ANALOG SYSTEMS : PROBLEM SET 8

Problem 1

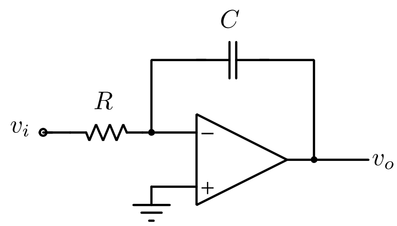


Figure 1: Circuit for Problem 1.

Fig. 1 shows an integrator. The opamp is ideal. The capacitor is initially uncharged. $v_i = \sin(\omega_o t)u(t)$, where $\omega_o = 1/RC$ and $u(t)$ is the unit step function. Draw to scale, on the same graph, v_i and v_o. Repeat with $v_i = \cos(\omega_o t)u(t)$.

Problem 2

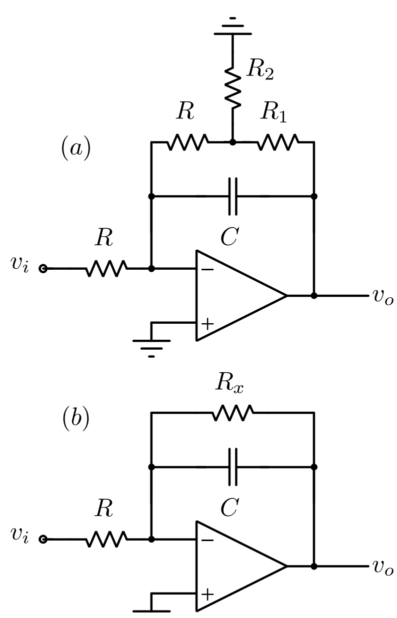


Figure 2: Circuit for Problem 2.

All opamps are ideal in Fig. 2. Determine the dc gain and 3-dB bandwidth of the circuit of Fig. 2(a). What R_x should be chosen in the circuit of Fig. 2(b) to obtain the same transfer function?

Evaluate R_x in the limiting case when $R_1, R_2 \ll R$. What might be the utility of the T-network in Fig. 2(a)?

Problem 3

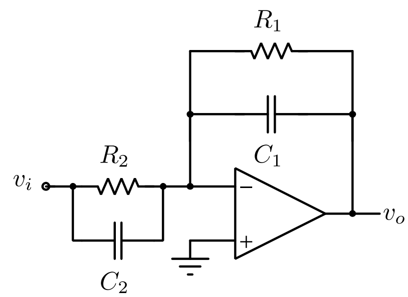


Figure 3: Circuit for Problem 3.

Determine the transfer function of the circuit of Fig. 3. Sketch a Bode plot.

Problem 4

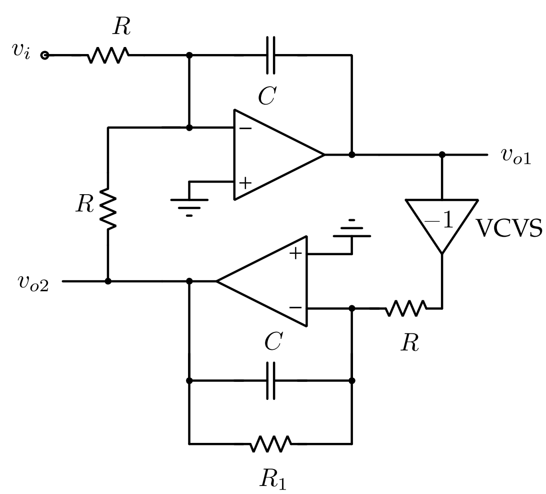


Figure 4: Circuit for Problem 4.

The opamps are ideal. Determine the transfer functions from the v_i to v_{o1} and v_{o2}.

Problem 5

The opamps are ideal. The initial conditions are marked. Plot the waveforms v_{o1} and v_{o2}.

