Question 2

Given, a coin that shows heads with a probability p, and tails with a probability (1 - p).

q_n represents the probability that after n-tosses, there are an even number of heads.

Two cases are possible; at a time n, there may already be an even number of heads; here, $p' = q_{n-1}(1-p)$

Alternatively, there may be an odd number of heads at a given time n. This would occur with a probability of $1 - q_{n-1}$; therefore the probability of even heads would be $p'' = (1 - q_{n-1})p$

$q_n = p' + p''$

$q_n = (1 - q_{n-1})p + q_{n-1}(1-p)$

$q_n = (1-2p)q_{n-1} + p$

$q_n = (1-2p)^n + p(1 + (1-2p) + (1-2p)^2 + (1-2p)^3 + ... + (1-2p)^n)$

$q_n = (1-2p)^n + \frac{p(1-(1-2p)^n)}{1-(1-2p)}$

$q_n = (1-2p)^n + \frac{1-(1-2p)^n}{2}$

$q_n = \frac{1+(1-2p)^n}{2}$