BITS Pilani

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## S1-25_DSECLZG530/SSCLZG599

# Natural Language Processing

## (Lecture #4 – Neural Network Basics)

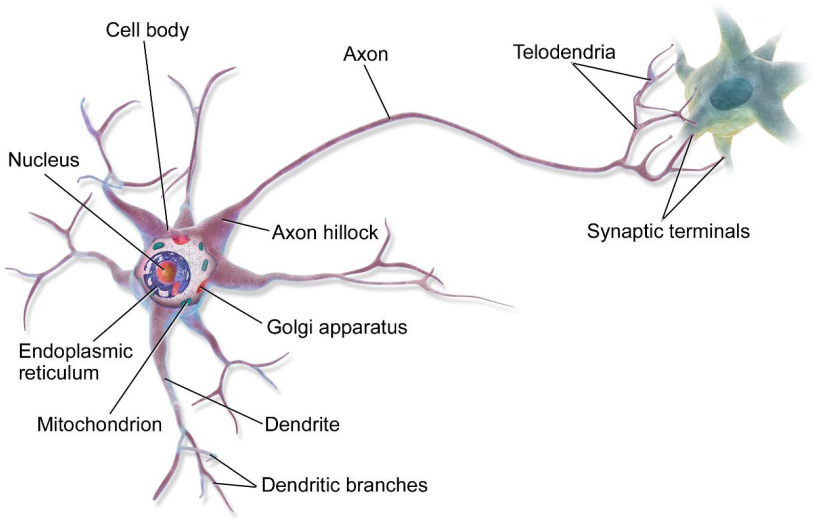

- *The slides presented here are obtained from the authors of the books and from various other contributors. I hereby acknowledge all the contributors for their material and inputs.*
- *I have added and modified a few slides to suit the requirements of the course.*

# Session Content

## Neural Networks and Neural Language Models

- Units in Neural Networks
- The XOR problem
- Feedforward Neural Networks
- Applying Feedforward Neural Networks to NLP tasks
- Training Neural Networks: Overview

# This is in your brain

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# Neural Network Unit

This is not in your brain

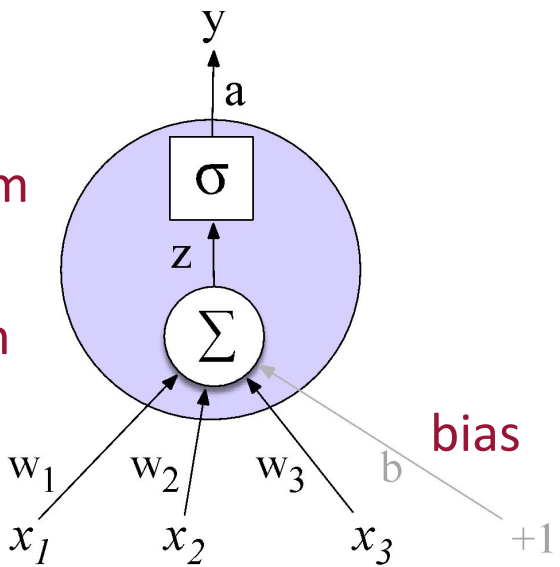
Output value

Non-linear transform

Weighted sum

Weights

Input layer



# Neural unit

- Take weighted sum of inputs, plus a bias

$$z = b + \sum_i w_i x_i$$

$$z = w \cdot x + b$$

- Instead of just using z, we'll apply a nonlinear activation function f:

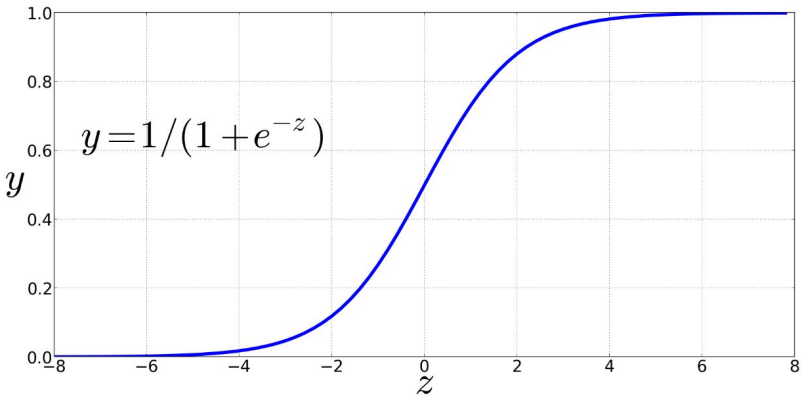
$$y = a = f(z$$

# Non-Linear Activation Functions

We've already seen the sigmoid for logistic regression:

## Sigmoid

$$y = s(z) = \frac{1}{1 + e^{-z}}$$



# Final function the unit is computing

$$y = s(w \cdot x + b) = \frac{1}{1 + \exp(-(w \cdot x + b))}$$

# Final unit again

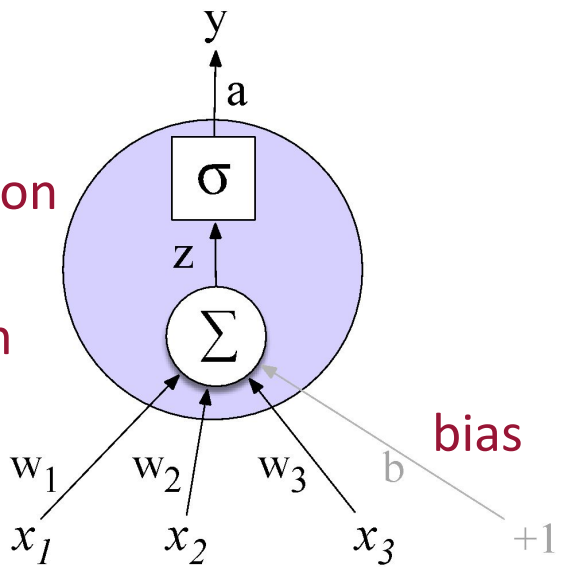
Output value

Non-linear activation function

Weighted sum

Weights

Input layer



# An example

- *Suppose a unit has:*
- $w = [0.2, 0.3, 0.9]$
- $b = 0.5$
- What happens with input x:
- $x = [0.5, 0.6, 0.1]$

$$y = s(w \cdot x + b) =$$

# An example

- *Suppose a unit has:*
  - $w = [0.2, 0.3, 0.9]$
  - $b = 0.5$
- What happens with the following input x?
- $x = [0.5, 0.6, 0.1]$

$$y = s(w \cdot x + b) = \frac{1}{1 + e^{-(w \cdot x + b)}} =$$

# An example

- *Suppose a unit has:*
- $w = [0.2, 0.3, 0.9]$
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- What happens with input x:
- $x = [0.5, 0.6, 0.1]$

