Tut-1

1. Write the boundary conditions that exist at the interface of free space and a magnetic material of infinite permeability.

Free space	Magnetic material
ε_0	ε
μ_0	$\mu = \infty$
H_0, B_0, E_0	B, E, μ

$E^{\parallel} - E_0^{\parallel} = 0$

$$\boxed{E^{\perp} - E_0^{\perp} = \sigma}$$

$J = 0$ at equilibrium for non-perfect conductor

$$\vec{H} = \frac{\vec{B}}{\mu} = \frac{\vec{B}}{\infty} = \boxed{\vec{0}}$$

$$\boxed{B^{\perp} = B_0^{\perp}}$$

2. Prove that Lorentz condition for potentials $\vec{\nabla} \cdot \vec{A} + \mu\varepsilon \frac{\partial V}{\partial t} = 0$ is consistent with the equation of continuity.

Assuming static field

To prove: $\nabla \cdot J = -\frac{\partial \rho_e}{\partial t}$

$$\nabla \cdot A + \mu\varepsilon \frac{\partial v}{\partial t} = 0$$

Taking ∇,

$$\nabla(\nabla \cdot A) + \mu\varepsilon \frac{\partial (\nabla V)}{\partial t} = 0$$

$$\nabla^2 A + \mu_0 J + \cancel{\frac{1}{c^2}\frac{\partial E}{\partial t}} - \cancel{\mu\varepsilon \frac{\partial \vec{E}}{\partial t}} = 0$$

Take $\nabla \cdot$

$$\nabla \cdot (\nabla^2 A) + \mu_0 (\nabla \cdot J) = 0 \quad - (1)$$

$$\nabla \times \vec{B} = \mu_0 \vec{J} + \frac{1}{c^2}\frac{\partial \vec{E}}{\partial t}$$

$$\nabla(\nabla \cdot \vec{A}) - \nabla^2 \vec{A} = \mu_0 J + \frac{1}{c^2}\frac{\partial E}{\partial t}$$

$\nabla \cdot A =$

$$\vec{\nabla}\cdot\vec{E} = \frac{\rho}{\varepsilon}$$

$$\nabla\cdot\left(\frac{\partial\vec{E}}{\partial t}\right) = \frac{\frac{\partial\rho}{\partial t}}{\varepsilon}$$

$$\nabla\cdot\frac{\left[\nabla\times B - \mu_0 J_e\right]}{\mu\cancel{\varepsilon}} = \frac{\frac{\partial\rho}{\partial t}}{\cancel{\varepsilon}}$$

$$\nabla\cdot\left(\nabla^2 A\right) = \frac{\partial\rho}{\partial t} \quad - \; 2$$

1 & 2

$$\Rightarrow \boxed{\nabla\cdot J + \frac{\partial\rho}{\partial t} = 0 \quad \Rightarrow \text{Eqn of Continuity}}$$

3. Find out $\vec{H}$ and β, if the electric field is given as $\vec{E} = 0.2\sin(10\pi y)\cos(6\pi 10^9 t - \beta z)\,\hat{x}$ in air.

$$\sin A \cos B = \frac{\sin(A+B) + \sin(A-B)}{2}$$

$$\vec{S} = \vec{E} \times \vec{B}$$
$$\vec{B} = \vec{S} \times \vec{E}$$

$$\vec{E} = 0.1\left[\sin\left(6\pi 10^9 t + 10\pi y - \beta z\right) + \sin\left(10\pi y + \beta z - 6\pi 10^9 t\right)\right]\hat{i}$$

$$\omega = 6\pi 10^9$$
$$k = \frac{\omega}{c} = \frac{6\pi 10^9}{3\times 10^8} = \boxed{20\pi}$$

$$\nu\lambda = c$$
$$k = \frac{2\pi}{\lambda} = \frac{2\pi\nu}{c} = \frac{\omega}{c}$$
$$\beta = \boxed{10\pi\sqrt{3}}$$

$$|B| = \frac{0.1}{c}$$

$$\vec{k}_1 = 10\pi\sqrt{3}\,\vec{k} - 10\pi\,\vec{j}$$
$$\vec{k}_2 = -10\pi\sqrt{3}\,\vec{k} - 10\pi\,\vec{j}$$
$$\vec{B}_1 = \frac{\sqrt{3}}{2}\vec{j} + \frac{1}{2}\vec{k}$$
$$\hat{B}_2 = -\frac{\sqrt{3}}{2}\vec{j} + \frac{1}{2}\vec{k}$$

$$\vec{B} = \frac{0.1}{3\times 10^8}\left[\sin\left(6\pi 10^9 t + 10\pi y - 10\pi\sqrt{3}z\right)\left(\frac{\sqrt{3}}{2}\vec{j} + \frac{1}{2}\vec{k}\right) - \sin\left(10\pi y + 10\pi\sqrt{3}z - 6\pi 10^9 t\right)\left(-\frac{\sqrt{3}}{2}\vec{j} + \frac{1}{2}\vec{k}\right)\right]$$

$$\sin A - \sin B = 2\cos\frac{A+B}{2}\sin\left(\frac{A-B}{2}\right)$$

$$H = \frac{B}{\mu_0} = \frac{3.33\times 10^{-10}(2)}{4\pi 10^{-7}}\left[\frac{\sqrt{3}}{2}\sin(10\pi y)\cos\left(6\pi 10^9 t - 10\pi\sqrt{3}z\right)\vec{j} + \frac{1}{2}\cos(10\pi y)\sin\left(6\pi 10^9 t - 10\pi\sqrt{3}z\right)\vec{k}\right]$$

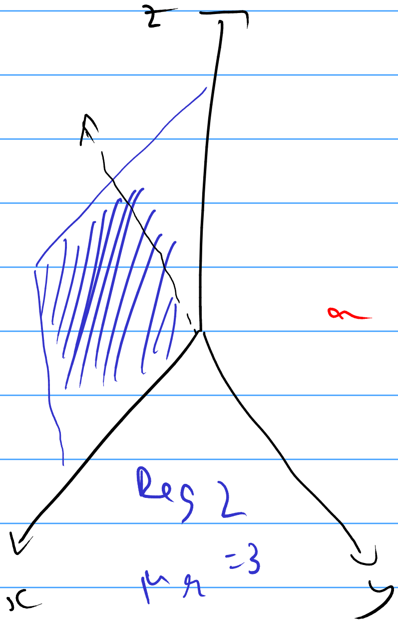
$$= 5.3\times 10^{-4}\left[\frac{\sqrt{3}}{2}\sin(10\pi y)\cos\left(6\pi 10^9 t - 10\pi\sqrt{3}z\right)\vec{j} + \frac{1}{2}\cos(10\pi y)\sin\left(6\pi 10^9 t - 10\pi\sqrt{3}z\right)\vec{k}\right]$$

4. A current sheet $\vec{K} = 9.0\,\hat{y}$ A/m, is located at z = 0, the interface between the region 1, z < 0, with $\mu_r = 4$, and region 2, z > 0, $\mu_r = 3$. Given that $\vec{H}_2 = 14.5\,\hat{x} + 8.0\,\hat{z}$ (A/m), find $\vec{H}_1$.

$$H_1^{\perp} = H_2^{\perp}$$

$$H_1^{\parallel} - H_2^{\parallel} = \vec{K} \times \vec{n}$$

Reg 1
$\mu_r = 4$



$$\hat{n} = -\vec{z}$$ [Normal goes from 2 to 1]

$$a\,\hat{x} - 14.5\,\vec{x} = -9.0\,\vec{x}$$

$$H_2 = a\vec{x} = \boxed{5.5\,\vec{x}}$$

$$H_1^{\perp} = H_2^{\perp} = 8.0\,\vec{z}$$

$$\boxed{\vec{H}_1 = 5.5\,\vec{x} + 8\,\vec{z}}\ \mathrm{Am^{-1}}$$

5. In a homogeneous nonconducting region where $\mu_r = 1$, find ε_r and ω if $\vec{E} = 30\,\pi e^{j(\omega t - 4/3\, y)}\hat{z}$ (V/m) and $\vec{H} = 1.0 e^{j(\omega t - 4/3\, y)}\hat{x}$ A/m.

$$B = \mu_0 \mu_r H = \mu_0 e^{j(\omega t - \frac{4}{3}y)}\vec{x}$$

$$= 4\pi \times 10^{-7} e^{j(\omega t - \frac{4}{3}y)}\vec{x}$$

$$\frac{2\pi}{\lambda} = \frac{4}{3} \qquad \lambda = \frac{6\pi}{4} = \boxed{\left(\frac{3\pi}{2}\right) m}$$

$$v = \frac{E}{B} = \frac{30\,\pi}{4\pi \times 10^{-7}} = \frac{30}{4} \times 10^{7}$$

$$f = \frac{v}{\lambda} = \frac{2 \cdot 30 \times 10^{7}}{3\pi (4)}\ 2$$

$$\omega = 2\pi f = 2\pi \frac{2}{3\pi}\left(\frac{30}{4}\right) \times 10^{7} = \boxed{10^{8}\ \mathrm{rad\,s^{-1}}}$$

$$\varepsilon_r = \frac{1}{\mu c^2} = \frac{1}{\frac{30}{4} \times \frac{30}{4} \times 10^{14} \times 1 \times 4\pi\, 10^{-7} \times 9 \times 10^{-12}}$$

$$= \boxed{15.79}$$

6. Prove that the electric field intensity $\vec{E}(x,y,z) = \vec{E}_o e^{-j(k_x x + k_y y + k_z z)}$ satisfied the homogeneous Helmoltz's equation provided that the condition $k_x^2 + k_y^2 + k_z^2 = \omega^2 \mu \varepsilon$ is satisfied

Given: $\nabla^2 \vec{E} = \vec{E}_o \left[e^{-j[k_x x + k_y y + k_z z]} (k_x^2 + k_y^2 + k_z^2)(-1) \right]$

$$= -\vec{E} \left[k_x^2 + k_y^2 + k_z^2 \right] = -\omega^2 \mu \varepsilon \vec{E}$$

$$\nabla^2 \vec{E} + \omega^2 \mu \varepsilon \vec{E} = 0$$ → Helmholtz eqn.

