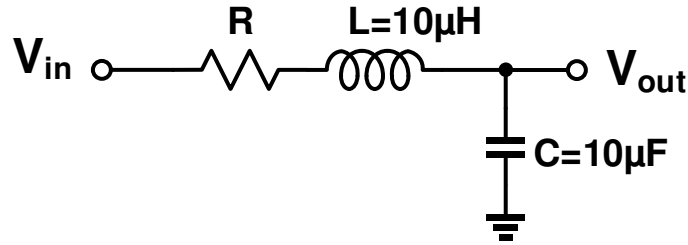


EE2019-Analog Systems and Lab: Tutorial-4

Aniruddhan S., Qadeer Khan, Saurabh Saxena

1. In the RLC circuit below,



derive the transfer function $H(s) = V_{out}(s)/V_{in}(s)$ and prove that the circuit is equivalent to a standard 2nd order system with transfer function:

$$H(s) = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

- Find the expressions for damping factor (ζ), quality factor ($Q = 1/2\zeta$), natural frequency (ω_n) and poles p_1 , p_2 (roots of s) and their respective frequencies, ω_{p1} and ω_{p2} in terms of R , L and C .
- Considering $L=10\mu\text{H}$ and $C=10\mu\text{F}$, fill the values in the following table for the corresponding values of R .
- Show p_1 and p_2 calculated in (b) on the s -plane and comment on the movement of poles w.r.t. damping factor (ζ).

$R (\Omega)$	ζ	$Q=1/2\zeta$	p_1 ($\sigma + j\omega$)	p_2 ($\sigma + j\omega$)	ω_{p1} (rad/s)	ω_{p2} (rad/s)
0.02						
0.1						
0.4						
1						
1.4						
2						
5						
10						
20						
100						

- Enter the circuit in LTspice and perform following simulations for all values of R given in the table:
 - Plot AC magnitude and phase response for $V_{out}(s)/V_{in}(s)$. Comment on the behavior of AC magnitude and phase response w.r.t. damping factor, ζ .
 - Plot the transient response by applying a unit step (0 to 1V with initial delay of 1ms and $T_{rise} = 1\text{ns}$) for the time span of 10ms. Comment on the effect of varying ζ on the transient response.
 - After observing the behavior of AC and transient behavior w.r.t. ζ in (i) & (ii), it is now understood that both AC and transient response are interrelated. How can you intuitively guess the approximate value of ζ by simply looking at either AC magnitude or transient response?

EE2019-Analog Systems and Lab: Tutorial-4

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2. For the non-inverting amplifier shown in Figure-A and considering op-amp model shown in Figure-B where $A(s)=V_o(s)/V_i(s)$

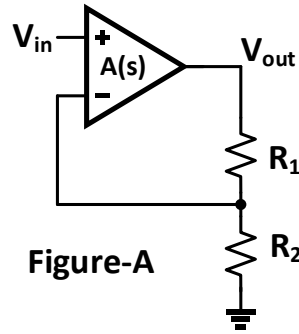


Figure-A

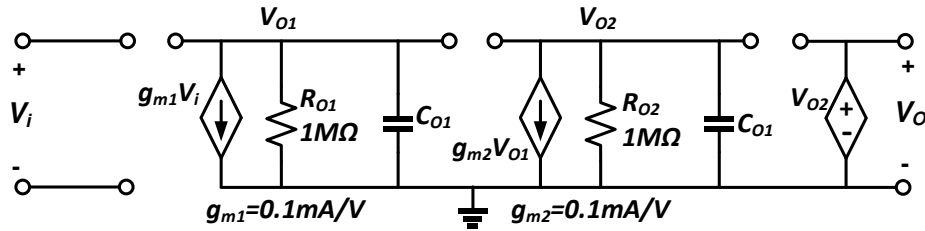


Figure-B

- Find the loop gain transfer function, $LG(s)$, DC gain A_o , poles p_{1_lg} , p_{2_lg} (roots of s) and their respective frequencies, w_{p1_lg} and w_{p2_lg} in terms of g_{m1} , R_{o1} , C_{o1} , g_{m2} , R_{o2} and C_{o2} .
- Find the closed loop transfer function, $H(s)=V_{out}(s)/V_{in}(s)$, poles p_{1_cl} , p_{2_cl} (roots of s) and their respective frequencies, w_{p1_cl} and w_{p2_cl} in terms of g_{m1} , R_{o1} , C_{o1} , g_{m2} , R_{o2} and C_{o2} .
- Prove that, the circuit in Figure-A behaves similar to the RLC circuit of problem-1 for feedback factor, $\beta=1$ (i.e. $R_1=0$ or $R_2=\infty$) and find the expressions for damping factor (ζ), quality factor (Q), natural frequency (w_n) in terms of loop gain pole frequencies, w_{p1_lg} and w_{p2_lg} and DC gain, A_o .
- Considering $g_{m1}=g_{m2}=0.1\text{mA/V}$ and $R_{o1}=R_{o2}=1\text{M}\Omega$, fill the values in following table for the corresponding values of C_{o1} and C_{o2}

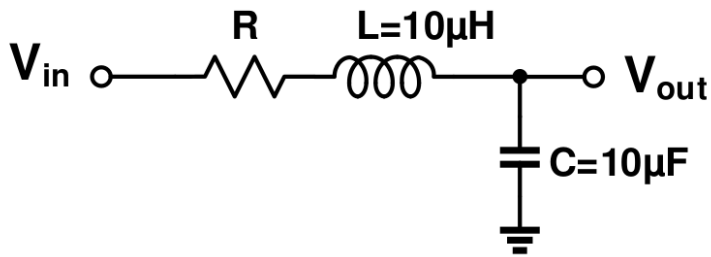
$\beta \approx 1$

C_{o1} (F)	C_{o2} (F)	Loop Gain				Closed Loop			
		w_{p1_lg} (rad/s)	w_{p2_lg} (rad/s)	w_{ugf} (rad/s)	PM (deg.)	ζ	$Q=1/2 \zeta$	w_{p1_cl} (rad/s)	w_{p2_cl} (rad/s)
1e-9	1e-9								
10e-9	1e-10								
4e-8	2.5e-11								
1e-7	1e-11								
1.41e-7	7.07e-12								
2e-7	5e-12								
5e-7	2e-12								
1e-6	1e-12								
2e-6	5e-13								
1e-5	1e-13								

