BABEŞ-BOLYAI UNIVERSITY
Faculty of Computer Science and Mathematics

ARTIFICIAL INTELLIGENCE

Intelligent systems

Rule-based systems – uncertainty

Topics

A. Short introduction in Artificial Intelligence (AI)

A. Solving search problems
   A. Definition of search problems
   B. Search strategies
      A. Uninformed search strategies
      B. Informed search strategies
      C. Local search strategies (Hill Climbing, Simulated Annealing, Tabu Search, Evolutionary algorithms, PSO, ACO)
      D. Adversarial search strategies

C. **Intelligent systems**
   A. Rule-based systems in certain environments
   B. **Rule-based systems in uncertain environments (Bayes, Fuzzy)**
   C. Learning systems
      A. Decision Trees
      B. Artificial Neural Networks
      C. Support Vector Machines
      D. Evolutionary algorithms
   D. Hybrid systems

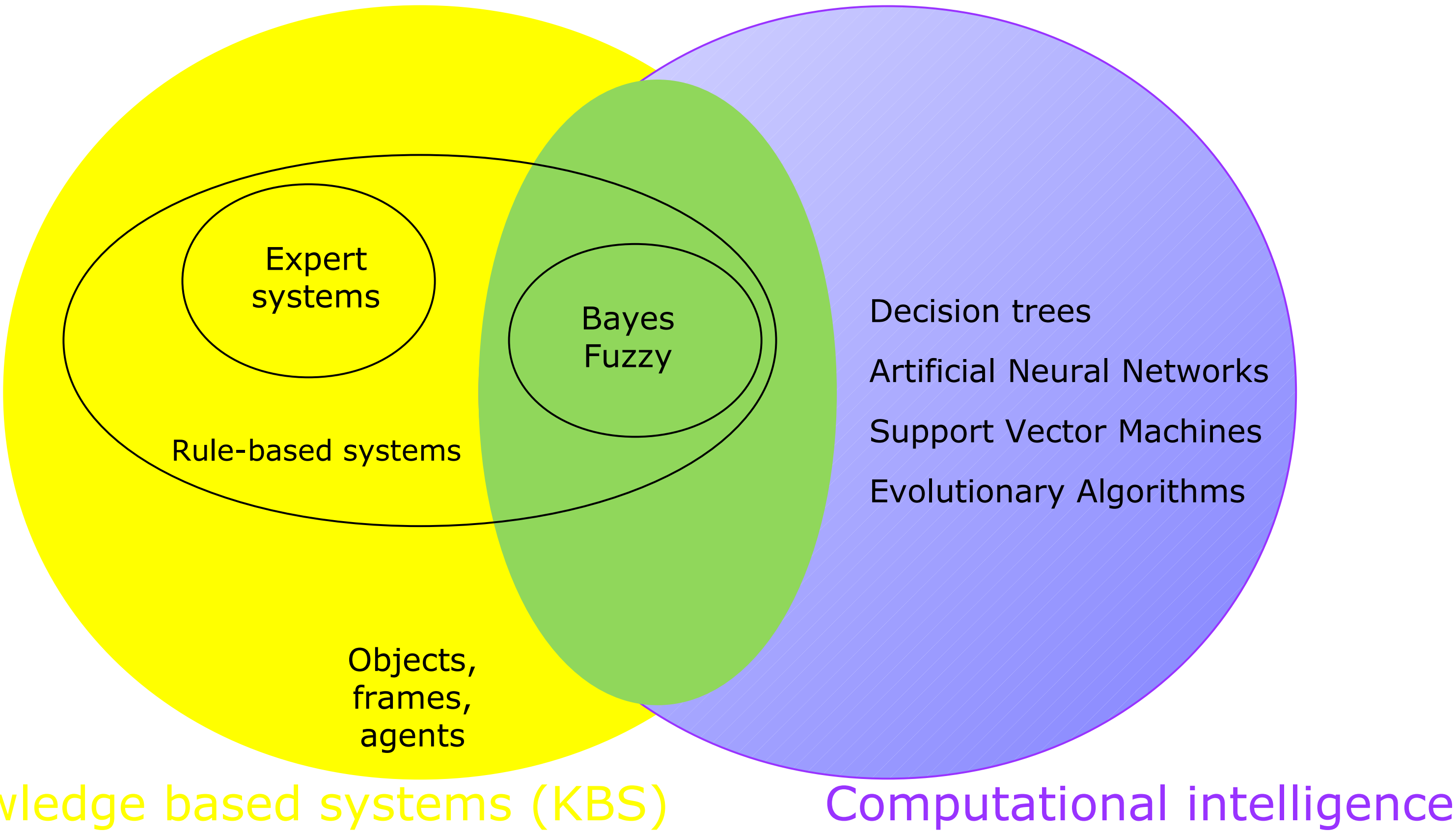
Useful information

- Chapter V of *S. Russell, P. Norvig, Artificial Intelligence: A Modern Approach, Prentice Hall, 1995*
- Chapter 3 of *Adrian A. Hopgood, Intelligent Systems for Engineers and Scientists, CRC Press, 2001*
- Chapters 8 and 9 of *C. Groşan, A. Abraham, Intelligent Systems: A Modern Approach, Springer, 2011*

Content

- Intelligent systems
  - Knowledge-based systems
    - Rule-based systems in uncertain environments

Intelligent systems



Intelligent systems – knowledge-based systems(KBS)

- Computational systems – composed of 2 parts:
  - Knowledge base (KB)
    - Specific information of the domain
  - Inference engine (IE)
    - Rules for generating new information
    - Domain-independent algorithms

Intelligent systems - KBS

Knowledge base

- Content
  - Information (in a given representation) about environment
  - Required information for problem solving
  - Set of propositions that describe the environment

- Typology
  - Perfect information
    - Classical logic
      - *IF A is true THEN A is ┐false*
      - *IF B is false THEN B is ┐true*
  - Imperfect information
    - Non-exact
    - Incomplete
    - Incommensurable

Intelligent systems - KBS

- Sources of uncertainty
  - Imperfection of rules
  - Doubt of rules
  - Using a vague (imprecise) language

- Modalities to express the uncertainty
  - Probabilities
  - Fuzzy logic
  - Bayes theorem
  - Theory of Dempster-Shafer

- Modalities to represent the uncertainty
  - By using a single value → certainty factors, confidence, truth value
    - How sure we are that the given facts are valid
  - By using more values → logic based on ranges
    - Min → lower limit of uncertainty (confidence, necessity)
    - Max → upper limit of uncertainty (plausibility, possibility)

Intelligent systems - KBS

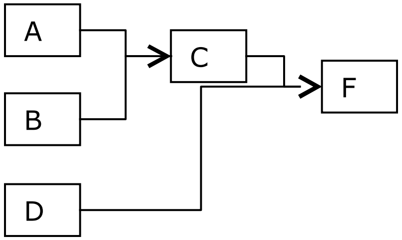
- Reasoning techniques for uncertainty
  - Teory of Bayes – probabilistic method
  - Theory of certainty
  - Theory of possibility (fuzzy logic)

Heuristic methods

Intelligent systems – KBS – certainty factors

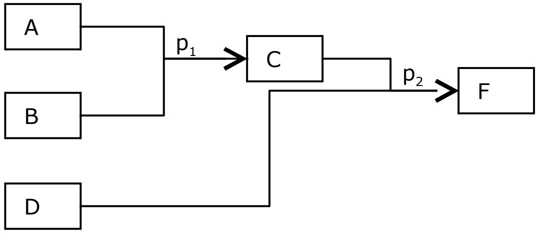
- Bayes systems
  - KBS with probabilistic facts and rules
- Systems based on certainty factors
  - KBS – facts and rules have associated a certainty factor (confidence factors)
  - A kind of Bayes systems with the probabilities replaced by certainty factors

- IF A and B then C
- IF C and D then F



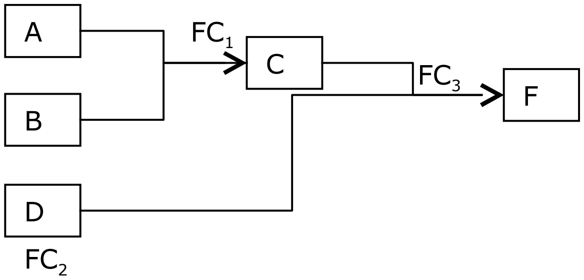
SBR classic

- If A and B then C [with prob p_1]
- If C and D then F [with prob p_2]



SBR de tip Bayes

- If A and B then C [CF_1]
- If C and $D_{[FC2]}$ then F [CF_3]



SBR cu FC

Intelligent systems – KBS – certainty factors

- Bayes KBSs *vs.* KBSs based on CFs

Bayes	CF
Theory of probabilities is old and has a mathematical foundation	Theory of CFs is new and without mathematical demonstrations
Require statistical information	Do not require statistical data
Certainty propagation exponentially increases	Information is quickly and efficiently passed
Require to *apriori* compute some probabilities	Do not require to *apriori* compute some probabilities
Hypothesis could be independent or not	Hypothesis are independent

Intelligent systems – KBS – Bayes systems

- Elements of probability theory
- Content and design
- Typology
- Tools
- Advantages and limits

Intelligent systems – KBS – Bayes systems

Amintim componenţa unui SBC

- Baza de cunoştinţe (BC) → Modalităţi de reprezentare a cunoştinţelor
  - Logica formală (limbaje formale)
    - Definiţie
      - Ştiinţa principiilor formale de raţionament
    - Componente
      - Sintaxă
      - Semantică
      - Metodă de inferenţă sintactică
    - Tipologie
      - În funcţie de numărul valorilor de adevăr:
        - logică duală
        - logică polivalentă
      - În funcţie de tipul elementelor de bază:
        - clasică → primitivele = propoziţii (predicate)
        - probabilistică → primitivele = variabile aleatoare
      - În funcţie de obiectul de lucru:
        - logica propoziţională → se lucrează doar cu propoziţii declarative, iar obiectele descrise sunt fixe sau unice (Ionică este student)
        - logica predicatelor de ordin I → se lucrează cu propziţii declarative, cu predicate şi cuantificări , iar obiectele descrise pot fi unice sau variabile asociate unui obiect unic (Toţi studenţii sunt prezenţi)
  - Reguli
  - Reţele semantice
- Modulul de control (MC – pentru inferenţă)

Intelligent systems – KBS – Bayes systems

Elemente de teoria probabilităţilor

- Teorii ale probabilităţilor
- Concepte de bază
  - Teoria clasică şi teoria modernă
  - Eveniment
  - Probabilitate simplă
  - Probabilitate condiţionată
  - Axiome

Elemente de teoria probabilităţilor

- Teorii ale probabilităţilor
  - Teoria clasică (*a priori*)
    - Propusă de Pascal şi Fermat în 1654
    - Lucrează cu sisteme ideale:
      - toate posibilele evenimente sunt cunoscute
      - toate evenimentele se pot produce cu aceeaşi probabilitate (sunt uniform distribuite)
    - evenimente discrete
    - metode combinatoriale
    - spaţiul rezultatelor posibile este continuu
  - Teoria modernă
    - evenimente continue
    - metode combinatoriale
    - spaţiul rezultatelor posibile este cuantificabil

Elemente de teoria probabilităţilor

- Concepte de bază
  - Considerăm un experiment care poate produce mai multe ieşiri (rezultate)
    - Ex. *Ev1: Aruncarea unui zar poate produce apariţia uneia din cele 6 feţe ale zarului (deci 6 rezultate)*
  - Eveniment
    - Definiţie
      - producerea unui anumit rezultat
      - Ex. *Ev2: Apariţia feţei cu nr 3*
      - Ex. *Ev3: Apariţia unei feţe cu un nr par (2,4,6)*
    - Tipologie
      - Evenimente independente şi mutual exclusive
        - Nu se pot produce simultan
        - Ex. *Ev4: Apariţia feţei 1 la aruncarea unui zar* şi *Ev5: Apariţia feţei 3 la aruncarea unui zar*
      - Dependente
        - Producerea unor evenimente afectează producerea altor evenimente
        - Ex. *Ev6: Apariţia feţei 6 la prima aruncare a unui zar* şi Ev7: *Apariţia unor feţe a căror numere însumate să dea 8 la 2 aruncări succesive ale unui zar*
  - Mulţimea tuturor rezultatelor = *sample space* al experimentului
    - Ex. pentru *Ev1*: (1,2,3,4,5,6)
  - Mulţimea tuturor rezultatelor tuturor evenimentelor posibile = *power set* (mulţimea părţilor)

Elemente de teoria probabilităţilor

- Concepte de bază
  - Probabilitate simplă *p(A)*
    - probabilitatea producerii unui eveniment A independent de alte evenimente (B)
    - şansa ca acel eveniment să se producă
    - proporţia cazurilor de producere a evenimnetului în mulţimea tuturor cazurilor posibile
    - nr cazurileor favorabile / nr cazurilor posibile
    - un număr real în [0,1]
      - 0 – imposibilitate absolută
      - 1 – posibilitate absolută
    - Ex. P(*Ev1*) =1/6, P(*Ev3*)=3/6
  - Probabilitate condiţionată *p(A|B)*
    - probabilitatea producerii unui eveniment A dependentă de producerea altor evenimente (B)
    - proporţia cazurilor de producere a evenimnetului A şi a evenimentului B în mulţimea tuturor cazurilor producerii evenimentului B
    - probabilitatea comună /proabilitatea lui B

$$p(A \mid B) = \frac{p(A \cap B)}{p(B)}$$

Intelligent systems – KBS – Bayes systems

Elemente de teoria probabilităţilor

- Concepte de bază
  - Axiome
    - $0 \le p(E) \le 1$ pentru orice eveniment E
    - $p(Adevărat) = 1$, $p(Fals) = 0$
    - $\sum_i p(E_i) = 1$
    - Dacă A şi B sunt independente
      - $p(A \cup B) = p(A) + p(B)$
      - $p(A \cap B) = p(A) * p(B)$
    - Dacă A şi B nu sunt independente
      - Dacă A depinde de B
        - $p(A \cup B)=p(A) + p(B)-p(A \cap B)$
        - $p(A \cap B) = p(A|B) * p(B)$
        - $p(B \cap A) = p(A \cap B)$
        - $p(A|B) = \frac{p(B|A)p(A)}{p(B)}$ (b)
      - Dacă A depinde de B_1, B_2, ..., B_n (evenimente mutual exclusive)
        - $p(A) = \sum_{i=1}^{n} p(A|B_i)p(B_i)$ (a)

Elemente de teoria probabilităţilor

- Concepte de bază
  - Exemplu
    - Dacă A depinde de 2 evenimente mutual exclusive (B şi ┐B), FC ec. $p(A) = \sum_{i=1}^{n} p(A \mid B_i) p(B_i)$ avem:
      - $p(B) = p(B|A)p(A) + p(B|\neg A)p(\neg A)$
    - Înlocuind pe $p(B)$ în ec. $p(A \mid B) = \dfrac{p(B \mid A) p(A)}{p(B)}$ se obţine ec.:

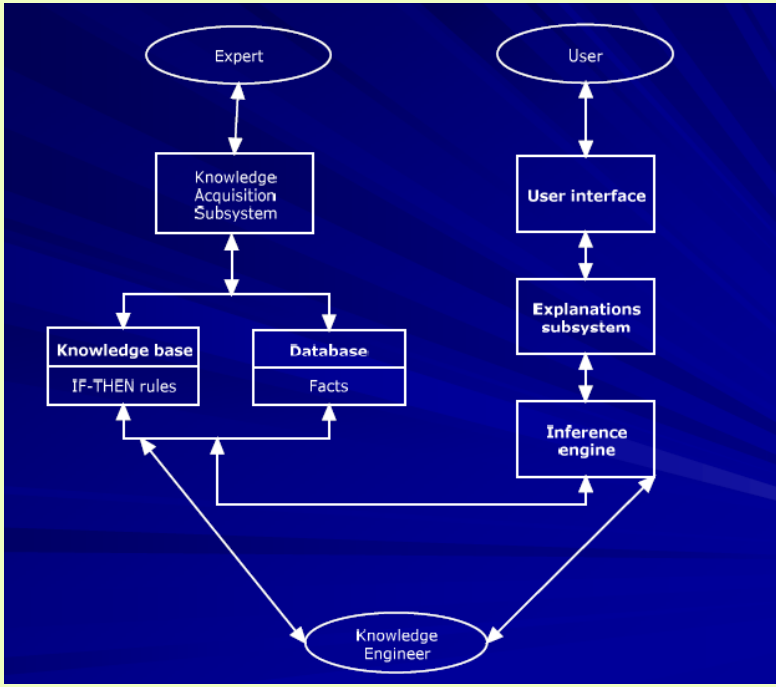
$$p(A \mid B) = \frac{p(B \mid A) p(A)}{p(B \mid A) p(A) + p(B \mid \neg A) p(\neg A)} \qquad \text{(c)}$$

    - Ecuaţia (c) se foloseşte pentru controlul incertitudinii în sistemele expert

Intelligent systems – KBS – Bayes systems

Reamintim ca un SBR are următoarea

Arhitectură



- Baza de cunoştinţe (BC)
  - Informaţiile specifice despre un domeniu
- Modulul de control (MC)
  - Regulile prin care se pot obţine informaţii noi
- Interfața cu utilizatorul
  - permite dialogul cu utilizatorii în timpul sesiunilor de consultare, precum și accesul acestora la faptele și cunoștințele din BC pentru adăugare sau actualizare
- Modulul de îmbogățire a cunoașterii
  - ajută utilizatorul expert să introducă în bază noi cunoștințe într-o formă acceptată de sistem sau să actualizeze baza de cunoștințe.
- Modulul explicativ
  - are rolul de a explica utilizatorilor atât cunoștințele de care dispune sistemul, cât și raționamentele sale pentru obținerea soluțiilor în cadrul sesiunilor de consultare. Explicațiile într-un astfel de sistem, atunci când sunt proiectate corespunzător, îmbunătățesc la rândul lor modul în care utilizatorul percepe și acceptă sistemul

Intelligent systems – KBS – Bayes systems

Reamintim: SBR – arhitectură

- baza de cunoştinţe
  - Conţinut
    - Informaţiile specifice despre un domeniu sub forma unor
      - fapte – afirmaţii corecte
      - reguli - euristici speciale care generează informaţii (cunoştinţe)
  - Rol
    - stocarea tuturor elementelor cunoașterii (fapte, reguli, metode de rezolvare, euristici) specifice domeniului de aplicație, preluate de la experții umani sau din alte surse

- modulul de control
  - Conţinut
    - regulile prin care se pot obţine informaţii noi
    - algoritmi independenţi de domeniu
    - creierul SBR – un algoritm de deducere bazat pe BC şi specific metodei de raţionare
      - un program în care s-a implementat cunoașterea de control, procedurală sau operatorie, cu ajutorul căruia se exploatează baza de cunoștințe pentru efectuarea de raționamente în vederea obținerii de soluții, recomandări sau concluzii.
    - depinde de complexitate şi tipul cunoştinţelor cu care are de-a face
  - Rol
    - cu ajutorul lui se exploatează baza de cunoștințe pentru efectuarea de raționamente în vederea obținerii de soluții, recomandări sau concluzii

