Mathematical foundations

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Q1) find the maximum & minimum value of the function $x^3-3x^2-9x+12$

Ans:- $f'(x) = 3x^2-6x-9$

$\frac{d}{dx} x^n = nx^{n-1}$

Put $f'(x)=0$

$3x^2-6x-9=0$

$x^2-2x-3=0$

$x^2-3x+x-3=0$

$x(x-3)+1(x-3)=0$

$(x-3)(x+1)=0$

$x=3 \quad x=-1$

$\to f''(3) = 6\times3-9 = 18-9 = 9$

$f''(3) > 0$

$f(x)$ is minimum at $x=3$

Minimum value $= f(3)$

$(3)^3-3(3)^2-9\times3+12$

$= 27-27-27+12$

$= -15$

Hence minimum value of $f(x) = -15$

Q2

Ans

Q3

when $x = -1$, we get

$\to f(-1) = 6\times(-1) - 9 = -6-9 = -15$

$f''(-1) < 0$

$f(x)$ is maximum at $x = -1$

$(-1)^3 - 3(-1)^2 - 9(-1) + 12$

$= -1 - 3 + 9 + 12$

$= 17$

Hence Maximum value of $f(x) = 17$

Q2) Calculate the slope & the equation of a line which passes through the points $(-1,-1), (3,8)$

Ans:- $m = \dfrac{y_2 - y_1}{x_2 - x_1}$

$= \dfrac{8-(-1)}{3-(-1)} = 9/4 = 2.25$

Q3) Solve for $w(z)$ when

$w(z) = \dfrac{4z-5}{2-z}$

$w(z) = 4z - 5 - 2 - z$

$\Rightarrow 4z - z - 5 - 2 = 0$

$3z - 7 = 0$

$z = 7/3$

$\boxed{z = 7/3}$ (or) $\boxed{z = 2.33}$

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Qu) Consider $y = 2x^3 + 6x^2 + 3x$. Identify the critical value & verify if it is the maxima (or) minima

Ans :- $\cancel{f(x) = 2x^3 - 3x^2 - 36x - 3}$

$\cancel{f'(x) = 6x^2 - 6x - 36}$

$\cancel{6x^2 - 6x - 36 = 0}$

$\cancel{x^2 - x - 6 = 0}$

$\cancel{(x-3)(x+2) = 0}$

$\cancel{x = 3 \quad x = -2}$

$y(x) = 2x^3 + 6x^2 + 3x$

$y'(x) = \dfrac{dy}{dx} = 6x^2 + 12x + 3$

$y''(x) = \dfrac{d^2y}{d^2x} = 12x + 12$

Critical point :-

$\dfrac{dy}{dx} = 0 \Rightarrow 6x^2 + 12x + 3 = 0$

$= 2x^2 + 4x + 1 = 0$

$x = -1 - \dfrac{1}{\sqrt{2}}, \; -1 + \dfrac{1}{\sqrt{2}}$

Maxima (or) minima

at $x = -1 - \dfrac{1}{\sqrt{2}}$

$= \dfrac{d^2y}{d^2x}_{x = -1 - \frac{1}{\sqrt{2}}} = 12\left[\cancel{-1} - \dfrac{1}{\sqrt{2}} + \cancel{1}\right]$

$= 12\left(\dfrac{-1}{\sqrt{2}}\right) < 0$

critical minima

$\therefore y(x)$ has maximum value at $x = -1 - \frac{1}{\sqrt{2}}$

max value is $y\left(-1 - \frac{1}{\sqrt{2}}\right) =$

at $x = -1 + \frac{1}{\sqrt{2}}$

$\Rightarrow \frac{d^2y}{dx^2}$

$x = -1 + \frac{1}{\sqrt{2}} = 12\left[\cancel{-1} + \frac{1}{\sqrt{2}} \cancel{+1}\right]$

$= 12\left(+\frac{1}{\sqrt{2}}\right) > 0$

$\therefore y(x)$ has minimum value at $x = -1 + \frac{1}{\sqrt{2}}$

min value is $y\left(-1 + \frac{1}{\sqrt{2}}\right) = \ldots$

Q5) Determine the critical point & obtain relative minima (or) maxima of a function defined by

$y = 2x_1^2 + 2x_1x_2 + 2x_2^2 + 6x_1$

$= 4x_1 + 2x_2 + 6$

$4x_1 + 2x_2 + 6 = 0$

$2x_1 + x_2 + 3 = 0$

$x_1 = 4 \; ; x_2 = 2 \qquad x_2 = 4x_2 + 2x_1$

$4x_2 + 2x_1 = 0$

$2x_2 + x_1 = 0$

$x_1 = 2x_2$

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$2x_1 + x_2 + 3 = 0$

$4x_2 + x_2 + 3 = 0$

$3x_2 = 3$

$x_2 = 1$

$x_1 = 2$

Critical point

$(-2, 1)$

Minima & maxima

$(4 \times 4) - 2^2 = 16 - 4 = 12 > 0$

$\therefore$ we get the critical point as $(2-1)$ which is the relative maxima of the function.

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