


```

In [10]: import numpy as np

def gradient_descent(f, initial_point, learning_rate, iterations):
    point = initial_point.copy()
    history = []

    for _ in range(iterations):
        # Compute gradient of the function at the current point
        gradient = np.array([2 * point[0], 2 * point[1]])

        # Update point using gradient descent update rule
        point -= learning_rate * gradient

        # Calculate the value of the function at the updated point
        value = f(point[0], point[1])

        # Save the history of points and function values
        history.append((point.copy(), value))

    print("Values of points and objective function value are: {} and {}".format(
        history, value))

    return point, history # Make sure this line is inside the function

def objective_function(x, y):
    return x**2 + y**2 + 4

# Define initial point, learning rate, and number of iterations
initial_point = np.array([1.0, 1.0])
learning_rate = 0.1
iterations = 100

# Run gradient descent
minimum_point, history = gradient_descent(objective_function, initial_point, learning_rate, iterations)

print("Minimum point (x, y):", minimum_point)
print("Minimum value of the function:", objective_function(*minimum_point))

```

Values of points and objective function value are: [0.8 0.8] and 5.28
 Values of points and objective function value are: [0.64 0.64] and 4.8192
 Values of points and objective function value are: [0.512 0.512] and 4.524288
 Values of points and objective function value are: [0.4096 0.4096] and 4.3355
 4432
 Values of points and objective function value are: [0.32768 0.32768] and 4.21
 47483648
 Values of points and objective function value are: [0.262144 0.262144] and 4.
 137438953472
 Values of points and objective function value are: [0.2097152 0.2097152] and
 4.08796093022208
 Values of points and objective function value are: [0.16777216 0.16777216] an
 d 4.056294995342131
 Values of points and objective function value are: [0.13421773 0.13421773] an
 d 4.036028797018964
 Values of points and objective function value are: [0.10737418 0.10737418] an
 d 4.023058430092137
 Values of points and objective function value are: [0.08589935 0.08589935] an
 d 4.014757395258967
 Values of points and objective function value are: [0.06871948 0.06871948] an
 d 4.009444732965739
 Values of points and objective function value are: [0.05497558 0.05497558] an
 d 4.006044629098073
 Values of points and objective function value are: [0.04398047 0.04398047] an
 d 4.003868562622767
 Values of points and objective function value are: [0.03518437 0.03518437] an
 d 4.0024758800785705
 Values of points and objective function value are: [0.0281475 0.0281475] and
 4.001584563250285
 Values of points and objective function value are: [0.022518 0.022518] and 4.
 001014120480183
 Values of points and objective function value are: [0.0180144 0.0180144] and
 4.0006490371073165
 Values of points and objective function value are: [0.01441152 0.01441152] an
 d 4.000415383748683
 Values of points and objective function value are: [0.01152922 0.01152922] an
 d 4.000265845599157
 Values of points and objective function value are: [0.00922337 0.00922337] an
 d 4.00017014118346
 Values of points and objective function value are: [0.0073787 0.0073787] and
 4.0001088903574145
 Values of points and objective function value are: [0.00590296 0.00590296] an
 d 4.000069689828745
 Values of points and objective function value are: [0.00472237 0.00472237] an
 d 4.000044601490397
 Values of points and objective function value are: [0.00377789 0.00377789] an
 d 4.000028544953854
 Values of points and objective function value are: [0.00302231 0.00302231] an
 d 4.000018268770467
 Values of points and objective function value are: [0.00241785 0.00241785] an
 d 4.000011692013099
 Values of points and objective function value are: [0.00193428 0.00193428] an
 d 4.000007482888383
 Values of points and objective function value are: [0.00154743 0.00154743] an
 d 4.000004789048565
 Values of points and objective function value are: [0.00123794 0.00123794] an
 d 4.000003064991081

Values of points and objective function value are: [0.00099035 0.00099035] and 4.000001961594292

Values of points and objective function value are: [0.00079228 0.00079228] and 4.000001255420347

Values of points and objective function value are: [0.00063383 0.00063383] and 4.000000803469022

Values of points and objective function value are: [0.00050706 0.00050706] and 4.000000514220174

Values of points and objective function value are: [0.00040565 0.00040565] and 4.000000329100912

Values of points and objective function value are: [0.00032452 0.00032452] and 4.0000002106245836

Values of points and objective function value are: [0.00025961 0.00025961] and 4.000000134799733

Values of points and objective function value are: [0.00020769 0.00020769] and 4.00000008627183

Values of points and objective function value are: [0.00016615 0.00016615] and 4.000000055213971

Values of points and objective function value are: [0.00013292 0.00013292] and 4.000000035336941

Values of points and objective function value are: [0.00010634 0.00010634] and 4.000000022615643

Values of points and objective function value are: [8.50705917e-05 8.50705917e-05] and 4.000000014474011

Values of points and objective function value are: [6.80564734e-05 6.80564734e-05] and 4.000000009263367

Values of points and objective function value are: [5.44451787e-05 5.44451787e-05] and 4.0000000059285545

Values of points and objective function value are: [4.3556143e-05 4.3556143e-05] and 4.000000003794275

Values of points and objective function value are: [3.48449144e-05 3.48449144e-05] and 4.000000002428336

Values of points and objective function value are: [2.78759315e-05 2.78759315e-05] and 4.0000000015541355