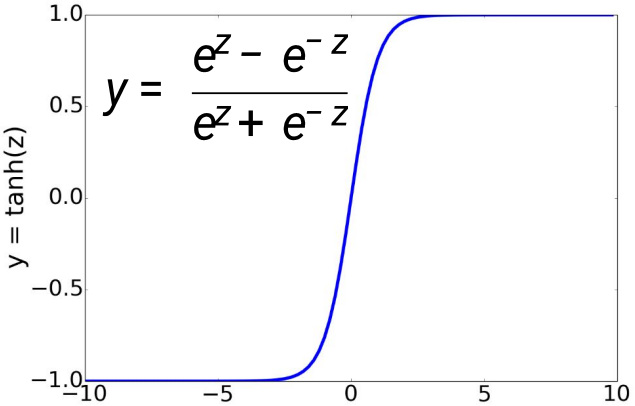
$$y = s(w \cdot x + b) = \frac{1}{1 + e^{-(w \cdot x + b)}} = \frac{1}{1 + e^{-(.5 \times .2 + .6 \times .3 + .1 \times .9 + .5)}} =$$

# An example

- *Suppose a unit has:*
- $w = [0.2, 0.3, 0.9]$
- $b = 0.5$
- What happens with input x:
- $x = [0.5, 0.6, 0.1]$

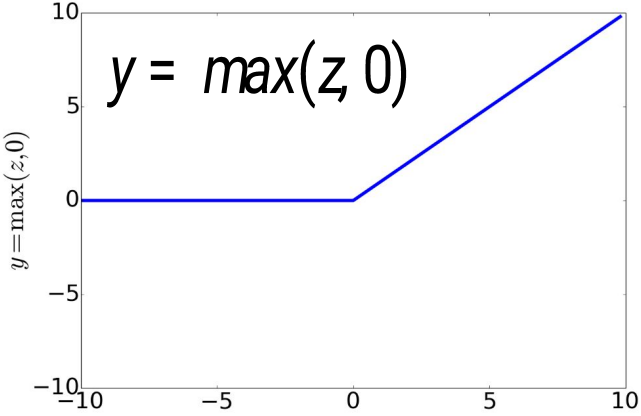
$$y = s(w \cdot x + b) = \frac{1}{1 + e^{-(w \cdot x + b)}} = \frac{1}{1 + e^{-(.5 \times .2 + .6 \times .3 + .1 \times .9 + .5)}} = \frac{1}{1 + e^{-0.87}} = .70$$

# Non-Linear Activation Functions besides sigmoid




tanh

Most Common:




ReLU
Rectified Linear Unit

# The XOR problem

Minsky and Papert (1969)

- Can neural units compute simple functions of input?

| AND | | | OR | | | XOR | | |
|---|---|---|---|---|---|---|---|---|
| x1 | x2 | y | x1 | x2 | y | x1 | x2 | y |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 1 | 0 | 0 | 1 | 1 | 0 | 1 | 1 |
| 1 | 0 | 0 | 1 | 0 | 1 | 1 | 0 | 1 |
| 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 0 |

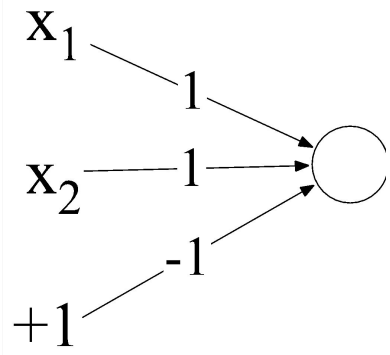
# Perceptrons

- A very simple neural unit
- Binary output (0 or 1)
- No non-linear activation function

$$y = \begin{cases} 0, & \text{if } w \cdot x + b \leq 0 \\ 1, & \text{if } w \cdot x + b > 0 \end{cases}$$

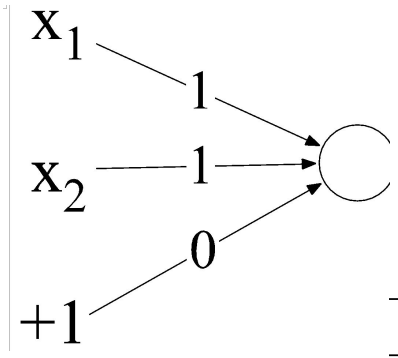
# Easy to build AND or OR with perceptrons

$$y = \begin{cases} 0, & \text{if } w \cdot x + b \leq 0 \\ 1, & \text{if } w \cdot x + b > 0 \end{cases}$$




AND

| AND | | |
|---|---|---|
| x1 | x2 | y |
| 0 | 0 | 0 |
| 0 | 1 | 0 |
| 1 | 0 | 0 |
| 1 | 1 | 1 |




OR

| OR | | |
|---|---|---|
| x1 | x2 | y |
| 0 | 0 | 0 |
| 0 | 1 | 1 |
| 1 | 0 | 1 |
| 1 | 1 | 1 |

# Not possible to capture XOR with perceptrons

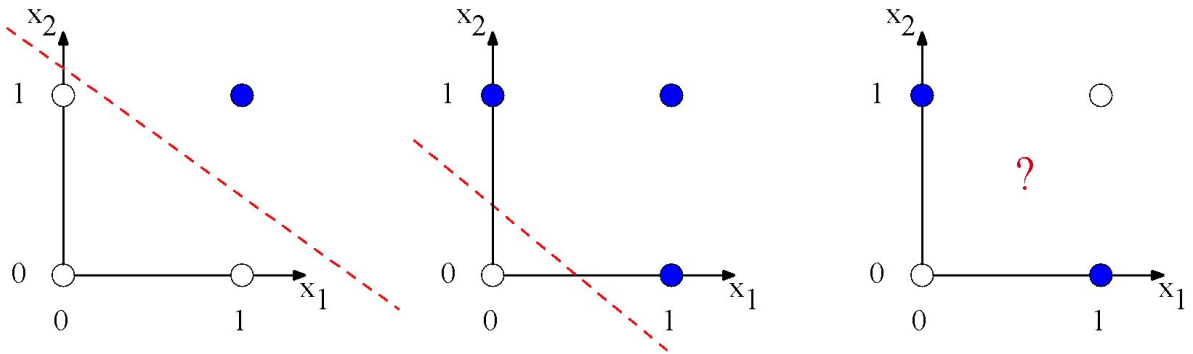
- Pause the lecture and try for yourself!

# Why? Perceptrons are linear classifiers

- Perceptron equation given $x_1$ and $x_2$, is the equation of a line
- $w_1 x_1 + w_2 x_2 + b = 0$
- (in standard linear format: $x_2 = (-w_1/w_2)x_1 + (-b/w_2)$ )

- This line acts as a **decision boundary**
- 0 if input is on one side of the line
- 1 if on the other side of the line

# Decision boundaries

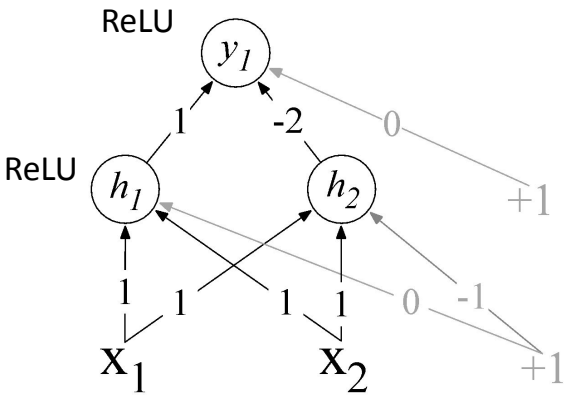



XOR is not a **linearly separable** function!

