EE2019–Analog Systems and Lab: Tutorial 5

Nagendra Krishnapura, Shanthi Pavan

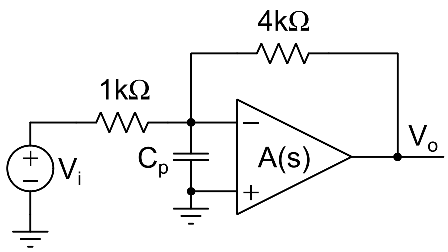


Figure 1: Circuit for problem 1

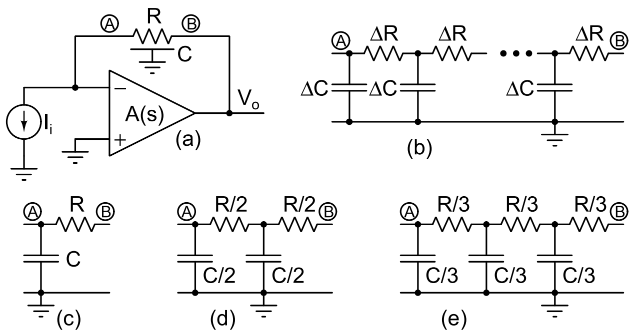


Figure 2: Circuit for problem 2

1. Fig. 1 shows an inverting amplifier. The opamp has a gain $A(s) = A_0/(1 + s/p_1)(1 + s/p_2)$ where $A_0 = 20000$, $p_1 = 1\,$krad/s, $p_2 = 10\,$Mrad/s. C_p is a parasitic capacitor.
   - What is the phase margin of the system with $C_p = 0$?
   - What is the closed loop bandwidth of the system? (Calculate this from (a) Unity loop gain frequency, (b) Natural frequency of the second order system, and (c) Exact calculation-computing the frequency at which the gain magnitude drops to $1/\sqrt{2}$ times the dc gain.; Compare the estimates so obtained)
   - What is the value of C_p for which the circuit becomes unstable?
   - With C_p being the value calculated in the previous part, can you change the circuit so that the phase margin is 60° without changing the opamp or the closed-loop dc gain V_o/V_i?

2. Fig. 2 shows a transimpedance amplifier. The opamp has a frequency independent gain A_0. The feedback resistor R has a parasitic capacitor C. C is distributed across the length of the resistor and should be modeled as shown in Fig. 2(b) where the infinite number of infinitesimal ΔR and ΔC sum up to R and C respectively. This cannot be analyzed easily, so we model it as shown in Fig. 2(c), (d), or (e). Analyze each case and comment on the effect of A_0 on stability or damping. (In addition to stability, this problem also tells you something about oversimplified models).

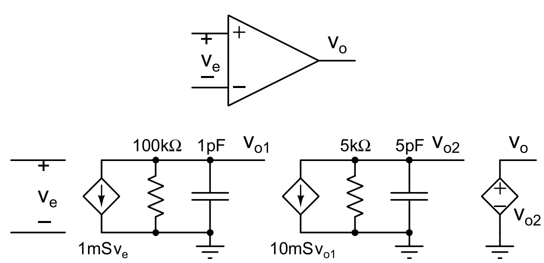
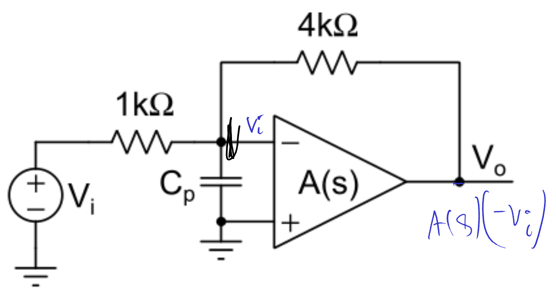


Figure 3: Circuit for problem 3

3. Fig. 3 shows the internal schematic of an opamp. This opamp is used to realize a unity gain, non-inverting amplifier.
   - What is the phase margin?
   - Connect a capacitor across one of the existing capacitors inside the opamp so that the phase margin is 60°.

Repeat the above if the opamp is used to realize an inverting amplifier of gain -4.