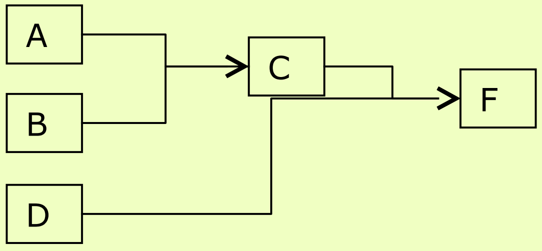
Intelligent systems – KBS – Bayes systems

Conţinut şi arhitectură

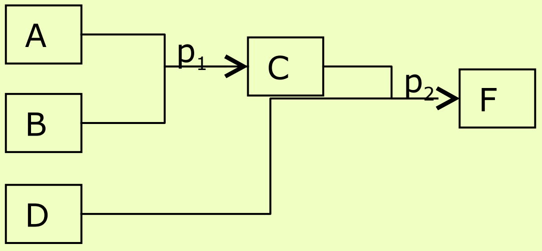
- Ideea de bază
  - SBR (Sisteme expert) în care faptele şi regulile sunt probabilistice

- Dacă A şi B atunci C
- Dacă C şi D atunci F

- Dacă A şi B atunci C [cu probabilitatea p_1]
- Dacă C şi D atunci F [cu probabilitatea p_2]



SBR classic



SBR de tip Bayes

Intelligent systems – KBS – Bayes systems

Conţinut şi arhitectură

- Regulile din BC sunt (în general) de forma:
  - Dacă evenimentul (faptul) I este adevărat, atunci evenimentul (faptul) D este adevărat [cu probabilitatea p]
  - Dacă evenimentul I s-a produs, atunci evenimentul D se va produce cu probabilitatea p
    - I – ipoteza (aserţiune, concluzie)
    - D – dovada (premisa) care susţine ipoteza

$$p(I \mid D) = \frac{p(D \mid I)\,p(I)}{p(D \mid I)\,p(I) + p(D \mid \neg I)\,p(\neg I)} \qquad \text{(d)}$$

  - unde:
    - p(I) – probabilitatea apriori ca ipoteza I să fie adevărată
    - p(D|I) – probabilitatea ca ipoteza I fiind adevărată să implice dovada D
    - p(┐I) – probabilitatea apriori ca ipoteza I să fie falsă
    - p(D|┐I) - probabilitatea găsirii dovezii D chiar dacă ipoteza I este falsă
- Cum şi cine calculează aceste probabilităţi? → modulul de control

Intelligent systems – KBS – Bayes systems

Conţinut şi arhitectură

- Cum calculează MC aceste probabilităţi într-un SBR?

$$p(I \mid D) = \frac{p(D \mid I)\,p(I)}{p(D \mid I)\,p(I) + p(D \mid \neg I)\,p(\neg I)} \qquad \text{(d)}$$

  - utilizatorul furnizează informaţii privind dovezile observate
  - experţii determină probabilităţile necesare rezolvării problemei
    - Probabilităţi apriori pentru posibile ipoteze (adevărate sau false) – p(I) şi p(┐I)
    - Probabilităţile condiţionate pentru observarea dovezii D dacă ipoteza I este adevărată p(D|I), respectiv falsă p(D|┐I)
  - SBR calculează probabilitatea posteriori p(I|D) pentru ipoteza I în condiţiile dovezilor D furnizate de utilizator

- Actualizare de tip Bayes
  - O tehnică de actualizare a probabilităţii *p* asociate unei reguli care susţine o ipoteză pe baza dovezilor (pro sau contra)
  - Inferenţă (raţionament) de tip Bayes

Intelligent systems – KBS – Bayes systems

Conţinut şi arhitectură

- Actualizare de tip Bayes
  - O tehnică de actualizare a probabilităţii *p* asociate unei reguli care susţine o ipoteză pe baza dovezilor (pro sau contra)
  - Actualizarea poate ţine cont de:
    - una sau mai multe (m) ipoteze (exclusive şi exhaustive)
    - una sau mai multe (n) dovezi (exclusive şi exhaustive)
  - Cazuri:
    - Mai multe ipoteze şi o singură dovadă

$$p(I_i \mid D) = \frac{p(D \mid I_i)p(I_i)}{\sum_{k=1}^{m} p(D \mid I_k)p(I_k)}$$

    - Mai multe ipoteze şi mai multe dovezi

$$p(I_i \mid D_1 D_2 ... D_n) = \frac{p(D_1 D_2 ... D_n \mid I_i)p(I_i)}{\sum_{k=1}^{m} p(D_1 D_2 ... D_n \mid I_k)p(I_k)} = \frac{p(D_1 \mid I_i)p(D_2 \mid I_i)...p(D_n \mid I_i)p(I_i)}{\sum_{k=1}^{m} p(D_1 D_2 ... D_n \mid I_k)p(I_k)}$$

Conţinut şi arhitectură

- Exemplu numeric
  - Pp. un SBR în care:
    - utilizatorul
      - furnizează 3 dovezi condiţionate independente D_1, D_2 şi D_3
    - expertul
      - crează 3 ipoteze mutual exclusive şi exhaustive I_1, I_2 şi I_3 şi stabileşte probabilităţile asociate lor – $p(I_1)$, $p(I_2)$ şi $p(I_3)$
      - determină probabilităţile condiţionate pentru observarea fiecărei dovezi pentru toate ipotezele posibile

probabilitatea	Ipotezele		
	i = 1	i = 2	i = 3
$p(I_i)$	0.40	0.35	0.25
$p(D_1\|I_i)$	0.30	0.80	0.50
$p(D_2\|I_i)$	0.90	0.00	0.70
$p(D_3\|I_i)$	0.60	0.70	0.90

Intelligent systems – KBS – Bayes systems

Conţinut şi arhitectură

probabilitatea	Ipotezele		
	i = 1	i = 2	i = 3
$p(I_i)$	0.40	0.35	0.25
$p(D_1\|I_i)$	0.30	0.80	0.50
$p(D_2\|I_i)$	0.90	0.00	0.70
$p(D_3\|I_i)$	0.60	0.70	0.90

- Exemplu numeric
  - Presupunem că prima dovadă observată este D_3
  - SE calculează probabilităţile posteriori $p(I_i|D_3)$ pentru toate ipotezele:

$$p(I_1 \mid D_3) = \frac{0.60 \cdot 0.40}{0.60 \cdot 0.40 + 0.70 \cdot 0.35 + 0.90 \cdot 0.25} = 0.34$$

$$p(I_2 \mid D_3) = \frac{0.70 \cdot 0.35}{0.60 \cdot 0.40 + 0.70 \cdot 0.35 + 0.90 \cdot 0.25} = 0.34$$

$$p(I_3 \mid D_3) = \frac{0.90 \cdot 0.25}{0.60 \cdot 0.40 + 0.70 \cdot 0.35 + 0.90 \cdot 0.25} = 0.32$$

  - După observarea dovezii D_3
    - încrederea în ipoteza I_2 este aceeaşi cu încrederea în ipoteza I_1
    - încrederea în ipoteza I_3 creşte

Intelligent systems – KBS – Bayes systems

Conţinut şi arhitectură

probabilitatea	Ipotezele		
	i = 1	i = 2	i = 3
$p(I_i)$	0.40	0.35	0.25
$p(D_1\|I_i)$	0.30	0.80	0.50
$p(D_2\|I_i)$	0.90	0.00	0.70
$p(D_3\|I_i)$	0.60	0.70	0.90

- Exemplu numeric
  - Presupunem că a doua dovadă observată este D_1
  - SE calculează probabilităţile posteriori $p(I_i|D_1D_3)$ pentru toate ipotezele:

$$p(I_1 | D_1D_3) = \frac{0.30 \cdot 0.60 \cdot 0.40}{0.30 \cdot 0.60 \cdot 0.40 + 0.80 \cdot 0.70 \cdot 0.35 + 0.50 \cdot 0.90 \cdot 0.25} = 0.19$$

$$p(I_2 | D_1D_3) = \frac{0.80 \cdot 0.70 \cdot 0.35}{0.30 \cdot 0.60 \cdot 0.40 + 0.80 \cdot 0.70 \cdot 0.35 + 0.50 \cdot 0.90 \cdot 0.25} = 0.52$$

$$p(I_3 | D_1D_3) = \frac{0.50 \cdot 0.90 \cdot 0.25}{0.30 \cdot 0.60 \cdot 0.40 + 0.80 \cdot 0.70 \cdot 0.35 + 0.50 \cdot 0.90 \cdot 0.25} = 0.29$$

  - După observarea dovezii D_1
    - încrederea în ipoteza I_1 scade
    - încrederea în ipoteza I_2 creşte (fiind cea mai probabilă de a fi adevărată)
    - încrederea în ipoteza I_3 creşte

Conţinut şi arhitectură

- Exemplu numeric
  - Presupunem că ultima dovadă observată este D_2

probabilitatea	Ipotezele		
	i = 1	i = 2	i = 3
$p(I_i)$	0.40	0.35	0.25
$p(D_1\|I_i)$	0.30	0.80	0.50
$p(D_2\|I_i)$	0.90	0.00	0.70
$p(D_3\|I_i)$	0.60	0.70	0.90

  - Se calculează probabilităţile posteriori $p(I_i|D_2D_1D_3)$ pentru toate ipotezele:

$$p(I_1 \mid D_2 D_1 D_3) = \frac{0.90 \cdot 0.30 \cdot 0.60 \cdot 0.40}{0.90 \cdot 0.30 \cdot 0.60 \cdot 0.40 + 0.00 \cdot 0.80 \cdot 0.70 \cdot 0.35 + 0.70 \cdot 0.50 \cdot 0.90 \cdot 0.25} = 0.45$$

$$p(I_2 \mid D_2 D_1 D_3) = \frac{0.00 \cdot 0.80 \cdot 0.70 \cdot 0.35}{0.90 \cdot 0.30 \cdot 0.60 \cdot 0.40 + 0.00 \cdot 0.80 \cdot 0.70 \cdot 0.35 + 0.70 \cdot 0.50 \cdot 0.90 \cdot 0.25} = 0.00$$

$$p(I_3 \mid D_2 D_1 D_3) = \frac{0.70 \cdot 0.50 \cdot 0.90 \cdot 0.25}{0.90 \cdot 0.30 \cdot 0.60 \cdot 0.40 + 0.00 \cdot 0.80 \cdot 0.70 \cdot 0.35 + 0.70 \cdot 0.50 \cdot 0.90 \cdot 0.25} = 0.55$$

  - După observarea dovezii D_2
    - încrederea în ipoteza I_1 creşte
    - încrederea în ipoteza I_2 e nulă (ipoteza e falsă)
    - Încrederea în ipoteza I_3 creşte

Intelligent systems – KBS – Bayes systems

Conţinut şi arhitectură

- Exemplu practic
  - Presupunem cazul unei maşini care nu porneşte când este accelerată, dar scoate fum
    - Dacă scoate fum, atunci acceleraţia este defectă [cu probabilitatea 0.7]
    - $P(I_1|D_1) = 0.7$
  - Pe baza unor observări statistice, experţii au constatat:
    - următoarea regulă:
      - Dacă acceleraţia este defectă, atunci maşina scoate fum [cu probabilitatea 0.85]
    - probabilitatea ca maşina să pornească din cauză că acceleraţia este defectă = 0.05 (probabilitate *apriori*)
    - deci avem
      - 2 ipoteze:
        - I_1: acceleraţia este defectă
        - I_2: acceleraţia nu este defectă
      - o dovadă
        - D_1: maşina scoate fum
      - probabilitatea că acceleraţia este defectă dacă maşina scoate fum
        - $P(I_1|D_1)=p(D_1|I_1)*p(I_1)/(p(D_1|I_1)*p(I_1)+p(D_1|I_2)*p(I_2))$
        - $P(I_1|D_1)=0.23 < 0.7$

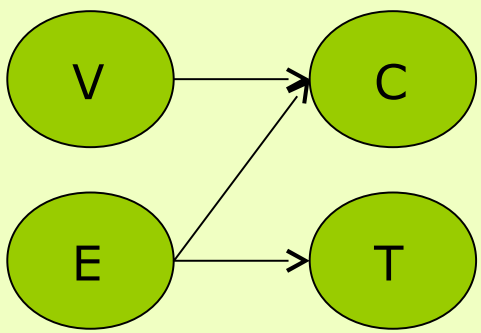
	I_1	I_2
$p(I_i)$	0.05	1-0.05=0.95
$P(D_1\|I_i)$	0.85	1-0.85=0.15

Intelligent systems – KBS – Bayes systems

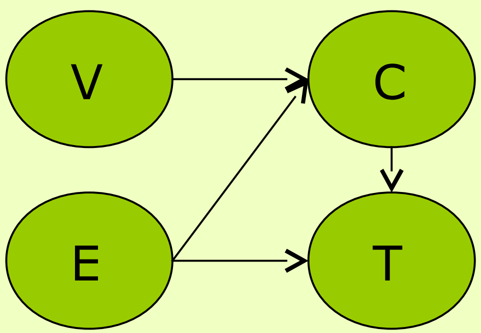
Tipologie

- Sisteme simple de tip Bayes
  - Consecinţele unei ipoteze nu sunt corelate
- Reţele de tip Bayes
  - Consecinţele unei ipoteze pot fi corelate

- De exemplu, reţinem informaţii despre vârsta (V), educaţia (E), câştigurile (C) şi preferinţa pentru teatru (T) ale unor persoane



Sistem Bayes simplu (naiv)



Reţea Bayes simplu

Intelligent systems – KBS – Bayes systems

- Tool-uri
  - MSBNx – view
  - JavaBayes – view
  - BNJ – view

Intelligent systems – KBS – Bayes systems

- Avantaje ale inferenţei de tip Bayes
  - Tehnică bazată pe teoreme statistice
  - Probabilitatea dovezilor (simptomelor) în ipotezele (cauzele) date sunt posibil de furnizat
  - Probabilitatea unei ipoteze se poate modifica datorită uneia sau mai multor dovezi

- Dezavantaje ale inferenţei de tip Bayes
  - Trebuie cunoscute (sau ghicite) probabilităţile apriori ale unor ipoteze

Intelligent systems – KBS

- Tehnici de raţionare în medii nesigure
  - Teoria Bayesiana – metodă probabilistică
  - **Teoria certitudinii**
  - Teoria posibilităţii (logica fuzzy)

Metode euristice

Intelligent systems – KBS – certainty factors

- Conţinut şi arhitectură
- Tipologie
- Tool-uri
- Avantaje şi dezavantaje

Intelligent systems – KBS – certainty factors

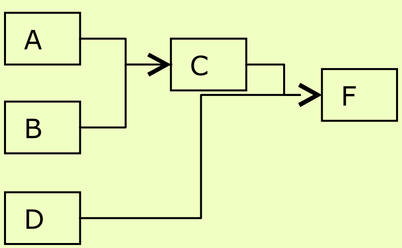
Conţinut şi arhitectură

- Ideea de bază
  - SBR (sisteme expert) în care faptele şi regulile au asociate câte un factor de certitudine (FC)/coeficienţ de încredere
  - Un fel de sisteme de tip Bayes în care probabilităţile sunt înlocuite cu factori de certitudine

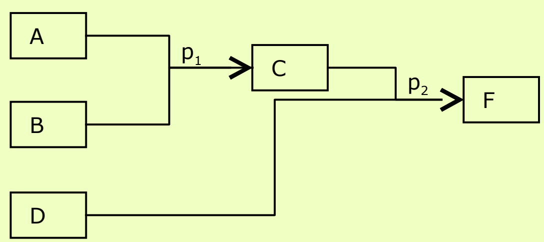
- Dacă A şi B atunci C
- Dacă C şi D atunci F

- Dacă A şi B atunci C [cu prob p_1]
- Dacă C şi D atunci F [cu prob p_2]

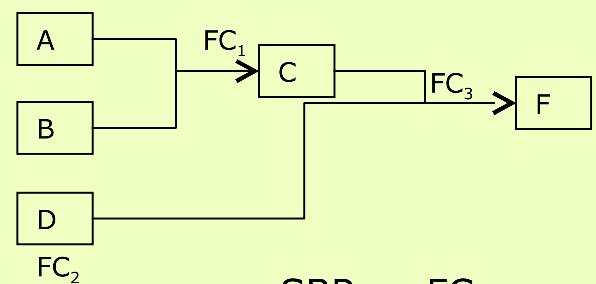
- Dacă A şi B atunci C [FC_1]
- Dacă C şi $D_{[FC2]}$ atunci F [FC_3]



SBR classic



SBR de tip Bayes



SBR cu FC

Intelligent systems – KBS – certainty factors

Conţinut şi arhitectură

- FC măsoară încrederea acordată unor fapte sau reguli
- Utilizarea FC → alternativă la actualizarea de tip Bayes

- FC pot fi aplicaţi
  - faptelor
  - regulilor (concluziei/concluzilor unei reguli)
  - fapte + reguli

- Într-un SBR (sistem expert) cu factori de certitudine
  - regulile sunt de forma:
    - dacă dovada atunci ipoteza [FC]
    - dacă dovada$_{[FC]}$ atunci ipoteza
    - dacă dovada$_{[FC]}$ atunci ipoteza [FC]
  - ipotezele susţinute de probe sunt independente

Intelligent systems – KBS – certainty factors

Conţinut şi arhitectură

- FC – mod de calcul
  - Măsura încrederii (measure of belief – MB)
    - măsura creşterii încrederii în ipoteza I pe baza dovezii D

$$MB(I,D)=\begin{cases}1, & \text{dacă } p(I)=1\\ \dfrac{p(I|D)-p(I)}{1-p(I)} & \text{dacă } p(I)<1\end{cases}$$

  - Măsura neîncrederii (measure of disbelief – MD)
    - măsura creşterii neîncrederii în ipoteza I pe baza dovezii D

$$MD(I,D)=\begin{cases}1, & \text{dacă } p(I)=0\\ \dfrac{p(I)-p(I|D)}{p(I)} & \text{dacă } p(I)>0\end{cases}$$

  - Pentru evitarea valorilor negative ale MB şi MD:

$$MB(I,D)=\begin{cases}1, & \text{dacă } p(I)=1\\ \dfrac{\max\{p(I|D),p(I)\}-p(I)}{1-p(I)} & \text{dacă } p(I)<1\end{cases}$$

$$MD(I,D)=\begin{cases}1, & \text{dacă } p(I)=0\\ \dfrac{\min\{p(I|D),p(I)\}-p(I)}{0-p(I)} & \text{dacă } p(I)>0\end{cases}$$

