ANALOG SYSTEMS : PROBLEM SET 3

IN ALL PROBLEMS IN THIS TUTORIAL, THE OPAMP IS NONIDEAL IN THE SENSE THAT IT HAS FINITE BANDWIDTH. The transfer function of the opamp is given by $A(s) = \frac{\omega_u}{s}$

Problem 1

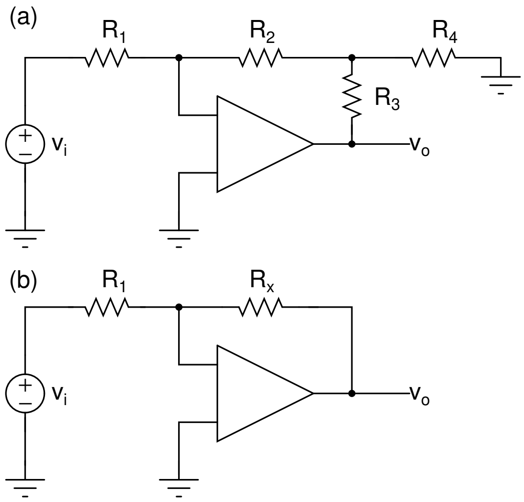


Figure 1: Circuits for Problem 1.

In the circuits above, determine the signs on the opamps for negative feedback operation, and determine the transfer function v_o/v_i. Consider the case where $R_x = 100\,R_1$, $R_2 = R_1 = R_3$. Determine R_4 so that both circuits have the same gain. Compare the 3-dB bandwidths achieved for both circuits.

Problem 2

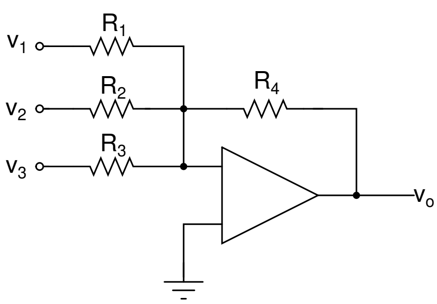


Figure 2: Circuit for Problem 2.

Determine the transfer functions from v_1, v_2 and v_3 to v_o. What is the 3-dB bandwidth of each of these transfer functions?

Problem 3

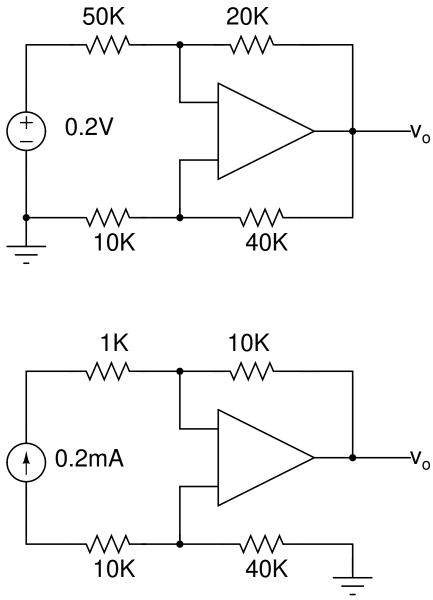


Figure 3: Circuits for Problem 3.

In the circuits above, mark the signs on the opamp for negative feedback operation, and determine the transfer functions from the input sources to v_o.

Problem 4

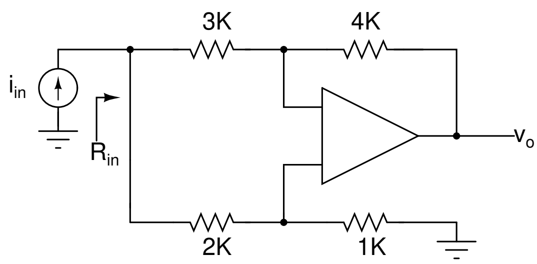


Figure 4: Circuits for Problem 4.

In the circuit above, mark the signs on the opamp for negative feedback operation, and determine the transfer function from i_{in} to v_o. Determine the input impedance (as a function of frequency) looking in, as denoted by R_{in}.

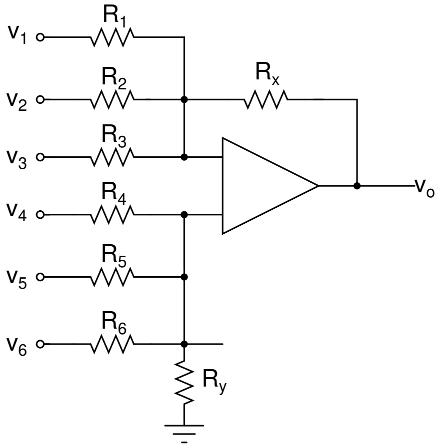


Figure 5: Circuit for Problem 5.

Problem 5

In the circuit above, mark the signs on the opamp for negative feedback operation, and determine the transfer functions to v_o from $v_1, \cdots, v_6$.