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   - What is the phase margin of the system with $C_p = 0$?

$$\frac{A_0}{\left(1+\frac{s}{p_1}\right)\left(1+\frac{s}{p_2}\right)}$$

$$Lh(s) = -\frac{\frac{1}{5}A_0}{\left(1+\frac{s}{p_1}\right)\left(1+\frac{s}{p_2}\right)}$$

$$= \left|\frac{-4000}{\left(1+\frac{s}{10^3}\right)\left(1+\frac{s}{10^7}\right)}\right| = 1$$

$$4000^2 = \left(1+\frac{\omega^2}{10^6}\right)\left(1+\frac{\omega^2}{10^{14}}\right)$$

$$\boxed{\omega_{ug} = 4 \times 10^6 \text{ rad/s}}$$

$$\angle \frac{-4000}{(1+j\,4 \times 10^3)(1+j\,0.4)} = \boxed{68^\circ}$$

- What is the closed loop bandwidth of the system? (Calculate this from (a) Unity loop gain frequency, (b) Natural frequency of the second order system, and (c) Exact calculation-computing the frequency at which the gain magnitude drops to $1/\sqrt{2}$ times the dc gain.; Compare the estimates so obtained)

a) $\boxed{\omega_{ug} = 4 \times 10^{6} \text{ rad/s}}$

b) 2nd order system

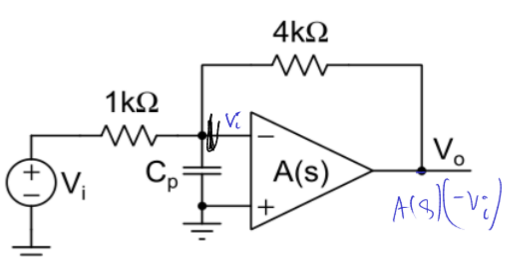
$$\frac{-4 A(s)}{\boxed{A(s)+5}} = s^2 + \underbrace{2\zeta s \omega_n}_{} + \omega_n^2$$

$$\left(1+\frac{s}{p_1}\right)\left(1+\frac{s}{p_2}\right)5 + A_0 = 0$$

$$s = -5\times10^{6} \pm 3.873\times10^{6}$$

$$\text{abs}(s) = \boxed{6.35 \times 10^{6} \text{ rad/s}}$$

c) Exact



$$V_o = A(s)(-V_1)$$

$$\frac{V_i - V_1}{1k} = \frac{V_1 - V_o}{4k}$$

$$4V_i - 4V_1 = V_1 - V_o$$

$$4V_i + V_o = 5V_1 = -\frac{V_o\, 5}{A(s)}$$

$$V_i = \frac{-V_o - \frac{V_o\, 5}{A(s)}}{4}$$

$$\frac{V_i}{V_o} = \frac{-1 - \frac{5}{A(s)}}{4} \qquad \frac{V_o}{V(s)} = \frac{4}{-1 - \frac{5}{A(s)}}$$

$$= \boxed{\frac{-4\, A(s)}{+A(s) + 5}}$$

$$\boxed{\left|\frac{V_i(s)}{V_o}\right| = \frac{1}{\sqrt{2}}}$$

$$\left| \frac{\frac{+4\, A_o}{\left(1+\frac{j\omega}{p_1}\right)\left(1+\frac{j\omega}{p_2}\right)}}{\frac{A_o}{\left(1+\frac{j\omega}{p_1}\right)\left(1+\frac{j\omega}{p_2}\right)} + 5} \right| = \frac{4}{\sqrt{2}}$$

$$\left| \frac{4\, A_o}{A_o + 5\left(1+\frac{j\omega}{p_1}\right)\left(1+\frac{j\omega}{p_2}\right)} \right| = \frac{4}{\sqrt{2}}$$

$$\text{abs}\left(4 * \frac{20000}{\left(20000 + 5\left(1 + \frac{iw}{1e3}\right)\left(1 + \frac{iw}{1e7}\right)\right)}\right) = \frac{4}{\text{sqrt}(2)}$$

NATURAL LANGUAGE MATH INPUT

POPULAR

Assuming i is the imaginary unit | Use i as a variable instead

Input interpretation

$$\left|4 \times \frac{20\,000}{20\,000 + 5\left(1 + \frac{i\,w}{1 \times 10^3}\right)\left(1 + \frac{i\,w}{1 \times 10^7}\right)}\right| = \frac{4}{\sqrt{2}}$$