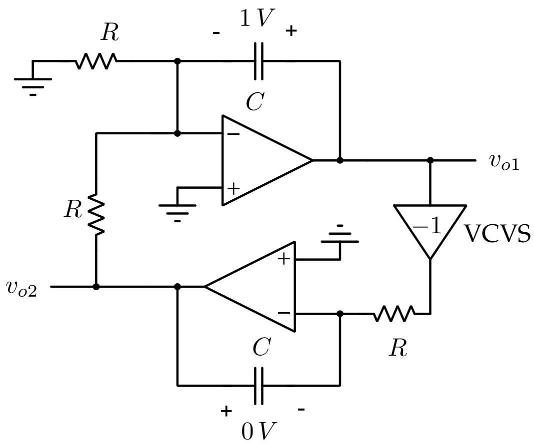


Figure 5: Circuit for Problem 5.

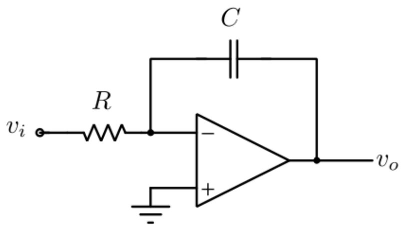
Problem 6

Consider the integrator of Fig. 1. The opamp is not ideal, but has a frequency dependent gain determined by GB/s, where GB denotes its gain-bandwidth product. Determine the integrator's transfer function, when a nonideal opamp is used.

Problem 7

Use the results of Problem 6 to evaluate the transfer function of the circuit of Fig. 4 when the opamps have a finite gain-bandwidth product. The VCVS can be assumed to be ideal.

Problem 1



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$$V_o sC = \frac{V_i}{R}$$

$$\frac{V_o}{V_i} = \frac{-1}{RsC}$$

Rough

$$\cos\omega_o t = \frac{e^{j\omega} + e^{-j\omega}}{2}$$

$$\frac{1}{2}\left(\frac{1}{s+j\omega} + \frac{1}{s-j\omega}\right)$$

$$\frac{1}{2}\left(\frac{2s}{s^2+\omega^2}\right) = \frac{s}{s^2+\omega_o^2}$$

$$\sin\omega_o t \longleftrightarrow \frac{\omega_o}{s^2+\omega_o^2}$$

$$V_i = \sin\omega_o t \, u(t)$$

$$V_i(s) = \frac{\omega_o}{s^2+\omega_o^2}$$

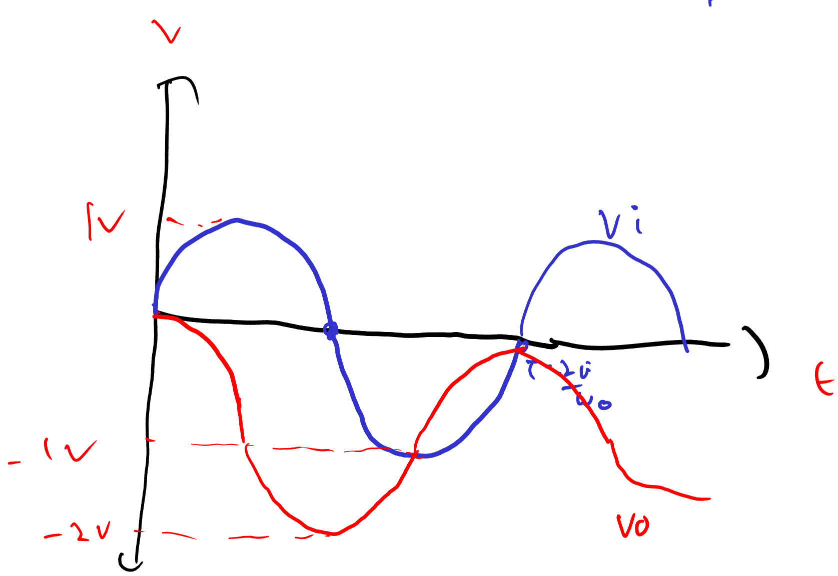
$$V_o = -\frac{1}{sRC}\frac{\omega_o}{s^2+\omega_o^2}$$

$$= -\left(\frac{\omega_o}{RC}\right)\frac{1}{s(s^2+\omega_o^2)}$$

$$= -\left(\frac{\omega_o}{RC}\right)\left[\frac{1}{s} - \frac{s}{s^2+\omega_o^2}\right]\frac{1}{\omega_o^2}$$

