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Interaction of an ultrashort, relativistically strong laser pulse with an overdense plasma

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The results of an analytical description and of a particle-in-cell simulation of the interaction of an ultrashort, relativistically intense laser pulse, obliquely incident on a nonuniform overdense plasma, are presented and several novel features are identified. The absorption and reflection of the ultraintense electromagnetic laser radiation from a sharp-boundary plasma, high harmonic generation, and the transformation into low-frequency radiation are discussed. In the case of weak plasma nonuniformity the excitation of nonlinear Langmuir oscillations in the plasma resonance region and the resulting electron acceleration are investigated. The vacuum heating of the electrons and the self-intersection of the electron trajectories are also studied. In the case of a sharp-boundary plasma, part of the energy of the laser pulse is found to be converted into a localized, relativistically strong, nonlinear electromagnetic pulse propagating into the plasma. The expansion of the hot electron cloud into the vacuum region and the action of the ponderomotive force of the laser pulse in the localized longitudinal electric field of the Langmuir oscillations lead to ion acceleration. The energy increase of a minority population of multicharged ions is found to be much greater than that of the ambient ions.

I. INTRODUCTION

Recent progress in the production of extremely powerful ultrashort laser pulses[1] has stimulated an increasing interest in the problem of laser–matter interaction. Among the different applications of such intense, short laser pulses, we mention the acceleration of charged particles, photon acceleration, and the production of x-ray sources.[2–4]

The interaction between the laser pulse and matter depends critically on the ratio of the pulse carrier frequency ω to the Langmuir frequency ω_p. In a transparent plasma, $\omega \gg \omega_p$, the main absorption mechanisms of an ultrashort, relativistically intense laser pulse arise from the losses due to the radiation of wake plasma waves behind the pulse and from its modulation due to stimulated forward and backward Raman scattering.[5,6] Instead, in the case of a laser pulse interacting with a collisionless overdense plasma, $\omega \ll \omega_p$, the main absorption mechanisms are determined by the ratio of the electron oscillation amplitude $r_E = eE_0/m_e\omega^2$ to the plasma nonuniformity scale length $L = (\partial \ln n/\partial x)^{-1}|_{n=n_{\rm cr}}$, and by the pulse polarization and incidence angle. Here E_0 is the amplitude of the electric field in the pulse and $n_{\rm cr}$ is the critical value of the plasma density for which $\omega = (4\pi n_{\rm cr} e^2/m_e)^{1/2}$. For $\eta = r_E/L \ll 1$ the main absorption mechanism for p-polarized laser pulses is due to nonlinear processes occurring in the plasma resonance region.[7,8] On the contrary, if the oscillation amplitude r_E is larger than the typical nonuniformity scale length ($\eta \gg 1$), the laser energy is absorbed through the mechanism of electron "vacuum heating."[9–11] Furthermore, in Ref. 5 it was shown that a laser pulse with an ultrarelativistically strong amplitude transforms part of its energy into relativistic electromagnetic solitons.[12] It is also known that the interaction between matter and extremely powerful laser pulses is accompanied by the generation of high harmonics of the electromagnetic radiation and by the production of fast particles.[13–15]

The aim of the present article is to investigate the interaction of a relativistically strong laser pulse of finite duration with an overdense plasma. The interest of this investigation stems from the fact that, while the interaction between a laser pulse and an overdense plasma has been widely studied for moderate intensities and relatively long pulses, see Refs. 7 and 9–11, new physical phenomena involving the pulse absorption and the penetration of the electromagnetic energy in the plasma are found to occur in the case of relativistically strong laser pulses.

This article is organized as follows. In Sec. II we show, following Ref. 16, that a Lorentz transformation to a reference frame moving along the plasma boundary reduces the problem of oblique incidence to that of normal incidence on the plasma. In this reference frame all variables are taken to depend only on time and on the coordinate along the direction of nonuniformity. Thus the usefulness of this transformation, which is very convenient when treating an obliquely incident pulse, is limited to processes that are one dimensional in the transformed frame. In order to understand which physical processes can be described in this frame, known analytical results on the propagation of a small amplitude electromagnetic wave in a nonuniform plasma and on the formation of surface waves are explicitly rederived in the "moving" frame in Sec. III. Section IV is devoted to the nonlinear plasma dynamics under the action of the incident electromagnetic radiation. In this section the results of both an analytical evaluation and a numerical simulation with the help of a "PIC" (particle-in-cell) relativistic electromagnetic code are pre-

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sented. First, we consider the case of a weakly nonuniform plasma and, with the help of an approximate analytical fluid model, discuss the nonlinear processes that occur in the plasma resonance region where large amplitude fields are excited by the ultrashort, relativistically strong laser pulse. These phenomena include wave breaking and the resulting acceleration of ions and electrons. A new regime is analyzed, corresponding to an extremely short pulse that excites finite amplitude, long-lasting waves which eventually break due to the increase in their wave number caused by the plasma nonuniformity. In the opposite case, when the plasma nonuniformity is large, we consider the limit of a sharp-boundary plasma[17] and study the process of "vacuum heating" of the electrons. Here, we present new results that are related to the space and time dependence of the density of the electron cloud in the vacuum region. We use this density to calculate the electric field in vacuum. Both the analytical treatment and the computer simulations show that this field leads to the acceleration of a minority of high charge ions and of ambient ions up to velocities of the order of the electron velocities. The strong oscillatory fields that are excited at resonance in the case of a weakly nonuniform plasma, or at the plasma boundary in the case of a strong nonuniformity, give rise to a ponderomotive force that acts mainly on electrons and leads to charge separation. We find that the electric field that arises from charge separation accelerates a minority of high charge ions much more effectively than the ambient ions. The energy spectrum of the accelerated ions is also computed. The phenomenon of high harmonic generation, which has been previously observed both in computer simulations and in laboratory experiments, is studied in this article for the case of a relativistically strong laser pulse. In this limit, high harmonic generation is found to be very effective in the case of p-polarized waves in a sharp-boundary plasma. We interpret it as due to the Doppler effect produced by a reflecting charge sheet, formed in a narrow region at the plasma boundary, oscillating under the action of the relativistically strong laser pulse. In the case of an s-polarized relativistically strong laser pulse, the excitation of a relativistically strong, nonlinear electromagnetic pulse is observed in the numerical simulation. This nonlinear pulse has a frequency smaller than the local plasma frequency and propagates inside the plasma at a speed smaller than the speed of light. This new result, together with the finding of a similar nonlinear wave in the case of a laser pulse propagating in an underdense plasma,[5] indicates that the formation of a localized electromagnetic pulse is likely to be an important channel of energy absorption and penetration into the plasma in the case of relativistically strong laser pulses. For the angles of incidence considered in the numerical simulations presented in this article, conversion of up to 10% of the laser pulse energy into nonlinear pulse energy is observed. Finally, Sec. V contains a discussion of the results obtained in this article and the conclusions.

II. OBLIQUE INCIDENCE ON A NONUNIFORM PLASMA

Let us consider an electromagnetic wave with carrier frequency ω, wave vector $\mathbf{k}$, and dimensionless amplitude $a=eE_0/m_e\omega c$. The limits $a\gg 1$ and $a\ll 1$ correspond to ultrarelativistic and nonrelativistic intensities, respectively. The wave is obliquely incident on a plasma (located in the region $x>0$) and forms an angle θ with respect to the normal to the plasma surface. At large distances from the plasma boundary ($x\rightarrow\infty$), the plasma is overdense, $\omega\ll\omega_p=(4\pi ne^2/m_e)^{1/2}$. In the vacuum region ($x<0$) the frequency ω and wave vector $\mathbf{k}$ are connected through the dispersion equation $\omega^2=c^2(k_x^2+k_y^2)$, where $k_x=k\cos\theta$, $k_y=k\sin\theta$. Following Refs. 10, 16, we perform a Lorentz transformation to the reference frame K' moving in the y direction with velocity V. In the K' frame, the frequency ω' and the components of the wave vector $\mathbf{k}'$ are

$$\omega'=\frac{\omega-k_y\beta c}{(1-\beta^2)^{1/2}},\quad k_x'=k_x,\quad k_y'=\frac{k_y-\omega\beta/c}{(1-\beta^2)^{1/2}},\tag{1}$$

where $\beta=V/c$. For $V=k_y c^2/\omega=c\sin\theta$ the y-component k_y' of the wave vector is equal to zero. Thus, in this frame, the electromagnetic wave propagates only in the x direction, and the problem under consideration is reduced to that of normal incidence of the wave on the plasma, where both ions and electrons move with velocity $-\mathbf{V}$ along the y direction.

In the K' frame, the frequency is $\omega'=\omega\cos\theta$ and wave vector has only one nonzero component $\mathbf{k}'=(k_x',0,0)=(k\cos\theta,0,0)$. In the case of p-polarized waves, the electric field is $\mathbf{E}'=(0,H_0\cos\theta,0)$ and the magnetic field is $\mathbf{H}'=(0,0,H_0\cos\theta)$. For s-polarized waves $\mathbf{E}'=(0,0,E_0\cos\theta)$, and $\mathbf{H}'=(0,E_0\cos\theta,0)$. The unperturbed plasma density n' in the K' frame is $n'=n\gamma=n/\cos\theta$, where the relativistic factor γ depends on $\beta=\sin\theta$ as follows: $\gamma=(1-\beta^2)^{-1/2}=1/\cos\theta$. Electrons and ions move in this frame with velocity $-\mathbf{V}$ and their unperturbed momentum and energy are given by $P_{ye,i}=m_{e,i}c\tan\theta$, $m_{e,i}'c^2=m_{e,i}c^2/\cos\theta$. The number of oscillations in the laser pulse $N_p=\omega t_p/2\pi$, where t_p is the pulse duration, and the Langmuir frequency $\omega_p=(4\pi n'e^2/m_e')^{1/2}$ are the Lorentz invariant. In the K' frame all quantities are taken to depend on the time t' and the coordinate $x=x'$ only. A sketch of the wave electric and magnetic fields in the K and K' frames is shown in Fig. 1.