$|z|$ is the absolute value of z

i is the imaginary unit

Result

$$\frac{80\,000}{\left|5\left(\frac{i\,w}{10\,000\,000} + 1\right)\left(\frac{i\,w}{1000} + 1\right) + 20\,000\right|} = 2\sqrt{2}$$

Real solutions More digits Exact forms Step-by-step solution

$w \approx -5\,587\,607$

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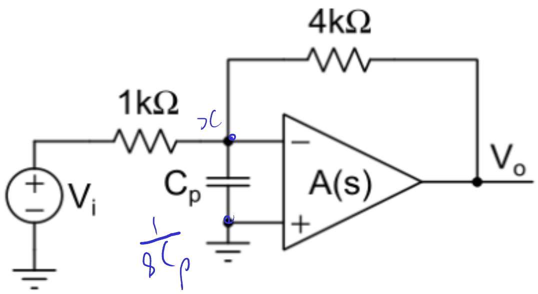
$$\boxed{\omega = 5.587 \times 10^6 \text{ rad/s}}$$

- What is the value of C_p for which the circuit becomes unstable?



$$\frac{V_i - x}{1k} + \frac{V_o - x}{4k} + \frac{0 - x}{1/sC_p} = 0$$

$$\boxed{V_o = -A(s)\,x}$$

$$\frac{-V_o}{A(s)}\left[\frac{1}{1k} + \frac{1}{4k} + sC_p\right] = \frac{V_i}{1k} + \frac{V_o}{4k}$$

$$-V_o\left[\frac{1}{A(s)}\left(\frac{1}{1k} + \frac{1}{4k} + sC_p\right) + \frac{1}{4k}\right] = \frac{V_i}{1k}$$

$$\frac{V_o}{V_i} = -\frac{1}{1k}\,\frac{1}{\boxed{\left(\frac{1}{A(s)}\left(\frac{1}{1k} + \frac{1}{4k} + sC_p\right) + \frac{1}{4k}\right)}}$$

$$= -\frac{1}{1k\left[\frac{(1+\frac{s}{p_1})(1+\frac{s}{p_2})}{A_0}\left(\frac{1}{800} + sC_p\right) + \frac{1}{4k}\right]}$$

$$= -\frac{1}{1k\left[\frac{(1+\frac{s}{p_1})(1+\frac{s}{p_2})(1+800sC_p)}{800\,(20000)} + \frac{16000}{(16000)\,4000}\right]}$$

$$= \frac{16000}{\left((1+\frac{s}{p_1})(1+\frac{s}{p_2})(1+800C_ps) + 4000\right)}$$

$$L_u(s) = \frac{4\times10^3}{\left(1+\frac{s}{p_1}\right)\left(1+\frac{s}{p_2}\right)\left(1+\frac{s}{p_c}\right)}$$

$$\boxed{p_c = \frac{1}{800\,C_p}}$$

Now, set $PM = 0°$

Assume $p_1 \ll p_c \ll p_2$

$$0 = 180 - \tan^{-1}\left(\frac{w_u}{p_1}\right) - \tan^{-1}\frac{w_u}{p_c} - \tan^{-1}\left(\frac{w_u}{p_2}\right)$$

Take $\boxed{w_u = 10\,p_c}$ (by Bode approximation)

set $|L_u(s)| = 1$

$$4\times10^3 = \left|\left(1+\frac{s}{p_1}\right)\underbrace{\left(1+\frac{s}{p_2}\right)}_{\simeq 1}\left(1+\frac{s}{p_c}\right)\right|$$

$$4\times10^3 = \sqrt{1+\frac{100p_c^2}{10^6}}\sqrt{1+100}$$

$$\frac{4\times10^3}{\sqrt{101}} = \sqrt{\cancel{1}+\frac{100p_c^2}{10^6}}$$

$$\frac{16\times10^6}{101} = \frac{\cancel{100}p_c^2}{\cancel{10^6}\,10^4}$$

$$p_c^2 = \frac{16\times10^{10}}{101}$$

$$= 16\times10^8$$

$$p_c = \boxed{4\times10^4} = \frac{1}{800C}$$

$$C = \frac{1}{800\times4\times10^4}$$

$$= \boxed{31.25\,nF}$$

$$\boxed{PM = 0°}$$

becomes unstable?