$$= -\frac{1}{\omega_o RC}\left[u(t) - \cos\omega_o t \, u(t)\right]$$

$$= \cancel{\frac{1}{\omega_o RC}}\left[\cos\omega_o t - 1\right]u(t)$$



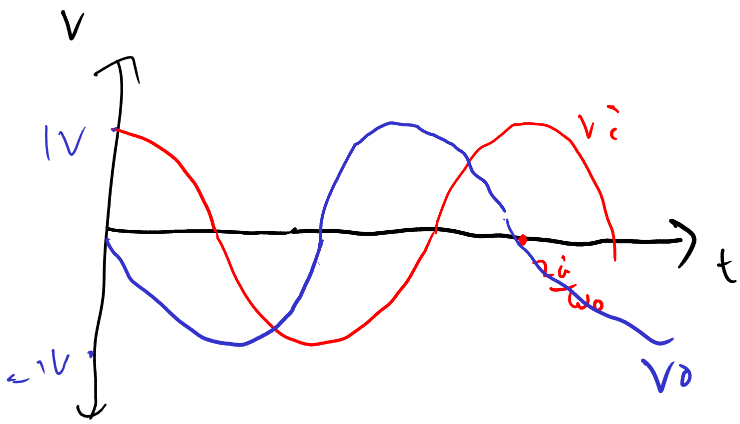
$v_i = (\cos\omega_0 t)u(t)$

$\mathcal{L}$ $V_i = \dfrac{s}{s^2+\omega_0^2}$

$V_0(s) = \left(\dfrac{1}{RC}\right)\dfrac{s}{s^2+\omega_0^2}\;\dfrac{1}{s}\;(-1)$

$= \left(\dfrac{1}{RC}\right)\dfrac{1}{s^2+\omega_0^2}\;(-1)$

$v_0(t) = \cancel{\left(\dfrac{-1}{RC}\right)}\dfrac{\sin\omega_0 t}{\omega_0} = -\sin\omega_0 t$



Problem 2

a) At node 1,

$$\frac{x}{R_2} + \frac{x - V_0}{R_1} + \frac{x}{R} = 0$$

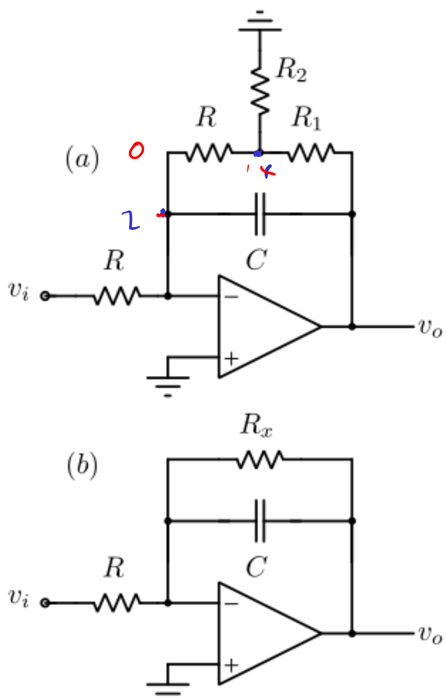


Figure 2: Circuit for Problem 2.

$$\frac{0 - v_i}{R} + (0 - V_0)sC + \frac{0 - x}{R} = 0$$

$$-\left(\frac{V_i}{R} + V_0 sC\right) = \frac{x}{R} = \frac{\left(\frac{V_0}{RR_1}\right)}{\frac{1}{R} + \frac{1}{R_1} + \frac{1}{R_2}}$$

$$-\frac{V_i}{R} = \frac{V_0\left(\frac{1}{RR_1}\right)}{\frac{1}{R} + \frac{1}{R_1} + \frac{1}{R_2}} + V_0 sC$$

All opamps are ideal in Fig. 2. Determine the dc gain and 3-dB bandwidth of the circuit of Fig. 2(a). What R_x should be chosen in the circuit of Fig. 2(b) to obtain the same transfer function?

