Project Assignment 2 EQ1220 Signal Theory

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I. Introduction

Our report delves into the significance of the receiver in a wireless digital communication system, precisely its equalizer and decoder. It is imperative to have a decoding key in place to prevent distortions during transmission and ensure that clear images are received through wireless channels. The recipient needs to have suitable equalizer and detector models to ensure the precise decoding of the encrypted image. The primary aim is to generate an unencrypted image that closely resembles the original one. We elaborate on selecting the appropriate filter order for the received signal, enabling us to determine the filter coefficients and reconstruct the disrupted key.

II. Problems and Solutions

Key reconstruction

Suitable filters are an absolute necessity when utilizing a wireless digital communication system. Without them, the transmitted signal will inevitably suffer from distortion and noise. To solve our problem, we consider a casual FIR Wiener filter. Given the output signal of the communication channel $r(k)$, or in this case the input of the Wiener filter, we obtain

$$z(k) = \sum_{l=0}^{L} w(l) r(k-l),$$

as the filter output. The filter coefficients are $w(l)$ for $l = 0, ..., L$, where L is the filter order. This output signal should be as close to the input signal on the communication channel $b(k)$. We must find the filter coefficients to design the appropriate casual FIR Wiener filter and filter order. One can build an estimator X based on a vector of observations Y, i.e., $\hat{X}(Y) = g(Y)$. Suppose w =$(w1, w2, \ldots, w_L)$ is the vector of weights (filter coefficients), then $\hat{X}$ is a linear estimator if $\hat{X}(Y) = w^T Y$. The optimal choice of the filter coefficients $w(l)$ is minimizing the mean square error: MSE$[\hat{X}(Y)] = E[(\hat{X}(Y) - X)^2]$. By taking the gradient of $E[(\hat{X}(Y) - X)^2]$ w.r.t $w(l)$, we get $E[(X - h^T Y)Y] = 0$. Following the last derivation, one can define the coefficients using the Wiener-Hopf equation [1] $w_{FIR} = R_Y^{-1} r_{xy}$, where R_Y is the $L+1$ by $L+1$ autocorrelation matrix of r(k), and r_{xy} is the cross-correlation vector between $z(k)$ and $r(k-l)$, for $l = 0, ..., L$.

$$w_{FIR} = \begin{pmatrix} r_Y(0) & \cdots & r_Y(L+1) \\ \vdots & \ddots & \vdots \\ r_Y(L+1) & \cdots & r_Y(0) \end{pmatrix}^{-1} \begin{bmatrix} r_{XY}(0) \\ r_{XY}(1) \\ \vdots \\ r_{XY}(L+1) \end{bmatrix}$$

To calculate the matrices, it is necessary to determine the autocorrelation function of variable $r(k)$ and the cross-correlation function between $z(k)$ and $r(k)$. The estimators can be formulated as follows:

$$\hat{r}_Y(k) = \frac{1}{N} \sum_{l=0}^{L-k-1} r(l) r(l+k), \quad \text{for } k = 0, 1, \ldots, N-1,$$

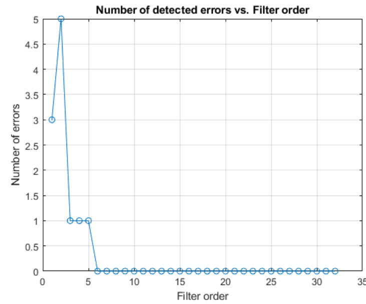


Fig. 1: Number of errors vs. filter order

$$\hat{r}_{YX}(k) = \frac{1}{N} \sum_{l=0}^{L-k-1} r(l) z(l+k), \quad \text{for } k = 0, 1, \ldots, N-1.$$

Initially, we calculate the ACF of the disturbed key and the cross-correlation of the undisturbed key (found in the training sequence) with the disturbed key. After calculating the number of detection errors for each filter order ranging from 1 to 32, we concluded that any order above 6 suffices for decoding the image. We can filter the image once we find the filter order number for which we obtain the minimum error. Figure 1 depicts the relationship between the number of detection errors and the filter order.