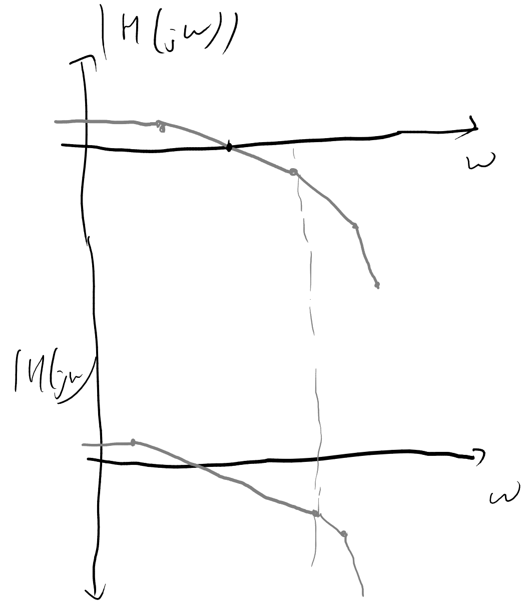
- With C_p being the value calculated in the previous part, can you change the circuit so that the phase margin is 60° without changing the opamp or the closed-loop dc gain V_o/V_i?

$\boxed{\text{Decrease } R_1 \text{ \& } R_2}$

Poles are $\frac{1}{R_1C_1}\uparrow$, $\frac{1}{R_2C_2}\uparrow$

Poles move to left

Wrong, so PM ↓



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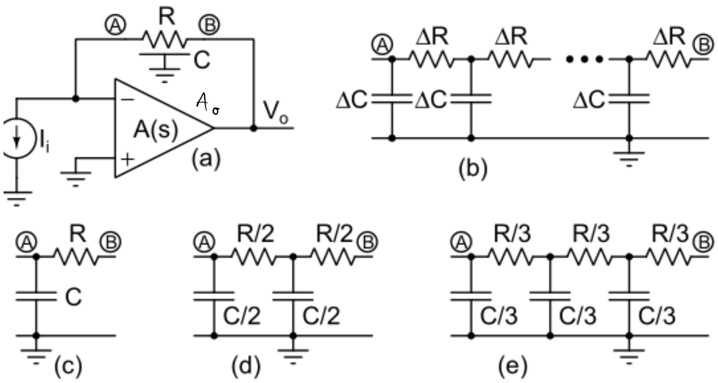
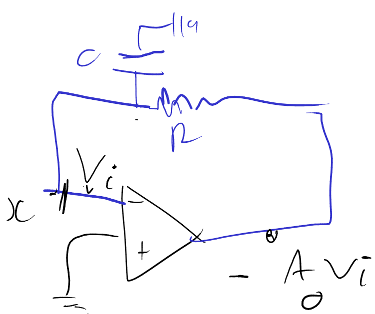


Figure 2: Circuit for problem 2

and C respectively. This cannot be analyzed easily, so we model it as shown in Fig. 2(c), (d), or (e). Analyze each case and comment on the effect of A_0 on stability or damping. (In addition to stability, this problem also tells you something about oversimplified models).



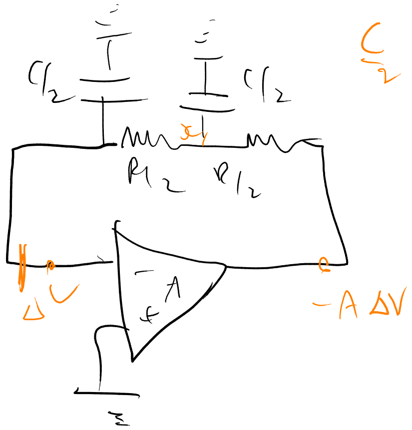
$$\frac{-A_o v_i - V_{fb}}{R} + \frac{0 - V_{fb}}{\left(\frac{1}{sC}\right)} = 0$$

$$\frac{-A_o v_i}{R} = V_{fb}\left(\frac{1}{R} + sC\right)$$

$$LL(s) = +A_o \frac{1}{R\left(\frac{1}{R} + sC\right)}$$

$$- \boxed{\frac{+A_o}{(1+RsC)}}$$

$$\boxed{s = \frac{-1}{RC}}$$



$\frac{C}{2} = C'$, $\frac{R}{2} = R'$

$$\frac{-A\Delta V - x}{R/2} + \frac{0 - x}{\frac{2}{sC}} + \frac{0 - x}{\frac{R}{2} + \frac{1}{sC/2}}$$