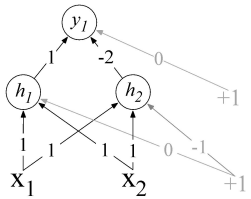
# Solution to the XOR problem

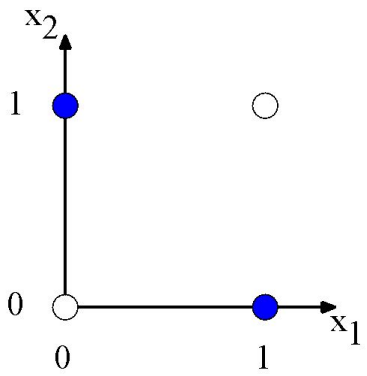
- XOR **can't** be calculated by a single perceptron
- XOR **can** be calculated by a layered network of units.

| XOR | | |
|---|---|---|
| x1 | x2 | y |
| 0 | 0 | 0 |
| 0 | 1 | 1 |
| 1 | 0 | 1 |
| 1 | 1 | 0 |

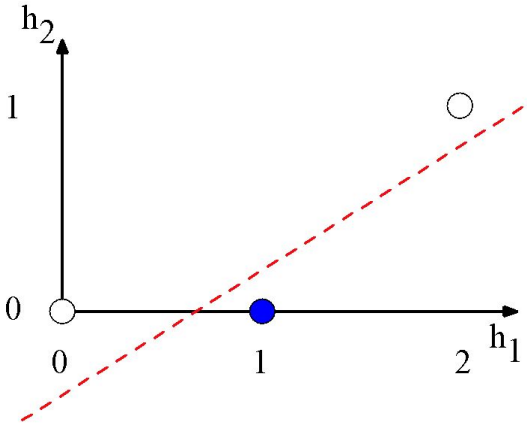


# The hidden representation h




a) The original *x* space

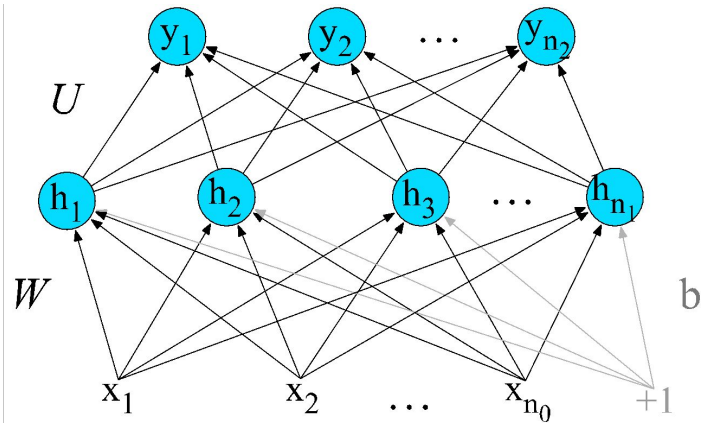



b) The new (linearly separable) *h* space

(With learning: hidden layers will learn to form useful representations)

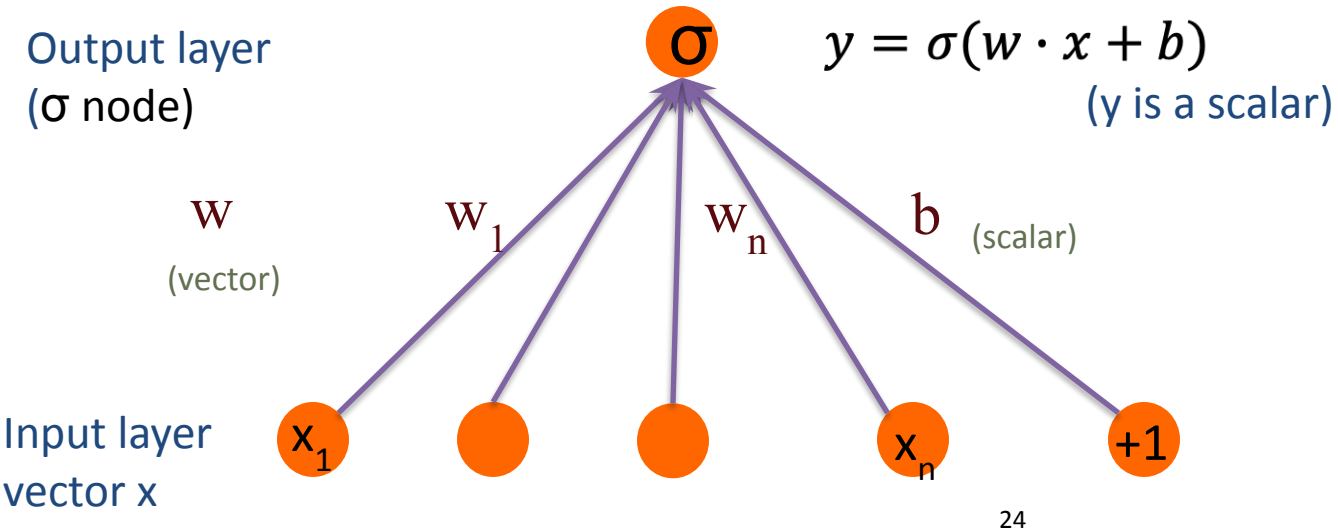
# Feedforward Neural Networks

- Can also be called **multi-layer perceptrons** (or **MLPs**) for historical reasons

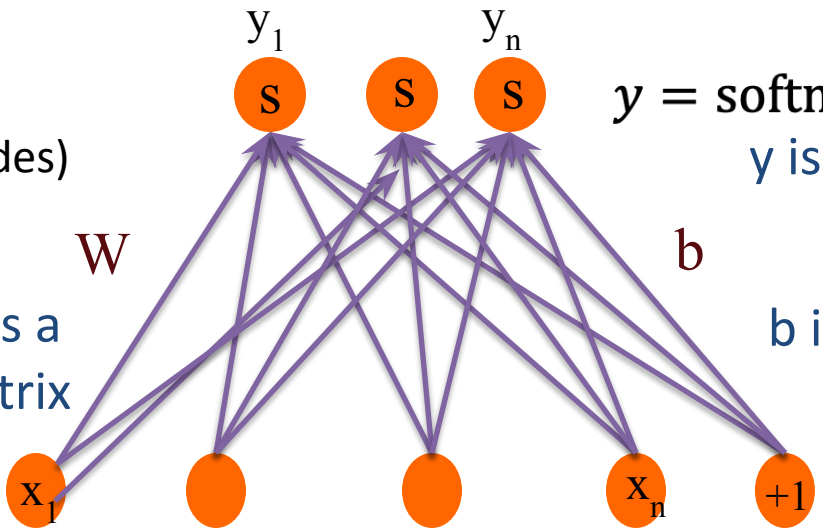


# Binary Logistic Regression as a 1-layer Network

(we don't count the input layer in counting layers!)