Decoding the image

The image was reconstructed using the $r(k)$. First, we calculate the filter coefficients, after which we use these coefficients to design a filter that minimizes the mean squared error between the received signal and the original signal. A filter order beyond six recovered the image in the process. We then map positive values to 1 and non-positive values to -1. The final image accurately portrays Rick from "Rick and Morty." This confirms the decoding procedure's validity. The image is located in Figure 2 on the following page. The equation to reconstruct the image is as follows:

$$\hat{b}(k) = \sum_{l=0}^{L} w(l) r(k-l).$$

Bit errors in the reconstructed key

The Bit errors can be described by the error rate, which determines the rate at which bits in the signal would be flipped to simulate transmission errors. Random indices represent the positions in the signal where errors were introduced. The bits at these indices are then flipped, simulating the effect of random transmission errors on the reconstructed signal.

Through this, we observed some distortions when there were about 1000 errors, but the image was still decodable. On the other hand, when the number of errors increased to roughly 2000, the decoded image became entirely distorted and unrecoverable. Figure 3 portrays the effect that bit errors had on the decoded image.

III. Conclusions

In our second project, we explored the practical mechanics of a fundamental wireless digital communication system. It was observed that signals frequently encounter distortion and noise as they traverse the communication channel. Once received, they require reconstruction through an equalizer and detector.
To accomplish this, we investigated the utilization of a casual FIR Wiener filter and analyzed the results of applying the appropriate version of the filter. Furthermore, we analyzed the effects of introducing random bit errors on the reconstructed key and experimented with different thresholds for the number of bit errors.

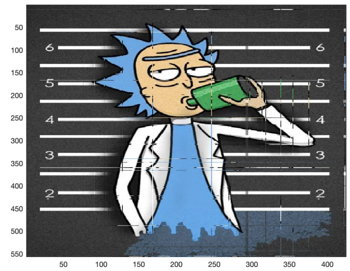
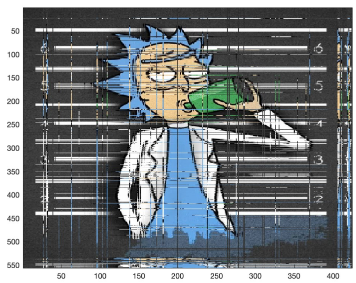
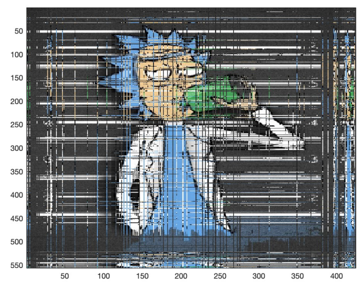


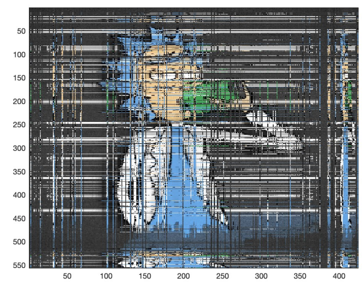
Fig. 2: The reconstructed image



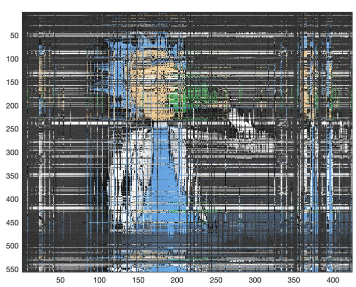
(a) 100 bit errors



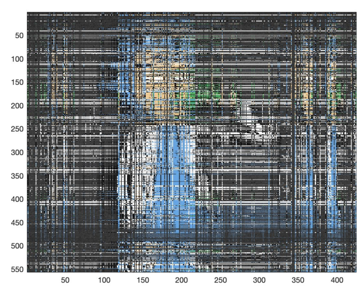
(b) 300 bit errors



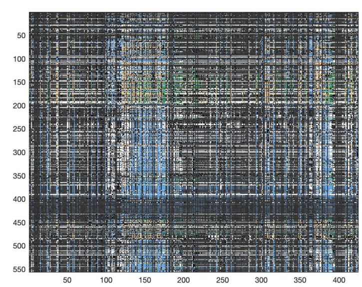
(c) 500 bit errors



(d) 700 bit errors



(e) 1000 bit errors



(f) 2000 bit errors

Fig. 3: Random bit errors in the reconstructed key.

References

[1] P. Handel, R. Ottoson, H. Hjalmarsson, *Signal Theory*, KTH, 2012