1. IEEE 802.11a (WLAN standard) uses 64 QAM modulation to achieve a data rate of 54 Mbps. Suppose an $\frac{Eb}{N_0}$ = 38 dB is required to achieve the target BER of 10^{-6}. For this data rate,
   a. What is the sensitivity of the receiver, if the noise figure = 8 dB.
   b. What is the sensitivity of receiver, if receiver has implementation loss of 3 dB

$$\frac{P_s}{P_n} = 38\,dB + 10\log(6) = \boxed{45.7\,dB}$$

a) $B_{eq} = \frac{1}{T_s}$

$T_b = \frac{1}{54\times10^6}$ sec $\qquad T_s = 6T_b = \boxed{\frac{1}{9\times10^6}\text{ sec}}$

$\Rightarrow B_{eq} = \boxed{9\,MHz}$

$P_n = k_B T B_{eq} F$

$P_n = -174 + B_{eq}(dB) + F$

$= -174 + 69.542 + 8 = \boxed{-96.5\,dBm}$

$\Rightarrow P_{s,min} = \boxed{-50.8\,dBm}$

b) Receiver implementation noise adds 3dB

$P_{s,min} = -50.8 + 3dB = \boxed{-47.8\,dBm}$

2. The complementary error function is given by $Q(z)=\frac{1}{\sqrt{2\pi}}\int_z^\infty e^{-\frac{x^2}{2}}dx$. The task is to develop appropriate bounds for $Q(z)$. Show that $\frac{1}{\sqrt{2\pi}}\frac{1}{z}e^{-\frac{z^2}{2}}\left(1-\frac{1}{z^2}\right)<Q(z)<\frac{1}{\sqrt{2\pi}}\frac{1}{z}e^{-\frac{z^2}{2}}$

Consider $g(z)=\frac{1}{\sqrt{2\pi}}\frac{1}{z}e^{-z^2/2}-Q(z)$

$\rightarrow g'(z)=\frac{1}{\sqrt{2\pi}}\left[\frac{1}{z}(-z)\,e^{-z^2/2}-\frac{e^{-z^2/2}}{z^2}\right]+\frac{1}{\sqrt{2\pi}}e^{-z^2/2}$

$=\frac{1}{\sqrt{2\pi}}\left[\frac{-e^{-z^2/2}}{z^2}\right]<0$ always $\rightarrow$ Decreasing

$g(\infty)=0-0=\boxed{0}$ $\quad g(z)\to 0$ as $z\to\infty$, $\Rightarrow$ g decreases then + value to zero

$g(x)>0 \quad \forall x$, decreasing function

$\rightarrow h(z)=Q(z)-\frac{1}{\sqrt{2\pi}}\left[\frac{1}{z}-\frac{1}{z^3}\right]e^{-z^2/2}$

$h'(z)=\frac{-1}{\sqrt{2\pi}}e^{-z^2/2}-\frac{1}{\sqrt{2\pi}}\left[e^{-z^2/2}\left[\frac{-1}{z^2}+\frac{3}{z^4}\right]+\left[\frac{1}{z}-\frac{1}{z^3}\right](-z)e^{-z^2/2}\right]$

$=\frac{1}{\sqrt{2\pi}}e^{-z^2/2}\left[\cancel{-1}+\cancel{\frac{1}{z^2}}-\frac{3}{z^4}\cancel{+1}\cancel{-\frac{1}{z^2}}\right]$

$=\frac{1}{\sqrt{2\pi}}e^{-z^2/2}\left[\frac{-3}{z^4}\right]<0$

Again $\boxed{h(\infty)=0}$

Using this, $h(z)>0$ always

Since $h(z)>0$ & $g(z)>0$, we get the reqd. bound.

3. With Nakagami-m fading, the pdf of the SNR $\gamma = v^2 \dfrac{E_b}{N_0}$ is given by $f_\gamma(\gamma) = \dfrac{m^m \gamma^{m-1}}{(m-1)!\Gamma^m} e^{-\frac{m\gamma}{\Gamma}}$ where $\Gamma = E[V^2]\dfrac{E_b}{N_0}$. Verify that $f_\gamma(\gamma)$ is a valid pdf.