Assignment - 2

1. For a harmonic uniform plane wave propagating in a simple medium, both E and H vary in accordance with the factor $exp(-j\vec{k}\cdot\vec{R})$ as indicated by $\vec{E}(R) = E_o e^{-j\vec{k}\cdot\vec{R}}$. Then show that the four Maxwell's equation for uniform plane wave in a source-free region reduce to the following i) $\vec{k}\times\vec{E} = \omega\mu\vec{H}$, ii) $\vec{k}\times\vec{H} = -\omega\varepsilon\vec{E}$, iii) $\vec{k}\cdot\vec{E} = 0$, and iv) $\vec{k}\cdot\vec{H} = 0$.

i) $\nabla\times E = \nabla\left(e^{-j\vec{k}\cdot\vec{R}}\right)\times\vec{E_o} = -j\omega\mu\vec{H}$

$\left(-j\vec{k}\, e^{-j\vec{k}\cdot\vec{R}}\right)\times E_o = -j\omega\mu\vec{H}$

$\boxed{\vec{k}\times\vec{E_o} = \omega\mu\vec{H}}$

ii) $\nabla\times\vec{H} = j\omega\varepsilon\vec{E}$

$\left(-j\vec{k}\, e^{-j\vec{k}\cdot\vec{R}}\right)\times\vec{H_0} = j\omega\varepsilon\vec{E}$

$\boxed{\vec{k}\times\vec{H} = -\omega\varepsilon\vec{E}}$

iii) $\nabla\cdot\vec{E} = 0$

$-j(\vec{k}\cdot\vec{E}) = 0 \Rightarrow \vec{k}\cdot\vec{E} = 0$ & $\boxed{\vec{k}\perp\vec{E}}$

iv) $\nabla\cdot\vec{H} = 0$ $\quad -j(\vec{k}\cdot\vec{H}) = 0 \Rightarrow \vec{k}\cdot\vec{H} = 0$ $\quad \boxed{\vec{k}\perp\vec{H}}$

2. The E-field of a uniform plane wave propagating in a dielectric medium is given by $\vec{E}(z,t) = \hat{x}2\cos\left(10^8 t - \frac{z}{\sqrt{3}}\right) - \hat{y}\sin\left(10^8 t - \frac{z}{\sqrt{3}}\right)$.

a) Determine the frequency and wavelength of the wave.
b) What is the dielectric constant of the medium?
c) Describe the polarization of the wave.
d) Find the corresponding H-field.

a) $\omega = 10^8$ rad $f = \frac{\omega}{2\pi} = \frac{10^8}{2\pi} = 1.57\times10^7$ Hz $= 15.7$ MHz

$\lambda = \frac{2\pi}{k}$ $k = \frac{1}{\sqrt{3}}$ $\boxed{\lambda = 2\pi\sqrt{3}\text{ m}}$

b) $v = \frac{\omega}{k} = \frac{10^8}{\frac{1}{\sqrt{3}}} = \sqrt{3}\times10^8$ m/s

$\frac{c}{v} = \sqrt{3} \Rightarrow \varepsilon_r = \sqrt{3}^2 = \boxed{3}$

c) At $t=0$

$E = 2\cos 10^8 t - \sin 10^8 t$

Elliptically polarized

d) $\eta = \sqrt{\frac{\mu}{\varepsilon}} = \boxed{217\,\Omega}$

$H = \frac{1}{\eta}\,\hat{z}\times\vec{E} = 0.0046\left[\sin\left(10^8 t - \frac{z}{\sqrt{3}}\right)\hat{x} + 2\cos\left(10^8 t - \frac{z}{\sqrt{3}}\right)\hat{y}\right]$

3. A 3 GHz, *y*-polarized uniform plane wave propagating in the +*x*-direction in a nonmagnetic medium having a dielectric constant 2.5 and a loss tangent 10^{-2}.

a) Determine the distance over which the amplitude of the propagating wave will be cut in to half.
b) Determine the intrinsic impedance, the wavelength, the phase velocity and the group velocity of the wave in the medium.
c) Assuming $\vec{E} = \hat{y}50\sin(6\pi 10^9 t + \pi/3)$ at $x = 0$, write the instantaneous expression for $\vec{H}$ for all t and x.

a) $\alpha = \frac{\omega\varepsilon''}{2}\sqrt{\frac{\mu}{\varepsilon'}} = \boxed{0.497\text{ m}^{-1}}$

$e^{-\alpha x} = \frac{1}{2}$ $x = \frac{\ln 2}{\alpha} = \boxed{1.395\text{ m}}$

b) $\eta = \sqrt{\frac{\mu}{\varepsilon}}\left(1 + j\frac{\varepsilon''}{2\varepsilon'}\right) = 238(1 + j0.005)$

$\beta = \omega\sqrt{\mu\varepsilon}\left[1 + \frac{1}{8}\left(\frac{\varepsilon''}{\varepsilon'}\right)^2\right] \boxed{99.27\ \text{rad}\,s^{-1}}$

$\lambda = \frac{2\pi}{\beta} = \boxed{0.063m}$

$u_p = \frac{\omega}{\beta} = 1.89 \times 10^8\ ms^{-1}$

$u_g = \frac{1}{d\beta/d\omega} = \frac{1}{\sqrt{\mu\varepsilon}}\left[1 + \frac{1}{8}\left(\frac{\varepsilon''}{\varepsilon'}\right)^2\right]^{-1} = 1.89\times10^8\ ms^{-1}$

$\boxed{u_g = u_p}$

c) $\vec{E} = 50 \sin\left(60\pi\, 10^9 t + \frac{\pi}{3}\right)$

$\vec{H} = 0.042\, e^{-0.497x} \sin\left(60\pi 10^9 t + 0.335 - 99.27x\right)$

4. Given that the skin depth for graphite at 100 MHz is 0.16 mm, determine i) the conductivity of graphite, and ii) the distance that a 1 GHz wave travels in graphite such that its field intensity is reduced by 30 dB.

i) skin depth $= \frac{1}{\sqrt{\pi f \mu\sigma}} = \frac{1}{\alpha} = 0.16mm$

$\sigma = \boxed{9.89\times10^4\ \Omega^{-1} m^{-1}}$

ii) $f = 1GHz = 10^9 Hz$ $\qquad \alpha = \sqrt{\pi f \mu\sigma} = 2\times10^4\ m^{-1}$

$z = \frac{3/2}{\alpha \log_{10} e} = \boxed{0.175\ mm}$

5. Consider a case of uniform plane wave, with polarization direction parallel to the plane of incidence, making oblique incidence at a plane conducting boundary. Obtain the expression for total electric and magnetic field in medium 1 (lossless dielectric medium) and discuss about the wave nature along the directions which are i) perpendicular and ii) parallel to the plane of the boundary between medium 1 and medium 2.

$$E_i = \vec{y}\, E_i\, e^{-j(x\sin\theta_i + z\cos\theta_i)}$$

$$E_r = \vec{y}\, E_r\, e^{-j(-z\cos\theta_r + x\sin\theta_r)}$$

Boundary Condn.

$$E_i\, e^{-j\sin\theta_i} + E_r\, e^{-j\sin\theta_r} = 0$$

$$E_r = -E_i \Rightarrow \boxed{\theta_i = \theta_r}$$

$$\vec{E}_T = \vec{E}_1 + \vec{E}_2$$

$$= \boxed{-\vec{y}\; 2E_i \cos(z\cos\theta_i)\, e^{-jx\cos\theta_i}}$$

$$\vec{H}_T = \vec{H}_i + \vec{H}_r = \frac{1}{R_c}\left[\vec{z}\,\sin\theta_i\, E_i \left(e^{-j(x\sin\theta_i + z\cos\theta_i)} + e^{-j(x\sin\theta_i - z\cos\theta_i)}\right) - \vec{x}\left(\cos\theta_i\, E_i\, e^{-j(x\sin\theta_i + z\cos\theta_i)} - e^{-j(x\sin\theta_i - z\cos\theta_i)}\right)\right]$$

$\vec{z}$ direction ~~the~~ only exists standing wave whereas wave propagation is happening along $\vec{x}$ direction

Tut - 3

1. A uniform plane wave in air $\vec{E}_i(z) = \hat{x}E_o \exp(-j\beta_o z)$ which impinges normally onto the surface at $z = 0$ of highly conducting medium having constitutive parameters ε_0, μ, and σ ($\sigma/\omega\varepsilon_o \gg 1$) . a) Find the reflection coefficient, b) derive the expression for the fraction of the incident power absorbed by the conducting medium, and c) obtain the fraction of the power absorbed at 1 (MHz) if the medium is iron (you can choose the relevant characteristic parameters of Fe).

a) $E_o e^{-j\beta_o z}\, \hat{x}$

$$\boxed{\Gamma = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1} \qquad \eta_1 = \sqrt{\frac{\mu_o}{\varepsilon_o}} \qquad \eta_2 = \sqrt{\frac{j\omega\mu}{\sigma}}}$$

b) $$|\Gamma|^2 = \left|\frac{\eta_2 - \eta_1}{\eta_2 + \eta_1}\right|^2 \simeq \left|1 - 2\frac{\eta_2}{\eta_1}\right|^2 = 1 - 4\frac{Re\{\eta_c\}}{\eta_o}$$

Fraction of power $= 1 - |\Gamma|^2 = \frac{4}{\eta_1}\sqrt{\frac{\omega\mu}{2\sigma}}$

c) For iron, $f = 10^6\,Hz$, $\mu_r = 4000$, $\sigma = 10^7$

$\boxed{F = 0.042\ \%}$ So, very less power is absorbed

2. A uniform plane wave is incident on the ionosphere at an angle of incidence θ_i = 60°. Assuming a constant electron density and a wave frequency equal to one-half of the plasma frequency of the ionosphere, determine a) Reflection coefficient and transmission coefficient for perpendicular polarization. b) Reflection coefficient and transmission coefficient for parallel polarization.

a) For $\perp$ to plane, everything is reflected,

$\Rightarrow \boxed{\Gamma_\parallel = -1}$ $\boxed{\tau_\parallel = 0}$

b) $\Gamma_\perp = -1$

$\tau_\perp = 1 + \Gamma_\perp = \boxed{0}$

3. An electromagnetic wave from an underwater source with perpendicular polarization is incident on water-air interface at $\theta_i = 20^o$. Using ε_r = 81 and μ_1 =1 for fresh water, find a) critical angle θ_C,, b) reflection coefficient, c) transmission coefficient, and d) attenuation in dB for each wavelength into the air.