Problem 6

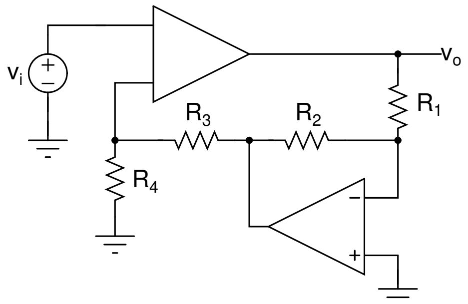


Figure 6: Circuit for Problem 6.

In the circuit above, mark the signs on the opamp for negative feedback operation, and determine $V_o(s)/V_i(s)$.

Problem 7

In the circuit above, mark the signs on the opamp for negative feedback operation, and determine the Norton equivalent for the circuit looking across the load resistor R_L. Remember, that the Norton equivalent will be frequency dependent.

Problem 8

The figure above shows three different ways of achieving an amplifier with a gain of n^2, where $n^2 \gg 1$. If $v_{off,1,2} = 0$ and the opamps have infinite gain, all three are equivalent.

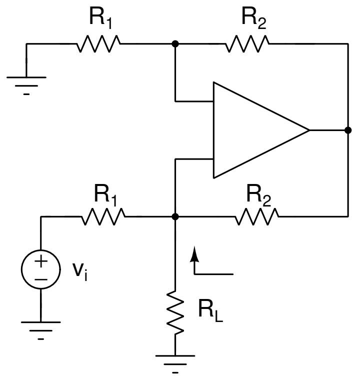


Figure 7: Circuit for Problem 7.

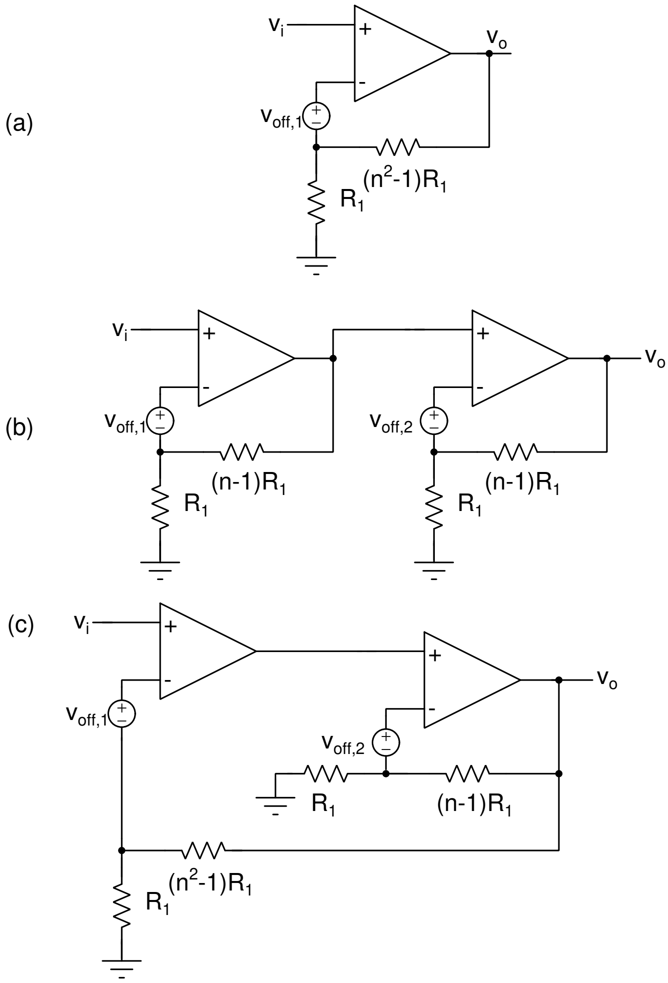


Figure 8: Circuit for Problem 8.

Determine the transfer functions of each of these amplifiers. Assume that the offset voltages are all zero. WHich of them has the highest bandwidth?

Problem 1

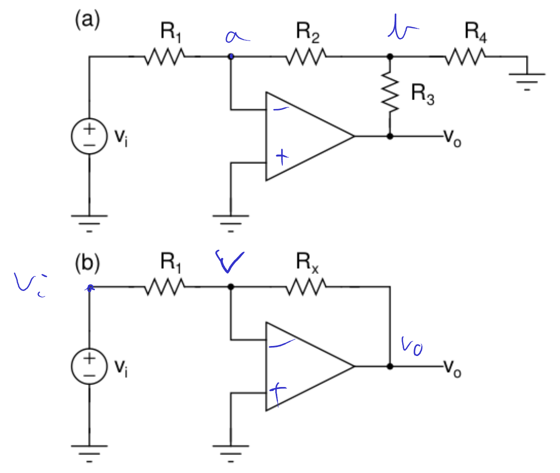


Figure 1: Circuits for Problem 1.

In the circuits above, determine the signs on the opamps for negative feedback operation, and determine the transfer function v_o/v_i. Consider the case where $R_x = 100\,R_1$, $R_2 = R_1 = R_3$. Determine R_4 so that both circuits have the same gain. Compare the 3-dB bandwidths achieved for both circuits.

a) To find sign convention, give ΔV as input. As potential at point a ↑, a is −ve terminal

b) To find sign convention, give ΔV as input. As potential at point V ↑, V is −ve terminal

$$\frac{v_i - a}{R_1} + \frac{b - a}{R_2} = 0$$

$$\frac{b - a}{R_2} + \frac{b - v_o}{R_3} + \frac{b}{R_4} = 0$$

$$b = \frac{\frac{a}{R_2} + \frac{v_o}{R_3}}{\frac{1}{R_2} + \frac{1}{R_3} + \frac{1}{R_4}}$$

$$\frac{b-a}{R_2} = \frac{1}{R_2}\left[\frac{\frac{v_o}{R_3} - \frac{a}{R_3} - \frac{a}{R_4}}{\frac{1}{R_2} + \frac{1}{R_3} + \frac{1}{R_4}}\right]$$

a)

$$\frac{v_i - a}{R_1} + \frac{\frac{1}{R_2}\left[\frac{v_o}{R_3} - \frac{a}{R_3} - \frac{a}{R_4}\right]}{\frac{1}{R_2} + \frac{1}{R_3} + \frac{1}{R_4}} = 0$$

$$\frac{v_i}{R}\cancel{\frac{\left(\frac{1}{R_2}+\frac{1}{R_3}+\frac{1}{R_4}\right)}{\left(\frac{1}{R_2}+\frac{1}{R_3}+\frac{1}{R_4}\right)}} + \frac{\frac{1}{R_2}\frac{v_o}{R_3}}{\frac{1}{R_2}+\frac{1}{R_3}+\frac{1}{R_4}} = \frac{\frac{a}{R_2}\left(\frac{1}{R_3}+\frac{1}{R_4}\right)}{\frac{1}{R_2}+\frac{1}{R_3}+\frac{1}{R_4}} + \frac{a}{R_1}\cancel{\frac{\left(\frac{1}{R_2}+\frac{1}{R_3}+\frac{1}{R_4}\right)}{\left(\frac{1}{R_2}+\frac{1}{R_3}+\frac{1}{R_4}\right)}}$$

$$\left(\frac{v_i}{R_1}\left[\frac{1}{R_1}+\frac{1}{R_2}+\frac{1}{R_3}\right] + \frac{v_o}{R_2 R_3}\right)\frac{1}{\frac{\frac{1}{R_3}+\frac{1}{R_4}}{R_2} + \frac{1}{R_1}\left(\frac{1}{R_2}+\frac{1}{R_3}+\frac{1}{R_4}\right)} = a$$

$$v_o = -\frac{\omega_a}{s}\,a = -\frac{\omega_a}{s}\left(\frac{v_i}{R_1}\left[\frac{1}{R_1}+\frac{1}{R_2}+\frac{1}{R_3}\right] + \frac{v_o}{R_2 R_3}\right)\frac{1}{\frac{\frac{1}{R_3}+\frac{1}{R_4}}{R_2} + \frac{1}{R_1}\left(\frac{1}{R_2}+\frac{1}{R_3}+\frac{1}{R_4}\right)}$$

$$v_o\left(\frac{1}{R_2}\left(\frac{1}{R_3}+\frac{1}{R_4}\right) + \frac{1}{R_1}\left(\frac{1}{R_2}+\frac{1}{R_3}+\frac{1}{R_4}\right)\right)\left(\frac{-s}{\omega_a}\right) = \frac{v_i}{R_1}\left[\frac{1}{R_1}+\frac{1}{R_2}+\frac{1}{R_3}\right]$$

$$\frac{-\frac{\omega_a}{s}\left(\frac{1}{R_1} + \frac{1}{R_2(R_1+R_2)}\left(\frac{1}{R_3}+\frac{1}{R_4}+\frac{1}{R_1+R_2}\right)\right)}{\left(\frac{1}{R_1}+\frac{1}{R_2}\right) + \frac{\omega_a}{s}\frac{1}{R_2 R_3}\left(\frac{1}{R_3}+\frac{1}{R_4}+\frac{1}{R_1+R_2}\right)}$$

F) $R_1 = R_2 = R_3$.

$$A_{CL}(s) = \frac{-1 - \frac{1}{3} + \frac{9R_1}{R_4}}{\frac{s}{\omega_u}\frac{3}{} + \frac{1}{R_1}\left(\frac{3}{2R_1} + \frac{1}{R_4}\right)}$$

b) $V_o = \frac{\omega_u}{s}(-V)$

$$V_o = -\frac{\omega_u}{s}\left[\frac{V_i}{R_1} + \frac{V_o}{R_x}\right] \Big/ \left(\frac{1}{R_1} + \frac{1}{R_x}\right)$$

$$\left(\frac{1}{R_1} + \frac{1}{R_x}\right)(-1)\frac{s V_o}{\omega_u} - \frac{V_o}{R_x} = \frac{V_i}{R_1}$$

$$\boxed{\frac{V_o}{V_i} = -\frac{1}{R_1}\frac{1}{\left(\left(\frac{s}{\omega_u}\right)\left(\frac{1}{R_1} + \frac{1}{R_x}\right) + \frac{1}{R_x}\right)}}$$

I) $R_x = 100R_1$

$$\boxed{\frac{R_1}{R_4} = 98}$$

a) $\omega_{-3dB} = \frac{\omega_u}{199} \longrightarrow$ Smaller

b) $\omega_{-3dB} = \frac{\omega_u}{101} \longrightarrow$ Greater

Problem 2

Sign conventions are taken to ensure -ve feedback.