Valid pdf

$\infty f_V(\infty) = 0$

$$\int_0^\infty \frac{m^m \gamma^{m-1}}{(m-1)!\Gamma^m} e^{-m\frac{\gamma}{\Gamma}} d\gamma$$

Let $\dfrac{m\gamma}{\Gamma} = t$

$\dfrac{m\,d\gamma}{\Gamma} = dt$

$$= \int_0^\infty \frac{t^{m-1} e^{-t}}{(m-1)!} dt = \frac{\Gamma(m)}{(m-1)!} = 1$$

4. The probability of error for BPSK in AWGN is given by $P_{e,BPSK} = Q\left(\sqrt{2\gamma}\right)$ where γ is the SNR. The probability of error in Rayleigh fading is given by

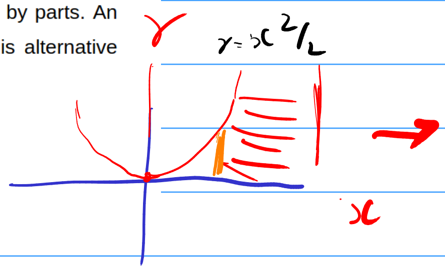
$$P_{e,BPSK,Rayleigh} = \int_0^\infty Q\left(\sqrt{2\gamma}\right) f_\gamma(\gamma)\, d\gamma \qquad (1)$$

where $f_\gamma(\gamma) = \begin{cases} \frac{1}{\Gamma} e^{-\frac{\gamma}{\Gamma}} & \gamma \geq 0 \\ 0 & otherwise \end{cases}$,

where Γ is the average SNR.

One approach is to evaluate the $\int$ given in eqn. (1) closed form using integration by parts. An alternative approach is via changing the order of integration in eqn. (1). Using this alternative approach, show that $P_{e,BPSK,Rayleigh} = \frac{1}{2}\left[1 - \sqrt{\frac{\Gamma}{1+\Gamma}}\right]$

(Hint : $\boxed{\int_0^\infty e^{-q^2 x^2}\, dx = \frac{\sqrt{\pi}}{2q},}\quad q > 0$



$$\int_0^\infty \int_{\sqrt{2\gamma}}^\infty \frac{e^{-\frac{x^2}{2}}}{\sqrt{2\pi}}\, dx \; \frac{1}{\Gamma} e^{-\frac{\gamma}{\Gamma}}\, d\gamma$$

Change order of integration

$$= \left(\frac{1}{\Gamma}\right) \int_0^\infty \int_0^{\frac{x^2}{2}} \frac{e^{-\frac{\gamma}{\Gamma}}}{\sqrt{2\pi}}\, d\gamma \; e^{-\frac{x^2}{2}}\, dx$$

$$= -\int_0^\infty \left(\frac{e^{-\frac{x^2}{2\Gamma}} - 1}{\sqrt{2\pi}}\right) e^{-x^2/2}\, dx$$

$$= \frac{1}{2} - \int_0^\infty \frac{e^{-x^2\left[\frac{1}{2\Gamma} + \frac{1}{2}\right]}}{\sqrt{2\pi}}\, dx$$

$$= \frac{1}{2} - \frac{\sqrt{\pi}}{\sqrt{2\pi}\; 2\sqrt{\frac{1}{2\Gamma} + \frac{1}{2}}} = \frac{1}{2}\left[1 - \frac{1}{\sqrt{\Gamma + 1/\Gamma}}\right]$$

$$= \frac{1}{2}\left[1 - \sqrt{\frac{\Gamma}{\Gamma+1}}\right]$$

5. For QPSK using Gray coding, the symbol error probability (P_s) is related to the bit error probability (P_e) as $\frac{P_s}{2} = P_b$ (2)

a. Assuming $\gamma_b = \frac{E_b}{N_0} = 7dB$, evaluate the P_e (BER) and P_s (SER) using exact expressions.

b. Compare the *Ps* obtained in (a) with the expression in eqn. (2)