While the Lorentz transformation to the K' frame simplifies the incidence problem from oblique to normal, a number of paradoxes remain to be understood. First comes the question of what is the driving force acting on the particles in the x direction in the case of p-polarized waves, since in the K' frame the x component of the electric field of the laser pulse vanishes. A second question is related to the location of the plasma resonance. In the K frame this resonance is located at the point $x=x_p$, where the frequency ω of the electromagnetic wave equals the local value of the Langmuir frequency. In the K' frame the value of the Langmuir frequency ω_p is unchanged, while the fre-

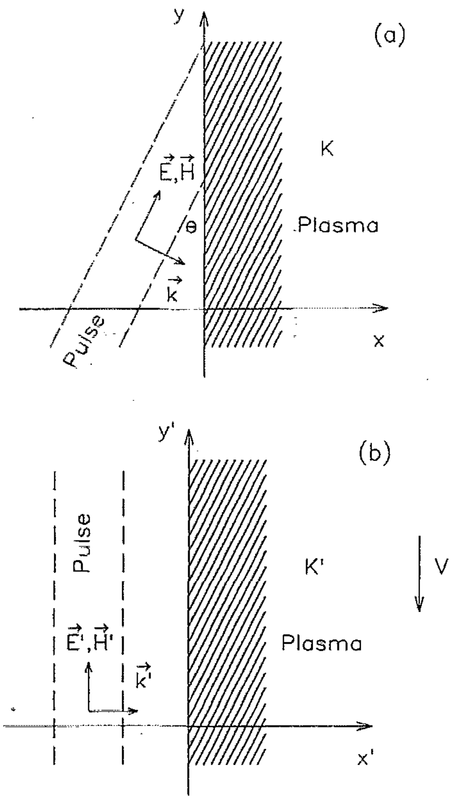

FIG. 1. Electromagnetic wave incident on a nonuniform plasma in the K frame (a) and in the K' frame (b). The plasma is located in the $x>0$ half-plane (shaded area). θ is the incidence angle in the K frame. For s-polarized (p-polarized) waves the magnetic (electric) field vector is in the (x,y) plane. In the K' frame the plasma moves in the y direction with velocity $-V=-c\sin\theta$.

quency ω of the laser pulse decreases $\omega'=\omega\cos\theta$. The next question is related to the time behavior of the ion energy in the electromagnetic wave: Numerical simulations presented in Sec. IV B show that the ion energy changes drastically, in spite of the large value of the ratio between the ion and electron mass. To elucidate these problems we shall first analyze the limit of a small amplitude electromagnetic wave in the K' frame.

III. SMALL AMPLITUDE ELECTROMAGNETIC WAVES IN NONUNIFORM PLASMAS

By linearizing the cold plasma fluid equations and Maxwell's equations, we obtain in the K' frame

$$\frac{d}{dx}\frac{1}{\epsilon'(\omega',x)}\frac{d}{dx}(H'_z-\beta E'_x)+\frac{\omega'^2-\omega_p^2}{c^2\epsilon'(\omega',x)}(H'_z-\beta E'_x)=0, \tag{2}$$

$$(E'_x-\beta H'_z)=-\frac{\beta}{\epsilon'(\omega',x)}(H'_z-\beta E'_x) \tag{3}$$

for p-polarized waves, and

$$\frac{d^2}{dx^2}E'_z+\frac{\omega'^2-\omega_p^2}{c^2}E'_z=0 \tag{4}$$

for s-polarized waves. Here the dielectric constant $\epsilon'(\omega',x)$ is equal to

$$\epsilon'(\omega',x)=1-\frac{\omega_p^2(x)}{\omega'^2}(1-\beta^2). \tag{5}$$

In the vacuum region, where $\epsilon'=1$, $E'_x=0$. From Eqs. (2)–(4) we see that the reflection point of the electromagnetic waves occurs where $\omega'^2=\omega_p^2$ and that, in the case of p-polarized waves, the plasma resonance point occurs where $\epsilon'(\omega',x_p)=0$ [i.e., where $\omega'^2=\omega_p^2(1-\beta^2)$].

In the vicinity of the resonance point $x=x_p$, where

$$\epsilon'(\omega',x)\approx(x-x_p)/L \tag{6}$$

$H'_z-\beta E'_x$ has a logarithmic term

$$H'_z-\beta E'_x\approx H_m\left[1+\frac{\beta^2\omega'^2(x-x_p)^2}{4c^2(1-\beta^2)}\ln\left(\frac{\beta^2\omega'^2(x-x_p)^2}{4c^2(1-\beta^2)}\right)\right] \tag{7}$$

and, from Eq. (3), it follows that the value of

$$E'_x-\beta H'_z\approx\frac{L\beta H_m}{x-x_p} \tag{8}$$

becomes infinite for $x\to x_p$. Here the constant H_m is of order H_0/γ if the wavelength is of the order of the scale of the plasma nonuniformity and, in general, can be found by matching Eq. (7) to the solution of Eq. (2) far from the resonant region.

Langmuir oscillations correspond to oscillations with a frequency $\omega'=\omega_p(1-\beta^2)^{1/2}=\omega_p\cos\theta$, and have a perturbed magnetic field in the z direction and an electric field in the x direction related by $H'_z=\sin\theta\, E'_x$. This relationship must be satisfied for the oscillations to be potential in the K frame. These expressions clarify the first two paradoxes mentioned above. Indeed in the K' frame the driving force is the $e\mathbf{v}\times\mathbf{H}'/c$ force, and the resonance occurs at the same x location both in the K and in the K' frame: In the K' frame the mode frequency changes while the Langmuir frequency remains invariant, but the expression of the dielectric constant ϵ' changes so that the resonance condition is now given by $\omega'=\omega_p(1-\beta^2)^{1/2}$.

Because of the assumption $k'_y=0$, surface waves localized near the boundary between the plasma and the vacuum cannot be found from Eqs. (2)–(4). Such waves could lead to enhanced absorption of the electromagnetic waves in the case of a plasma with a sharp boundary (see Ref. 18 and references therein). Instead, in the case of a sharp boundary between two plasmas with the densities n_1 for $x<0$ and n_2 for $x>0$ ($n_1<n_2$), Eqs. (2) and (3) permit surface waves. From Eq. (2) it follows that both $(1/\epsilon')[d(H'_z-\beta E'_x)/dx]$ and $H'_z-\beta E'_x$ are continuous functions at the boundary. In regions 1 ($x<0$) and 2($x>0$), we have, respectively,

$$H'_z-\beta E'_x=[H'_z(0)-\beta E'_x(0)]_{1,2}\exp(\pm\kappa_{1,2}x), \tag{9}$$

where the signs $\pm$ apply, respectively, to regions 1 and 2, and

$$\kappa_{1,2}=(\omega_{p1,2}^2-\omega'^2)^{1/2}/c, \tag{10}$$

with $\omega_{p1,2}^2=4\pi n_{1,2}e^2/m_e$. From the boundary conditions we have

$$[H_z'(0)-\beta E_x'(0)]_1=[H_z'(0)-\beta E_x'(0)]_2 \tag{11}$$

and

$$\frac{\kappa_1}{\epsilon_1'(\omega')}=-\frac{\kappa_2}{\epsilon_2'(\omega')}. \tag{12}$$

For the surface waves to exist, it is necessary that both κ_1 and κ_2 are positive. This requires that $\epsilon_1'(\omega')>0$ and $\epsilon_2'(\omega')<0$, that is,

$$\omega_{p1}^2<\omega'^2/(1-\beta^2)<\omega_{p2}^2. \tag{13}$$

Then, the dispersion relation of the surface waves has the form with frequency ω'

$$\frac{(\omega_{p1}^2-\omega'^2)^{1/2}}{1-\omega_{p1}^2(1-\beta^2)/\omega'^2}=\frac{(\omega_{p2}^2-\omega'^2)^{1/2}}{\omega_{p2}^2(1-\beta^2)/\omega'^2-1}. \tag{14}$$

If $\omega'\ll\omega_{p1}$, ω_{p2}, we have

$$\omega'^2\approx\omega_{p1}\omega_{p2}(1-\beta^2). \tag{15}$$

From Eq. (3) we see that $E_x'-\beta H_z'$ changes sign with x, while from Eq. (9) it follows that $H_z'-\beta E_x'$ is largest at the boundary $x=0$.

IV. NONLINEAR PLASMA DYNAMICS NEAR THE SHARP BOUNDARY

The nonlinear, self-consistent behavior of the plasma and of the electromagnetic field can be investigated by combining the results obtained with the help of a "PIC" relativistic electromagnetic code, developed earlier and used in Ref. 5, with analytical evaluations. In the numerical code all quantities depend on $x=x'$ and t', both electrons and ions are described by three-component velocities with the proper electron–ion mass ratio. The numerical simulations are performed for different incidence angles and wave polarizations (p-, s-, elliptic, and circularly polarized waves). The normalized wave amplitude varies in the range $0.1\leqslant a\leqslant 3$. The length of the laser pulse is described in terms of the number N_p of periods per pulse. Most simulations are performed with 16 000 particles per species. The typical length of the problem, c/ω', is equal to eight times the cell size δx. The number of particles per cell is 10–20.

