EE2019–Analog Systems and Lab: Tutorial 6

Nagendra Krishnapura, Shanthi Pavan

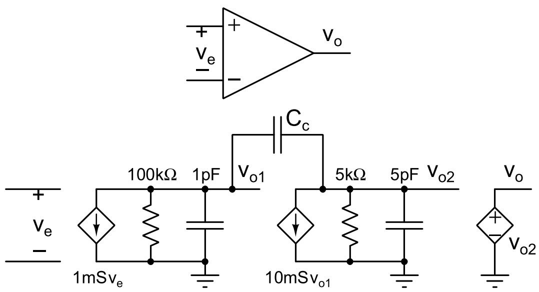


Figure 1: Circuit for problem 1

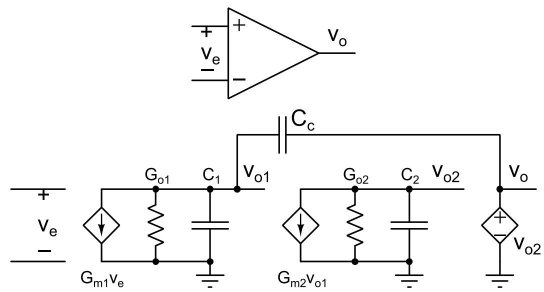


Figure 2: Circuit for problem 2

1. Fig. 1 shows the internal schematic of a Miller-compensated opamp. This opamp is used to realize a unity gain, non-inverting amplifier.
   - What is the phase margin?
   - Determine C_c so that the phase margin is 60°.
   - If the same opamp is used without any change to realize an inverting amplifier of gain -4, what are the phase margin and the closed loop bandwidth?
   - Re-design the opamp (value of C_c) so that when an inverting amplifier of gain -4 is realized using it, the phase margin is 60°. Compare the three cases wrt the following aspects: (a) Closed loop bandwidth, (b) phase margin, (c) phase lag contributed by the right-half-plane zero at the unity loop gain frequency.
   - Compare the bandwidths you obtain to the ones in the previous tutorial in which you simply increased C_1.

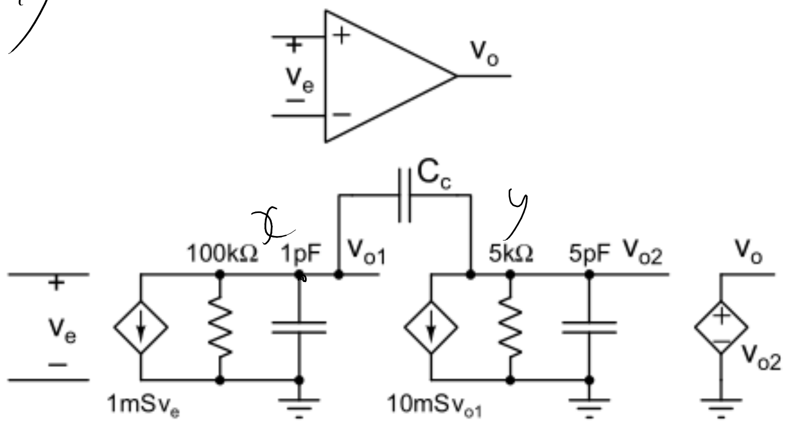
   While determining the unity loop gain frequency, phase margin, and C_c, do the calculations with and without the approximation $C_c \gg C_{1,2}$.

2. Determine the transfer function of the opamp in Fig. 2. How does it differ from the conventional Miller compensated opamp in the previous problem?

3. It is common to approximate the unity loop gain frequency as $\omega_{u,loop} \approx L_0 p_1$ where L_0 is the dc loop gain and p_1 is the dominant pole. If the loop gain is a second order function $L(s) = L_0/(1 + s/p_1)(1 + s/p_2)$, determine the exact unity loop gain frequency and the phase margin for the following cases: (a) $p_2 = 4L_0p_1$, (b) $p_2 = 2L_0p_1$, and (c) $p_2 = L_0p_1$. Compare them to the values obtained using the approximation above. $L_0 \gg 1$.

   (This approximation is very commonly used for hand calculations, but you should know how much error you end up with while doing so.)

