Dynamic Communication with Trading Commissions

Hargungeet Singh

New York University

University of Utah, February 7, 2020

Motivation

- Large **investors** (insurance firms, asset managers, Buy-side) deal with **Brokerage firms** (banks, Sell-side) who charge **trading commissions** for executing trades.
- Along with execution of trades, brokers also provide **research and advice**.
- Analysis of information provision with commissions has been restricted to static settings.

Motivation

- To study dynamic environments, need to take into account:
    1. Broker choosing *when* to provide advice,
    2. Investor can trade as a response to stochastic environment *before* receiving advice.

- **How does this influence Investor-optimal equilibrium** and **Overall commissions?**

Motivation

Objective

Study a Dynamic Communication game in which

1. Investor's payoff depends on the value of an adjustable (portfolio) position,
2. and a broker
   - knows the optimal target position ,
   - earns the adjustment costs (commissions),
   - can communicate using cheap-talk messages

Preview of Results

1. In informative equilibria -
    - sender's truth-telling incentives depend on the position, and
    - receiver adjusts position to get information.

Preview of Results

1. In informative equilibria -
   - sender's truth-telling incentives depend on the position, and
   - receiver adjusts position to get information.
2. When the sender can withhold information -
   - set of informative positions becomes smaller,
   - and there is on-path 'delay' in communication.

Motivation

Commissions in Practice:

- Brokers often 'bundle' research and execution services in exchange for commissions in 'soft-dollar arrangements'.
- Retail investors can hire 'full-service' brokers who provide advice.
- Related issues and conflicts-of-interest: Excessive churning, or inducements to trade.
- *"soft-dollar arrangements create a conflict ... by inducing the manager to direct trades to broker-dealers that offer research the manager wants ... and encourage overtrading of client portfolios."*
  - Christopher Cox, Former SEC Chairman. Details

Motivation

Theoretical Approach:

A Dynamic portfolio allocation problem -

The Basics:

- Risk Averse investor with wealth W,
- invests W in 2 assets- risky and riskless,
- and is allowed to trade continuously over time,
- as Price of risky asset follows a stochastic process.

The Result:

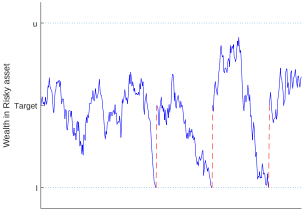
Invest a target amount in risky asset -
trade continuously as price of risky asset evolves

Motivation

Add fixed transaction costs-

Investor uses an (s, S) **policy** -

only trading when the asset value leaves an interval around an optimal benchmark or target.



Benchmark depends on risk-return trade-off between assets.

Motivation

Motivating question:

What if **a Broker**:

- was privately informed of optimal asset mix,
- earned the transaction cost as commission
- could communicate but without verifiable evidence?

Related Literature

- **Dynamic Communication or Information Provision :** Madsen (2019), Renault et al. (2013), Ivanov (2015), Golosov et al. (2014) and Grenadier et al. (2016)

  Contribution: Focus on role of commissions

- **Advice and Information with Commissions :** Inderst and Ottaviani (2012), Lipnowski and Ravid (2018), Chakraborty and Harbaugh (2010) Brennan and Chordia (1993), Admati and Pfleiderer (1990)

  Contribution: Extend to Dynamic setting with delay option

- **Dynamic Portfolio Allocation:** Merton (1969), Davis and Norman (1990), Liu (2004)

  Contribution: Add Asymmetric Information

1. Motivation
2. Model
3. Asymmetric Information
4. Result
5. Extensions
   - Proportional Commissions
   - Stochastic θ
6. Conclusion

Motivating Portfolio Problem

- An **Investor (or Receiver)** has
  - starting wealth W,
  - CARA utility over terminal wealth, and
  - can invest in either a Risky or Riskless asset.

Motivating Portfolio Problem

- An **Investor (or Receiver)** has
  - starting wealth W,
  - CARA utility over terminal wealth, and
  - can invest in either a Risky or Riskless asset.
- Both assets pay once-and-for-all at a common date, occurring randomly at rate ρ.
- **Payoffs**:
  - Risky: Amount invested $y \rightarrow (R + \hat{\theta})y$, where

$$R \sim \mathbf{N}(\bar{R}_t, \sigma_y)$$
$$d\bar{R} = \hat{\sigma} dB$$
$$\hat{\theta} \in \{\hat{\theta}_H, \hat{\theta}_L\}$$

  - Riskless: Amount invested $W - y \rightarrow r(W - y)$

Motivating Portfolio Model

Under Full Information and without transaction costs:

Investor's problem:

$$V(\bar{R}) = \max_{(y_s)_t^\infty} \int_t^\infty \rho e^{-\rho s} \mathbb{E}_t(-e^{-\gamma((W-y)r+(R_s+\hat{\theta})y_s)})ds$$