Values of points and objective function value are: [2.23007452e-05 2.23007452e-05] and 4.000000000994646

Values of points and objective function value are: [1.78405962e-05 1.78405962e-05] and 4.0000000006365735

Values of points and objective function value are: [1.42724769e-05 1.42724769e-05] and 4.000000000407407

Values of points and objective function value are: [1.14179815e-05 1.14179815e-05] and 4.000000000260741

Values of points and objective function value are: [9.13438523e-06 9.13438523e-06] and 4.000000000166874

Values of points and objective function value are: [7.30750819e-06 7.30750819e-06] and 4.000000000106799

Values of points and objective function value are: [5.84600655e-06 5.84600655e-06] and 4.0000000000683515

Values of points and objective function value are: [4.67680524e-06 4.67680524e-06] and 4.0000000000437455

Values of points and objective function value are: [3.74144419e-06 3.74144419e-06] and 4.000000000027997

Values of points and objective function value are: [2.99315535e-06 2.99315535e-06] and 4.000000000017918

Values of points and objective function value are: [2.39452428e-06 2.39452428e-06] and 4.000000000011467

Values of points and objective function value are: [1.91561943e-06 1.91561943e-06]

e-06] and 4.00000000007339
 Values of points and objective function value are: [1.53249554e-06 1.53249554e-06] and 4.00000000004697
 Values of points and objective function value are: [1.22599643e-06 1.22599643e-06] and 4.000000000030065
 Values of points and objective function value are: [9.80797146e-07 9.80797146e-07] and 4.00000000001924
 Values of points and objective function value are: [7.84637717e-07 7.84637717e-07] and 4.00000000001231
 Values of points and objective function value are: [6.27710174e-07 6.27710174e-07] and 4.00000000000788
 Values of points and objective function value are: [5.02168139e-07 5.02168139e-07] and 4.000000000005045
 Values of points and objective function value are: [4.01734511e-07 4.01734511e-07] and 4.00000000000322
 Values of points and objective function value are: [3.21387609e-07 3.21387609e-07] and 4.00000000000207
 Values of points and objective function value are: [2.57110087e-07 2.57110087e-07] and 4.00000000000132
 Values of points and objective function value are: [2.0568807e-07 2.0568807e-07] and 4.00000000000084
 Values of points and objective function value are: [1.64550456e-07 1.64550456e-07] and 4.00000000000054
 Values of points and objective function value are: [1.31640365e-07 1.31640365e-07] and 4.00000000000035
 Values of points and objective function value are: [1.05312292e-07 1.05312292e-07] and 4.00000000000022
 Values of points and objective function value are: [8.42498333e-08 8.42498333e-08] and 4.00000000000014
 Values of points and objective function value are: [6.73998667e-08 6.73998667e-08] and 4.00000000000009
 Values of points and objective function value are: [5.39198933e-08 5.39198933e-08] and 4.00000000000006
 Values of points and objective function value are: [4.31359147e-08 4.31359147e-08] and 4.000000000000036
 Values of points and objective function value are: [3.45087317e-08 3.45087317e-08] and 4.00000000000003
 Values of points and objective function value are: [2.76069854e-08 2.76069854e-08] and 4.00000000000002
 Values of points and objective function value are: [2.20855883e-08 2.20855883e-08] and 4.00000000000001
 Values of points and objective function value are: [1.76684706e-08 1.76684706e-08] and 4.00000000000001
 Values of points and objective function value are: [1.41347765e-08 1.41347765e-08] and 4.0
 Values of points and objective function value are: [1.13078212e-08 1.13078212e-08] and 4.0
 Values of points and objective function value are: [9.04625697e-09 9.04625697e-09] and 4.0
 Values of points and objective function value are: [7.23700558e-09 7.23700558e-09] and 4.0
 Values of points and objective function value are: [5.78960446e-09 5.78960446e-09] and 4.0
 Values of points and objective function value are: [4.63168357e-09 4.63168357e-09] and 4.0
 Values of points and objective function value are: [3.70534686e-09 3.70534686e-09] and 4.0

Values of points and objective function value are: [2.96427748e-09 2.96427748e-09] and 4.0
Values of points and objective function value are: [2.37142199e-09 2.37142199e-09] and 4.0
Values of points and objective function value are: [1.89713759e-09 1.89713759e-09] and 4.0
Values of points and objective function value are: [1.51771007e-09 1.51771007e-09] and 4.0
Values of points and objective function value are: [1.21416806e-09 1.21416806e-09] and 4.0
Values of points and objective function value are: [9.71334446e-10 9.71334446e-10] and 4.0
Values of points and objective function value are: [7.77067557e-10 7.77067557e-10] and 4.0
Values of points and objective function value are: [6.21654046e-10 6.21654046e-10] and 4.0
Values of points and objective function value are: [4.97323236e-10 4.97323236e-10] and 4.0
Values of points and objective function value are: [3.97858589e-10 3.97858589e-10] and 4.0
Values of points and objective function value are: [3.18286871e-10 3.18286871e-10] and 4.0
Values of points and objective function value are: [2.54629497e-10 2.54629497e-10] and 4.0
Values of points and objective function value are: [2.03703598e-10 2.03703598e-10] and 4.0
Minimum point (x, y): [2.03703598e-10 2.03703598e-10]
Minimum value of the function: 4.0

In []: