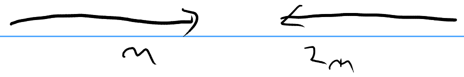
1. A particle of mass m and velocity u_1 makes a head-on collision with another particle of mass $2m$ at rest. If the coefficient of restitution is such to make the loss of the total K.E. a maximum, what are velocities v_1 and v_2 after the collision.



Stick together after collision

$$m u_1 = (3m) v_1 \qquad \boxed{v_1 = u_1 \frac{1}{3}, \quad v_2 = u_1 \frac{1}{3}}$$

$b = K \cot\frac{\theta}{2}$

$\boxed{b = Z_\alpha Z_\beta e^2 \cot\frac{\theta}{2}}$

2. A parallel beam of energetic alpha particles (helium nuclei from radium decay) of kinetic energy E is sent towards a thin gold foil, scattering off of individual gold nuclei.

(a) Assume that the potential is that of a "point-like" scatterer, so that

$$V(r) = Z_{Au} Z_\alpha e^2 / r$$

down to the smallest values of r accessible by the experimental conditions. Starting with the formula relating the impact parameter b to the scattering angle θ, derive the differential cross section for Rutherford scattering:

$$\frac{d\sigma}{d\Omega} = \left(\frac{Z_{Au} Z_\alpha e^2}{4E}\right)^2 \frac{1}{\boxed{\sin^2\frac{\theta}{2}}}$$

You can assume that the gold nucleus target is so much heavier than the beam alpha particle that it is at rest in the center of mass (neglect recoil), and the total center-of-mass energy is equal to the kinetic energy of the beam alpha particle, E.

(b) Sketch plot the differential cross section as a function of θ.

(c) How close (minimum distance of approach) will an alpha particle with kinetic energy E come to the gold nucleus, in terms of all of the parameters of the problem?

$$\theta = \int_{r_m}^{\infty} \frac{\frac{b}{r^2}\, dr}{\sqrt{1 - \frac{b^2}{r^2} - \frac{a}{r_0}}} = \int_{r_m}^{\infty}$$

$$\cos\theta = \frac{\frac{K}{b}}{\sqrt{1+\left(\frac{K}{b}\right)^2}}$$

$$\theta = \frac{\pi - \theta}{2}$$

$$\tan\theta = \frac{K}{b}$$

$$\boxed{K = \frac{k}{2T_0}}$$

$$b = K \tan\theta$$

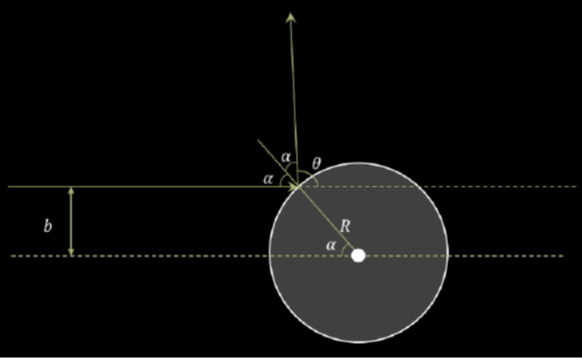
$$\boxed{b = K \cot\frac{\theta}{2}} \qquad \boxed{b = K \cot\frac{\theta}{2}}$$

$$\sigma(\theta) = \frac{b}{\sin\theta} \frac{db}{d\theta}$$

$$= \frac{K\left(\cot\frac{\theta}{2}\right)}{\sin\theta} \; K\left(\frac{1}{2}\right) \frac{1}{\sin^2 \frac{\theta}{2}}$$

$$= K^2 \frac{\frac{\cos\frac{\theta}{2}}{\sin\frac{\theta}{2}}}{2\cos\frac{\theta}{2}\sin\frac{\theta}{2}} \left(\frac{1}{2}\right) \frac{1}{\sin^2\frac{\theta}{2}} = \frac{K^2}{4\sin^4\frac{\theta}{2}}$$

3. A point projectile scatters off of a target rigid sphere of radius R, as depicted below from the side. Compute the differential scattering cross section. Integrate your result over all scattering angles to find the total cross section.



$$\sin\alpha = \frac{b}{R}$$

$$\Delta\theta = \int_a^\infty \frac{\left(\frac{b}{r}\right) dr}{\sqrt{1 - \left(\frac{b}{r}\right)^2}} \qquad \frac{b}{r} = \cos u$$

$$= \int_a^\infty \frac{\cos u}{\sin u}$$

$$\sin\theta = \frac{b}{a}$$

$$\theta = \frac{\pi - \theta}{2}$$

$$\boxed{\cos\frac{\theta}{2} = \frac{b}{a}} \qquad \boxed{b = a\cos\frac{\theta}{2}}$$

$$\sigma(\theta) = \frac{b}{\sin\theta}\left|\frac{db}{d\theta}\right| = \frac{b}{\sin\theta}\, a \sin\left(\frac{\theta}{2}\right) \frac{1}{2}$$

$$= \frac{b a}{2} \frac{1}{2\cos\frac{\theta}{2}} = \boxed{\frac{b a}{4\cos\frac{\theta}{2}}}$$

$$= \frac{a^2 \cos}{4} \qquad \boxed{\frac{a^2}{8}}$$

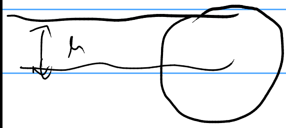
4. A fixed (infinitely massive) force center scatters a particle of mass m and initial velocity v_0 according to the force law $F(r) = k/r^3$, with $k > 0$.

(a) Find the relation between the impact parameter b to the magnitude of the total angular momentum l.

(b) Write down an expression for the total energy, in terms of $u(\theta) = 1/r(\theta)$, and l.

(c) Show that the differential scattering cross section is:

$$\sigma(\theta) = \frac{k\pi^2(\pi - \theta)}{mv_0^2\theta^2(2\pi - \theta)^2 \sin\theta}$$



$$\frac{1}{2}\frac{k}{r^2} = \boxed{\frac{k}{2r^2}}$$

a) $l = m b v_0$

b) $E = \frac{k}{2r^2} + \frac{1}{2} m \dot{r}^2 + \frac{l^2}{2mr^2}$

c) $$\int_{r_{min}}^{\infty} \frac{\frac{l}{r^2}\, dr}{\sqrt{1 - \frac{k}{2r^2 E} - \frac{l^2}{r^2}}}$$

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(c) How close (minimum distance of approach) will an alpha particle with kinetic energy E come to the gold nucleus, in terms of all of the parameters of the problem?

$b(\theta) = k \tan$

$= k \cot \theta/2 = \boxed{\frac{k}{2\tau_\theta}}$

$\sigma(\theta) = \left|\frac{db}{d\theta}\right| \frac{b}{\sin\theta} = \frac{(\csc^2\frac{\theta}{2})(k)\left(\frac{1}{2}\right)}{\sin\theta} \frac{1}{\sin^2(\theta/2)}$

$= \frac{k^2}{4} \frac{1}{\sin^4\frac{\theta}{2}}$

$\psi = \frac{\pi}{2}$

$\frac{k^2}{4} \frac{1}{\sin^4\frac{\theta}{2}} (4) \cos\psi = \boxed{\frac{k^2 \cos\psi}{\sin^4\psi}}$

$\frac{k^2}{4\cos^4\psi}$