Mathematics Presentation

Euler's Equation

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Outline

Euler's Formula

The Euler's formula in which I'm interested in is

$$e^{ix} = \cos x + i \sin x \tag{1}$$

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Here, I'll use basic arithmatic to prove this.

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4. Using this assumption and the rule of complex multiplication, I'll show that we do get Euler's equation 1.

Step 1

First, I'll show that for a small number x, we have:

$$10^x = 1 + 2.3026x \tag{2}$$

So, how to show this with just arithmatic? What I'll do is that I'll start with 10 and keep taking the square root of it as we already know how to take square root of any number. Doing this, I get the following table:

Step 1

10^x	$\frac{10^x - 1}{x}$	x
10.0	9.0	1
3.16227766	4.32455532	1/2
1.77827941	3.11311764	1/4
1.33352143	2.66817145	1/8
1.15478198	2.47651175	1/16
1.07460782	2.38745050	1/32
1.03663292	2.34450742	1/64
1.01815172	2.32342037	1/128
1.00903504	2.31297147	1/256
1.00450736	2.30777049	1/512
1.00225114	2.30517585	1/1024
1.00112494	2.30387998	1/2048
1.00056231	2.30323241	1/4096
1.00028111	2.30290872	1/8192
1.00014054	2.30274690	1/16384
1.00007027	2.30266599	1/32768
1.00003513	2.30262554	1/65536
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As we can see, as x gets smaller, the value of $\frac{10^x - 1}{x}$ goes to a constant value of 2.3026. Which suggests that for a small x, we'll have:

$$\begin{aligned} \frac{10^x - 1}{x} &= 2.3026 \\ 10^x &= 1 + 2.3026x \end{aligned} \tag{3}$$