# Fully connected single layer network




Output layer
(softmax nodes)

$y = \text{softmax}(Wx + b)$

y is a vector

W is a matrix

b is a vector

Input layer
scalars

# softmax: a generalization of sigmoid

- For a vector *z* of dimensionality *k*, the softmax is:

$$\text{softmax}(z) = \left[ \frac{\exp(z_1)}{\sum_{i=1}^{k} \exp(z_i)}, \frac{\exp(z_2)}{\sum_{i=1}^{k} \exp(z_i)}, \ldots, \frac{\exp(z_k)}{\sum_{i=1}^{k} \exp(z_i)} \right]$$

$$\text{softmax}(z_i) = \frac{\exp(z_i)}{\sum_{j=1}^{k} \exp(z_j)} \quad 1 \leq i \leq k$$

- Example:

$$z = [0.6, 1.1, -1.5, 1.2, 3.2, -1.1]$$

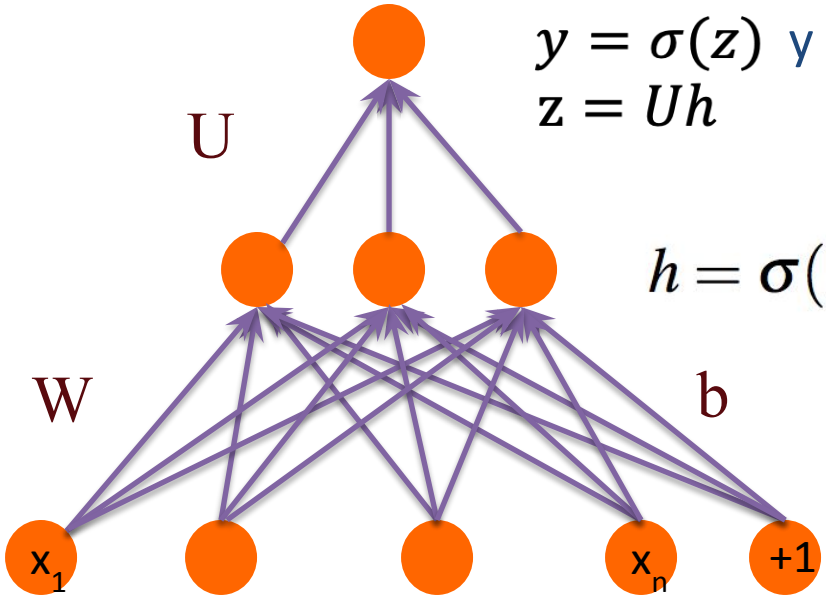
$$\text{softmax}(z) = [0.055, 0.090, 0.006, 0.099, 0.74, 0.010]$$

# Two-Layer Network with scalar output

Output layer
(σ node)

hidden units
(σ node)

Input layer
(vector)




$y = \sigma(z)$ y is a scalar

$z = Uh$

$h = \sigma(Wx + b)$

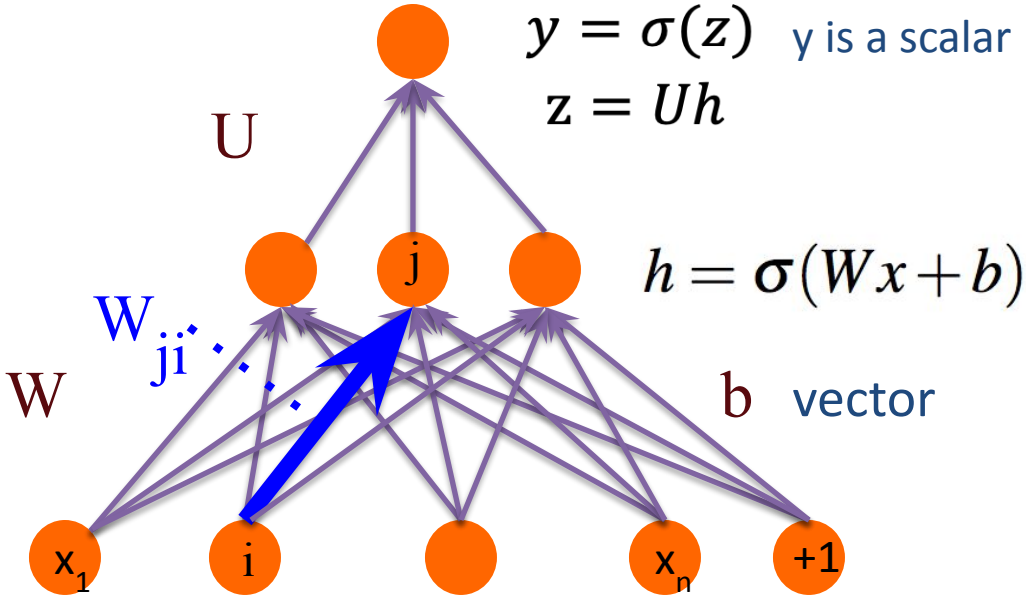
Could be ReLU
Or tanh

# Two-Layer Network with scalar output

Output layer
(σ node)

hidden units
(σ node)

Input layer
(vector)

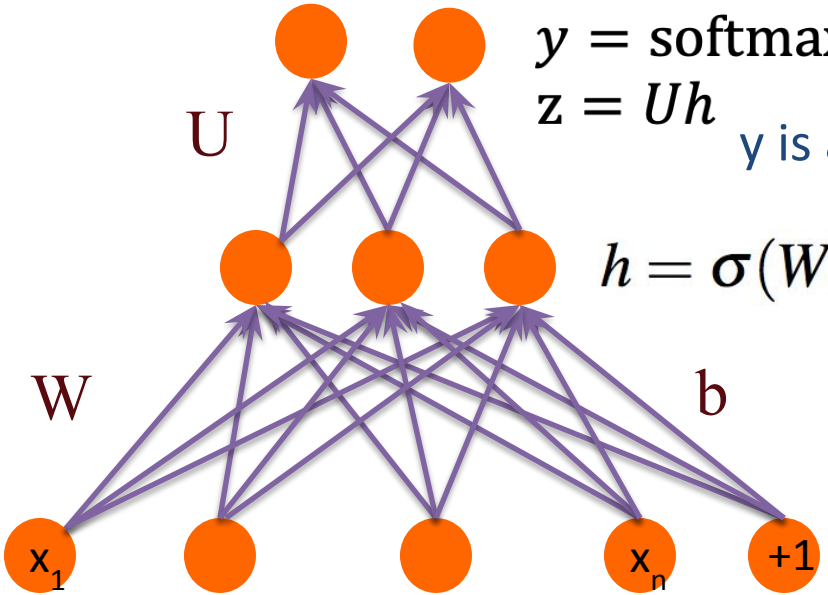


# Two-Layer Network with softmax output

Output layer
(σ node)

hidden units
(σ node)

Input layer
(vector)



