Dynamic Information Acquisition with Trading Commissions

Hargungeet Singh

New York University

Preliminary

Motivation

- Large **investors** (insurance firms, asset managers, Buy-side) deal with **Brokerage firms** (banks, Sell-side) who charge **trading commissions** for executing trades.
- Along with execution of trades, brokers also provide **research and advice**.
- Analysis of information provision with commissions has been restricted to static settings.

Motivation

- To study dynamic environments, need to take into account:
  1. Broker choosing *when* to acquire information, and
  2. Investor trading behaviour depends on broker's providing information frequently over time.
- **How does this influence Investor-optimal equilibrium** and **Overall portfolio adjustments?**

Motivation

Objective

Study a Dynamic Information generation game in which

1. Investor's (Receiver) payoff depends on the value of an adjustable (portfolio) position,
2. and a Broker (advisor)
   - earns the adjustment costs (commissions),
   - can choose when to generate information

Summary of Results

- I compare two environments:
  1. Investor is able to both adjust portfolio position and acquire information at some cost.
  2. Myopic investor delegates information acquisition to an advisor who is paid the adjustment costs.

  More adjustments and less information is generated in the second case than the first case.

Motivation

Commissions in Practice:

- Brokers often 'bundle' research and execution services in exchange for commissions in 'soft-dollar arrangements'.
- Retail investors can hire 'full-service' brokers who provide advice.
- Related issues and conflicts-of-interest: Excessive churning, or inducements to trade.
- *"soft-dollar arrangements create a conflict ... by inducing the manager to direct trades to broker-dealers that offer research the manager wants ... and encourage overtrading of client portfolios."*
  - Christopher Cox, Former SEC Chairman. Details

Motivating Portfolio Problem

- An **Investor (or Receiver)** has
  - starting wealth W,
  - CARA utility over terminal wealth, and
  - can invest in either a Risky or Riskless asset.

Motivating Portfolio Problem

- An **Investor (or Receiver)** has
  - starting wealth W,
  - CARA utility over terminal wealth, and
  - can invest in either a Risky or Riskless asset.
- Both assets pay once-and-for-all at a common date, occurring randomly at rate ρ.
- **Payoffs**:
  - Risky: Amount invested $y \rightarrow Ry$, where

$$
\begin{aligned}
R \sim & \mathbf{N}(\bar{R}_t, \sigma_y) \\
dR = & \hat{\sigma} dB
\end{aligned}
$$

  - Riskless: Amount invested $W - y \rightarrow r(W - y)$

Motivating Portfolio Model

Under Full Information and without transaction costs:

Investor's problem:

$$V(\bar{R}) = \max_{(y_s)_t^{\infty}} \int_t^{\infty} \rho e^{-\rho s} \mathbb{E}_t(-e^{-\gamma((W-y)r + R_s y_s)}) ds$$

Motivating Portfolio Model

Under Full Information and without transaction costs:

Investor's problem:

$$V(\bar{R}) = \max_{(y_s)_t^\infty} \int_t^\infty \rho e^{-\rho s} \mathbb{E}_t(-e^{-\gamma((W-y)r+R_s y_s)})ds$$

can be written as

$$V(\bar{R}) = \max_{(y_s)_t^\infty} \int_t^\infty \rho e^{-\rho s} \mathbb{E}_t(-e^{-\gamma r W - \gamma(\bar{R}_s - r)y_s + \gamma^2 \sigma_y^2 y_s^2/2})ds$$

Model

Under Transaction costs:

Suppose it costs α to trade between risky or riskless assets.

$$V(\bar{R}, y, W) = \max_{T, y_T} \int_t^T \rho e^{-\rho s} \mathbb{E}_t(-e^{-\gamma r W - \gamma(\bar{R}_s - r)y + \gamma^2 \sigma_y^2 y^2/2}) ds + e^{-\rho T} V(\bar{R}_T, y_T, W - \alpha)$$

Model

To simplify: Normalize $x = y - \frac{\bar{R}_s - r}{\gamma \sigma_y^2}$

Model:

- x is unobservable optimal *position* and follows BM $dx = \sigma dB$.
- x can also be adjusted by Receiver, by paying a fixed cost α.
- In the **Single-Agent Model** considered, the exact value of x_t can be observed by the principal at a fixed cost of c at time t.
- **Receiver:**
  Earning once-and-for-all payoffs at a stochastic rate ρ:

$$-\exp\left\{-\gamma r(W - \text{no. of transactions} \times \alpha) - \left(\frac{\bar{R}_s - r}{\sigma_y}\right)^2 + 2\gamma^2\sigma^2 \left(x_s\right)^2\right\}$$

Model

- In the **Model with Delegation** considered, the sender can generate a public report of the precise value of x_t at fixed cost of c at time t.
- The adjustment cost α is a commission paid to sender.