Motivating Portfolio Model

Under Full Information and without transaction costs:

Investor's problem:

$$V(\bar{R}) = \max_{(y_s)_t^\infty} \int_t^\infty \rho e^{-\rho s} \mathbb{E}_t(-e^{-\gamma((W-y)r+(R_s+\hat{\theta})y_s)}) ds$$

can be written as

$$V(\bar{R}) = \max_{(y_s)_t^\infty} \int_t^\infty \rho e^{-\rho s} \mathbb{E}_t(-e^{-\gamma r W - \gamma(\bar{R}_s+\hat{\theta}-r)y_s + \gamma^2 \sigma_y^2 y_s^2/2}) ds$$

The optimal policy is the same as if minimizing

$$-\gamma r W - \left(\frac{\bar{R}_s + \hat{\theta} - r}{\sigma_y}\right)^2 + 2\gamma^2 \sigma_y^2 \left(y_s - \frac{\bar{R}_s - r}{\gamma \sigma_y^2} - \frac{\hat{\theta}}{\gamma \sigma_y^2}\right)^2$$

Model

Under Transaction costs:

Suppose it costs α to trade between risky or riskless assets.

$$V(\bar{R}, y, W) = \max_{T, y_T} \int_t^T \rho e^{-\rho s} \mathbb{E}_t(-e^{-\gamma r W - \gamma(\bar{R}_s + \hat{\theta} - r)y + \gamma^2 \sigma_y^2 y^2/2}) ds + e^{-\rho T} V(\bar{R}_T, y_T, W - \alpha)$$

The optimal policy is same as minimizing

$$\mathbb{E}_t \left(-\gamma r (W - \text{no. of transactions} \times \alpha) - \left(\frac{\bar{R}_s + \hat{\theta} - r}{\sigma_y} \right)^2 + 2\gamma^2 \sigma_y^2 \left(y_s - \frac{\bar{R}_s - r}{\gamma \sigma_y^2} - \frac{\hat{\theta}}{\gamma \sigma_y^2} \right)^2 \right)$$

Model

To simplify: Normalize $x = y - \frac{\bar{R}_s - r}{\gamma\sigma_y^2}$ and $\theta = \frac{\hat{\theta}}{\gamma\sigma_y^2}$

Model:

- x is observable *position* and follows BM $dx = \sigma dB$.
- x can also be adjusted by Receiver, by paying a fixed cost α.
- θ - a state unobserved by Receiver.
- **Receiver:**
  Earning once-and-for-all payoffs at a stochastic rate ρ:

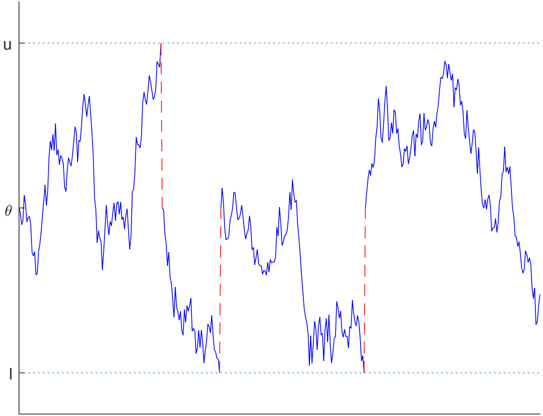
$$-(x_t - \theta)^2$$

Model

- **Sender** is privately informed of $\theta \in \{\theta_H, \theta_L\}$.
- Common prior: Prob$(\theta = \theta_H) = p$
- The adjustment cost is a commission paid to sender.
- The Sender can reveal θ to Receiver through cheap-talk messages.

Full Information

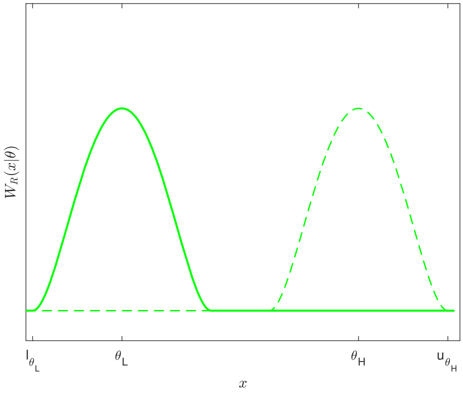
If θ were known, the Receiver follows an (s, S) policy -