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- e) Verify that increasing the spacing between loop gain poles, w_{p1_lg} and w_{p2_lg} has similar effect on damping factor as increasing R has in the RLC circuit of problem-1.
- f) Show the locations of both loop gain poles p_{1_lg} , p_{2_lg} and corresponding closed loop poles, p_{1_cl} , p_{2_cl} on s-plane for different values of ζ calculated in the above table.
- g) Plot phase margin (PM) vs. ζ and find the range of ζ for which phase margin can be approximated as 100 times of ζ with +/-10% inaccuracy.
- h) Enter the circuit in LTspice and perform following simulations for all the values of w_{p1_lg} , w_{p2_lg} :
 - i. Plot the AC magnitude and phase response of the loop gain transfer function for all values of w_{p1_lg} , w_{p2_lg} and corresponding AC magnitude and phase response of the closed loop transfer function. Comment on effect of increasing and decreasing phase margin on the closed loop AC magnitude and phase response.
 - ii. Plot the step response by applying a unit step (0 to 1V with initial delay of 1ms and $T_{rise} = 1ns$) for the time span of 10ms. Comment on the effect of varying phase margin and ζ on the transient response. Find the phase margin and corresponding value of ζ for the fastest settling (when output settles within 95% of the final value).
 - iii. Change the value of feedback factor, β from 1 to 1/10 (i.e. $R_1=10R_2$) and observe explain the effect of increasing closed loop gain ($k=1/\beta$) on phase margin, damping factor and unity gain frequency (w_{ugf}).



$$\frac{V_{out}}{V_{in}} = \frac{\frac{1}{sC}}{\frac{1}{sC} + sL + R}$$

$$= \frac{1}{1 + s^2 LC + R s C}$$

$$= \frac{\frac{1}{LC}}{\frac{1}{LC} + s^2 + \frac{R s}{L}}$$

$$\omega_n = \sqrt{\frac{1}{LC}}$$

$$\frac{1}{\sqrt{LC}} \cdot 2\eta = \frac{R}{L}$$

$$\eta = \frac{1}{2} R \sqrt{\frac{C}{L}}$$

$$Q = \frac{1}{2\eta}$$

$$= \frac{1}{R} \sqrt{\frac{L}{C}}$$

$$\frac{1}{s^2 + \omega_n^2 + 2\eta\omega_n s}$$

$$s = \frac{-2\eta\omega_n \pm \sqrt{4\eta^2\omega_n^2 - 4\omega_n^2}}{2}$$

$$= \omega_n \left(-\eta \pm \sqrt{\eta^2 - 1} \right)$$

$$\omega_{p1}, \omega_{p2} = \text{mag}(p_1), \text{mag}(p_2)$$

5)

```
from cmath import *
from tabulate import tabulate

R = [0.02, 0.1, 0.4, 1, 1.4, 2, 5, 10, 20, 100]

L = 1e-5
C = 1e-5

w_n = sqrt(1/(L * C))

def eta(R):
    return R/2 * sqrt(C/L)

def Q(R):
    return 1/(2 * eta(R))

def s_1(R):
    return w_n * (-eta(R) + sqrt(eta(R)**2 - 1))

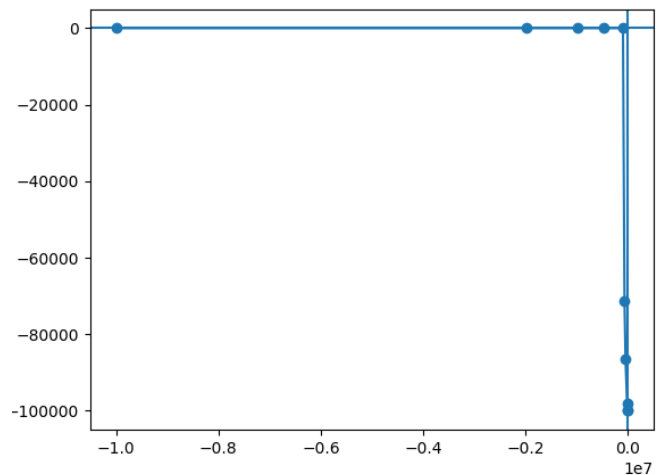
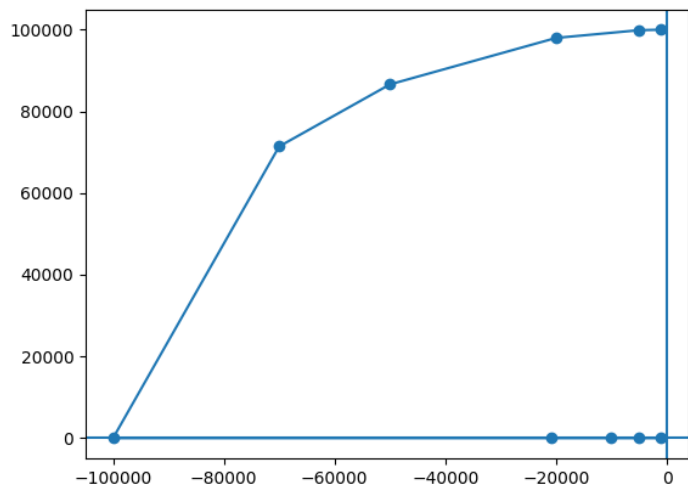
def s_2(R):
    return w_n * (-eta(R) - sqrt(eta(R)**2 - 1))

lis = [[r, eta(r), Q(r), s_1(r), s_2(r), abs(s_1(r)), abs(s_2(r))] for r in R]

print(tabulate(lis, headers = ["R", "eta", "Q", "s1", "s2", "abs(s1)", "abs(s2)"]))
```

R	eta	Q	s1	s2	abs(s1)	abs(s2)
0.02	(0.01+0j)	(50+0j)	(-999.9999999999999+99994.99987499374j)	(-999.9999999999999-99994.99987499374j)	100000	100000
0.1	(0.05+0j)	(10+0j)	(-5000+99874.92177719087j)	(-5000-99874.92177719087j)	100000	100000
0.4	(0.2+0j)	(2.5+0j)	(-20000+97979.5897113271j)	(-20000-97979.5897113271j)	100000	100000
1	(0.5+0j)	(1+0j)	(-49999.99999999999+86602.54037844385j)	(-49999.99999999999-86602.54037844385j)	100000	100000
1.4	(0.7+0j)	(0.7142857142857143+0j)	(-69999.99999999999+71414.28428542848j)	(-69999.99999999999-71414.28428542848j)	100000	100000
2	(1+0j)	(0.5+0j)	(-99999.99999999999+0j)	(-99999.99999999999-0j)	100000	100000
5	(2.5+0j)	(0.2+0j)	(-20871.215252208007+0j)	(-479128.7847477919-0j)	20871.2	479129
10	(5+0j)	(0.1+0j)	(-10102.051443364422+0j)	(-989897.9485566354-0j)	10102.1	989898
20	(10+0j)	(0.05+0j)	(-5012.562893380056+0j)	(-1994987.4371066198-0j)	5012.56	1.99499e+06
100	(50+0j)	(0.01+0j)	(-1000.1000200048792+0j)	(-9998999.899979994-0j)	1000.1	9.999e+06

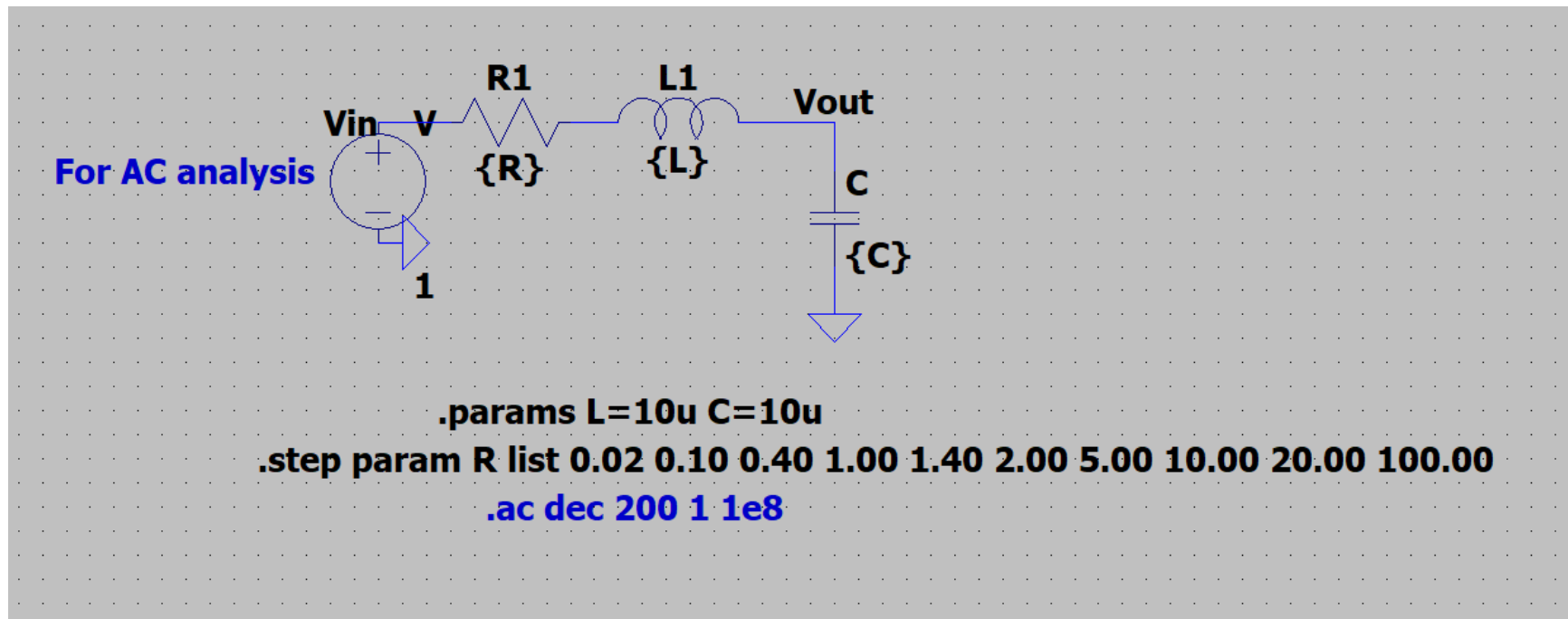
6)