Single-Agent Model

The Agent's Problem:

$$\max_{x_t^\infty, \{\tau_{ci}, \tau_{\alpha i}\}_{i=0}^\infty} \int_0^\infty \mathbb{E}\left(-e^{-\rho t} \exp\left\{-\gamma r(W - n_{tc}c - n_{t\alpha}\alpha) + 2\gamma^2\sigma^2 x_t^2\right\}\right)$$

such that

$$\begin{aligned}
\{\tau_{ci}\}_{i=0}^\infty &:= \{t \mid \mathbb{V}(x_t|\mathcal{F}_t) = 0\} \\
n_{tc} &:= \text{No. of times information observed till } t \\
\{\tau_{\alpha i}\}_{i=0}^\infty &:= \{t \mid x_{t^-} \neq x_{t^+}\} \\
n_{t\alpha} &:= \text{No. of transactions till } t
\end{aligned}$$

Single-Agent Model

Dynamic Problem:

- Let $v_t := \mathbb{V}(x_t|\mathcal{F}_\sqcup)$
- State : (x_t, v_t).
- Solution separates state-space into three regions:
  1. No-action region
  2. Change region - In which agent changes a_t and incurs cost α
  3. Information region - in which agent observes x_t and incurs cost c.

Single-Agent Model

- In **No-Action** region, Value function satisfies

$$
\begin{aligned}
V(\hat{x}_t, v_t) =& \mathbb{E}\left(-e^{-\rho t} \exp\left\{2\gamma^2\sigma^2 x_t^2\right\}\right) - V_v'\sigma \\
=& -\frac{1}{\sqrt{1-4\gamma^2\sigma^2 v_t}} \exp\left\{\frac{2x^2\gamma^2\sigma^2}{1-4\gamma^2\sigma^2 v_t}\right\} - V_v'\sigma
\end{aligned}
$$

- In **Change** region, the agent changes x_t to 0:

$$
V(\hat{x}_t, v_t) = V(0, v_t) e^{\gamma r \alpha}
$$

- In **Information** region, agent observes $\hat{x}_t$ precisely,

$$
V(\hat{x}_t, v_t) = \mathbb{E}\left(V(\tilde{x}_t, 0) \mid \tilde{x}_t \sim \mathbf{N}(\hat{x}_t, v_t)\right) e^{\gamma r c}
$$

Single-Agent Model

Optimality conditions

- For every $\hat{x}$ in the no-action region, the agent chooses a threshold $\bar{v}$ at which to acquire information.
- This implies

$$\lim_{v \uparrow \bar{v}} V'(\hat{x}, v) = 0$$

- At $v = 0$, let the value of not changing $\hat{x}$ till the time of optimal information acquisition be $V_0(\hat{x})$.
- Then boundaries $\underline{\hat{x}}, \bar{\hat{x}}$ chosen such that

$$\max_{\bar{v}(\hat{x}), \underline{\hat{x}}, \bar{\hat{x}}} V(\hat{x}, v) = \begin{array}{cc} V_0(\hat{x}) & \hat{x} \in (\underline{\hat{x}}, \bar{\hat{x}}) \\ V_0(0) - \alpha & \text{otherwise} \end{array}$$

Single-Agent Model

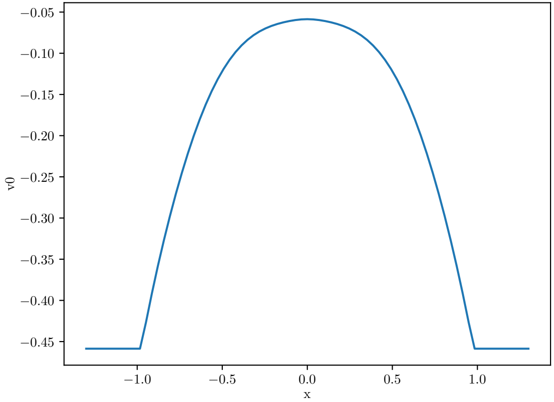

Figure: value function at $v = 0$

Single-Agent Model

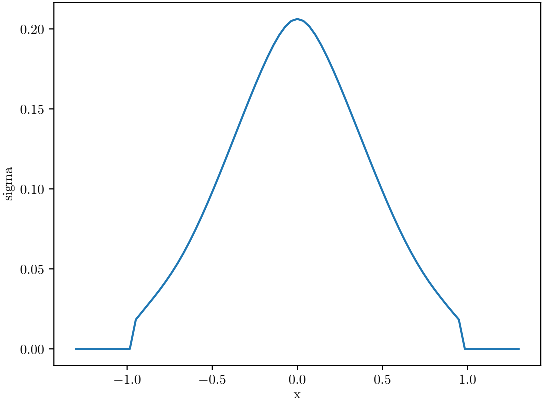

Figure: Information threshold variance values as a function of $\hat{x}$

Model with Delegation

- Consider a second agent - an **advisor** - who is able to generate information for the first agent - the **Principal**
- He can choose to generate a signal to make x_t perfectly observed at a cost c.
- He earns the adjustment cost α.
- Myopic utility.