Evaluate R_x in the limiting case when $R_1, R_2 \ll R$. What might be the utility of the T-network in Fig. 2(a)?

$$V_i = -V_0 \frac{\left[\frac{1}{RR_1} + sC\left[\frac{1}{R} + \frac{1}{R_1} + \frac{1}{R_2}\right]\right]}{\left(\frac{1}{R} + \frac{1}{R_1} + \frac{1}{R_2}\right)} R$$

$$\frac{V_o}{V_i} = -\frac{1}{R}\frac{\left(\frac{1}{R} + \frac{1}{R_1} + \frac{1}{R_2}\right)}{\frac{1}{RR_1} + sC\left(\frac{1}{R} + \frac{1}{R_1} + \frac{1}{R_2}\right)}$$

b) $V_0\left(sC + \frac{1}{R_x}\right) = -\frac{V_i}{R}$ $\qquad \frac{V_o}{V_i} = -\left(\frac{1}{R}\right)\frac{1}{\left(sC + \frac{1}{R_x}\right)}$

$$\boxed{\frac{1}{R_x} = \frac{1}{RR_1}\frac{1}{\left(\frac{1}{R} + \frac{1}{R_1} + \frac{1}{R_2}\right)}}$$

DC gain = $\boxed{-R_1\left[\frac{1}{R}+\frac{1}{R_1}+\frac{1}{R_2}\right]}$

$$\omega_{3dB} = \left(\frac{1}{CR_x}\right) = \frac{1}{CRR_1\left(\frac{1}{R}+\frac{1}{R_1}+\frac{1}{R_2}\right)}$$

Limiting Case

$$R_x = RR_1\left(\frac{1}{R}+\frac{1}{R_1}+\frac{1}{R_2}\right) \simeq \boxed{R\left(1+\frac{R_1}{R_2}\right)}$$

Same as inverting amplifier, so if $R_2 \ll R_1$,

R_x effective can be very large.

Problem 3

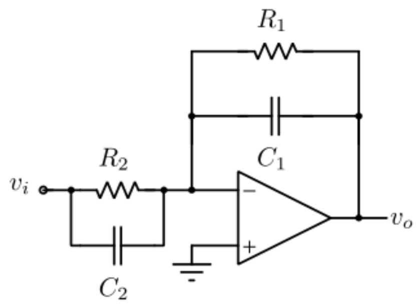


Figure 3: Circuit for Problem 3.

Determine the transfer function of the circuit of Fig. 3. Sketch a Bode plot.

$$Z_1 = \frac{1}{\frac{1}{R_1} + sC_1} = \frac{R_1}{1+sCR_1}$$

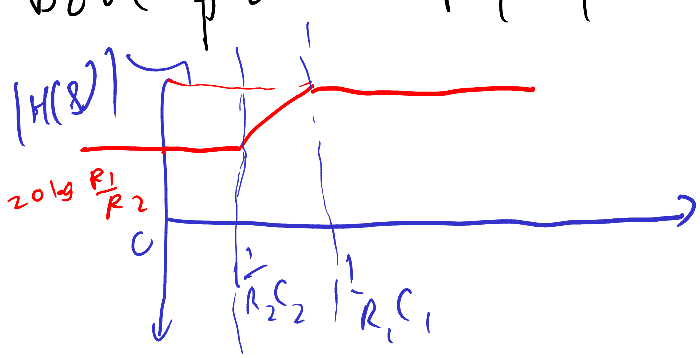
$$Z_2 = \frac{R_2}{1+sCR_2}$$

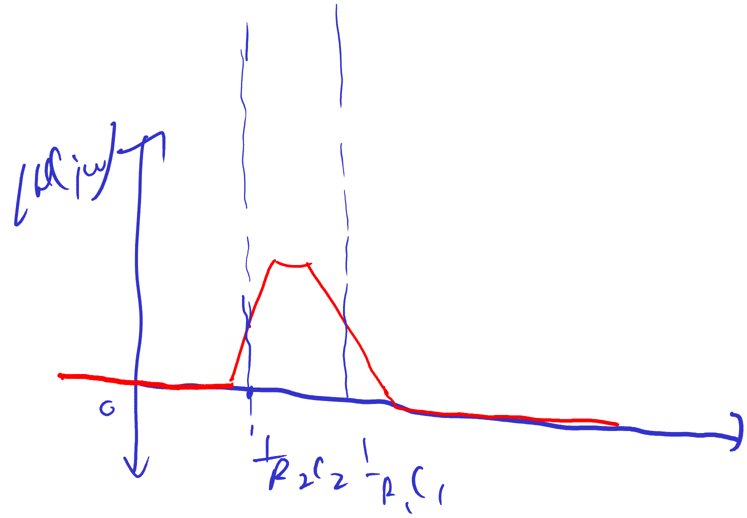
$$\frac{V_i}{Z_2} = -\frac{V_o}{Z_1}$$

$$V_o = -\frac{Z_1}{Z_2} V_i$$

$$H(s) = -\frac{\frac{R_1}{1+sC_1R_1}}{\frac{R_2}{1+sC_2R_2}} = -\frac{R_1}{R_2}\left(\frac{1+sC_2R_2}{1+sC_1R_1}\right)$$