$$\frac{-A\Delta V}{R/2} = x\left[\frac{2}{R} + \frac{sC}{2} + \frac{1}{\frac{R}{2} + \frac{1}{sC/2}}\right]$$

$$x = \frac{\frac{-A\Delta V}{R/2}}{\left[\frac{2}{R} + \frac{sC}{2} + \frac{1}{\frac{R}{2} + \frac{1}{sC/2}}\right]}$$

$$\frac{V_{out}}{\Delta V} = \frac{\frac{-A}{R/2}}{\left[\frac{2}{R} + \frac{sC}{2} + \frac{1}{\frac{R}{2} + \frac{2}{sC}}\right]} \cdot \frac{\frac{1}{sC/2}}{\frac{1}{sC/2} + \frac{R}{2}}$$

$$LG(s) = \left(\frac{A}{\left(1 + \frac{sRC}{4} + \frac{RsC}{RsC + 4}\right)}\right)\left(\frac{4}{4 + RsC}\right)$$

$$= \frac{4A}{\left(1 + \frac{sRC}{4} + \frac{RsC}{RsC + 4}\right)\left[4 + RsC\right]}$$

$$= \frac{4A}{4 + RsC + sRC + \frac{s^2R^2C^2}{4} + RsC}$$

$$= \frac{16A}{\boxed{16 + 12RsC + s^2R^2C^2}}$$

144
64

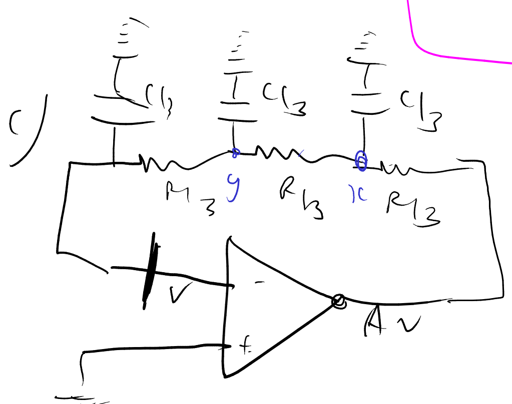
$$sRC = \frac{-12 \pm \sqrt{144 - 4(16)}}{2}$$

$$= \frac{-12 \pm \sqrt{80}}{2}$$

$$= \boxed{-6 \pm 2\sqrt{5}}$$

$$\boxed{s = \frac{(-6 \pm 2\sqrt{5})}{RC}} \simeq \frac{-1.527}{RC}, \frac{-10.4}{RC}$$

c)



$$\frac{Av - x}{R/3} + \frac{y - x}{R/3} + \frac{0 - x}{3/sC} = 0$$

$$\frac{0 - y}{3/sC} + \frac{x - y}{R/3} + \frac{0 - y}{\frac{R}{3} + \frac{3}{sC}} = 0$$

$$x = \frac{R}{3} y \left[\frac{sC}{3} + \frac{3}{R} + \frac{1}{\frac{R}{3} + \frac{3}{sC}} \right]$$

$$\frac{y}{R/3} = -\frac{Av}{3} + y \left[\frac{3}{R} + \frac{3}{R} + \frac{sC}{3}\right] \frac{R}{3} \left[\frac{sC}{3} + \frac{3}{R} + \frac{1}{\frac{R}{3} + \frac{3}{sC}}\right]$$

$$-\frac{Av}{3} = y \left[\frac{3}{R} - \frac{R}{3}\left(\frac{6}{R} + \frac{sC}{3}\right)\left(\frac{sC}{3} + \frac{3}{R} + \frac{1}{(R/3 + 3/(sC))}\right)\right]$$

$$\frac{V_O}{V} = \frac{-\frac{A}{3}}{\left[\frac{3}{R} - \frac{R}{3}\left(\frac{6}{R} + \frac{sC}{3}\right)\left(\frac{sC}{3} + \frac{3}{R} + \frac{1}{(R/3 + 3/(sC))}\right)\right] \frac{3}{(9 + RsC)}}$$