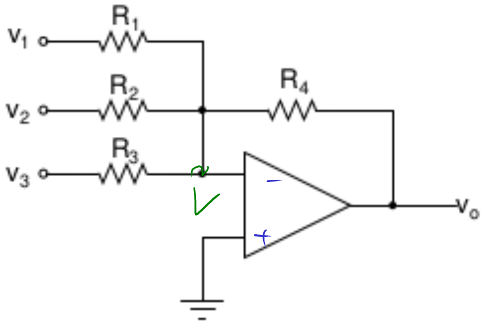


Figure 2: Circuit for Problem 2.

Determine the transfer functions from v_1, v_2 and v_3 to v_o. What is the 3-dB bandwidth of each of these transfer functions?

$$\frac{V_1 - V}{R_1} + \frac{V_2 - V}{R_2} + \frac{V_3 - V}{R_3} + \frac{V_4 - V}{R_4} = 0$$

$$\frac{\frac{V_1}{R_1} + \frac{V_2}{R_2} + \frac{V_3}{R_3} + \frac{V_4}{R_4}}{\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \frac{1}{R_4}} = V$$

$$v_o = -\frac{\omega_u}{s} V = \frac{\left(\frac{V_1}{R_1} + \frac{V_2}{R_2} + \frac{V_3}{R_3} + \frac{V_o}{R_4}\right)\left(-\frac{\omega_u}{s}\right)}{\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \frac{1}{R_4}}$$

$$-\frac{s}{\omega_u}\left[\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \frac{1}{R_4}\right] V_o = \frac{V_1}{R_1} + \frac{V_2}{R_2} + \frac{V_3}{R_3} + \frac{V_o}{R_4}$$

$$V_o\left[-\frac{s}{\omega_u}\left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \frac{1}{R_4}\right) - \frac{1}{R_4}\right] = \frac{V_1}{R_1} + \frac{V_2}{R_2} + \frac{V_3}{R_3}$$

$$\left.\frac{V_0}{V_1}\right|_{\substack{v_2=0\\ v_3=0}} = -\frac{1}{R_1}\,\frac{1}{\left(\frac{s}{\omega_u}\left(\frac{1}{R_1}+\frac{1}{R_2}+\frac{1}{R_3}\right)+\frac{1}{R_4}\right)}$$

$\underline{\omega_{3dB}}$

$$\boxed{s = -\frac{\omega_u}{R_4\left(\frac{1}{R_1}+\frac{1}{R_2}+\frac{1}{R_3}+\frac{1}{R_4}\right)}}$$

By symmetry,

$$\omega_{-3dB}(A) = \omega_{3dB}(B) = \omega_{-3dB}(C)$$

Problem 3

Consider ΔV input · AΔV is caused at the output : As potential at pt a ↑, it is at higher potential

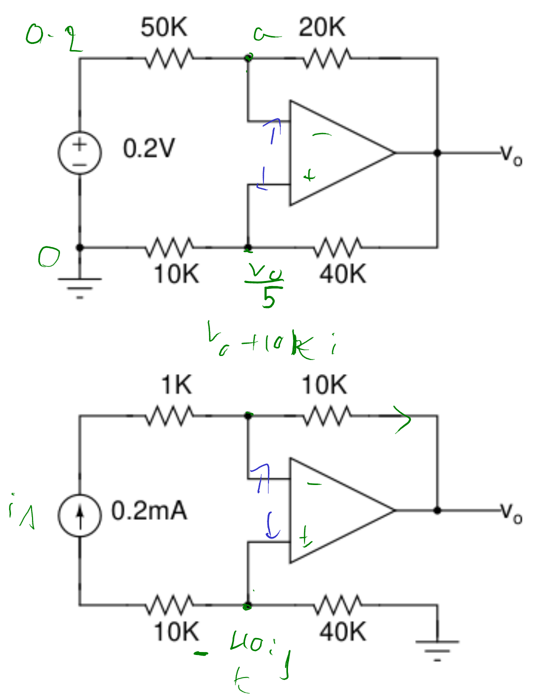


Figure 3: Circuits for Problem 3.

