

Test Function 4 :  $f(x_1, x_2) = (x_1 + 2x_2 - 7)^2 + (2x_1 + x_2 - 5)^2$   $f(k)$ : func. value at  $x(k)$   
 $g(k)$ : grad. of  $f(x)$  at  $x(k)$   
 Minimum :  $(1, 3)^T$  Starting Point :  $[2, 2]^T$

Steepest Descent	Convergence Criterion: $\ g(k)\  < 0.01 \rightarrow \text{STOP}$ BFGS	Fletcher Reeves
Iteration # : 3	Iteration # : 2	Iteration # : 2
$x_{\text{final}} = [0.99, 3.00]^T$	$x_{\text{final}} = [0.99, 3.00]^T$	$x_{\text{final}} = [0.99, 3.00]^T$
$f_{\text{final}} = 0.0001$	$f_{\text{final}} = 0.0001$	$f_{\text{final}} = 0.0001$

Conjug. Gradient: $f(k) - f(k+1) < 10^{-6}$
Iteration # : 5
$x_{\text{final}} = [1.00, 2.99]^T$
$f_{\text{final}} = 0.000$

Test Function - 3 :  $f(x_1, x_2) = (x_1)^2 [x_1 - 10]^2 + [(x_2)^2 \times (x_2 - 10)^2]$   $f(x)$  true value @  $x(k)$

Critical Points : (5,0), (5,10), (5,5), (10,0), (10,10), (10,5), (0,0), (0,10), (0,5)  $g(k) = \text{grad } f(x(k))$

Steepest Descent

BFGS

Fletcher Reeves

Starting Point : (2,2)<sup>T</sup>

Convergence Criterion :

$f(k) - f(k+1) < 10^{-6} \rightarrow \text{STOP}$

Iteration # : 2

Iteration # : 1

Iteration # : 5

$x_{\text{final}} = [9.10, 9.10]$

$x_{\text{final}} = [9.10, 9.10]$

$x_{\text{final}} = [9.99, 9.99]$

$f_{x_{\text{final}}} = 133.34$

$f_{x_{\text{final}}} = 133.34$

$f_{x_{\text{final}}} = 0.0000$

$\|g(k)\| < 0.01 \rightarrow \text{STOP}$

Iteration # :

Iteration # : 31

Iteration # : 1

Iteration # : 6

$x_{\text{final}} = [0.00, 9.99]$

$x_{\text{final}} = [9.10, 9.10]$

$x_{\text{final}} = [9.99, 9.99]$

$f_{x_{\text{final}}} = 0.0000$

$f_{x_{\text{final}}} = 133.34$

$f_{x_{\text{final}}} = [0.0000]$

Test Function - 2 :  $f(x_1, x_2, \dots, x_{10}) = \sum_{i=1}^9 [100 \times (x(i+1) - x(i)^2)^2 + (1 - x(i))^2]$   $f(k)$ : function value at  $x(k)$   
 MINIMUM :  $(1, 1, 1, 1, 1, 1, 1, 1, 1, 1)^T$  [St. point :  $[2, 2, 2, 2, 2, 2, 2, 2, 2, 2]^T$ ]  $g(k)$ : Gradient of  $f(x)$  at  $x^k$   $z(k)$

Steepest Descent	BFGS	Fletcher - Reeves
Starting point : $[2, 2, 2, 2, 2, 2, 2, 2, 2, 2]^T$ Convergence Criterion - I : $\ g(k)\  \leq 0.01 \rightarrow \text{STOP}$ Iteration # : 1419 $X_{\text{final}} = [1.00, 1.00, 1.00, 1.00, 1.00, 1.00, 1.01, 1.04, 1.08, 1.17]^T$ $f(X_{\text{final}}) = 0.0090$	Convg. Criterion - I : $\ g(k)\  \leq 0.01 \rightarrow \text{STOP}$ Iteration # : 37 $X_{\text{final}} = [1.00, 1.00, 1.00, 1.00, 1.00, 1.00, 1.00, 1.01, 1.03]^T$ $f(X_{\text{final}}) = 0.0003$ PLEASE CONSIDER CRITERION - II on (precision - 1)	Convg. Criterion - I : $\ g(k)\  \leq 0.01 \rightarrow \text{STOP}$ Iteration # : 1022 $X_{\text{final}} = [0.86, 0.83, \text{NaN}, \text{NaN}, \text{NaN}, 0.92, 0.96, 1.15, 1.43, 2.05]^T$
Criteria-II : $f(k+1) \rightarrow f(k) \rightarrow \text{STOP}$ $f(k) - f(k+1) < 10^{-6}$ Iteration # : 1864 $X_{\text{final}} = [1.00, 1.00, 1.00, 1.00, 1.00, 1.00, 1.01, 1.02, 1.05]^T$ $f(X_{\text{final}}) = 0.0008$	Criteria-II : $f(k) - f(k+1) < 10^{-6}$ Iteration # : 48 $X_{\text{final}} = [1.00, 1.00, 1.00, 1.00, 1.00, 1.00, 1.00, 1.00, 1.00]^T$ $f(X_{\text{final}}) = 0.0000$	Criteria-II : $f(k) - f(k+1) < 10^{-6}$ Iteration # : 4 $X_{\text{final}} = [0.86, 0.83, 0.87, 0.91, 0.92, 0.92, 0.96, 1.15, 1.43, 2.05]^T$ $f(X_{\text{final}}) = 14.897$



Test - Function 1 :  $f(x_1, x_2, \dots, x_n) = \sum_{i=1}^{10} i \times x(i)^2$  ; Minimum =  $[0, 0, \dots, 0]^T$   $f(k)$  : funct. value @  $x(k)$   
 $g(k)$  : grad of  $f(x)$  at  $x(k)$

Starting Pt :  $[2, 2, \dots, 2]^T$

Steepest Descent

BFGS

Fletcher - Reeves

Convg. Cnd - I :  $\|g(k)\| < 0.01 \rightarrow \text{STOP}$

Iteration # : 15

$x_{\text{final}} =$

$[0.06, -0.00, 0.00, 0.00, -0.00, -0.00, 0.00, 0.00, -0.00, 0.00, 0.05]$

$f(x_{\text{final}}) = 0.0011$

Iteration # : 10

$x_{\text{final}} =$

$[0.00, 0.00, -0.00, -0.00, -0.00, -0.00, 0.00, 0.00, 0.00, 0.00]$

$f(x_{\text{final}}) = 0.0002$

Iteration # : 1025

$x_{\text{final}} =$

$[NaN, NaN, NaN, NaN, -0.27, -0.24, -0.18, -0.14, -0.13, -0.19]$

PLEASE  
 $\rightarrow$  CONSIDER THIS (PRIORITY 1)

Convg. Cnd - II :  $|f(k) - f(k+1)| < 10^{-6} \rightarrow \text{STOP}$

Iteration # : 25

$x_{\text{final}} =$

$[0.09, 0.00, \dots, 0.00]$

$f(x_{\text{final}}) = 0.0000$

Iteration # : 15

$x_{\text{final}} =$

$[0.00, -0.00, 0.00, -0.00, -0.00, 0.00, 0.00, -0.00, -0.00, 0.00]$

$f(x_{\text{final}}) = 0.0000$

Iteration # : 7

$x_{\text{final}} =$

$[0.99, 0.34, -0.03, 0.22, -0.27, -0.24, -0.18, -0.14, -0.13, -0.19]$

$f(x_{\text{final}}) = 3.1186$