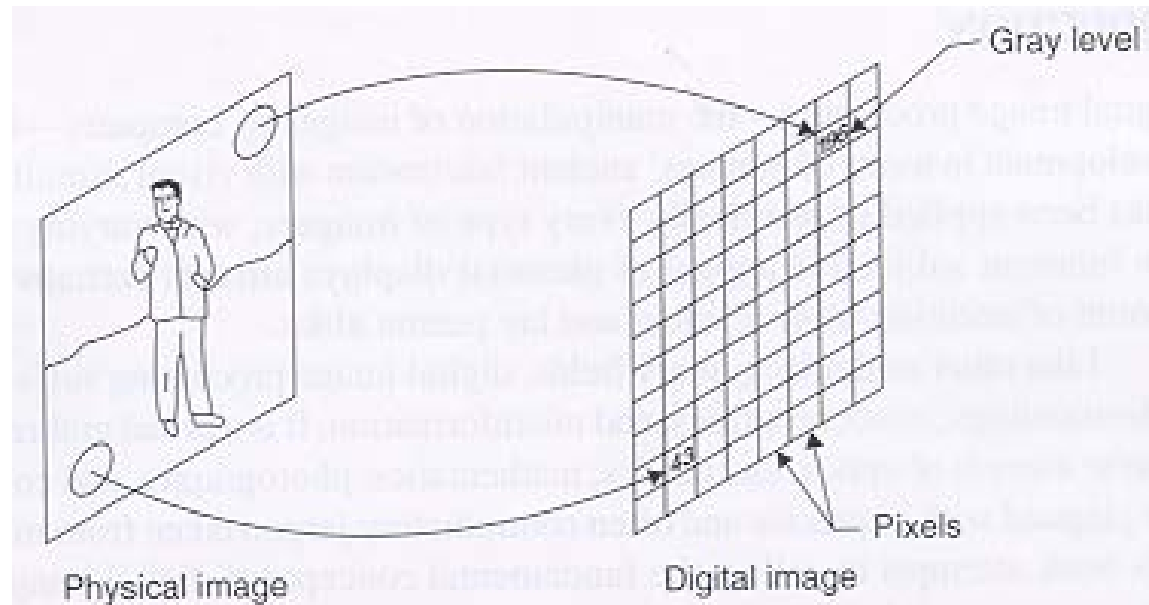
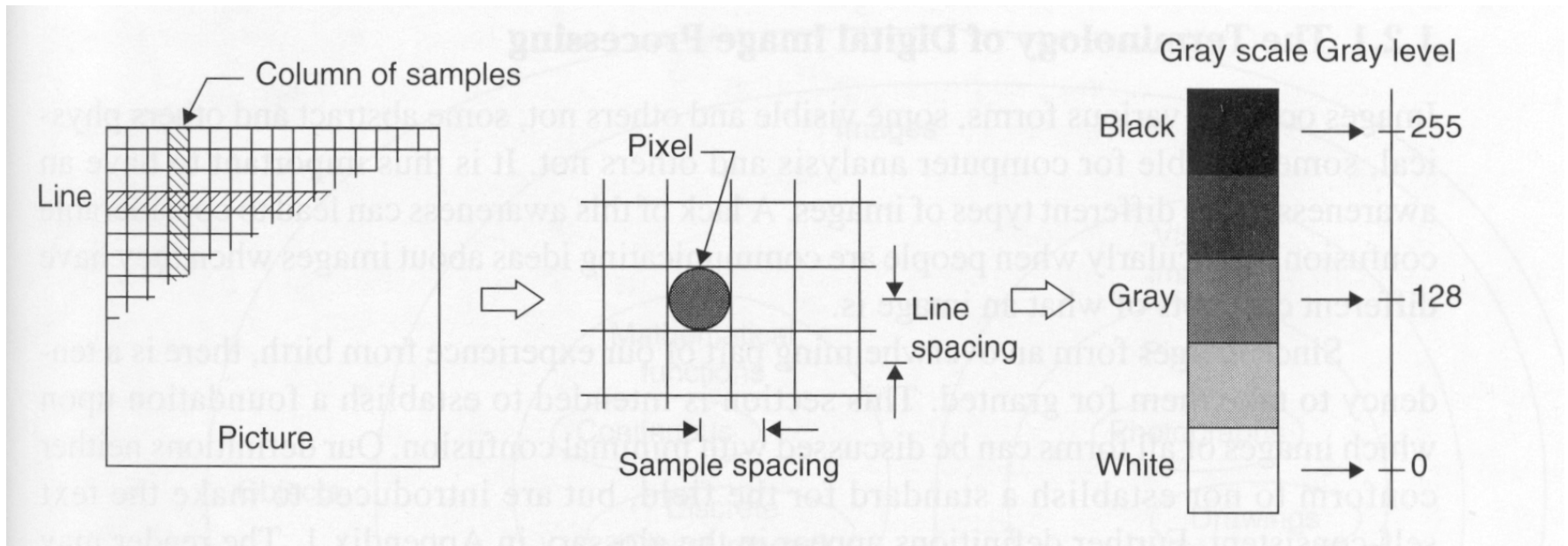


Image digitization



- **Sampling** means measuring the value of an image at a finite number of points.
- **Quantization** is the representation of the measured value at the sampled point by an integer.

Image digitization



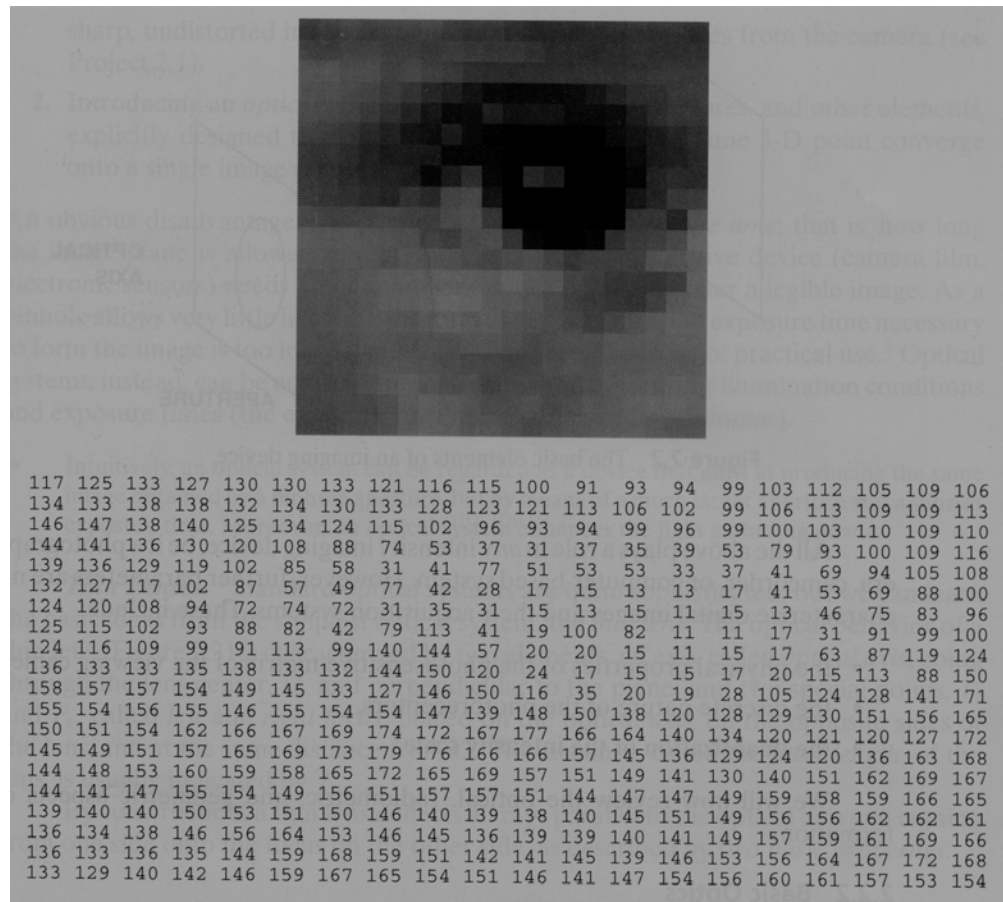
Digital image

- An image is represented by a rectangular array of integers.
- An integer represents the brightness or darkness of the image at that point.
- N: # of rows, M: # of columns

$$\begin{array}{cccc} f(0,0) & f(0,1) & \dots & f(0,M-1) \\ f(1,0) & f(1,1) & \dots & f(1,M-1) \\ \dots & \dots & \dots & \dots \\ f(N-1,0) & f(N-1,1) & \dots & f(N-1,M-1) \end{array}$$

How are images represented in the computer?

**Grey images
(0 to 255,
8 bits*1channel)**



**Color images
(0 to 255,
8 bits*3channel)**

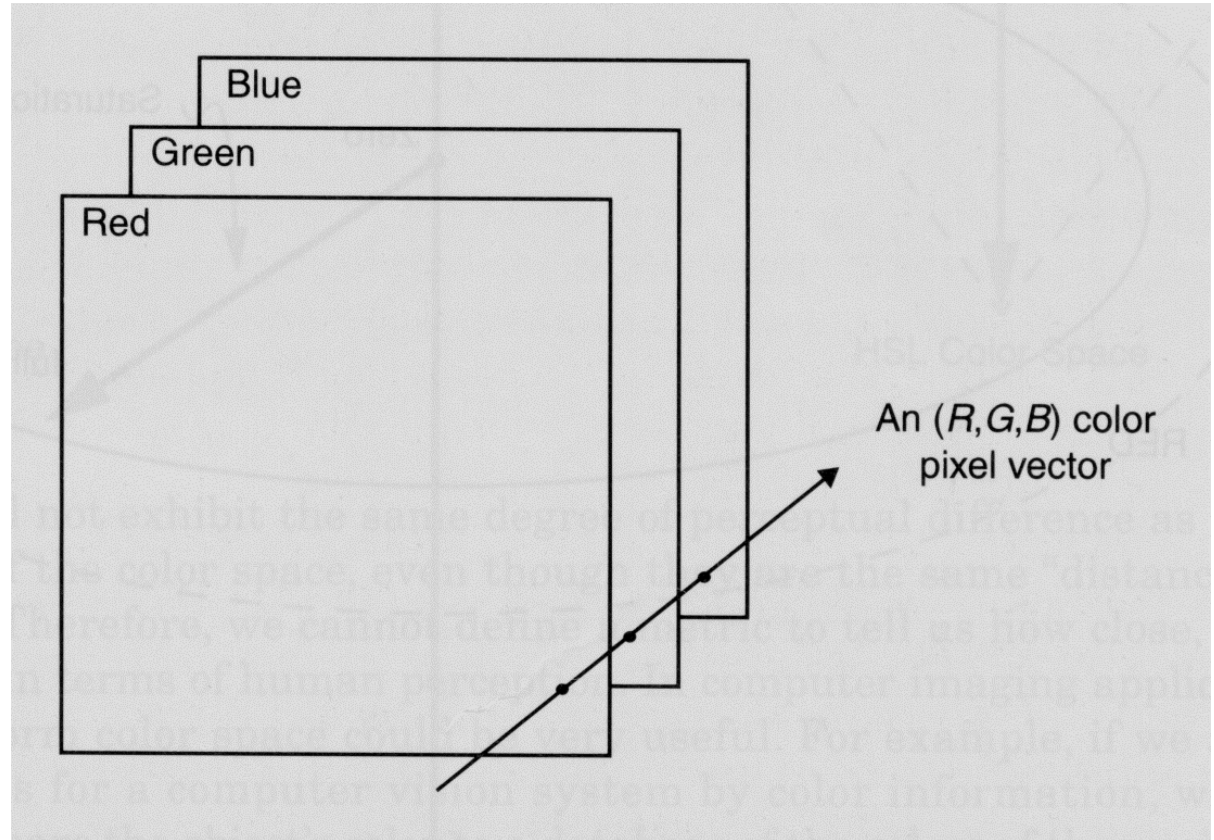
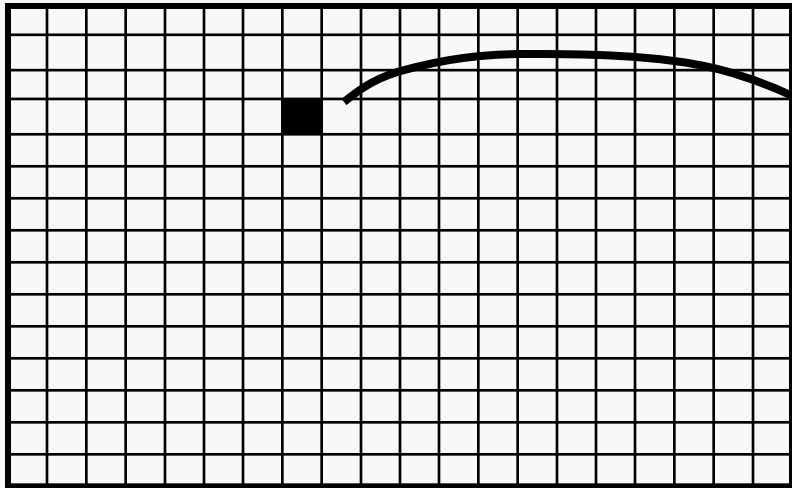