Full Information

If θ were known, the Receiver follows an (s, S) policy -

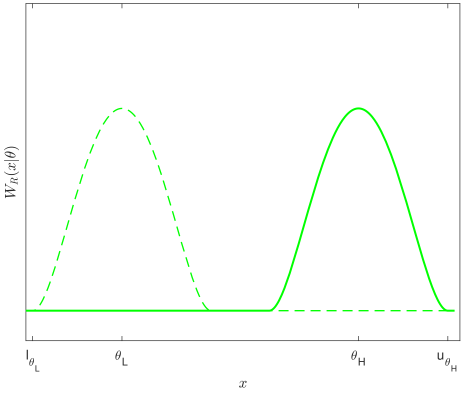
Receiver's value function (with optimal boundaries (l_θ, u_θ))



Full Information

If θ were known, the Receiver follows an (s, S) policy -

Receiver's value function (with optimal boundaries (l_θ, u_θ))



Full Information

If θ were known, the Receiver follows an (s, S) policy -

Receiver's value function: with optimal boundaries (l_θ, u_θ)

HJB Equation:

$$\rho W_R(x|\theta) = -(x-\theta)^2 + \frac{1}{2}\sigma^2 W_R''(x\theta)$$

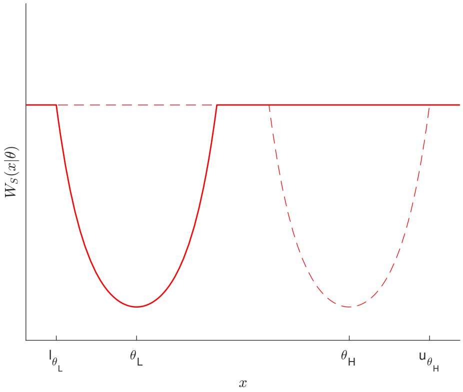
for $x \in (l_\theta, u_\theta)$.

Boundary conditions:

$$\begin{aligned} W_R(l_\theta|\theta) =& W_R(u_\theta|\theta) = W_R(\theta|\theta) - \alpha \\ \lim_{x \downarrow l_\theta} W_R'(x|\theta) =& \lim_{x \uparrow u_\theta} W_R'(x|\theta) = W_R'(x|\theta)|_{x=\theta} = 0 \end{aligned}$$

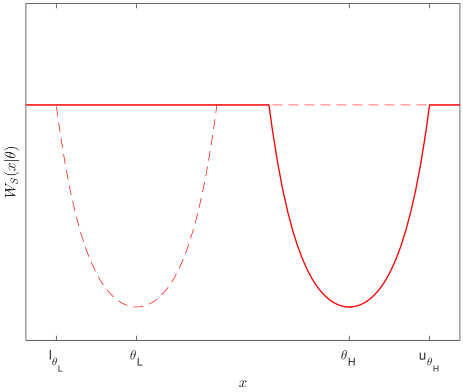
Full Information

Sender's value function: Sender is a passive agent who earns commissions.



Full Information

Sender's value function: Sender is a passive agent who earns commissions.



Full Information

Sender's value function: Sender is a passive agent who earns commissions.:

$W_S(x|\theta)$ satisfies the *HJB Equation*

$$\rho W_S(x|\theta) = \frac{1}{2}\sigma^2 W_S''(x|\theta)$$

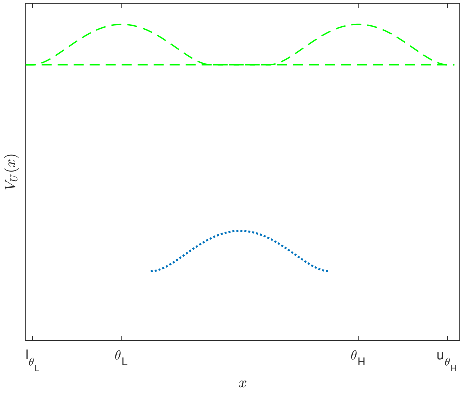
for $x \in (l_\theta, u_\theta)$, with the boundary conditions

$$W_S(l_\theta|\theta) = W_S(u_\theta|\theta) = W_S(\theta|\theta) + \alpha$$

Asymmetric Information

Without Sender (or the Babbling eq'm):

Receiver still follows (s, S) policy around target $\mathbb{E}(\theta)$.

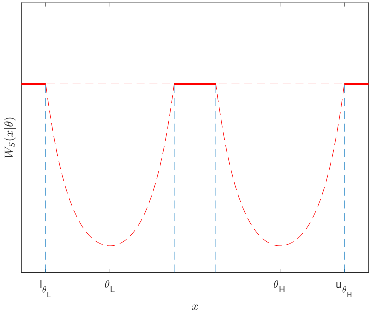


Asymmetric Information

Time-0 communication: Suppose Sender was allowed to communicate only at time 0, and never again.