a) $\theta_c = \sin^{-1}\left(\frac{\eta_2}{\eta_1}\right) = \sin^{-1}\left(\sqrt{\frac{\varepsilon_2}{\varepsilon_1}}\right) = \sin^{-1}\left(\frac{1}{9}\right) = \boxed{6.38^\circ}$

b) $\frac{\sin\theta_i}{\sin\theta_t} = \frac{\eta_2}{\eta_1}$

$\sin\theta_t = 9\frac{\sin 20^\circ}{1} \simeq 3.07$

$\cos\theta_t = \sqrt{1-3^2} = \boxed{-2.9j}$

$\cos\theta_i = 0.94$

$$\Gamma = \frac{\eta_2\cos\theta_i - \eta_1\cos\theta_t}{\eta_2\cos\theta_i + \eta_1\cos\theta_t} = \frac{9\times 0.93 - (1)\times 2.9j}{9\times 0.93 + 1\times 2.9j}$$

$$= \boxed{0.78 - 0.61j} = \boxed{e^{-0.66j}}$$

c) $\tau = 1+\Gamma = 1.88\,\underline{|-0.33} = \boxed{1.78 - 0.61j}$

d) $E_t = a_x E_{to}\, e^{-j\beta_2(x\sin\theta_t + z\cos\theta_t)}$

$\cos\theta_t$ is complex

Attenuation coeff $= \beta_2\, \mathrm{Im}\{\cos\theta_t\} = \beta_2 \times 2.9$

$= \frac{2\pi}{\lambda}\, 2.9 = \boxed{18.212 \text{ per } \lambda}$

Attenuation / wavelength

$= e^{-18.212}$ $\boxed{\simeq 158.18 \text{ dB}}$

4. Consider a transmission line made of two parallel brass strips of conductivity $\sigma_c = 1.6 \times 10^7$ S/m, of width 20 mm and separated by a lossy dielectric slab ($\mu = \mu_0$, $\varepsilon_r = 3$, $\sigma = 10^{-3}$ S/m, of thickness 2.5 mm. The operating frequency is 500 MHz. a) Calculate the R, L, G and C per unit length. b) compare the magnitude of the axial and transverse components of the electric field. c) Also find out γ and Z_0.

a) $R = \frac{2}{w}\sqrt{\frac{\pi f \mu_c}{\sigma_c}} = \frac{2}{20\times10^{-3}}\sqrt{\frac{\pi\times500\times10^{6}\times4\times\pi\times10^{-7}}{1.6\times10^{7}}}$

$= \boxed{1.11\ \Omega\, m^{-1}}$

$G = \sigma\frac{w}{d}$

$= 10^{-3}\times\frac{20\times10^{-3}}{2.5\times10^{-3}} = \boxed{8\times10^{-3}\ S\,m^{-1}}$

$L = \mu\frac{d}{w} = 4\pi10^{-7}\times\frac{2.5}{20} = \boxed{0.15\ \mu H\, m^{-1}}$

$C = \varepsilon\frac{w}{d} = \boxed{0.212\ nF\,m^{-1}}$

b) $\frac{E_{Axial}}{E_{transverse}} = \sqrt{\frac{2\pi f\,\varepsilon_0\varepsilon_r}{\sigma_c}} = \sqrt{\frac{2\pi\, 500\times10^{6}\times3\times8.85\times10^{-12}}{1.6\times10^{7}}}$

$= \boxed{7.7\times10^{-5}}$

We observe $\boxed{E_{axial} \ll E_{transverse}}$

c) $\gamma = \omega\sqrt{LC}\left[1+\left[\frac{1}{2j}\left[\frac{R}{\omega L}+\frac{G}{\omega C}\right]\right]\right]$

$= \boxed{0.11 + 1.15\times10^{-10}j\ \Omega}$

$Z_0 = \frac{d}{w}\sqrt{\frac{\mu}{\varepsilon}} = \frac{2.5\times10^{-3}}{20\times10^{-3}}\sqrt{\frac{4\pi10^{-7}}{3\times8.85\times10^{-12}}}$

$= \boxed{27\ \Omega}$

5. Prove that, under the condition of no reflection at an interface, the sum of the Brewster angle and the angle of refraction is $\pi/2$ for a) perpendicular polarization ($\mu_1 \neq \mu_2$) and b) parallel polarization ($\varepsilon_1 \neq \varepsilon_2$).

a) $$\Gamma_\perp = \frac{\eta_2 \cos\theta_i - \eta_1 \cos\theta_t}{\eta_2 \cos\theta_i + \eta_1 \cos\theta_t} = 0$$

$$\frac{\eta_2}{\eta_1} = \frac{\cos\theta_t}{\cos\theta_i} = \frac{\sin\theta_i}{\sin\theta_t}$$

$$\Rightarrow \sin 2\theta_i = \sin 2\theta_t$$

$$\underbrace{\theta_i = \theta_t}_{\text{Invalid}} \quad \text{or} \quad 2\theta_i = \pi - 2\theta_t$$

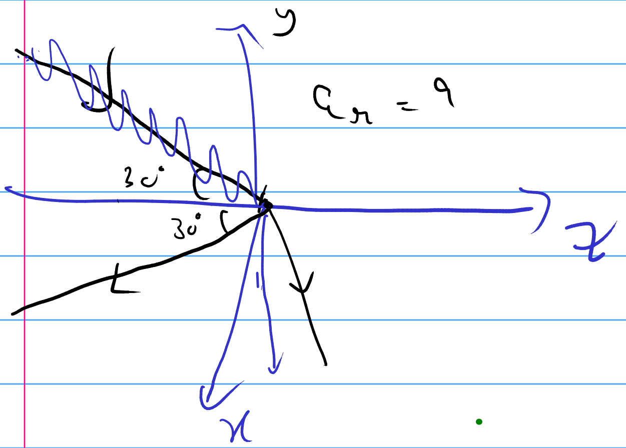
$$\boxed{\theta_i + \theta_t = \frac{\pi}{2}} \Rightarrow \boxed{\theta_B + \theta_t = \frac{\pi}{2}}$$

b) $$\Gamma_\parallel = \frac{\eta_2 \cos\theta_t - \eta_1 \cos\theta_i}{\eta_2 \cos\theta_t + \eta_1 \cos\theta_i} = 0$$

$$\frac{\eta_2}{\eta_1} = \frac{\sin\theta_i}{\sin\theta_t} = \frac{\cos\theta_i}{\cos\theta_t}$$

$$\tan\theta_t = \tan\theta_i$$

6. The electric field intensity of the uniform plane wave propagating in free space is known to be $\hat{x}377e^{-j0.866z}e^{-j0.5y}$ V/m. It strikes a dielectric medium with $\varepsilon_r = 9$ at 30° with respect to the normal to the plane interface. Determine i) the frequency of the wave, ii) expression for the electric and magnetic field phasor in both media, and iii) the average power density of the wave in the dielectric medium. Assume that the permeability of the medium is the same as that of free space.



i) $\beta = 1 \quad m^{-1}$

$\omega = kc$

$f = \dfrac{kc}{2\pi}$

$= \boxed{47.7 \text{ MHz}}$

ii) $\vec{E}_i = 377 e^{-j[0.5y + 0.866z]} \vec{x} \quad Vm^{-1}$

$\vec{H}_i = \dfrac{377}{377} e^{-j(0.5y+0.866z)} \cdot [0.866\vec{j} - 0.5\vec{k}]$

$= \dfrac{377}{1} [0.806\vec{j} - 0.5\vec{k}] e^{-j(0.5y+0.866z)} \quad Am^{-1}$

$\cos\theta_i = \dfrac{\sqrt{3}}{2} \qquad \cos\theta_t = \dfrac{\sqrt{35}}{36}$

Note that wave is $\perp$r polarized

$\Gamma_{\perp} = \dfrac{\eta_2 \cos\theta_i - \eta_1 \cos\theta_t}{\eta_2 \cos\theta_i + \eta_1 \cos\theta_t} = 0.44$

$E_{r\perp} = 0.44 \times 377 \; e^{-j[0.5y - 0.866z]} \vec{x}$

$= \boxed{165 e^{-j(0.5y - 0.866z)} \vec{x}} \quad Vm^{-1}$

$H_{r\perp} = 0.44 \; e^{-j(0.5y - 0.866z)} [-0.806\vec{j} - 0.5\vec{k}] \quad Am^{-1}$

$\tau_{\perp} = \boxed{1.44}$

$\vec{E}_t = 542 \; e^{-j(0.5y + 0.866z)} \vec{x} \quad Vm^{-1}$

$\vec{H}_t = 1.44 \; e^{-j(0.5y+0.866z)} (0.866\vec{j} - 0.5\vec{k})$

c)

Power Density $= \frac{1}{2}\frac{E^2}{Z} = \frac{1}{2} \times \frac{542 \times 542}{125.66} = \boxed{1168 \ Wm^{-2}}$

EM formulae

$$\nabla\cdot D = \rho \qquad \nabla\times H = J + \frac{\partial D}{\partial t}$$

$$\nabla\times E = -\frac{\partial B}{\partial t} \qquad \nabla\cdot B = 0$$

$$D = \varepsilon E \qquad H = \frac{B}{\mu}$$

1) Lorentz Gauge

$\nabla\times A = B \qquad \nabla\cdot A =$

$$\nabla\times E = -\nabla\times\frac{\partial A}{\partial t} \implies \boxed{E = -\frac{\partial A}{\partial t} - \nabla V} \quad -(1)$$

$$\nabla\times(\nabla\times A) = \mu J + \mu\frac{\partial D}{\partial t}$$

$$\underbrace{\nabla(\nabla\cdot A)}\ - (\nabla^2 A) = \mu J + \mu\varepsilon\frac{\partial}{\partial t}\underbrace{\left(-\frac{\partial A}{\partial t} - \nabla V\right)}$$

$$\nabla^2 A - \mu\varepsilon\frac{\partial^2 A}{\partial t^2} = -\mu J$$

$$\nabla^2 V - \mu\varepsilon\frac{\partial^2 V}{\partial t^2} = -\frac{\rho}{\varepsilon}$$

2) Coulomb Gauge $\nabla\cdot A = 0$

3)

(i) $\epsilon_1 E_1^{\perp} - \epsilon_2 E_2^{\perp} = \sigma_f$, (iii) $\mathbf{E}_1^{\parallel} - \mathbf{E}_2^{\parallel} = \mathbf{0}$,

(ii) $B_1^{\perp} - B_2^{\perp} = 0$, (iv) $\frac{1}{\mu_1}\mathbf{B}_1^{\parallel} - \frac{1}{\mu_2}\mathbf{B}_2^{\parallel} = \mathbf{K}_f \times \hat{\mathbf{n}}$.