1)



$G_{m_1} = 1e^{-3}$

$G_{m_2} = 10^{-2}$

$R_1 = 10^5$

$C_1 = 10^{12}$

$R_2 = 5 \times 10^3$

$C_2 = 5 \times 10^{-12}$

- What is the phase margin?

$$\frac{x}{R_1} + x s C_1 + G_{m_1} V_e + (x-y)(sC_c) = 0$$

$$x\left[\frac{1}{R_1} + sC_1 + sC_c\right] = y sC_c - G_{m_1} V_e$$

$$(y-x) sC_c + \frac{y}{R_2} + y sC_2 + G_{m_2} x = 0$$

$$y\left[sC_c + \frac{1}{R_2} + sC_2\right] = x\left[sC_c - G_{m_2}\right]$$

$$y = \frac{x(sC_c - G_{m_2})}{\left[sC_c + \frac{1}{R_2} + sC_2\right]}$$

$$y = \frac{(sC_c - G_{m_2})}{\left(sC_c + \frac{1}{R_2} + sC_2\right)} \frac{(y sC_c - G_{m_1} V_e)}{\left[\frac{1}{R_1} + sC_1 + sC_c\right]}$$

$$= \frac{(g_{m1} g_{m2})\left(1 - \frac{s C_C}{g_{m2}}\right)}{s^2\left[C_1C_2 + C_cC_1 + C_cC_2\right] + s\left[\frac{C_1}{R_2} + \frac{C_2}{R_1} + C_c\left[\frac{1}{R_1} + \frac{1}{R_2} + g_{m2}\right]\right] + \frac{1}{R_1R_2}}$$

Dominant pole

$$\frac{1}{s_1} = \frac{\left[\frac{C_1}{R_2} + \frac{C_2}{R_1} + \left(\frac{1}{R_1} + \frac{1}{R_2} + g_{m2}\right)C_c\right]}{\frac{1}{R_1R_2}}$$

$$= g_{m2} C_c R_1 R_2$$

Approx

$$\frac{1}{cs^2 + bs + a}$$

Dominant pole $= \frac{c}{b}$

$g_{m1} = 10^{-3}$
$g_{m2} = 10^{-2}$
$R_1 = 10^5$
$C_1 = 10^{-12}$
$R_2 = 5 \times 10^3$
$C_2 = 5 \times 10^{-12}$

$$s_1 = \frac{1}{g_{m2} C_c R_1 R_2} = \frac{1}{5 \times 10^8 \times 10^{-2} \times C_c}$$

$$= \frac{10}{5 \times 10^8 \times C_c} = \boxed{\frac{2 \times 10^{-7}}{C_c}}$$

Other pole

$$P_2 = \frac{b}{a}$$

$$= \frac{g_{m2} C_c}{C_c\left[C_1 + C_2\right] + C_1C_2}$$

$$= \frac{C_c\, 10^{-2}}{C_c\left[6\times10^{-12}\right] + 5\times10^{-24}}$$

$$w_{cg} = 5\, m_1 g_{m2}\, r_{o1} r_{o2}\, P_1$$

$$= \frac{5\times10^{3}\times 2\times10^{-7}}{C_c} = \boxed{\frac{1}{10^{3} C_c}}$$

$$PM = 180 - \tan^{-1} 800f - \tan^{-1}\frac{\frac{1}{10^{3}C_c}}{\frac{C_c 10^{-2}}{C_c(6\times10^{-12})+5\times10^{-24}}} - \tan^{-1}\frac{w_{cg} C_c}{g_{m2}}$$

$$= 84.2^{0} - \tan^{-1}\frac{\frac{1}{10^{3}C_c}}{\frac{C_c 10^{-2}}{C_c 6\times10^{-12}+5\times10^{-24}}}$$

$$= 84.2^{\circ} - \tan^{-1}\frac{6\times10^{-12}C_c + 5\times10^{-24}}{C_c^{2}\, 10}$$

- Determine C_c so that the phase margin is 60°.

