Experiment- 4
Simulation of M/M/1 Queues

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***Abstract*—In this experiment, we simulate M/M/1 queues in the Python language. We study work-conserving queueing disciplines like FIFO, LIFO, PS and analyze parameters like the average waiting time, average sojourn time, and average queue length and compare it with the theoretically observed value.**

INTRODUCTION

M/M/1 Queues are queues in which both the arrival process and the service process are Poisson processes for continuous time (the "M" there indicates that it is memoryless). There is also only one server present (that is the "1", and the buffer size is assumed to be infinite.

PROCEDURE AND METHODOLOGY

- **FIFO (First-in-first-out) Queue:** The FIFO queue is maintained using the queue data structure. In this, every time a new packet enters the system, it is added to the end of the queue and is served last.
- **LIFO (Last-in-first-out) Queue:** The LIFO queue is maintained using a stack data structure. If a new packet arrives while a packet is currently being served, the packet is finished serving, and then the new packet is served.
- **PS (Processor Sharing):** The resources in the server are equally shared among all the packets in the system. For example, if there are 3 packets entering the system at the same time, all of them are served simultaneously, and it takes thrice the amount of time to serve them.

It is noted here that the quantities plotted and simulated are actually the average sojourn times (waiting time + service time) and average number of packets in the system instead of waiting times and queue length. This is to ensure uniformity across all the systems. The average number of packets in the system is approximately the queue length (neglecting the one packet that is served currently), while the sojourn time is approximately the same as waiting time (neglecting the serving time). Little's theorem can also be verified using the data.

THEORY FOR FIFO QUEUES

The FIFO Queue can be modelled as a CTMC (continuous-time markov chain), in which the inter-arrival times are exponential with parameter λ and the service times are exponential with parameter μ. Let us define $\rho = \frac{\lambda}{\mu}$. Since we can't serve the packets if the arrival rate is greater than the service rate, let us assume $\lambda < \mu \implies \rho < 1$.

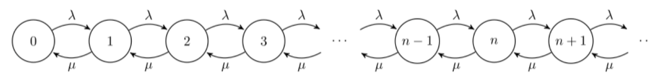

Fig. 1. State Space Diagram

Solving this Markov chain using the balance equations, if the probability that there are i packets in the system would be $\pi_i = (1-\rho)\rho^i$. It is noted here that this is a geometric distribution, and hence the average number of packets in the system is expected value of geometric i.e. $\frac{1}{1-\rho} - 1 = \boxed{\frac{\rho}{1-\rho}}$.

Using Little's law (not proven here), we observe that average sojourn time is $\frac{1}{\lambda} \cdot \frac{\rho}{1-\rho} = \boxed{\frac{1}{\mu - \lambda}}$.

OBSERVATIONS

Note: For better plots, instead of 10 *arrival rates (as required in the question),* 20 *values of arrival rates are considered.* For clarity, the theoretical plots are also plotted.

A. FIFO Scheduling

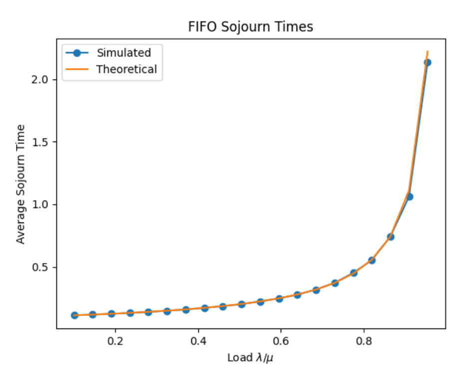

Fig. 2. FIFO Sojourn Times

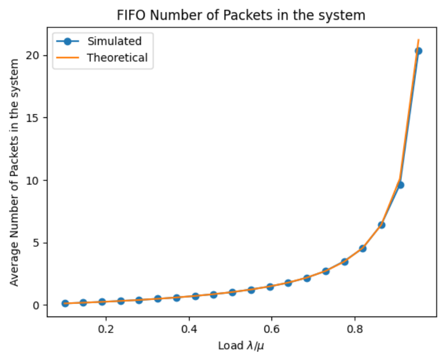

Fig. 3. FIFO Number of packets

B. LIFO Scheduling

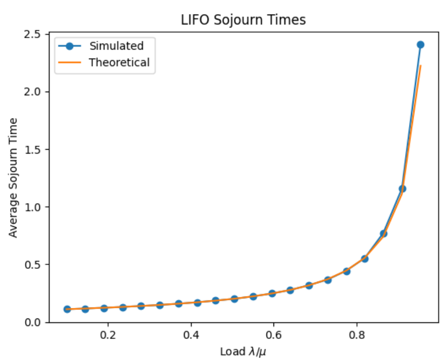

Fig. 4. LIFO Sojourn Times

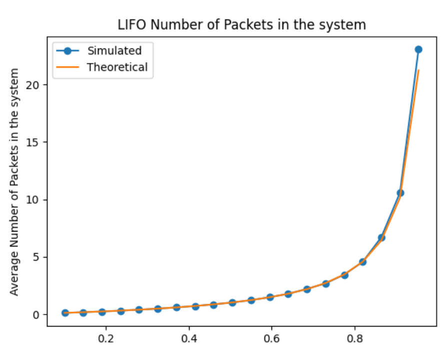

Fig. 5. LIFO Number of packets

C. PS Scheduling

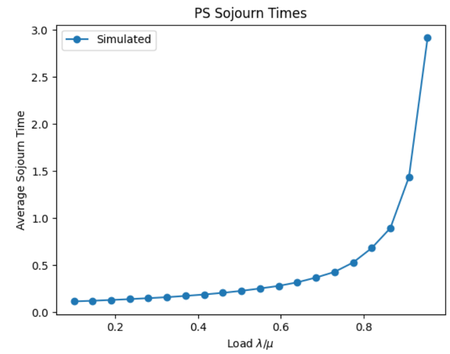

Fig. 6. PS Sojourn Times

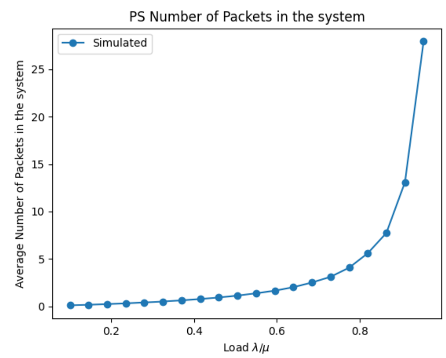

Fig. 7. PS Number of packets

CONCLUSIONS

- We observe that the average queue length and waiting time is the same for both FIFO and LIFO queues. This is expected, since it is a work-conserving system.
- In all of the cases, we observe that as the load is close to 1, the waiting time and the number of packets in the system tend to infinity.
- We can also observe Little's theorem, i.e. the number of packets in the system is directly proportional to the sojourn time.

REFERENCES

[1] https://www.win.tue.nl/ĩadan/que/h4.pdf
[2] https://en.wikipedia.org/wiki/M/M/1_queue