In the circuits above, mark the signs on the opamp for negative feedback operation, and determine the transfer functions from the input sources to v_o.

$$\frac{a-0.2}{5} + \frac{a-V_o}{2} = 0$$

$$V_o = +\frac{\omega_u}{s}\left[\frac{V_o}{5} - a\right]$$

$$V_o = \frac{\omega_u}{s}\left[\frac{V_o}{5} - \frac{\left(\frac{V_i}{5}+\frac{V_o}{2}\right)}{\left(\frac{1}{5}+\frac{1}{2}\right)}\right]$$

$$V_o\left[\frac{7}{10}\cdot\frac{s}{\omega_u}\right] = \frac{V_o}{5}\left(\frac{7}{10}\right) - \left(\frac{V_i}{5}+\frac{V_o}{2}\right)$$

$$V_o\,\frac{7}{10}\,\frac{s}{\omega_u} = V_o\left[\frac{-3}{10}\right] - \frac{V_i}{5}$$

$$V_o\left[\frac{7}{10}\frac{s}{\omega_u} + \frac{3}{10}\right] = -\frac{V_i}{5}$$

$$= \boxed{+\frac{10\,\omega_u}{35s+18\omega_u}}$$

$$V_o = \frac{\omega_u}{s}\left[-40k\,i_s - (V_o + 10k\,i)\right]$$

$$\frac{sV_o}{\omega_u} + V_o = i_s\left[-50k\right]$$

$$V_o\left[\frac{s}{\omega_u}+1\right] = i_s(-50k)$$

$$\frac{V_o}{i_s} = \boxed{\frac{-50k\,\omega_u}{s+\omega_u}}$$

Problem 4

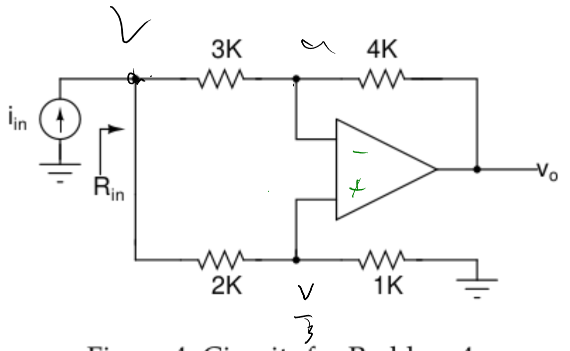


Figure 4: Circuits for Problem 4.

To find sign convention, assume $i_{in}=0$, so potential at point a ↑, so a is −ve terminal

$$\frac{a-v_o}{4} + \frac{a-v}{3} = 0$$

In the circuit above, mark the signs on the opamp for negative feedback operation, and determine the transfer function from i_{in} to v_o. Determine the input impedance (as a function of frequency) looking in, as denoted by R_{in}.

$$v_o = A\left[\frac{v}{3} - a\right] = A\left[\frac{v}{3} - \frac{\frac{v_o}{4}+\frac{v}{3}}{\frac{1}{4}+\frac{1}{3}}\right]$$

$$= A\left(\frac{v}{3} - \frac{(3v_o+4v)}{7}\right)$$

$$v_o = A\left[\frac{7v - 9v_o - 12v}{21}\right] = \frac{A}{21}\left[-5v - 9v_o\right]$$

$$i_{in} = \frac{v}{3k} + \frac{v-v_o}{7k}$$

$$T_1(s) = \frac{v_o}{i_{in}} = \frac{-\frac{78}{143}K}{\frac{6}{71}+\frac{1}{A}}$$

$$\boxed{A = \frac{\omega_u}{s}}$$

$$R_{in} = \frac{v_x}{i_{in}} = \frac{1638K}{71} + \frac{27}{58}T_1(s)$$

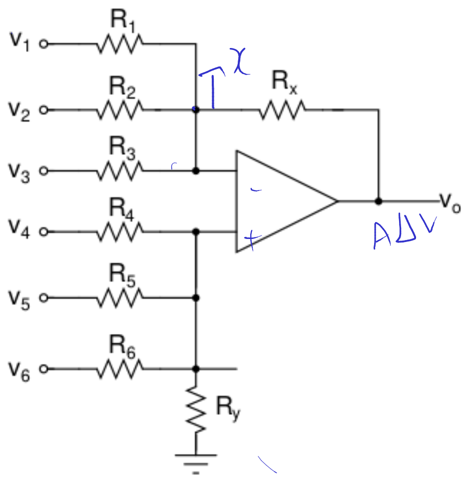


Figure 5: Circuit for Problem 5.

Problem 5

In the circuit above, mark the signs on the opamp for negative feedback operation, and determine the transfer functions to v_o from $v_1, \cdots, v_6$.

Assume ΔV input. $v_o = A\Delta V$,

so potential at pt. x ↑,

so x has to be -ve terminal

$$\frac{x - v_1}{R_1} + \frac{x}{R_2} + \frac{x}{R_3} + \frac{x - v_o}{R_x} = 0$$

$$x = \frac{\frac{v_1}{R_1} + \frac{v_o}{R_x}}{\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \frac{1}{R_x}}$$

$$V_o = -Ax$$

$$V_o = -A\left(\frac{\frac{v_1}{R_1} + \frac{v_o}{R_x}}{\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \frac{1}{R_x}}\right)$$