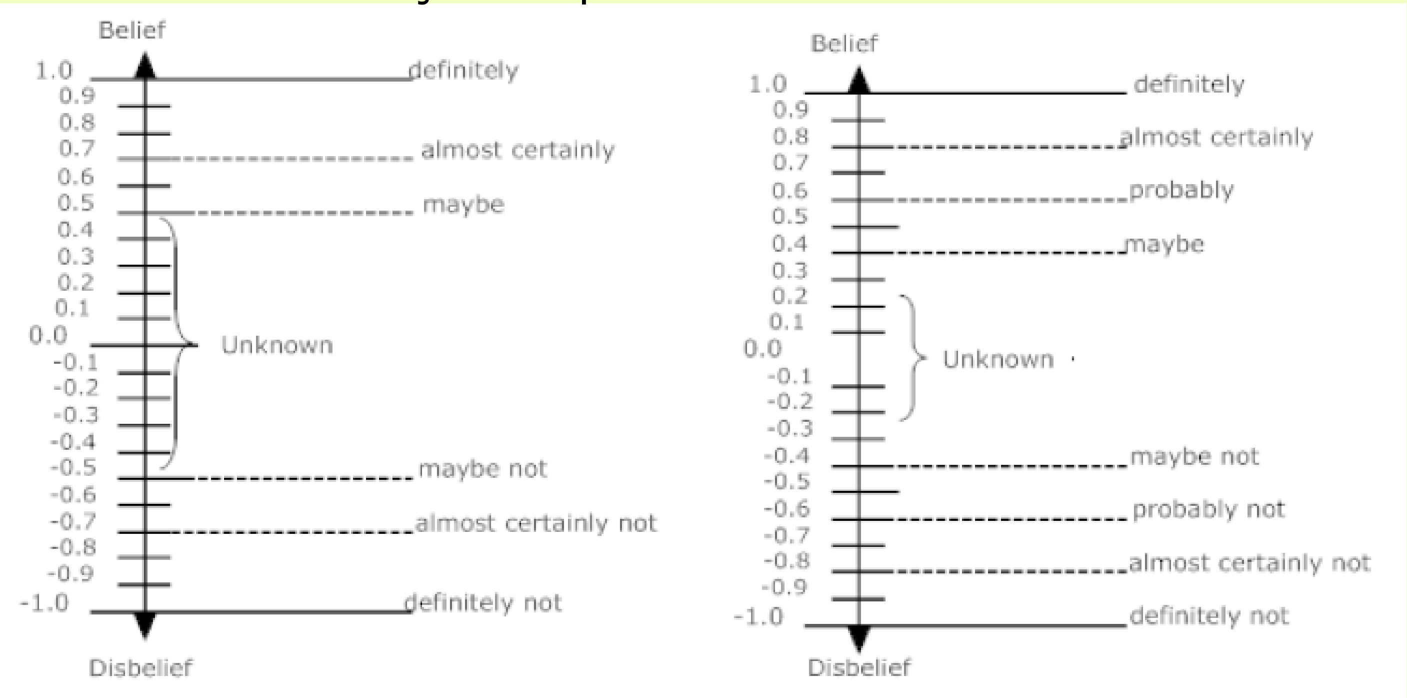
  - FC – încrederea în ipoteza *I* dată fiind dovada *D*
    - Număr din [-1,1]
    - FC=-1 dacă se ştie că ipoteza *I* este falsă
    - FC=0 dacă nu se ştie nimic despre ipoteza *I*
    - FC=1 dacă se ştie că ipoteza *I* este adevărată

$$FC(I,D)=\frac{MB(I,D)-MD(I,D)}{1-\min\{MB(I,D),MD(I,D)\}}$$

Intelligent systems – KBS – certainty factors

Conţinut şi arhitectură

- FC – mod de calcul
  - încrederea în ipoteza I dată fiind dovada D
    - FC=-1 dacă se ştie că ipoteza este falsă
    - FC=0 dacă nu se ştie nimic despre ipoteză
    - FC=1 dacă se ştie că ipoteza este adevărată



Intelligent systems – KBS – certainty factors

Conţinut şi arhitectură

- FC – mod de calcul
  - încrederea în ipoteza *I* dată fiind dovada *D*
  - ipoteza *I* poate fi:
    - simplă (ex. *Dacă D atunci I*)
    - compusă (ex. *Dacă D atunci* I_1 *şi* I_2 *şi ...* I_n)
  - dovada *D* poate fi
    - dpdv al compoziţiei:
      - simplă (ex. Dacă D atunci I)
      - compusă (ex. Dacă D1 şi D2 şi ... Dn atunci I)
    - dpdv al incertitudinii (încrederii în dovadă):
      - sigură (ex. Dacă D atunci I)
      - nesigură (ex. Dacă D[FC] atunci I)

Intelligent systems – KBS – certainty factors

Conţinut şi arhitectură

- FC – mod de calcul pentru combinarea încrederii
  - o dovadă incertă care susţine sigur o ipoteză
  - mai multe dovezi incerte care susţin sigur o singură ipoteză
  - o dovadă incertă care susţine incert o ipoteză
  - mai multe dovezi incerte care susţin incert o ipoteză

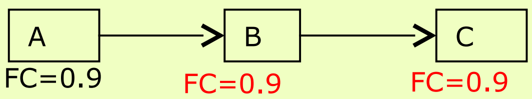
Intelligent systems – KBS – certainty factors

Conţinut şi arhitectură

- FC – mod de calcul pentru combinarea încrederii
  - O dovadă incertă care susţine sigur o ipoteză

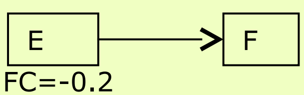
$$FC(I)=\begin{cases} FC(D), & \text{dacă } FC(D)>0 \\ 0, & \text{altfel} \end{cases}$$

  - Exemplul 1
  - R_1: *Dacă $A_{[FC=0.9]}$ atunci B*
  - R_2: *Dacă B atunci C*



  - FC(B)=FC(A)=0.9
  - FC(C)=FC(B)=0.9

- Exemplul 2
  - R_1: *Dacă $E_{[FC=-0.2]}$ atunci F*



  - FC(E este adevărat) = -0.2 ➔ dovadă negativă ➔ nu putem spune nimic despre faptul că E este adevărat ➔ nu se poate spune nimic despre F

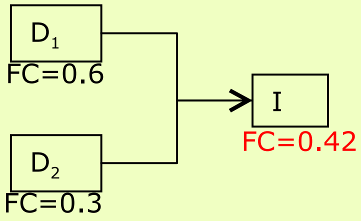
Intelligent systems – KBS – certainty factors

Conţinut şi arhitectură

- FC – mod de calcul pentru combinarea încrederii
  - Mai multe dovezi incerte care susţin sigur o singură ipoteză
    - Dovezi (probe) adunate incremental
    - Mai multe reguli care, pe baza unor dovezi diferite, furnizează aceeaşi concluzi
    - Aceeaşi ipoteză (valoare de atribut) I este obţinută pe două căi de deducţie distincte, cu două perechi diferite de valori pentru *FC, FC[I,D_1] si FC[I,D_2]*
    - Cele doua cai de deductie distincte, corespunzatoare dovezilor (probelor) D_1 şi D_2 pot fi:
      - ramuri diferite ale arborelui de cautare generat prin aplicarea regulilor
      - dovezi (probe) indicate explicit sistemului

$$FC(I, D_1 \wedge D_2) = \begin{cases} CF(D_1) + CF(D_2)(1 - CF(D_1)), & \text{dacă } CF(D_1),\ CF(D_2) > 0 \\ CF(D_1) + CF(D_2)(1 + CF(D_1)), & \text{dacă } CF(D_1),\ CF(D_2) < 0 \\ \dfrac{\mathrm{CF}(D_1) + \mathrm{CF}(D_2)}{1 - \min\{|CF(D_1|, |CF(D_2)|\}}, & \text{dacă } sign(CF(D_1)) \neq sign(CF(D_2)) \end{cases}$$

- Exemplu
  - R_1: Dacă $D_{1\ [FC=0.6]}$ atunci I
  - R_2: Dacă $D_{2[FC=-0.3]}$ atunci I
  - $FC(I,D_1 \wedge D_2)=(0.6+(-0.3))/(1-0.3)=0.42$



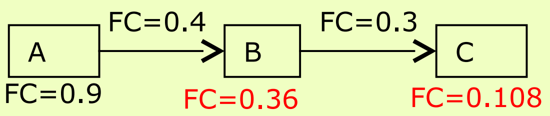
Intelligent systems – KBS – certainty factors

Conţinut şi arhitectură

- FC – mod de calcul pentru combinarea încrederii
  - O dovadă incertă care susţine incert o ipoteză

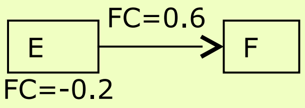
$$FC(I)=\begin{cases} FC(D) * FC(regulă), & \text{dacă } FC(D)>0 \\ 0, & \text{altfel} \end{cases}$$

  - Exemplul 1
    - R_1: *Dacă $A_{[FC=0.9]}$ atunci B [FC=0.4]*
    - R_2: *Dacă B atunci C [FC=0.3]*



    - $FC(B)=FC(A)*FC(R_1)=0.9*0.4=0.36$
    - $FC(C)=FC(B)*FC(R_2)=0.36*0.3=0.108$

- Exemplul 2
  - R_1: *Dacă $E_{[FC=-0.2]}$ atunci F [FC=0.6]*



  - FC(E este adevărat) = -0.2 ➔ dovadă negativă ➔ nu putem spune nimic despre faptul că E este adevărat ➔ nu se poate spune nimic despre F

Intelligent systems – KBS – certainty factors

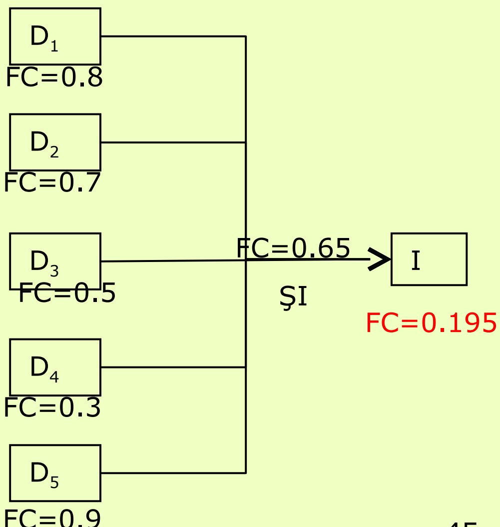
Conţinut şi arhitectură

- FC – mod de calcul pentru combinarea încrederii
  - Mai multe dovezi incerte care susţin incert o ipoteză
    - Dovezile sunt legate prin ŞI logic

$$CF(I)=\begin{cases}\min\{CF(D_1),CF(D_2),...,CF(D_n),\}*CF(regulă), & dacă\ \mathrm{CF(D_i)}>0, \mathrm{i}=1,2,...,\mathrm{n}\\ 0, & \text{altfel}\end{cases}$$

  - Una sau mai multe dintre dovezile incerte care susţin incert o ipoteză
    - Dovezile sunt legate prin SAU logic

$$CF(I)=\begin{cases}\max\{CF(D_1),CF(D_2),...,CF(D_n),\}*CF(regulă), & dacă\ \mathrm{CF(D_i)}>0, \mathrm{i}=1,2,...,\mathrm{n}\\ 0, & \text{altfel}\end{cases}$$



  - Exemplul 1
    - R_1: *Dacă* $D_{1[FC = 0.8]}$ *şi* $D_{2[FC = 0.7]}$ *şi* $D_{3[FC = 0.5]}$ *şi* $D_{4[FC = 0.3]}$ *şi* $D_{5[FC = 0.9]}$ *atunci I [FC = 0.65]*
    - *FC(I) = 0.3 * 0.65 = 0.195*

Intelligent systems – KBS – certainty factors

Conţinut şi arhitectură

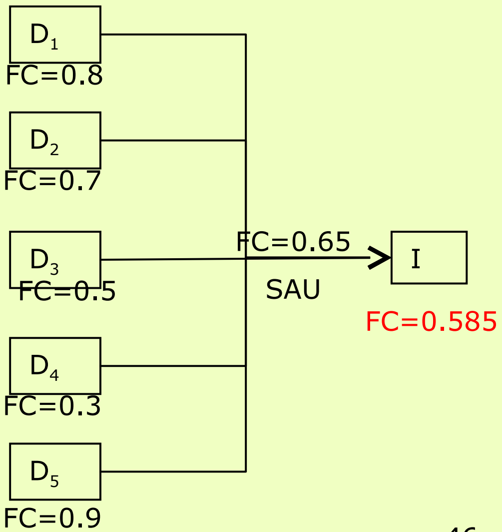
- FC – mod de calcul pentru combinarea încrederii
  - Mai multe dovezi incerte care susţin incert o ipoteză
    - Dovezile sunt legate prin ŞI logic

$$CF(I)=\begin{cases}\min\{CF(D_1),CF(D_2),...,CF(D_n),\}*CF(regulă), & dacă\ \mathrm{CF(D_i)>0, i=1,2,...,n}\\ 0, & \mathrm{altfel}\end{cases}$$

  - Una sau mai multe dintre dovezile incerte susţin incert o ipoteză
    - Dovezile sunt legate prin SAU logic

$$CF(I)=\begin{cases}\max\{CF(D_1),CF(D_2),...,CF(D_n),\}*CF(regulă), & dacă\ \mathrm{CF(D_i)>0, i=1,2,...,n}\\ 0, & \mathrm{altfel}\end{cases}$$

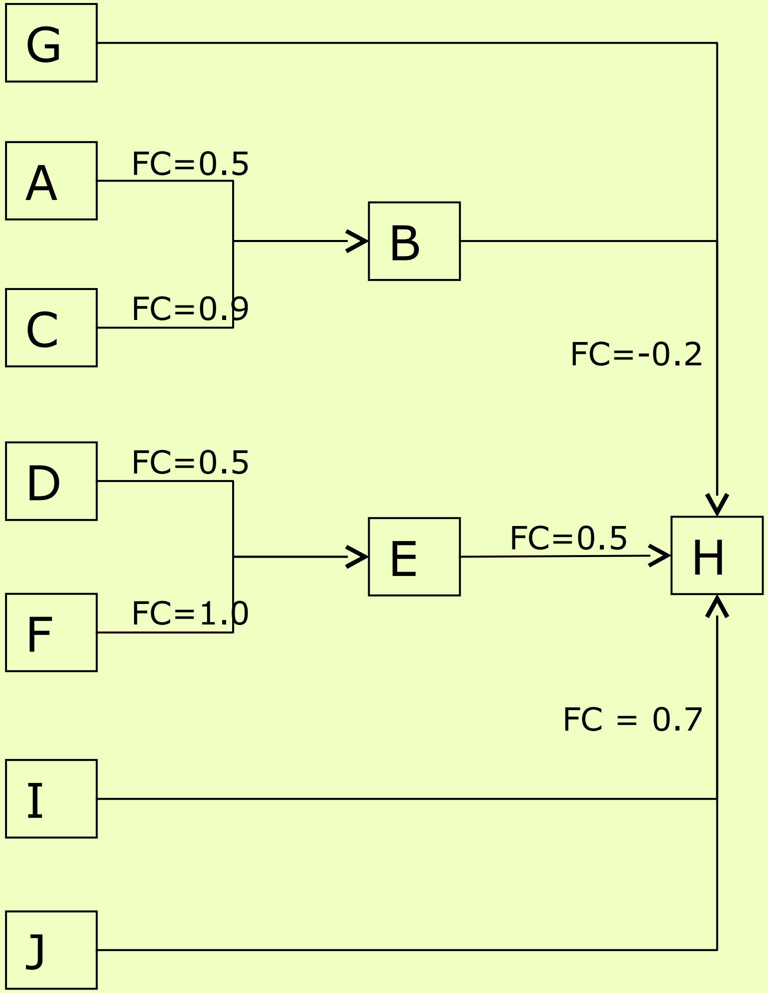
  - Exemplul 2
    - R_1: *Dacă* $D_{1[FC = 0.8]}$ *sau* $D_{2[FC = 0.7]}$ *sau* $D_{3[FC = 0.5]}$ *sau* $D_{4[FC = 0.3]}$ *sau* $D_{5[FC = 0.9]}$ *atunci I [FC = 0.65]*
    - *FC(I) = 0.9 * 0.65 = 0.585*



Intelligent systems – KBS – certainty factors

Exemplu

- Sistem expert pentru diagnosticarea unei răceli
  - Fapte în baza de date:
    - Febra pacientului 37.4
    - Pacientul tuşeşte de mai puţin de 24 ore
    - Pacientul nu are expectoraţii
    - Pacientul are o durere de cap cu FC = 0.4
    - Pacientul are nasul înfundat cu FC = 0.5
  - Reguli:
    - R_1: Dacă *A: febra < 37.5* atunci *B: simptomele de răceală sunt prezente* [FC=0.5]
    - R_2: Dacă *C: febra > 37.5* atunci *B: simptomele de răceală sunt prezente* [FC=0.9]
    - R_3: Dacă *D: tuşeşte > 24 ore* atunci *E: durerea de gât e prezentă* [FC=0.5]
    - R_4: Dacă *F: tuşeşte > 48 ore* atunci *E: durerea de gât e prezentă* [FC=1.0]
    - R_5: Dacă *B: are simptome de răceală* şi *G: nu expectorează* atunci *H: a răcit* [FC=-0.2]
    - R_6: Dacă *E: îl doare gâtul* atunci *H: a răcit* [FC=0.5]
    - R_7: Dacă *I: îl doare capul* şi *J: are nasul înfundat* atunci *H: a răcit* [FC=0.7]
  - Concluzia:
    - Pacientul este sau nu răcit?



Intelligent systems – KBS – certainty factors

Exemplu

- Sistem expert pentru diagnosticarea unei răceli
  - Fapte în baza de date:
    - Febra pacientului 37.4
    - Pacientul tuşeşte de mai puţin de 24 ore
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    - R_2: Dacă *C: febra > 37.5* atunci *B: simptomele de răceală sunt prezente* [FC=0.9]
    - R_3: Dacă *D: tuşeşte > 24 ore* atunci *E: durerea de gât e prezentă* [FC=0.5]
    - R_4: Dacă *F: tuşeşte > 48 ore* atunci *E: durerea de gât e prezentă* [FC=1.0]
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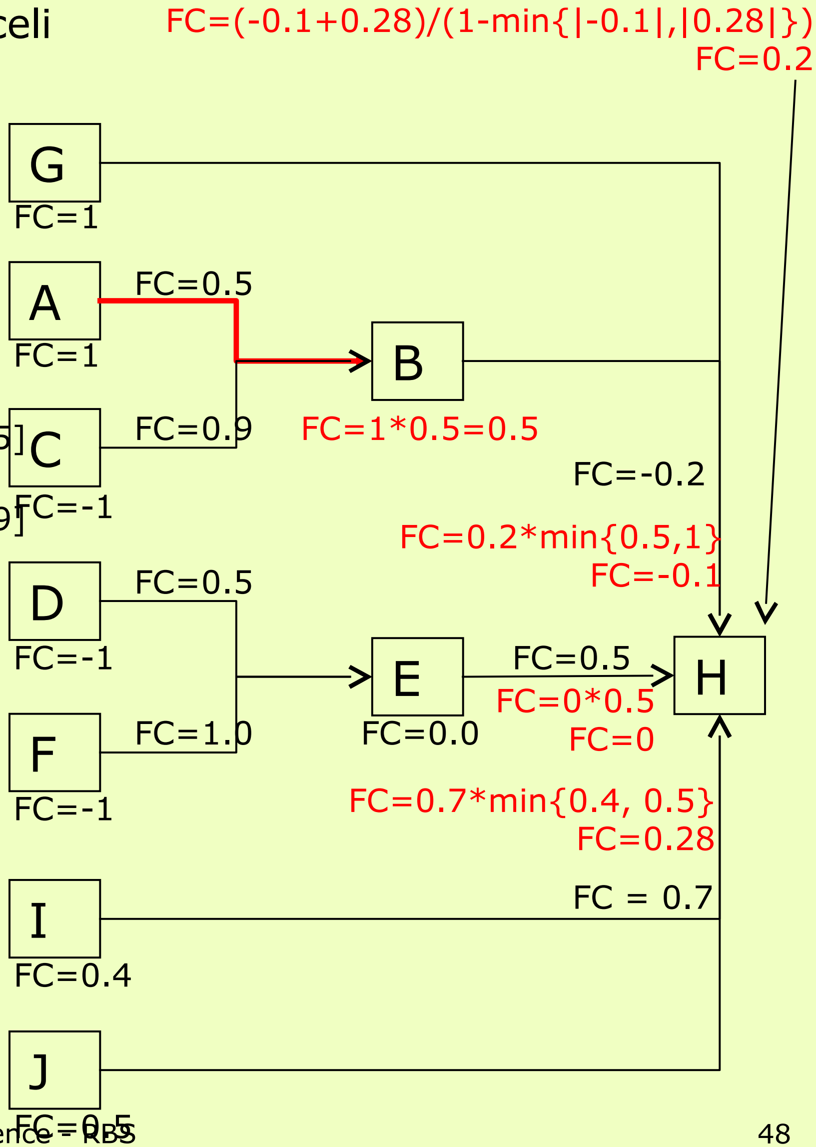
    - R_7: Dacă *I: îl doare capul* şi *J: are nasul înfundat* atunci *H: a răcit* [FC=0.7]
  - Concluzia:
    - Pacientul este sau nu răcit?