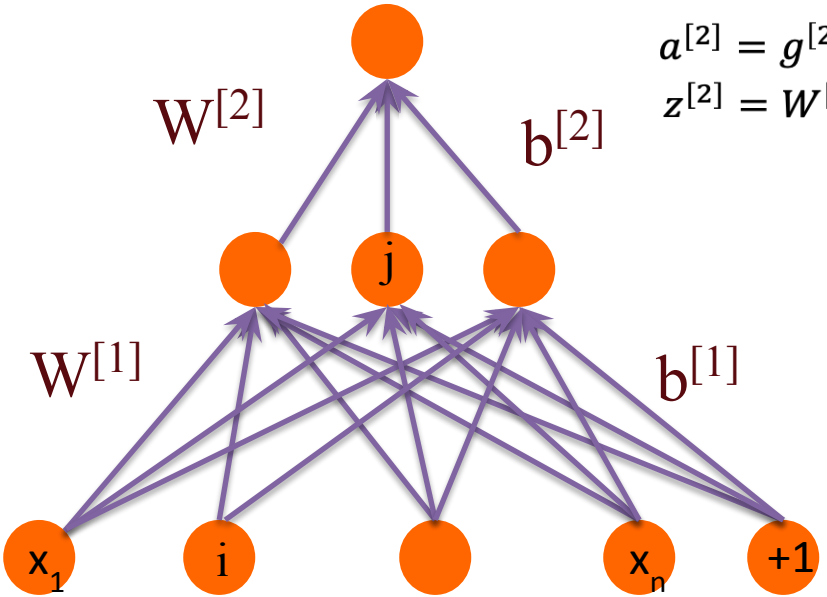

$$y = \text{softmax}(z)$$
$$z = Uh$$

y is a vector

$$h = \sigma(Wx + b)$$

Could be ReLU
Or tanh

# Multi-layer Notation




$$y = a^{[2]}$$

$$a^{[2]} = g^{[2]}(z^{[2]})$$ sigmoid or softmax

$$z^{[2]} = W^{[2]}a^{[1]} + b^{[2]}$$

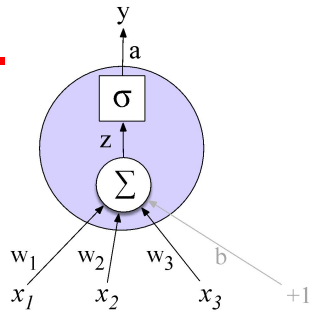
$$a^{[1]} = g^{[1]}(z^{[1]})$$ ReLU

$$z^{[1]} = W^{[1]}a^{[0]} + b^{[1]}$$

$a^{[0]}$

# Multi Layer Notation

$$
\begin{aligned}
z^{[1]} &= W^{[1]} a^{[0]} + b^{[1]} \\
a^{[1]} &= g^{[1]}(z^{[1]}) \\
z^{[2]} &= W^{[2]} a^{[1]} + b^{[2]} \\
a^{[2]} &= g^{[2]}(z^{[2]}) \\
\hat{y} &= a^{[2]}
\end{aligned}
$$




**for** $i$ **in** 1..n

$$
\begin{aligned}
z^{[i]} &= W^{[i]} a^{[i-1]} + b^{[i]} \\
a^{[i]} &= g^{[i]}(z^{[i]}) \\
\hat{y} &= a^{[n]}
\end{aligned}
$$

# Replacing the bias unit

- Let's switch to a notation without the bias unit
- Just a notational change

1. Add a dummy node $a_0=1$ to each layer
2. Its weight $w_0$ will be the bias
3. So input layer $a^{[0]}_0=1$,
   - And $a^{[1]}_0=1$ , $a^{[2]}_0=1$,…

# Replacing the bias unit

- Instead of:

$$x = x_1, x_2, \ldots, x_{n0}$$

$$h = \sigma(Wx + b)$$

$$h_j = \sigma\left(\sum_{i=1}^{n_0} W_{ji} x_i + b_j\right)$$

We'll do this:

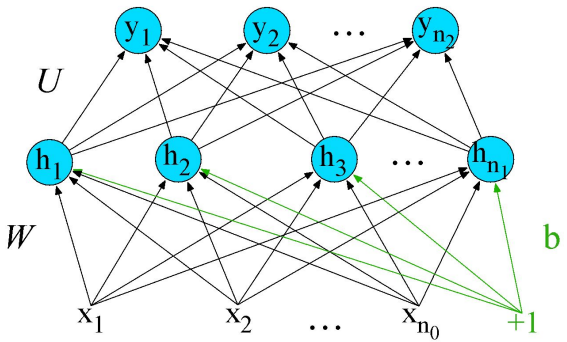
$$x = x_0, x_1, x_2, \ldots, x_{n0}$$

$$h = \sigma(Wx)$$

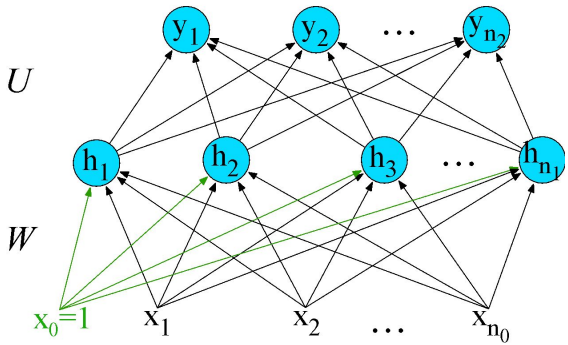
$$\sigma\left(\sum_{i=0}^{n_0} W_{ji} x_i\right)$$

# Replacing the bias unit

Instead of:




We'll do this:



# Use cases for feedforward networks

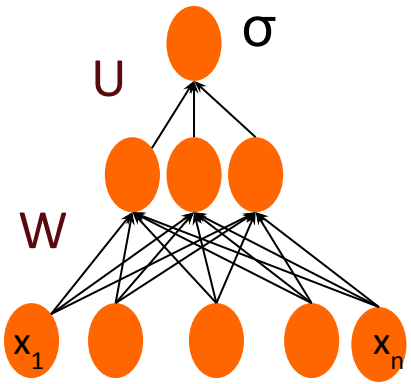
- Let's consider 2 (simplified) sample tasks:

1. Text classification
2. Language modeling

- State of the art systems use more powerful neural architectures, but simple models are useful to consider!

# Classification: Sentiment Analysis

- We could do exactly what we did with logistic regression
- Input layer are binary features as before
- Output layer is 0 or 1

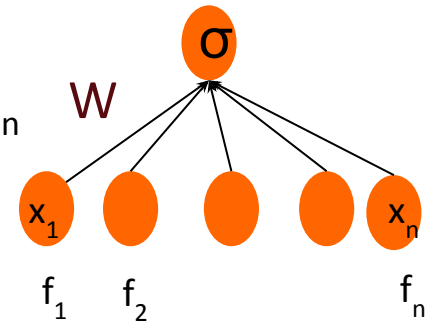


# Sentiment Features

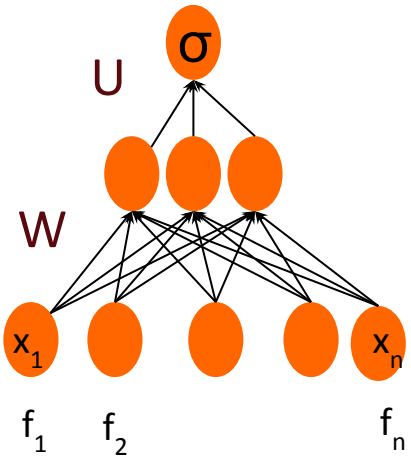
| Var | Definition |
|---|---|
| $x_1$ | count(positive lexicon) $\in$ doc) |
| $x_2$ | count(negative lexicon) $\in$ doc) |
| $x_3$ | $\begin{cases} 1 & \text{if } \text{“no”} \in \text{doc} \\ 0 & \text{otherwise} \end{cases}$ |
| $x_4$ | count(1st and 2nd pronouns $\in$ doc) |
| $x_5$ | $\begin{cases} 1 & \text{if } \text{“!”} \in \text{doc} \\ 0 & \text{otherwise} \end{cases}$ |
| $x_6$ | log(word count of doc) |

