EE6143: Advanced Topics in Communications
Assignment
MIMO

Haricharan B

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1 Information-Theoretic Limits

We know that the information-theoretic channel capacity for an AWGN (equivalent to a single Tx antenna and a single Rx antenna) is:

$$C = \log_2(1 + SNR) \approx \log_2(SNR)_{\text{at high SNR}}$$

It is not possible to send information at a rate faster than this capacity C.

However, if there are N_t transmit antennae and N_r receive antennae, it is a $N_r \times N_t$ MIMO channel. The interesting thing is that at high SNR, this channel has a channel capacity approximately equal to:

$$C = \min(N_r, N_t) \log_2(SNR)$$

Thus, MIMO helps us overcome the information-theoretic limits posed by a single-antenna system.

2 Mathematical Framework of MIMO

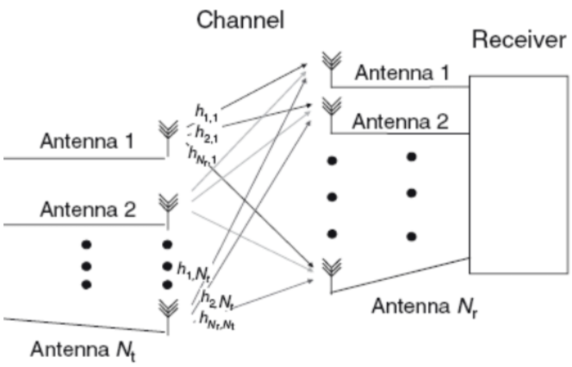

Figure 1: Schematic diagram of MIMO

Let the N_t Tx antennas transmit a vector $z \in \mathbb{R}^{N_t}$. Let the ith Rx antenna receive a signal $y_i = \sum_{j=1}^{N_t} h_{ij} z_j + \eta_i$ for $1 \leq i \leq N_r$. Now, consider the channel matrix $H = [h_{ij}]_{N_r \times N_t}$.

At the output, the signal we receive in matrix form is:

$$y_{N_r \times 1} = H_{N_r \times N_t} z_{N_t \times 1} + \eta_{N_r \times 1}$$

Assume that at the receiver, we know H. From this, we can get back an estimate for z, $\hat{z} \in \mathbb{R}^{N_t}$:

$$\hat{z} = H^{-1}Hz + H^{-1}\eta \tag{1}$$

2.1 Rank of H

The problem with the above equation is that it assumes that H is invertible. In general, H might not be invertible, in fact H might not even be a square matrix. In the above equation, H^{-1} might be replaced with the pseudoinverse of H that is $H^{\dagger}$. Either way, it is impossible to obtain more than $\texttt{rank}(H)$ many independent signals of z from y.

2.1.1 Pseudoinverse

The pseudoinverse being talked about here is the Moore-Penrose inverse. When a matrix A has linearly independent columns, $A^{\dagger} = (A^*A)^{-1}A^*$ and when a matrix A has linearly independent rows, $A^{\dagger} = A^*(AA^*)^{-1}$ (here A^* denotes the conjugate transpose of A).

2.2 Precoder Matrix

A layer is an independent data stream. Assume that there are N_l many layers. Now, we need a precoder matrix $P_{N_t \times N_l}$ that maps each of the layers to the transmit antennae.

From the argument above, we note that:

$$N_l \leq \texttt{rank}(H) \leq \min(N_r, N_t)$$

Since P maps the layers to transmit antennae,

$$z_{N_t \times 1} = P_{N_t \times N_l} x_{N_l \times 1}$$

Using this,

$$y = Hz + \eta = HPx + \eta = \tilde{H}x + \eta$$

The effective channel matrix $\tilde{H}$ is a $N_r \times N_l$ matrix. Thus, the number of layers and the precoder we choose is crucial in determining the *effective* channel characteristics.

2.3 Inverting using the Pseudoinverse

As mentioned before, $\tilde{H}$ might not be invertible. A more sophisticated treatment of x as described in section 2.1.1 using the pseudoinverse yields:

$$\hat{x} = (\tilde{H}^{\dagger}\tilde{H})^{-1}\tilde{H}^{\dagger}y = (\tilde{H}^{\dagger}\tilde{H})^{-1}\tilde{H}^{\dagger}\tilde{H}x + \eta'$$

3 OFDM + MIMO

MIMO applies for each Resource Element (RE) of OFDM. That is, for each symbol and each timeslot, MIMO is done independently.

We can see this schematically (assume that we have two layers, two antennae, let P_0 be the first row of the precoder matrix and P_1 be the second row of the precoder matrix):

$$\text{Input Bits} \longrightarrow \text{Modulation} \longrightarrow \text{Layer mapper} \longrightarrow \text{Precoder} \longrightarrow \text{IFFT} \longrightarrow \text{Tx}$$

$$\begin{array}{cccccc}
 & & \begin{bmatrix} s_0 & s_4 & \dots \\ s_2 & s_6 & \dots \\ \vdots & \vdots & \ddots \end{bmatrix} & \begin{bmatrix} P_0 s_0 & P_0 s_4 & \dots \\ P_0 s_2 & P_0 s_6 & \dots \\ \vdots & \vdots & \ddots \end{bmatrix} & c_0 & \\
x_0, x_1, x_2, x_3, \dots & s_0, s_1, \dots & & & & \\
 & & \begin{bmatrix} s_1 & s_5 & \dots \\ s_3 & s_7 & \dots \\ \vdots & \vdots & \ddots \end{bmatrix} & \begin{bmatrix} P_1 s_1 & P_1 s_5 & \dots \\ P_1 s_3 & P_1 s_7 & \dots \\ \vdots & \vdots & \ddots \end{bmatrix} & c_1 &
\end{array}$$

Figure 2: Schematic Diagram which I created to describe the process involved.

The reverse process will be observed from the Rx side.