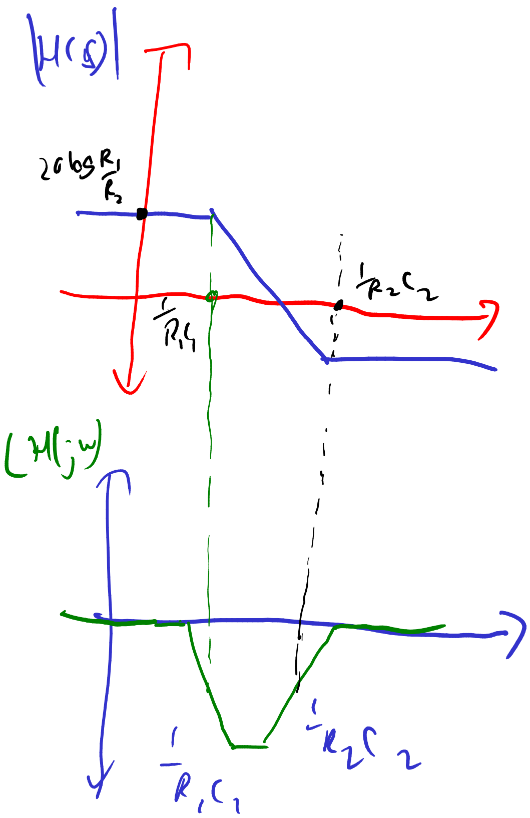
Bode plot: $R_1C_1 < R_2C_2$





Bode plot

$R_1C_1 \gg R_2C_2$



Problem 4

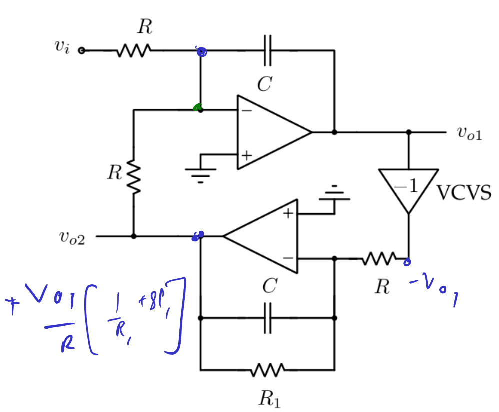


Figure 4: Circuit for Problem 4.

The opamps are ideal. Determine the transfer functions from the v_i to v_{o1} and v_{o2}.

$$\frac{V_{o1}}{R} = +V_{o2}\left[\frac{1}{R_1}+sC\right]$$

$$\frac{V_i}{R} + \frac{V_{o2}}{R} + (V_{o1})sC = 0$$

$$\frac{V_i}{R} + \frac{V_{o1}}{R^2\left[\frac{1}{R_1}+sC\right]} + \frac{RV_{o1}(sC)}{R} = 0$$

$$V_i = -V_{o1}\left[\frac{1}{R\left(\frac{1}{R_1}+sC\right)} + RsC\right]$$

$$= -V_{o1}\left[\frac{1}{\frac{R}{R_1}+RsC} + RsC\right]$$

$$V_i = -\frac{V_{o1}}{\frac{R}{R_1}+RsC}\left[1+(RsC)\left(\frac{R}{R_1}+RsC\right)\right]$$