$\vec{n}$ is from '2' to 1

Wave equations

$$\nabla^2 V - \frac{1}{u}\frac{\partial^2 V}{\partial t^2} = 0$$

Assume spherical symmetry

$$V = \frac{1}{4\pi\epsilon_0}\int_V \frac{\rho(t - R/u)}{R}\, dV \qquad \text{Retarded potential}$$

Wave eqn

$$\nabla^2 E - \frac{1}{u^2}\frac{\partial^2 E}{\partial t^2} = 0$$

$$\nabla^2 H - \frac{1}{u^2}\frac{\partial^2 H}{\partial t^2} = 0$$

$$\eta_0 = \sqrt{\frac{\mu_0}{\epsilon_0}} \rightarrow \text{intrinsic imp of free space}$$

Polarization

Consider waves E_1, E_2 along z direction

$$E_1^0 e^{(\omega t - kz)} \vec{i} + E_2^0 e^{(\omega t - kz - \theta)} \vec{j}$$

$\theta = \pi/2$ gives ellipse

Amplitude same = circular polarized
Else = elliptically polarized

RH Hand

$$\boxed{} \cos\omega t \, \vec{i} + \boxed{} \sin\omega t \, \vec{j}$$

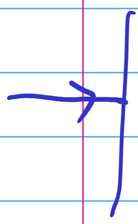
$\vec{P} = \vec{E} \times \vec{H}$

$-\oint \vec{P} \cdot d\vec{s} = \frac{\partial}{\partial t} \int (w_e + w_m)\, dv + \underbrace{\int p_\sigma \, dv}_{\text{ohmic power density}}$ $\quad \sigma E^2$

$w_e = \frac{1}{2}\varepsilon E^2 \quad w_m = \frac{1}{2}\mu H^2$

$P_{av} = \frac{1}{2} \mathrm{Re} \left\{ \vec{E} \times \vec{H} \right\}$

Plane boundary



$z=0$

$E_i = E_{i0}\, e^{-j\beta_1 z} \quad \vec{x}$ $\qquad E_2 = H_2 = 0$

$H_i = \frac{E_{i0}}{\eta}\, e^{-j\beta_1 z} \quad \vec{y}$

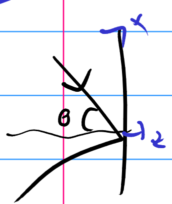
$E_{r0} = -E_{i0}$ $\qquad E_1 = E_{i0}\, e^{-j\beta_1 z} - E_{i0}\, e^{j\beta_1 z} \quad \vec{x}$

$H_1 = \frac{E_{i0}}{\eta}\, e^{-j\beta_1 z} + \frac{E_{i0}}{\eta}\, e^{j\beta_1 z} \quad \vec{y}$

Standing wave

Plane Conducting Boundary

$\perp$ polarization



$E_i = \mathbf{a}_y E_{i0} e^{-j\beta_1(x \sin\theta_i + z\cos\theta_i)}$,

$E_r = a_y E_{r0}\, e^{-j\beta_1 (x \sin\theta_i - z\cos\theta_i)}$

$E_1 = -a_y 2j\, E_{i0}\, e^{-j\beta \sin\theta_i} \sin(\beta_1 z \cos\theta_i)$

$H_i = \frac{E_{i0}}{\eta_1}\left[-a_x \cos\theta_i + a_z \sin\theta_i\right] e^{-(\ \ )}$

$H_r = \frac{E_{i0}}{\eta_1}\left[a_x \cos\theta_i - a_z \sin\theta_i\right] e^{-(\ \ )}$

$H_1 =$

z dir No power conducted $\boxed{\beta_1 \cos\theta_i}$ $\boxed{u_{1x} = \frac{u_1}{\sin\theta_i}}$

$z = -\frac{m\lambda_1}{2\cos\theta_i}$,

(1)

$$\mathbf{E}_i(x, z) = E_{i0}(\mathbf{a}_x \cos\theta_i - \mathbf{a}_z \sin\theta_i)e^{-j\beta_1(x\sin\theta_i + z\cos\theta_i)},$$

$$\mathbf{H}_i(x, z) = \mathbf{a}_y \frac{E_{i0}}{\eta_1} e^{-j\beta_1(x\sin\theta_i + z\cos\theta_i)}.$$

$$\mathbf{E}_r(x, z) = E_{r0}(\mathbf{a}_x \cos\theta_r + \mathbf{a}_z \sin\theta_r)e^{-j\beta_1(x\sin\theta_r - z\cos\theta_r)},$$

$$\mathbf{H}_r(x, z) = -\mathbf{a}_y \frac{E_{r0}}{\eta_1} e^{-j\beta_1(x\sin\theta_r - z\cos\theta_r)}.$$

z dir → $\beta = \beta_1 \cos\theta_i$

x dir → $\frac{u}{\sin\theta_i}$

insert at add $\boxed{\beta\cos\theta = m u}$

Dielectric Boundary

Normal

$$\mathbf{E}_i(z) = \mathbf{a}_x E_{i0} e^{-j\beta_1 z},$$

$$\mathbf{H}_i(z) = \mathbf{a}_y \frac{E_{i0}}{\eta_1} e^{-j\beta_1 z}.$$

Ref coeff

$$\boxed{\Gamma = \frac{E_{r0}}{E_{i0}} = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1} \quad \text{(Dimensionless)}}$$

Trans coeff

$$\boxed{\tau = \frac{E_{t0}}{E_{i0}} = \frac{2\eta_2}{\eta_2 + \eta_1} \quad \text{(Dimensionless).}}$$

$\boxed{1 + \Gamma = \tau}$

SWR

$$S = \frac{|E|_{max}}{|E|_{min}} = \frac{1 + |\Gamma|}{1 - |\Gamma|}$$

Oblique

$$\Gamma_\perp = \frac{E_{r0}}{E_{i0}} = \frac{\eta_2\cos\theta_i - \eta_1\cos\theta_t}{\eta_2\cos\theta_i + \eta_1\cos\theta_t} = \frac{(\eta_2/\cos\theta_t) - (\eta_1/\cos\theta_i)}{(\eta_2/\cos\theta_t) + (\eta_1/\cos\theta_i)}$$

$$\sin^2\theta_{B\perp} = \frac{1 - \mu_1\epsilon_2/\mu_2\epsilon_1}{1 - (\mu_1/\mu_2)^2}.$$

$$\tau_\perp = \frac{E_{t0}}{E_{i0}} = \frac{2\eta_2\cos\theta_i}{\eta_2\cos\theta_i + \eta_1\cos\theta_t} = \frac{2(\eta_2/\cos\theta_t)}{(\eta_2/\cos\theta_t) + (\eta_1/\cos\theta_i)}.$$

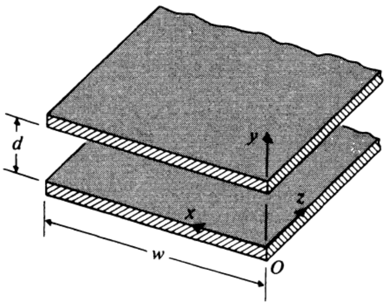
$$\Gamma_{\parallel} = \frac{E_{r0}}{E_{i0}} = \frac{\eta_2 \cos\theta_t - \eta_1 \cos\theta_i}{\eta_2 \cos\theta_t + \eta_1 \cos\theta_i}$$

$$\tau_{\parallel} = \frac{E_{t0}}{E_{i0}} = \frac{2\eta_2 \cos\theta_i}{\eta_2 \cos\theta_t + \eta_1 \cos\theta_i}.$$

$$\boxed{\sin^2\theta_{B\parallel} = \frac{1 - \mu_2\epsilon_1/\mu_1\epsilon_2}{1 - (\epsilon_1/\epsilon_2)^2}.}$$

$$\boxed{1 + \Gamma_{\parallel} = \tau_{\parallel}\left(\frac{\cos\theta_t}{\cos\theta_i}\right).}$$

TEM of ||l plate cap



Parameter	Formula	Unit
R	$\frac{2}{w}\sqrt{\frac{\pi f \mu_c}{\sigma_c}}$	Ω/m
L	$\mu \frac{d}{w}$	H/m
G	$\sigma \frac{w}{d}$	S/m
C	$\epsilon \frac{w}{d}$	F/m

Surface impedance $= \frac{E_t}{J_s} = \boxed{Z_c}$

$$\boxed{\frac{|E_z|}{|E_y|} = \sqrt{\frac{\epsilon}{\mu}}\, Z_c}$$

Trans. Line eqn :

$$\boxed{\begin{aligned} -\frac{dV(z)}{dz} &= (R + j\omega L)I(z), \\ -\frac{dI(z)}{dz} &= (G + j\omega C)V(z). \end{aligned}}$$

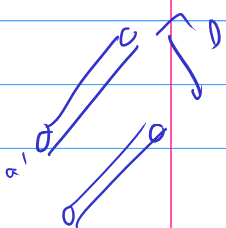
$$\gamma = \alpha + j\beta = \sqrt{(R + j\omega L)(G + j\omega C)}$$

$$Z_0 = \frac{R + j\omega L}{\gamma} = \frac{\gamma}{G + j\omega C} = \sqrt{\frac{R + j\omega L}{G + j\omega C}}$$

$\boxed{\text{Assume } R \simeq 0}$

$$\gamma = j\omega\sqrt{LC}\left(1 + \frac{G}{j\omega C}\right)^{1/2}.$$

$$LC = \mu\epsilon.$$



Parameter	Two-Wire Line	Coaxial Line	Unit
R	$\frac{R_s}{\pi a}$	$\frac{R_s}{2\pi}\left(\frac{1}{a} + \frac{1}{b}\right)$	Ω/m
L	$\frac{\mu}{\pi}\cosh^{-1}\left(\frac{D}{2a}\right)$	$\frac{\mu}{2\pi}\ln\frac{b}{a}$	H/m
G	$\frac{\pi\sigma}{\cosh^{-1}(D/2a)}$	$\frac{2\pi\sigma}{\ln(b/a)}$	S/m
C	$\frac{\pi\epsilon}{\cosh^{-1}(D/2a)}$	$\frac{2\pi\epsilon}{\ln(b/a)}$	F/m

a b

$$\alpha = \frac{P_L(z)}{2P(z)}$$

Waveguides

$\nabla^2 E + \gamma^2 E = 0$

$$\boxed{h^2 = \gamma^2 + k^2}$$

$$H_x^0 = -\frac{1}{h^2}\left(\gamma\frac{\partial H_z^0}{\partial x} - j\omega\epsilon\frac{\partial E_z^0}{\partial y}\right),$$
$$H_y^0 = -\frac{1}{h^2}\left(\gamma\frac{\partial H_z^0}{\partial y} + j\omega\epsilon\frac{\partial E_z^0}{\partial x}\right),$$
$$E_x^0 = -\frac{1}{h^2}\left(\gamma\frac{\partial E_z^0}{\partial x} + j\omega\mu\frac{\partial H_z^0}{\partial y}\right),$$
$$E_y^0 = -\frac{1}{h^2}\left(\gamma\frac{\partial E_z^0}{\partial y} - j\omega\mu\frac{\partial H_z^0}{\partial x}\right),$$