7dB → 5 linear scale

a) $P_{e,BER}(\gamma) = Q(\sqrt{2\gamma_b}) = \boxed{0.0007814}$

$$P_{e\ SER}(\gamma) = 1 - \left[1 - Q(\sqrt{2\gamma_b})\right]^2$$

$$= \boxed{0.001562}$$

b) $\frac{P_s}{2} = \boxed{0.0007810} \simeq P_b$, so the approximation holds to a good degree

This is expected:

$$P_{s,QPSK} = 1 - (1 - P_{e,BPSK})^2$$

$$= 2P_{b,BPSK} - P_{e,BPSK}^2 \simeq 2P_{b,BPSK}$$

6. Estimate the BER of QPSK if SNR = 9.5 dB. Use the upper bound approximation for $Q(z)$

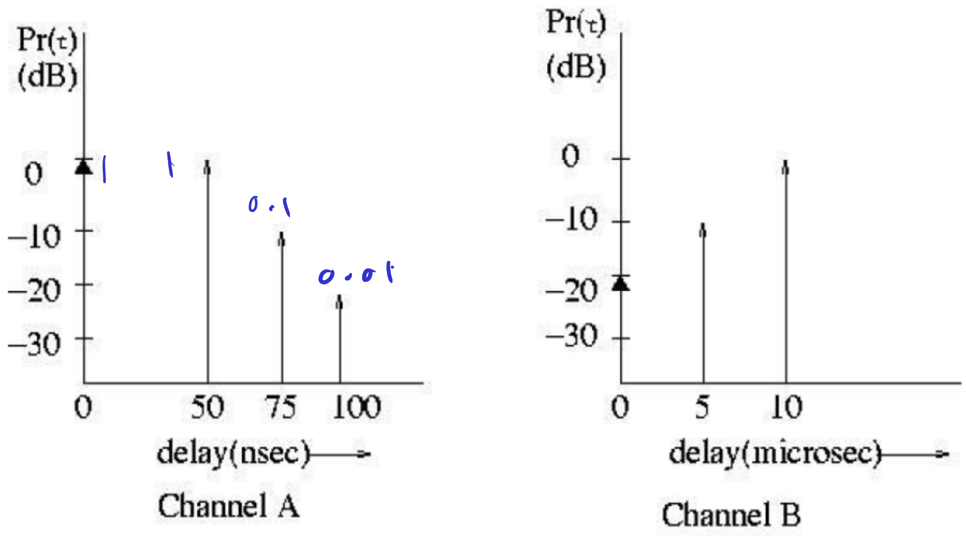
$\boxed{SNR = \frac{P_s}{P_n}}$. If we take E_b/N_0 we will get different answer.

$$Q(z) \le \frac{1}{\sqrt{2\pi}\, z} e^{-z^2/2}$$

$$SNR = 9.5\,dB = \boxed{9} = 2\gamma_b$$

$$BER = Q(\sqrt{2\gamma_b}) = Q(\sqrt{SNR}) = \frac{1}{\sqrt{2\pi\, SNR}} e^{-\frac{SNR}{2}} = \frac{1}{\sqrt{2\pi(9)}} e^{-4.5} = \boxed{4.9 \times 10^{-4}}$$

7. Consider the following channels whose power-delay profiles ($\Pr(\tau)$ vs. τ) are given below. (Note: In class, we used the notation $R_h(\tau)$ for the power delay profile). Assuming that the receiver (without equalizer) meets the minimum BER specification if $\frac{\sigma_\tau}{T_s} \leq 0.1$ where σ_τ is the RMS delay spread and T_s is the symbol duration. For each of the given channels, estimate the maximum data rate that may be transmitted while meeting the BER specification.



Channel A

$\bar{\tau} = 27.72$ ns , $\sigma_\tau = \boxed{27.02 \text{ ns}}$

$T_s \geq 10\,\sigma_\tau$ for no equalizer

$\Rightarrow$ Max data rate $= \frac{1}{10 T_s}$ symbols $= \boxed{3.7 \text{ Msymb/sec}}$

Channel B :

$\bar{\tau} = 9.45\ \mu s$, $\sigma_\tau = 1.69\ \mu s$

Max data rate

$= \frac{1}{10 T_s} = \boxed{59.17 \text{ ksymb/sec}}$

Note that this is the symbol rate, and the corresponding bit rate can be calculated depending upon the modulation scheme.