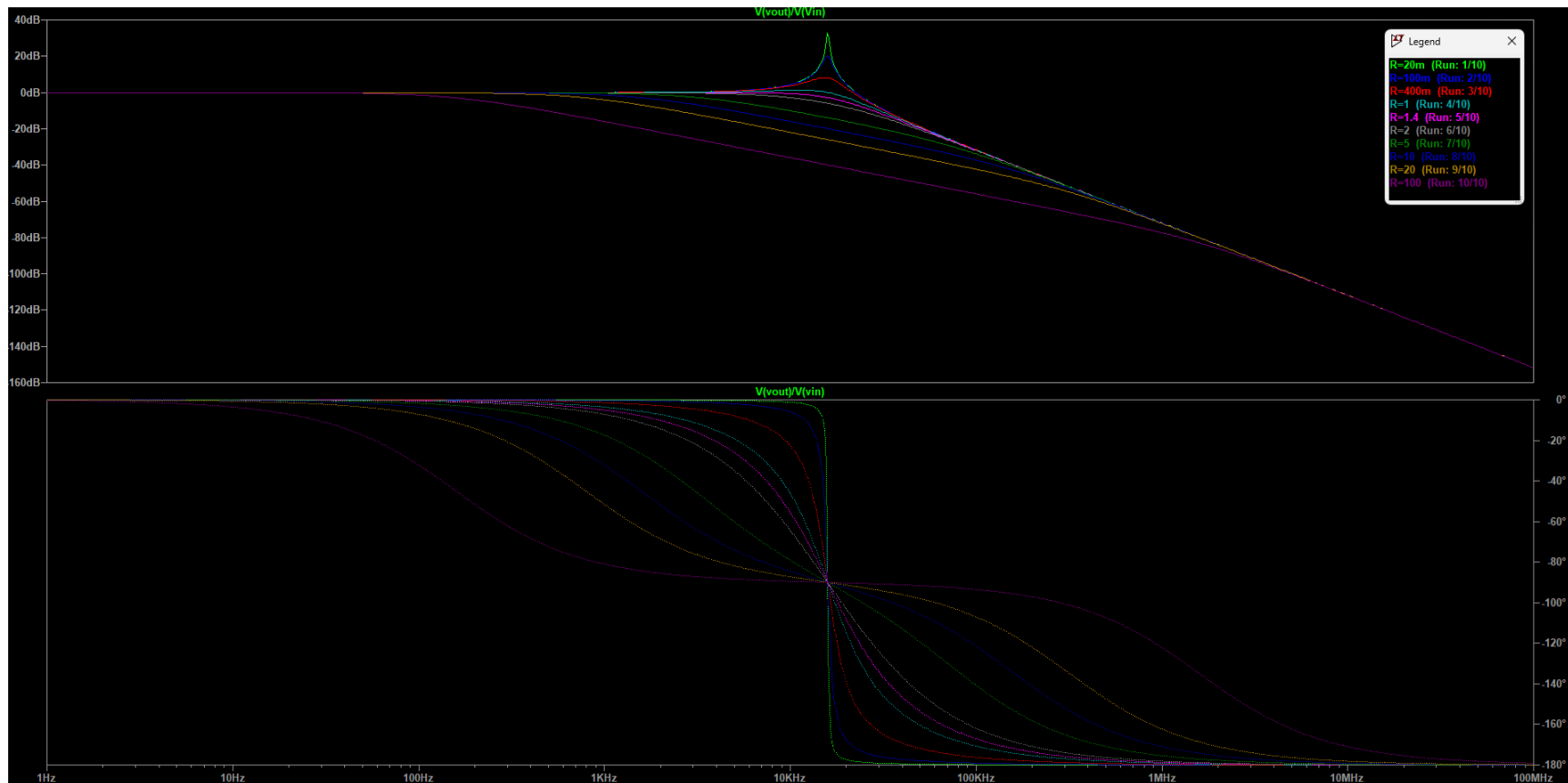


Question 1d

i)

Circuit Diagram from LTSpice



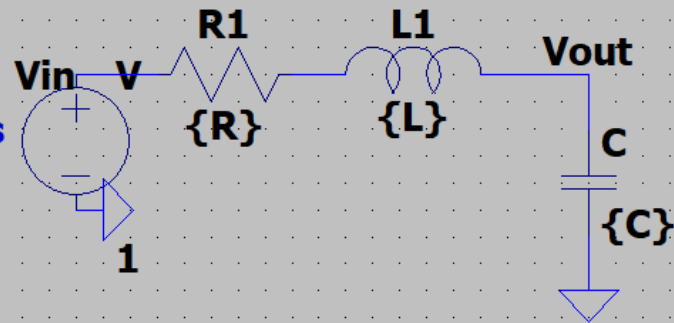


Simulation files in LTSpice have been attached.

Clearly, the damping factor is proportional to R . As we increase the damping factor, the $|H(s)|$ becomes more smoother, and the sharp peak is no longer seen. Also, $\text{phase}(H(s))$ becomes smoother.

ii)

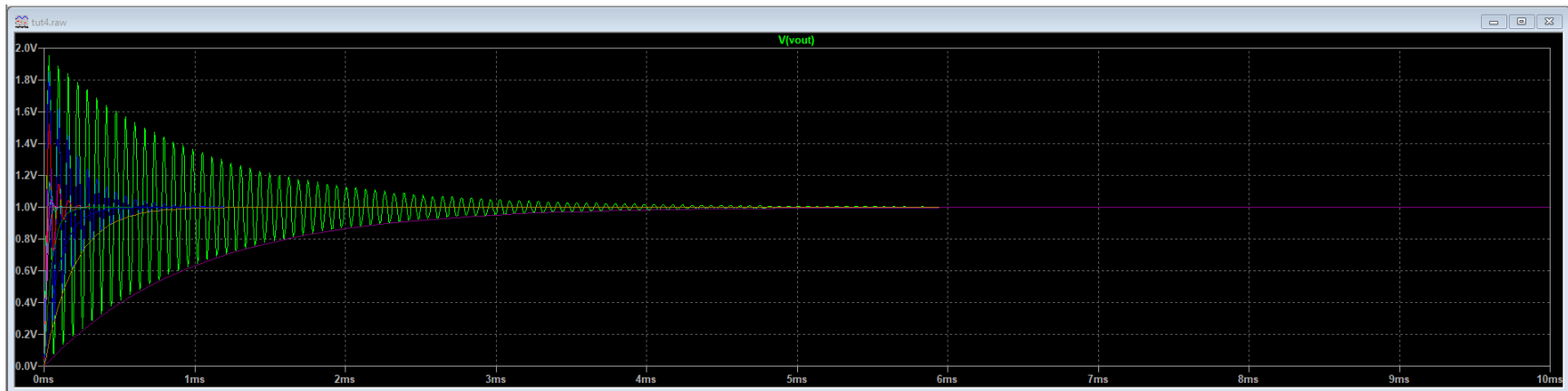
For AC analysis



```
.params L=10u C=10u
```

```
.step param R list 0.02 0.10 0.40 1.00 1.40 2.00 5.00 10.00 20.00 100.00
```

```
.tran 0 10m 0
```

As we increase the damping factor, the ringing decreases, then it reaches critical damping, after which there is no ringing.

iii)

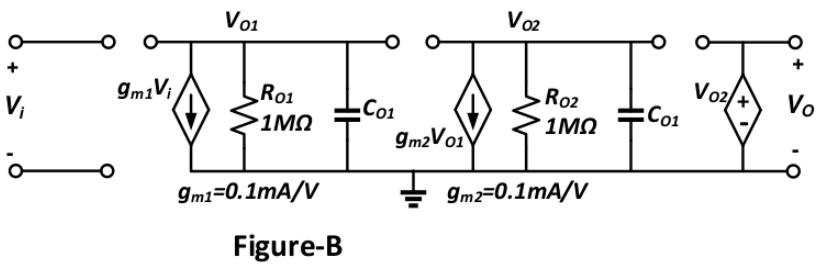
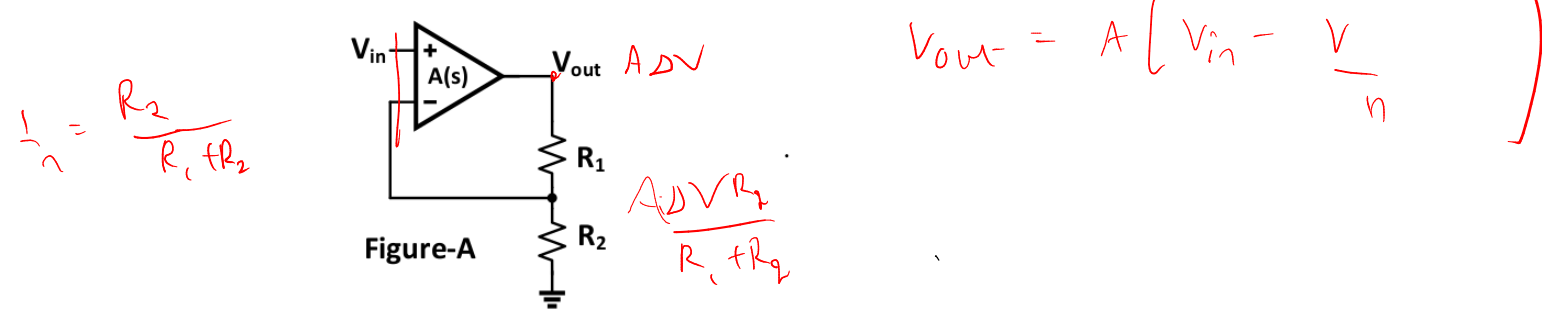
By looking at AC response, we can see the quality factor. If the quality factor is large, the damping factor is small and vice versa. If the damping factor is equal to 1, the quality factor is $\frac{1}{2}$.