$\nabla^2_{xy} \vec{E} + h^2 \vec{E} = 0$

$\nabla^2_{xy} \vec{H} + h^2 \vec{H} = 0$

TEM wave

$h = 0 \Rightarrow \gamma = jk$

$v_{phase} = \frac{\omega}{k} = \frac{1}{\sqrt{\mu\epsilon}}$

$\frac{E_x^0}{H_y^0} = Z_{TEM} = \boxed{Z}$

$$\boxed{\vec{H} = \frac{\vec{a}_z \times \vec{E}}{Z}}$$

TM $\boxed{H_z = 0}$

$$(\mathbf{E}_T^0)_{TM} = \mathbf{a}_x E_x^0 + \mathbf{a}_y E_y^0 = -\frac{\gamma}{h^2}\nabla_T E_z^0$$

$$\boxed{\mathbf{H} = \frac{1}{Z_{TM}}(\mathbf{a}_z \times \mathbf{E}) \qquad \text{(A/m)}.}$$

$$\boxed{Z_{TM} = \frac{E_x^0}{H_y^0} = -\frac{E_y^0}{H_x^0} = \frac{\gamma}{j\omega\epsilon} \qquad (\Omega).}$$

$h^2 = k^2 = \omega^2\mu\epsilon$

$$\boxed{\omega = \frac{h}{\sqrt{\mu\epsilon}}}$$

$\gamma = h\sqrt{1 - \left(\frac{f}{f_c}\right)^2} \longrightarrow$ Img $\rightarrow$ Propagating modes

$\longrightarrow$ Real $\rightarrow$ Dies

H.P. filter

$$\boxed{\frac{1}{\lambda^2} = \frac{1}{\lambda_g^2} + \frac{1}{\lambda_c^2}.}$$

$$f_c = \frac{h}{2\pi\sqrt{\mu\epsilon}}$$

TE

$$(\mathbf{H}_T^0)_{TE} = \mathbf{a}_x H_x^0 + \mathbf{a}_y H_y^0 = -\frac{\gamma}{h^2}\nabla_T H_z^0$$

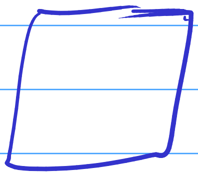
$$\boxed{Z_{TE} = \frac{E_x^0}{H_y^0} = -\frac{E_y^0}{H_x^0} = \frac{j\omega\mu}{\gamma} \qquad (\Omega).}$$

$$\boxed{\mathbf{E} = -Z_{TE}(\mathbf{a}_z \times \mathbf{H}) \qquad (\text{V/m}).}$$

r γ is the same as that given in

$$\gamma = j\beta = jk\sqrt{1-\left(\frac{f_c}{f}\right)^2}.$$

|| plate



infinite in x

$z=0$ $\quad z=b$

$$\boxed{h = \frac{n\pi}{b}}$$

TM

$H_z = 0$

$$E_z = A_n \sin\left(\frac{n\pi y}{b}\right)$$

$$\frac{d^2E_z^0(y)}{dy^2} + h^2 E_z^0(y) = 0.$$

$E_y = 0$

$$j\ E_y = -\frac{\gamma}{h} A_n \cos\left(\frac{n\pi y}{b}\right)$$

$$f_c = \frac{n}{2b\sqrt{\mu\varepsilon}}$$

$n=0 \to$ TEM

$n>0 \to$ TM

TE waves

$$\boxed{TE_0 \text{ d } ne}$$

$$H_z = B_n \cos\left(\frac{n\pi}{b}y\right)$$

$$H_y = \frac{+\gamma}{h} B_n \sin\left(\frac{n\pi y}{b}\right) \qquad E_x = \frac{j\omega\mu}{\gamma} H_y$$

Energy Transport vel

$$(P_z)_{av} = \int_S \mathscr{P}_{av}\cdot d\mathbf{s},$$

over the guide cross section:

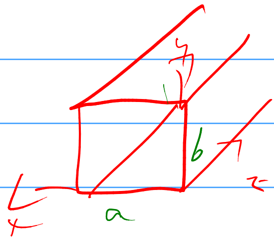
$$W'_{av} = \int_S [(w_e)_{av} + (w_m)_{av}]\, ds.$$

$$u_{en} = \frac{(P_z)_{av}}{W'_{av}} = \text{Wecp vel.}$$

$\square$ r waveguides

TM

$\boxed{H_z = 0}$



$E_z(x,y,z) = E_z^o(x,y)e^{-\gamma z}$

$$(\mathbf{E}_T^0)_{TM} = \mathbf{a}_x E_x^0 + \mathbf{a}_y E_y^0 = -\frac{\gamma}{h^2}\nabla_T E_z^0$$

$$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + h^2\right)E_z^o(x,y) = 0.$$

$$E_z^o = E_0 \sin\left(\frac{m\pi}{a}\right)\sin\left(\frac{n\pi}{b}\right)e^{-\gamma z}$$

$$h^2 = \left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2$$

$$\gamma = \sqrt{h^2 - k^2} = j\sqrt{k^2 - h^2}$$

$$= j\sqrt{\omega^2\mu\varepsilon - \left(\frac{m\pi}{a}\right)^2 - \left(\frac{n\pi}{b}\right)^2}$$

$$f_{cutoff} = \frac{1}{2\sqrt{\mu\varepsilon}}\sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2}$$

$m, n \geq 1$

$$E_x^0(x, y) = -\frac{\gamma}{h^2}\left(\frac{m\pi}{a}\right)E_0 \cos\left(\frac{m\pi}{a}x\right)\sin\left(\frac{n\pi}{b}y\right),$$

$$E_y^0(x, y) = -\frac{\gamma}{h^2}\left(\frac{n\pi}{b}\right)E_0 \sin\left(\frac{m\pi}{a}x\right)\cos\left(\frac{n\pi}{b}y\right),$$

$$H_x^0(x, y) = \frac{j\omega\epsilon}{h^2}\left(\frac{n\pi}{b}\right)E_0 \sin\left(\frac{m\pi}{a}x\right)\cos\left(\frac{n\pi}{b}y\right),$$

$$H_y^0(x, y) = -\frac{j\omega\epsilon}{h^2}\left(\frac{m\pi}{a}\right)E_0 \cos\left(\frac{m\pi}{a}x\right)\sin\left(\frac{n\pi}{b}y\right),$$

TE waves

Same idea,

$$\boxed{H_z^0(x, y) = H_0 \cos\left(\frac{m\pi}{a}x\right)\cos\left(\frac{n\pi}{b}y\right) \qquad \text{(A/m).}}$$

Same idea

$$f_c = \frac{1}{2a\sqrt{\mu\varepsilon}} = \boxed{\frac{u}{2\max(a,b)}}$$

$$\lambda_g = \frac{\lambda}{2\sqrt{1-\left(\frac{f_c}{f}\right)^2}}$$

TE_{10}

$P_{av} = \omega\mu\beta ab \left(\frac{aH_0}{2\pi}\right)^2$

$P_{loss}(z) = \left[b + \frac{a}{2}\left(\frac{f}{f_c}\right)^2\right]H_0^2 R_s.$

$$(\alpha_c)_{TE_{10}} = \frac{R_s[1 + (2b/a)(f_c/f)^2]}{\eta b\sqrt{1-(f_c/f)^2}} = \frac{1}{\eta b}\sqrt{\frac{\pi f \mu_c}{\sigma_c[1-(f_c/f)^2]}}\left[1 + \frac{2b}{a}\left(\frac{f_c}{f}\right)^2\right]$$

$$(\alpha_c)_{TM_{11}} = \frac{2R_s(b/a^2 + a/b^2)}{\eta ab\sqrt{1-(f_c/f)^2}(1/a^2 + 1/b^2)}.$$

Circular

$$J_n(hr) = \sum_{m=0}^{\infty} \frac{(-1)^m (hr)^{n+2m}}{m!(n+m)!2^{n+2m}}$$

TM

$$\boxed{E_z^0 = C_n J_n(hr)\cos n\phi. \qquad \text{(TM modes)}}$$

$$E_r^0 = -\frac{j\beta}{h} C_n J_n'(hr)\cos n\phi,$$

$$E_\phi^0 = \frac{j\beta n}{h^2 r} C_n J_n(hr)\sin n\phi,$$

$$H_r^0 = -\frac{j\omega\epsilon n}{h^2 r} C_n J_n(hr)\sin n\phi,$$

$$H_\phi^0 = -\frac{j\omega\epsilon}{h} C_n J_n'(hr)\cos n\phi,$$

$$H_z^0 = 0,$$

$J_n(ha) = 0$

$\Rightarrow$ ha is zero of J_n

$\Rightarrow h = \frac{2.405}{a}$

$f_{c_{TM_{01}}} = \frac{h}{2\pi\sqrt{\mu\varepsilon}}$

TE

$$\boxed{H_z^0 = C_n' J_n(hr)\cos n\phi.}$$

$$H_r^0 = -\frac{j\beta}{h} C_n' J_n'(hr)\cos n\phi,$$

$$H_\phi^0 = \frac{j\beta n}{h^2 r} C_n' J_n(hr)\sin n\phi,$$

$$E_r^0 = \frac{j\omega\mu n}{h^2 r} C_n' J_n(hr)\sin n\phi,$$

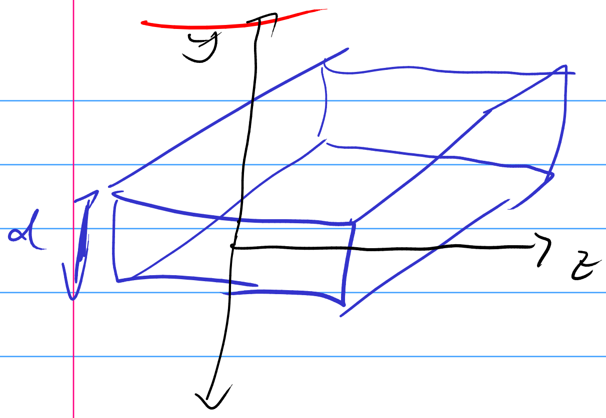
$$E_\phi^0 = \frac{j\omega\mu}{h} C_n' J_n'(hr)\cos n\phi,$$

$$E_z^0 = 0.$$

$$\boxed{J_n'(ha) = 0. \qquad \text{(TE modes)}}$$

$$(h)_{TE_{11}} = \frac{1.841}{a},$$

Dielectric Slab



No change in x

E_z^0 is fn of 'y' only.