8. Consider an input data stream with a bit rate Rb = 100 Kbps, modulated on a RF carrier using BPSK modulation.
   a. Estimate the maximum value of RMS delay spread (σ_τ) such that received signal may be treated as a flat-fading signal.
   b. If the carrier freq f_c = 5.8 GHz, what is the coherence time (T_c) of the channel, assuming that the receiver is on a vehicle moving at 30 Kmph. (Estimate T_c for correlation = 0.5)
   c. For the scenario in (b), is the channel experiencing 'fast-fading' or 'slow-fading'?

   d. Using scenario in (b), approx. how many symbols are transmitted during the time interval appears "static" (Corresponding to $\frac{T_c}{2}$)

$$T_s = T_b = 1/10^5 = \boxed{10\mu s}$$

a) $\sigma_\tau \ll \frac{T_s}{10} = \boxed{1\mu s}$ → No equalizer

b) $1.125 = 2\pi f_D T_c$ $\quad f_D = \frac{5.8 \times \cancel{10^9}}{3 \times \cancel{10^8}} \times \frac{\cancel{30} \times 5}{18} = \boxed{161\,Hz}$ ($\frac{100}{3}$)

$$T_c = \frac{1.125}{2\pi\, 161} = \boxed{1.11\,ms}$$

c) $T_s = 1\mu s$ $\quad T_s \ll T_c$ is time, true slow fading

d) # of symbols = $\frac{T_c}{2T_s} = \frac{1.11 \times 1000}{2 \times 10} = \boxed{55}$

9. A vehicle receives a 900 MHz transmission while moving at a constant velocity for 10 secs. The average fade duration for a signal level 10 dB below RMS value is found to be 1 msec.
   (a) Evaluate how far the vehicle travels during the 10 sec interval.
   (b) Approx. how many fades does the signal undergo at this threshold level.

a) $\rho = \frac{V_{th}}{V_{RMS}} = \boxed{\frac{1}{\sqrt{10}}}$

$$\frac{e^{\frac{1}{10}} - 1}{\sqrt{2\pi}\, f_D \sqrt{\frac{1}{10}}} = 10^{-3} \Rightarrow \boxed{f_D = 132 \text{ Hz}}$$

$$\Rightarrow \boxed{v = 44 \text{ m/s}}$$

$\Rightarrow$ Dist travelled in 10 s $= \boxed{441 \text{ m}}$

b)

$\#$ of fades $= \sqrt{2\pi}\, f_D\, \rho\, e^{-\rho^2} =$ $\boxed{95 \text{ fades per second}}$

10. For maximal ratio combining (MRC) diversity with M -antennas, $\gamma_{MRC} = \sum_{k=1}^{M} \gamma_k$ wherein each γ_k has a Chi-square pdf with 2 two degrees of freedom and γ_{MRC} has a Chi-square pdf with $2M$ degrees of freedom given by $f_{\gamma_{MRC}}(\gamma) = \frac{\gamma^{M-1}}{\Gamma^M (M-1)!} e^{-\frac{\gamma}{\Gamma}}$, $\gamma \ge 0 \quad \cdots \quad eqn\,(1)$

(c) Verify that the pdf in eqn. (1) satisfies $\int_0^{\infty} f_{\gamma_{MRC}}(\gamma)\, d\gamma = 1$

(d) Show that the outage probability with MRC diversity

$$P(\gamma_{MRC} < \gamma_T) = 1 - e^{-\frac{\gamma_T}{\Gamma}} \sum_{k=1}^{M} \frac{\left(\frac{\gamma_T}{\Gamma}\right)^{k-1}}{(k-1)!}$$, where γ_T is the SNR threshold for outage.