Truthtelling is Incentive Compatible if

$$x_0 \in \bar{R} := \{x \mid \text{Sender's value equal from both reports}\}$$



Strategies

- Consider only Pure strategies.
- **Sender's Strategy:** Pure Markov Strategy in messages

$$(p, x, \theta) \to m \in M$$

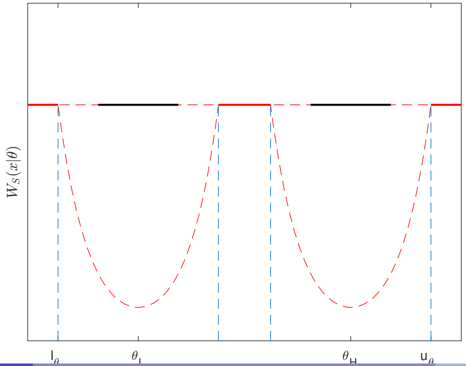
- Pure strategy implies: belief remains at p and immediately jump to 0 or 1.
- The game can be treated as if it is divided into two stages - Uninformed and informed.

Strategies

Receiver's strategy

Chooses states at which to exercise control and 'change' x, and the value which she changes to - B and s respectively;

as a response to states where she believes truth will be revealed -R.



Receiver Optimal Equilibrium

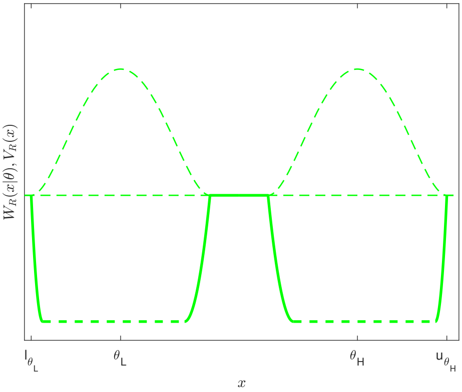
Receiver-optimal Equilibrium

- $R = \bar{R}$,
- $\forall x \in R, \forall m \in M$, updated posterior belief $\hat{p}(p, x, m) = 0$ or 1.

I.e. When truth-telling is incentive-compatible and the Sender can only choose informative messages.

Receiver Optimal Equilibrium

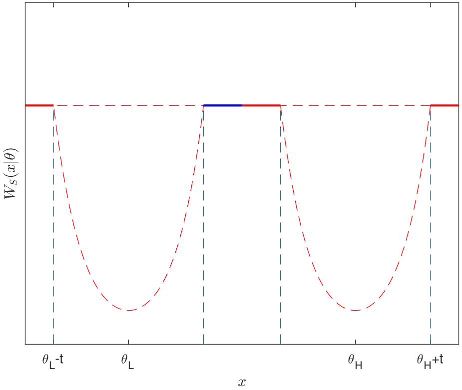
In the **Receiver-optimal Equilibrium**:



Equilibria with Delay

Focus on Equilibria where Sender is allowed to send no information.

Sender's Strategy: choose a set of states $R \subseteq \bar{R}$ at which to credibly reveal the truth.



Equilibria with Delay

Equilibrium

A **Markov Perfect Equilibrium in Pure strategies with Delay-option** is the collection of sets B, R and the target value s such that

- they solve the Sender and Receiver's problems in the uninformed stage, RP
- $R \subseteq \bar{R}$,
- $\exists\, m \in M$ such that $\hat{p}(p, m) = p$, for all $p \in [0, 1]$.

Result

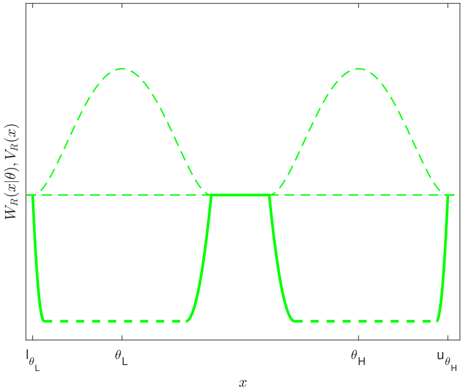
Proposition

- For intermediate $\theta_H - \theta_L$ i.e. when info is valuable enough, $R \subsetneq \bar{R}$:
  There is on-path delay of communication relative to one-shot communication.
- For large $\theta_H - \theta_L$, $R = \emptyset$, and the only pure-strategy MPE is the babbling equilibrium.