$$\frac{d^2}{dy^2} E_z^0 + h^2 E_z^0 = 0$$

$$h^2 = k^2 + \gamma^2 = \gamma^2 + \omega^2 \mu \epsilon \qquad k = \omega\sqrt{\mu\epsilon} \qquad \gamma = j\beta$$

$$E_z^0(y) = E_o \sin k_y y + E_e \cos k_y y, \qquad |y| \le \frac{d}{2},$$

$$\boxed{k_y^2 = \omega^2 \mu_d \epsilon_d - \beta^2 = h_d^2.}$$

$$E_z^0(y) = \begin{cases} C_u e^{-\alpha(y-d/2)}, & y \ge \frac{d}{2}, \\ C_l e^{\alpha(y+d/2)}, & y \le -\frac{d}{2}, \end{cases}$$

$$\boxed{\alpha^2 = \beta^2 - \omega^2 \mu_0 \epsilon_0 = -h_0^2.}$$

$$\boxed{\frac{\alpha}{k_y} = \frac{\epsilon_0}{\epsilon_d} \tan \frac{k_y d}{2} \qquad \text{(Odd TM modes).}} \qquad Z_s = -\frac{E_z^0}{H_x^0} = j\frac{\alpha}{\omega\epsilon_0}$$

Even Mode

$$\frac{\alpha}{k_y} = -\frac{\epsilon_0}{\epsilon_d} \cot \frac{k_y d}{2}$$

Odd TM Modes	Even TM Modes
$\tan\left(\frac{\omega_{co} d}{2}\sqrt{\mu_d\epsilon_d - \mu_0\epsilon_0}\right) = 0$	$\cot\left(\frac{\omega_{ce} d}{2}\sqrt{\mu_d\epsilon_d - \mu_0\epsilon_0}\right) = 0$
$\pi f_{co} d\sqrt{\mu_d\epsilon_d - \mu_0\epsilon_0} = (n-1)\pi,$ $n = 1, 2, 3, \ldots$	$\pi f_{ce} d\sqrt{\mu_d\epsilon_d - \mu_0\epsilon_0} = (n-\frac{1}{2})\pi,$ $n = 1, 2, 3, \ldots$
$\boxed{f_{co} = \frac{(n-1)}{d\sqrt{\mu_d\epsilon_d - \mu_0\epsilon_0}}}$ (10–264)	$\boxed{f_{ce} = \frac{(n-\frac{1}{2})}{d\sqrt{\mu_d\epsilon_d - \mu_0\epsilon_0}}}$ (10–265)

TE

$$\boxed{\frac{\alpha}{k_y} = \frac{\mu_0}{\mu_d} \tan \frac{k_y d}{2} \qquad \text{(Odd TE modes)},}$$

$$\boxed{\frac{\alpha}{k_y} = -\frac{\mu_0}{\mu_d} \cot \frac{k_y d}{2} \qquad \text{(Even TE modes)}.}$$

Mode		Characteristic Relation	Cutoff Frequency
TM	Odd	$(\alpha/k_y) = (\epsilon_0/\epsilon_d) \tan (k_y d/2)$	$f_{co} = (n-1)/d\sqrt{\mu_d\epsilon_d - \mu_0\epsilon_0}$
	Even	$(\alpha/k_y) = -(\epsilon_0/\epsilon_d) \cot (k_y d/2)$	$f_{ce} = (n-\frac{1}{2})/d\sqrt{\mu_d\epsilon_d - \mu_0\epsilon_0}$
TE	Odd	$(\alpha/k_y) = (\mu_0/\mu_d) \tan (k_y d/2)$	$f_{co} = (n-1)/d\sqrt{\mu_d\epsilon_d - \mu_0\epsilon_0}$
	Even	$(\alpha/k_y) = -(\mu_0/\mu_d) \cot (k_y d/2)$	$f_{ce} = (n-\frac{1}{2})/d\sqrt{\mu_d\epsilon_d - \mu_0\epsilon_0}$

† $\alpha = [\omega^2(\mu_d\epsilon_d - \mu_0\epsilon_0) - k_y^2]^{1/2}$.

Cavity Resonator

TM Mode

$$E_z(x, y, z) = E_0 \sin\left(\frac{m\pi}{a}x\right)\sin\left(\frac{n\pi}{b}y\right)\cos\left(\frac{p\pi}{d}z\right), \tag{10–295}$$

$$E_x(x, y, z) = -\frac{1}{h^2}\left(\frac{m\pi}{a}\right)\left(\frac{p\pi}{d}\right)E_0 \cos\left(\frac{m\pi}{a}x\right)\sin\left(\frac{n\pi}{b}y\right)\sin\left(\frac{p\pi}{d}z\right), \tag{10–296}$$

$$E_y(x, y, z) = -\frac{1}{h^2}\left(\frac{n\pi}{b}\right)\left(\frac{p\pi}{d}\right)E_0 \sin\left(\frac{m\pi}{a}x\right)\cos\left(\frac{n\pi}{b}y\right)\sin\left(\frac{p\pi}{d}z\right), \tag{10–297}$$

$$H_x(x, y, z) = \frac{j\omega\epsilon}{h^2}\left(\frac{n\pi}{b}\right)E_0 \sin\left(\frac{m\pi}{a}x\right)\cos\left(\frac{n\pi}{b}y\right)\cos\left(\frac{p\pi}{d}z\right), \tag{10–298}$$

$$H_y(x, y, z) = -\frac{j\omega\epsilon}{h^2}\left(\frac{m\pi}{a}\right)E_0 \cos\left(\frac{m\pi}{a}x\right)\sin\left(\frac{n\pi}{b}y\right)\cos\left(\frac{p\pi}{d}z\right), \tag{10–299}$$

where

$$h^2 = \left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2. \tag{10–300}$$

$$\omega_{mnp} = \frac{1}{\sqrt{\mu\epsilon}}\sqrt{\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2 + \left(\frac{p\pi}{d}\right)^2}$$

$$\boxed{f_{mnp} = \frac{u}{2}\sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2 + \left(\frac{p}{d}\right)^2} \qquad \text{(Hz)}.}$$

TE Mode

$$H_z(x, y, z) = H_0 \cos\left(\frac{m\pi}{a}x\right)\cos\left(\frac{n\pi}{b}y\right)\sin\left(\frac{p\pi}{d}z\right),$$

$$E_x(x, y, z) = \frac{j\omega\mu}{h^2}\left(\frac{n\pi}{b}\right)H_0 \cos\left(\frac{m\pi}{a}x\right)\sin\left(\frac{n\pi}{b}y\right)\sin\left(\frac{p\pi}{d}z\right),$$

$$E_y(x, y, z) = -\frac{j\omega\mu}{h^2}\left(\frac{m\pi}{a}\right)H_0 \sin\left(\frac{m\pi}{a}x\right)\cos\left(\frac{n\pi}{b}y\right)\sin\left(\frac{p\pi}{d}z\right),$$

$$H_x(x, y, z) = -\frac{1}{h^2}\left(\frac{m\pi}{a}\right)\left(\frac{p\pi}{d}\right)H_0 \sin\left(\frac{m\pi}{a}x\right)\cos\left(\frac{n\pi}{b}y\right)\cos\left(\frac{p\pi}{d}z\right)$$

$$H_y(x, y, z) = -\frac{1}{h^2}\left(\frac{n\pi}{b}\right)\left(\frac{p\pi}{d}\right)H_0 \cos\left(\frac{m\pi}{a}x\right)\sin\left(\frac{n\pi}{b}y\right)\cos\left(\frac{p\pi}{d}z\right),$$

TM_{110}, TE_{011}, and TE_{101}.

The resonant frequency for both TM and TE modes is given by Eq. (10–301).

a) For $a > b > d$: The lowest resonant frequency is

$$f_{110} = \frac{c}{2}\sqrt{\frac{1}{a^2} + \frac{1}{b^2}}, \tag{10–310}$$

where c is the velocity of light in free space. Therefore TM_{110} is the dominant mode.

b) For $a > d > b$: The lowest resonant frequency is

$$f_{101} = \frac{c}{2}\sqrt{\frac{1}{a^2} + \frac{1}{d^2}}, \tag{10–311}$$

and TE_{101} is the dominant mode.

c) For $a = b = d$, all three of the lowest-order modes (namely, TM_{110}, TE_{011}, and TE_{101}) have the same field patterns. The resonant frequency of these degenerate modes is

$$f_{110} = \frac{c}{\sqrt{2}a}. \tag{10–312}$$

$$\boxed{Q = 2\pi \frac{\text{Time-average energy stored at a resonant frequency}}{\text{Energy dissipated in one period of this frequency}}.}$$

$$Q_{101} = \frac{\pi f_{101}\mu_0 abd(a^2 + d^2)}{R_s[2b(a^3 + d^3) + ad(a^2 + d^2)]}$$

$$\boxed{R_s = \sqrt{\frac{\pi f \mu_0}{\sigma}}}$$

$$\begin{aligned} W_e &= \frac{\epsilon_0}{4}\int |E_y|^2\, dv \\ &= \frac{\epsilon_0\omega^2\mu_0^2\pi^2}{4h^4a^2} H_0^2 \int_0^d\int_0^b\int_0^a \sin^2\left(\frac{\pi}{a}x\right)\sin^2\left(\frac{\pi}{d}z\right) dx\,dy\,dz \\ &= \frac{\epsilon_0\omega_{101}^2\mu_0^2a^2}{4\pi^2} H_0^2\left(\frac{a}{2}\right)b\left(\frac{d}{2}\right) = \frac{1}{4}\epsilon_0\mu_0^2a^3bdf_{101}^2H_0^2, \end{aligned}$$

W_m $= \frac{\mu_0}{16} abd\left(\frac{a^2}{d^2} + 1\right) H_0^2.$

Circular Cavity Resonator

$$E_z = C_0 J_0(hr) = C_0 J_0\left(\frac{2.405}{a} r\right),$$

$$H_\phi = -\frac{jC_0}{\eta_0} J_0'(hr) = \frac{jC_0}{\eta_0} J_1\left(\frac{2.405}{a} r\right),$$

$$Q = \left(\frac{\eta_0}{R_s}\right) \frac{2.405}{2(1 + a/d)} \qquad (\text{TM}_{010} \text{ mode}),$$