$$60 = 84.2^\circ - \tan^{-1}\frac{6\times10^{-12}C_c + 5\times10^{-24}}{C_c^2\,10}$$

$$24^\circ = \tan^{-1}\frac{6\times10^{-12}C_c + 5\times10^{-24}}{C_c^2\,10}$$

$$0.44 = \frac{6\times10^{-12}C_c + 5\times10^{-24}}{C_c^2\,10}$$

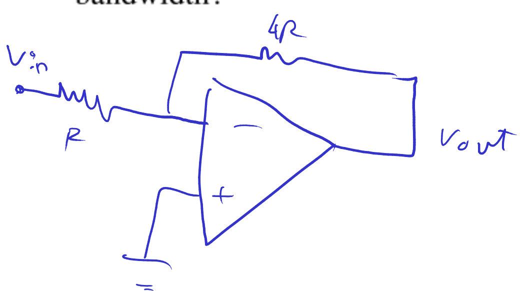
$$C_c \simeq 1.9\times10^{-12}\,F = \boxed{1.9\,pF}$$

$$\boxed{B.W = 5.26\times10^{8}\ rad\,s^{-1}}$$

- If the same opamp is used without any change to realize an inverting amplifier of gain -4, what are the phase margin and the closed loop bandwidth?

$$LG(s) = -\beta A(s) = \boxed{\frac{-A(s)}{5}}$$

$$a + bs + cs^2$$



$$\omega_{ug} = A_0 p_1$$

$$= \left(\frac{1}{5\times10^{3}C}\right) = \frac{1}{5\times10^{3}\times1.9\times10^{-12}}$$

$$= \boxed{1.05\times10^{8}\ rad/s}$$

$$P.M = 180 - \tan^{-1}\frac{\omega_u}{p_1} - \tan^{-1}\frac{\omega_u}{p_2} - \tan^{-1}\frac{\omega_u C_c}{g_{m2}}$$

$$p_1 = \frac{2\times10^{-7}}{C_c} = \boxed{1.05\times10^{5}}$$

$$p_2 = \frac{C_c\,10^{-2}}{C_c(6\times10^{-12}) + 5\times10^{-24}}$$

$$= \boxed{1.1\times10^{9}}$$

$$PM = 180 - 90^\circ - 5.45 - 1.14 = \boxed{83^\circ}$$

$$B.W = \omega_{ug} = \boxed{1.05 \times 10^{8} \ rad\ s^{-1}}$$

- Re-design the opamp (value of C_c) so that when an inverting amplifier of gain -4 is realized using it, the phase margin is 60°. Compare the three cases wrt the following aspects: (a) Closed loop bandwidth, (b) phase margin, (c) phase lag contributed by the right-half-plane zero at the unity loop gain frequency.

$$60 = 180 - \left(90 + 1.14 + \tan^{-1}\frac{\omega_{ug}}{p_2}\right)$$

$$\frac{\omega_{ug}}{p_2} = 0.55$$

$$p_2 = \frac{\omega_{ug}}{0.55} = \frac{1}{5000 C_c \times 0.55}$$

$$= \frac{C_c 10^{-2}}{C_c(6\times10^{-12}) + 5\times10^{-24}}$$

solve for x 1/(5000x * 0.55) = x * 1e-2/(x * 6e-12 + 5e-24)

NATURAL LANGUAGE MATH INPUT EXTENDED KEYBOARD EXAMPLES UPLOAD RANDOM

Input interpretation

solve $\frac{1}{5000\,x \times 0.55} = x \times \frac{1 \times 10^{-2}}{x \times 6 \times 10^{-12} + 5 \times 10^{-24}}$ for x

Results Step-by-step solution

$x = -3.31044 \times 10^{-13}$

$x = 5.49226 \times 10^{-13}$

$$\boxed{C_c = 0.54\ pF}$$

P.M = 60°

B.W = w_{ug} = $\boxed{3.7 \times 10^{8} \text{ rads}^{-1}}$

Comparison

CLBW

b → 5.2×10^{8} rads^{-1}

c → 1×10^{8} rads^{-1}

d → 3.7×10^{8} rads^{-1}

BW

b → 60

c → 83

d → 60

Phase Lag

$= \tan^{-1} \frac{w_{ug} C_c}{g_2}$

b) 5.6°

c) 1°

d) 1°

- Compare the bandwidths you obtain to the ones in the previous tutorial in which you simply increased C_1.