# Feedforward nets for simple classification

Logistic Regression



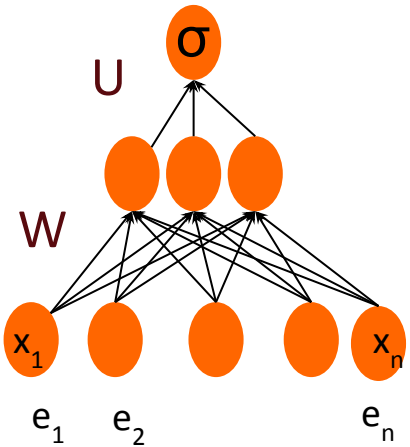

2-layer feedforward network



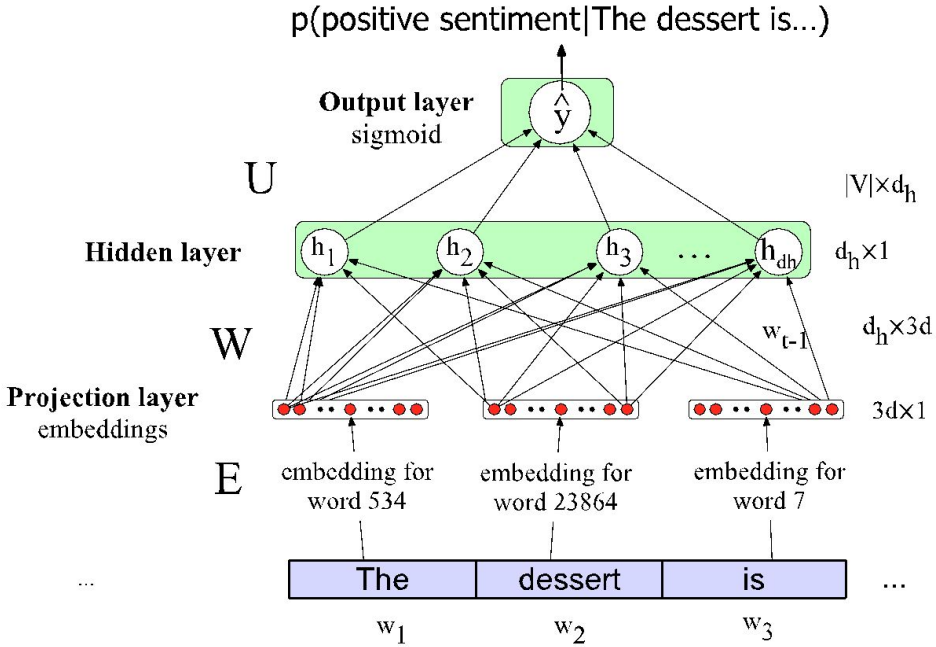

- Just adding a hidden layer to logistic regression
- allows the network to use non-linear interactions between features
- which may (or may not) improve performance.

# Even better: representation learning

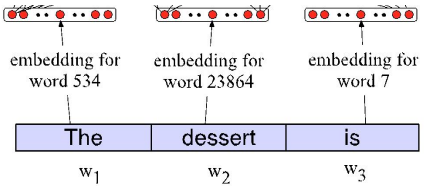
- The real power of deep learning comes from the ability to **learn** features from the data
- Instead of using hand-built human-engineered features for classification
- Use learned representations like embeddings!



# Neural Net Classification with embeddings as input features!



# Issue: texts come in different sizes



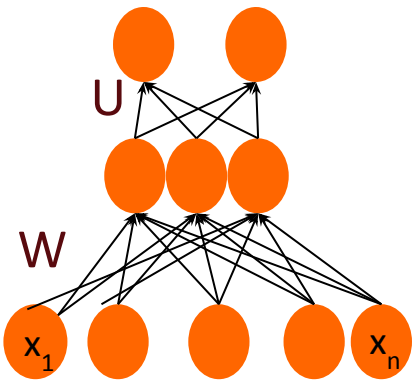

- This assumes a fixed size length (3)!
- Kind of unrealistic.
- Some simple solutions (more sophisticated solutions later)

1. Make the input the length of the longest review
   - If shorter then pad with zero embeddings
   - Truncate if you get longer reviews at test time
2. Create a single "sentence embedding" (the same dimensionality as a word) to represent all the words
   - Take the mean of all the word embeddings
   - Take the element-wise max of all the word embeddings
     - For each dimension, pick the max value from all words

# Reminder: Multiclass Outputs

- What if you have more than two output classes?
  - Add more output units (one for each class)
  - And use a "softmax layer"

$$\text{softmax}(z_i) = \frac{e^{z_i}}{\sum_{j=1}^{k} e^{z_j}} \quad 1 \leq i \leq D$$



# Neural Language Models (LMs)

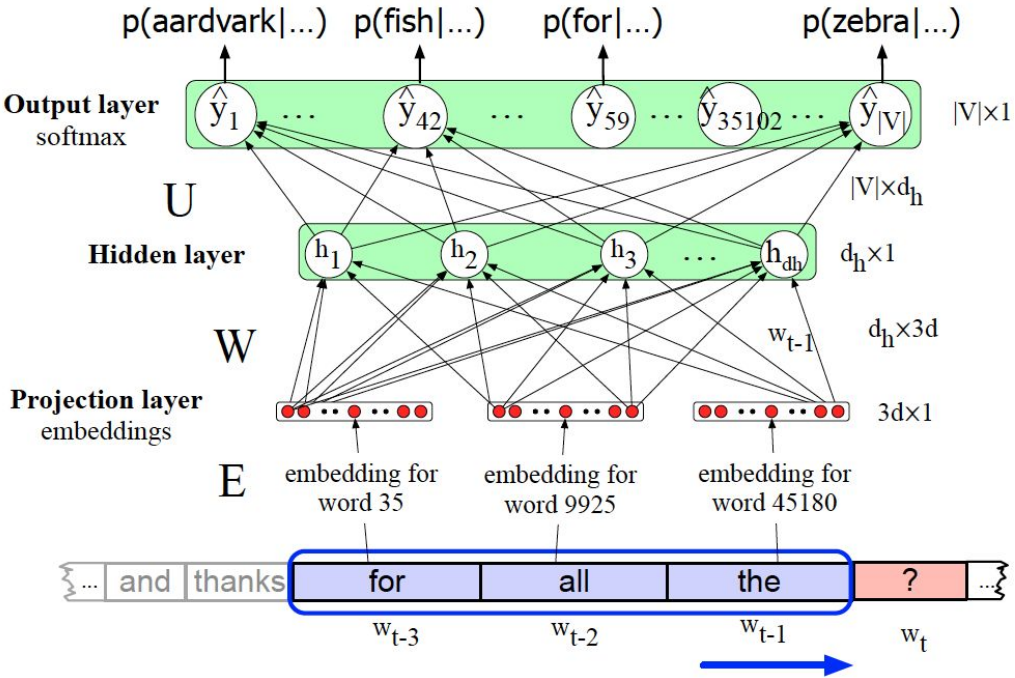
- **Language Modeling**: Calculating the probability of the next word in a sequence given some history.
  - We've seen N-gram based LMs
  - But neural network LMs far outperform n-gram language models
- State-of-the-art neural LMs are based on more powerful neural network technology like Transformers
- But **simple feedforward LMs** can do almost as well!

