19ECE301- Control Theory

Report submitted for Term project of 19ECE301 Control Theory

Date: 07/01/2023
Title: Ball & Beam: System Modeling
Group Members:

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Marks (to be filled by the faculty)

S.No	Description	Excellent (10)	Good (5)	Not acceptable (0)
1	Design and analysis using CAD tools			
2	Understanding of the problem (To be decided on seeing the quality of the report and if necessary, by interacting with the students)			
3	Relationship of the problem to the topics taught in the syllabus			
4	Contribution by all the team members (To be decided, if necessary, by interacting with the students)			
5	Report structure			
	Total marks (50)			
	Marks (5)			

Problem Statement:

The Ball and Beam problem is a classic control engineering problem that involves designing a control system to balance a ball on a beam. The system consists of a ball rolling along a horizontal beam, which is mounted on a vertical support and can rotate about a pivot point. The goal is to design a control system that can maintain the ball at a desired position on the beam, despite external disturbances or changes in the system's parameters.

The Ball and Beam system consists of a beam mounted on a support block, with a ball that can roll along the beam. The beam can be tilted about its axis by a **DC brush motor**, which causes the ball to roll along the beam with an acceleration proportional to the angle of the beam. The motor has a **built-in rotary encoder** that provides feedback on the current position of the motor shaft, and there is a **linear potentiometer sensor** along the beam that senses the current position of the ball on the beam. Both measured positions are fed back to the control system to form a closed loop control. [As shown in Fig.1]

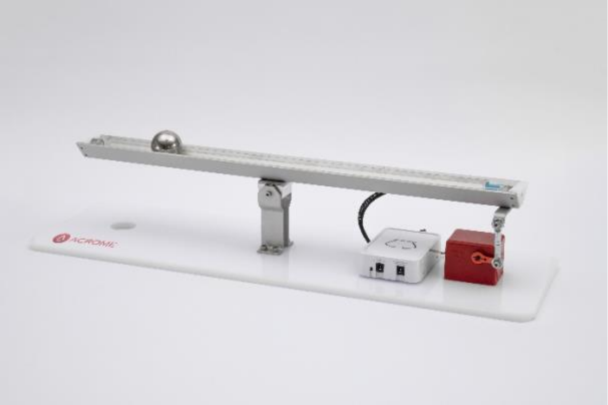


Fig.1

The problem is to control the position of the ball on the beam by **changing the angle of the beam**. The control task is to regulate the position of the ball on the beam by **controlling the acceleration** of the ball. The system is open loop unstable, so feedback control is needed to keep the ball in a desired position on the beam. A suitable controller must be designed based on the dynamics of the ball and beam system.

Design a system with Overshoot % 0.5 and Settling time of 1 sec

Solution:

1) Modelling:

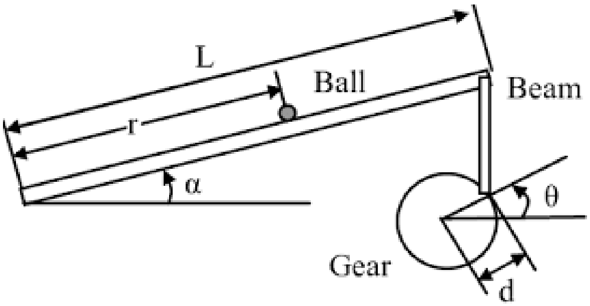


Fig.2

The system has been modelled as described and the final transfer function is :

$$\left(m+\frac{J_b}{R^2}\right)r'' + \frac{J_b}{R^2\alpha''} - mr\alpha^{2'} + mg\sin\alpha = 0$$

By linearizing (1) about the beam angle, $\sin\alpha = \alpha$ for small α, gives us the following linear approximation of the system.

$$\left(m+\frac{J_b}{R^2}\right)r'' = mg\alpha$$

$$\alpha = \left(\frac{d}{l}\right)\theta$$

By solving above two equations, we obtain:

$$\left(m+\frac{J_b}{R^2}\right)r'' = mg\left(\frac{d}{l}\right)\theta$$

$$\left(m+\frac{J_b}{R^2}\right)s^2\, r(s) = mg\left(\frac{d}{l}\right)\theta$$

Therefore, ***transfer function*** obtained as:

$$\boxed{\boldsymbol{\frac{r(s)}{\theta(s)} = \frac{mgd}{L\left(m+\frac{J_b}{R^2}\right)s^2}}}$$

Parameters:

Symbol	Description	Values
m	Mass of the ball	0.11
R	Radius of the ball	0.015m
d	Lever arm offset	0.04
g	Gravitational acceleration	9.8 m/s^2
L	Length of the ball	40 cm
J_b	moment of the ball	$2mR^2/5$ Kgm^2

By substituting the values of parameters given in Table I in above equation, the **transfer function obtained** as:

$$\boxed{\frac{r(s)}{\theta(s)} = \frac{0.7}{s^2}}$$

It shows, the ball and beam system are a double integrating unstable process.