(e) Using Matlab, plot the outage probability for MRC combining for M = 1, 2, and 3 antennas in the probability range [0.0001, 1] and $\frac{\gamma_T}{\Gamma}$ in the range [−30 dB, 10 dB]

a) $\int_0^{\infty} \frac{\gamma^{M-1}}{\Gamma^M (M-1)!} e^{-\frac{\gamma}{\Gamma}} d\gamma = \int_0^{\infty} \frac{e^{-t}\, t^{M-1}}{(M-1)!} dt = \frac{\Gamma(M)}{(M-1)!} = \boxed{1}$

b)

$t = \frac{\gamma}{\Gamma}$

$$- \frac{t^{M-1}}{(M-1)} t^{M-2} e^{-t} \quad - e^{-t}$$

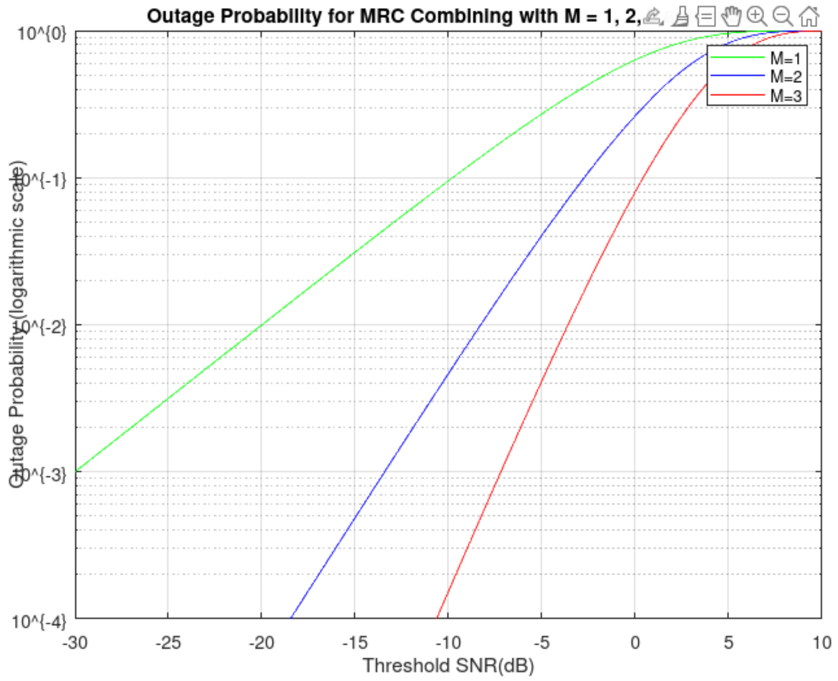
$P(\gamma_{MRC} < \gamma_T)$

$$= \int_0^{\gamma_T} \frac{\gamma^{M-1} e^{-\frac{\gamma}{\Gamma}}}{\Gamma^M (M-1)!} d\gamma = \int_0^{\gamma_T/\Gamma} \frac{t^{M-1}}{(M-1)!} e^{-t}\, dt$$

$$= \left[\frac{-t^{M-1} e^{-t}}{(M-1)!} \right]_0^{\gamma_T/\Gamma} + \int_0^{\gamma_T/\Gamma} \frac{t^{M-2}}{(M-2)!} dt$$

$$= 1 - e^{-\gamma_T/\Gamma} \left[\sum_{k=0}^{M-1} \frac{(\gamma_T/\Gamma)^k}{k!} \right] \rightarrow \text{Same as question}$$

c)



MATLAB code is attached.

```matlab
% Let the ratio of gamma_T and Gamma be "ratio"
ratio_dB = linspace(-30, 10, 1000);

ratio = 10.^(ratio_dB / 10);

M = [1, 2, 3];

Outage_probability = zeros(length(M), length(ratio));

for index = 1:length(M)
    m = M(index);

    for i = 1:length(ratio)
        sum_term = 0;
        for k = 1:m
            sum_term = sum_term + (1/factorial(k - 1)) * (ratio(i))^(k - 1);
        end
        Outage_probability(index, i) = 1 - exp(-ratio(i)) * sum_term;
    end
end

figure;

semilogy(ratio_dB, Outage_probability(1,:), 'g');

hold on;
semilogy(ratio_dB, Outage_probability(2,:), 'b');
semilogy(ratio_dB, Outage_probability(3,:), 'r');

grid on;

xlabel('Threshold SNR(dB)');
ylabel('Outage Probability (logarithmic scale)');
title('Outage Probability for MRC Combining with M = 1, 2, 3 antennas');
legend('M=1', 'M=2', 'M=3');
axis([-30 10 1e-4 1]);
```