Image Processing

- **Image Fundamental**
- **Digital Image Processing**

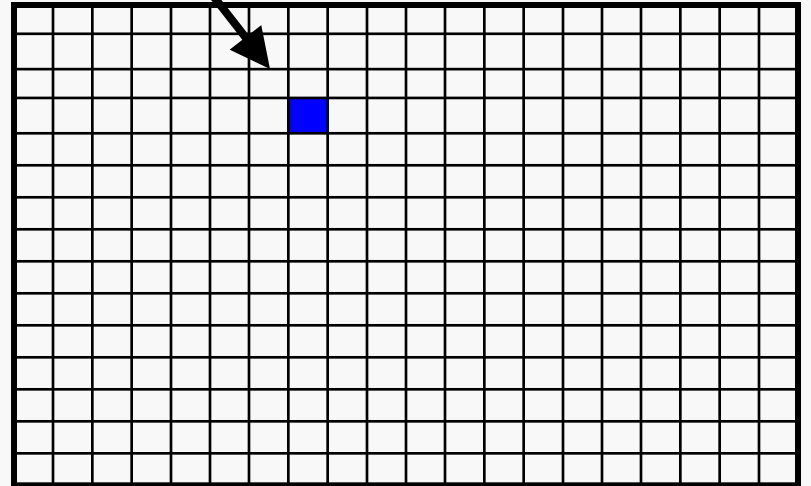
Point Processing

Input $r(x,y)$



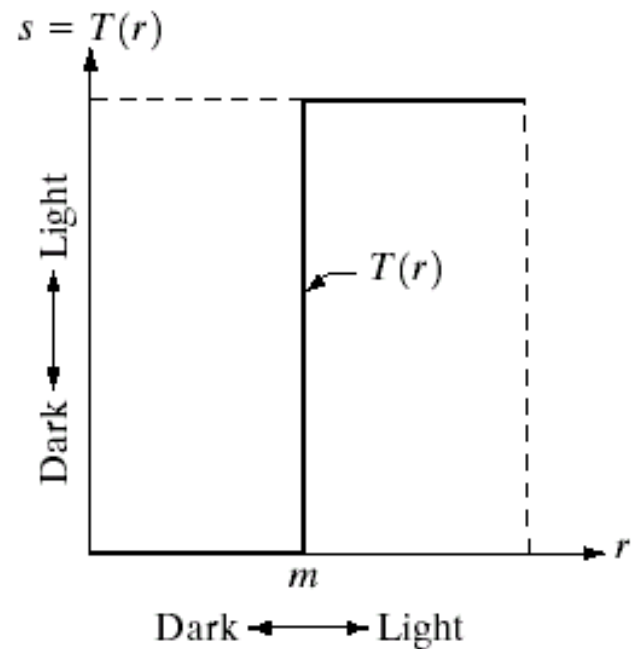
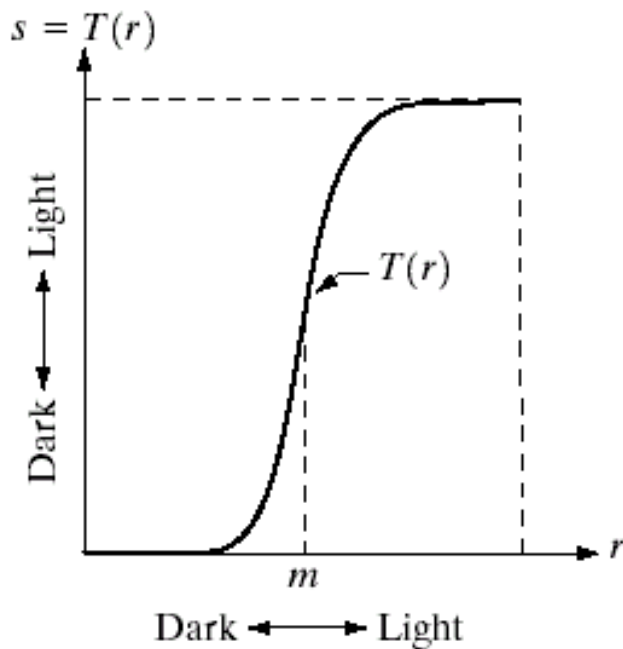
$$s(x,y) = T(r(x,y))$$

Output $s(x,y)$

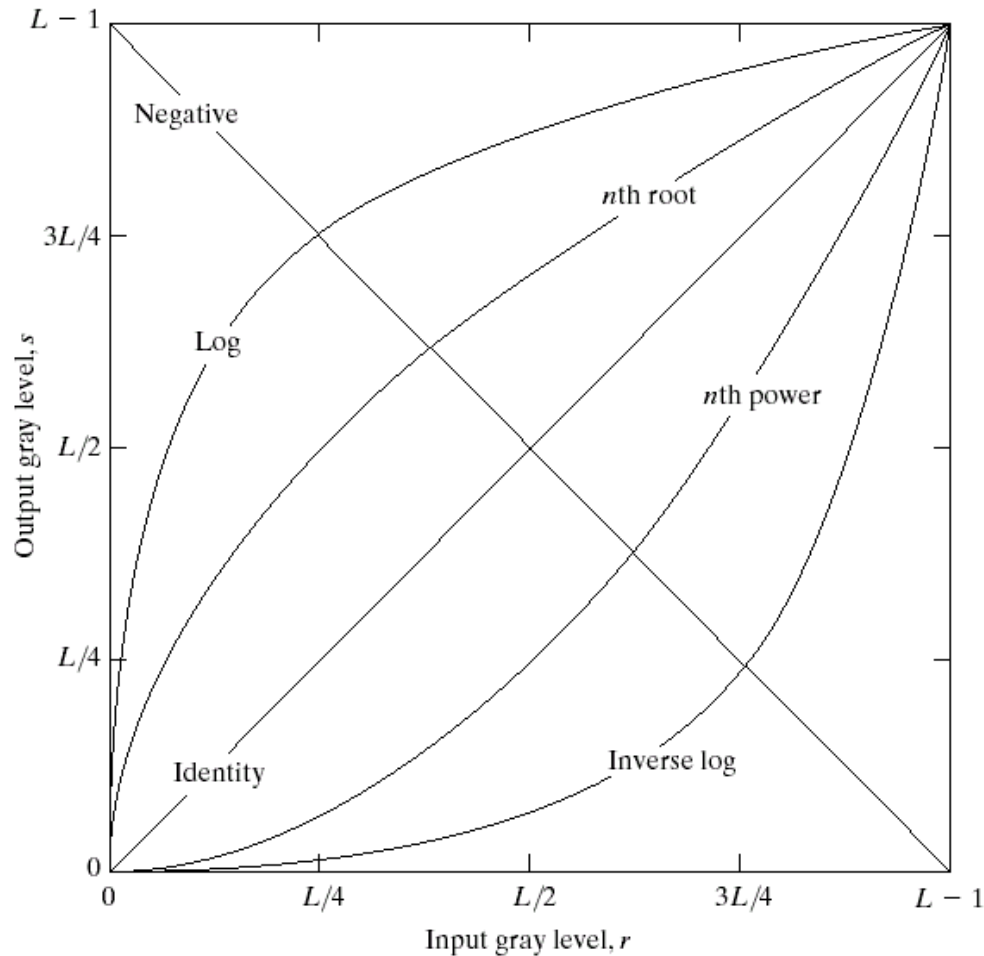


Contrast Enhancement

- Put a suitable curve to map r to s .
- Curve should be monotonically increasing.



Some Curves



Negative:

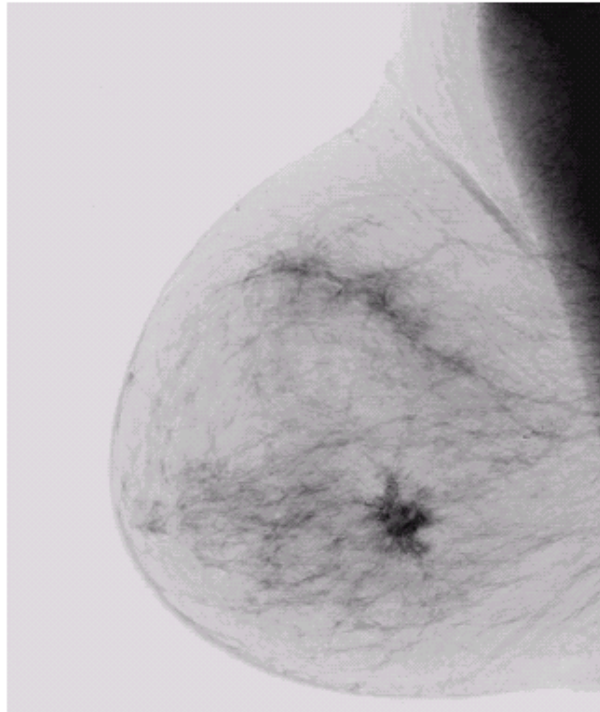
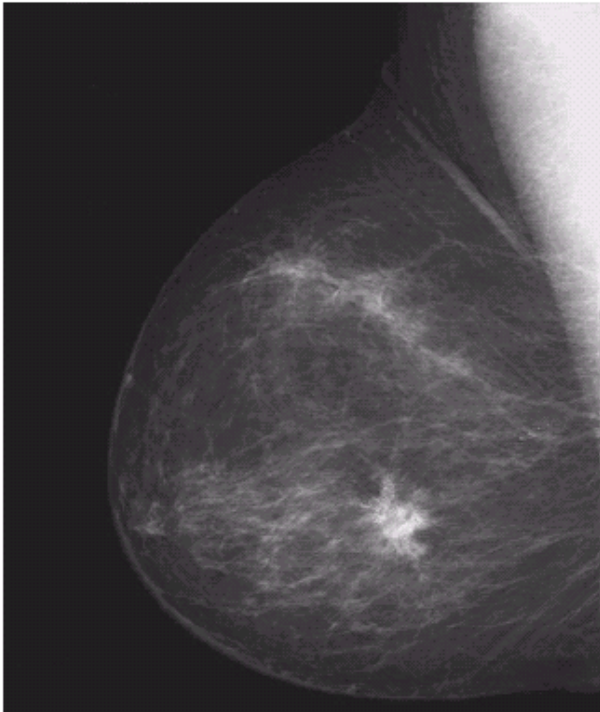
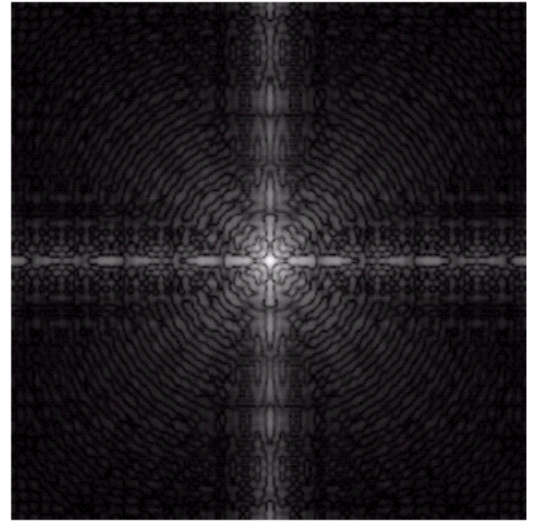
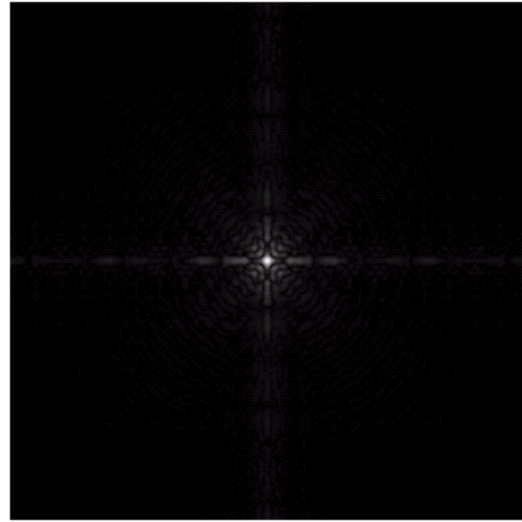
$$s = L - 1 - r$$