Intelligent systems – KBS – certainty factors

- Avantaje
  - Nu este necesar calculul apriori a probabilităţilor

- Limite
  - ipotezele sustinute de probe sunt independente.

  - exemplu:
    - Fie următoarele fapte:
      - A: Aspersorul a funcţionat noaptea trecută
      - U: Iarba este udă dimineaţă
      - P: Noaptea trecută a plouat.
    - şi următoarele două reguli care leagă între ele aceste fapte:
      - R_1: dacă aspersorul a funcţionat noaptea trecută atunci există o încredere puternică (0.9) că iarba este udă dimineaţa
      - R_2: dacă iarba este udă dimineaţa atunci există o încredere puternică (0.8) că noaptea trecută a plouat

    - Deci:
      - FC[U,A] = 0.9 - deci proba aspersor sustine iarba uda cu 0.9
      - FC[P,U] = 0.8 - deci iarba uda sustine ploaie cu 0.8
      - FC[P,A] = 0.8 * 0.9 = 0.72 - deci aspersorul sustine ploaia cu 0.72

Intelligent systems – KBS – certainty factors

- SBR de tip Bayes *vs.* SBR cu FC

Bayes	FC
Teorie probabilităţilor este veche şi fundamentată matematic	Teoria FC este nouă şi fără demostrţii matematice
Necesită existenţa unor informaţii statistice	Nu necesită existenţa unor date statistice
Propagarea încrederii creşte în timp exponenţial	Informaţia circulă repede şi eficient în SBR

Intelligent systems - KBS

- Reasoning techniques for uncertainty
  - Teory of Bayes – probabilistic method
  - Theory of certainty
  - Theory of possibility (fuzzy logic)

Heuristic methods

Intelligent systems – KBS – Fuzzy systems

- Theory of possibility
- Content and design
- Typology
- Tools
- Advantages and limits

Intelligent systems – KBS – Fuzzy systems

Teoria posibilităţii (logica fuzzy)

- Why fuzzy?
  - Problem: translate in C++ code the following sentences:
    - Georgel is tall.
    - It is cold outside.

- When fuzzy is important?
  - Natural queries
  - Knowledge representation for a KBS
  - Fuzzy control – then we dead by imprecise phenomena (noisy phenomena)

Intelligent systems – KBS – Fuzzy systems

Remember the components of a KBS

- Knowledge base → knowledge representation
  - Formal logic (formal languages)
    - Definition
      - Science of formal principles for rationing
    - Components
      - Syntax – atomic symbols used by language and the constructing rules of the language
      - Semantic – associates a meaning to each symbol and a truth value (true or false) to each rule
      - Syntactic inference – rules for identifying a subset of logic expressions → theorems (for generating new expressions)
    - Typology
      - True value
        - Dual logic
        - Polyvalent logic
      - Basic elements
        - Classic → primitives = sentences (predicates)
        - Probabilistic → primitives = random variables
      - Working manner
        - Propositional logic → declarative propositions and fix or unique objects (Ionica is student)
        - First-order logic → declarative propositions, predicates and quantified variables, unique objects or variables associated to a unique object
  - Rules
  - Semantic nets
- Inference engine

Intelligent systems – KBS – Fuzzy systems

Theory of possibility – a little bit of history

- Parminedes (400 B.C.)
- Aristotle
  - "Law of the Excluded Middle‟ – every sentence must be True or False
- Plato
  - A third region, between True and False
  - Forms the basis of fuzzy logic
- Lukasiewicz (1900)
  - Has proposed an alternative and sistematic approach related to bi-valent logic of Aristotle – trivalent logic: true, false or possible
- Lotfi A. Zadeh (1965)
  - Mathematical description of fuzzy set theory and fuzzy logic: truth functions takes values in [0,1] (instead of {True, False})
    - He as proposed new operations in fuzzy logic
    - He has considered the fuzzy logic as a generalisation of the classic logic
  - He has written the first paper about fuzzy sets

Intelligent systems – KBS – Fuzzy systems

Theory of possibility

- Fuzzy logic
  - Generalisation of Boolean logic
  - Deals by the concept of partial truth
    - Classical logic – all things are expressed by binary elements
      - 0 or 1, white or black, yes or no
    - Fuzzy logic – gradual expression of a truth
      - Values between 0 and 1

- Logic vs. algebra
  - Logical operators are expressed by using mathematical terms (George Boole)
    - Conjunction = minimum ➔ $a \wedge b = \min(a, b)$
    - Disjunction = maximum ➔ $a \vee b = \max(a, b)$
    - Negation = difference ➔ $\neg a = 1 - a$

Intelligent systems – KBS – Fuzzy systems

Remember: KBS - design

- Knowledge base
  - Content
    - Specific information
      - Facts – correct affirmations
      - Rules – special heuristics that generate knowledge
  - Aim
    - Store all the information (facts, rules, solving methods, heuristics) about a given domain (taken from some experts)
- Inference engine
  - Content
    - Rules for generating new information
    - Domain-independent algorithms
    - Brain of a KBS
  - Aim
    - Help to explore the KB by reasoning for obtaining solutions, recommendations or conclusions

Intelligent systems – KBS – Fuzzy systems

Content and design

- Main idea
  - Cf. to certainty theory:
    - *Popescu is tall*
  - Cf. to uncertainty theory
    - Cf. to probability theory
      - *There is 80% chance that Popescu is young*
    - Cf. fuzzy logic

  - Cf. teoriei informaţiilor certe
    - *Popescu este tânăr*
  - Cf. teoriei informaţiilor incerte
    - Cf. teoriei probabilităţilor:
      - *Există 80% şanse ca Popescu să fie tânăr*
    - Cf. logicii fuzzy:
      - *Popescu's degree of membership to the group of young people is 0.80*

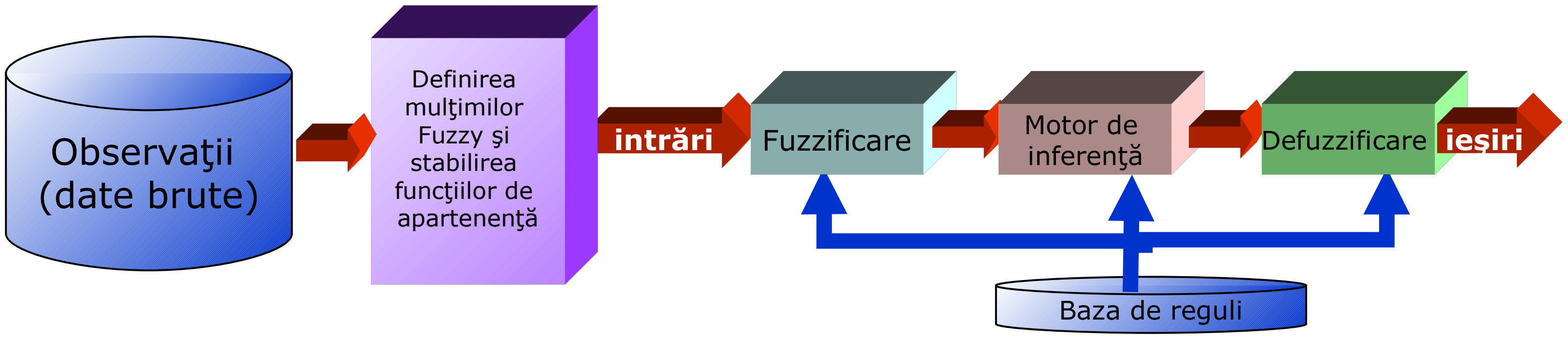
- Necessity
  - Real phenomena involve fuzzy sets
  - Example
    - *The room's temperature can be:*
      - *low,*
      - *Medium or*
      - *high*

    - These sets of possible temperatures can overlap
      - A temperature can belong to more classes (groups) depends on the person that evaluates that temperature

Intelligent systems – KBS – Fuzzy systems

Content and design

- Steps for constructing a fuzzy system
  - Define the inputs and the outputs – by an expert
    - Raw inputs and outputs
    - Fuzzification of inputs and outputs
      - Fix the fuzzy variables and fuzzy sets based on membership functions
  - Construct a base of rules – by an expert
    - Decision matrix
  - Evaluate the rules
    - Inference – transform the fuzzy inputs into fuzzy outputs by applying all the rules
  - Aggregate the results
  - Defuzzificate the result
  - Interpret the result



Intelligent systems – KBS – Fuzzy systems

Content and design → fuzzification of input data

- Elements from probability theory (fuzzy logic)
  - Fuzzy facts (fuzzy sets)
    - Definition
    - Representation
    - Operations – complements, containment, intersection, reunion, equality, algebraic product, algebraic sum
    - Properties – associativity, commutativity, distributivity, transitivity, idempotency, identity, involution
    - Hedges
  - Fuzzy variables
    - Definition
    - Properties

- Establish the fuzzy variables and the fuzzy sets based on membership functions

Intelligent systems – KBS – Fuzzy systems

Content and design → fuzzification of input data

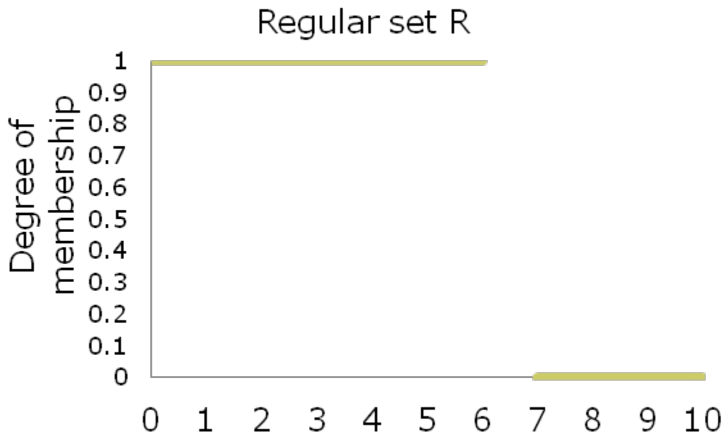
- Elements from probability theory (fuzzy logic) → Fuzzy facts (fuzzy sets) → **definition**
  - Set definition – 2 possibilities:
    - By enumeration of elements
      - Ex. *Set of students = {Ana, Maria, Ioana}*
    - By specifying a property of elements
      - Ex. Set of even numbers =*{x| x = 2n, where n = 2k}*
  - Characteristic function μ for a set
    - Let X a universal set and x an element of this set (x∈X)
    - Classical logic
      - Let R a sub-set of X: R⊂X, R – regular set
      - Every element x belong to set R
      - μ_R : X → {0, 1}, where $$\mu_R(x) = \begin{cases} 1, & x \in R \\ 0, & x \notin R \end{cases}$$
    - Fuzzy logic
      - Let F a sub-set of X (a univers) : F⊂X, F – fuzzy set
      - Every elemt x belongs to F by a given degree of membership $\mu_F(x)$
      - μ_F : X → [0, 1], $\mu_F(x)=g$, where g∈ [0,1] – membership degree of x to F
      - g = 0 → not-belong
      - g = 1 → belong
      - A fuzzy set = a pair (F, μ_F), where $$\mu_F(x) = \begin{cases} 1, & \text{if } x \text{ is totaly in } F \\ 0, & \text{if } x \text{ is not in } F \\ \in (0,1) & \text{if } x \text{ is part of } F (x \text{ is a fuzzy number}) \end{cases}$$

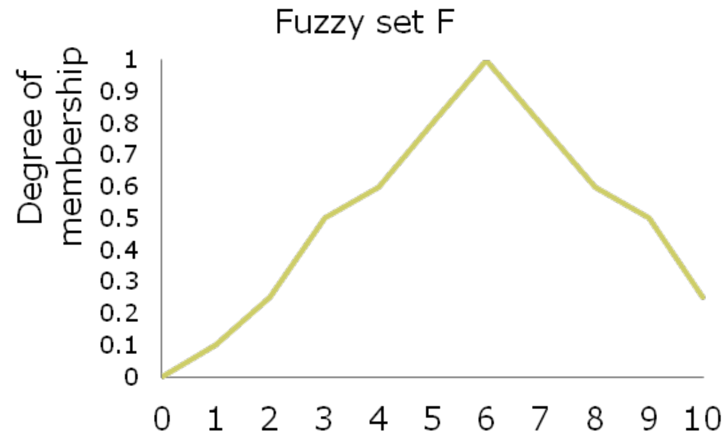
Intelligent systems – KBS – Fuzzy systems

Content and design → fuzzification of input data

- Elements from probability theory (fuzzy logic) → Fuzzy facts (fuzzy sets) → **definition**
  - Example 1
    - X –set of natural numbers < 11
    - R – set of natural numbers < 7
    - F – set of natural numbers that are neighbours of 6

x	$\mu_R(x)$	$\mu_F(x)$
0	1	0
1	1	0.1
2	1	0.25
3	1	0.5
4	1	0.6
5	1	0.8
6	1	1
7	0	0.8
8	0	0.6
9	0	0.5
10	0	0.25

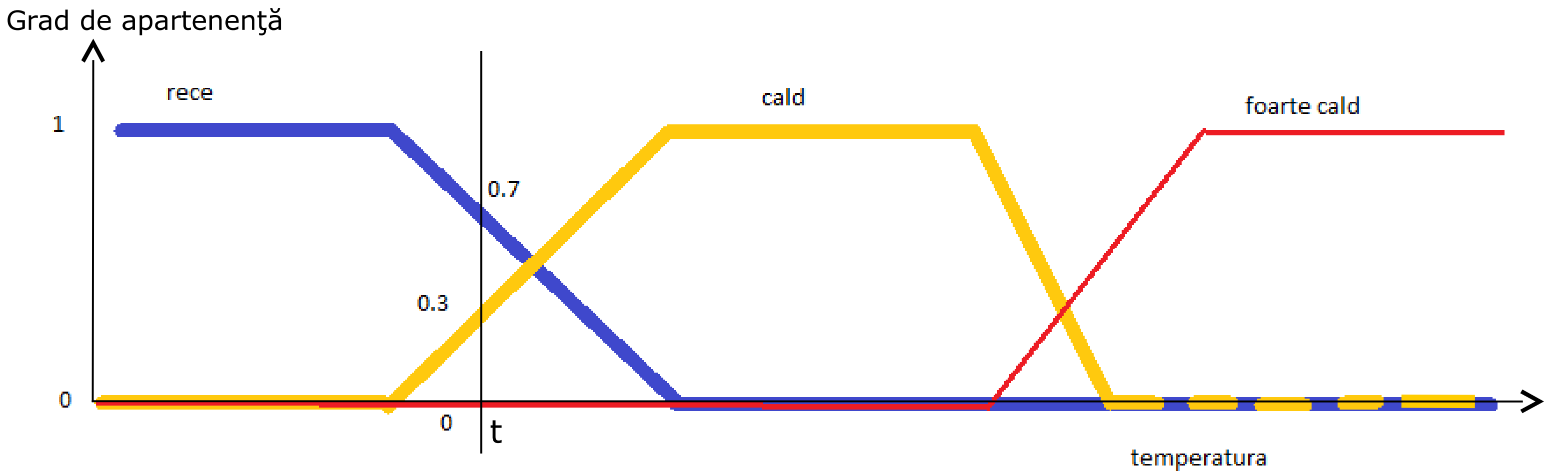




Intelligent systems – KBS – Fuzzy systems

Content and design → fuzzification of input data

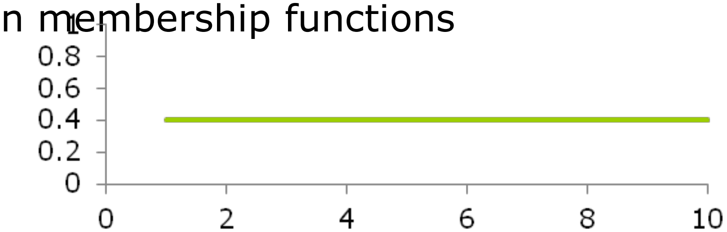
- Elements from probability theory (fuzzy logic) → Fuzzy facts (fuzzy sets) → **definition**
  - Example 2
    - A temperature *t* can have 3 truth values:
      - Red (0): is not hot
      - Orange (0.3): warm
      - Blue (0.7): cold

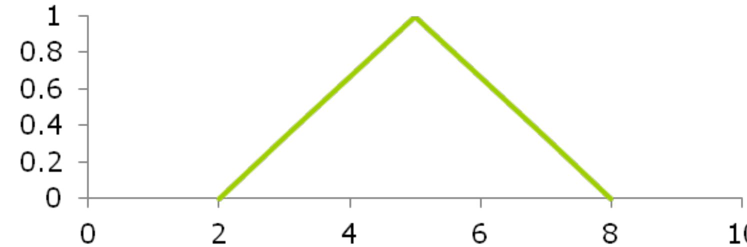


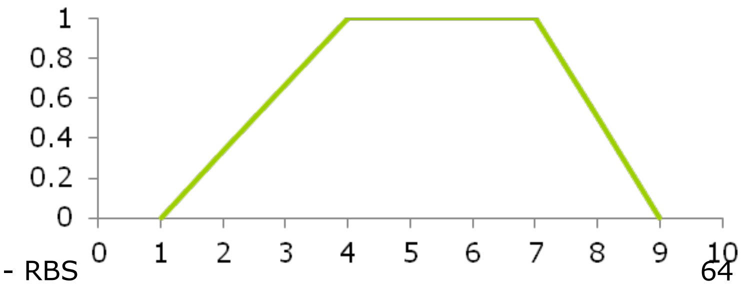
Intelligent systems – KBS – Fuzzy systems