$$= \frac{-A}{\left(\frac{3}{R} - \frac{R}{3}\left(\frac{6}{R} + \frac{sC}{3}\right)\right)\left(\frac{sC}{3} + \frac{3}{R} + \frac{3sC}{RsC+9}\right)\left(9+RsC\right)}$$

$$= \frac{-A}{\left(\frac{3(9+RsC)}{R} - \frac{R}{3}\left(\frac{6}{R} + \frac{sC}{3}\right)\right)\left[3sC + \frac{Rs^2c^2}{3} + \frac{27}{R} + 3sC + 3sC\right]}$$

$$= \frac{-A}{(9+RsC)\frac{3}{R} - \frac{1}{3}\left(\frac{18+RsC}{3}\right)\left(\frac{Rs^2c^2}{3} + \frac{27}{R} + 9sC\right)}$$

$$= \frac{A}{\frac{81(9+RsC)}{27R} - (18+RsC)\left(\frac{R^2s^2c^2 + 81 + 27sCR}{27R}\right)}$$

$$= \frac{A(27R)}{729 + 81RsC - (18+RsC)(R^2s^2c^2 + 81 + 27sCR)}$$

$$\begin{array}{r} 27 \\ 18 \\ \hline 45 \end{array}$$

$$= \frac{27AR}{729 + 81RsC - \left(R^3c^3s^3 + 81RsC + 27s^2c^2R^2 + 18s^2c^2R^2 + (18)81 + (18)27sCR\right)}$$

$$= \frac{-27AR}{729 + 486sCR + 45s^2c^2R^2 + R^3c^3s^3}$$

$$\boxed{s = \frac{-29.2}{RC},\ \frac{-13.9}{RC},\ \frac{-1.78}{RC}}$$

We can see that, as more R & C are added, poles become larger & larger.

DC gain increases by large value, but DC gain is lesser, so P.M. ↓

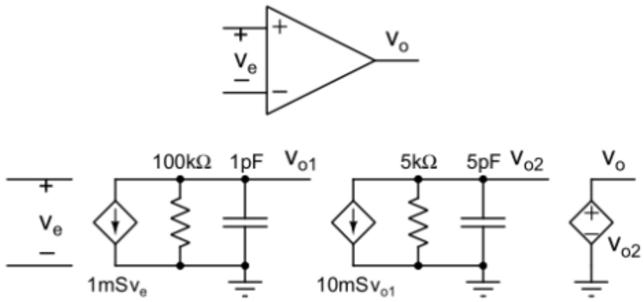


Figure 3: Circuit for problem 3

3. Fig. 3 shows the internal schematic of an opamp. This opamp is used to realize a unity gain, non-inverting amplifier.
   - What is the phase margin?
   - Connect a capacitor across one of the existing capacitors inside the opamp so that the phase margin is 60°.

$$V_o = V_{o2}$$

$$= \frac{(10mS\, V_{o1})}{\left(sC_2 + \frac{1}{R_2}\right)}$$

$$A(s) = \frac{10mS \;\; 1mS}{\left(sC_1 + \frac{1}{R_1}\right)\left(sC_2 + \frac{1}{R_2}\right)}$$

$$= \frac{10^{-7} \times R_1 R_2}{(R_1 C_1 s + 1)(R_2 C_2 s + 1)}$$

$$= \frac{5000}{\left(\frac{s}{10^7} + 1\right)\left(1 + \frac{s}{4 \times 10^7}\right)}$$

$$5000 = \sqrt{\frac{\omega^2}{10^{14}} + 1}\sqrt{\frac{\omega^3}{16 \times 10^{14}} + 1}$$

$$25 \times 10^6 = \left(\frac{t}{10^{14}} + 1\right)\left(\frac{t}{16 \times 10^{14}} + 1\right)$$

$$\boxed{\omega_{ug} = 1.4 \times 10^{9}}$$

$$\angle A(s) = \angle\left(\frac{50}{\left(\frac{s}{10^{7}}+1\right)\left(\frac{s}{4\times10^{7}}+1\right)}\right)$$

$$= 180^{\circ} - \left(\tan^{-1}\left(\frac{\omega_{ug}}{p_1}\right) + \tan^{-1}\frac{\omega_{ug}}{p_1}\right)$$

$$= 2^{\circ}$$

- Connect a capacitor across one of the existing capacitors inside the opamp so that the phase margin is 60°.