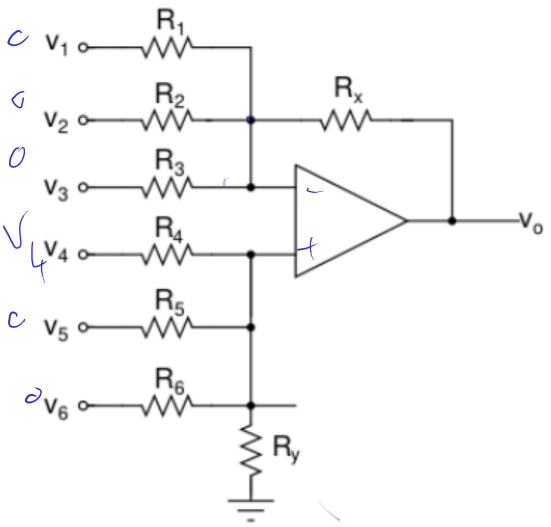
$$V_d \left[\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \frac{1}{R_x} \right] = -A \left(\frac{V_i}{R_1} + \frac{V_o}{R_x} \right)$$

$$V_o \left[\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \frac{1}{R_x} + \frac{A}{R_x} \right] = -A \frac{V_i}{R_1}$$

$$\frac{V_o}{V_1} = \boxed{\frac{-\frac{A}{R_1}}{\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \frac{1}{R_x} + \frac{A}{R_x}}}$$

$$\frac{V_o}{V_2} = \frac{-\frac{A}{R_2}}{\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \frac{1}{R_x} + \frac{A}{R_x}}$$

$$\frac{V_o}{V_3} = \frac{-\frac{A}{R_3}}{\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \frac{1}{R_x} + \frac{A}{R_x}}$$



$$\boxed{R_{11} = R_5 \| R_6 \| R_y}$$

$$R' = R_1 \| R_2 \| R_3$$

$$\boxed{V_+ = V_4 \frac{R_{11}}{R_{11} + R_4}}$$

$$\boxed{V_- = V_o \frac{R_x}{R' + R_x}}$$

$$\frac{V_o}{A} = \left[\frac{V_4 R_{11}}{R_{11} + R_4} - \frac{V_o R_x}{R' + R_x} \right]$$

$$V_o \left[\frac{1}{A} + \frac{R_x}{R' + R_x} \right] = \frac{V_4 R_{11}}{R_{11} + R_4}$$

$$\frac{V_o}{V_4} = \frac{\frac{R_{11}}{R_{11} + R_4}}{\frac{1}{A} + \frac{R_x}{R' + R_x}}$$

$$R' = R_1 \| R_2 \| R_3$$

$$R_{11} = R_5 \| R_6 \| R_y$$

$$\frac{V_o}{V_5} = \frac{\frac{R_{11}}{R_{11} + R_5}}{\frac{1}{A} + \frac{R_x}{R' + R_x}}$$

$$R' = R_1 \| R_2 \| R_3$$

$$R_{11} = R_4 \| R_6 \| R_y$$

$$\frac{V_o}{V_6} = \frac{\frac{R_{11}}{R_{11} + R_6}}{\frac{1}{A} + \frac{R_x}{R' + R_x}}$$

$$R' = R_1 \| R_2 \| R_3$$

$$R_{11} = R_4 \| R_5 \| R_y$$

Problem 6

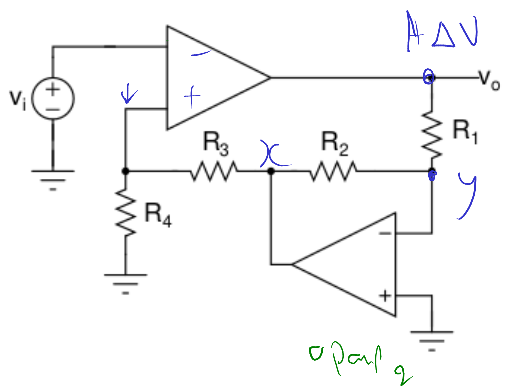


Figure 6: Circuit for Problem 6.

To find sign, assume ideality. Give ΔV input. As opamp 2 is inverting, potential at pt. x ↓, so V_+ ↓, so V_+ is the terminal.

In the circuit above, mark the signs on the opamp for negative feedback operation, and determine $V_o(s)/V_i(s)$.

$$\frac{y - V_o}{R_1} + \frac{y - x}{R_2} = 0$$

$$x = +A(-y) = -(A_0 y)$$

$$-x = +A \left[\frac{\frac{V_o}{R_1} + \frac{x}{R_2}}{\frac{1}{R_1} + \frac{1}{R_2}} \right]$$

$$-\frac{x}{A}\left[\frac{1}{R_1} + \frac{1}{R_2}\right] = \frac{V_o}{R_1} + \frac{x}{R_2}$$

$$-x\left[\frac{1}{A}\left[\frac{1}{R_1} + \frac{1}{R_2}\right] + \frac{1}{R_2}\right] R_1 = V_o$$

$$V_o = -A\left[V_i - \frac{R_4 x}{R_3 + R_4}\right]$$

$$V_o = -\left\{ A V_i + \left(\frac{R_4}{R_3 + R_4}\right) \frac{V_o}{\frac{1}{A}\left[\frac{1}{R_1} + \frac{1}{R_2}\right] + \frac{1}{R_2}} \frac{1}{R_1} \right\}$$