Content and design → fuzzification of input data

- Elements from probability theory (fuzzy logic) → Fuzzy facts (fuzzy sets) → **representation**
  - Regular sets
    - Exact limits → Venn diagrams
  - Fuzzy sets
    - Gradual limits → representations based on membership functions
      - Singular
        - $\mu(x) = s$, where s is a scalar
      - Triangular
        - $\mu(x) = \max\left\{0, \min\left\{\frac{x-a}{b-a}, 1, \frac{c-x}{c-b}\right\}\right\}$
      - Trapezoidal
        - $\mu(x) = S(x) = \max\left\{0, \min\left\{\frac{x-a}{b-a}, 1, \frac{d-x}{d-c}\right\}\right\}$
      - Z function
        - $\mu(x) = 1 - S(x)$
      - Π function
        - $\mu(x) = \Pi(x) = \begin{cases} S(x), & \text{if } x \le c \\ Z(x), & \text{if } x > c \end{cases}$



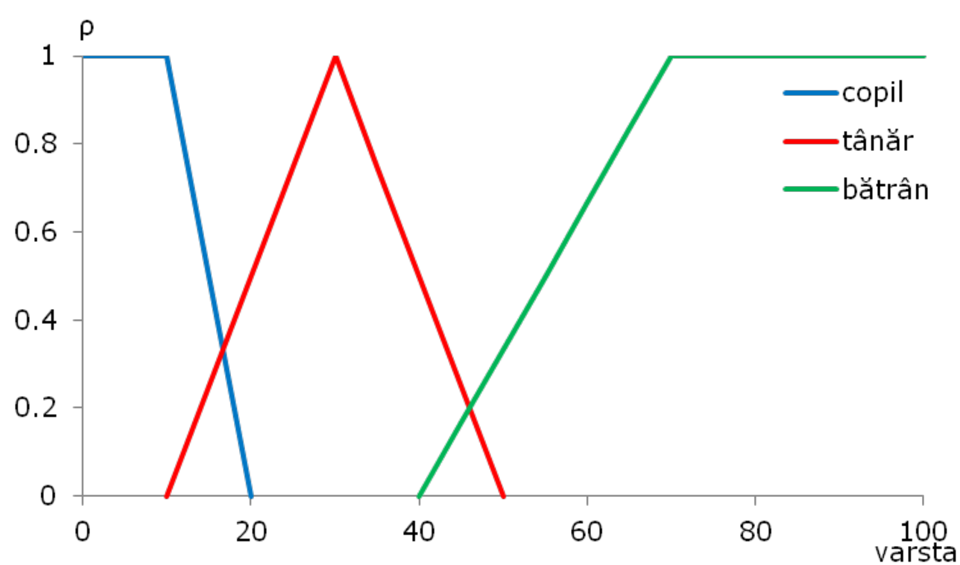




Intelligent systems – KBS – Fuzzy systems

Content and design → fuzzification of input data

- Elements from probability theory (fuzzy logic) → Fuzzy facts (fuzzy sets) → **representation**
  - Example
    - *Age of a person*



Intelligent systems – KBS – Fuzzy systems

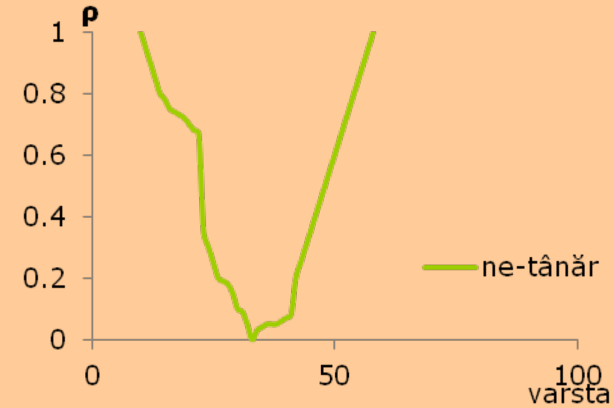
Content and design → fuzzification of input data

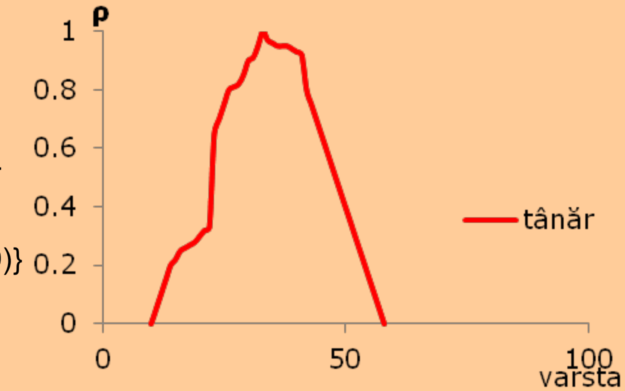
- Elements from probability theory (fuzzy logic) → Fuzzy facts (fuzzy sets) → **operations**
  - complement
  - Containment
  - Intersection
  - Union
  - Equality
  - Algebraic product
  - Algebraic sum

Intelligent systems – KBS – Fuzzy systems

Content and design → fuzzification of input data

- Elements from probability theory (fuzzy logic) → Fuzzy facts (fuzzy sets) → **operations**
  - Complement
    - X – a universe
    - A – a fuzzy set (with universe X)
    - B – a fuzzy set (with universe X)
    - B is complement of A ($B=\neg A$) if:
      - $\mu_B(x)=\mu_{\neg A}(x)=1-\mu_A(x)$ for all $x \in X$
    - Example:
      - *Old persons (based on their age)*
        - A={(30,0), (40, 0.2), (50, 0.4), (60, 0.6), (70, 0.8), (80, 1)}
      - *Young persons (that are not old) (based on their age)*
        - ┐A={(30,1), (40, 0.8), (50, 0.6), (60, 0.4), (70, 0.2), (80, 0)}





Intelligent systems – KBS – Fuzzy systems

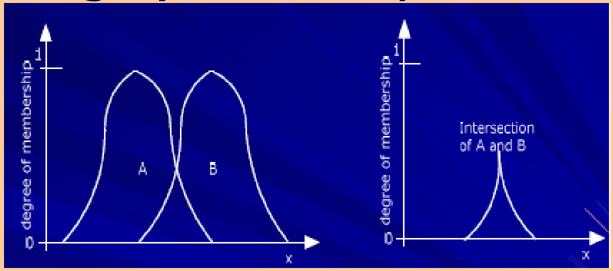
Content and design → fuzzification of input data

- Elements from probability theory (fuzzy logic) → Fuzzy facts (fuzzy sets) → **operations**
  - Containment
    - X – a universe
    - A – a fuzzy set (with universe X)
    - B – a fuzzy set (with universe X)
    - B is a subset of A ($B \subset A$) if:
      - $\mu_B(x) \leq \mu_A(x)$ for all $x \in X$
    - Example
      - *Old persons (based on their age)*
        - A={(60, 0.6), (65, 0.7) (70, 0.8), (75, 0.9), (80, 1)}
      - *Very old persons (based on their age)*
        - B={(60, 0.6), (65, 0.67) (70, 0.8), (75, 0.8), (80, 0.95)}

Intelligent systems – KBS – Fuzzy systems

Content and design → fuzzification of input data

- Elements from probability theory (fuzzy logic) → Fuzzy facts (fuzzy sets) → **operations**
  - intersection
    - X – a universe
    - A – a fuzzy set (with universe X)
    - B – a fuzzy set (with universe X)
    - C – a fuzzy set (with universe X)
    - **C** is an intersection of A and B if:
      - $\mu_C(x)=\mu_{A\cap B}(x)=\min\{\mu_A(x), \mu_B(x)\}=\mu_A(x)\cap\mu_B(x)$ for all $x \in X$

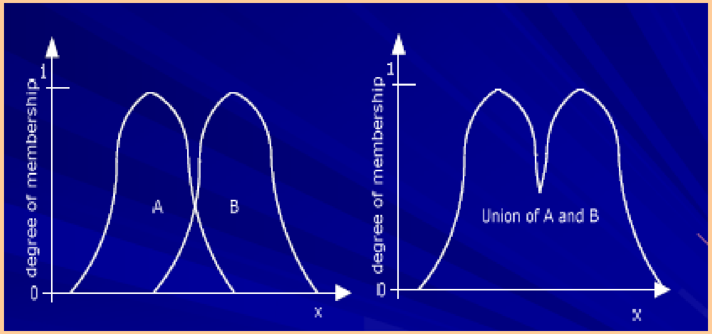


    - Example
      - *Old persons (based on their age)*
        - A={(30,0) (40, 0.1) (50, 0.2) (60, 0.6), (65, 0.7) (70, 0.8), (75, 0.9), (80, 1)}
      - *Middle-age persons*
        - B={(30,0.1) (40, 0.2) (50, 0.6) (60, 0.5), (65, 0.2) (70, 0.1), (75, 0), (80, 0)}
      - *Old and middle age persons*
        - C={(30,0) (40, 0.1) (50, 0.2) (60, 0.5), (65, 0.2) (70, 0.1), (75, 0), (80, 0)}

Intelligent systems – KBS – Fuzzy systems

Content and design → fuzzification of input data

- Elements from probability theory (fuzzy logic) → Fuzzy facts (fuzzy sets) → **operations**
  - union

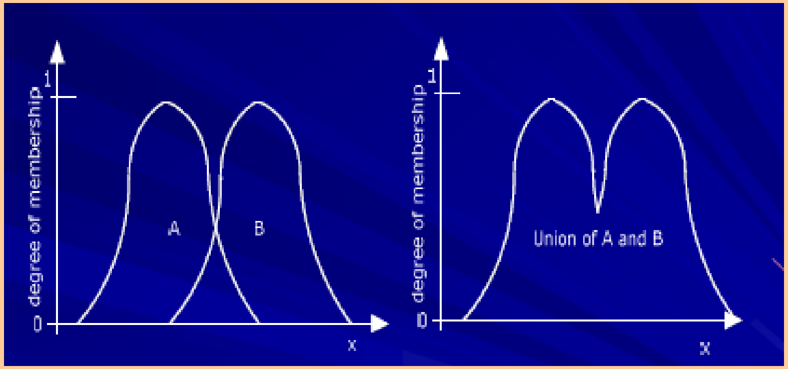


    - X – a universe
    - A – a fuzzy set (with universe X)
    - B – a fuzzy set (with universe X)
    - C – a fuzzy set (with universe X)
    - C is the union of A nad B if:
      - $\mu_C(x)=\mu_{A\cup B}(x)=\max\{\mu_A(x), \mu_B(x)\}=\mu_A(x) \cup \mu_B(x)$ for all $x \in X$
    - Example
      - *Old persons (based on their age)*
        - A={(30,0) (40, 0.1) (50, 0.2) (60, 0.6), (65, 0.7) (70, 0.8), (75, 0.9), (80, 1)}
      - *Middle-age persons*
        - B={(30,0.1) (40, 0.2) (50, 0.6) (60, 0.5), (65, 0.2) (70, 0.1), (75, 0), (80, 0)}
      - *Old or middle-age persons*
        - C={(30,0.1) (40, 0.2) (50, 0.6) (60, 0.6), (65, 0.7) (70, 0.8), (75, 0.9), (80, 1)}

Intelligent systems – KBS – Fuzzy systems

Content and design → fuzzification of input data

- Elements from probability theory (fuzzy logic) → Fuzzy facts (fuzzy sets) → **operations**
  - Equality, product and algebraic sum
    - X – a universe
    - A – a fuzzy set (with universe X)
    - B – a fuzzy set (with universe X)
    - C – a fuzzy set (with universe X)



    - B is equal to A (B=A) if:
      - $\mu_B(x)=\mu_A(x)$ for all $x \in X$
    - C is the product of A and B (C=A*B) if:
      - $\mu C(x)=\mu A*B(x)=\mu A(x)*\mu B(x)$ for all $x \in X$
    - C is the sum of A and B (C=A+B) if:
      - $\mu C(x)=\mu A+B(x)=\mu A(x)+\mu B(x)$ for all $x \in X$

Intelligent systems – KBS – Fuzzy systems

Content and design → fuzzification of input data

- Elements from probability theory (fuzzy logic) → Fuzzy facts (fuzzy sets) → **properties**
  - Asociativity
  - Commutativity
  - Distributivity
  - Transitivity
  - Idem potency
  - Identity
  - Involution

Intelligent systems – KBS – Fuzzy systems

Content and design → fuzzification of input data

- Elements from probability theory (fuzzy logic) → Fuzzy facts (fuzzy sets) → **hedges**

  - Main idea
    - Modifiers, adjectives or adverbs that change the truth values of sentences
      - Ex. *Very, less, much, more, close*, etc.
    - Change the shape of fuzzy sets
    - Can act on
      - Fuzzy numbers
      - Truth values
      - Membership functions
    - Heuristics

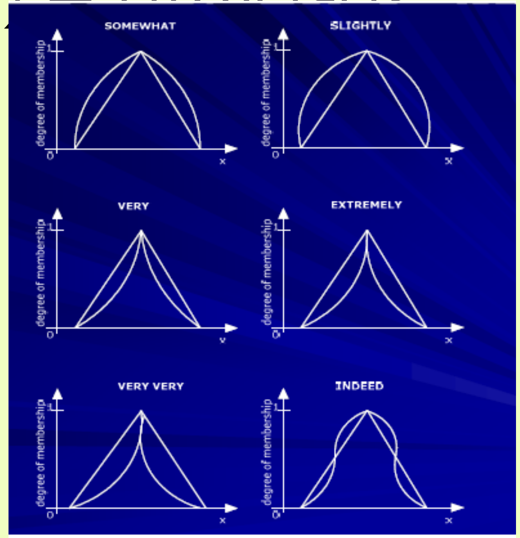
  - Utility
    - Closer to the natural language → subjectivism
    - Evaluation of linguistic variables

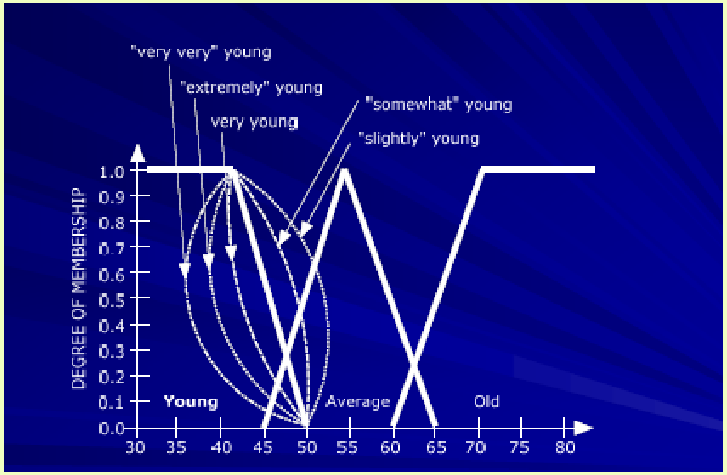
Intelligent systems – KBS – Fuzzy systems

Content and design → fuzzification of input data

- Elements from probability theory (fuzzy logic) → Fuzzy facts (fuzzy sets) → **hedges**
  - Typology
    - *Hedges* that reduce the truth value (produce a concentration)
      - *Very* $\mu_{A\_very}(x)=(\mu_A(x))^2$
      - *Extremly* $\mu_{A\_extremly}(x)=(\mu_A(x))^3$
      - *Very very* $\mu_{A\_very\_very}(x)=(\mu_{A\_foarte}(x))^2=(\mu_A(x))^4$
    - *Hedges* that increase the truth value (produce a dilatation)
      - *Somewhat* $\mu_{A\_somewhat}(x)=(\mu A(x))^{1/2}$
      - *slightly* $\mu_{A\_slightly}(x)=(\mu A(x))^{1/3}$
    - *Hedges* cthat intensify the truth value
      - *indeed*

$$\mu_{A\_indeed}(x)=\begin{cases}2(\mu_A(x))^2, & \text{if } 0\le\mu_A(x)\le 0.5\\ 1-2(1-\mu_A(x))^2, & \text{if } 0.5\le\mu_A(x)\le 1\end{cases}$$





Intelligent systems – KBS – Fuzzy systems

Content and design → fuzzification of input data

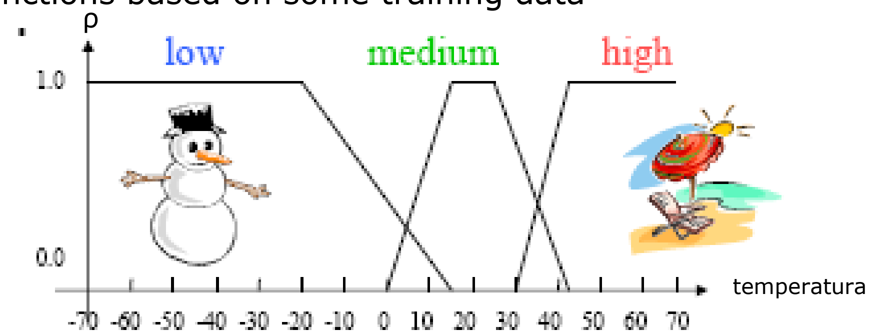
- Elements from probability theory (fuzzy logic)
  - Fuzzy facts (fuzzy sets)
    - Definition
    - Representation
    - Operations – complements, containment, intersection, reunion, equality, algebraic product, algebraic sum
    - Properties – associativity, commutativity, distributivity, transitivity, idempotency, identity, involution
    - Hedges
  - **Fuzzy variables**
    - **Definition**
    - **Properties**

    - Establish the fuzzy variables and the fuzzy sets based on membership functions

Intelligent systems – KBS – Fuzzy systems

Content and design → fuzzification of input data

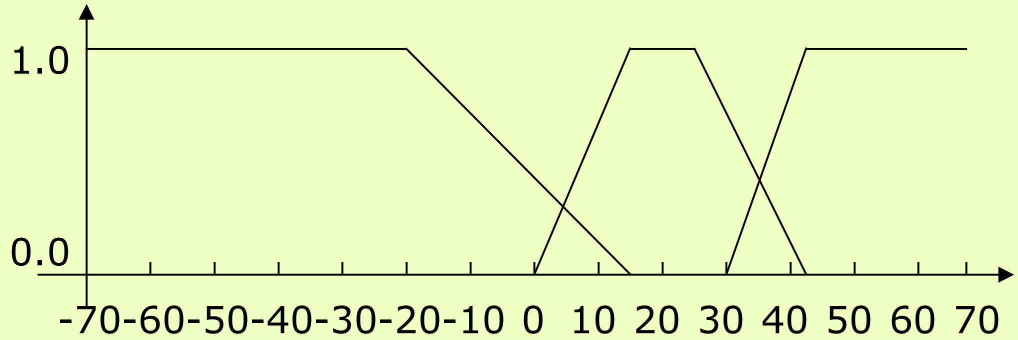
- Elements from probability theory (fuzzy logic) → Fuzzy variables → **definition**
  - A fuzzy variable is defined by *V = {x, l, u, m}*, where:
    - x – name of symbolic variable
    - L – set of possible labels for variable x
    - U – universe of the variable
    - M – semantic regions that define the meaning of labels from L (membership functions)
  - Membership functions
    - Subjective assessment
      - The shape of functions is defined by experts
    - Ad-hoc assessment
      - Simple functions that can solve the problem
    - Assessment based on distributions and probabilities of information extracted from measurements
    - Adapted assessment
      - By testing
    - Automated assessment
      - Algorithms utilised for defining functions based on some training data
  - Example
    - X = Temperature
    - L = {low, medium, high}
    - U = {x∈X | -70° ≤ x ≤+70°}
    - M =