By looking at the transient response:

- Damping factor > 1 : Overdamped, so there is no ringing
- Damping factor < 1 : Underdamped, ringing is present
- Damping factor $= 1$: Critically damped. Fastest settling time is for damping factor $= 1$.

So, we can find the value of R at which settling time is fastest, that will be damping factor $= 1$.

2. For the non-inverting amplifier shown in Figure-A and considering op-amp model shown in Figure-B where $A(s)=V_o(s)/V_i(s)$



a) Find the loop gain transfer function, $LG(s)$, DC gain A_o , poles p_{1_lg} , p_{2_lg} (roots of s) and their respective frequencies, ω_{p1_lg} and ω_{p2_lg} in terms of g_{m1} , R_{o1} , C_{o1} , g_{m2} , R_{o2} and C_{o2} .

$A(s) = \frac{g_{m1}}{\left(\frac{1}{R_{o1}} + s C_{o1} \right)} \cdot \frac{g_{m2}}{\left(\frac{1}{R_{o2}} + s C_{o2} \right)}$

$LG(s) = \frac{A(s) R_2}{R_1 + R_2}$

DC gain = $g_{m1} g_{m2} R_{o1} R_{o2}$

Poles are $-\frac{1}{R_{o1} C_{o1}}$ & $-\frac{1}{R_{o2} C_{o2}}$

$\omega_1 = \frac{1}{R_{o1} C_{o1}}$ $\omega_2 = \frac{1}{R_{o2} C_{o2}}$

b) Find the closed loop transfer function, $H(s) = V_{out}(s)/V_{in}(s)$, poles p_{1_cl} , p_{2_cl} (roots of s) and their respective frequencies, ω_{p1_cl} and ω_{p2_cl} in terms of g_{m1} , R_{o1} , C_{o1} , g_{m2} , R_{o2} and C_{o2} .

$$\frac{1}{n} = \frac{R_2}{R_1 + R_2}$$

$$H(s) = \frac{1}{\frac{1}{A} + \frac{1}{n}}$$

$$= \frac{1}{\frac{1}{A} + \frac{R_2}{R_1 + R_2}}$$

$$\frac{g_{m1}}{\left(\frac{1}{R_{o1}} + sC_{o1}\right)}$$

$$\frac{g_{m2}}{\left(\frac{1}{R_{o2}} + sC_{o2}\right)}$$

$$= \frac{1}{\left(\frac{1}{R_{o1}} + sC_{o1}\right)\left(\frac{1}{R_{o2}} + sC_{o2}\right) + \frac{R_2}{R_1 + R_2}}$$

$$= \frac{g_{m1} g_{m2}}{\left(\frac{1}{R_{o1}} + sC_{o1}\right)\left(\frac{1}{R_{o2}} + sC_{o2}\right) + \frac{R_2}{R_1 + R_2}}$$

$$s_{1,2} = \frac{-\left(\frac{C_{o1}}{R_{o1}} + \frac{C_{o2}}{R_{o2}}\right) \pm \sqrt{\left(\frac{C_{o1}}{R_{o1}} + \frac{C_{o2}}{R_{o2}}\right)^2 - 4C_{o1}C_{o2}}}{2C_{o1}C_{o2}}$$

$$= \frac{-(p_1 + p_2) \pm \sqrt{(p_1 - p_2)^2 - 4p_1 p_2}}{2}$$

c)

- c) Prove that, the circuit in Figure-A behaves similar to the RLC circuit of problem-1 for feedback factor, $\beta=1$ (i.e. $R_1=0$ or $R_2=\infty$) and find the expressions for damping factor (ζ), quality factor (Q), natural frequency (ω_n) in terms of loop gain pole frequencies, ω_{p1_lg} and ω_{p2_lg} and DC gain, A_o .

$$\beta = \frac{1}{A} = 1$$

$$\begin{aligned}
 A(s) &= \frac{1}{1 + \frac{1}{A}} = \frac{A}{1 + \frac{1}{A}} = \frac{g_{m1}g_{m2}}{(s - \omega_{p1_lg})(s - \omega_{p2_lg}) + \frac{1}{R_1R_2g_{m1}g_{m2}}} \\
 &= \frac{g_{m1}g_{m2}}{s^2 - s(\omega_{p1_lg} + \omega_{p2_lg}) + \frac{1}{R_1R_2g_{m1}g_{m2}}} \\
 &= \frac{1}{1 + \frac{s^2 - s(\omega_{p1_lg} + \omega_{p2_lg})}{s_{n1}s_{n2}} + \frac{1}{s_{n1}s_{n2}R_1R_2g_{m1}g_{m2}}}
 \end{aligned}$$

- d) Considering $g_{m1}=g_{m2}=0.1\text{mA/V}$ and $R_{o1}=R_{o2}=1\text{M}\Omega$, fill the values in following table for the corresponding values of C_{o1} and C_{o2}

C_{o1} (F)	C_{o2} (F)	Loop Gain				Closed Loop			
		w_{p1_lg} (rad/s)	w_{p2_lg} (rad/s)	w_{ugf} (rad/s)	PM (deg.)	ζ	$Q=1/2\zeta$	w_{p1_cl} (rad/s)	w_{p2_cl} (rad/s)
1e-9	1e-9								
10e-9	1e-10								
4e-8	2.5e-11								
1e-7	1e-11								
1.41e-7	7.07e-12								
2e-7	5e-12								
5e-7	2e-12								
1e-6	1e-12								
2e-6	5e-13								
1e-5	1e-13								

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c1	c2	w1	w2	wug	pm	eta	Q	pole1	pole2
1e-09	1e-09	1000	1000	99995	1.14593	0.01	50	100000	100000
1e-08	1e-10	100	10000	99750.3	5.78223	0.0505	9.90099	100000	100000
4e-08	2.5e-11	25	40000	96083	22.6172	0.200125	2.49844	100000	100000
1e-07	1e-11	10	100000	78615.1	51.8346	0.50005	0.9999	100000	100000
1.41e-07	7.07e-12	7.0922	141443	64524.9	65.4842	0.706141	0.708074	100157	100157
2e-07	5e-12	5	200000	48586.8	76.3513	1.00002	0.499988	99295.4	100710
5e-07	2e-12	2	500000	19984	87.7169	2.50001	0.199999	20871.1	479131
1e-06	1e-12	1	1e+06	9999.5	89.4328	5	0.0999999	10102	989899
2e-06	5e-13	0.5	2e+06	4999.98	89.8625	10	0.05	5012.56	1.99499e+06
1e-05	1e-13	0.1	1e+07	1000	90	50	0.01	1000.1	9.999e+06

- e) Verify that increasing the spacing between loop gain poles, w_{p1_lg} and w_{p2_lg} has similar effect on damping factor as increasing R has in the RLC circuit of problem-1.