Model with Delegation

Dynamic Problem:

- Myopic advisor only bears information cost when it is equal to expected adjustment gain.
- State space remains $\{(x_t, v_t)\}$.
- Advisor chooses **Information region** I- in which both agents observe x_t and advisor incurs cost c.
- Principal chooses **change region** C- In which principal changes x_t and incurs cost α

Model with Delegation

As before, for the prinicpal:In **No-Action** region, Value function satisfies

$$V(\hat{x}_t, v_t) = \mathbb{E}\left(-e^{-\rho t}\exp\left\{2\gamma^2\sigma^2 x_t^2\right\}\right) - V_v'\sigma$$
$$= -\frac{1}{\sqrt{1-4\gamma^2\sigma^2 v_t}}\exp\left\{\frac{2x^2\gamma^2\sigma^2}{1-4\gamma^2\sigma^2 v_t}\right\} - V_v'\sigma$$

In **Change** region, the agent changes x_t to 0:

$$V(\hat{x}_t, v_t) = V(0, v_t)e^{\gamma r\alpha}$$

In **Information** region, agent observes $\hat{x}_t$ precisely,

$$V(\hat{x}_t, v_t) = \mathbb{E}\left(V(\tilde{x}_t, 0) \mid \tilde{x}_t \sim \mathbf{N}(\hat{x}_t, v_t)\right) e^{\gamma rc}$$

Single-Agent Model

Optimality conditions

- For every x in the no-action region, the advisor chooses a threshold $\bar{v}$ at which to acquire information.
- This implies
$$\Pr((x,0) \in C) \times \alpha = c$$
- At $v = 0$, let the value of not changing x till the time of optimal information acquisition be $V_0(\hat{x})$.
- Then boundaries $\underline{\hat{x}}, \bar{\hat{x}}$ chosen such that
$$\max_{\bar{v}(\hat{x}),\underline{\hat{x}},\bar{\hat{x}}} V(\hat{x}, v) = \begin{matrix} V_0(\hat{x}) & \hat{x} \in (\underline{\hat{x}}, \bar{\hat{x}}) \\ V_0(0) - \alpha & \text{otherwise} \end{matrix}$$

Model with Delegation

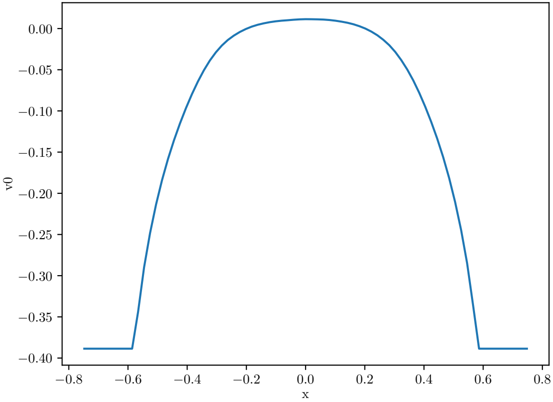

Figure: value function at $v = 0$

Model with Delegation

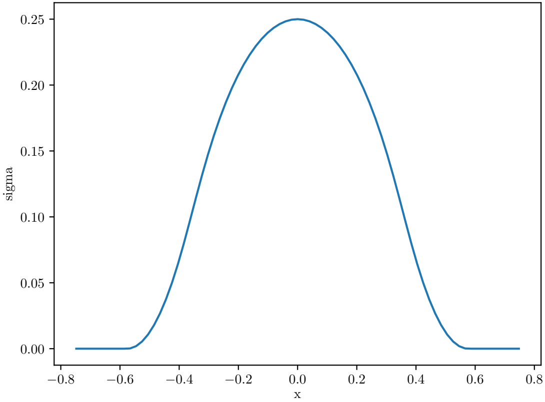

Figure: Information threshold variance values as a function of $\hat{x}$

Overview of Commissions

Source: Goldstein et al. (2009)

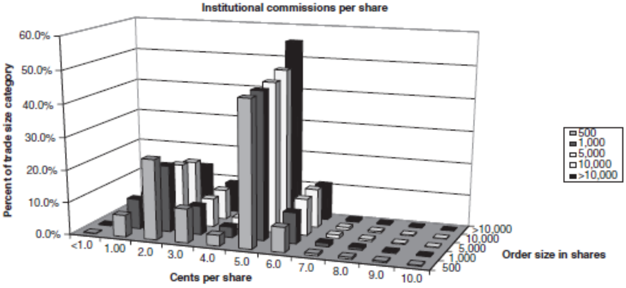

Figure 4
Per share institutional commissions for the NYSE-listed stocks in 1999–2003 by trade size
All commissions per share are rounded to the nearest cent. Zero cents per share commissions are not included, and the distribution is truncated above ten cents per share, losing only a few observations. Overall frequency of trades at each commission price is presented for five trade-size categories.

References I

Goldstein, M. A., P. Irvine, E. Kandel, and Z. Wiener (2009): "Brokerage commissions and institutional trading patterns," *The Review of Financial Studies*, 22, 5175–5212.