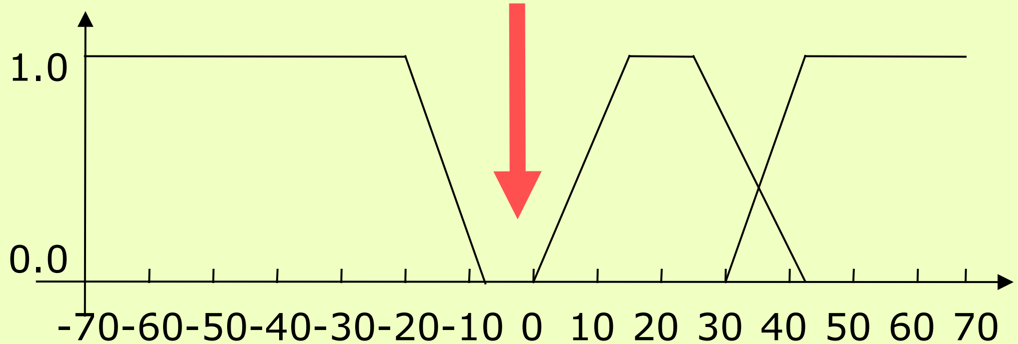
Intelligent systems – KBS – Fuzzy systems

Content and design → fuzzification of input data

- Elements from probability theory (fuzzy logic) → Fuzzy variables → **properties**
  - Completeness
    - A fuzzy variable V is complete if for all $x \in X$ there is a fuzzy set A such as $\mu_A(x)>0$



Complete

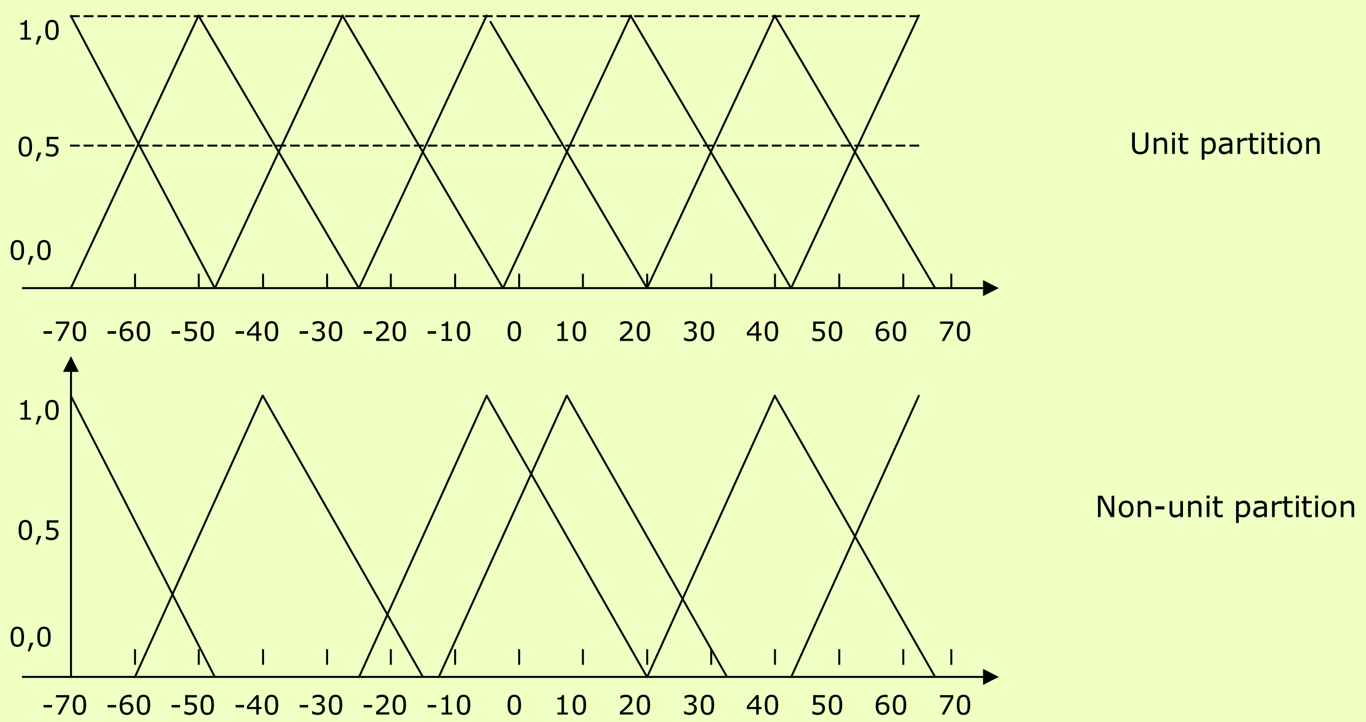


Incomplete

Intelligent systems – KBS – Fuzzy systems

Content and design → fuzzification of input data

- Elements from probability theory (fuzzy logic) → Fuzzy variables → **properties**
  - Unit partition
    - A fuzzy variable V forms a unit partition if for all input values x we have $\sum_{i=1}^{p} \mu_{A_i}(x) = 1$
    - where p is the number of sets that x belongs to
    - There are no rules for defining 2 neighbour sets
      - Usually, the overlap is between 25% şi 50%



Intelligent systems – KBS – Fuzzy systems

Content and design → fuzzification of input data

- Elements from probability theory (fuzzy logic) → Fuzzy variables → **properties**
  - Unit partition
    - A complete fuzzy variable can be transformed into a unit partition:

$$\mu_{\hat{A}_i}(x) = \frac{\mu_{A_i}(x)}{\sum_{j=1}^{p} \mu_{A_j}(x)} \; for \; i = 1, \ldots, p$$

Intelligent systems – KBS – Fuzzy systems

Content and design → fuzzification of input data

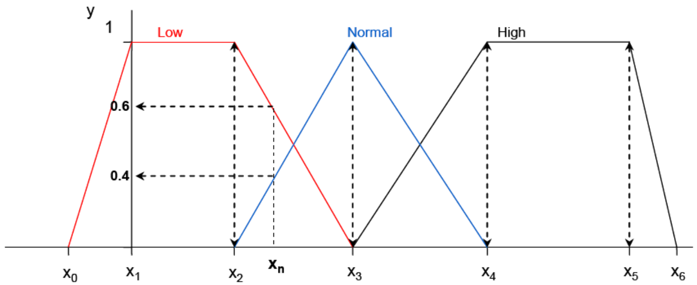
- Elements from probability theory (fuzzy logic)
  - Fuzzy facts (fuzzy sets)
    - Definition
    - Representation
    - Operations – complements, containment, intersection, reunion, equality, algebraic product, algebraic sum
    - Properties – associativity, commutativity, distributivity, transitivity, idempotency, identity, involution
    - Hedges
  - Fuzzy variables
    - Definition
    - Properties

- **Establish the fuzzy variables and the fuzzy sets based on membership functions**

Intelligent systems – KBS – Fuzzy systems

Content and design → fuzzification of input data

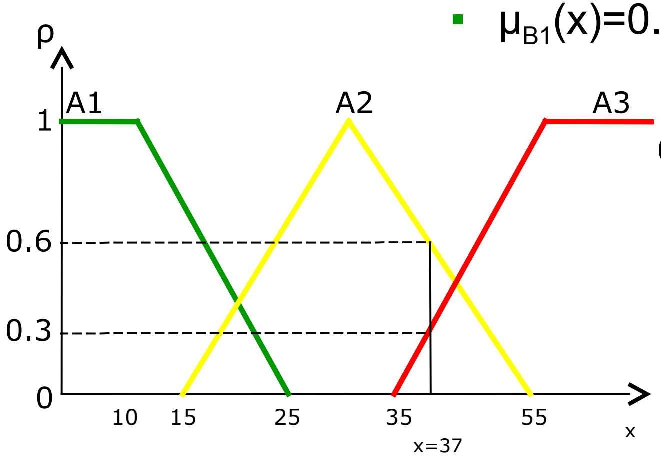
- Mechanism
  - Establish the raw (input and out[put) data of the system
  - Define membership functions for each input data
    - Each membership function has associated a quality label – linguistic variable
    - A raw variable can have associated one or more linguistic variables
    - Example
      - Raw variable: temperature T
      - Linguistic variable: law →A1, medium → A2, high → A3
  - Transform each raw input data into a linguistic data → fuzzification
    - Establish the fuzzy set of that raw input data
    - How?
      - For a given raw input determine the membership degree for each possible set
    - Example
      - T ($=x_n$) $=5°$
      - $A_1 \rightarrow \mu_{A1}(T) = 0.6$
      - $A_2 \rightarrow \mu_{A2}(T) = 0.4$

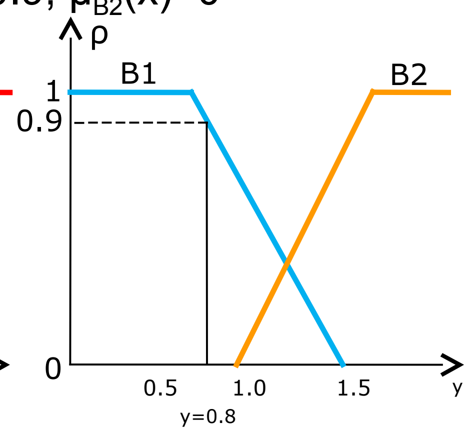


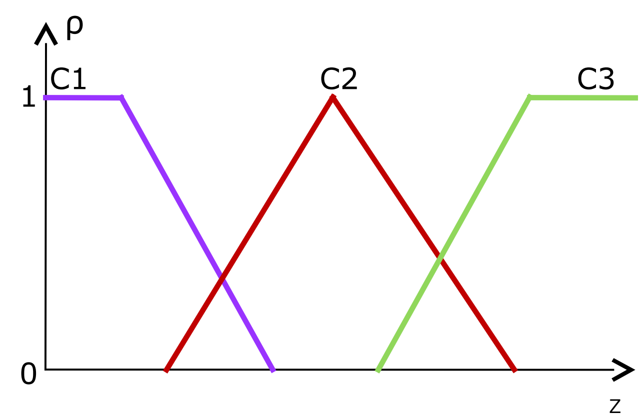
Intelligent systems – KBS – Fuzzy systems

Content and design → fuzzification of input data

- **Mechanism**
  - Example - air conditioner device
    - Inputs :
      - x (temperature – cold, normal, hot) and
      - y (humidity – small, large)
    - Outputs:
      - z (machine power – law, medium, high)
    - Input data:
      - Temperature x = 37
        - $\mu_{A1}(x)=0$, $\mu_{A2}(x)=0.6$, $\mu_{A3}(x)=0.3$
      - Humidity y = 0.8
        - $\mu_{B1}(x)=0.9$, $\mu_{B2}(x)=0$



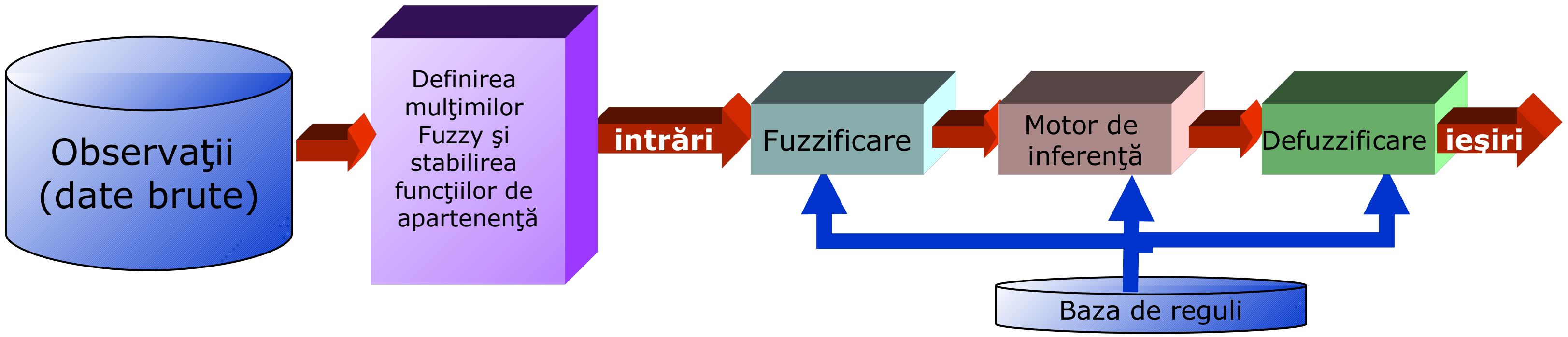




Intelligent systems – KBS – Fuzzy systems

Content and design

- Steps for constructing a fuzzy system
  - Define the inputs and the outputs – by an expert
    - Raw inputs and outputs
    - Fuzzification of inputs and outputs
      - Fix the fuzzy variables and fuzzy sets based on membership functions
  - **Construct a base of rules – by an expert**
    - **Decision matrix**
  - Evaluate the rules
    - Inference – transform the fuzzy inputs into fuzzy outputs by applying all the rules
  - Aggregate the results
  - Defuzzificate the result
  - Interpret the result



Intelligent systems – KBS – Fuzzy systems

Content and design → Construct a base of rules – by an expert

- Rules
  - Definition
    - Linguistic constructions
      - Affirmative sentences: A
      - Conditional sentences: if A then B
    - Where A and B are (collections of) sentences that contain linguistic variables
      - A – premise of the rule
      - B – consequence of the rule
  - Typology
    - Non-conditional
      - x is (in) A_i
      - Eg. *Save the energy*
    - Conditional
      - If x is (in) A_i then z is (in) C_k
      - If x is (in) A_i and y is (in) B_j, then z is (in) C_k
      - If x is (in) A_i or y is (in) B_j, then z is (in) C_k

Intelligent systems – KBS – Fuzzy systems

Content and design → Construct a base of rules – by an expert

- Rules
  - Example

	Rules of classical logic	Rules of fuzzy logic
R_1	*If temperature is -5, then is cold*	*If temperature is law, then is cold*
R_2	*If temperature is 15, then is warm*	*If temperature is medium, then is warm*
R_3	*If temperature is 35, then is hot*	*If temperature is high, then is hot*

Intelligent systems – KBS – Fuzzy systems

Content and design → Construct a base of rules – by an expert

- Rules
- Database of fuzzy rules
    - R_{11}: if x is A_1 and y is B_1 then z is C_u
    - R_{12}: if x is A_1 and y is B_2 then z is C_v
    - ...
    - R_{1n}: : if x is A_1 and y is B_n then z is C_x
    - R_{21}: if x is A_2 and y is B_1 then z is C_x
    - R_{22}: if x is A_2 and y is B_2 then z is C_z
    - ...
    - R_{2n}: if x is A_2 and y is B_n then z is C_v
    - ...
    - R_{m1}: if x is A_m and y is B_1 then z is C_x
    - R_{m2}: if x is A_m and y is B_2 then z is C_v
    - ...
    - R_{mn}: if x is A_m and y is B_n then z is C_u

Intelligent systems – KBS – Fuzzy systems

Content and design → Construct a base of rules – by an expert

- Rules
- Properties
  - Completeness
    - A database of fuzzy rules is complete
      - If all input values have associated a value between 0 and 1
      - If all fuzzy variable are complete
      - If used fuzzy sets have a non-compact support
  - Consistency
    - A set of fuzzy rules is inconsistent if two rules have the same premises and different consequences
      - If x in A and y in B then z in C
      - If x in A and y in B then z in D
- Problems of the database
  - Rule's explosion
    - #of rules increases exponential whit the # of input variables
    - # of input set combinations is
      - Where the i^{th} variable is composed by p_i sets

$$P = \prod_{i=1}^{n} p_i$$

Intelligent systems – KBS – Fuzzy systems

Content and design → Construct a base of rules – by an expert

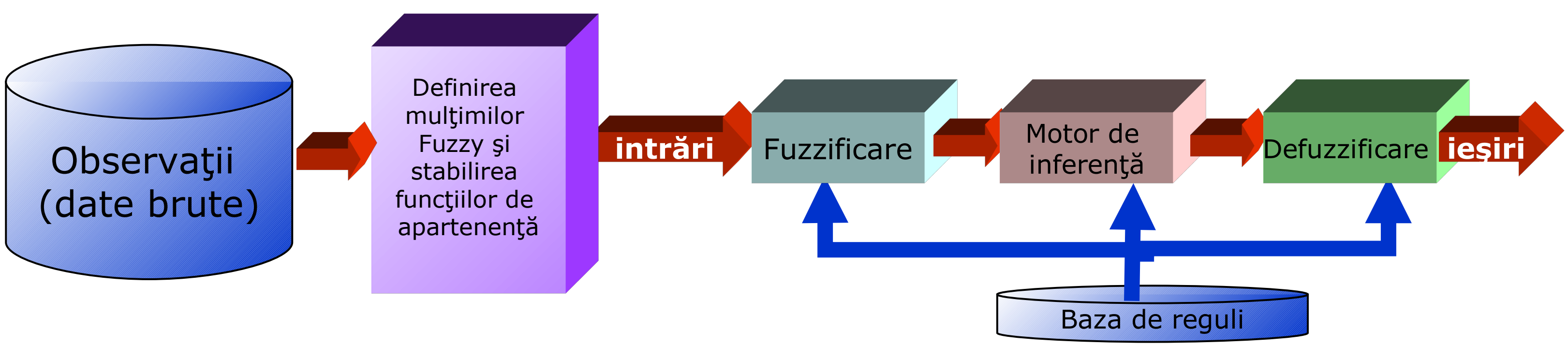
- Decision matrix of the knowledge database
- Example – air conditioner device
  - Inputs :
    - x (temperature – cold, normal, hot) and
    - y (humidity – small, large)
  - Outputs:
    - z (machine power – law, constant, high)
  - Rules:
    - *If temperature is normal and humidity is small then the power is constant*

		Input data y	
		Small	Large
Input data x	Cold	Law	Constant
	Normal	Constant	High
	Hot	High	High

Intelligent systems – KBS – Fuzzy systems

Content and design

- Steps for constructing a fuzzy system
  - Define the inputs and the outputs – by an expert
    - Raw inputs and outputs
    - Fuzzification of inputs and outputs
      - Fix the fuzzy variables and fuzzy sets based on membership functions
  - Construct a base of rules – by an expert
    - Decision matrix
  - **Evaluate the rules**
    - **Inference – transform the fuzzy inputs into fuzzy outputs by applying all the rules**
  - Aggregate the results
  - Defuzzificate the result
  - Interpret the result



Intelligent systems – KBS – Fuzzy systems

Content and design → rule evaluation (fuzzy inference)

- Which rules are firstly evaluated?
  - Fuzzy inference
    - Rules are evaluated in **parallel** , each rules contributing to the shape of the final result
    - Resulted fuzzy sets are de-fuzzified **after all the rules** have been evaluated

  Remember
  - Forward inference
    - For a given state of problem, collect the required information and apply the possible rules
  - Backward inference
    - Identify the rules that determine the final state and apply only that rules (if it is possible)

- How the rules are evaluated?
  - Evaluation of causes
  - Evaluation of consequences

Intelligent systems – KBS – Fuzzy systems

Content and design → rule evaluation (fuzzy inference)