Log:

$$s = c \log(1 + r)$$

Examples

Fourier spectra:
log image

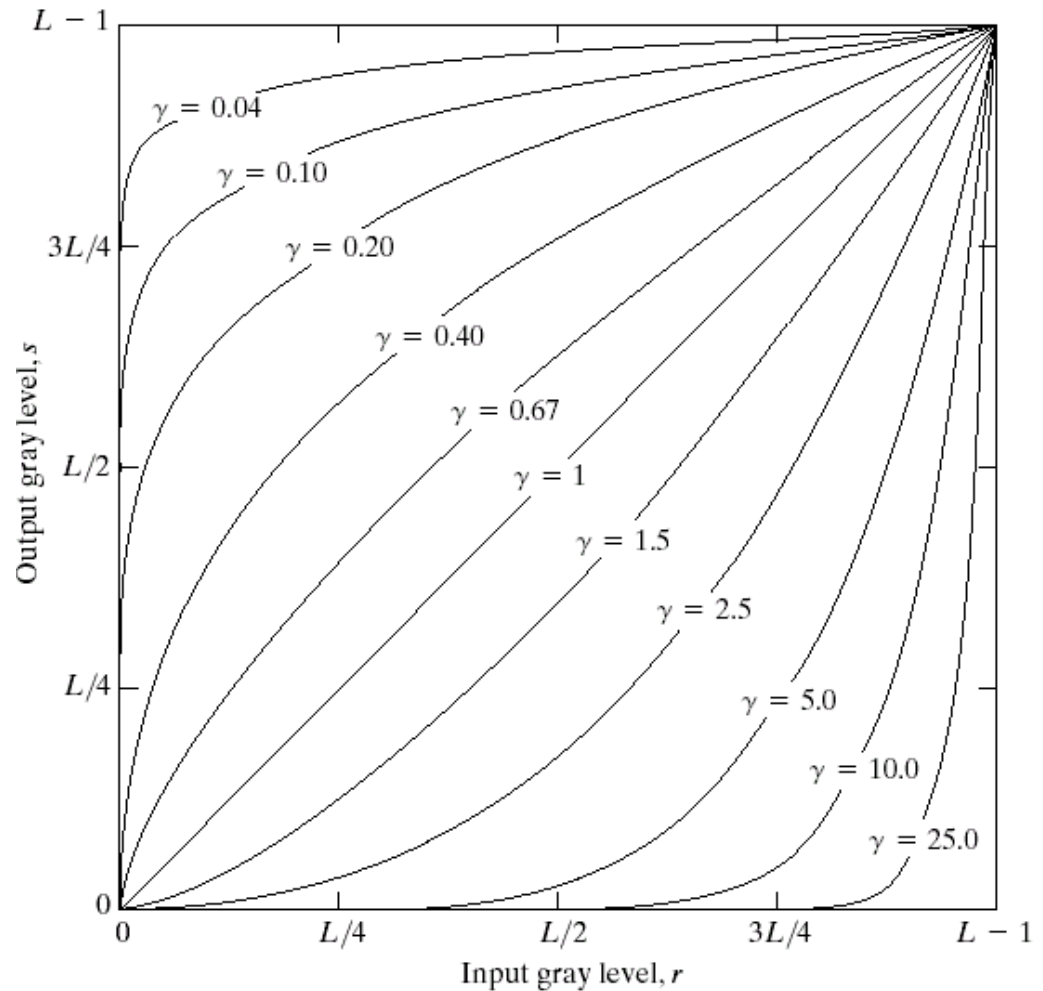


Mammogram:
negative image

Power Curve

Power:

$$s = c r^\gamma$$



Examples

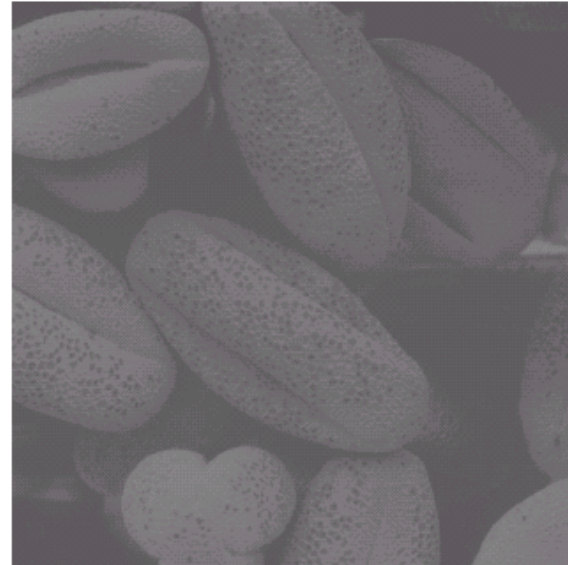
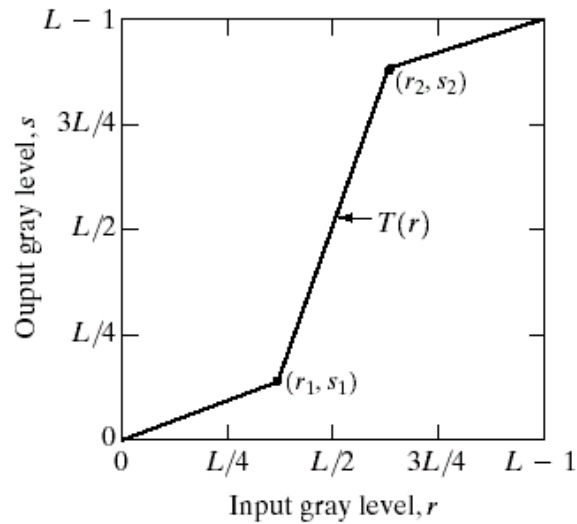
a b
c d

FIGURE 3.9

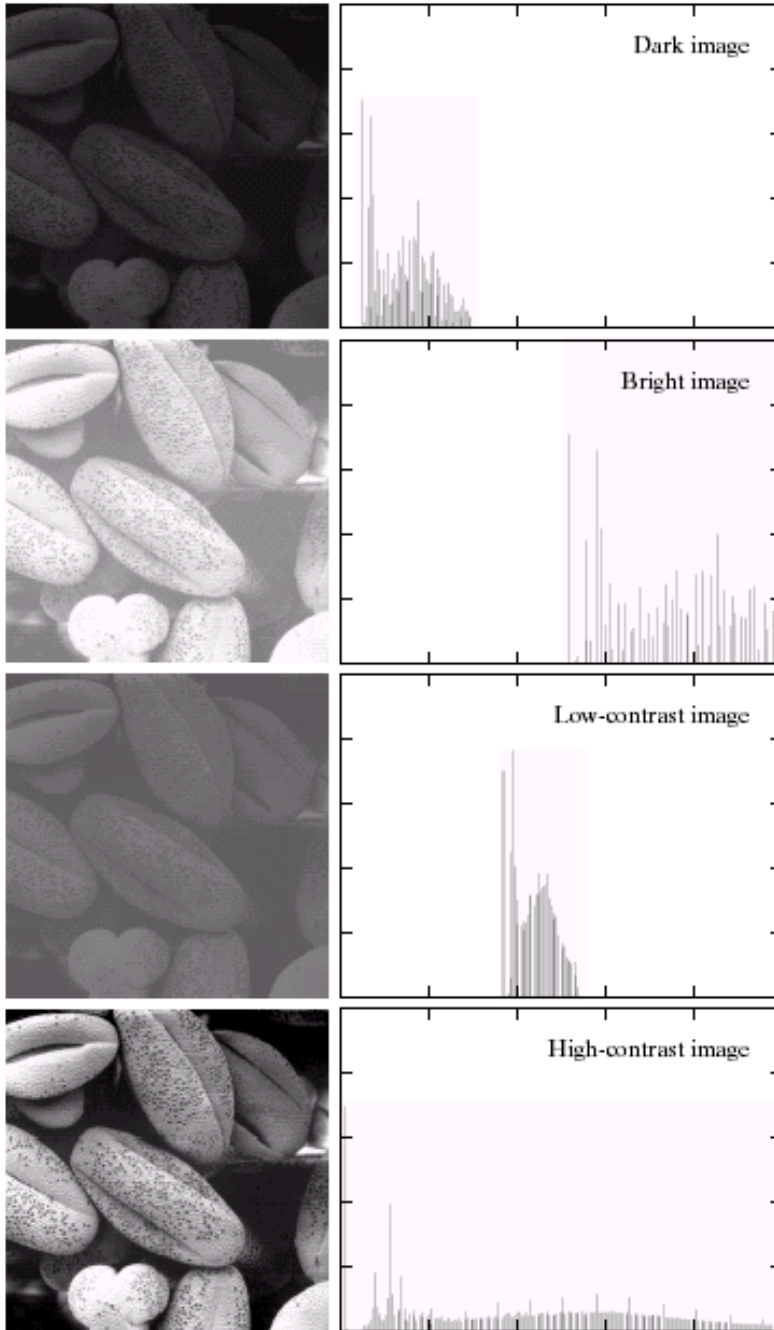
(a) Aerial image.
(b)–(d) Results of
applying the
transformation in
Eq. (3.2-3) with
 $c = 1$ and
 $\gamma = 3.0, 4.0$, and
 5.0 , respectively.
(Original image
for this example
courtesy of
NASA.)



Piecewise Linear Curves



Histogram



- Number of pixels with a particular value v .
 - for all v
- Becomes probability density function (pdf) when divided by total num. of pixels n .