Sketch of proof

Details

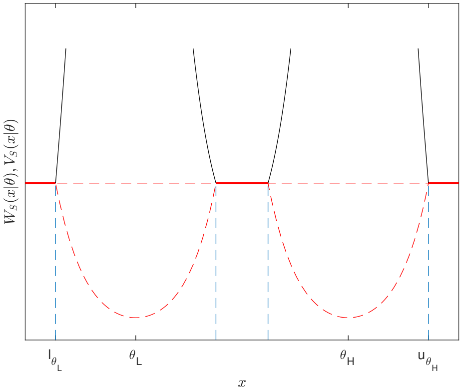
Suppose $R = \bar{R}$. The Receiver's value function:



Sketch of proof

Details

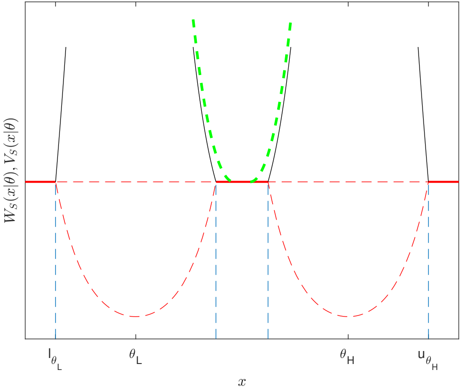
Sender has a profitable deviation from delaying.



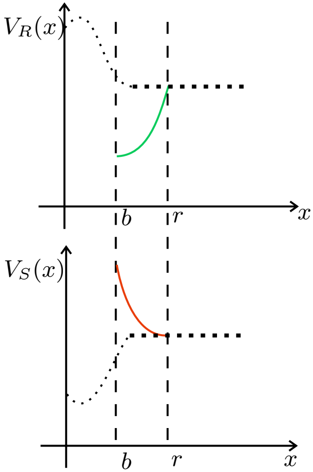
Sketch of proof

Details

Sender has a profitable deviation from delaying.



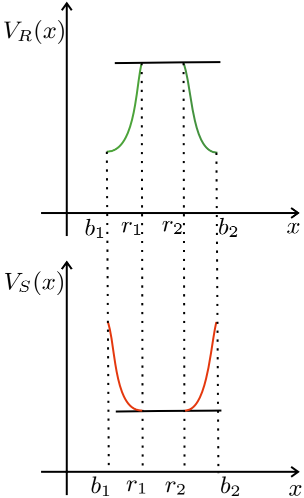
Equilibrium



In Delay equilibrium with maximal set R,

- the sender's value function must be 'smooth pasted' at $r \in \partial R$,
- the receiver's value function must be 'smooth pasted' at $b \in \partial B$.

Equilibrium



In Delay equilibrium with maximal set R,

- the sender's value function must be 'smooth pasted' at $r \in \partial R$,
- the receiver's value function must be 'smooth pasted' at $b \in \partial B$.

Equilibrium

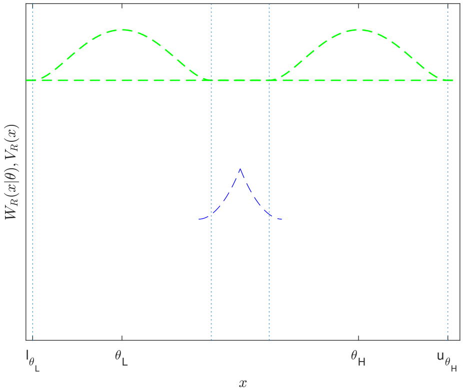
Equilibrium with Sender-mixing :

Proposition

- In the class of equilibria where sender mixes between delay and sending perfectly informative message (no partial revelation),
- in the receiver-optimal mixed strategy equilibrium,
- at $x = \mathbb{E}(\theta) \in \bar{R}$, sender informs with probability ν.

Equilibrium

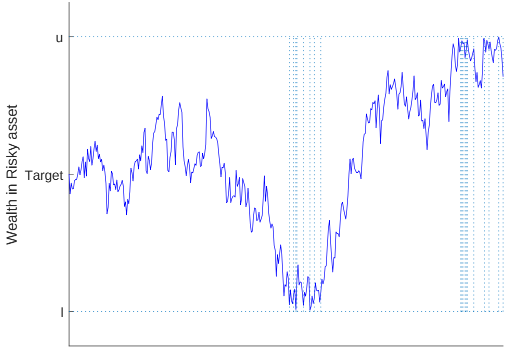
Equilibrium with Sender-mixing :



1. Motivation
2. Model
3. Asymmetric Information
4. Result
5. Extensions
   - Proportional Commissions
   - Stochastic θ
6. Conclusion

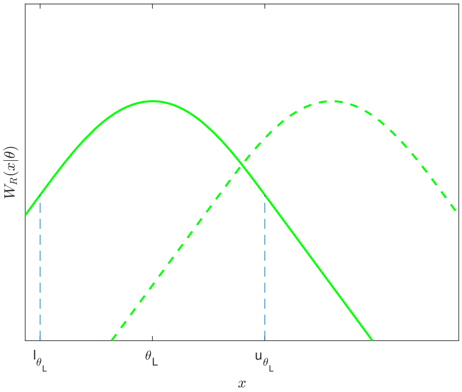
Proportional Commissions

- Suppose change of x_1 to x_2 entails cost $\alpha \times |x_2 - x_1|$.
- *Full Information:*
  Optimal Policy: If Receiver knows θ, she chooses **reflecting boundaries** l_θ and u_θ