2) Analysis:

Since the system is unstable, we bring it to stable state by using the PID controller.

Code:

```
>> %defining the transfer function;
>> g1=tf([0.7],[1 0 0]);
>> %analysis;
>> step(g1);
```

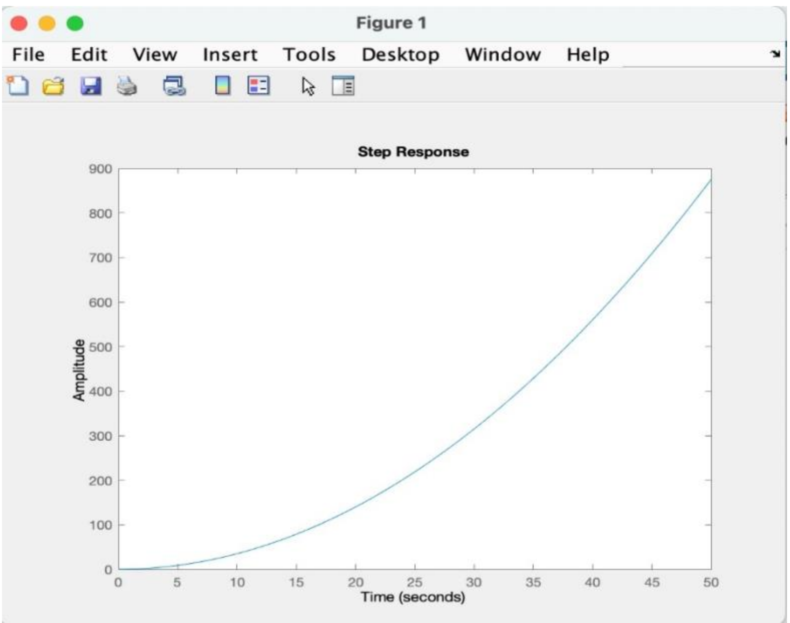


Fig.3

From this plot it is clear that the system is unstable in open-loop causing the ball to roll right off the end of the beam. Therefore, some method of controlling the ball's position in this system is required.

```
%design to a marginally stable state by a PID controller
Kp = 1;
C = pid(Kp);
sys_cl=feedback(g1,1);
step(sys_cl)
axis([0 70 0 2])
```

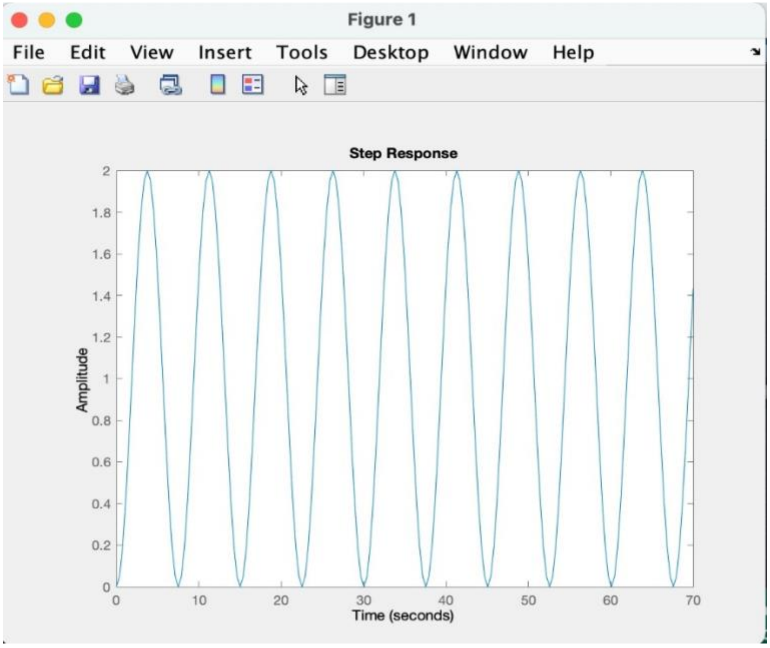


Fig.4

```
%designing a PD controller
Kp = 10;
Kd = 10;
C = pid(Kp,0,Kd);
sys_cl=feedback(g1*C,1);
step(sys_cl)
```