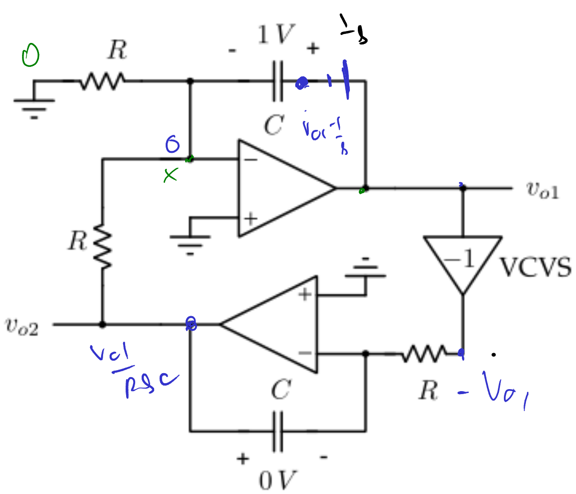
$$\boxed{\frac{V_{o1}}{V_i} = \frac{-\left(\frac{R}{R_1} + RsC\right)}{1+(RsC)\left(\frac{R}{R_1}+RsC\right)}}$$

$$\frac{V_{o2}}{V_i} = \frac{V_{o2}}{V_{o1}}\frac{V_{o1}}{V_i} = \frac{1}{-R\left(\frac{1}{R_1}+sC\right)} \cdot \frac{-\left(\frac{R}{R_1}+RsC\right)}{1+(RsC)\left(\frac{R}{R_1}+RsC\right)}$$

$$= \boxed{\frac{-1}{1+RsC\left(\frac{R}{R_1}+RsC\right)}}$$

Problem 5

The opamps are ideal. The initial conditions are marked. Plot the waveforms v_{o1} and v_{o2}.



$$\left(V_{o1}-\frac{1}{s}\right)sC + \frac{V_{o2}}{R} = 0$$

$$sC\,V_{o1} - C + \frac{V_{o2}}{R} = 0$$

$$sC\,V_{o1} - C + \frac{V_{o1}}{R^2sC} = 0$$

$$V_{o1}\left[sC + \frac{1}{R^2sC}\right] = C$$

$$V_{o1} = \frac{C}{sC+\frac{1}{R^2sC}} = \frac{CR^2sC}{R^2s^2C^2+1}$$

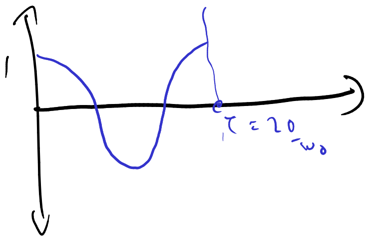
$$= RC\,\frac{RsC}{R^2s^2C^2+1}$$

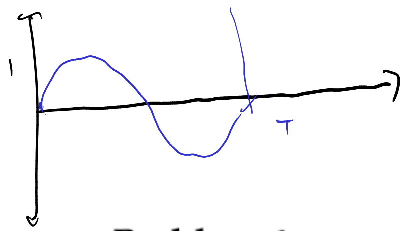
$$v_{o1}(t) = \boxed{+\cos\left(\frac{t}{RC}\right)}$$

$$V_{o2} = \frac{V_{o1}}{-RsC} = \frac{RC}{R^2s^2C^2+1}$$

$$v_{o2} = \boxed{\sin\left(\frac{t}{RC}\right)}$$

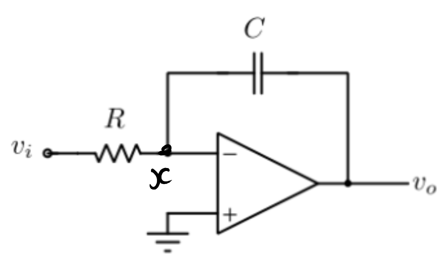
$\omega_0 = \frac{1}{RC}$





Problem 6

Consider the integrator of Fig. 1. The opamp is not ideal, but has a frequency dependent gain determined by GB/s, where GB denotes its gain-bandwidth product. Determine the integrator's transfer function, when a nonideal opamp is used.



$$V_o = -Ax$$

$$\frac{x - V_i}{R} + (x - V_o)sC = 0$$

$$x - V_i + (x - V_o)RsC = 0$$

$$x(1+RsC) - V_i - V_o RsC = 0$$

$$-\frac{V_o}{A}(1+RsC) - RsCV_o = V_i$$

$$\frac{V_o}{V_i} = \frac{-1}{\left(\frac{1+RsC}{A}\right) + RsC}$$

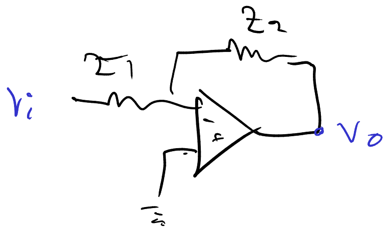
$$= \frac{-1}{\frac{(1+RsC)(s)}{GB} + RsC}$$