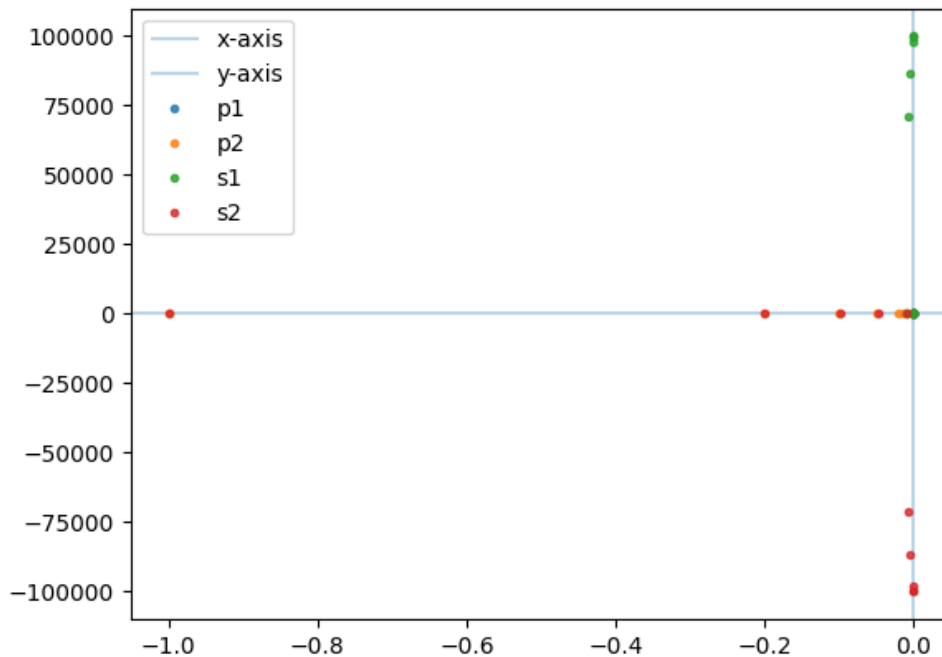
$$\gamma = \frac{1}{2} \sqrt{A_0} \left[\sqrt{\frac{L_1}{P_2}} + \sqrt{\frac{L_2}{P_1}} \right]$$

$$\text{As } P_1 \uparrow \text{ or } P_2 \uparrow, \quad \gamma \uparrow$$

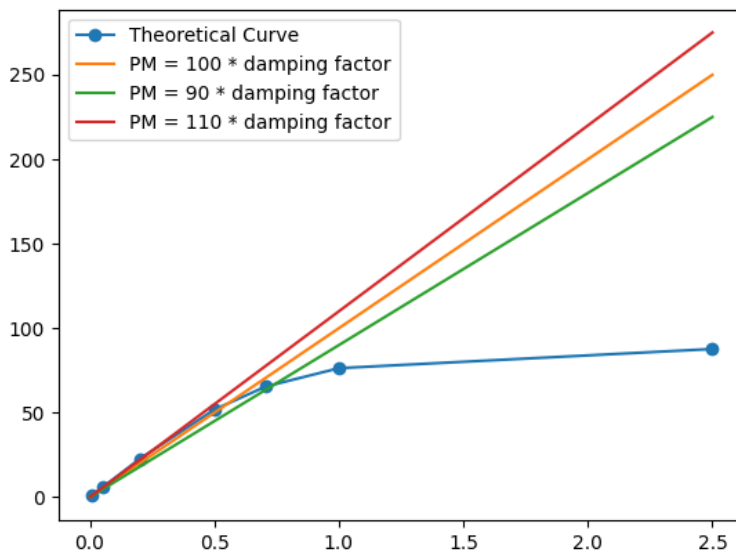
Clearly, as in above table, as γ increases,

$$w_{p2_lg} - w_{p1_lg} \uparrow$$

- f) Show the locations of both loop gain poles p_{1_lg} , p_{2_lg} and corresponding closed loop poles, p_{1_cl} , p_{2_cl} on s-plane for different values of ζ calculated in the above table.



- g) Plot phase margin (PM) vs. ζ and find the range of ζ for which phase margin can be approximated as 100 times of ζ with $\pm 10\%$ inaccuracy.



Range of values is approx
 $[0.01, 0.70]$

Question 2

h)

Explanation:

As the damping factor changes to 1, the oscillations gradually decrease. At damping factor = 1, it is critically damped. For damping factor > 1, it takes a longer time to reach the final value.

Graphically, phase margin is approx 75 degrees.

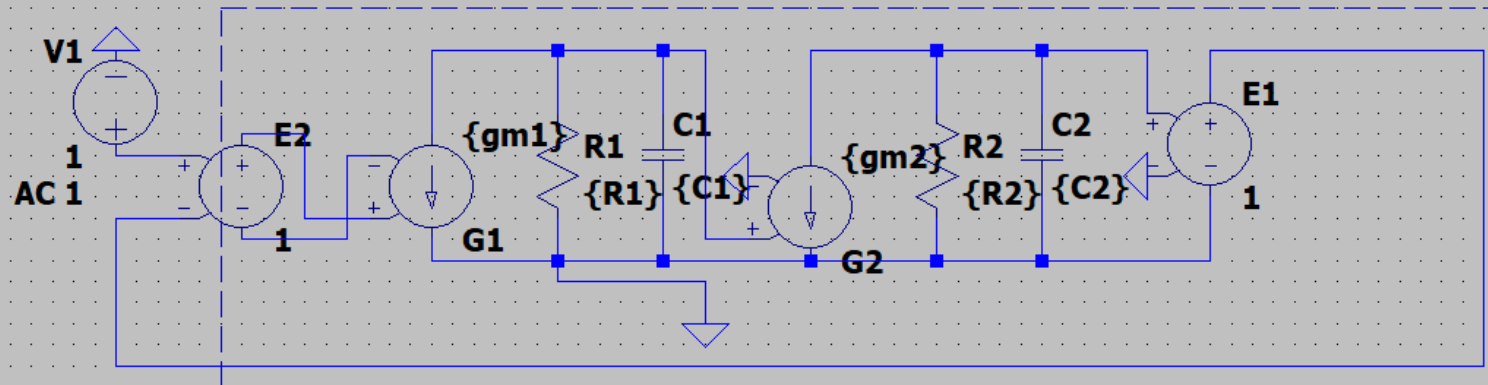
Damping factor is proportional to $\beta^{1/2}$.