In order to obtain analytical estimates of the physical processes that are involved, we will adopt a simplified one-fluid, cold electron model and compare its results with those of the computer simulation. Instead of the Eulerian coordinates x, t' we use the Lagrangian variables x_0 and τ, where

$$x=x_0+\xi(x_0,\tau), \quad v_x=\frac{\partial\xi}{\partial\tau} \tag{16}$$

and $\xi(x_0,\tau)$ is the displacement of the elementary volume of the electron fluid from the initial position x_0.

As is well known, the solution of the continuity equation for the electron density n' can be written as a function of these variables in the form

$$n'(x,t')=n'(x_0,\tau)=\frac{n'(x_0,0)}{1+\partial\xi/\partial x_0} \tag{17}$$

and the solution of Maxwell's equations for the longitudinal electric field E_x' is

$$E_x'=4\pi e\int_0^{\xi(x_0,\tau)} n_i'(x_0+s)ds, \tag{18}$$

where $n_i'(x)=n'(x,0)$ is the ion density distribution which, in the present case, is assumed to be given.

If we consider a linear density profile $n_i'=n_{cr}'x/L$, where the scale length L of the plasma nonuniformity is chosen such that the plasma resonance $\omega'=\omega_p(x_p)(1-\beta^2)^{1/2}$ occurs at $x_p=L$, E_z' is given by

$$E_x'=4\pi en_{cr}'\left(\frac{x_0\xi}{L}+\frac{\xi^2}{2L}\right) \tag{19}$$

for $x_0+\xi>0$, that is, inside the region occupied by the plasma, and by

$$E_x'=-4\pi en_{cr}'\left(\frac{x_0^2}{2L}\right) \tag{20}$$

for $x_0+\xi<0$, that is, outside the plasma.

In the case of a step-like density profile for which $n'=n_0'$ for $x_0>0$ and $n'=0$ for $x_0<0$, we obtain

$$E_x'=4\pi en_0'\xi \tag{21}$$

for $x_0+\xi>0$, and

$$E_x'=-4\pi en_0'x_0 \tag{22}$$

for $x_0+\xi<0$.

Here we consider a linear density profile. Then, we obtain for the displacement $\xi(x_0,\tau)$ driven by an electromagnetic wave with a weakly relativistic amplitude, $a=eE_0/m_e\omega c<1$, the equations

$$\frac{\partial^2\xi}{\partial\tau^2}+\omega'^2\left(\frac{x_0}{L}\xi+\frac{\xi^2}{2L}\right)+\frac{\partial^2\xi}{\partial\tau^2}\left(\frac{\partial\xi}{\partial\tau}\right)^2\frac{3\gamma^2}{2c^2}=\frac{eH_0}{m_e\gamma^2}\sin\omega'\tau \tag{23}$$

for $x_0+\xi>0$, and

$$\frac{\partial^2\xi}{\partial\tau^2}-\omega'^2\frac{x_0^2}{2L}+\frac{\partial^2\xi}{\partial\tau^2}\left(\frac{\partial\xi}{\partial\tau}\right)^2\frac{3\gamma^2}{2c^2}=\frac{eH_0}{m_e\gamma^2}\sin\omega'\tau \tag{24}$$

in the vacuum region where $x_0+\xi<0$. In these equations the Langmuir frequency depends on the coordinate x_0, and the nonlinear terms arise from the plasma nonuniformity, $(\xi^2/2L)\omega'^2$, and from relativistic effects, $(3\gamma^2/2c^2)(\partial^2\xi/\partial\tau^2)(\partial\xi/\partial\tau)^2$. To obtain these equations we have assumed that the function $H_z'-\beta E_x'$ is given and is of order H_0/γ. Then, we have expressed H_z' as a function of E_x' and H_0, we have inserted it into the expression of the Lorentz force, and we have used Eqs. (19) and (20) to express E_x' as a function of ξ and x_0 where we have substituted ω'^2 for $\omega_p^2(x_p)(1-\beta^2)$. Furthermore, we have neglected the change in the electron velocity in the y direction due to the E_y component of the electric field.

A. Plasma resonance

For $\eta=r_E/L\ll 1$, with η the ratio between the electron oscillation amplitude r_E defined in the Introduction [see also Eq. (26)] and the scale L of the plasma nonuniformity, that is, for $\xi/L\ll 1$, a strong plasma resonance occurs near the point $x=x_p$ where $\omega'^2=\omega_p^2(1-\beta^2)$. In this region strong Langmuir oscillations are excited, leading to a large amplification of the x component of the electric field E'_x, up to values for which the saturation effects become important.[7,8,19–21] These effects can be characterized in terms of a dimensionless parameter S, small compared to unity, which plays the role of an inverse quality factor of the Langmuir oscillations. In terms of this parameter, the maximum value of the amplitude of the electric field is approximately $E'_{\max}\approx H_0/S$, and the width of the region where the strong electric field E'_x is localized is given by $\Delta x\approx SL$ while the characteristic time of the resonance saturation equals $\Delta t'\approx 1/(\omega' S)$.

In the case of an ultrashort femtosecond laser pulse, with an intensity of the order of 10^{18} W/cm^2, the main mechanisms of resonance saturation that determine the value of S are the self-intersection of the electron trajectories, the relativistic detuning of the oscillations, and the finite duration of the laser pulse. At the initial stage of the growth, when the amplitude of the electric field is still small, we can neglect the nonlinear terms on the left-hand side of Eq. (23). Then, we find for the displacement $\xi(x_0,\tau)$ as a function x_0,τ

$$\xi(x_0,\tau)\approx-\frac{r_E L}{x_0-L}\left(\frac{\omega'\sin\Omega\tau}{\Omega}-\sin(\omega'\tau)\right), \qquad (25)$$

where Ω is a function of the Lagrangian coordinate x_0

$$\Omega=\omega'\left(\frac{x_0}{L}\right)^{1/2}, \quad \text{and } r_E\equiv\frac{eH_0}{m_e\gamma^2\omega'^2}=\frac{eH_0}{m_e\omega_p^2(x_p)}. \qquad (26)$$

At the resonant point $x_0=x_p=L$, where $\Omega=\omega'$, the amplitude of the oscillation displacement grows linearly with time as

$$\xi(x_p,\tau)\approx-(r_E/2)\omega'\tau\cos\omega'\tau, \qquad (27)$$

while the resonance width decreases with time as $1/\tau$. At time $t'\approx 2/(\omega'\eta^{1/2})$, the amplitude of the oscillations becomes equal to the resonance width $\Delta x\approx L\eta^{1/2}$, the Jacobian $|\partial x/\partial x_0|$ vanishes, and the electron trajectories start to self-intersect. The maximum oscillation amplitude is of order $\xi_{\max}\approx\Delta x\approx(r_e L)^{1/2}$, and the dimensionless parameter S is equal to $S=\eta^{1/2}$.

The relativistic nonlinearity leads to a frequency detuning $\delta\omega'=-3\gamma^2\omega'^3\xi_0^2/8c^2$, where ξ_0 is the oscillation amplitude. The dependence of this amplitude in the plasma resonance region as a function of the Lagrangian coordinate x_0 can be found from the algebraic equation[21]

$$\xi_0\left(\frac{\omega'}{2}\frac{x_0-L}{L}-\frac{3\gamma^2\omega'^3}{16c^2}\xi_0^2\right)=\frac{r_E\omega'}{2}. \qquad (28)$$

We see that for $\eta^{1/2}<a^{2/3}$, where a is the dimensionless amplitude of the laser pulse, the maximum value of the oscillation amplitude is given by $\xi_{\max}\approx r_E/a^{2/3}$, the width of the resonance is of order $a^{2/3}L$, and the saturation time is $\Delta t'\approx 1/\delta\omega'\approx 1/(\omega' a^{2/3})$. We also see that in this regime the nonlinearity arising from the plasma nonuniformity can be neglected.

If the duration $\Delta t'=(2\pi/\omega')N_p$ of the laser pulse is sufficiently short, the maximum oscillation amplitude is $\xi_{\max}\approx r_E/(\omega'\Delta t')=r_E\pi N_p$, and the width of the resonance is $\Delta x\approx L/(\pi N_p)$.

Hence, depending on the values of the pulse parameters under consideration, the dimensionless parameter S is given by the largest among

$$\{S_N=\eta^{1/2},\quad S_R=a^{2/3},\quad S_T=1/(\pi N_p)\}, \qquad (29)$$

where S_N is related to the self-intersection of the electron trajectories ($\eta=r_E/L$), S_R to the relativistic detuning of the frequency, and S_T to the finite duration of the laser pulse.