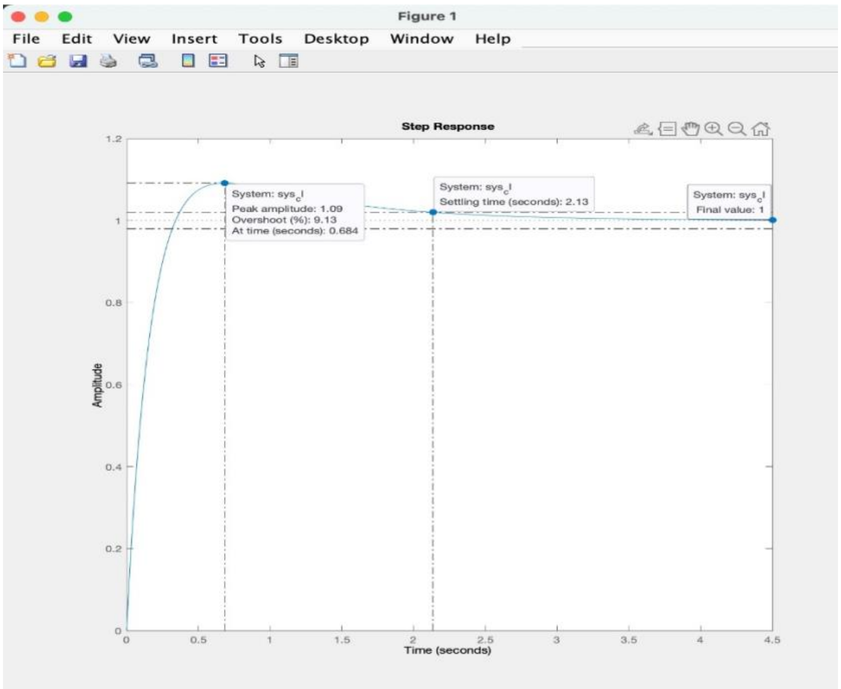
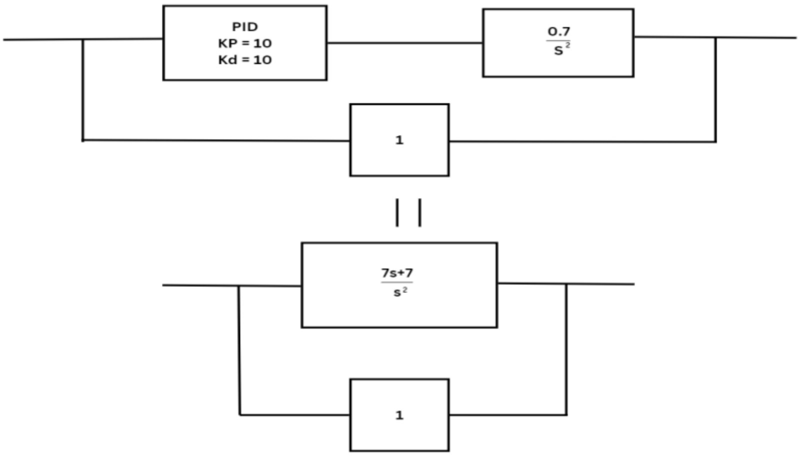


Fig.5

The system has become **stable** and it has steady state error of **1%**.

Block Diagram:



3) Design:

Design Specifications:

- System with Overshoot %**0.5**
- Settling time of 1 sec.

```
>> bode(g1*C)
>> margin(g1*C)
```