Antennae

$$\mathbf{A} = \frac{\mu}{4\pi}\int_{V'} \frac{\mathbf{J}e^{-jkR}}{R}\,dv',$$

$$= \left(\frac{\mu_0}{4\pi}\, I\,dl\, \frac{e^{-jkR}}{R}\right)\hat{z}$$

$$\mathbf{a}_z = \mathbf{a}_R \cos\theta - \mathbf{a}_\theta \sin\theta,$$

$$H = -\mathbf{a}_\phi \frac{I\,d\ell}{4\pi}\beta^2 \sin\theta\left[\frac{1}{j\beta R} + \frac{1}{(j\beta R)^2}\right]e^{-j\beta R}.$$

$$E = \frac{1}{j\omega\epsilon_0}\left[\mathbf{a}_R \frac{1}{R\sin\theta}\frac{\partial}{\partial\theta}(H_\phi \sin\theta) - \mathbf{a}_\theta \frac{1}{R}\frac{\partial}{\partial R}(RH_\phi)\right],$$

$$E_R = -\frac{I\,d\ell}{4\pi}\eta_0\beta^2 2\cos\theta\left[\frac{1}{(j\beta R)^2} + \frac{1}{(j\beta R)^3}\right]e^{-j\beta R},$$

$$E_\theta = -\frac{I\,d\ell}{4\pi}\eta_0\beta^2 \sin\theta\left[\frac{1}{j\beta R} + \frac{1}{(j\beta R)^2} + \frac{1}{(j\beta R)^3}\right]e^{-j\beta R},$$

$$H_\phi = j\frac{I\,d\ell}{4\pi}\left(\frac{e^{-j\beta R}}{R}\right)\beta \sin\theta \qquad \text{(A/m)},$$

$$E_\theta = j\frac{I\,d\ell}{4\pi}\left(\frac{e^{-j\beta R}}{R}\right)\eta_0\beta \sin\theta \qquad \text{(V/m)}.$$

Magnetic Dipole: $I\pi r^2 = m$

$$\cong e^{-j\beta R}[1 - j\beta(R_1 - R)].$$

$$\mathbf{A} = \frac{\mu_0 I}{4\pi}e^{-j\beta R}\left[(1 + j\beta R)\oint \frac{d\ell'}{R_1} - \cancel{j\beta \oint d\ell'}\right].$$

$$\mathbf{A} = \mathbf{a}_\phi \frac{\mu_0 m}{4\pi R^2}(1 + j\beta R)e^{-j\beta R}\sin\theta.$$

$$E_\phi = \frac{j\omega\mu_0 m}{4\pi}\beta^2 \sin\theta\left[\frac{1}{j\beta R} + \frac{1}{(j\beta R)^2}\right]e^{-j\beta R},$$

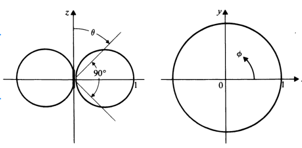
$$H_R = -\frac{j\omega\mu_0 m}{4\pi\eta_0}\beta^2 2\cos\theta\left[\frac{1}{(j\beta R)^2} + \frac{1}{(j\beta R)^3}\right]e^{-j\beta R},$$

$$H_\theta = -\frac{j\omega\mu_0 m}{4\pi\eta_0}\beta^2 \sin\theta\left[\frac{1}{j\beta R} + \frac{1}{(j\beta R)^2} + \frac{1}{(j\beta R)^3}\right]e^{-j\beta R}.$$

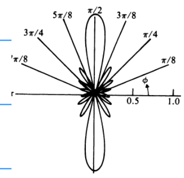
Duality:

$$\mathbf{E}_e = \eta_0 \mathbf{H}_m \qquad I\,d\ell = j\beta m,$$

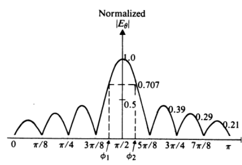
$$\mathbf{H}_e = -\frac{\mathbf{E}_m}{\eta_0}$$



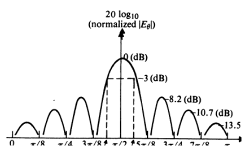
(a) E-plane pattern. (b) H-plane pattern.



(a) A typical radiation pattern in polar coordinates.



(b) Radiation pattern in rectangular coordinates.



$$P_r = \oint \mathscr{P}_{av}\cdot d\mathbf{s} = \oint U\,d\Omega \qquad G_D(\theta,\phi) = \frac{U(\theta,\phi)}{P_r/4\pi} = \frac{4\pi U(\theta,\phi)}{\oint U\,d\Omega}.$$

Hertzian dipole => 3/2

$$\boxed{G_P = \frac{4\pi U_{max}}{P_i}}$$

$$\boxed{D = \frac{U_{max}}{U_{av}} = \frac{4\pi U_{max}}{P_r}} \qquad \boxed{\eta_r = \frac{G_P}{D} = \frac{P_r}{P_i} \quad \text{(Dimensionless).}}$$

$$\boxed{R_r = 80\pi^2\left(\frac{d\ell}{\lambda}\right)^2 \qquad (\Omega).}$$

$$\eta_r = \frac{1}{1 + \frac{R_s}{160\pi^3}\left(\frac{\lambda}{a}\right)\left(\frac{\lambda}{d\ell}\right)}. \qquad R_\ell = R_s\left(\frac{d\ell}{2\pi a}\right),$$

Thin Linear Length 2h

$$I(z) = I_m \sin \beta(h - |z|),$$
$$= \begin{cases} I_m \sin \beta(h - z), \\ I_m \sin \beta(h + z), \end{cases}$$

$$F(\theta) = \frac{\cos(\beta h \cos\theta) - \cos \beta h}{\sin\theta}.$$

$$E_\theta = \frac{j60I_m}{R} e^{-j\beta R} F(\theta),$$

Half wave

$2h = \lambda/2$

$$F(\theta) = \frac{\cos[(\pi/2)\cos\theta]}{\sin\theta}.$$

$$E_\theta = \frac{j60I_m}{R} e^{-j\beta R}\left\{\frac{\cos[(\pi/2)\cos\theta]}{\sin\theta}\right\}$$

$$H_\phi = \frac{jI_m}{2\pi R} e^{-j\beta R}\left\{\frac{\cos[(\pi/2)\cos\theta]}{\sin\theta}\right\}.$$

$$\mathscr{P}_{av} = \frac{1}{2}E_\theta H_\phi^* = \frac{15I_m^2}{\pi R^2}\left\{\frac{\cos[(\pi/2)\cos\theta]}{\sin\theta}\right\}^2.$$

$$P_r = \int_0^{2\pi}\int_0^{\pi} \mathscr{P}_{av} R^2 \sin\theta \, d\theta \, d\phi$$
$$= 30I_m^2 \int_0^{\pi} \frac{\cos^2[(\pi/2)\cos\theta]}{\sin\theta} d\theta.$$

$$P_r = 36.54 I_m^2$$

$$R_r = \frac{2P_r}{I_m^2} = 73.1$$

$$D = \frac{4\pi U_{max}}{P_r} = \frac{60}{36.54} = 1.64,$$

Effective length

$$\ell_e(\theta) = \frac{\sin\theta}{I(0)} \int_{-h}^{h} I(z) e^{j\beta z \cos\theta}\, dz$$

$$\ell_e(\theta) = \frac{2}{\beta}\left[\frac{\cos\left(\frac{\pi}{2}\cos\theta\right)}{\sin\theta}\right].$$

$$\boxed{\ell_e(\theta) = -\frac{V_{oc}}{E_i},}$$

Tut - 4

1. An air-filled rectangular waveguide operates at 1GHz and has $a = 5$ cm and $b = 2$ cm. i) Show that the TM_{21} mode cannot propagate at this frequency (evanescent mode). ii) Determine the distance from the source at which the z-component of the electric field reduces to 0.5 % of its amplitude at $z = 0$. The amplitude of the z-component of the electric field at $z = 0$ is 1 kV/m.

TM_{21} $\quad f_c = \frac{1}{2\sqrt{\mu\varepsilon}}\sqrt{\left(\frac{2}{a}\right)^2 + \left(\frac{1}{b}\right)^2}$ $\quad \frac{4}{25} + \frac{1}{4}$

$= \frac{3\times10^8}{2}\sqrt{10^4\left[\left(\frac{2}{5}\right)^2 + \left(\frac{1}{2}\right)^2\right]}$

$= \frac{3\times10^8}{2}(100)\sqrt{\frac{41}{100}} = 1.5\times10^9\sqrt{41}$

$f < f_c$, does not propagate

ii) $\gamma = j\sqrt{\omega^2\mu\varepsilon - \left(\frac{m\pi}{a}\right)^2 - \left(\frac{n\pi}{b}\right)^2}$ $\quad \omega = 2\pi 10^9$

$= j\sqrt{\frac{10^{18}}{9\times10^{16}} - \left(\frac{2\pi 100}{5}\right)^2 - \left(\frac{\pi 100}{2}\right)^2}$

$= \boxed{-201}$

$e^{-201z} = \frac{0.5}{100} = \frac{1}{200}$

$j\sqrt{\frac{\omega^2}{c^2} - (\ )^2 - (\ )^2}$

$= \sqrt{\frac{(2\pi)^2}{c^2}f^2 - \left(\frac{2\pi f_c}{c}\right)^2}$

$\boxed{z = 2.63\times10^{-2}\ m}$

2. Determine the energy-transport velocity of the TE_{10} mode in a lossless a x b rectangular waveguide in terms of its cut-off frequency.

$u_{en} = u\sqrt{1 - \left(\frac{f_c}{f}\right)^2}$ $\quad f_{10} = \frac{1}{2a\sqrt{\mu\varepsilon}}\ [TE_{10}]$

$$u_{en} = u\sqrt{1-\left(\frac{1}{\varepsilon\mu}\right)\left(\frac{1}{2a f}\right)^2}$$

3. Calculate and compare the values of β, u_p, λ_g, and Z_{TE10} for a 2.5 cm x 1.5 cm rectangular waveguide operating at 7.5 GHz: a) if the waveguide is hallow and b) if the waveguide is filled with a dielectric medium characterized by $\varepsilon_r = 2$, $\mu_r = 1$, and $\sigma = 0$.

a) Hollow

$$\lambda = \frac{c}{f} = 4\times10^{-2}\,m$$

$$F_1 = \sqrt{1-\left(\frac{\lambda}{2a}\right)^2} = \sqrt{1-\left(\frac{4\times10^{-2}}{2\times2.5\times10^{-2}}\right)^2} = \boxed{0.6}$$

$$\lambda_g = \frac{\lambda}{F_1} = 6.7\times10^{-2}\,m$$

$$\beta = \frac{2\pi}{\lambda_g} = \frac{2\pi}{6.7\times10^{-2}} = \boxed{94.2\ rad\ m^{-1}}$$

$$u_p = \frac{c}{F_1} = 5\times10^{8}\,m s^{-1}$$

$$Z_{E10} = \frac{\eta_0}{F_1} = \frac{377}{0.6} = \boxed{628\,\Omega}$$

b)

$$\lambda = \frac{u}{f} = \frac{c}{f}\frac{1}{\sqrt{2}} = \boxed{2.8\times10^{-2}\,m}$$

$$F = \sqrt{1-\left(\frac{\lambda}{2a}\right)^2} = \sqrt{1-\left(\frac{0.028}{0.05}\right)^2} = \boxed{0.825}$$

$$\lambda_g = \frac{\lambda}{F_2} = \frac{2.8\times10^{-2}}{0.825} = \boxed{3.4\times10^{-2}\,m}$$

$$\beta = \frac{2\pi}{\lambda_g} = 183\ rad\,m^{-1}$$

$$u_p = \frac{u}{F_2} = \frac{3\times10^{8}}{0.825\sqrt{2}} = \boxed{2.57\times10^{8}\,m s^{-1}}$$

$$Z_{E10} = \frac{\eta_0}{\sqrt{2}F_2} = \frac{377}{\sqrt{2}(0.825)} = \boxed{323\,\Omega}$$

4. Given an air-filled rectangular cavity resonator with dimensions 8 cm x 6 cm x 5 cm, find the first twelve lowest-order modes and their resonant frequencies.