In the case of rather short pulses, with $S_T=1/\pi N_p>S_R,S_N$, the amplitude of the Langmuir oscillations stops growing after the pulse ends. These oscillations remain localized around the resonance point since their group velocity, $v_g\approx v_{\text{the}}(\lambda_{De}/\Delta x)$ with v_{the} the electron thermal velocity and λ_{De} their Debye length, is small. However, their effective wave number increases and their phase velocity decreases according to the formula

$$\frac{\partial k}{\partial t'}=-(1-\beta^2)^{1/2}\frac{\partial\omega_p}{\partial x}, \qquad (30)$$

which describes the change of the wave vector k in a nonuniform plasma in the geometrical optics approximation. Hence $k(t')\approx k_0-[\omega_p(1-\beta^2)^{1/2}/L]t'$ tends to $-\infty$ when $t'\to\infty$. This increase of the wave vector can also be seen directly, without the assumption $kL\gg 1$ from the expression of ξ, as a function of x_0 and of τ, for $\tau>2\pi N_p/\omega'$

$$\xi(x_0,\tau)\approx-\frac{r_E L\omega'}{(x_0-L)\Omega}[\sin(\Omega\tau)-\sin(\Omega\tau-\theta)]. \qquad (31)$$

Here we have supposed that the magnetic field in the laser pulse has the form $H(\tau)=H_0\sin(\omega'\tau)$ for $0<\tau<2\pi N_p/\omega'$ and $H(\tau)=0$ for $\tau>2\pi N_p/\omega'$, and $\theta=2\pi N_p\Omega/\omega'\approx 2\pi N_p[1-(x_0-L)/L]$. From Eq. (31) we see that the maximum electric field is $E_{\max}\approx H_0\pi N_p$ and that the field is localized in a region of width $\Delta x\approx L/\pi N_p$. In this region

$$\xi(x_0,\tau)\approx\frac{r_E L\omega'}{(x_0-L)\Omega}\left[\sin\left(\frac{\pi N_p(x_0-L)}{D}\right)\cos(\Omega\tau)\right]. \qquad (32)$$

In Eq. (32) we may disregard, in the dependence of $\Omega(x_0)$ on the Lagrangian variable x_0, the difference between the Eulerian variable x and x_0. This is consistent with the fact that the nonlinearities arising from the nonuniformity of the plasma density have been neglected. Then we find that the points of constant phase $\phi=\Omega(x_0)\tau$ move with the wave phase velocity

$$v_{\text{ph}}\approx\left.\frac{\partial x_0}{\partial\tau}\right|_\phi=\frac{-\Omega}{\tau\partial\Omega/\partial x_0}=\frac{-\Omega^2}{\phi\partial\Omega/\partial x_0}, \qquad (33)$$

which is negative for positive values of the gradient of Ω. Differentiation of Eq. (32) with respect to x_0 shows that for large τ the gradient of the oscillation amplitude has a term that is a growing function of time

$$\frac{\partial \xi(x_0,\tau)}{\partial x_0} \approx -\xi_{\max}\frac{\partial \Omega}{\partial x_0}\tau \sin(\Omega\tau), \tag{34}$$

where

$$\xi_{\max}=\frac{r_E L\omega'}{(x_0-L)\Omega}\sin\left(\frac{\pi N_p(x_0-L)}{L}\right)\approx r_E\pi N_p. \tag{35}$$

Wave breaking occurs when $\partial\xi/\partial x_0=-1$, which corresponds to the time $\tau_{\rm br}\approx(\xi_{\max}\partial\Omega/\partial x_0)^{-1}$ and to the value of the wave phase $\phi\approx\Omega\tau_{\rm br}=\Omega(\xi_{\max}\partial\Omega/\partial x_0)^{-1}$ such that $\sin\phi\approx 1$. Following Ref. 22 we can describe the structure of the wave near the wave-breaking point $x=x_{\rm br}$ and look for the dependence of $v=dx/dt'$ on x in phase space. From Eq. (16) it follows that in the vicinity of the point $x=x_{\rm br}$, we have for $\delta x=x-x_{\rm br}$ and $\delta x_0=x_0-x_{\rm br}$

$$\delta x\approx\delta x_0+\frac{\partial\xi}{\partial x_0}\delta x_0+\frac{\partial^2\xi}{\partial x_0^2}\frac{(\delta x_0)^2}{2}+\frac{\partial^3\xi}{\partial x_0^3}\frac{(\delta x_0)^3}{6}+\cdots. \tag{36}$$

Due to the conditions $\partial\xi/\partial x_0=-1$ and $\sin(\Omega\tau_{\rm br})\approx 1$, the first three terms in Eq. (36) vanish and

$$\delta x\approx\frac{\partial^3\xi}{\partial x_0^3}\frac{(\delta x_0)^3}{6}\approx\left(\frac{\partial\Omega}{\partial x_0}\tau\right)^2\frac{(\delta x_0)^3}{6}. \tag{37}$$

The velocity v of the electrons in the neighborhood of the wave-breaking point can be written as

$$v=\frac{dx}{dt'}=\frac{\partial\xi}{\partial\tau}=-\xi_{\max}\Omega\sin\left[\left(\Omega(x_{\rm br})+\frac{\partial\Omega}{\partial x_0}\delta x_0\right)\tau\right], \tag{38}$$

which gives

$$\begin{aligned} v&\approx-\xi_{\max}\Omega\sin(\Omega\tau)\cos\left[\frac{\partial\Omega}{\partial x_0}\delta x_0\tau\right]\\ &\approx-r_E\pi N_p\Omega\left[1-\frac{1}{2}\left(\tau\delta x_0\frac{\partial\Omega}{\partial x_0}\right)^2\right]. \end{aligned} \tag{39}$$

Using Eq. (37) we can express v as a function of δx in the form

$$v=-r_E\pi N_p\Omega\left(1-\frac{[6\tau(\partial\Omega/\partial x_0)]^{2/3}}{2}(\delta x)^{2/3}\right). \tag{40}$$

We see from this equation that the spatial dependence of the electron velocity has a cusp at the wave breaking point.

Figures 2(a)–2(d) show the results of the computer simulation of the amplification of the electric field near the plasma resonance. In these simulations, the value of the amplitude in vacuum of the electromagnetic wave is $a=0.1$, the number of the field periods per laser pulse is $N_p=30$, and the plasma nonuniformity length is $L=50c/\omega'$. The incidence angle of the wave corresponds to the "optimal" value.[7] In our case, this value is $\theta=15°$. Under these conditions the maximum amplitude of the electric field in the plasma resonance region is determined by the relativistic detuning of the frequency $S=S_R=a^{2/3}$. After the time $\tau_{\rm br}\approx 2L/\Omega\xi_{\max}=2LS_R/\Omega r_E=2S_R/S_N^2\Omega$ electron trajectories start to self-intersect, as discussed above, due to the growth of the wave number of the Langmuir oscillations in a nonuniform plasma. This is shown in Fig. 2(d). As a result part of the electrons are thrown out of the resonance region in the direction of decreasing plasma density because in this region the phase velocity of the Langmuir oscillations, $v_{\rm ph}=\Omega/k\approx-2L/t'\approx-\Omega\xi_{\max}$, is negative. At this time the velocity of the fast electrons is equal to the phase velocity of the Langmuir oscillations and their energy gain is approximately given by $eH_0LS_R/(\gamma S_N^2)$: Actually it can even exceed this value due to a further acceleration in the plasma wave.

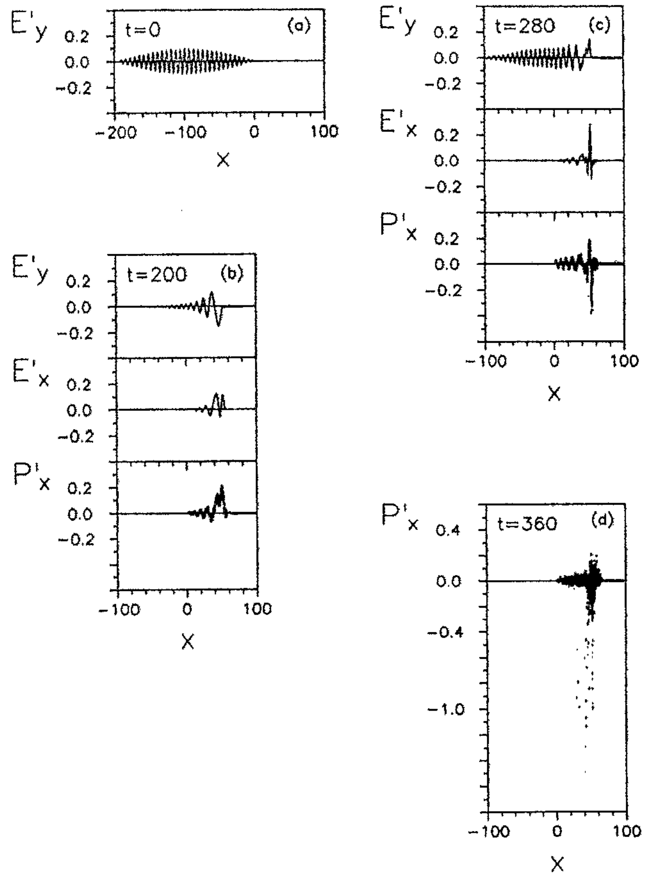

FIG. 2. Interaction in the K' frame of a weakly relativistic, p-polarized laser pulse with a weakly nonuniform plasma. The pulse parameters are dimensionless amplitude $a=0.1$, number of periods in the pulse $N_p=30$, incidence angle $\theta=15°$, and nonuniformity length $L=50c/\omega'$. The x dependence of the normalized electric field components in the x and y directions are shown, together with the (P_x',x) electron phase plane for $t=0$ (a), $t=200/\omega'$ (b), $t=280/\omega'$ (c), $t=360/\omega'$ (d). The particle momenta are normalized on m_ec.