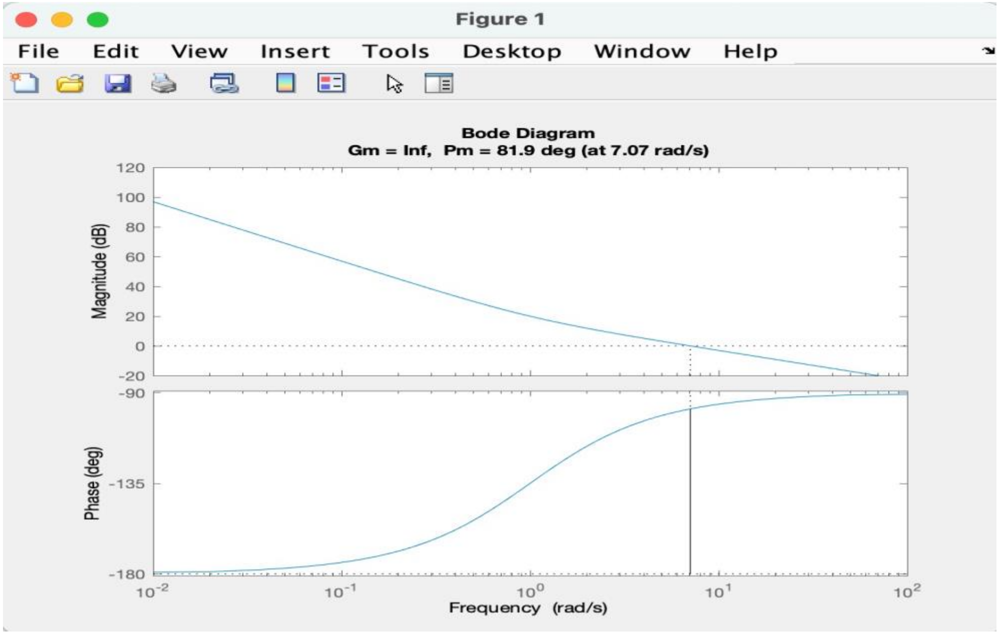


Fig.5

Bode of uncompensated system has **PM of 81.9**

Design for Overshoot and Settling time:

$\%\mathrm{OS} = 0.5$

$\mathrm{T_s} = 1sec$

$\mathrm{PM} = 81.9$

$$\%\mathrm{OS} = \mathrm{e}^{-\left(\frac{\zeta\pi}{\sqrt{1-\zeta^2}}\right)} * 100$$

$$\%\mathrm{OS} = 0.5$$

$$\mathrm{e}^{-\left(\frac{\zeta\pi}{\sqrt{1-\zeta^2}}\right)} * 100 = 0.5$$

Take log on both sides

$$\zeta = 0.96$$

$$PM = 100 \times 0.96$$

Phase margin of required system is 96.

$$\varphi_\mathrm{m} = 96 - 81.9 + 9.6$$

$$\boxed{\boldsymbol{\varphi_\mathrm{m} = 22.7}}$$

$$\alpha = \frac{1 + \sin\varphi_m}{1 - \sin\varphi_m} = 2.25$$

Magnitude $= -10\log_{10}\alpha = -3.5dB$

At $\boldsymbol{-3.5} \;\rightarrow\; \boldsymbol{\omega_m = 10.5\, rad/s}$

$$T = \frac{1}{10.5\sqrt{2.26}} = 1.33\ s$$

Compensated transfer function is :

$$\boxed{\frac{1 + \alpha T_s}{1 + T_s}}$$

Replacing the values of α and T, we obtain :

$$\boxed{\frac{\mathbf{1 + 3.005s}}{\mathbf{1 + 1.33s}}}$$

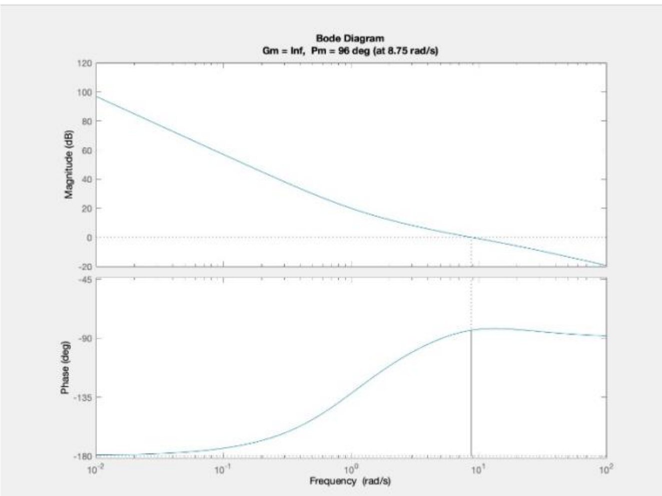
Transfer function after adding compensator in cascade with original transfer function:

```
  21.04 s^2 + 28.04 s + 7
  -----------------------
      1.33 s^3 + s^2

Continuous-time transfer function.
```