B.W is greater in this case

2)

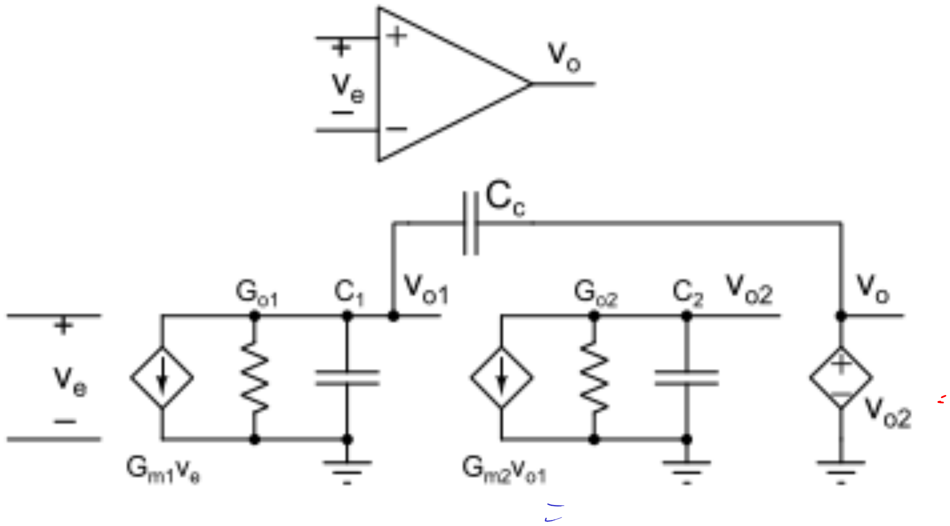


Figure 2: Circuit for problem 2

2. Determine the transfer function of the opamp in Fig. 2. How does it differ from the conventional Miller compensated opamp in the previous problem?

$$G_{m1}V_e + (V_{o1} - 0)G_{o1} + V_{o1}sC_1 + (V_{o1} - V_{o2})sC_c = 0$$

$$G_{m1}V_e + V_{o1}[G_{o1} + sC_1 + sC_c] - V_{o2}sC_c = 0$$

$$G_{m1}V_e - \frac{V_{o2}}{G_{m2}}[G_{o2} + sC_2][G_{o1} + sC_1 + sC_c] - V_{o2}sC_c = 0$$

$$\frac{V_{o2}}{V_e} = \frac{G_{m1}}{sC_c + \frac{(G_{o2} + sC_2)(G_{o1} + s(C_1 + C_c))}{G_{m2}}}$$

$$= \frac{G_{m_1} G_{m_2}}{s C_c G_{m_2} + (G_{o_2} + s C_2)(G_{o_1} + s(C_1 + C_c))}$$

$$= \boxed{\frac{G_{m_1} G_{m_2}}{G_{o_2} G_{o_1} + s\left[C_c G_{m_2} + C_2 G_{o_1} + G_{o_2}(C_1 + C_c)\right] + s^2 (C_1 + C_c) C_2}}$$

No zeros, unlike miller capacitor

3. It is common to approximate the unity loop gain frequency as $\omega_{u,loop} \approx L_0 p_1$ where L_0 is the dc loop gain and p_1 is the dominant pole. If the loop gain is a second order function $L(s) = L_0/(1 + s/p_1)(1 + s/p_2)$, determine the exact unity loop gain frequency and the phase margin for the following cases: (a) $p_2 = 4L_0 p_1$, (b) $p_2 = 2L_0 p_1$, and (c) $p_2 = L_0 p_1$. Compare them to the values obtained using the approximation above. $L_0 \gg 1$.

(This approximation is very commonly used for hand calculations, but you should know how much error you end up with while doing so.)