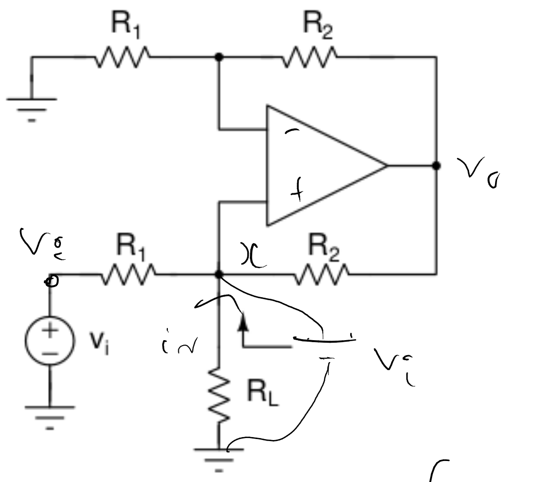
$$-\frac{v_o}{A} = v_i + \left(\frac{R_4}{R_3+R_4}\right)\frac{v_o}{R_1}\cdot\frac{1}{\frac{1}{A}\left(\frac{1}{R_1}+\frac{1}{R_2}\right)+\frac{1}{R_2}}$$

$$-v_o\left[\frac{1}{A}+\left(\frac{R_4}{R_3+R_4}\right)\frac{1}{R_1}\,\frac{1}{\left(\frac{1}{A}\right)\left(\frac{1}{R_1}+\frac{1}{R_2}\right)+\frac{1}{R_2}}\right] = v_i$$

$$\frac{v_o}{v_i} = -1 \Big/ \left[\frac{1}{A}+\left(\frac{R_4}{R_3+R_4}\right)\frac{1}{R_1}\,\frac{1}{\left(\frac{1}{A}\right)\left(\frac{1}{R_1}+\frac{1}{R_2}\right)+\frac{1}{R_2}}\right]$$

Problem 7

In the circuit above, mark the signs on the opamp for negative feedback operation, and determine the Norton equivalent for the circuit looking across the load resistor R_L. Remember, that the Norton equivalent will be frequency dependent.



i_N

$$\frac{x - V_i}{R_1} + \frac{x - V_o}{R_2} + \frac{x}{R_3} = 0$$

$$V_o = A\left[x - \frac{R_1}{R_1+R_2} V_o\right]$$

$$V_o\left[\frac{1}{A} + \frac{R_1}{R_1+R_2}\right] = x$$

$$x\left[\frac{1}{R_1}+\frac{1}{R_2}+\frac{1}{R_3}\right] = \frac{V_i}{R_1} + \frac{V_o}{R_2}$$

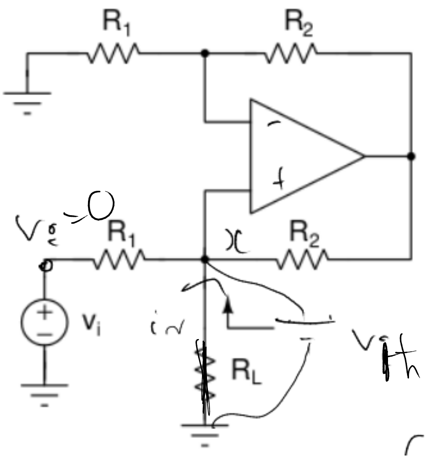
$$\left[\frac{1}{R_1}+\frac{1}{R_2}+\frac{1}{R_3}\right]x = \frac{V_i}{R_1} + \frac{1}{R_2}\left(\frac{x}{\frac{1}{A}+\frac{R_1}{R_1+R_2}}\right)$$

$$x\left[\frac{1}{R_1}+\frac{1}{R_2}+\frac{1}{R_3} - \frac{1}{R_2\left(\frac{1}{A}+\frac{R_1}{R_1+R_2}\right)}\right] = \frac{V_i}{R_1}$$

$$i = \frac{x}{R_3} = \frac{V_i}{R_1 R_3}\,\frac{1}{\frac{1}{R_1}+\frac{1}{R_2}+\frac{1}{R_3}-\frac{1}{R_2\left(\frac{1}{A}+\frac{R_1}{R_1+R_2}\right)}}$$

$R_3 \to 0$,

$$i_N = \boxed{\frac{V_i}{R_1}}$$



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$$V_0 = A\left[V_{th} - \frac{R_1}{R_1+R_2}V_0\right]$$

$$V_0\left[\frac{1}{A} + \frac{R_1}{R_1+R_2}\right] = V_{th}$$

$$-i_N + \frac{V_{th}-V_0}{R_2} + \frac{V_{th}}{R_1} = 0$$

$$i_N = \frac{V_{th}-V_0}{R_2} + \frac{V_{th}}{R_1}$$

$$i_N = V_{th}\left[\frac{1}{R_2} - \frac{1}{R_2}\left[\frac{1}{\frac{1}{A}+\frac{R_1}{R_1+R_2}}\right] + \frac{1}{R_1}\right]$$