Histogram Equalization

- Suppose input image has histogram:

$$p_r(w), \quad w = 0, \dots, 255$$

- Use the following curve:

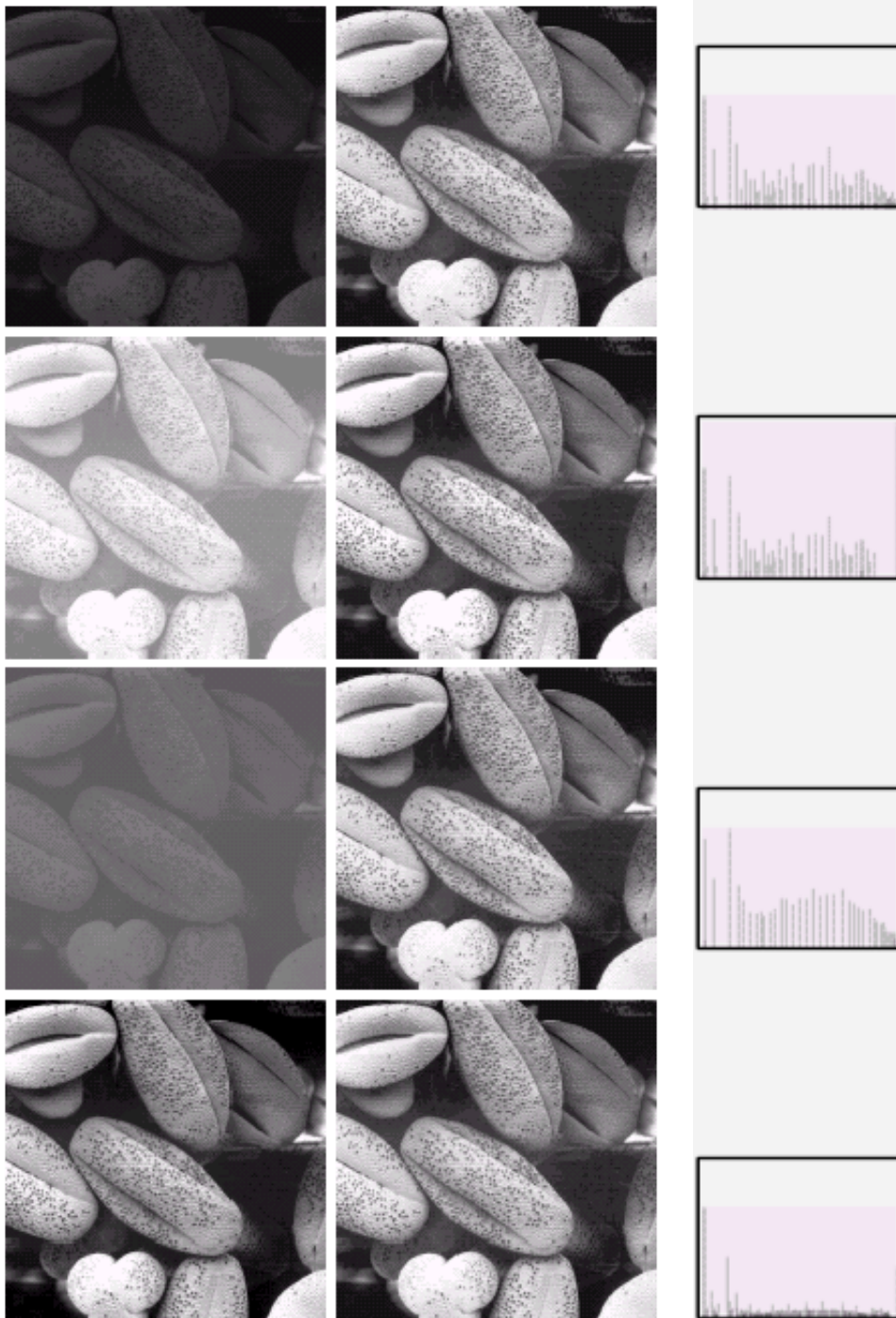
$$s = T(r) = \int_0^r p_r(w) dw$$

- Then it can be shown that output image histogram is flat:

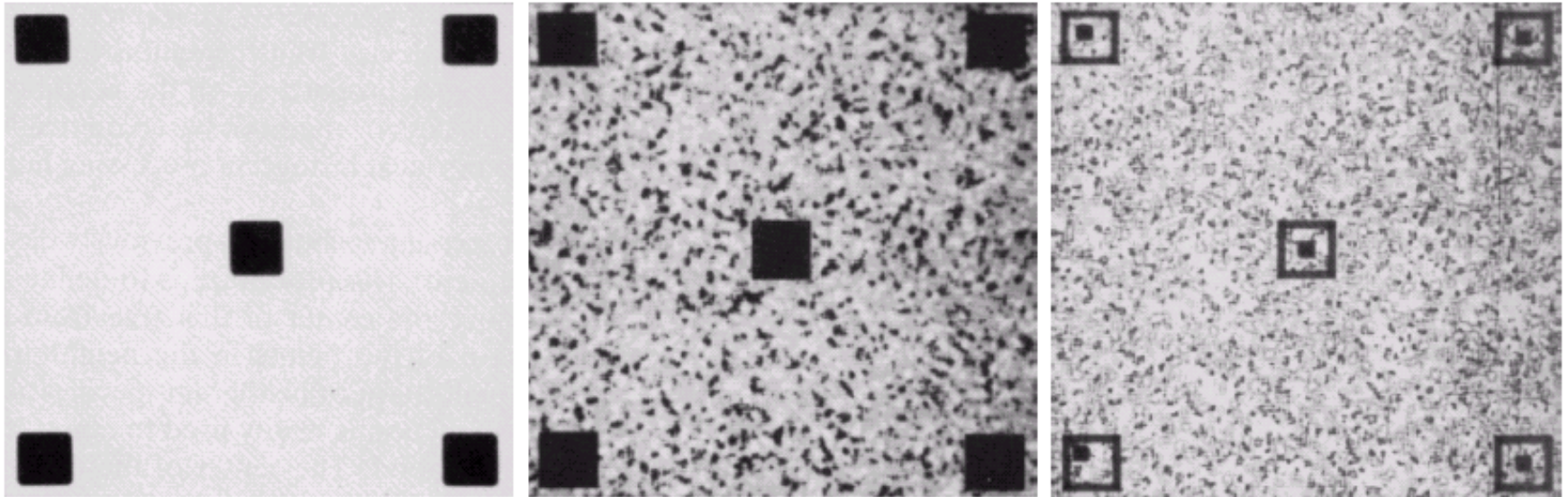
$$p_s(w) = 1, \quad w = 0, \dots, 255$$

Example

- Histogram equalization results.
- Note that discrete histograms are not completely flat.



Local Enhancement

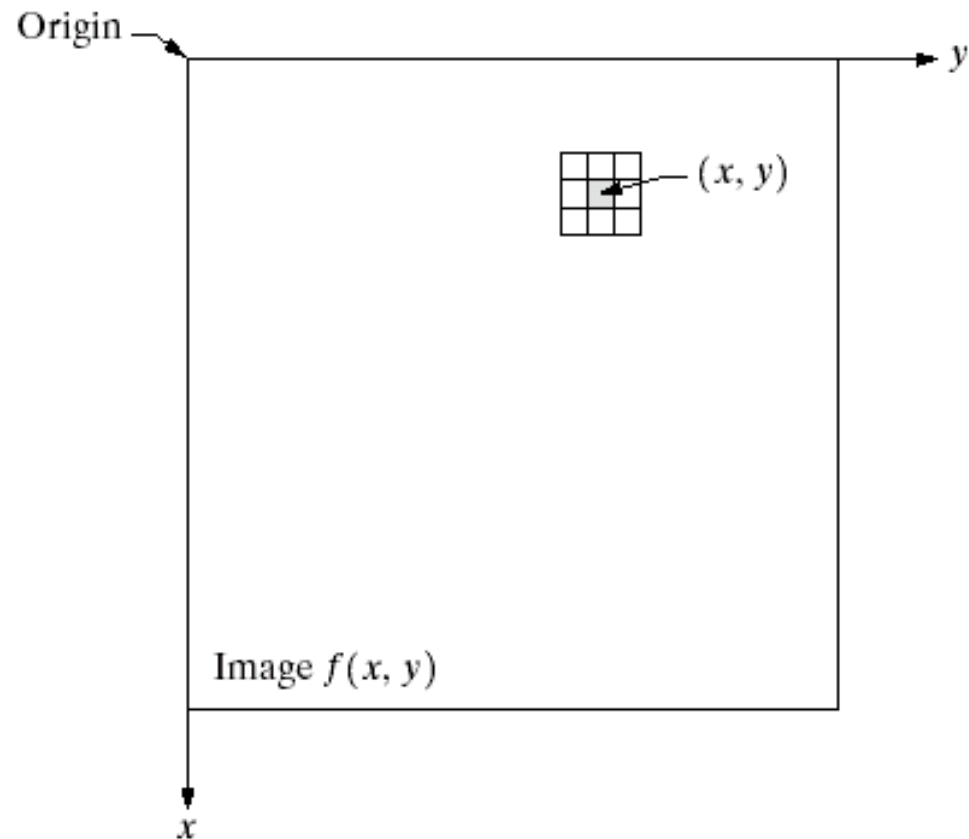


a b c

FIGURE 3.23 (a) Original image. (b) Result of global histogram equalization. (c) Result of local histogram equalization using a 7×7 neighborhood about each pixel.

Neighborhood Processing

- A 3 x 3 window w around (x, y)

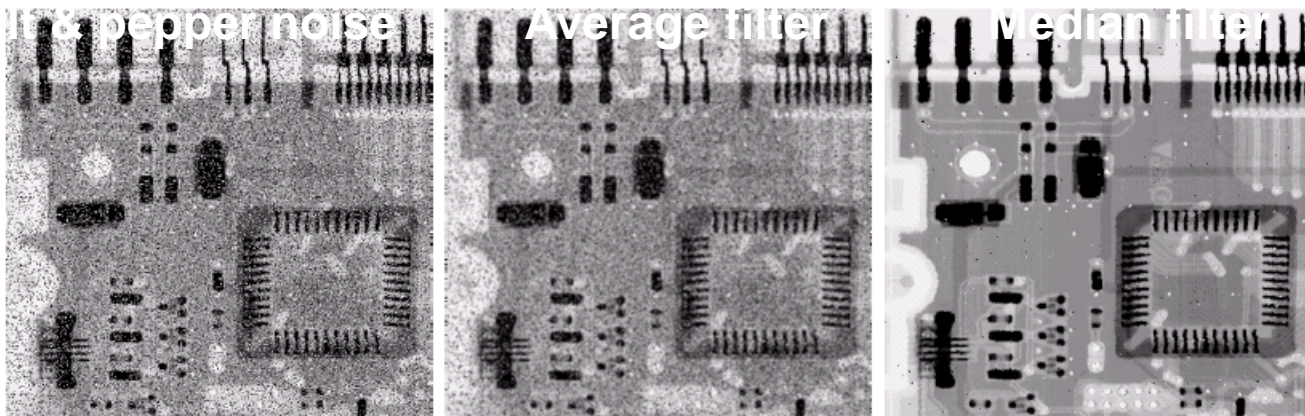


Neighborhood Processing

$$s(\bullet) = T\left(\begin{array}{|c|c|c|}\hline \bullet & \bullet & \bullet \\ \hline \bullet & \bullet & \bullet \\ \hline \bullet & \bullet & \bullet \\ \hline\end{array}\right)$$

Examples:

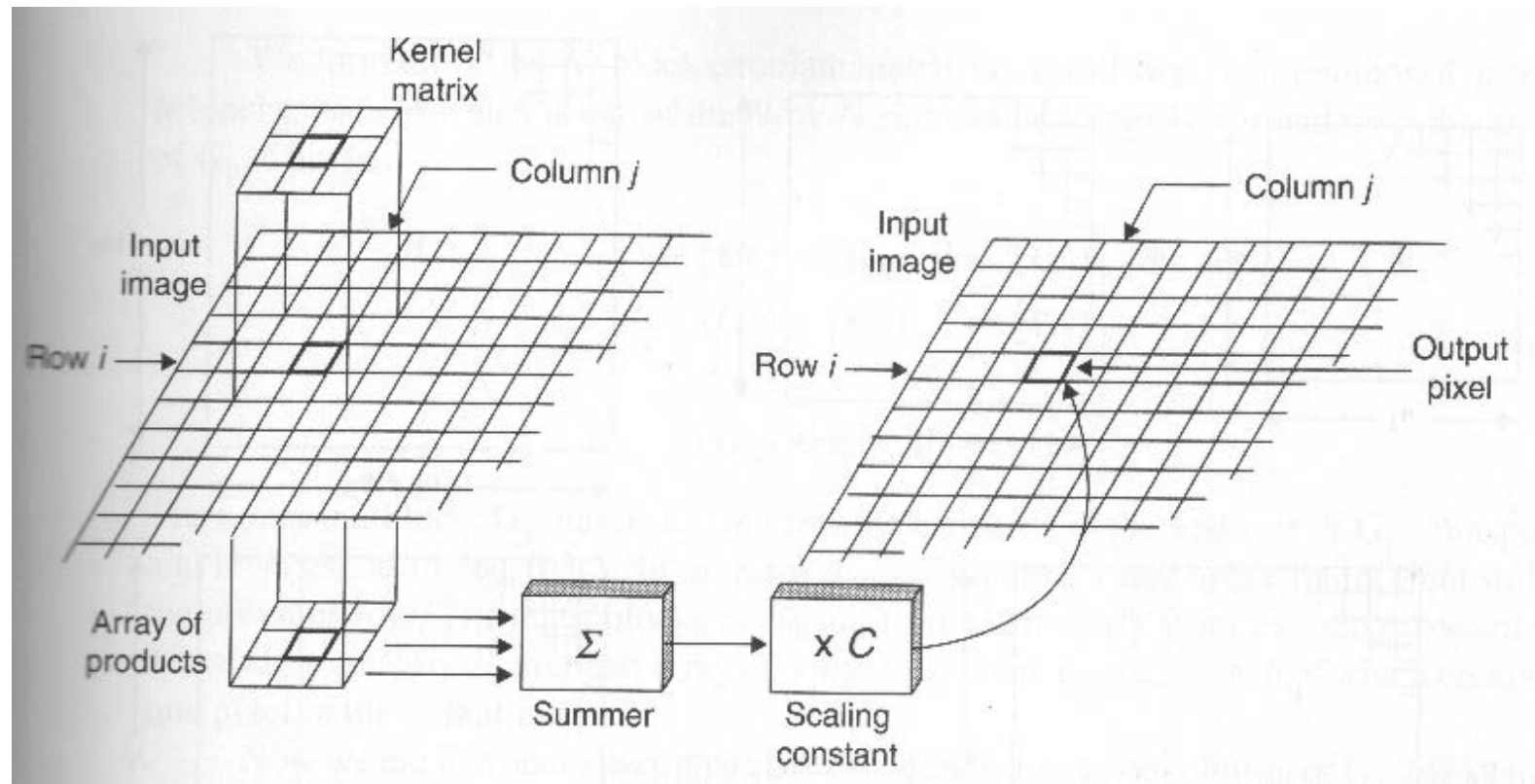
- $T(\) = \text{average}$
- $T(\) = \text{median (middle of sort listed list of values)}$
- $T(\) = \text{min, or max}$



Filtering

- Filter: a.k.a. mask, kernel

$$s(x, y) = C \sum_{(u,v) \in W} w(u, v) r(x + u, y + v)$$



Averaging as filtering

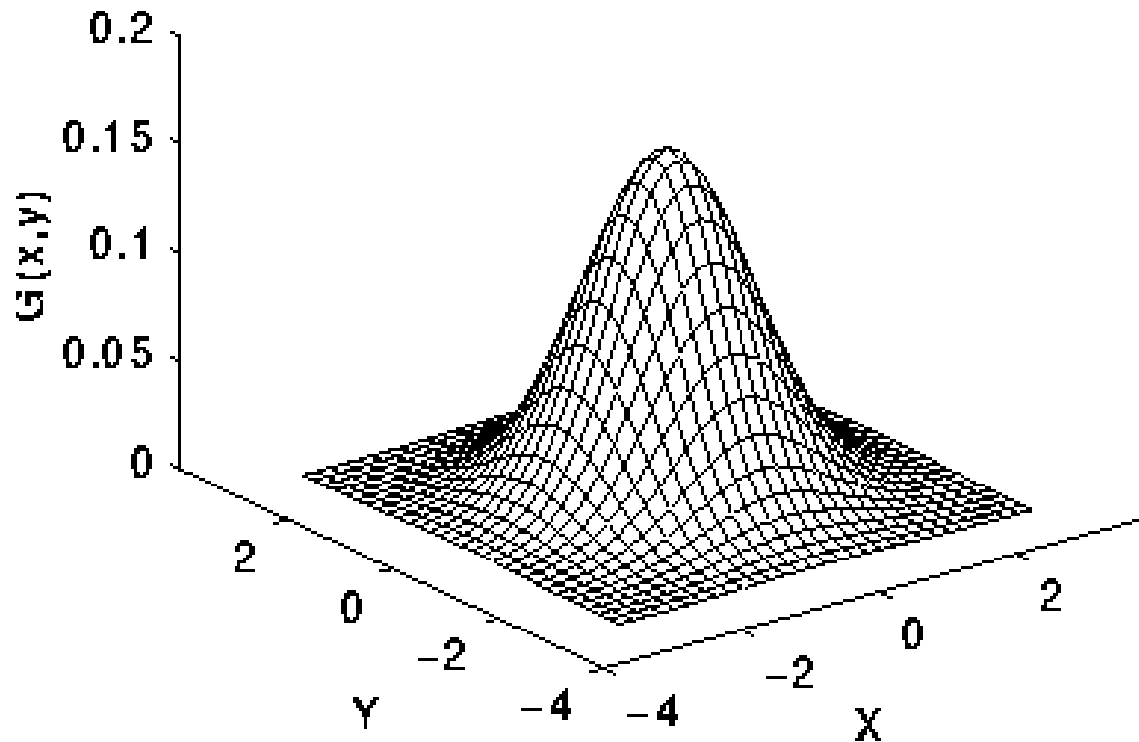
- 3x3 averaging as a mask:

1/9	1/9	1/9
1/9	1/9	1/9
1/9	1/9	1/9

- 5x5 mask:

1	1	1	1	1
1	1	1	1	1
1	1	1	1	1
1	1	1	1	1
1	1	1	1	1

Gaussian Mask



2D Gaussian function with mean = $(0,0)$ and $\sigma = 1$

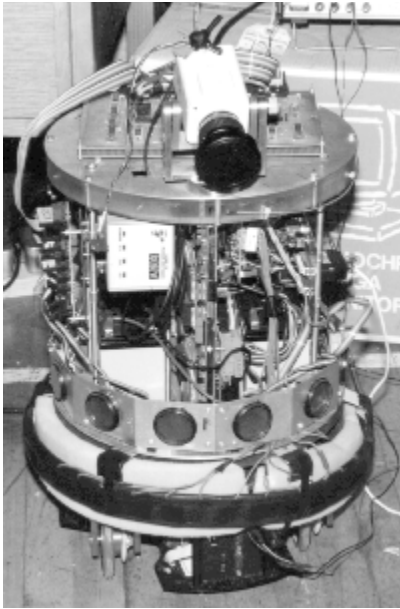
Gaussian Mask

- Discrete approximation, $\sigma = 1.4$

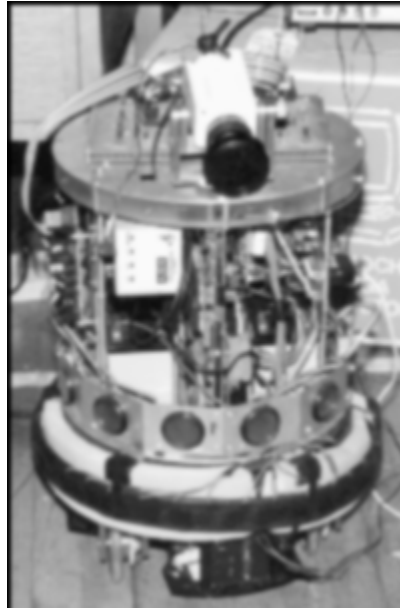
$$\frac{1}{115}$$

2	4	5	4	2
4	9	12	9	4
5	12	15	12	5
4	9	12	9	4
2	4	5	4	2

Gaussian Blurring



original
image



$\sigma = 1.0$
(5×5 kernel)



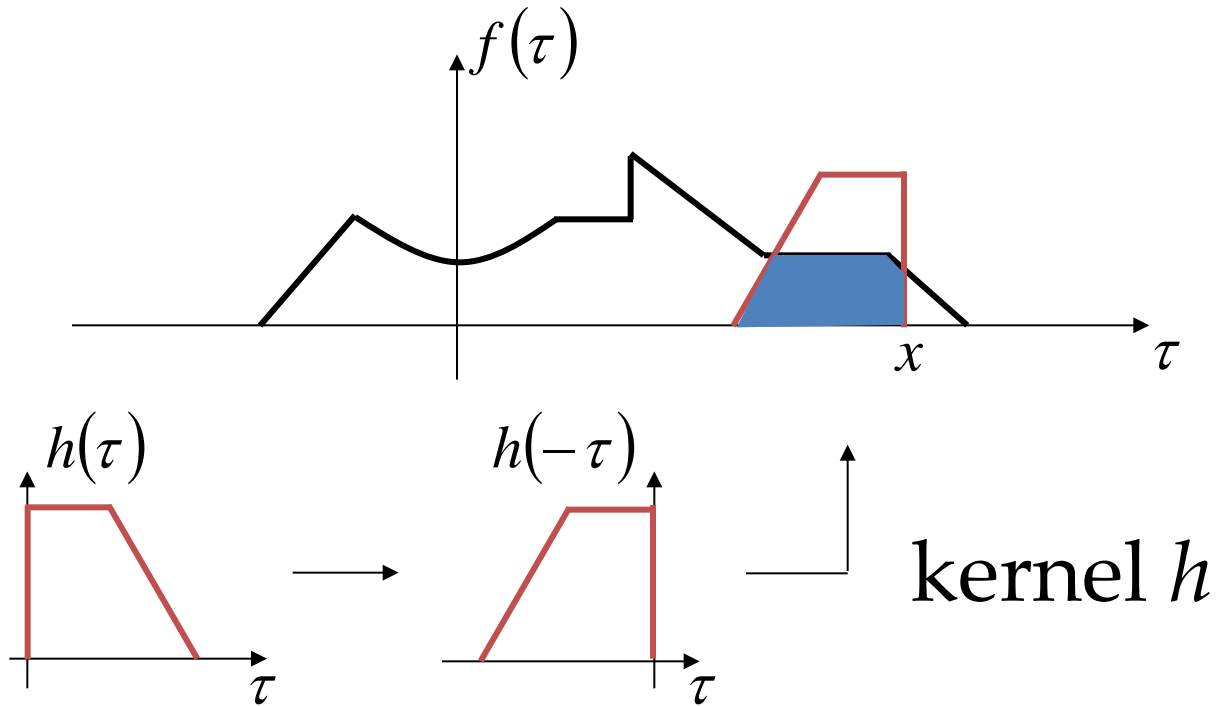
$\sigma = 2.0$
(9×9 kernel)



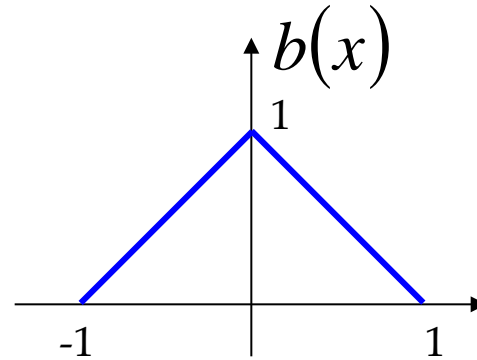
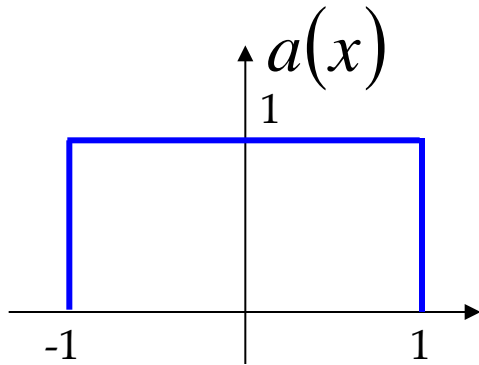
$\sigma = 4.0$
(15×15 kernel)