a) $$\frac{L_0}{\left(1 + \frac{s}{p_1}\right)\left(1 + \frac{s}{p_2}\right)}$$

$$L_0^2 = \left(1 + \frac{\omega_u^2}{p_1^2}\right)\left(1 + \frac{\omega_u^2}{p_2^2}\right)$$

$$1 + \omega_u^2 \left[\frac{1}{p_1^2} + \frac{1}{p_2^2}\right] + \frac{\omega_u^4}{p_1^2 p_2^2} - L_0^2 = 0$$

$$1 - L_0^2 + \omega_u^2 \left[\frac{1}{p_1^2} + \frac{1}{16 L_0^2 p_1^2}\right] + \frac{\omega_u^4}{p_1^2\, 16 L_0^2 p_1^2} = 0$$

$$1-L_o^2 + \left(\frac{w_u^2}{p_1^2}\right)\left[1+\frac{1}{16L_o^2}\right] + \frac{w_u^4}{p_1^4}\left[\frac{1}{16L_o^2}\right] = 0$$

$$\frac{w_u^2}{p_1^2} = \frac{-\left(1+\frac{1}{16L_o^2}\right) \pm \sqrt{1+\frac{1}{16^2L_o^4}+\frac{1}{8L_o^2}-\frac{4}{16L_o^2}\left(1-L_o^2\right)}}{(2)\frac{1}{16L_o^2}}$$

$$\frac{w_u}{p_1} = \frac{1}{\sqrt{2}}\sqrt{8\sqrt{5}L_o^2\sqrt{1-\frac{1}{16L_o^2}} - 1 - 16L_o^2}$$

$$= \sqrt{4L_o^2\left(\sqrt{5}-2\right)}$$

$$= \boxed{0.97\,L_o p_1}$$

P.M (assume $\tan^{-1} L_o \approx 90^\circ$) $= \tan^{-1}\left(\frac{4}{0.97}\right) = \boxed{76.3^\circ}$

b) $\boxed{p_2 = 2L_o p_1}$

$$0 = \left(1-L_o^2\right) + \frac{w_{ug}^2}{p_1^2}\left[1+\frac{1}{4L_o^2}\right] + \frac{w_{ug}^4}{p_1^4\,4L_o^2}$$

$$\frac{w_{ug}^2}{p_1^2} = \frac{-1-\frac{1}{4L_o^2} \pm \sqrt{1+\frac{1}{16L_o^4}+\frac{1}{2L_o^2}-\frac{(1-L_o^2)}{L_o^2}}}{2\,\frac{1}{4L_o^2}}$$

$$= \frac{\sqrt{1+\frac{1}{16L_o^4}+\frac{1}{2L_o^2}-\frac{1}{L_o^2}+1} - 1 - \frac{1}{4L_o^2}}{\frac{1}{2}L_o^2}$$

$$= \left[\frac{\sqrt{2L_0^2 + \cancel{\frac{1}{16L_0^2}} - \frac{1}{2}} - L_0^2 - \frac{1}{4}}{(x-y)} \right]^{1/2}$$

$$= 2L_0^2 \left(\sqrt{2 - \frac{1}{2L_0^2}} - 1 - \frac{1}{4L_0^2} \right)$$

$$\frac{\omega}{\sqrt{k}} = \sqrt{2}\, L_0 \sqrt{\sqrt{2}\left(1 - \frac{1}{4L_0^2}\right)^{1/2} - 1 - \frac{1}{4L_0^2}}$$

$$= \sqrt{2}\, L_0 \sqrt{\sqrt{2}\left[1 - \frac{1}{8L_0^2}\right] - 1 - \frac{1}{4L_0^2}}$$

$$= L_0 \sqrt{2(\sqrt{2}-1)} = 0.904 L_0$$

P.M. = 65°

c) $P_2 = L_0 p_1$

$$0 = 1 - L_0^2 + \frac{\omega_{vg}^2}{P_1^2}\left[1 + \frac{1}{L_0^2}\right] + \frac{\omega_{vg}^4}{p_1^4 L_0^2}$$

$$0 = L_0^2 - L_0^4 + \frac{\omega_{vg}^2}{P_1^2}(L_0^2 + 1) + \frac{\omega_{vg}^4}{P_1^4}$$

$$\frac{\omega_{vg}^2}{P_1^2} = \frac{-L_0^2 - 1 + \sqrt{L_0^4 + 2L_0^2 + 1 - 4L_0^2 + 4L_0^4}}{2}$$

$$= L_0^2 \left[\frac{-1 - \frac{1}{L_0^2} + \sqrt{5 - \frac{2}{L_0^2}}}{2} \right]$$

$$\frac{\omega_{vg}}{P_1} = L_0 \sqrt{\frac{\sqrt{5}-1}{2}}$$

$$= 0.78 L_0$$

P.M. = 52°