$$= \frac{-GB}{(1+RsC)s + RsC(GB)}$$

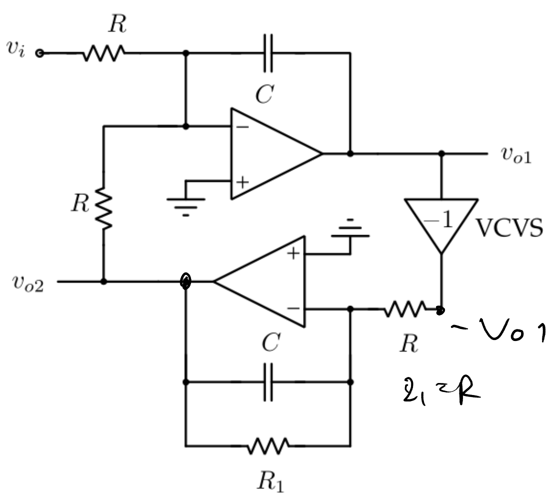
$$= \frac{-A}{(1+RsC) + ARsC}$$

Problem 7

Use the results of Problem 6 to evaluate the transfer function of the circuit of Fig. 4 when the opamps have a finite gain-bandwidth product. The VCVS can be assumed to be ideal.



$$\boxed{V_o = -Ax}$$



$$Z_2 = \frac{1}{sC + \frac{1}{R_1}} = \boxed{\frac{R_1}{R_1 sC + 1}}$$

$$\frac{V_i - x}{Z_1} + \frac{V_o - x}{Z_2} = 0$$

$$\frac{V_i}{Z_1} = -\frac{V_o}{Z_2} + x\left[\frac{1}{Z_1} + \frac{1}{Z_2}\right]$$

$$= -\frac{V_o}{Z_2} - \frac{V_o}{A}\left(\frac{1}{Z_1} + \frac{1}{Z_2}\right)$$

$$\frac{V_i}{Z_1} = -V_o\left[\frac{1}{Z_2} + \frac{1}{A}\left(\frac{1}{Z_1} + \frac{1}{Z_2}\right)\right]$$

$$\frac{V_o}{V_i} = \frac{-1}{Z_1\left(\frac{1}{Z_2} + \frac{1}{A}\left(\frac{1}{Z_1} + \frac{1}{Z_2}\right)\right)}$$

$$V_{o2} = \frac{+1}{R\left[\frac{R_1 sC + 1}{R_1} + \frac{1}{A}\left[\frac{1}{R} + \frac{R_1 sC + 1}{R_1}\right]\right]} V_{o1}$$

$$\frac{V_{o2} - x}{R} + \frac{V_i - x}{R} + (V_{o1} - x)sC = 0$$

$$\boxed{V_{o1} = -Ax}$$

$$V_{o2} - x + V_i - x + (V_{o1} - x)RsC = 0$$

$$V_{o2} + V_i + V_{o1}RsC = x\left[2 + RsC\right] = -\frac{V_{o1}}{A}\left[2 + RsC\right]$$

$$\left[\frac{1}{R\left[\frac{R_1 sC + 1}{R_1} + \frac{1}{A}\left[\frac{1}{R} + \frac{R_1 sC + 1}{R_1}\right]\right]} + RsC\right] V_{o1} + V_i = -\frac{V_{o1}}{A}\left[2 + RsC\right].$$

$$\frac{V_{o1}}{V_i} = \frac{-\frac{A}{R}}{\left(\frac{2}{R} + sC(1 + A)\right) - \frac{A^2/R^2}{\frac{1}{R^2} + (A+1)\left(sC + \frac{1}{R_1}\right)}}$$

$$\boxed{A \approx \frac{GB}{s}}$$

$$\frac{V_{o2}}{V} = \frac{A^2/R^2}{\left[\frac{2}{R} + sC(1+A)\right]\left(\frac{1}{R} + (A+1)\left(sC + \frac{1}{R_1}\right)\right) - \frac{A^2}{R^2}}$$

$$\boxed{A = \frac{4B}{s}}$$