Let us connect across first capacitor

Let C be new capacitance

$$1 = \left|\frac{50}{\left(1+\frac{s}{p_1}\right)\left(1+\frac{s}{p_2}\right)}\right|$$

$$\left(1+\frac{s}{p_2}\right) \simeq 1$$

$$\boxed{p_1 << p_2}$$

$$W_{ug} = 50 \times p_1$$

$$= 5000 \times \frac{1}{(10^5)\,C} = \boxed{\frac{1}{20C}}$$

$$60 = 180 - \tan^{-1}\left(\frac{\frac{1}{20C}}{10^5\,C}\right) - \tan^{-1}\left(\frac{\frac{1}{20C}}{4\times10^{-7}}\right)$$

$$4 \times 10^{-7} \times 20C = \sqrt{3}$$

$$C = \sqrt{3}\,\frac{10^7}{80} = \boxed{1.25 \times 10^{-9}\sqrt{3}\ nF}$$

$$C_{eq} = C_1 + C_2, \quad C_1 = \boxed{1.25 \times 10^{-9}\sqrt{3}\ nF}$$

2nd Cap

$P_1 \ggg P_2$

$$1 = \frac{50\,\omega_{ug}}{\cancel{\left(1+\frac{s}{P_1}\right)}\left(1+\frac{s}{P_2}\right)}$$

$$\omega_{ug} = \frac{5\times10^3 \times}{5\times10^3\,(C)} = \boxed{\frac{1}{C}}$$

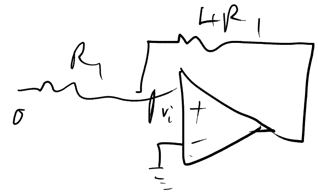
$$60 = 180 - \tan^{-1}\left(\frac{\omega_{ug}}{P_1}\right) - \underbrace{\tan^{-1}\left(\frac{\omega}{P_2}\right)}_{\simeq 90^\circ}$$

$$\frac{\omega_{ug}}{P_1} = \frac{1}{\sqrt{3}} \qquad \frac{\frac{1}{C}}{10^7} = \frac{1}{\sqrt{3}}$$

$$\boxed{C = \sqrt{3}\times10^{-7}} = \boxed{173.2\,nF}$$

$$\boxed{C' \simeq 17.32\,nF}$$

c) Repeat the above if the opamp is used to realize an inverting amplifier of gain −4.



$$LG = -\beta A(s) = \boxed{-\frac{1}{5} A(s)}$$

$$LG(s) = \frac{1000}{\left(1+\frac{s}{10^7}\right)\left(1+\frac{s}{4\times 10^7}\right)}$$

$$\omega_{ug} = \boxed{6.31 \times 10^8 \text{ rad/s}}$$

$$PM = 180 - \tan^{-1}\left(\frac{\omega_{ug}}{P_1}\right) - \tan^{-1}\left(\frac{\omega_{ug}}{P_2}\right)$$

$$\boxed{4.44^\circ}$$

* Connect across 1st capacitor

Let C be effective capacitance

Assume $\boxed{P_1 << P_2}$ $\boxed{P_1 = \frac{1}{RC}}$

$$1 = \frac{1000}{\left(1+\frac{s}{P_1}\right)\left(1+\frac{s}{P_2}\right)} \simeq \frac{1000}{\left(1+\frac{s}{P_1}\right)}$$

$$\omega_{ug} = 1000 \, P_1 = \frac{1000}{10^5 \, C} = \boxed{\frac{1}{100C}}$$

$$PM = 60^\circ = 180 - \underbrace{\tan^{-1}\left(\frac{\omega_{ug}}{p_1}\right)}_{90^\circ} - \tan^{-1}\left(\frac{\omega_{ug}}{p_2}\right)$$

$$\omega_{ug} = \frac{1}{\sqrt{3}} p_2$$

$$\frac{1}{100C} = \frac{1}{\sqrt{3}} \cdot 4\times10^7$$

$$C = \frac{\sqrt{3}}{4\times10^9} = \boxed{\frac{\sqrt{3}}{4}\,nF}$$

$$C_{actual} \simeq \boxed{\frac{\sqrt{3}}{4}\,nF - 1\,pF} = \boxed{432\,pF}$$