So, the new value of damping factor will be $\sqrt{10} * \beta$. This can be also observed in the simulations.

i)

Haricharan B EP21B015

Op Amp



.ac dec 100 10 1G

.params r1=1000k r2=1000k gm1=0.1m gm2=0.1m

.tran 0 1m 0 startup

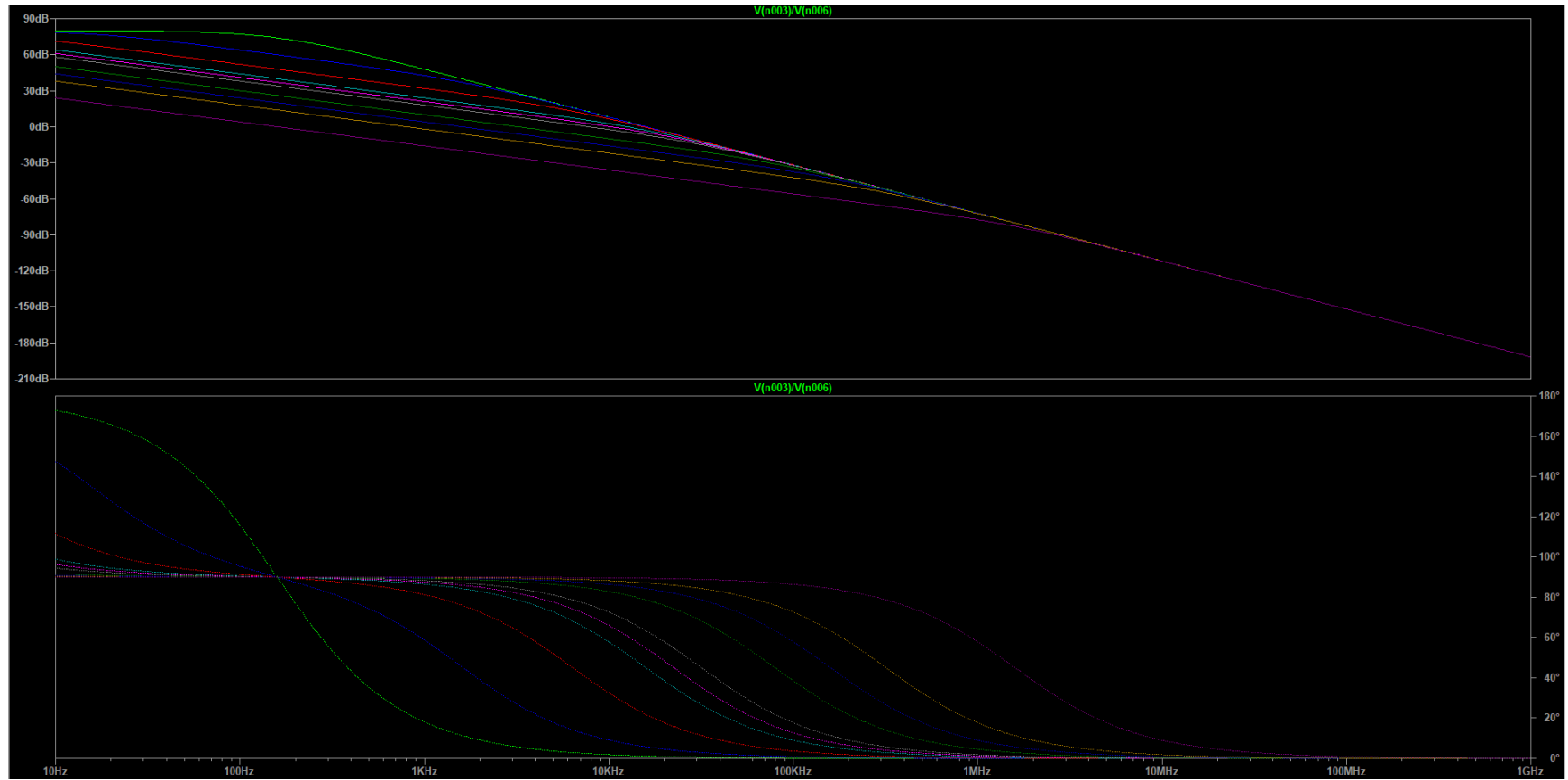
.step param K list 1 2 3 4 5 6 7 8 9 10