# Simple feedforward Neural Language Models

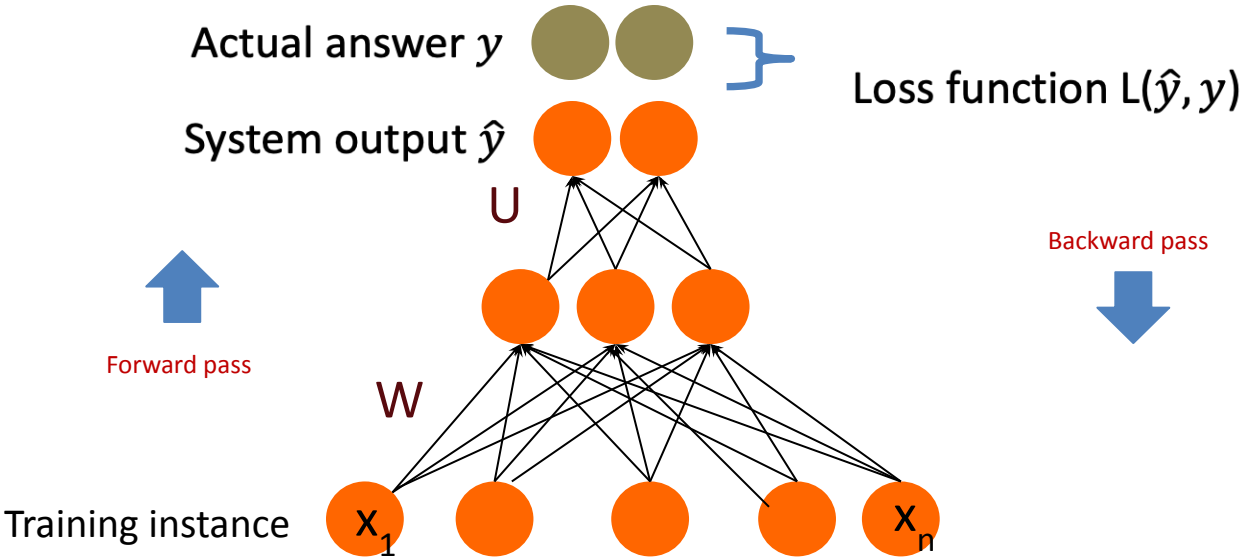
- **Task**: predict next word $w_t$ given prior words $w_{t-1}$, $w_{t-2}$, $w_{t-3}$, …
- **Problem**: Now we're dealing with sequences of arbitrary length.
- **Solution**: Sliding windows (of fixed length)

$$P(w_t|w_1^{t-1}) \approx P(w_t|w_{t-N+1}^{t-1})$$

# Neural Language Model



# Intuition: training a 2-layer Network



# Intuition: Training a 2-layer network

- For every training tuple $(x, y)$
  - Run *forward* computation to find our estimate $\hat{y}$
  - Run *backward* computation to update weights:
    - For every output node
    - Compute loss $L$ between true $y$ and the estimated $\hat{y}$
    - For every weight $w$ from hidden layer to the output layer
      - » Update the weight
    - For every hidden node
    - Assess how much blame it deserves for the current answer
    - For every weight $w$ from input layer to the hidden layer
      - » Update the weight

# Reminder: Loss Function for binary logistic regression

- A measure for how far off the current answer is to the right answer
- Cross entropy loss for logistic regression:

$$L_{\text{CE}}(\hat{y}, y) = -\log p(y|x) = -[y \log \hat{y} + (1-y)\log(1-\hat{y})]$$

$$= -[y \log \sigma(w \cdot x + b) + (1-y)\log(1-\sigma(w \cdot x + b))]$$

# Reminder: gradient descent for weight updates

- Use the derivative of the loss function with respect to weights

$$\frac{d}{dw} L(f(x;w),y)$$

- To tell us how to adjust weights for each training item
  - Move them in the opposite direction of the gradient

$$w^{t+1} = w^t - h \frac{d}{dw} L(f(x;w),y)$$

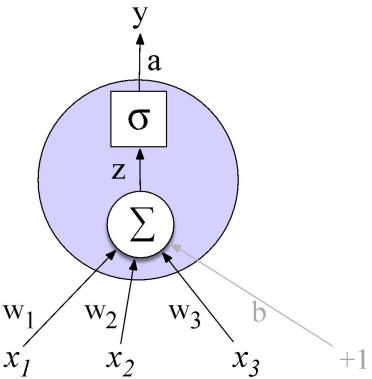
  - For logistic regression

$$\frac{\partial L_{\text{CE}}(\hat{y},y)}{\partial w_j} = [\sigma(w \cdot x + b) - y] x_j$$

# Where did that derivative come from?

- Using the chain rule! $f(x) = u(v(x))$
- Intuition (see the text for details)

$$\frac{df}{dx} = \frac{du}{dv} \cdot \frac{dv}{dx}$$




Derivative of the Activation

$$\frac{\partial L}{\partial w_i} = \frac{\partial L}{\partial y} \frac{\partial y}{\partial z} \frac{\partial z}{\partial w_i}$$

Derivative of the Loss

Derivative of the weighted sum

# How can I find that gradient for every weight in the network?

- These derivatives on the prior slide only give the updates for one weight layer: the last one!
- What about deeper networks?
- Lots of layers, different activation functions?
- Solution in the next lecture:
- Even more use of the chain rule!!
- Computation graphs and backward differentiation!

# Summary

- For training, we need the derivative of the loss with respect to weights in early layers of the network
- But loss is computed only at the very end of the network!
- Solution: **backward differentiation**
- Given a computation graph and the derivatives of all the functions in it we can automatically compute the derivative of the loss with respect to these early weights.