Let us now consider the acceleration of electrons in the plasma resonance region when $S=S_T$. The electron acceleration when the resonance is saturated due to the self-intersection of their trajectories has been discussed in Refs. 8, 19, and 20. When the Langmuir oscillations are described by Eq. (31), we can obtain the expression of their electric field from Eq. (19) in the approximation $\xi\ll x_0$ and $x_0\approx L$. In the K' frame, the energy that a test particle with charge e, moving along the x direction with velocity v, acquires in the wave is given by

$$\Delta \mathscr{E}' \approx e(1-\beta^2)^{1/2} \int_{-\infty}^{+\infty} E_x'(x,\tau)dx, \tag{41}$$

where the integration is performed along the particle trajectory and the time τ is expressed in terms of the particle position x and its initial position x_{in}, at the time when it is injected into the wave, by $\tau=(x-x_{\text{sin}})/v$. Then we obtain

$$\Delta \mathscr{E}' \approx \text{sign}(v) \frac{eH_0 L\pi}{\gamma} \left\{ \left[\text{sign}\left(\frac{\Omega}{v} + \frac{\pi N_p}{L} \right) - \text{sign}\left(\frac{\Omega}{v} \right) \right] \cos\left(\frac{\Omega(x_{\text{in}}-L)}{v} \right) + \frac{1}{\pi} \ln \frac{\Omega}{\Omega+\pi N_p v/L} \sin\left(\frac{\Omega(x_{\text{in}}-L)}{v} \right) \right\}. \tag{42}$$

When transformed into K frame, Eq. (42) shows that the typical value of the energy of fast particles is of the order of eH_0L. For the particles to acquire this energy they have to move at a negative velocity, not too large compared to $(x_{\text{in}}-L)\Omega$, and must have the correct phase (which is determined by $x_{\text{in}}-L$) in the wave. The maximum energy gain is achieved by particles with velocity $v=-\Omega L/\pi N_p$. This relation is the wave–particle resonance condition, since, as seen from Eq. (33) with $\tau=\pi N_p/\Omega$ and $\partial\Omega/\partial x_0 \approx \Omega/L$, the phase velocity of the plasma wave is equal to $-\Omega L/\pi N_p$. For this velocity $\Delta \mathscr{E}' \to \infty$ and it becomes necessary to take into account the dependence of the frequency of the Langmuir waves on the x coordinate. In the neighborhood of the resonance point we have $\Omega \approx \omega'[1+(x-L)/2L]$. Hence, from Eqs. (19), (31), and (41) we find that the leading contribution to the energy gain is given by

$$\Delta \mathscr{E}' \approx \text{sign}(v) \frac{eH_0 L}{\gamma} \sin\left(\frac{\pi N_p (x_{\text{in}}-L)}{L} \right) \times \int_{-\infty}^{+\infty} \frac{\cos(x^2 \pi N_p/2L^2) - \cos(x\pi N_p/L)}{x} dx, \tag{43}$$

which gives

$$\Delta \mathscr{E}' \approx \text{sign}(v) \frac{eH_0 L}{2\gamma} \sin\left(\frac{\pi N_p (x_{\text{in}}-L)}{L} \right) \ln(2\pi N_p). \tag{44}$$

This value can be much larger than the quiver energy of the electrons in the plasma wave.

The case discussed above corresponds to a relatively low value, $a \ll 1$, of the amplitude of the incident electromagnetic laser pulse. In the opposite limit the nonharmonicity of the Langmuir oscillations is very strong and the resonance amplification of the electric field is not possible. It is however of interest to study the behavior of a relativistically strong electromagnetic wave in a nonuniform, overdense plasma and the corresponding acceleration of electrons and ions and the generation of high harmonics. In Fig. 3 the results of the computer simulation are shown for a p-polarized wave with $a=3$ and incidence angle $\theta=15°$. In this case no well-defined resonance region inside the plasma can be identified, due to the large nonharmonicity of the electron oscillations. Nevertheless we can identify a relatively wide interaction region between the electromagnetic wave and the plasma localized around the position where $\omega' \approx \omega_p(1-\beta^2)^{1/2}$. In this region strong electron plasma oscillations are excited with amplitudes of the order of the wave amplitude a. The self-intersection of the electron orbits is seen in their (P_x',x) phase space. This orbit self-intersection occurs either because of the nonuniformity of the ion density distribution at the boundary of the plasma near $x=0$, or to the increase of the wave vector of the plasma wave discussed above. As a result breaking of the plasma wave occurs and leads to the heating of the electrons up to values that can be many times their quiver energy. The shape of the nonlinear electron oscillations in the lower half-plane of phase space (see Fig. 3) exhibits the cusp structure discussed before. In addition we see from the space dependence of $E_y'(x)$ that the electromagnetic wave penetrates into the plasma beyond the point $x=50c/\omega'$ where $\omega'=\omega_p(x)(1-\beta^2)^{1/2}$, due to the relativistic decrease of the plasma frequency.

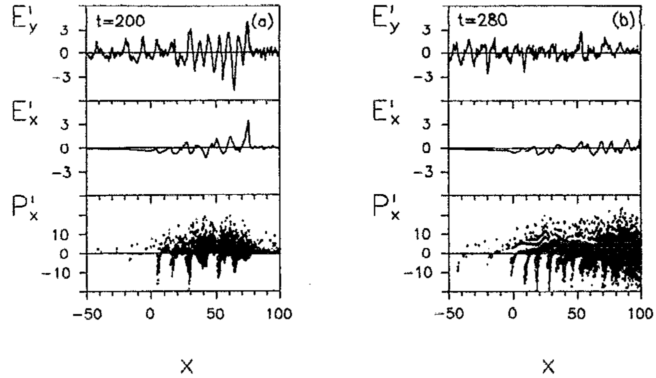

FIG. 3. Interaction in the K' frame of a relativistically strong, p-polarized laser pulse with a weakly nonuniform plasma. The value of the dimensionless amplitude is $a=3$, the other parameters are as in Figs. 2(b) and 2(c).

B. "Vacuum heating" of electrons

If $\eta \gg 1$, the electrons in the region near the plasma boundary are expelled from the plasma into the vacuum region. In our analysis of the motion of these electrons we will disregard the effect of the driving term in Eq. (24), which will only appear through the initial value of the electron velocity. We will also neglect relativistic terms. After one-half of the period of the electron motion, when the electrons come back into the plasma, their trajectories self-intersect and we can no longer use the cold fluid equation employed to obtain Eq. (24). Nevertheless the solution we will present can be useful in order to obtain estimates of the typical time scales of the process, the values of the electric field, and that of the electron density.

If $\eta=r_E/L \approx \xi/L \gg 1$, as a first approximation we can neglect the part of the electron motion inside the plasma, $x_0+\xi>0$, and consider only the motion of the electron cloud in the vacuum region. In the following we will consider two different density profiles that are chosen in order to represent the leading front and the bulk of the electron cloud, respectively. First we take a linear density profile and, from Eq. (24), we have

$$\xi=-v_0\tau+\frac{x_0^2\tau^2}{4L}\,\omega'^2, \tag{45}$$

where $v_0\approx r_E\omega'$ is the initial value of the electron velocity. Electrons come back into the plasma after the time interval $4v_0L/\omega'^2x_0^2$. For $x_0\to 0$, i.e., in the case of electrons from the region near the boundary, this time tends to infinity. From Eq. (45) we can express x_0 in terms of the Eulerian variables t' and $x=x_0+\xi$

$$x_0=\frac{2L\{[1+(x+v_0t')t'^2\omega'^2/L]^{1/2}-1\}}{\omega'^2t'^2}. \tag{46}$$

We can thus compute the Jacobian $J=|\partial x/\partial x_0|$

$$\left|\frac{\partial x}{\partial x_0}\right|=\left(1+\frac{(x+v_0t')t'^2\omega'^2}{L}\right)^{1/2} \tag{47}$$

and the density distribution of the electrons moving out of the plasma boundary

$$n'=n'_{\rm cr}\frac{2\{[1+(x+v_0t')t'^2\omega'^2/L]^{1/2}-1\}}{\omega'^2t'^2[1+(x+v_0t')t'^2\omega'^2/L]^{1/2}}. \tag{48}$$

Near the leading front, where $x\approx -v_0t'$, the electron density has a linear profile

$$n'\approx n'_{\rm cr}(x+v_0t')/L. \tag{49}$$

For electrons far from the front ($x_0\gg L$) it is more convenient to refer to the case of a step-like ion density profile. In this case the electric field for $x_0+\xi<0$ is given by Eq. (22) and is equal to $-4\pi e n'_0x_0$. Then, from the equation of motion in such a field, we see that the Lagrangian and Eulerian electron coordinates are related by

$$x=x_0-v_0t'+x_0(1-\beta^2)\omega_p^2t'^2/2. \tag{50}$$

Here $v_0\approx r_E\omega'(\gamma\omega'/\omega_p)^2\ll r_E\omega'$, and $\omega_p\gg\omega'$ is the plasma frequency corresponding to the plasma density inside the step-like profile. For the electron density in the cloud we obtain a constant profile that decreases with time as

$$n'(x,t')=\frac{n'_0}{1+\omega_p^2t'^2(1-\beta^2)/2}. \tag{51}$$

In Fig. 4, the results of the computer simulations are shown in the case of a step-like ion density profile. This figure shows the expansion of the electrons into the vacuum region. In these simulations, the amplitude of the laser pulse is $a=3$, and the number of periods $N_p=8$, while the plasma density is step-like with $n'=4n'_{\rm cr}$. In Fig. 4(a) we see the effective absorption of the wave energy due to vacuum heating. The largest absorption occurs for p-polarized waves, with an incidence angle of the order of $\theta=35°$, and increases with the wave amplitude. In Fig. 4(b) the process of "vacuum heating" is shown in the projection of electron phase space on the phase plane (P'_x,x).