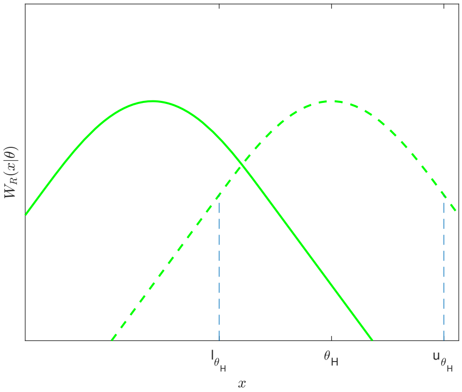
Proportional Commissions

- *Full Information:*
  Receiver's Value Function:



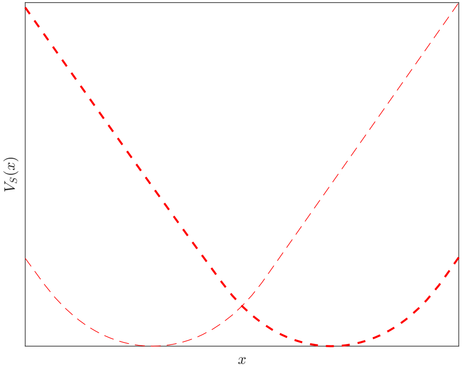
Proportional Commissions

- *Full Information:*
  Receiver's Value Function:



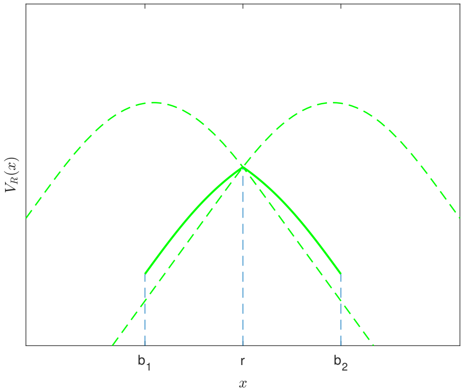
Proportional Commissions

- *Full Information:*
  Sender's Value Function:



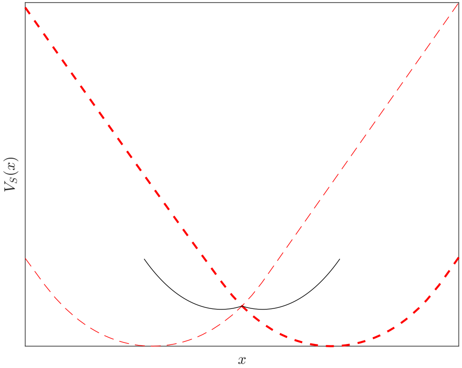
Proportional Commissions

- *Asymmetric Information:*
  Receiver's Value Function:



Proportional Commissions

- *Asymmetric Information:*
  Sender's Value Function:



Proportional Commissions

Proposition

For large $\theta_H - \theta_L$ and under proportional commissions, the only pure-strategy equilibrium with delay is the Babbling equilibrium.

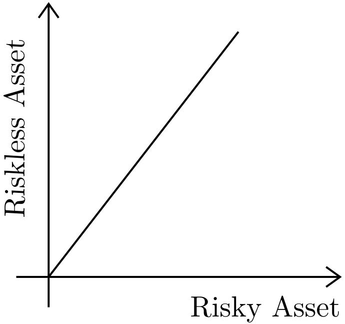
Portfolio Example Re-visited

(simplified Merton(1969))

- Investor with wealth W_t,
- maximizes $u(W_T) = \frac{W_T^{1-\gamma}}{1-\gamma}$,
- Risky Asset: $\frac{dP}{P} = \mu dt + \sigma dB$
- Risk-less asset - constant growth rate r.

Result: Invest **constant proportion of wealth** in risky asset.

Details

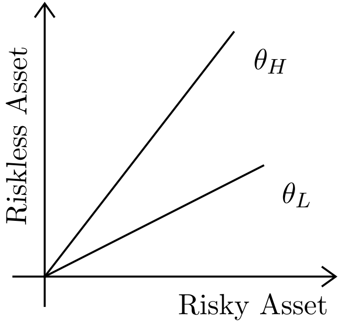


Portfolio Example Re-visited

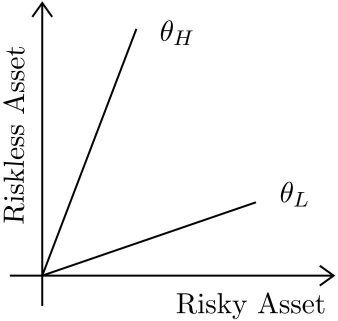
(simplified Merton(1969))

Result: Invest **constant proportion of wealth** in risky asset.