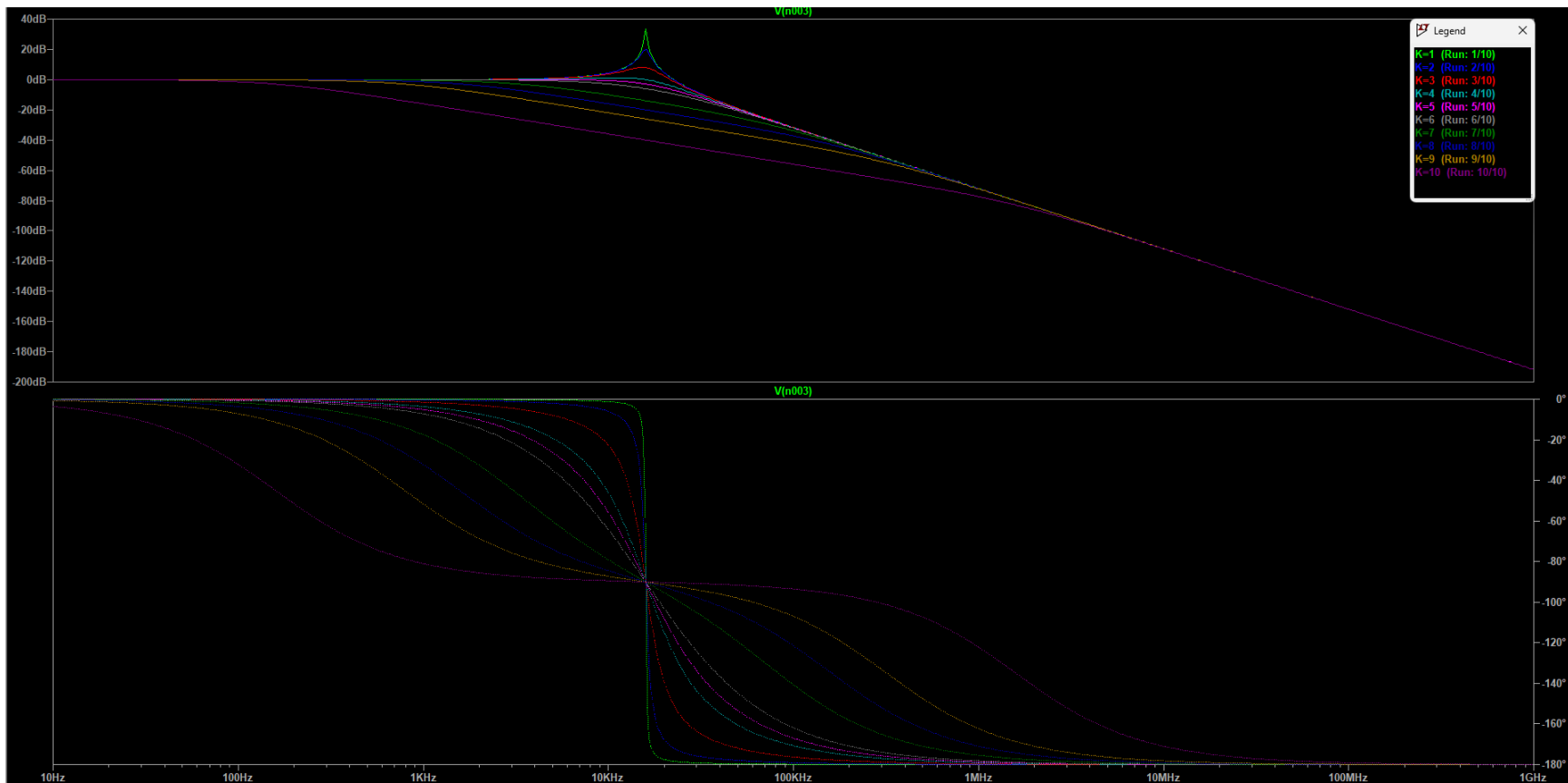
.param c1=table(K, 1, 1n, 2, 10n, 3, 40n, 4, 100n, 5, 141.42n, 6, 200n, 7, 500n, 8, 1u, 9, 2u, 10, 10u)

.param c2=table(K, 1, 1n, 2, 0.1n, 3, 25p, 4, 10p, 5, 7.07p, 6, 5p, 7, 2p, 8, 1p, 9, 0.5p, 10, 0.1p)

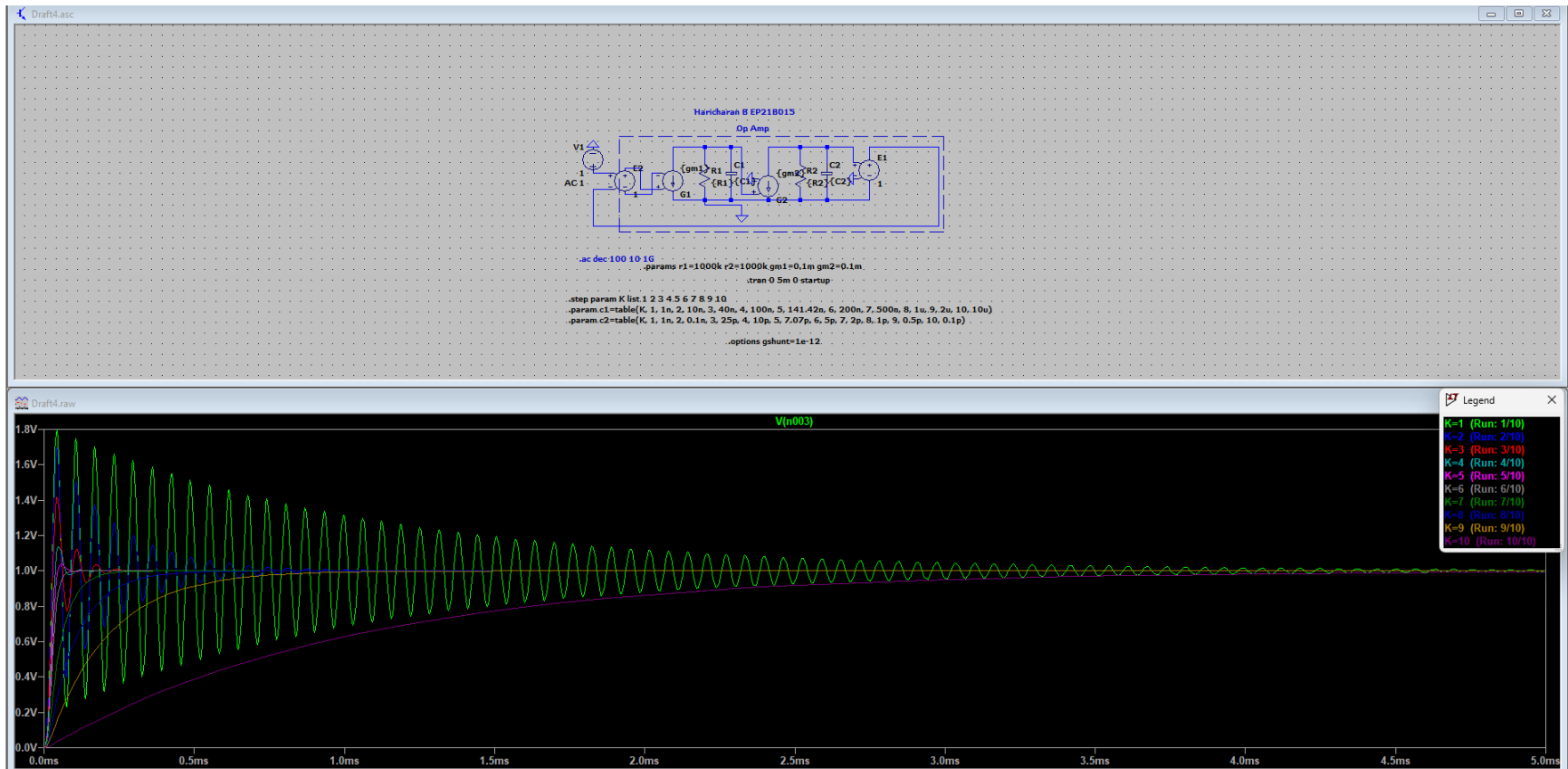
.options gshunt=1e-12

Loop gain Transfer function





ii)



iii)

