COL352 Assignment

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1 Problem

Let L be a infinite Turing recognizable language.Let M enumerates L Take another language L' where $L' = \{s_1, s_2, ...\}$ where s_1 is the first string enumerated by M and $s_i > s_{i-1}$ lexicographically $\forall i \geq 1$.
such that s_i is the just next string of s_{i-1} enumerated by M.
Claim1: $L \subseteq L$ and L is infinite.
Proof: Let us assume the opposite L' is not infinite.$\Rightarrow$ L' is finite of size n.Then s_n would be the last string of L'.Hence s_n must also be the last string enumerated by M.This implies that L is finite as all stings enumerated by M are lexicographically less than s_n.
Contradiction.
Hence L' is infinite. Every string present in L' is enumerated by M.Hence that must also be present in L. Hence $L' \subseteq L$. Our claim is true.
Claim2: L' is decidable.
Proof: If we can construct a enumerator M' for L' then L' would be decidable. We can construct M' as follows for an input x,run M' and compare x to string y produced.
If $x = y$, then accept the input.otherwise,when $|y| > |x|$,reject it. We will reach a condition out of these two after finite no. of steps.
Hence M' is enumerator for L'.Hence our claim is true.
Hence L' is infinite decidable subset.Hence every infinite Turing recognizable language has an infinite decidable set.

2 Problem

To Prove: Single tape TMs that cannot write on portion of the tape containing the input recognize only regular language.

Proof: Let T be the Turing machine as mentioned then $T = (Q, \Sigma, \tau, q_0, q_{acc}, q_{rej})$.It works by switching from input tape to output tape and then back from output tape to input tape.

Now we know by pumping lemma that regular language has a partition abc such that $\forall i > 0$ ab^ic also belongs to set of regular languages if abc belongs to set of regular languages.

Now in our Turing Machine M if we cannot write on portion of tape containing input then we know that input length is fixed.Let's say n and for that we cannot only pump a portion only when there is a change of head pointer i.e head pointer move from input to output or vice-versa.But that is not the case as we cannot write on input tape.Thus all the string formed in this case can be pumped and hence form a set of regular language only.

3 Problem

To Prove: C is Turing recognizable iff $\exists$ a decidable language D such that $C = \{x| \ \exists \ y((x, y) \in D)\}$

Proof: Here we will be proving both sides.Let's start in backward direction i.e. if $\exists$ a decidable language A such that $C = \{x| \ \exists \ y((x, y) \in D)\}$ then we will form a Turing Machine T and will take every possible string y so as to check $(x, y) \in D$. If $\exists$ atleast one such string y then T will accept that particular $x \in C$.Thus C will be Turing recognizable.

Now we will move the other way i.e if C is Turing recognizable then $\exists$ decidable language D such that $C = \{x| \ \exists \ y((x, y) \in D)\}$.

As C is Turing recognizable.Let's say that Turing Machine be T'.Then we define D as $((x, y)|T$ only accepts x in atmost y steps)

Claim: D is decidable.

Proof: This is because we only move up to y steps and check whether it has been accepted before and thus its decidebality is checked.

Thus, we have $x \ \exists \ C$ only when the above condition is satisfied for atleast one decidable language D. Hence $\exists$ D such that $C = \{x| \ \exists \ y((x, y) \in D)\}$.

4 Problem

5 Problem

Given: M Turing Machine, w input string when its head is on the left-most tape cell
Language: M moves its head left on input string w when its head is on the left-most tape cell.
To Prove: Show L is undecidable.
Proof: Let's say $\exists$ a Turing Machine T for which L is decidable.
Now make a Turing Machine T' from T such that T' move tape right by one position and then mark the left point with . Now as we run T' on our language L then there is two cases:
A.) If the head reads
B.) If M halts or accept the string w.
In case A.) the Machine stays at the same state where in the case B.) the head keeps moving to the left. Now let's say we runs X on Machine $T \Rightarrow T$ accepts if X accept otherwise reject.M shifts the head to the left if it accepts w and the head is in the leftmost tape cell. If T decide the halting problem $\Rightarrow$ decidable where as halting problem is undecidable. This contradicts our result.
Hence the language L is Turing undecidable.

6 Problem

Given: M Turing Machine, w input string
Language: $L = \{(M, w)$ where M moves its head to the left when input w is received$\}$
To Prove: Show L is decidable.
Proof: We can show that this language is decidable if we can find a Turing machine for it.
We can construct Turing machine T such that T on receiving the input (M, w) first runs for $n + |w| + 1$ steps and checks if the head of M is moved towards left or not (where n is the number of states). If it is moved towards left, then accepts otherwise rejects. Hence we can see that there exists a turing machine T for language L. Hence language L is decidable.