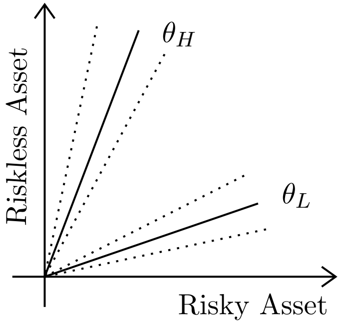
- Suppose with unknown probability $\theta \in \{\theta_L, \theta_H\}$, value of asset is revealed to be zero and P drops to 0.
- Optimal proportion in Risky Asset is decreasing in θ.



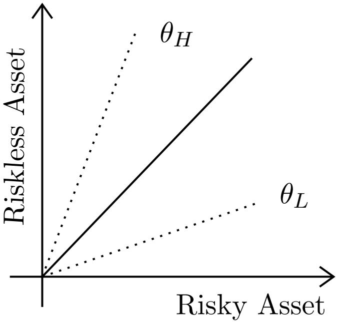
Portfolio Example Re-visited



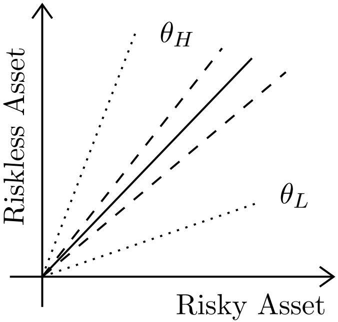
Portfolio Example Re-visited



Portfolio Example Re-visited



Portfolio Example Re-visited



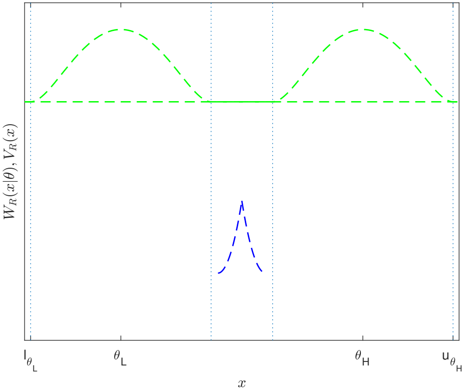
Stochastic θ

- At rate ν, θ is re-drawn i.i.d. from $\{\theta_H, \theta_L\}$, with $\text{Prob}(\theta = \theta_H) = p$.
- Receiver observes the shock but *not* the new value of θ.
- Sender observes the new value.
- **Note:** If Receiver is informed, she becomes uninformed at rate ν.
- *HJB Equation*:

$$\rho W_R(x|\theta) = -(x-\theta)^2 + \nu(V_R(x) - W_R(x|\theta)) + \frac{1}{2}\sigma^2 W_R''(x\theta)$$

Stochastic θ

Receiver's Value Functions:



Conclusion

- Study a Modified Dynamic Portfolio Allocation model where an investor does not know optimal portfolio as a target of an (s, S) policy.
- Information content of broker's report depends on investor's portfolio position.
- Receiver optimally adjusts portfolio position to obtain accurate information.
- When information is valuable, truth-telling in dynamic case is delayed and increased adjustment is induced endogenously.

Overview of Commissions

Source: Goldstein et al. (2009)

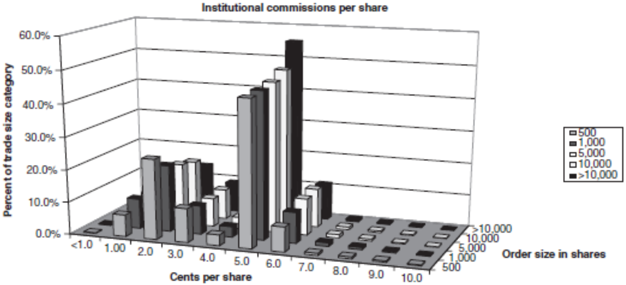


Figure 4
Per share institutional commissions for the NYSE-listed stocks in 1999–2003 by trade size
All commissions per share are rounded to the nearest cent. Zero cents per share commissions are not included, and the distribution is truncated above ten cents per share, losing only a few observations. Overall frequency of trades at each commission price is presented for five trade-size categories.