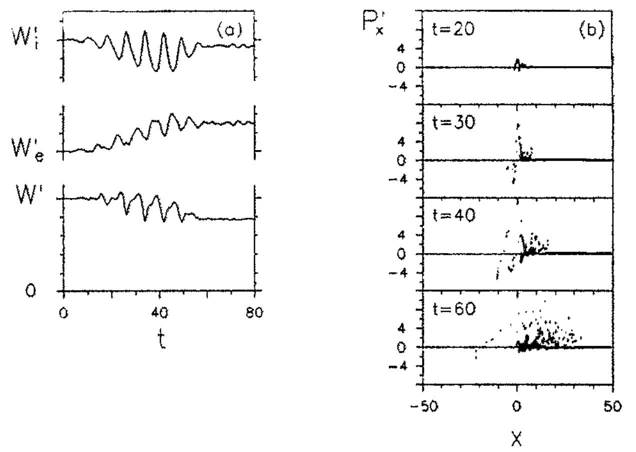

FIG. 4. Interaction in the K' frame of a relativistically strong, p-polarized laser pulse with a sharp-boundary plasma. The parameters are dimensionless amplitude $a=3$, number of periods in the pulse $N_p=8$, incidence angle $\theta=35°$, and ion density $n'_0=4n'_{\rm cr}$. The time dependence of the laser pulse energy W' and of the change in the ion energy (W'_i) and of the electron energy (W'_e) are shown in (a). The electron phase plane (P'_x,x) is shown in (b) at times $t=20/\omega'$, $t=30/\omega'$, $t=40/\omega'$, $t=60/\omega'$.

In Fig. 4(a) the time dependence of the ion and electron energy in the K' frame and of the energy of the electromagnetic field are presented. We find oscillations within the electromagnetic laser pulse, heating of the electrons, and losses of the electromagnetic energy. We also see a fast change with time of the ion energy, in spite of the small value of the electron to ion mass ratio. This result is related to the fact that in the K' frame, ions have a large unperturbed velocity $-\mathbf{V}$ in the y direction. Thus the largest contribution to the change of the ion kinetic energy, during the interaction time between the laser pulse and the plasma, arises from the term $\delta\mathscr{E}'=-\mathbf{P}'\cdot\mathbf{V}$ where $\mathbf{P}'$ is the y component of the momentum acquired from the wave. For a p-polarized electromagnetic wave, the value of $\mathbf{P}'$ is given by $\mathbf{P}'\approx(eE_y/\omega')\mathbf{e}_y$ and does not depend on the particle mass. This contribution to the ion energy oscillates with the frequency of the p-polarized wave. In the case of an s-polarized wave (not shown in the figure), the electric field is along the z axis. The y component of the force acting on the particles is proportional to the square of the electric field. As a result, for this polarization, the y component of the particle momentum and the energy of the particles oscillate with frequency $2\omega'$.

C. Ion acceleration in the localized high-frequency field

The nature of the acceleration of the ions and of that of the electrons is different due to their large mass difference. Electrons gain their energy approximately in a period of the laser radiation due to the mechanism of "vacuum heating," or as a result of the wave–particle interaction with the strong Langmuir wave excited in the plasma resonance region. Because of their large mass, ions only react to the slowly varying electric field due to charge separation. This field is produced by the ponderomotive force that arises from the high-frequency fields localized either at the plasma resonance region or in the vicinity of a sharp boundary in the plasma where the electromagnetic wave becomes evanescent. In the case of electrons the ponderomotive force is compensated by the electric field due to the charge separation. Ions instead can gain energy from their interaction with this localized electric field.

In order to demonstrate this, let us calculate the motion in the K' frame of a test particle with mass m_α and electric charge $Z_\alpha e$ in a high-frequency electric field $E'_x(x_0,\tau)$ which is assumed to be a given function of the electron Lagrangian variable x_0 and of time, as given at the plasma resonance by Eq. (19) or by Eq. (21) for a steplike ion density profile. We introduce Lagrangian variables for the electrons $x=x_0+\xi(x_0,\tau)$ and for the test particles $x=x_\alpha+\xi_\alpha(x_\alpha,\tau)$. Following Ref. 23, we separate the motion of the test particles into a slowly varying part X and a fast oscillating part $\tilde{\eta}$ so that $x=\tilde{\eta}+X=x_\alpha+\xi_\alpha(x_\alpha,\tau)\equiv x_0+\xi(x_0,\tau)$. Then we have

$$x_0=X+(\tilde{\eta}-\xi). \tag{52}$$

Inserting these expressions into the equation of motion of the test particles in the electric field $E'_x(x_0,t)$ we obtain

$$\frac{d^2X}{dt'^2}+\frac{d^2\tilde{\eta}}{dt'^2}\approx\frac{Z_\alpha e}{m_\alpha\gamma^3}\left(E'_x(X,t')+(\tilde{\eta}-\xi)\frac{dE'_x(X,t')}{dX}\right). \tag{53}$$

By averaging this equation over the high-frequency part, we obtain

$$\frac{d^2X}{dt'^2}=\frac{Z_\alpha e}{m_\alpha\gamma^3}\left\langle(\tilde{\eta}-\xi)\frac{dE'_x}{dX}\right\rangle. \tag{54}$$

For $\tilde{\eta}(X,t')$ we obtain

$$\frac{d^2\tilde{\eta}}{dt'^2}\approx\frac{Z_\alpha e}{m_\alpha\gamma^3}E'_x(X,t'). \tag{55}$$

Using the approximation $|\xi-\tilde{\eta}|\ll|X|$, we see that $E'_x(X,t')\approx E'_x(x_0t')$ and we can write

$$\tilde{\eta}\approx-\frac{Z_\alpha m_e}{m_\alpha}\xi \tag{56}$$

so that

$$\frac{d^2X}{dt'^2}\approx-\frac{Z_\alpha e^2}{2m_em_\alpha\gamma^4\omega_p^2}\left(1+\frac{Z_\alpha m_e}{m_\alpha}\right)\frac{d\langle E'^2_x(X,t')\rangle}{dX}. \tag{57}$$

The force term on the right-hand side of Eq. (57) is a slowly varying function of time. If the test particles are electrons ($Z_\alpha=-1$ and $m_\alpha=m_e$) this force vanishes, since for electrons the ponderomotive force is compensated for by the electric field due to the charge separation. On the contrary, Eq. (57) indicates that ions can be accelerated. Let us assume that their initial velocity along the x direction vanishes, that is, that the characteristic time of excitation of the high-frequency is much shorter than the time scale of the ion motion. Then from Eq. (57) the change of the ion kinetic energy $\Delta\mathscr{E}'_\alpha$ in the K' frame due to the ponderomotive force is given by

$$\begin{aligned}\Delta\mathscr{E}'_\alpha&=\frac{m_\alpha\gamma}{2}\left(\frac{dX}{dt'}\right)^2\\&=\frac{Z_\alpha e^2}{2m_e\gamma^3\omega_p^2}\left(1+\frac{Z_\alpha m_e}{m_\alpha}\right)(\langle E'_x(X_\alpha)^2\rangle-\langle E'_x(X,t')^2\rangle),\end{aligned} \tag{58}$$

where X_α is the initial coordinate. Under the action of the electric field the ions leave the region where the high-frequency field is localized and their final energy depends on their initial position and is given by

$$\Delta\mathscr{E}'_\alpha=\frac{Z_\alpha e^2}{2m_e\gamma^3\omega_p^2}\left(1+\frac{Z_\alpha m_e}{m_\alpha}\right)\langle E'_x(X_\alpha)^2\rangle. \tag{59}$$

Typical values of the ion energy are $\Delta\mathscr{E}'_\alpha\approx Z_\alpha\mathscr{E}'_e$ with $\mathscr{E}'_e$ the electron quiver energy. The energy spectrum $N'_\alpha(\Delta\mathscr{E}'_\alpha)$ of the fast ions, i.e., the distribution of the number of fast ions accelerated up to the energy $\Delta\mathscr{E}'_\alpha$, can be found from the flux conservation in velocity space

$$\frac{dN'_\alpha}{d\Delta\mathscr{E}'_\alpha}=n'_\alpha\left|\frac{dX_\alpha}{d\Delta\mathscr{E}'_\alpha}\right|, \tag{60}$$

where n'_α is the density of the ion of the α species.

If the high-frequency field is localized in the vicinity of the plasma boundary and decays exponentially with evanescence length c/ω_p, the energy spectrum of the fast ions has the form

$$\frac{dN'_\alpha}{d\Delta\mathscr{E}'_\alpha}=\frac{2n'_\alpha c}{\omega_p\Delta\mathscr{E}'_\alpha}. \tag{61}$$

In the neighborhood of the plasma resonance region the high-frequency field decays as $E'_{\max}\Delta x/[(x-x_p)^2+\Delta x^2]^{1/2}$, with Δx the resonance width. Then the energy of the fast ions depends on Δx, and on $X_\alpha-x_p$ according to

$$\Delta\mathscr{E}'_\alpha\approx\frac{Z_\alpha\mathscr{E}'_e\Delta x^2}{(X_\alpha-x_p)^2+\Delta x^2} \tag{62}$$

and the energy spectrum can be written as

$$\frac{dN'_\alpha}{d\Delta\mathscr{E}'_\alpha}=\frac{n'_\alpha\Delta x}{2}\frac{Z_\alpha\mathscr{E}'_e}{\Delta\mathscr{E}'^{3/2}_\alpha(Z_\alpha\mathscr{E}'_e-\Delta\mathscr{E}'_\alpha)^{1/2}}. \tag{63}$$

Figure 5 shows the acceleration of a minority population of high charge ions ($Z_\alpha=10$) in the plasma resonant region for $a=0.1$ [Fig. 5(a)] and for $a=3$ [Fig. 5(b)]. The phase plane $(P'_{x\alpha},x)$ is shown at different times. We see both the ion acceleration under the action of the ponderomotive force due to the localized high-frequency field up to energies of the order of the ponderomotive potential, and the acceleration due to the vacuum heating of the electrons.

