Problem Set 1

Mathematics Club, IITM

3rd November 2022

1 Problems from the Presentation

1. For the configuration of Bishops in a chessboard, we found that the maximum number of Bishops $n \times n$ chessboard was $2n - 2$. In how many ways can you do this?
2. For the configuration of Knights in a chessboard, how many knights can you place in a $n \times n$ chessboard and in how many ways?
3. Prove that $\lim_{n\to\infty} \left(1 + \frac{r}{n}\right)^n = e^r$
4. Solving for Lucas Numbers: Take $F_0 = 2, F_1 = 1, F_k = F_{k-1} + F_{k-2}, k \geq 2$
   (Brownie Points for finding out its relationship with Fibonacci Numbers)
5. Prove Strong induction using regular induction. While using strong induction, is one base case always enough, or do you need more?

A sidenote: We obtained the chessboard problems from Challenging Mathematical Problems with Elementary Solutions

2 More Problems

1. Differential Equations using Matrices: Just like the Fibonacci Sequence, it's possible to solve differential equations using matrices. This arises from an interesting definition: For a matrix A, define

$$e^A = I + A\frac{1}{1!} + A^2\frac{1}{2!} + A^3\frac{1}{3!} + \dots$$

   Here, we can write $A = PDP^{-1}$, for a diagonalizable matrix A.

   Using this, solve the differential equation for simple harmonic oscillator:

$$\frac{d^2x(t)}{dx^2} + w^2x(t) = 0$$

We'll sign off with a hint about the integration bee: Feynman Integration!