Convolution

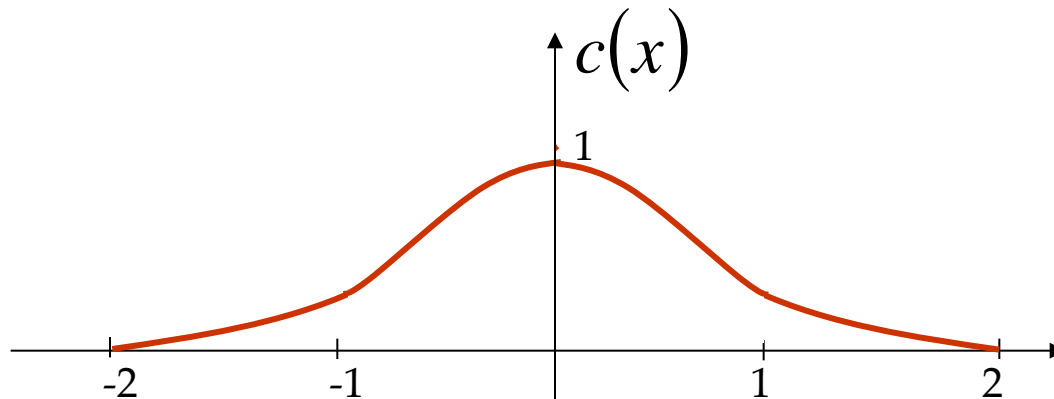
$$g(x) = \int_{-\infty}^{\infty} f(\tau)h(x-\tau)d\tau \quad g = f * h$$



Convolution - Example

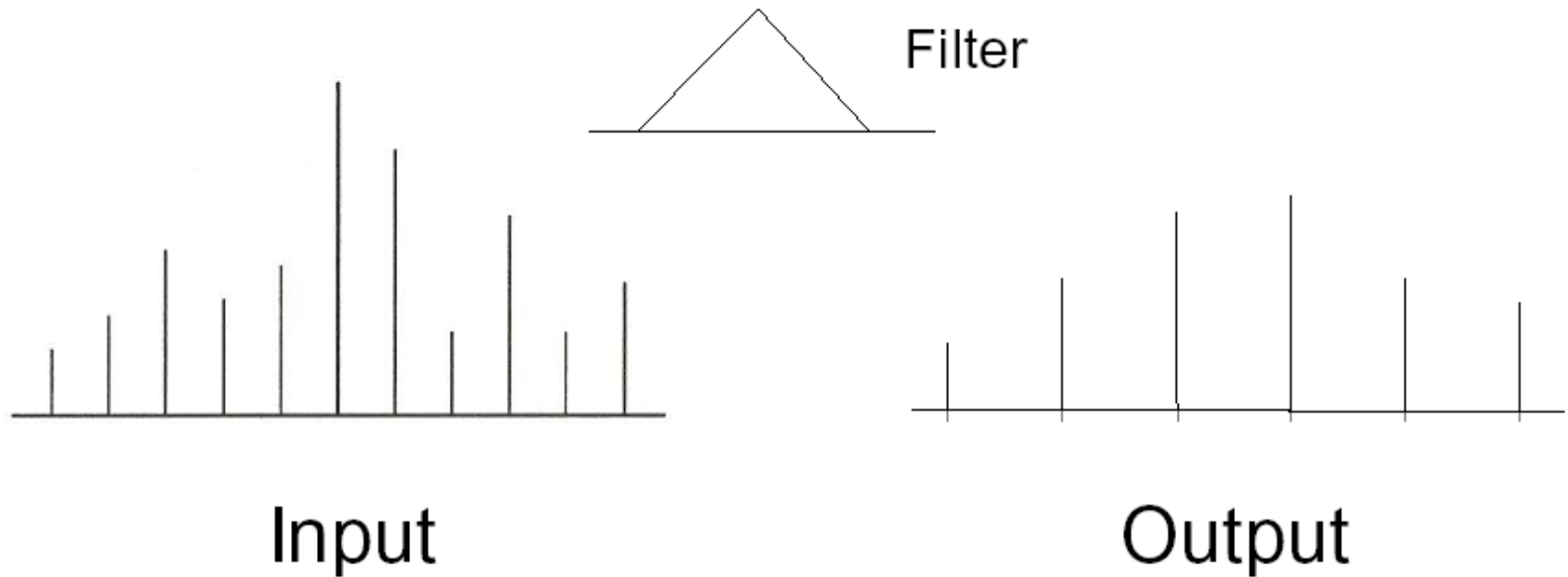


$$\downarrow c = a * b$$



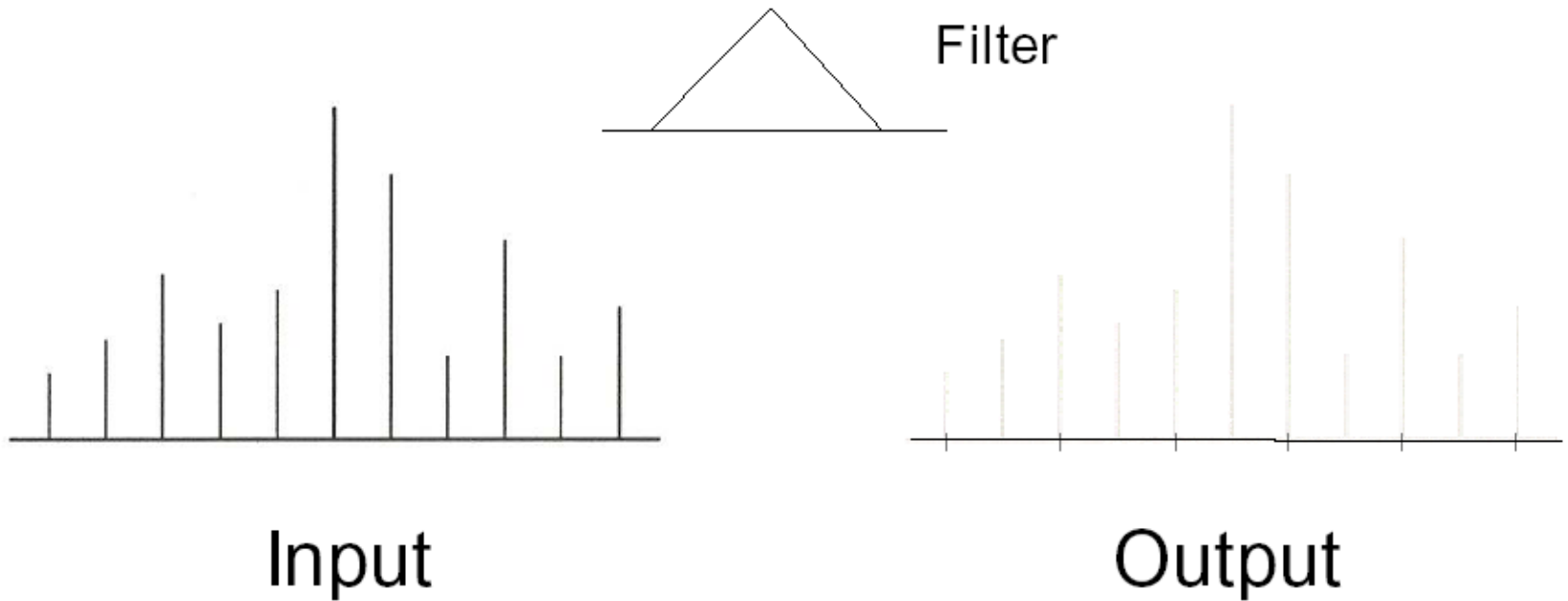
Convolution

- Spatial domain: output pixel is weighted sum of pixels in neighborhood of input image
 - Pattern of weights is the “filter”



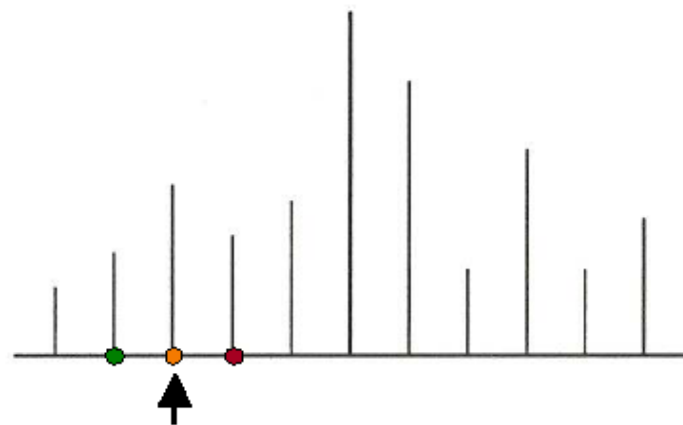
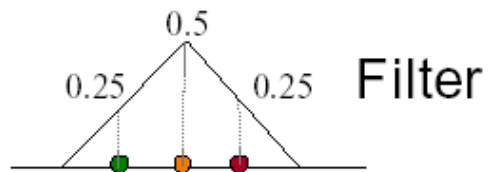
Convolution

- Example 1:

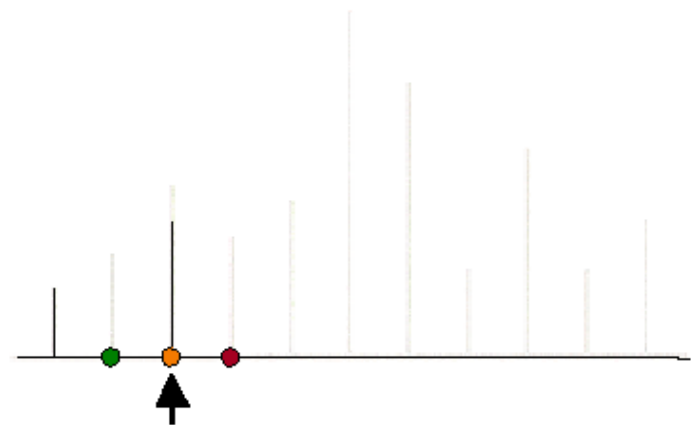


Convolution

- Example 1:



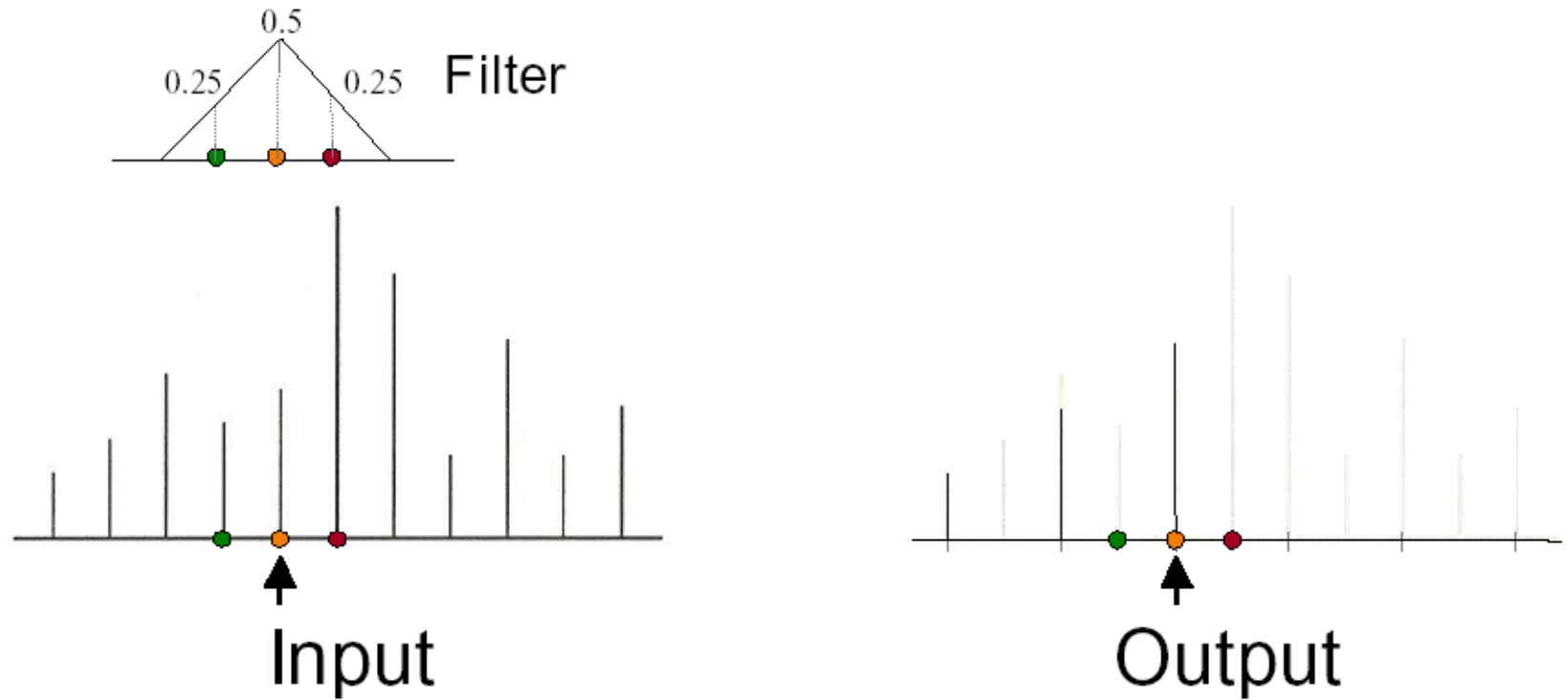
Input



Output

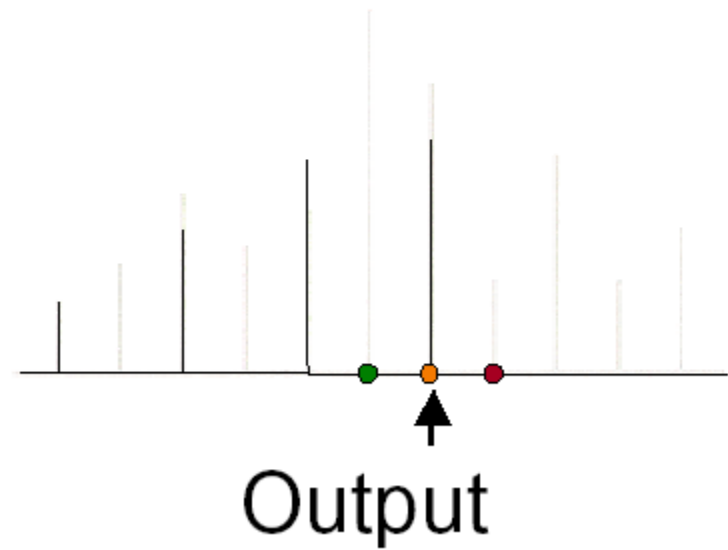
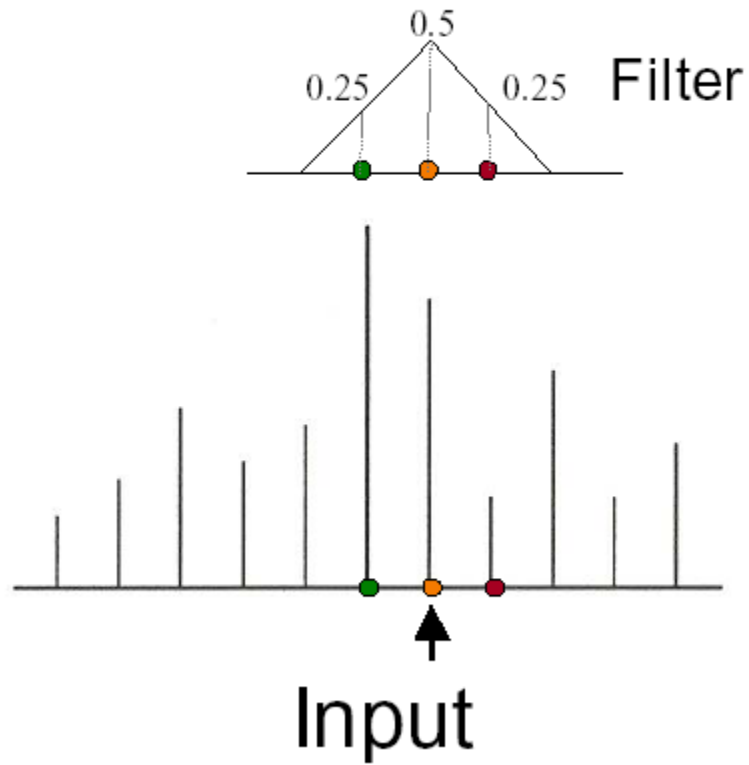
Convolution

- Example 1:



Convolution

- Example 1:



Discrete Convolution

Pixel gets sum of nearby pixels weighted by filter/mask

$$I_{new}(a,b) = \sum_{x=a-width}^{a+width} \sum_{y=b-width}^{b+width} f(x-a, y-b) I_{old}(x,y)$$

2	0	-7
5	4	9
1	-6	-2

Normalization

If you don't want overall brightness change, entries of filter must sum to 1. You may need to normalize by dividing

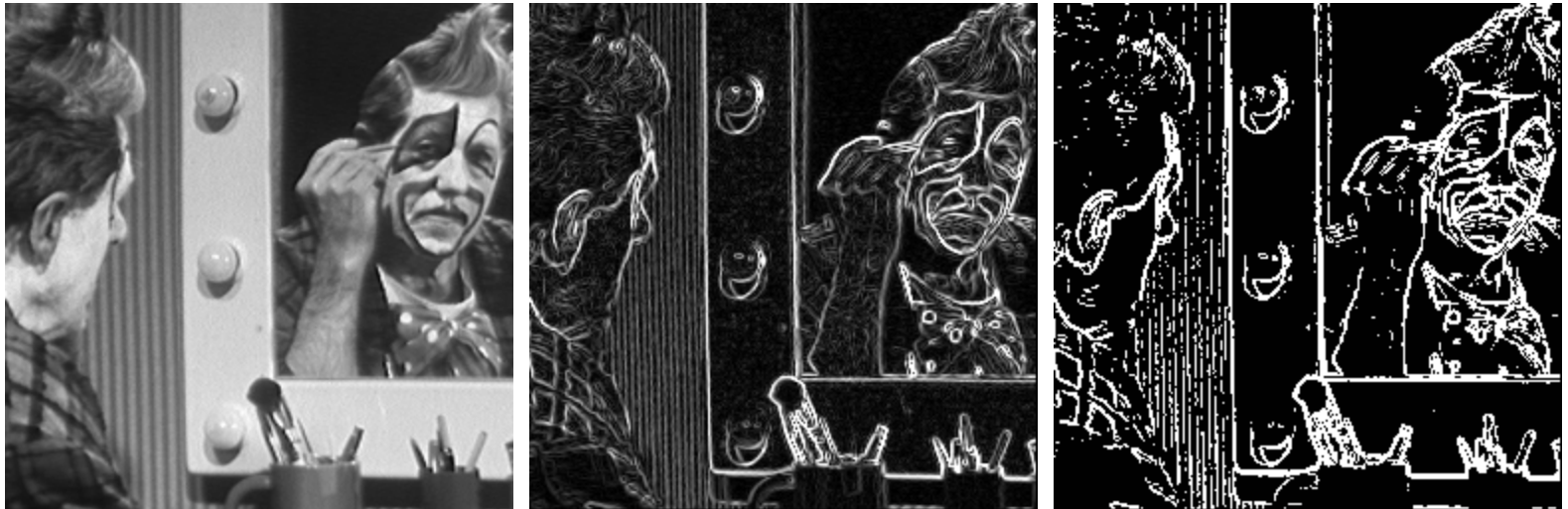
Edge Detection

+1	0
0	-1

Gx

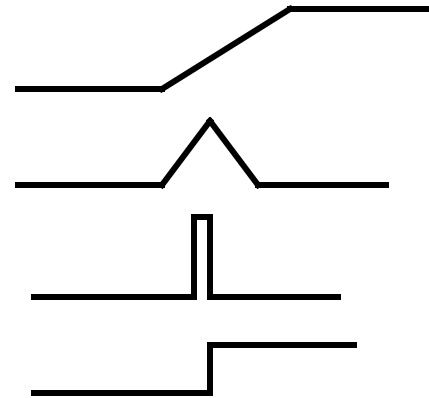
0	+1
-1	0

Gy



Edge Models

- Edges are places in the image with strong intensity contrast.
- Edges often occur at image locations representing object boundaries.
- Types of edges :
 - Ramp (1D profile)
 - Roof (1D profile)
 - Line (1D profile)
 - Step (1D profile)

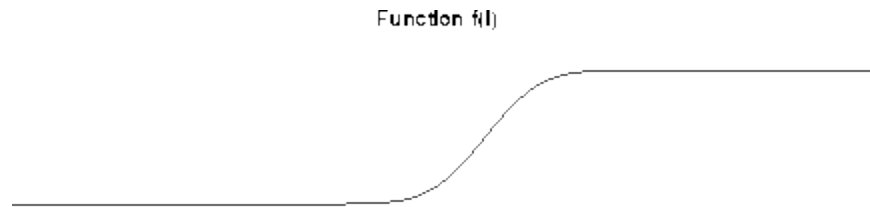


The Step Edge Model

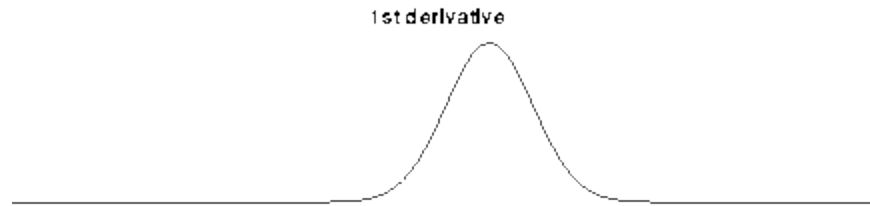
Step edge exist for artificially generated images.

Real images **DO NOT** have step edges because anti-aliasing filter used in the imaging system.

Real edge →



1st derivative →



2nd derivative →



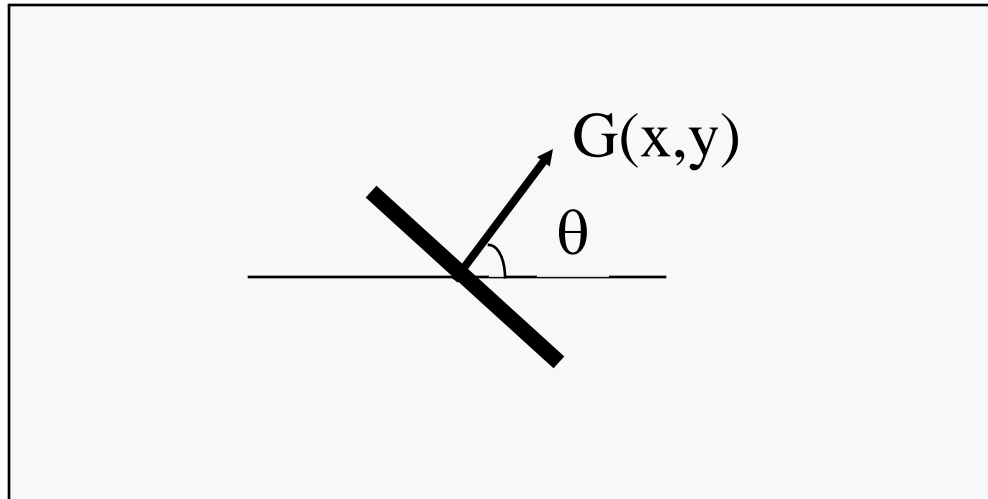
First derivative operator

Also known as Gradient operator.

For an edge in a 2D image,

$$G(x, y) = \frac{\partial F(x, y)}{\partial x} \cos(\theta) + \frac{\partial F(x, y)}{\partial y} \sin(\theta)$$

where $G(x,y)$ is the gradient normal to the edge.



First derivative operator

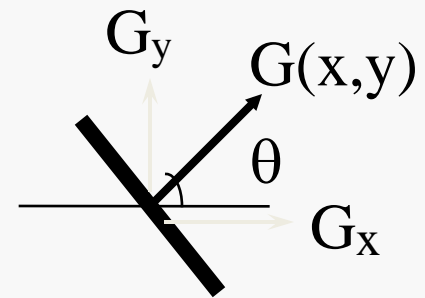
Usually, we compute

$$\nabla f = \begin{bmatrix} G_x \\ G_y \end{bmatrix} = \begin{bmatrix} \partial f / \partial x \\ \partial f / \partial y \end{bmatrix}$$

where G_x is the horizontal gradient,
 G_y is the vertical gradient.

$$|\nabla f| = \left[G_x^2 + G_y^2 \right]^{1/2}$$

$$|\nabla f| \approx |G_x| + |G_y| \quad ; \quad \theta(x, y) = \tan^{-1} \left(\frac{G_y}{G_x} \right)$$



Sobel's gradient operator

-1	0	+1
-2	0	+2
-1	0	+1

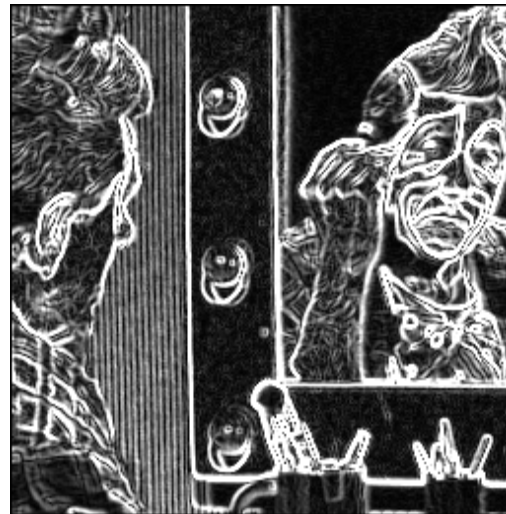
Gx

+1	+2	+1
0	0	0
-1	-2	-1

Gy



original



Convolved with Sobel's operator

Second derivative operator

Laplacian operator:

The Laplacian $L(x,y)$ of an image at $I(x,y)$ is:

$$L(x, y) = \frac{\partial^2 I}{\partial x^2} + \frac{\partial^2 I}{\partial y^2}$$

Laplacian kernels:

0	1	0
1	-4	1
0	1	0

1	1	1
1	-8	1
1	1	1

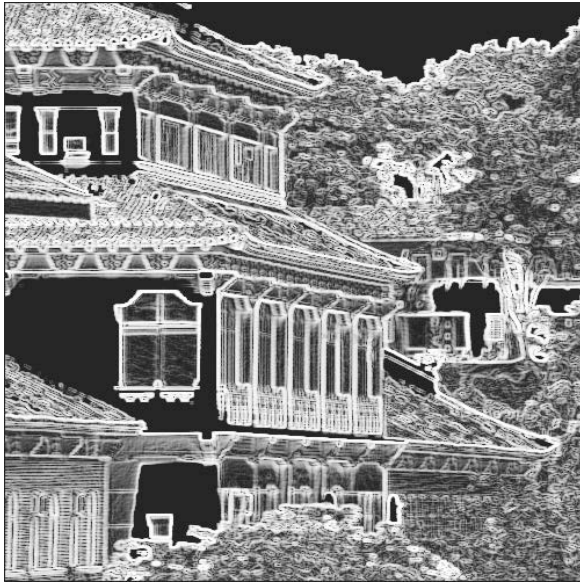
-1	2	-1
2	-4	2
-1	2	-1

Laplacian operator usually produce closed contour

original



Sobel output



Laplacian output



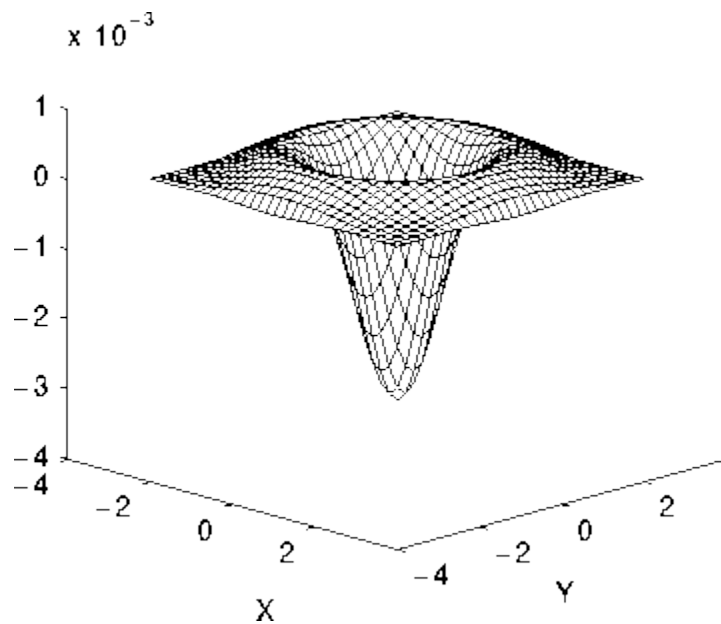
Second derivative operator

Laplacian of Gaussian (LoG) operator:

This is equivalent to Gaussian smoothing followed by Laplacian.

$$\begin{aligned} G(x, y) &= \nabla^2 [g(x, y) * I(x, y)] \\ &= \nabla^2 g(x, y) * I(x, y) \end{aligned}$$

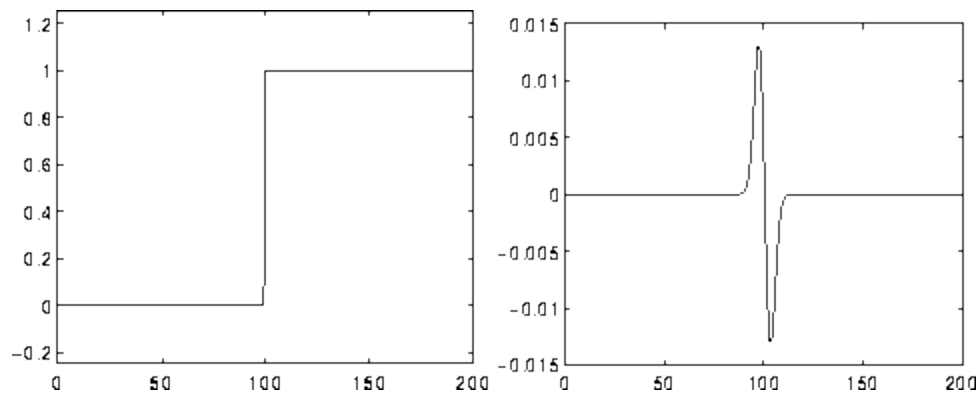
$$\nabla^2 g(x, y) = -\frac{1}{2\pi\sigma^4} \left(2 - \frac{x^2 + y^2}{\sigma^2} \right) \exp \left[-\frac{x^2 + y^2}{2\sigma^2} \right]$$



2D LoG function

0	0	3	2	2	2	3	0	0
0	2	3	5	5	5	3	2	0
3	3	5	3	0	3	5	3	3
2	5	3	-12	-23	-12	3	5	2
2	5	0	-23	-40	-23	0	5	2
2	5	3	-12	-23	-12	3	5	2
3	3	5	3	0	3	5	3	3
0	2	3	5	5	5	3	2	0
0	0	3	2	2	2	3	0	0

2D LoG kernel



Response of the LoG to a step edge

Edge Detection

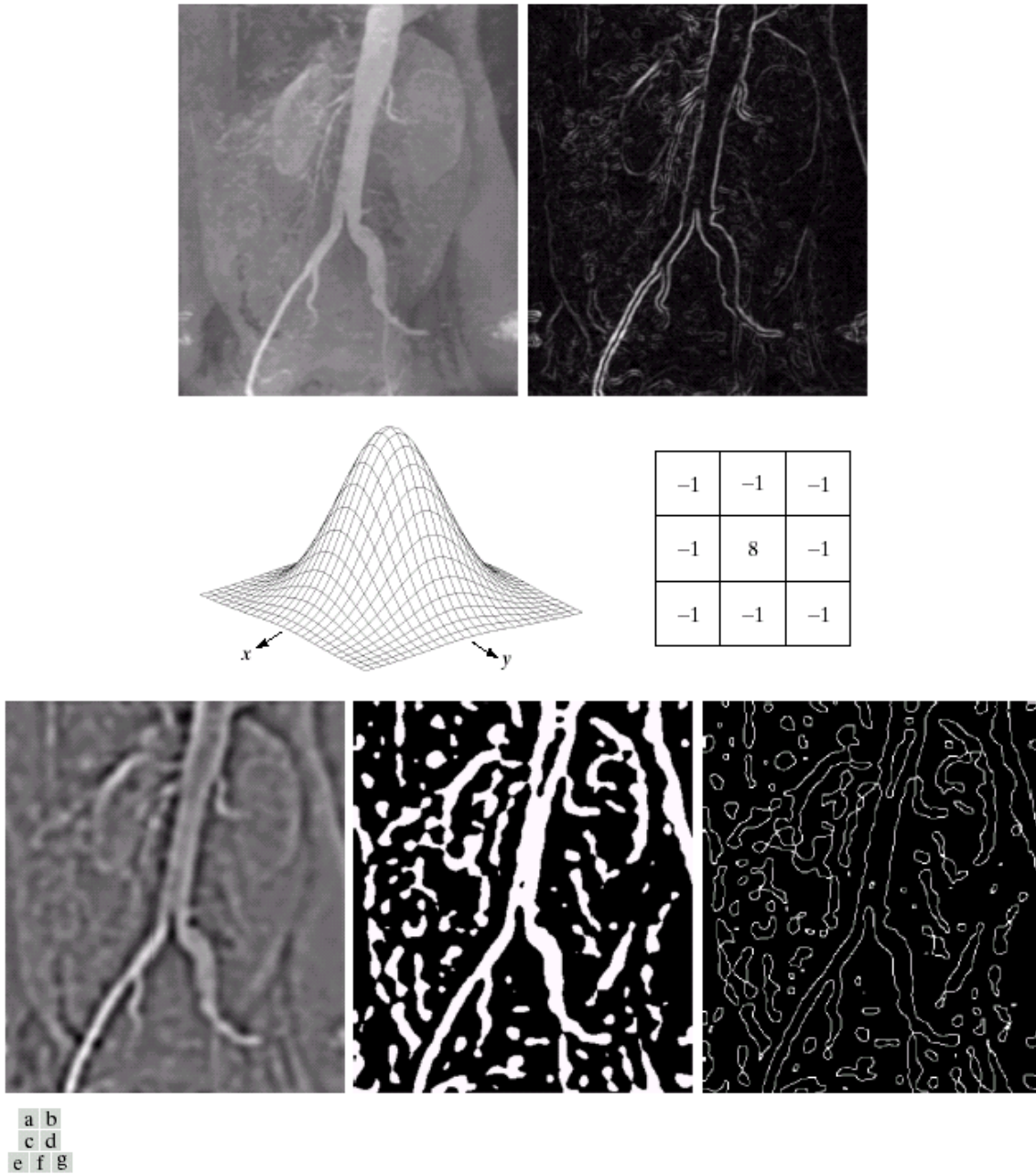


FIGURE 10.15 (a) Original image. (b) Sobel gradient (shown for comparison). (c) Spatial Gaussian smoothing function. (d) Laplacian mask. (e) LoG. (f) Thresholded LoG. (g) Zero crossings. (Original image courtesy of Dr. David R. Pickens, Department of Radiology and Radiological Sciences, Vanderbilt University Medical Center.)

Summary

- Point processing computes an output pixel as a function of the corresponding input pixel.
- Neighborhood processing computes an output pixel as a function of the *neighborhood* around the input pixel.
- One type of neighborhood processing is called filtering, via a mask.
- Edge detection can be done using masks.
- Same technique can be used to detect corners, arcs, lines.