D. Ion acceleration during "vacuum heating" of the electrons

As shown in Sec. IV B, the density of the electron cloud expanding in the vacuum region after a time larger than $\gamma\omega_p^{-1}$ is approximately constant behind the leading

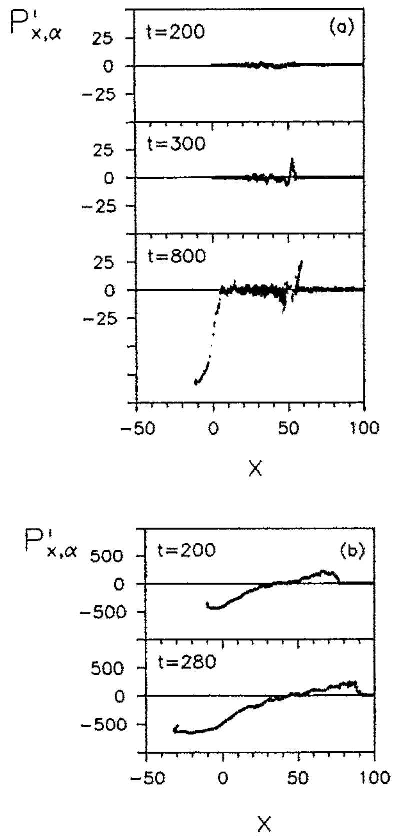

FIG. 5. Acceleration of high charge minority ions in a weakly nonuniform plasma in the case of a weakly relativistic laser pulse (a), $a=0.1$, all the parameters as in Fig. 2, and for a relativistically strong pulse (b), $a=3$, all other parameters as in Fig. 3.

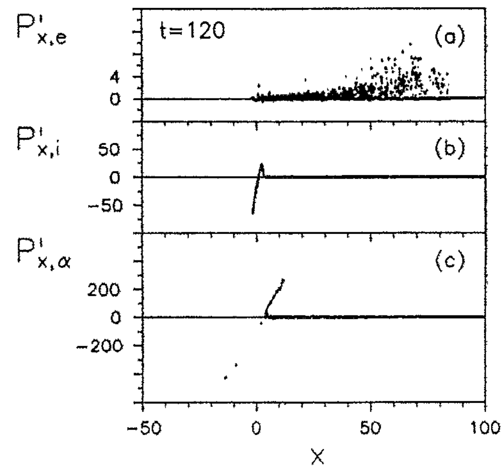

FIG. 6. Heating of electrons (a), acceleration of ambient ions (b), and of high minority ions (c) in a sharp-boundary plasma for a relativistically strong laser pulse, $a=3$, all parameters as in Fig. 4.

front and its value decreases as t'^{-2}. From Eqs. (22) and (51) we find that the electric field in the cloud is given by

$$E'_x \approx -8\pi e n'_0 \frac{x+v_0 t'}{(1-\beta^2)\omega_p^2 t'^2}. \tag{64}$$

This field decelerates electrons and accelerates positively charged ions. Their equation of motion can be written as

$$\frac{d^2x}{dt'^2} = -2\frac{Z_\alpha m_e}{m_\alpha}\frac{x+v_0 t'}{t'^2}, \tag{65}$$

where m_α is their mass and $Z_\alpha e$ their charge. This equation gives

$$x(t') = -v_0 t' + A_1 t'^{\alpha_1} + A_2 t'^{\alpha_2}, \tag{66}$$

where the constants A_1 and A_2 are determined by the initial conditions and $\alpha_{1,2}$ are given by

$$\alpha_{1,2} = [1 \pm (1-8Z_\alpha m_e/m_\alpha)^{1/2}]/2. \tag{67}$$

Since the ratio $Z_\alpha m_e/m_\alpha$ is positive, the ions which start behind the leading front of the electron cloud, which expands with velocity $-v_0$, are accelerated by the electric field given by Eq. (64), but never pass the electrons. Their energy gain can reach values of up to order $m_\alpha \gamma v_0^2/2$.

In Fig. 6 the phase plane (P'_x, x) is presented for electrons [Fig. 6(a)], ambient ions [Fig. 6(b)] and for ions with high electric charge, $m_\alpha = m_i = 1800\, m_e$, $Z_\alpha = 10$ [Fig. 6(c)], at time $t' = 200(2\pi/\omega')$. The high charge minority ions are treated as test particles. These results refer to the simulation of the interaction of a p-polarized laser pulse, with $a=3$, incidence angle $\theta = 35°$, and a number of oscillations per pulse $N_p = 8$. The laser pulse interacts with a plasma slab of thickness $50c/\omega'$. This incidence angle corresponds to the maximum wave absorption. We see the process of "vacuum heating" of the electrons in the vicinity of the left boundary and the acceleration of both ambient and minority ions in the direction of the expansion of the electron cloud into the vacuum region. Such an acceleration process has also been found in the simulation presented in Ref. 24 for a plasma with a single ion species as well as in Ref. 25 for moderate amplitude of the electromagnetic wave simulated in quasistatic approximation. We see that the acceleration of high charge ions is more effective than that of the ambient ions. We also see effective acceleration of the minority ions in the vicinity of the left hand side boundary which can be related to the energy gain in the high-frequency fields localized in the region of field evanescence near this boundary. In Fig. 6(b) the beginning of the motion of the ambient ions towards the center of the plasma slab is shown. This motion too results from the ponderomotive force of the high-frequency field. By comparing Figs. 5(b) and 6 we see that ion acceleration due to electron vacuum heating is more effective in the case of weakly nonuniform plasma. However, the localized high-frequency field accelerates ions towards the dense plasma more effectively in the case of a step-like density profile. From Figs. 5 (linear ion density profile) and 6 (step-like density) we see that in the case of an ultrarelativistic amplitude ($a \approx 3$) the energy of the fast ions accelerated inside the plasma is (in dimensionless units) of the order of 200. This value is larger than that of the ponderomotive potential. It can be related to the effective heating of the electrons up to energies of the order of 10–16 due to the self-intersection of their orbits shown in Figs. 3(a)

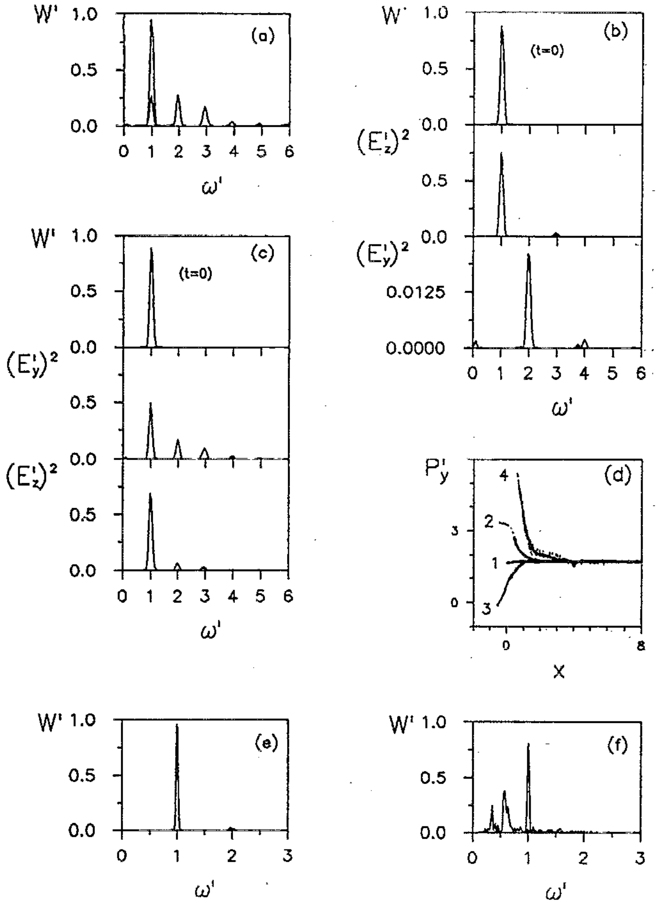

FIG. 7. High harmonic and low-frequency generation for a sharp-boundary plasma and a relativistically strong laser pulse $a=3$ with $N_p=8$, incidence angle $\theta=60°$, and ion density $n_0'=4n_{cr}'$(a)–(d). The incident wave is p-polarized in (a), s polarized in (b), and elliptically polarized in (c). The x dependence of the y component of the electron momentum is shown at different times during the first three periods of the pulse for the same parameters of (a). In (e)–(f) a weakly nonuniform plasma is considered for $a=0.1$ in (e), same parameters as in Fig. 2, and for $a=3$ in (f), same parameters as in Fig. 3.

and 3(b). This in turn leads to the formation, due to charge separation, of a spatially regular electric field which corresponds to an electrostatic potential of the order of the electron energy.