Receiver's Problem

Given a set of states R_{R} where the Receiver believes truth is credibly revealed, the *Receiver's problem* is to find a set $B \subseteq \mathbb{R}$ and target value s, such that they solve

$$V_R(x) = \sup_{B,s} \mathbb{E}_x[\int_0^T e^{-\rho t} u(x_t)dt + e^{-\rho T}\mathrm{Prob}(x_T \in R_{\mathrm{R}}) W_R(x_T|\theta) + \\ e^{-\rho T}\mathrm{Prob}(x_T \in B)(V_R(s) - \alpha)]$$

where, $T = \tau_R \wedge \tau_B$ and

$$u(x) = -p(x - \theta_H)^2 - (1-p)(x - \theta_L)^2$$

Sender's Problem

Given the sets R_R - where the receiver believes truth is credibly revealed - B and s, the *Sender's problem* is to find a set $R_\mathrm{S} \subseteq R_R$, such that it solves

$$V_S(x) = \sup_{R_\mathrm{S} \subseteq R_R} \mathbb{E}_x[e^{-\rho T}\mathrm{Prob}(x_T \in R_\mathrm{S})W_S(x_T|\theta) + e^{-\rho T}\mathrm{Prob}(x_T \in B)(V_S(s) + \alpha)]$$

Proof of Proposition

- Consider the interval $[s, b]$ where $[s, b] \cap B = b$.
- If the sender is to choose a threshold $r \in [s, b]$ when to reveal the truth, and end the uninformed stage,
$$\lim_{x \downarrow r} V_S'(x|\theta)|_{x=r} \leq 0$$
where it holds with inequality if $[r, b] \cap \bar{R} = r$.
- The above implies $b - r > \psi > 0$ for some $\psi > 0$ independent of θ_H or θ_L.

Proof of Proposition

- Case: $\theta_H - \theta_L$ large enough for $u_{\theta_L} < l_{\theta_H}$.
- Let $W_R(x|\theta) = w$ for $x \in [\theta_L + t, \theta_H - t]$.
- Now consider $[u_{\theta_L}, l_{\theta_H}] \subset \bar{R}$.
- Suppose, for contradiction, $[u_{\theta_L}, l_{\theta_H}] \subset R_B$. Receiver chooses $b > \theta_H - t$ at which the exercise control and adjust x.
- Consider the policy $b = l_{\theta_H} + \psi$ and the value to the receiver. For large $\theta_H - \theta_L$, $\lim_{x \uparrow b} V_R(x) > 0$, indicating optimal $b < l_{\theta_H} + \psi$.

Portfolio Example Re-visited

(simplified Merton(1969))

$$V(W_t) = \max_x \int_t^\infty e^{-\rho s} \mathbf{E}[u(W_s)]ds$$

s.t. $dW = (rW + (\mu - r)xW)dt + \sigma xW dB.$[1]

HJB Equation:

$$\rho V(W) = \max_x u(W) + (rW + (\mu - r)xW) V'(W) + \frac{1}{2}\sigma^2 x^2 W^2 V''(W)$$

[1] When the value function is well-defined, i.e., when $\rho > (1-\gamma)\left(r + (\mu - r)^2/\sigma^2\gamma\right)$

References I

Admati, A. R. and P. Pfleiderer (1990): "Direct and indirect sale of information," *Econometrica: Journal of the Econometric Society*, 901–928.

Brennan, M. J. and T. Chordia (1993): "Brokerage commission schedules," *The Journal of Finance*, 48, 1379–1402.

Chakraborty, A. and R. Harbaugh (2010): "Persuasion by cheap talk," *American Economic Review*, 100, 2361–82.

Davis, M. H. and A. R. Norman (1990): "Portfolio selection with transaction costs," *Mathematics of operations research*, 15, 676–713.

Goldstein, M. A., P. Irvine, E. Kandel, and Z. Wiener (2009): "Brokerage commissions and institutional trading patterns," *The Review of Financial Studies*, 22, 5175–5212.

Golosov, M., V. Skreta, A. Tsyvinski, and A. Wilson (2014): "Dynamic strategic information transmission," *Journal of Economic Theory*, 151, 304–341.

References II

Grenadier, S. R., A. Malenko, and N. Malenko (2016): "Timing decisions in organizations: Communication and authority in a dynamic environment," *American Economic Review*, 106, 2552–81.

Inderst, R. and M. Ottaviani (2012): "Competition through commissions and kickbacks," *American Economic Review*, 102, 780–809.

Ivanov, M. (2015): "Dynamic information revelation in cheap talk," *The BE Journal of Theoretical Economics*, 15, 251–275.

Lipnowski, E. and D. Ravid (2018): "Cheap talk with transparent motives," *Available at SSRN 2941601*.

Liu, H. (2004): "Optimal consumption and investment with transaction costs and multiple risky assets," *The Journal of Finance*, 59, 289–338.

Madsen, E. (2019): "Optimal project termination with an informed agent," .

References III

Merton, R. C. (1969): "Lifetime portfolio selection under uncertainty: The continuous-time case," *The review of Economics and Statistics*, 247–257.

Renault, J., E. Solan, and N. Vieille (2013): "Dynamic sender–receiver games," *Journal of Economic Theory*, 148, 502–534.