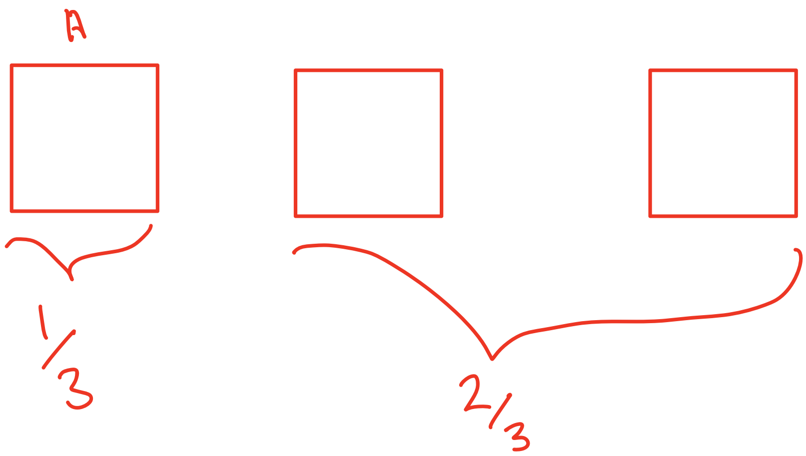
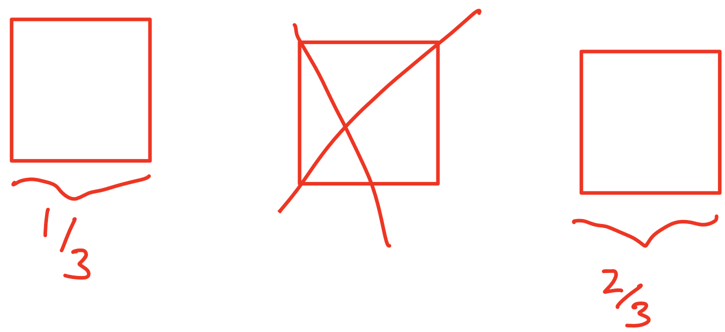
Monty hall problem.
Let us consider the 3 door case first.
Now when the contestant chooses a door(say A) there is a 1/3 probability that door A has the prize and there is a 2/3 probability that one of the other 2 doors has the prize.



Now when the host opens one door which is empty the 2/3 probability goes to the remaining door



So it's better to switch.

<u>Alternate explanation:</u>

If the door chosen by the contestant does not have the prize (p=2/3).then switching will get him the prize.
If the door chosen by the contestant has the prize then not switching will get him the prize.
Since 2/3>1/3. It is always better to switch.
So if the contestant always switches he/she will have a 2/3 chance of winning.

<u>Extension to n door problem</u>
Now if the problem is extended to n doors and the host opens n-2 empty doors after the contestant chooses a door.

If the contestant chooses a empty door initially (p=n-1/n) switching will get him the prize.
If the contestant initially chooses the door with the prize(p=1/n) then not switching will give him the prize.

Since n-1>1. The contestant should switch.

If the contestant switches his probability of winning the prize is n-1/n.

<u>If the host opens only p empty doors after contestant chooses a door:</u>

If the contestant initially chooses an empty door(probability =n-1/n). After this the host will open p empty doors. So now switching will give him the prize with a probability of

$$= \frac{\text{no of doors with prize}}{\text{no of doors to choose from}} = \frac{1}{n-p-1}$$

If the contestant initially chooses the door with the prize(probability =1/n). After this the host will open p empty doors. So now not switching will give him the prize with a probability of

$$\frac{1}{1} = 1$$

So the probability of winning by switching is $= \frac{n-1}{n}\left(\frac{1}{n-p-1}\right)$

The probability of winning by not switching is $= \frac{1}{n} \times 1 = \frac{1}{n}$

Since $\frac{n-1}{n}\left(\frac{1}{n-p-1}\right) > \frac{1}{n}$

It is always better to switch !