Code:

```
>> g4=tf([3.005 1],[1.33 1])

>> g2=tf([7 7],[1 0 0])

>> g5=g4*g2

>>bode(g5)
```

Bode plot of Compensated system:



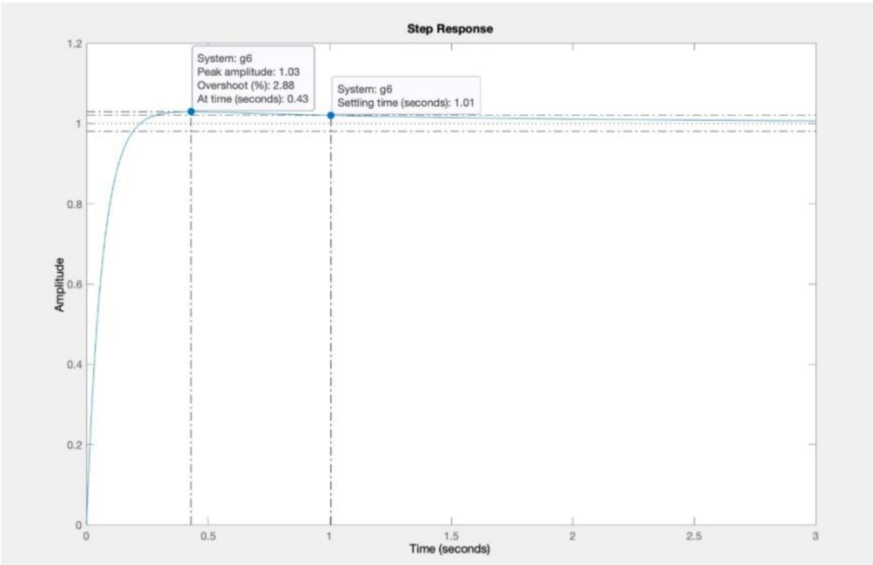
Transfer function after taking unity gain feedback:

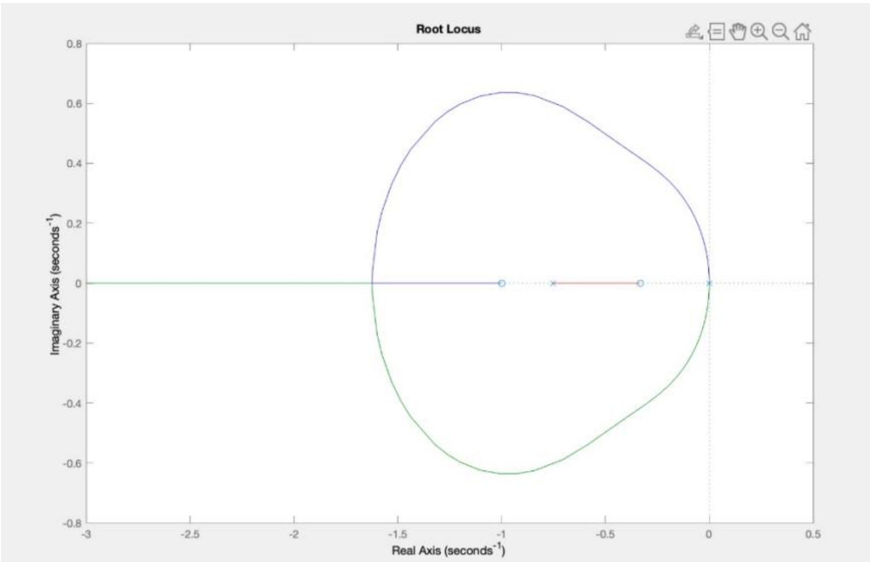
```
      21.04 s^2 + 28.04 s + 7
  ----------------------------------
  1.33 s^3 + 22.04 s^2 + 28.04 s + 7

Continuous-time transfer function.
```

Code:

```
>> g6=feedback(g5,1)
>> step(g6)
>> rlocus(g5)
```

Step response of Compensated system:





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Work Done by each member of the group:

CB.EN.U4ECE20215 **G Sai Karthik** ------ Analysis

CB.EN.U4ECE20219 **Hareekeshav S S** ------ Design: Lead compensator and Matlab

CB.EN.U4ECE20226 **K S S Pranav** ------ Analysis

CB.EN.U4ECE20264 **V Chandra Harsha** ------ Design and Documentation