Generd formula

$$f = \frac{c}{2}\sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2 + \left(\frac{p}{c}\right)^2}$$

$$= 1.5\times10^{10}\sqrt{\frac{m^2}{64} + \frac{n^2}{36} + \frac{p^2}{25}}$$

	Freq ($\times10^9$ Hz)
TM_{110}	3.125
TE_{101}	3.54
TE_{011}	3.4
TE_{111}	4.33
TM_{210}	4.51
TE_{201}	4.81
TM_{120}	5.34
TE_{211}	5.61
TE_{021}	5.33
TE_{121}	6.13

5)

5. A rectangular cavity resonator of length d is constructed from an a x b rectangular waveguide. It is to be operated at the TE_{101} mode.

a. For a fixed b, determine the relative magnitude of a and d such that the cavity Q is maximized.

b. Obtain an expression for Q as a function of a/b under the above conditions.

$$f = \frac{c}{2}\sqrt{\frac{1}{a^2} + \frac{1}{d^2}}$$

$$Q_{101} = \frac{\pi f_{101}\mu_0\, abd(a^2+d^2)}{R\left(2b(a^3+d^3)+ad(a^2+d^2)\right)}$$

$$Q_{101} = \frac{\pi\eta_0}{2R}\,\frac{b(a^2+d^2)^{3/2}}{\left(2b(a^3+d^3)+ad(a^2+d^2)\right)}$$

$$\boxed{a = d}$$

$a = d$

$\Rightarrow$

$$Q_{cal} = \frac{\bar{u}\,\eta_0}{2Rg}\;\frac{b\,2\,a^3}{4ba^3 + 2a^4}$$

$$= \boxed{\frac{\bar{u}\,\eta_0}{2Rg}\;\frac{\sqrt{2}\left(\frac{b}{a}\right)}{2\left(\frac{b}{a}\right) + 1}}$$

Assignment

1. 1. Using both $\vec{A}$ and V, obtain the expression for the electric and magnetic field intensity of a Hertzian dipole.

$$q(t) = q\, e^{j\omega t}$$

$\vec{z}$

$$J = l\, I\, \delta(\vec{r})\, \vec{z}$$

$$\frac{dq}{dt} = I$$

$$Il = \left(\frac{dq}{dt}\right) l = j\omega q\, l = \boxed{j\omega p}$$

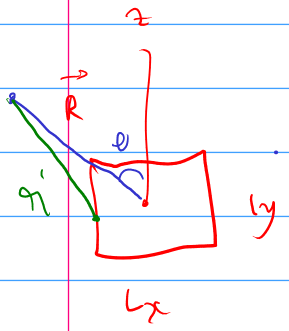
$$\vec{A}(r) = \frac{\mu}{4\pi} \int \underbrace{J(r')}_{\vec{z}\, I\, \delta(\vec{r}')} \, dr' \, \frac{e^{-j\beta |r - r'|}}{|r - r'|}$$

$$= \frac{\mu}{4\pi} (\vec{z}\, I)\, l\, \frac{e^{-j\beta r}}{r} = \boxed{\frac{\mu}{4\pi} \, \frac{I l\, e^{-j\beta r}}{r}\, \vec{z}}$$

$$H = \frac{1}{\mu} \nabla \times A = \boxed{\left(\frac{Il}{4\pi r^2} (1 + j\beta r)\, e^{-j\beta r} \sin\theta \right) \vec{\phi}}$$

$$E = \frac{\nabla \times H}{j\omega\varepsilon} = \frac{I l\, e^{-j\beta r}}{j\omega\varepsilon\, 4\pi r^3} \left[\left(2\cos\theta\, (1 + j\beta r) \right) \vec{r} + \sin\theta \left(1 + j\beta r - \beta^2 r^2 \right) \vec{\theta} \right]$$

2. Consider a small rectangular loop of dimensions Lx and Ly lies in the *xy*-plane with its centre at the origin and sides parallel to the *x*- and *y*-axes which carries a current $i(t) = I_o \cos \omega t$. Assuming that the dimensions of the loop is less than the wavelength, find the instantaneous expression for a) $\vec{A}$, b) $\vec{E}$, and c) $\vec{H}$ at a point in the far zone. Also obtain the radiation resistance and radiation efficiency of the magnetic dipole by considering that the rectangular loop is made of copper wire of radius '*a*'.



$$\vec{A} = \frac{\mu_0 I e^{-j\beta l}}{4\pi} \oint \frac{e^{-j\beta r'}}{r'} e^{+j\beta l} \, d\vec{l}'$$

$$= \frac{\mu_0 I}{4\pi} e^{-j\beta l} \left[\left(1 + e^{j\beta l}\right) \oint \frac{d\vec{l}}{r'} \cancel{- j\beta \oint d\vec{l}} \right]$$

$r' =$

$$\oint \frac{d\vec{l}}{r'} = \int \frac{1}{R} \left[1 + \frac{x}{R} \sin\theta \cos\phi + \frac{L_x}{2R} \sin\theta \sin\phi \right] dl$$

$$\boxed{m = I L_x L_y}$$

$$\vec{A} = \frac{\mu_0 m e^{-j\beta R}}{4\pi R^2} (1 + j\beta R) \sin\theta \left[-\sin\phi \, \vec{a_x} + \cos\phi \, \vec{a_y} \right]$$

$$\boxed{= \frac{\mu_0 m}{4\pi R^2} e^{-j\beta R} (1 + j\beta R) \sin\theta \, \vec{\phi}}$$

b) $E = \frac{\nabla \times \vec{A}}{\mu_0}$

$$E_\phi = \frac{j\omega\mu_0}{4\pi} m \beta^2 \sin\theta \left[\frac{1}{j\beta R} + \frac{1}{(j\beta R)^2} \right] e^{-j\beta R}$$

c) $$H_R = \frac{-j\omega\mu_0 m}{4\pi \eta_0} \beta^2 \, 2\cos\theta \left[\frac{1}{(j\beta R)^2} + \frac{1}{(j\beta R)^3} \right] e^{-j\beta R}$$

$$H_\theta = \frac{-j\omega\mu_0 m}{4\pi \eta_0} \beta^2 \sin\theta \left[\frac{1}{j\beta R} + \frac{1}{(j\beta R)^2} + \frac{1}{(j\beta R)^3} \right] e^{-j\beta R}$$

Far Zone

$$\vec{A} = \frac{\mu_0 m}{4\pi R} \beta \sin\theta \sin(\omega t - \beta R) \hat{\phi}$$

$$\vec{E} = \frac{\mu_0 m}{4\pi R} \beta \sin\theta \cos(\omega t - \beta R) \hat{\phi}$$

$$\vec{B} = \frac{-\mu_0 m}{4\pi R} \beta^2 \sin\theta \cos(\omega t - \beta R) \hat{\theta}$$

3. A 1 MHz uniform current flows in a vertical antenna of length 15 m. The antenna is a centre-fed copper rode having a radius 2 cm. Find a) the radiation resistance, and b) radiant efficiency. c) Also obtain the maximum electric field intensity at a distance of 20 km if the radiated power of the antenna is 1.6 kW.

$$\frac{dl}{\lambda} = \frac{15}{300} = \boxed{\frac{1}{20}} \qquad \boxed{\lambda = 300\,m}$$

a) $R_r = 80\pi^2 \cdot \left(\frac{dl}{\lambda}\right)^2$

$= \boxed{1.97\,\Omega}$

b) $\text{Efficiency} = \frac{R_r}{R_r + R_l}$

$$R_l = R_s \frac{dl}{2\pi a} = \left(\sqrt{\frac{\omega \mu_0}{2\sigma}}\right) \frac{dl}{2\pi a} = \boxed{0.031\,\Omega}$$

$$= \frac{0.031}{0.031 + 1.97} = \boxed{98.5\%}$$

c) $E_{max} = \frac{1}{R}\sqrt{90 P_r} = \frac{1}{R}\sqrt{\frac{\eta_0 I^2 (dl)^2 \eta_0 \beta^2}{12\pi}} = \boxed{19 \times 10^{-3}\,V/m}$

4. The transmitting antenna of a radio navigation system is a vertical metal pole 40 m in height insulated from the earth. A 180 kHz source sends a current having an amplitude of 100 A into the base of the pole. Assuming the current amplitude in the antenna to decrease linearly toward zero at the top of the pole and the earth to be a perfectly conducting plane, determine

$\beta = 0$

a) The effective length of the antenna

b) The maximum field intensity at a distance 160 km from the antenna, $\frac{30\, I(0)}{R} \beta e^{-j\beta R}$ l_{eff}

c) The time-average radiated power,

d) The radiation resistance. $k = \frac{\omega}{c}$

a) $$l_e = \int_0^h \left(1 - \frac{z}{h}\right) dh = h - \frac{h}{2} = \frac{h}{2} = \boxed{20\,m}$$

b) $$|E_\theta|_{max} = \frac{I_0}{4\pi R} \eta_0 \beta \underbrace{(2 l_e)}_{h} = \boxed{2.52\,V\,m^{-1}}$$

c) $$P = \frac{1}{2} \int_0^{\pi/2} \frac{|E_\theta|^2}{\eta_0} 2\pi R^2 \sin\theta\, d\theta = \frac{2\pi}{3} \eta_0 \left(\frac{I_0 \beta h}{4\pi}\right)^2 = \boxed{1.1 \times 10^3\,W}$$

d) $$R_{r} = \frac{2P}{I_0^2} = \boxed{0.227\,\Omega}$$

5. Find the radiation resistance of an antenna with unidirectional power pattern $P = 8 \sin^2\theta \sin^3\varphi$, where $0° \le \theta \le 180°$ and $0° \le \varphi \le 180°$, if the antenna terminal current is 3 A.

$$P_{av} = \int_0^{\pi}\int_0^{\pi} P \sin\theta\, d\theta\, d\varphi = 8 \cdot \frac{4}{3} \cdot \frac{4}{3} = \frac{3 \times 3}{2}(R)$$