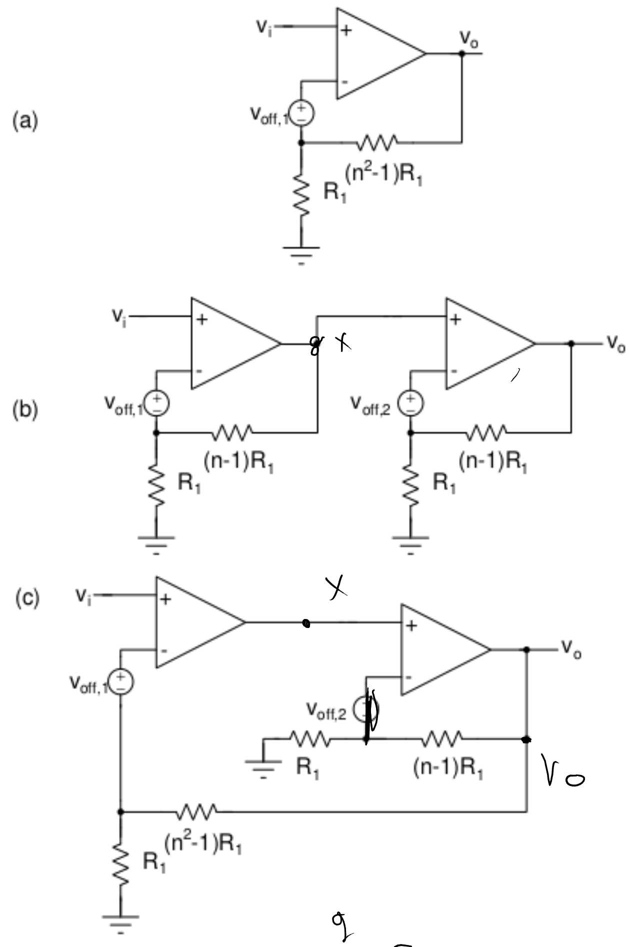
$$R_{th} = \boxed{\frac{1}{\frac{1}{R_2} - \frac{1}{R_2}\left[\frac{1}{A}+\frac{R_1}{R_1+R_2}\right] + \frac{1}{R_1}}}$$

Problem 8

The figure above shows three different ways of achieving an amplifier with a gain of n^2, where $n^2 \gg 1$. If $v_{off,1,2} = 0$ and the opamps have infinite gain, all three are equivalent.

2

Determine the transfer functions of each of these amplifiers. Assume that the offset voltages are all zero. WHich of them has the highest bandwidth?



for
ga-
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a) $v_o = A\left[v_i - \frac{v_o}{n^2}\right]$

$v_o\left[\frac{1}{A}+\frac{1}{n^2}\right] = v_i$

$\frac{v_o}{v_i} = \frac{1}{\frac{1}{A}+\frac{1}{n^2}}$ $\boxed{\omega = \frac{\omega_u}{n^2}}$

b) $x = v_i \Big/ \left[\frac{1}{A}+\frac{1}{n}\right]$

$v_o = x \Big/ \left[\frac{1}{A}+\frac{1}{n}\right]$

$v_o = \frac{v_i}{\left(\frac{1}{A}+\frac{1}{n}\right)^2}$ $\boxed{\omega = \frac{\omega_u}{n}\left(\sqrt{2}-1\right)^{1/2}}$

c) $\frac{v_o}{v_i} = \frac{1}{\left(\frac{1}{A}+\frac{1}{n}\right)^2}$

$v_o = A\left[x - \frac{v_o}{n}\right]$

$x = A\left[v_i - \frac{v_o}{n^2}\right]$

$v_o = A\left[\left(v_i - \frac{v_o}{n^2}\right) - \frac{v_o}{nA}\right]$

$v_o\left[\frac{1}{A^2}+\frac{1}{n^2}+\frac{1}{nA}\right] = v_i$

$\frac{v_o}{v_i} = \frac{1}{A^2+n^2+nA}$

$\frac{\omega_u^2}{s^2} + n^2 + \frac{\omega_u}{s}n = 0$

$$w_u^2 + s^2 n^2 + s w_u n = 0$$

$$w_u^2 - w^2 n^2 + jw w_u n = 0$$

$$- w w_u n = w_u^2 - w^2 n^2$$

$$w^2 n^2 - w w_u n - w_u^2 = 0$$

$$w = \frac{+ w_u \cancel{n} \pm \sqrt{w_u^2 \cancel{n^2} + 4 w_u^2 \cancel{n^2}}}{2 n^{\cancel{2}}}$$

$$w = \frac{w_u}{n} \left[\frac{\pm 1 + \sqrt{5}}{2} \right]$$

Assume Large value of n, so $\boxed{w_1 < w_2 < w_3}$

3 Has highest 3dB bandwidth