- Evaluation of causes
  - For each premise of a rule (*if s is (in) A*) establish the membership degree of raw input data to all fuzzy sets
  - A rule can have more premises linked by logic operators *AND, OR* or *NOT* → use fuzzy operators
    - Operator *AND* → intersection (minimum) of 2 sets
      - $\mu_{A \cap B}(x) = min\{\mu_A(x), \mu_B(x)\}$
    - Operator *OR* → union (maximum) of 2 sets
      - $\mu_{A \cup B}(x) = max\{\mu_A(x), \mu_B(x)\}$
    - Operator *NOT* → negation (complement) of a set
      - $\mu_{\neg A}(x) = 1 - \mu_A(x)$
  - The result of premise's evaluation
    - Degree of satisfaction
    - Other names:
      - Rule's firing strength
      - Degree of fulfillment

Intelligent systems – KBS – Fuzzy systems

Content and design → rule evaluation (fuzzy inference)

- Evaluation of consequences
  - Determine the results
    - Establish the membership degree of variables (involved in the consequences) to different fuzzy sets
  - Each output region must be de-fuzzified in order to obtain crisp value
  - Based on the consequence's type
    - Mamdani model – consequence of rule: "output variable belongs to a fuzzy set"
    - Sugeno model – consequence of rule: "output variable is a crisp function that depends on inputs"
    - Tsukamoo model – consequence of rule: "output variable belongs to a fuzzy set folowing a monotone membership function"

Intelligent systems – KBS – Fuzzy systems

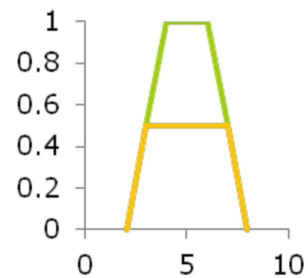
Content and design → rule evaluation (fuzzy inference) →
Evaluation of consequences

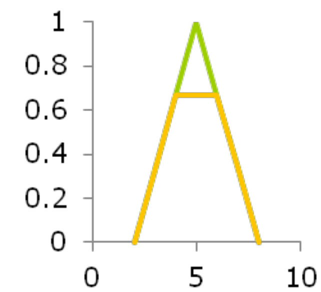
- Mamdani model
  - Main idea:
    - consequence of rule: "output variable belongs to a fuzzy set"
    - Result of evaluation is applied for the membership function of the consequence
    - Example
      - ***if x is in A and y is in B, then z is in C***
  - Typology (based on how the results is applied on the membership function of the consequence)
    - Clipped fuzzy sets
    - Scaled fuzzy sets

Intelligent systems – KBS – Fuzzy systems

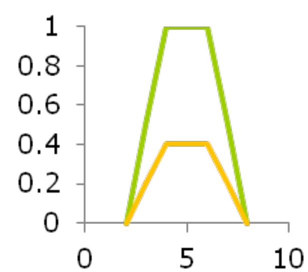
Content and design → rule evaluation (fuzzy inference) →
Evaluation of consequences

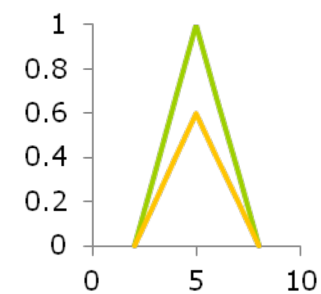
- Mamdani model
  - Typology (based on how the results is applied on the membership function of the consequence)
    - Clipped fuzzy sets
      - Membership function of the consequence is cut at the level of the result's truth value
      - Advantage → easy to compute
      - Disadvantage → some information are lost





-
  -
    - Scaled fuzzy sets
      - Membership function of the consequence is adjusted by scaling (multiplication) at the level of the result's truth value
      - Advantage → few information is lost
      - Disadvantage → complicate computing





Intelligent systems – KBS – Fuzzy systems

- Content and design → rule evaluation (fuzzy inference) → **Evaluation of consequences** → Mamdani model
  - Example – air conditioner device
    - Inputs :
      - x (temperature – cold, normal, hot) and
      - y (humidity – small, large)
    - Outputs:
      - z (machine power – law, constant, high)
    - Input data:
      - Temperature x = 37
        - $\mu_{A1}(x)=0$, $\mu_{A2}(x)=0.6$, $\mu_{A3}(x)=0.3$
      - Humidity y = 0.8
        - $\mu_{B1}(x)=0.9$, $\mu_{B2}(x)=0$

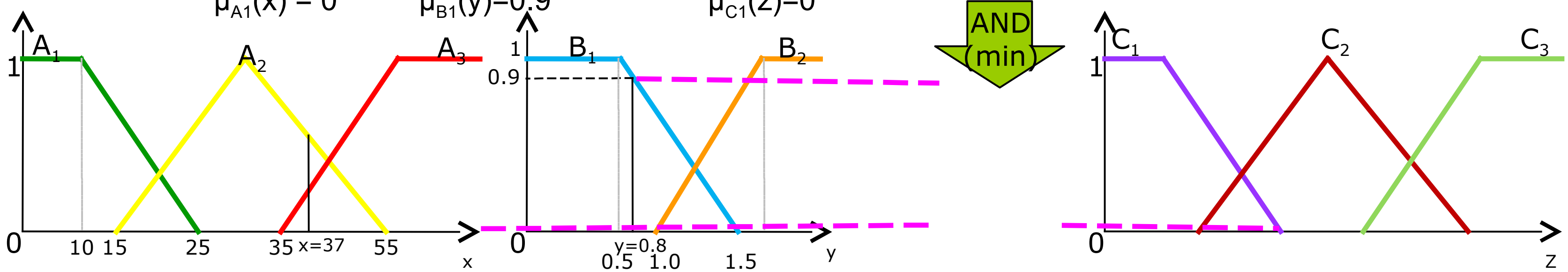
		Input data y	
		Small	Large
Input data x	Cold	Law	Constant
	Normal	Constant	High
	Hot	High	High

Intelligent systems – KBS – Fuzzy systems

Content and design → rule evaluation → Evaluation of consequences → Mamdani model

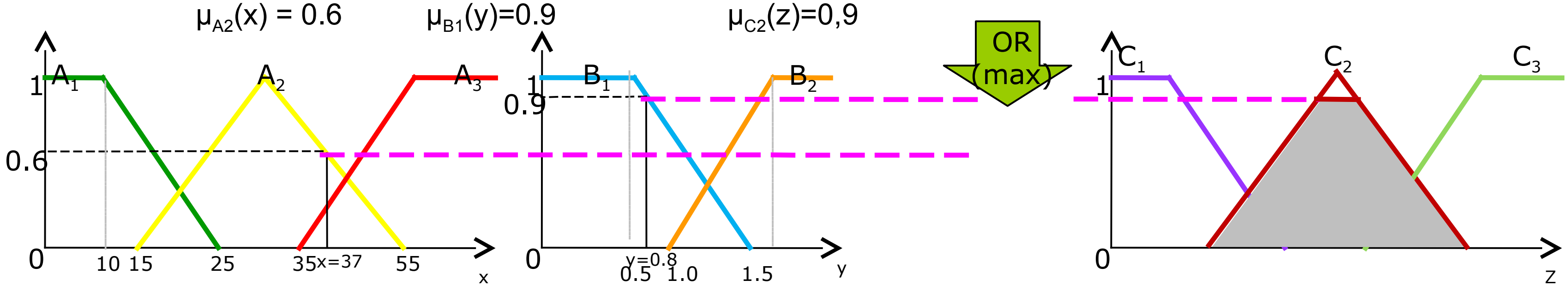
R1: if x is in A_1 and y is in B_1 then z is in C_1

$\mu_{A1}(x) = 0$ $\mu_{B1}(y)=0.9$ $\mu_{C1}(z)=0$



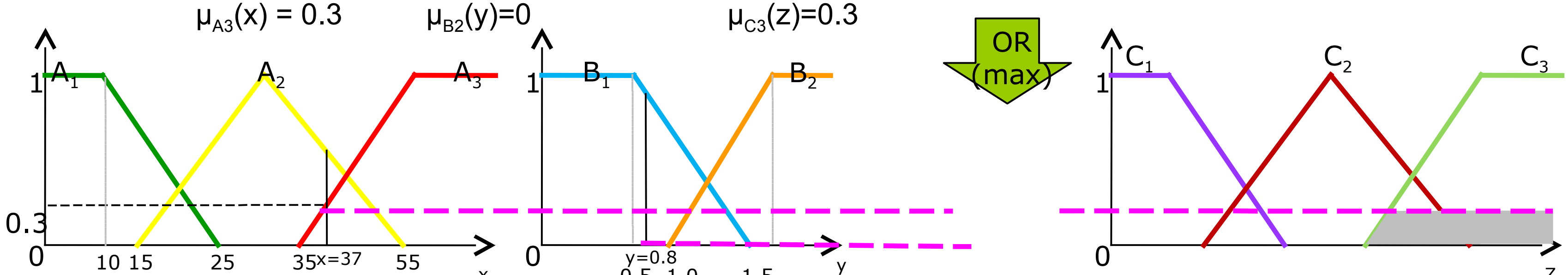
R2: if x is in A_2 or y is in B_1 then z is in C_2

$\mu_{A2}(x) = 0.6$ $\mu_{B1}(y)=0.9$ $\mu_{C2}(z)=0{,}9$



R3: if x is in A_3 or y is in B_2 then z is in C_3

$\mu_{A3}(x) = 0.3$ $\mu_{B2}(y)=0$ $\mu_{C3}(z)=0.3$



Intelligent systems – KBS – Fuzzy systems

Content and design → rule evaluation (fuzzy inference) → **Evaluation of consequences**

- Sugeno model
  - Main idea
    - consequence of rule: “output variable is a crisp function that depends on inputs”
    - Example

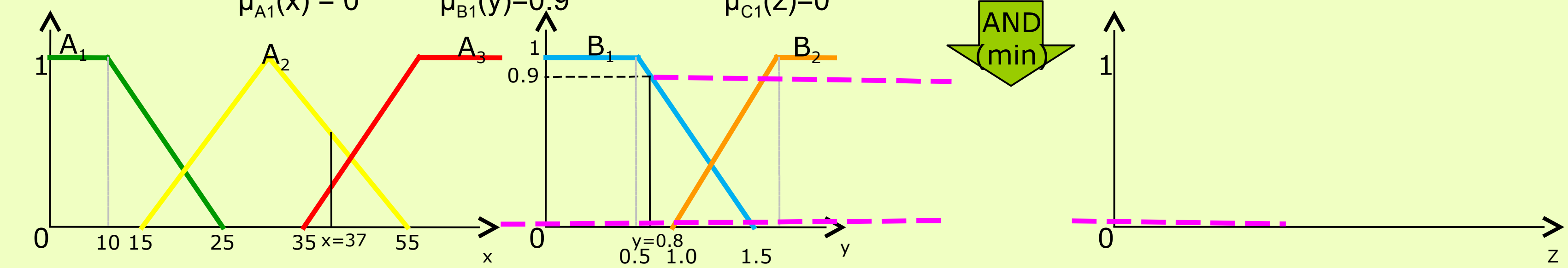
      If x is in A and y is in B then z is f(x,y)
    - Typology (based on charactersitics of f(x,y))
      - Sugeno model of degree 0 → if (f(x,y) = k – constant (membership function of the consequences are singleton – a fuzzy set whose membership functions have value 1 for a single (unique) point of the universe and 0 for all other points)
      - Sugeno model of degree 1→ if f(x,y) = ax + by+c

Intelligent systems – KBS – Fuzzy systems

Content and design → rule evaluation → Evaluation of consequences → Sugeno model

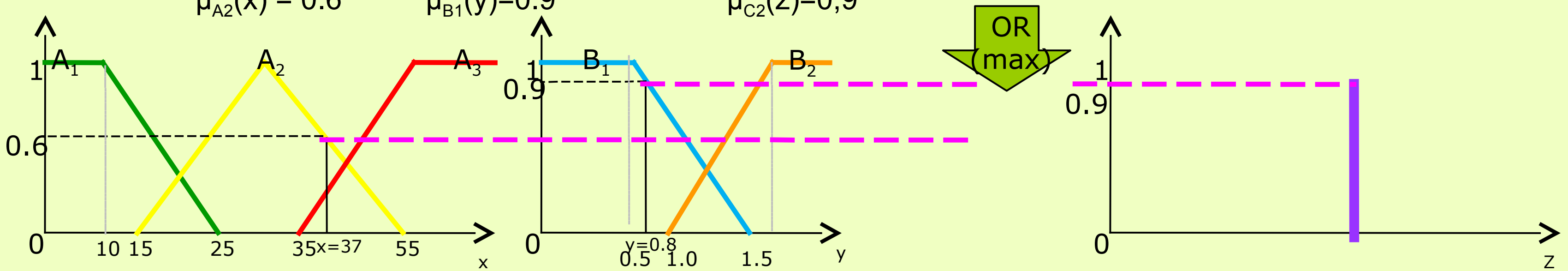
R1: if x is in A_1 and y is in B_1 then z is in C_1

$\mu_{A1}(x) = 0$ $\mu_{B1}(y)=0.9$ $\mu_{C1}(z)=0$



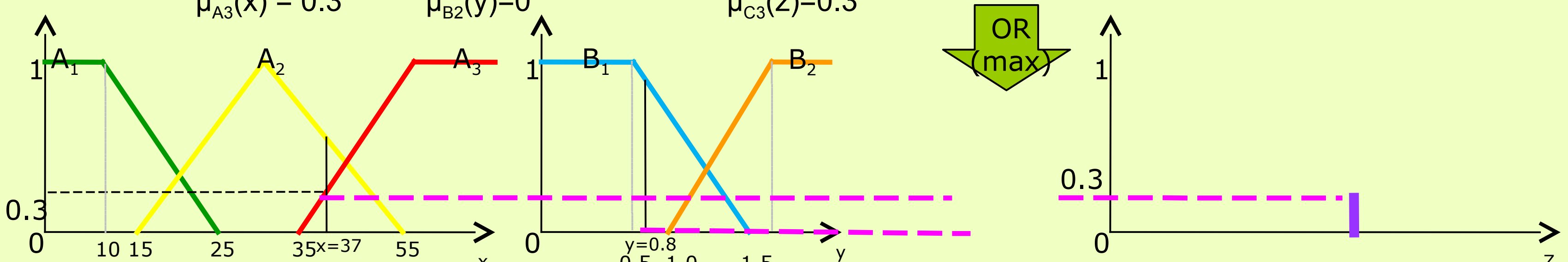
R2: if x is in A_2 or y is in B_1 then z is in C_2

$\mu_{A2}(x) = 0.6$ $\mu_{B1}(y)=0.9$ $\mu_{C2}(z)=0,9$



R3: if x is in A_3 or y is in B_2 then z is in C_3

$\mu_{A3}(x) = 0.3$ $\mu_{B2}(y)=0$ $\mu_{C3}(z)=0.3$



Intelligent systems – KBS – Fuzzy systems

Content and design → rule evaluation (fuzzy inference) →
Evaluation of consequences

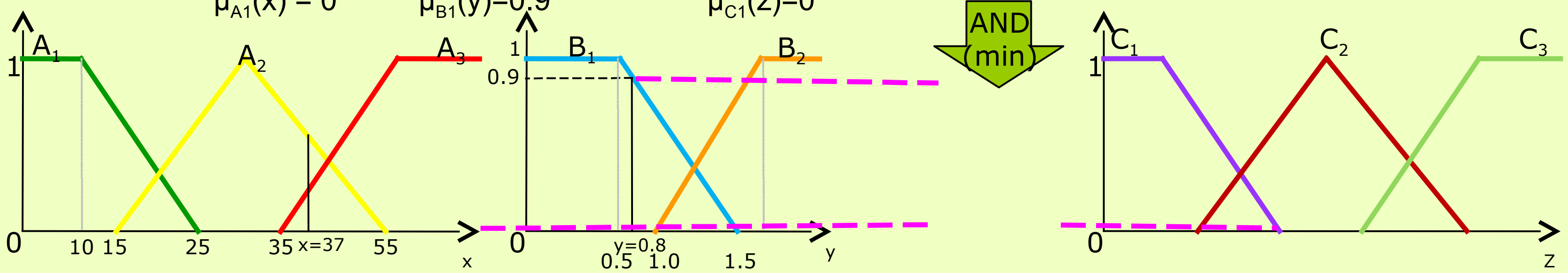
- Tsukamoto model
  - Main idea
    - consequence of rule: "output variable belongs to a fuzzy set folowing a monotone membership function"
      - A crisp value is obtained as output → *rule's firing strength*

Intelligent systems – KBS – Fuzzy systems

Content and design → rule evaluation → Evaluation of consequences → Tsukamoto model

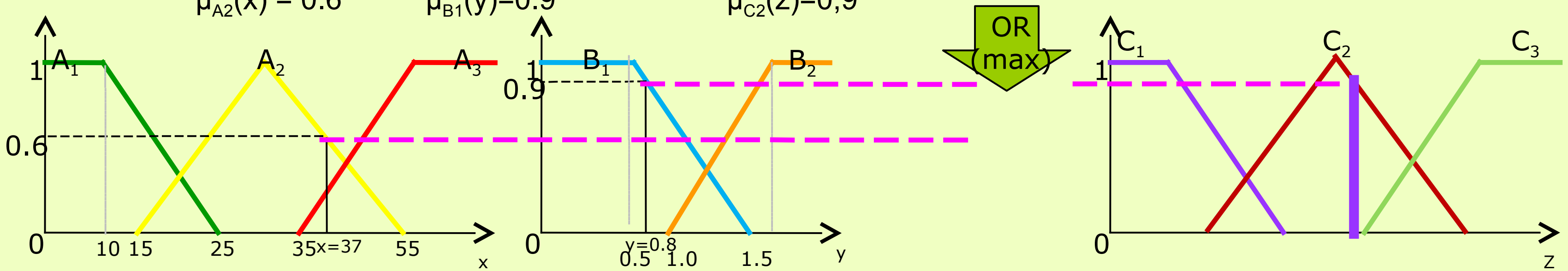
R1: if x is in A_1 and y is in B_1 then z is in C_1

$\mu_{A1}(x) = 0$ $\mu_{B1}(y)=0.9$ $\mu_{C1}(z)=0$



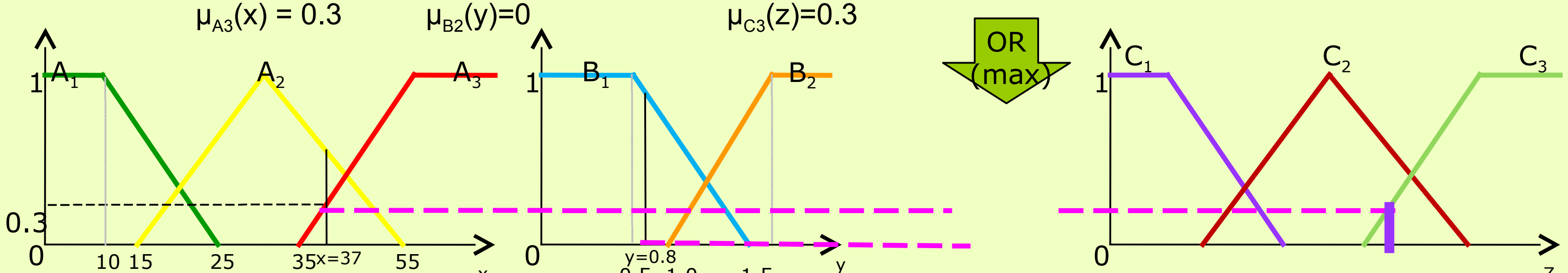
R2: if x is in A_2 or y is in B_1 then z is in C_2