E. High harmonic generation and excitation of relativistic electromagnetic nonlinear pulses near the plasma boundary

We shall now discuss the mechanisms that can lead to the generation of high harmonics of the electromagnetic radiation. In our case, relativistic effects cannot explain high harmonic generation since they give rise to odd harmonics only, while our numerical results indicate that all high harmonics are formed. In addition, energy is also transferred into low-frequency radiation as shown in Fig. 7. In this run a p-polarized wave, with dimensionless amplitude $a=3$ [Fig. 7(a)] interacts with a sharp-boundary plasma with $n_0'=4n_{cr}$. The incidence angle is $\theta=60°$ and $N_p=8$. The reflected wave contains high-frequency harmonics, all with comparable amplitudes and a low-frequency component with frequencies of the order of the inverse of the duration of the laser pulse. In these simulations the electromagnetic radiation reflected from the plasma is produced by a source near the plasma boundary which has the form of a flat charge sheet [Fig. 7(d)]. From the one-dimensional Lienard–Wiechert potential we obtain for the y component of the electric field

$$E_y=\frac{2\pi n'elv_y(t^*)\operatorname{sign}[x-x_0-\xi(t^*)]}{c-(\partial\xi/\partial t^*)}, \tag{68}$$

where v_y is the quiver velocity of the electrons in the y direction, $l\approx c/\omega'$ is the thickness of the reflecting charge sheet, and the retarded time t^* obeys the equation

$$c(t'-t^*)=x-x_0-\xi(x_0,t^*). \tag{69}$$

By expanding $v(t^*)$ and the denominator of Eq. (68) into a series of the ratio $(\partial\xi/\partial\tau)(1/c)$, we see that the reflected electromagnetic radiation contains all harmonics of the wave frequency ω'. For the p-polarized wave shown in Fig. 7(a) ($a=3$, $N_p=8$, $n=4n_{cr}$, and incidence angle $\theta=60°$) $v_x \gg v_y,v_z$, and this ratio is about unity. By comparing Figs. 7(b) and 7(c) we see that the generation of high harmonics by s-polarized waves is much less efficient. In the case of a weakly nonuniform plasma and $a \ll 1$, the efficiency of the high harmonic generation is much lower, as shown in Fig. 7(e). In this case the main effect is the generation of the second harmonic in agreement with the results of Ref. 26. In the case of large electromagnetic amplitudes, Fig. 7(f) shows the frequency spectrum of the radiation reflected from a weakly nonuniform plasma for $a=3$. We see a large shift towards lower frequencies of the carrier frequency and a broadening of the spectrum in the high-frequency range. This effect can be explained by the excitation of wake-field plasma waves[5,6] and by the development of stimulated backward Raman scattering in the underdense plasma region, similarly to the processes investigated both analytically and experimentally in Ref. 27.

An s-polarized wave interacting with a sharp-boundary plasma can excite oscillations localized at the initial time in the vicinity of the boundary. From Fig. 8 we see that these oscillations have the form of a regular electromagnetic nonlinear pulse with relativistic amplitude slowly propagating into the plasma. Figure 9 shows the time evolution of the minimum and maximum values of the electron transverse momentum in this wave. The oscillation frequency is below the Langmuir frequency even when the relativistic correction to the electron mass is taken into account. For the parameters of Fig. 8, the nonlinear electromagnetic pulse propagates inside the plasma, at a speed smaller than the speed of light, for a time interval of order $320/\omega'$, i.e., well beyond the interaction time of the laser pulse, and does not appear to be reflected.

Here we wish to emphasize that a similar excitation of localized relativistically strong electromagnetic waves has been observed in computer simulations of the interaction of a short laser pulse with an underdense plasma.[5] In the case of Ref. 5, these localized waves were produced during the

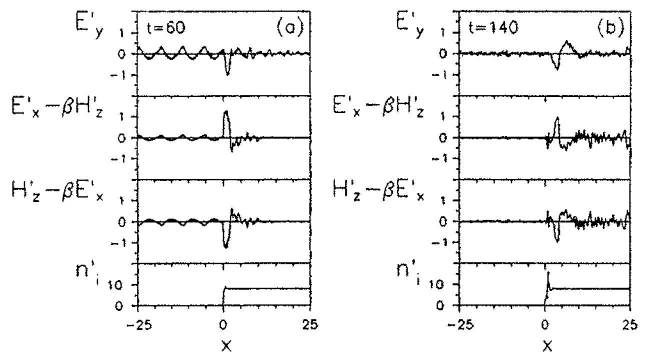

FIG. 8. Excitation of a nonlinear electromagnetic pulse in the vicinity of $x=0$ in a sharp-boundary plasma for an s-polarized laser pulse. ($a=3$, $N_p=8$, $\theta=60°n'_0=4n'_{cr}$) at $t=60/\omega'$ (a) and $t=140/\omega'$ (b). The x dependences of E'_x, E'_y, P'_x and of the combinations $E'_x-\beta H'_z, H'_z-\beta E'_x$ and the profile of the ion density are shown.

downshift of the carrier frequency of the laser pulse in the region where its value became comparable to the local value of the Langmuir frequency. In the case under discussion the s-polarized wave has an amplitude $a=3$ and the carrier frequency is $\omega'=\omega_p/2$. A possible explanation of the excitation of such waves could be the interaction with the plasma resonance at double the carrier frequency.

The analysis of the mode structure, similar to that made in Ref. 5, suggests that this nonlinear electromagnetic pulse is of the type considered in Ref. 12. We want to stress that its mode structure does not correspond to that of the surface waves considered in Sec. III and that it propagates inside a plasma wave. About 10% of the laser pulse energy is transformed into the energy of the nonlinear electromagnetic pulse. The fact that the excitation of relativistic nonlinear electromagnetic pulses has been observed under different conditions, i.e., for different frequencies of the laser radiation and scales of the plasma nonuniformity, leads to the conclusion that, in the case of short laser pulses with ultrarelativistic amplitudes, this process can be regarded as one of the main channels of transformation of the laser radiation and an important mechanism of penetration inside the plasma.

V. CONCLUSIONS

The interaction of intense laser radiation with underdense as well as with overdense plasmas is well understood

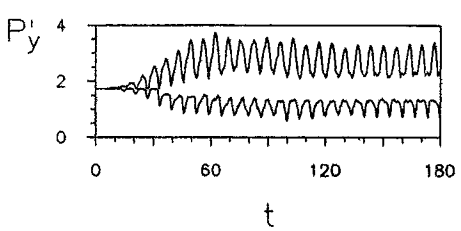

FIG. 9. Time dependence of the minimum and maximum values of the electron momentum in the nonlinear electromagnetic pulse. All parameters as in Fig. 8.

for fairly long laser pulses. However the investigation of the interaction of an ultrashort pulse highlights new physical processes. In the case considered in the present article, where the carrier frequency of the laser pulse is small in comparison with the characteristic value of the Langmuir frequency in the plasma, the polarization of the laser radiation and its incidence angle play an important role, and the processes that occur in the vicinity of the plasma boundary are the most important.

We have shown that, if the laser pulse is fairly short, the Langmuir oscillations excited at the plasma resonance cannot attain a sufficiently large amplitude at the end of the pulse for nonlinear effects to become important. Nevertheless the plasma nonuniformity leads to a change in the wave vector of the plasma oscillations. The phase velocity of the plasma waves decreases down to the value at which self-intersection of the electron trajectories takes place. As a result some of the electrons are thrown out of the plasma resonance region and are accelerated in the process of their interaction with the plasma waves. In the case of a laser pulse with an ultrarelativistic amplitude, this process leads to fast heating of the electrons.

The interaction of the laser pulse with a sharp-boundary plasma leads to the "vacuum heating" of the electrons, i.e., to the expulsion from the plasma into the vacuum region of part of the electrons which leads to strong absorption of the electromagnetic radiation. We have derived an analytical expression for the density of the electron cloud produced by the process of "vacuum heating," and consequently for the time and space dependence of the electric field in the cloud, and have shown that this nonuniform and nonstationary electric field accelerates ions up to a velocity equal to that of the electrons in the cloud front. Minority ions with high electric charge are accelerated to similar velocities in a much shorter time.

Due to the extremely short duration of the laser pulse, ambient ions under the action of the ponderomotive force do not have enough time to move out of the region where the high-frequency fields are localized, either in the plasma resonance region or close to the sharp-boundary plasma. However a small minority of high charge ions can be accelerated in these fields up to energies higher than the quiver energy of the electrons.

Close to a sharp-boundary plasma, a relativistically strong laser pulse produces high harmonic radiation as well as low-frequency radiation. In order to explain the generation of the high harmonics we have taken into account the oscillatory motion of a reflecting charge sheet of electrons in the direction perpendicular to the plasma boundary. The low-frequency part of the radiation has a typical frequency of the order of the reciprocal of the time duration of the laser pulse.

An essentially new phenomenon has been observed in our simulations: a relativistic nonlinear electromagnetic pulse is excited close to the boundary of an overdense plasma. This type of nonlinear electromagnetic pulse had previously been found in the case of the interaction of a short laser pulse with an underdense plasma.[5] Thus we can conclude that the transformation of a significant amount of the energy of the laser pulse into nonlinear electromagnetic pulses may turn out to be an important channel for the absorption of a relativistically strong laser pulse.

Finally we observe that the efficiencies of the processes considered in the present analysis have different dependencies on the incidence angle. This is shown in the figures presented in this article where, for the parameters considered, resonant absorption is optimal at 15°, vacuum heating of the electrons at 35°, and high harmonic generation at 60°.

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