# Probabilistic Language Models

- Probability of a sequence of words: $P(W) = P(w_1, w_2, ..., w_{t-1}, w_T)$
- **Conditional probability** of an upcoming word: $P(w_T | w_1, w_2, ..., w_{t-1})$
- Chain rule of probability: $P(w_1, w_2, ..., w_{t-1}, w_T) = P(w_1)P(w_2 \mid w_1)P(w_3 \mid w_1, w_2)...P(w_T \mid w_1, w_2, .., w_T)$
- $(n\text{-}1)^{\text{th}}$ order Markov assumption $P(w_1, w_2, ..., w_{t-1}, w_T) = \prod_{t=1}^{T} P(w_t \mid w_1, w_2, ..., w_{t-1})$
- Each $p(w_i|w_{i-4}, w_{i-3}, w_{i-2}, w_{i-1})$ may not have enough statistics to estimate
  - we back off to $p(w_i|w_{i-3}, w_{i-2}, w_{i-1})$, $p(w_i|w_{i-2}, w_{i-1})$, etc., all the way to $p(w_i)$

# Neural Probabilistic Language Models

- Instead of treating words as tokens, exploit semantic similarity
  - Learn a distributed representation of words that will allow sentences like these to be seen as similar

    The cat is walking in the bedroom.
    A dog was walking in the room.
    The cat is running in a room.
    The dog was running in the bedroom.
    etc.
  - Use a neural net to represent the conditional probability function $\boldsymbol{P(w_t|w_{t-n}, w_{t-n+1}, \cdots, w_{t-1})}$
  - Learn the word representation and the probability function simultaneously

# Neural Language Models (LMs)

- **Language Modeling**: Calculating the probability of the next word in a sequence given some history.
- We've seen N-gram based LMs
- But neural network LMs far outperform n-gram language models
- State-of-the-art neural LMs are based on more powerful neural network technology like **Transformers**
- But **simple feedforward LMs** can do almost as well!

# Simple feedforward Neural Language Models

- **Task**: predict next word $w_t$

  given prior words $w_{t-1}$, $w_{t-2}$, $w_{t-3}$, …

- **Output** : probability distribution over possible next words.

$$P(w_t|w_1,\ldots,w_{t-1}) \approx P(w_t|w_{t-N+1},\ldots,w_{t-1})$$

- **Problem**: Now we're dealing with sequences of arbitrary length.
- **Solution**: Sliding windows (of fixed length)

# Word Embeddings

- one-hot vector representation , e.g., dog = (0,0,0,0,1,0,0,0,0,....), cat = (0,0,0,0,0,0,0,1,0,....)
- Represent each of the N previous words as a one-hot vector of length |V| one-hot vector , i.e., with one dimension for each word in the vocabulary
- word "toothpaste", supposing it is $V_5$, i.e., index 5 in the vocabulary, $x_5 = 1$, and $x_i = 0$ i $\forall \neq 5$,

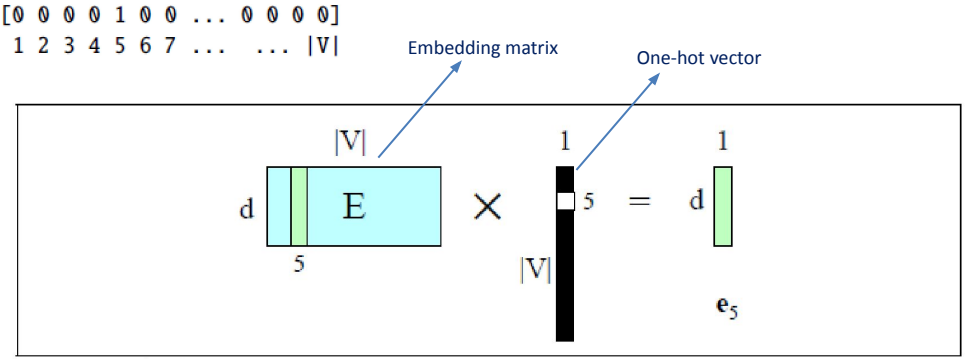



**Figure 7.12** Selecting the embedding vector for word $V_5$ by multiplying the embedding matrix **E** with a one-hot vector with a 1 in index 5.

# Why Neural LMs work better than N-gram LMs

## embeddings

- Neural language models represent words in this prior context by their embeddings, rather than just by their word identity as used in n-gram language models.
- Using embeddings allows neural language models to generalize better to unseen data.

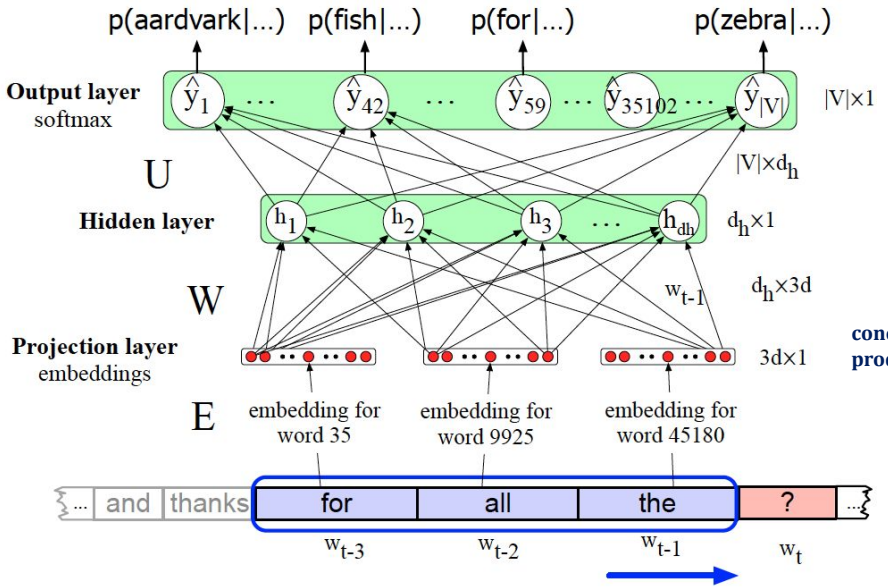
**Training data:**

- We've seen: I have to make sure that the cat gets fed.
- Never seen: dog gets fed

**Test data:**

- I forgot to make sure that the dog gets ___
- N-gram LM can't predict "fed"!
- Neural LM can use similarity of "cat" and "dog" embeddings to generalize and predict "fed" after dog

# Neural Language Model




Equations:

$$
\begin{aligned}
\mathbf{e} &= [\mathbf{E}x_{t-3}; \mathbf{E}x_{t-2}; \mathbf{E}x_{t-1}] \\
\mathbf{h} &= \sigma(\mathbf{W}\mathbf{e} + \mathbf{b}) \\
\mathbf{z} &= \mathbf{U}\mathbf{h} \\
\hat{\mathbf{y}} &= \text{softmax}(\mathbf{z})
\end{aligned}
$$

**concatenate 3 embeddings for the 3 context words to produce the embedding layer e**

# Training the neural language model

- Two ways:
- Freeze the embedding layer E with initial word2vec values.
  - —Freezing means we use **word2vec or some other pretraining algorithm** to compute the initial embedding matrix E, and then hold it constant while we only Modify W, U, and b, i.e., we don't update E during language model training
- Learn the embeddings simultaneously with training the network.
  - —Useful when the task the network is designed for (like sentiment classification, translation, or parsing) places strong constraints on what makes a good representation for words.

# N-gram vs NLM

- Compared to n-gram models, NLM can
  - —handle much longer histories
  - —generalize better over contexts of similar words,
  - —Are more accurate at word-prediction.
- NLM are
  - —much more complex,
  - —slower
  - —need more energy to train,
  - —less interpretable than n-gram models, so for many (especially smaller) tasks an n-gram language model is still the right tool.

# Thank You

## References


| | Author(s), Title, Edition, Publishing House |
|---|---|
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| T2 | Foundations of statistical Natural language processing by Christopher D.Manning and Hinrich schutze |