$\mu_{A2}(x) = 0.6$ $\mu_{B1}(y)=0.9$ $\mu_{C2}(z)=0,9$



R3: if x is in A_3 or y is in B_2 then z is in C_3

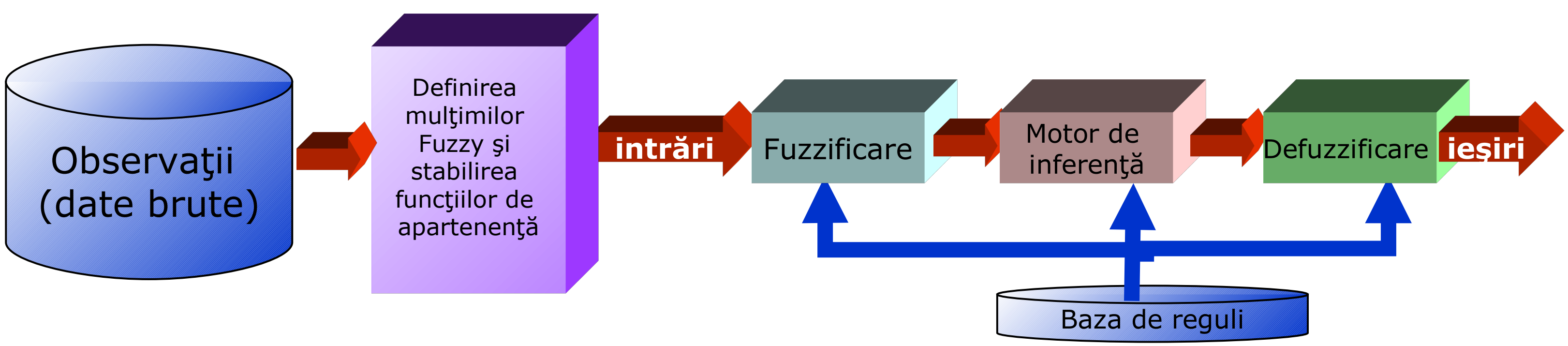
$\mu_{A3}(x) = 0.3$ $\mu_{B2}(y)=0$ $\mu_{C3}(z)=0.3$



Intelligent systems – KBS – Fuzzy systems

Content and design

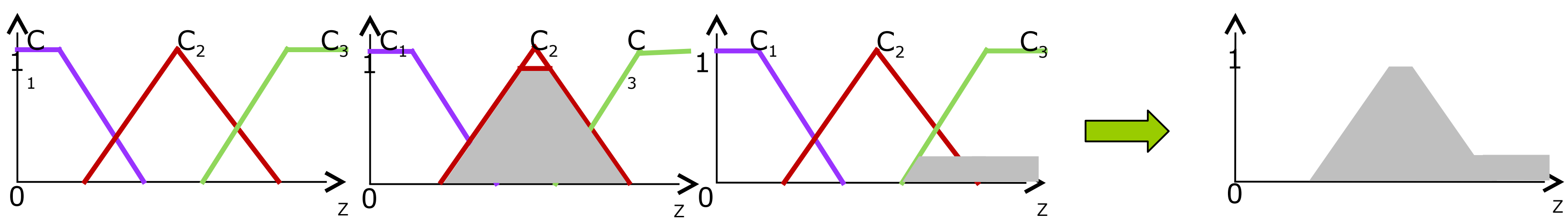
- Steps for constructing a fuzzy system
  - Define the inputs and the outputs – by an expert
    - Raw inputs and outputs
    - Fuzzification of inputs and outputs
      - Fix the fuzzy variables and fuzzy sets based on membership functions
  - Construct a base of rules – by an expert
    - Decision matrix
  - Evaluate the rules
    - Inference – transform the fuzzy inputs into fuzzy outputs by applying all the rules
  - **Aggregate the results**
  - Defuzzificate the result
  - Interpret the result



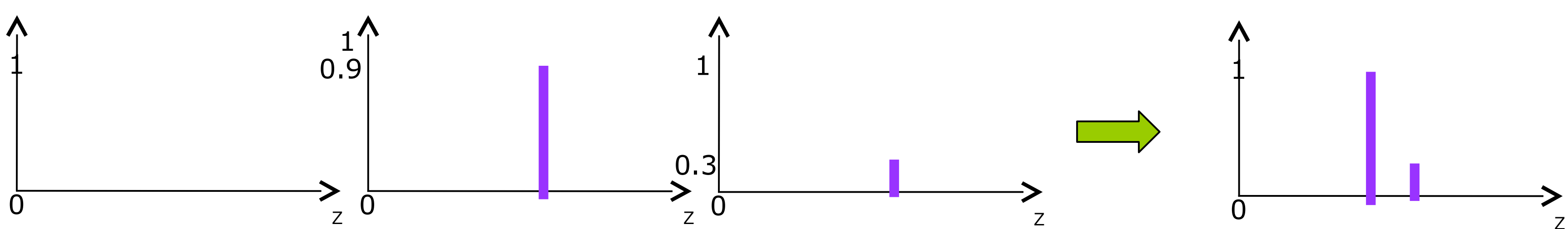
Intelligent systems – KBS – Fuzzy systems

Content and design → **Aggregate the results**

- Union of outputs for all the applied rules
- Consider the membership functions for all the consequences and combine them into a single fuzzy set (a single result)
- Aggregation process have as
  - Inputs → membership functions (clipped or scaled) of the consequences
  - Outputs → a fuzzy set of the output variable
- Example
  - Mamdani



  - Sugeno



Intelligent systems – KBS – Fuzzy systems

Content and design

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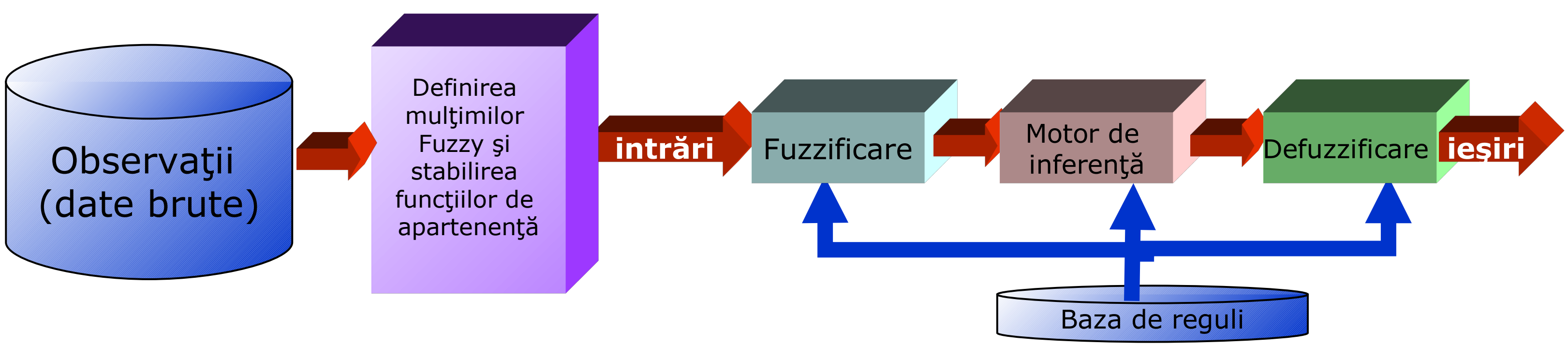
    - Inference – transform the fuzzy inputs into fuzzy outputs by applying all the rules
  - Aggregate the results
  - **Defuzzificate the result**
  - Interpret the result



Intelligent systems – KBS – Fuzzy systems

Content and design → defuzzification

- Main idea
  - Transform the fuzzy result into a crisp (raw) value
  - Inference → obtain some fuzzy regions for each output variable
  - Defuzzification → transform each fuzzy region into a crisp value
- Methods
  - Based on the gravity center
    - COA – Centroid Area
    - BOA – *Bisector of area*
  - Based on maximum of membership function
    - MOM - *Mean of maximum*
    - SOM - *Smallest of maximum*
    - LOM - *Largest of maximum*

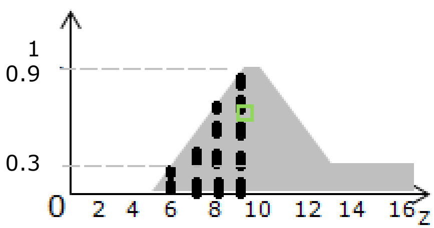
Intelligent systems – KBS – Fuzzy systems

Content and design → defuzzification → methods

- COA – Centroid Area
  - Identify the z point from the middle of aggregated set

$$COG = \frac{\sum_{i=0}^{n} x_i \mu_A(x_i)}{\sum_{i=0}^{n} \mu_A(x_i)} \text{ sau } COG = \frac{\int_{i=0}^{n} x_i \mu_A(x_i)}{\int_{i=0}^{n} \mu_A(x_i)}$$

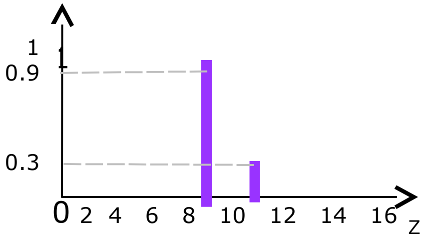
  - Example
    - Mamdani model → estimation of COA by using a sample of n points (x_i, i =1,2,..., n) of the resulted fuzzy set



$$COA = \frac{5*0+6*0.3+7*0.5+8*0.7+9*0.9+10*0.9+11*0.7+12*0.5+13*0.3+14*0.3+15*0.3+16*0.3}{0+0.3+0.5+0.7+0.9+0.9+0.7+0.5+0.3+0.3+0.3+0.3}$$

$$COA \cong 13.7$$

    - Sugeno or Tsukamoto model → COA becomes a weighted average of m crisp values obtained by applying all m rules



$$COA = \frac{9*0.9+11*0.3}{0.9+0.3}$$

$$COA \cong 9.5$$

Intelligent systems – KBS – Fuzzy systems

Content and design → defuzzification → methods

- BOA – Bisector of area
  - Identify the point z that determine the splitting of aggregated set in 2 parts of equal area

$$BOA = \int_{\alpha}^{z} \mu_A(x)dx = \int_{z}^{\beta} \mu_A(x)dx,$$

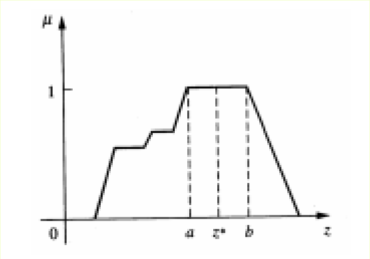
$$\text{where } \alpha = \min\{x \mid x \in A\} \text{ and } \beta = \min\{x \mid x \in A\}$$

Intelligent systems – KBS – Fuzzy systems

Content and design → defuzzification → methods

- MOM - *Mean of maximum*
  - Identify the point z that represents the mean of that points (from the aggregated set) that have a maximum membership function

$$MOM = \frac{\sum_{x_i \in \max \mu} x_i}{|max\mu|}, \text{where } max\mu = \mu^* = \{x \mid x \in A, \mu(x) = maxim\}$$



- SOM - *Smallest of maximum*
  - Identify the smallest point z (from the aggregated set) that have a maximum membership function

- LOM - *Largest of maximum*
  - Identify the largest point z (from the aggregated set) that have a maximum membership function

Intelligent systems – KBS – Fuzzy systems

Content and design → defuzzification

- Main idea
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  - Inference → obtain some fuzzy regions for each output variable
  - Defuzzification → transform each fuzzy region into a crisp value
- Methods
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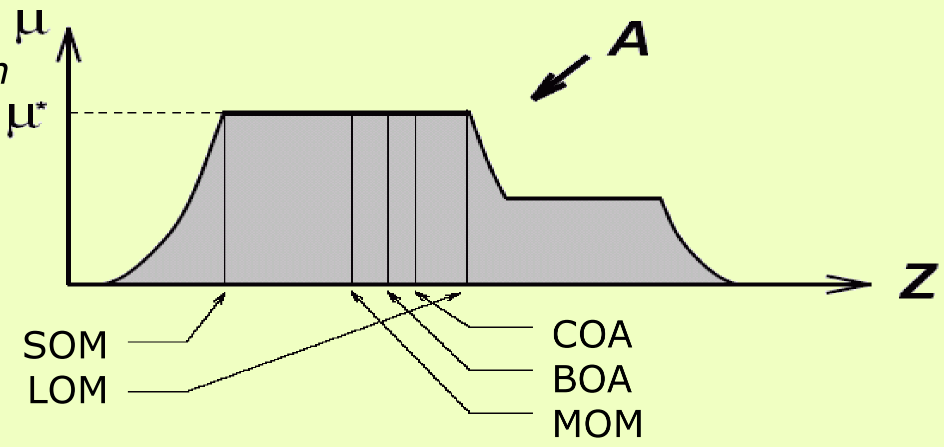

Intelligent systems – KBS – Fuzzy systems

Content and design

- Steps for constructing a fuzzy system
  - Define the inputs and the outputs – by an expert
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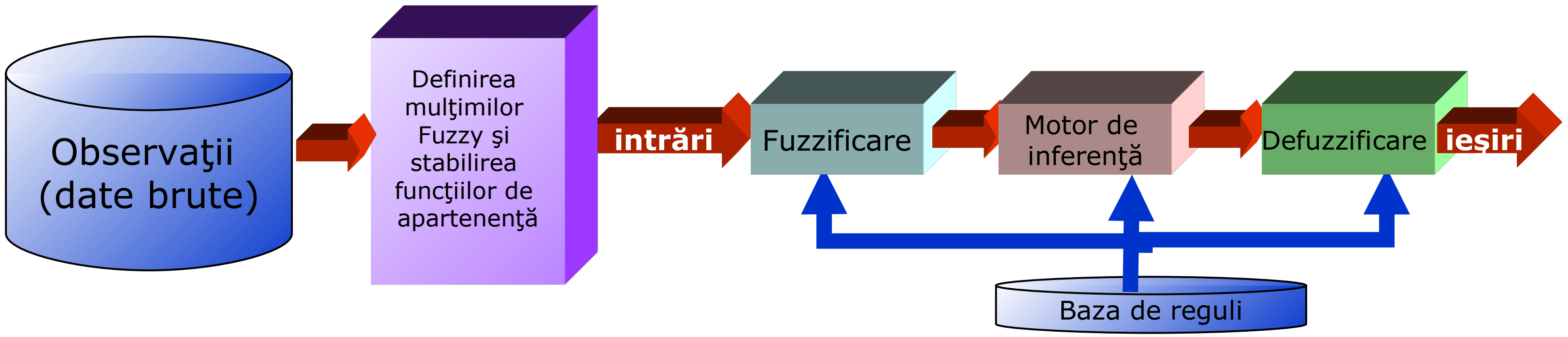
    - Fuzzification of inputs and outputs
      - Fix the fuzzy variables and fuzzy sets based on membership functions
  - Construct a base of rules – by an expert
    - Decision matrix
  - Evaluate the rules
    - Inference – transform the fuzzy inputs into fuzzy outputs by applying all the rules
  - Aggregate the results
  - Defuzzificate the result
  - **Interpret the result**



Intelligent systems – KBS – Fuzzy systems

Content and design

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  - Evaluate the rules
  - Aggregate the results
  - Defuzzificate the result
  - Interpret the result

Domain of expert

Inference engine

Intelligent systems – KBS – Fuzzy systems

- Advantages
  - Imprecision and real-world approximations can be expressed through some rules
  - Easy to understand, to test and to maintain
  - Robustness → can operate when rules are not so clear
  - Require few rules then other KBSs
  - Rules are evaluated in parallel

- Disadvantages
  - Require many simulations and tests
  - Do not automatically learn
  - It is difficult to identify the most correct rules
  - There is not mathematical model

Intelligent systems – KBS – Fuzzy systems

Applications

- Space control
  - Altitude of satellites
  - Setting the planes
- Auto-control
  - Automatic transmission, traffic control, anti-breaking systems
- Business
  - Decision systems, personal evaluation, fond management, market predictions, etc
- Industry
  - Energy exchange control, water purification control
  - pH control, chemical distillation, polymer production, metal composition
- Electronic devices
  - Camera exposure, humidity control. Air conditioner, shower setting
  - Freezer setting
  - Washing machine setting

Intelligent systems – KBS – Fuzzy systems

Applications

- Nourishment
  - Cheese production
- Military
  - Underwater recognition, infrared image recognition, vessel traffic decision
- Navy
  - Automatic drivers, route selection
- Medical
  - Diagnostic systems, pressure control during anesthesia, modeling the neuropathology results of Alzheimer patients
- Robotics
  - Kinematics (arms)

Review

- KBSs
  - Computation systems where knowledge database and inference engine overlap
- KBSs can work
  - In certainty environment
    - LBS
    - RBS
  - In uncertainty environments
    - Bayes systems
      - Rules have associated some probabilities
    - Systems based on certainty factors
      - Fact and rules have associated certainty factors
    - Fuzzy systems
      - Fact have associated degree of membership to some sets

Next lecture

A. Short introduction in Artificial Intelligence (AI)

A. Solving search problems
   A. Definition of search problems
   B. Search strategies
      A. Uninformed search strategies
      B. Informed search strategies
      C. Local search strategies (Hill Climbing, Simulated Annealing, Tabu Search, Evolutionary algorithms, PSO, ACO)
      D. Adversarial search strategies

C. **Intelligent systems**
   A. Rule-based systems in certain environments
   B. Rule-based systems in uncertain environments (Bayes, Fuzzy)
   C. **Learning systems**
      A. **Decision Trees**
      B. **Artificial Neural Networks**
      C. Support Vector Machines
      D. Evolutionary algorithms
   D. Hybrid systems

Next lecture – useful information

- Chapter VI (18 and 19) of *S. Russell, P. Norvig, Artificial Intelligence: A Modern Approach, Prentice Hall, 1995*
- Chapter 8 of *Adrian A. Hopgood, Intelligent Systems for Engineers and Scientists, CRC Press, 2001*
- Chapters 10, 11, 12 and 13 of *C. Groşan, A. Abraham, Intelligent Systems: A Modern Approach, Springer, 2011*
- Chapter V of *D. J. C. MacKey, Information Theory, Inference and Learning Algorithms, Cambridge University Press, 2003*
- Chapters 3 and 4 of *T. M. Mitchell, Machine Learning, McGraw-Hill Science, 1997*

- Presented information have been inspired from different bibliographic sources, but also from past AI lectures taught by:
  - PhD. Assoc. Prof. Mihai Oltean – www.cs.ubbcluj.ro/~moltean
  - PhD. Assoc. Prof. Crina Groşan - www.cs.ubbcluj.ro/~cgrosan
  - PhD. Prof. Horia F. Pop - www.cs.ubbcluj.ro/~hfpop