Strategy Wars

Mathematics Club, $C\Phi$

September 14th 2023

1 A primer on Dice Questions

a) What is the probability when two dice A and B are rolled, die A will show a number strictly greater than the die B?

b) Three dice are rolled together. What is the probability of getting a sum greater than or equal to 6?

c) There is a higher probability of getting 3 consecutive sixes in 3 throws than getting sum of numbers equal to 10 in 3 throws. Is this statement true or false?

d) I throw a die and keep rethrowing it until I get an odd number. What is the probability that I make 3 or more consecutive throws?

2 A chance at redemption

2c plays a game of dice. The game goes as follows: 2c rolls the die once. His score is what he rolls.

Now, the Grand Master offers him a chance: if 2c is not happy with his roll, he can roll the die again. His old score will be ignored and his new score will be what he rolls. This re-rolling is not compulsory.

What does 2c do?

The *optimal* strategy in this case is as follows: 2c rolls the die once. If he rolls a 1, 2, or a 3, he's not happy with it and re-rolls, while he's perfectly happy with a 4, 5, or a 6 and does not re-roll.

a) What is the expected score if the optimal strategy is used?

b) Now, the grand master says he can re-roll one more time (there are three rolls in total, including the first one). What is the optimal strategy, i.e. when should you re-roll?

c) For the above question, what is the expected value?

d) Now, we roll for a total of n times. We see that as n gets larger and larger, you can keep re-rolling until you get a 6 (and your expected score keeps approaching 6). What is the minimum value of n, for which your strategy is to "re-roll until you get a 6"?

3 Stones Game

There is a single pile of n stones. There are two players, Krishna and Skandan. In each turn, they can remove $1, 2$ or 3 coins. Krishna goes first. The player who removes the last coin wins.

A sample game with $n = 7$: Krishna removes 3 coins, now there are 4 coins remaining. Skandan removes 2 coins, now there are 2 coins removing. Krishna removes 2 coins and wins the game.

It turns out that there exists an optimal strategy, and if both players play optimally, the winner depends on the value of n only. Find the winner for n from 1 to 10. Write "K" if Krishna wins and write "S" if Skandan wins.

Hint: Start from $n = 1$ and see what happens.

4 Stones Game: 2

There is a single pile of n stones. There are two players, Krishna and Skandan. In each turn, they can remove $1, 3, 4$, or 5 coins. Krishna goes first. The player who removes the last coin wins.

A sample game with $n = 14$: Krishna removes 3 coins, now there are 11 coins remaining. Skandan removes 5 coins, now there are 6 coins removing. Krishna removes 1 coin, now there are 5 coins removing. Skandan removes all 5 coins and wins the game!

It turns out that there exists an optimal strategy, and if both players play optimally, the winner depends on the value of n only. Find the winner for n from 11 to 20. Write "K" if Krishna wins and write "S" if Skandan wins.

Hint: Start from $n = 1$ and see what happens.

5 Nim

We're given n piles of stones. Karthikeya and Atreya play the game, with Karthikeya going first. In each turn, they can remove how many ever stones they want from one particular pile. Atreya and Karthikeya can remove coins from the same pile if they want to. The configuration $(10, 5, 3)$ means there are 3 piles: one with 10 stones, one with 5 stones, and one with 3 stones.

For the given configurations, who wins? The number of stones in each pile is represented with brackets. If Karthikeya wins, write "K" and if Atreya wins, write "A".

a) (10)

b) $(1, 1)$

c) $(10, 1)$

d) $(10, 10)$

e) $(10, 15)$

f) $(10, 5, 6)$

g) $(10, 5, 6, 9)$

6 The Dacoit-Merchant Problem

There are n *dacoits* and 1 *merchant* in Dholakpur. *Chota Bheem* has placed an enchantment such that if a Dacoit robs a *merchant* he will become a Merchant himself, however if two or more *dacoits* rob a *merchant*, the *dacoit* who will first rob the *merchant* will become a *merchant* (nothing happens to the other dacoits.). *Dacoits* would love to become a *merchant* but do not want to get robbed after they become a merchant. All *merchants* of Dholakpur are intelligent. Assume that the *dacoits* are equally skilled in robbing people. Will the initial merchant be robbed or not? Comment for general n.

7 Bumble

Consider the number line of integers from 0 to n. Some points on the number line have coins on them. Gorlaid and Lex play a game, in which Gorlaid goes first. In each turn, exactly one coin can be moved to any point to its left on the number line. Note that one coin has to be moved to the left, i.e. you cannot skip a turn. Once all the coins are at $x = 0$, the game ends and the player who moved the last coin wins. Assuming they both play optimally, who wins for the below configurations?

a) There is a single coin at $x = 1$

b) There are two coins, one at $x = 5$ and the other at $x = 8$.

c) There are 6 coins, at $x = 1, 2, 5, 7, 10, 11$.

8 Game of Queens

A queen in a chessboard can move horizontally, vertically, or diagonally. Let's change it up a bit. We have a chessboard, and the Queen can go any number of squares to the left, down, or to the bottom-left diagonally only. Both players sit on the same side of the chessboard. In our chessboard the tiles are labeled as an ordered pair (a, b), starting from $(0, 0)$. Sreejaa and 2C take turns in playing the game, with Sreejaa going first. The person who moves the queen first to the point $(0, 0)$ wins.

It turns out that the winner depends only on the starting position. For instance, if the Queen was at (m, m) or $(0, m)$ or $(m, 0)$ initially, Sreejaa will always win because she can move it to $(0, 0)$ in one step.

For what all starting positions (a, b), $a \leq 5, b \leq 5$ does Sreejaa win?

9 Simple Dice Game

Suppose we play a game with a die where we roll and sum our rolls. We can stop any time and take the sum as our score, but if we roll a face we've rolled before then we lose everything (i.e. our score becomes 0) and stop the game. What strategy will maximize our expected score?

10 More Dice Games

Consider a game on creating a two-digit number in which the player rolls the die once and decides which of the two digits they want that roll to represent, i.e. "ones" or "tens" digit. Then, the player rolls a second time and this determines the other digit. For instance, the player might roll a 5, and decide this should be the "tens" digit, and then roll a 6, so their resulting number is 56.

What strategy should be used to create the largest number on average, i.e. what strategy should be used to get the greatest expected value of a number?

Strategy Wars

Answer Sheet

Name of player 1: _____

Roll number of player 1: _____

Name of player 2: _____

Roll number of player 2: _____

Contact Number (Any one): _____

1. a)

   b)

   c) True / False

   d)

2. a)

   b)

   c)

   d)

3.

n	1	2	3	4	5
Ans:					
n	6	7	8	9	10
Ans:					

4.

n	11	12	13	14	15
Ans:					
n	16	17	18	19	20
Ans:					

5. a)

   b)

   c)

   d)

   e)

   f)

   g)

6.

7. a)

   b)

   c)

8.

$(0, 0)$					

9.

10.