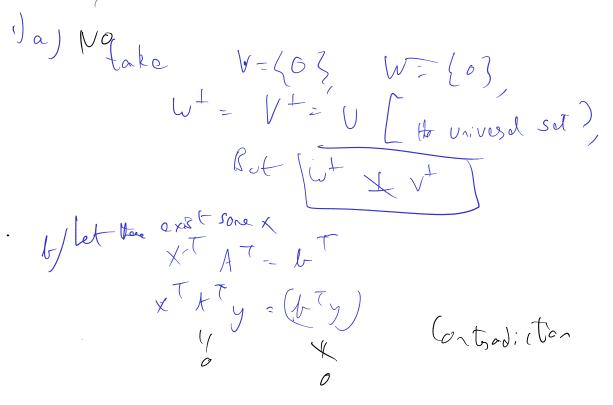
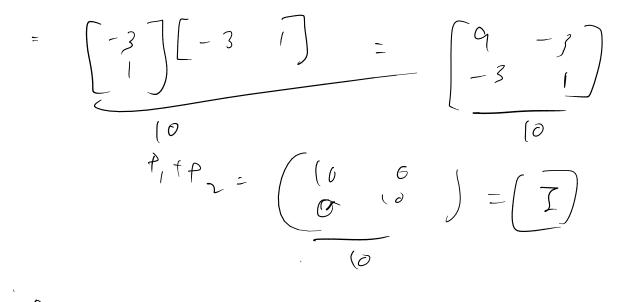
- 1. (a) Let V and W be vector spaces and  $V^{\perp}$  and  $W^{\perp}$  be the corresponding complementary orthogonal spaces. If V is orthogonal to W (if  $v \in V$  and  $w \in W$ ,  $v^T w = 0$ ), is  $V^{\perp}$  orthogonal to  $W^{\perp}$ ?
  - (b) Given an  $m \times n$  matrix A and  $m \times 1$  vectors y and b, such that  $A^Ty = 0$  and  $y^Tb \neq 0$ . Does the system of equations Ax = b have a solution?



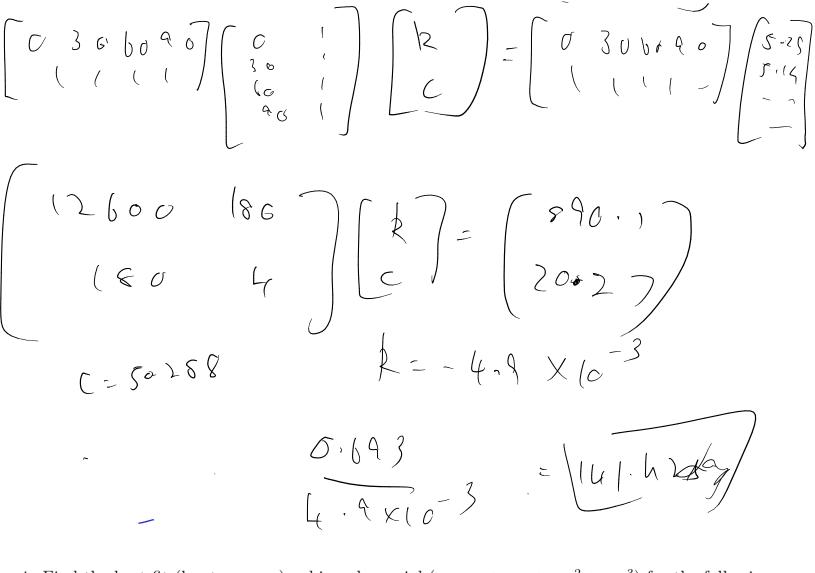
2. Find the projection matrix  $P_1$  that projects a matrix along the vector  $a = \begin{bmatrix} 1 \\ 3 \end{bmatrix}$  and the projection matrix  $P_2$  that projects onto a line perpendicular to a. Compute  $P_1 + P_2$  and  $P_1P_2$ .

$$P_{1} = \begin{bmatrix} 1 \\ 3 \end{bmatrix} \begin{pmatrix} R & 3 \\ 3 \end{pmatrix} \begin{pmatrix} 1 \\ 3 \end{pmatrix} \begin{pmatrix}$$



3. A 200 mg sample of radioactive polonium-210 is observed as it decays. The mass remaining at various times is as follows

Use an exponential model  $m(t) = ce^{kt}$  and do a least square fit to find the half-life of polonium-210.



4. Find the best fit (least squares) cubic polynomial  $(y = c_0 + c_1x + c_2x^2 + c_3x^3)$  for the following data.

$$(x,y) = [(-1,-2),(-\frac{1}{2},\frac{1}{4}),(\frac{1}{4},\frac{7}{4}),(\frac{1}{2},\frac{13}{4})]$$

$$(x,y) = [(-1,-2),(-\frac{1}{2},\frac{1}{4}),(\frac{1}{4},\frac{7}{4}),(\frac{1}{4},\frac{7}{4}),(\frac{1}{4},\frac{7}{4})]$$

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$$(x,y) = [(-1,-2),(-\frac{1}{$$

6. If  $x^*$  is the minimum-norm solution to Ax = b, show that  $x^{*^T}y = 0$  where  $y \in Null(A)$ .



X = A (A A ) b () = -2(A M) -1 7. Let f(x) denote a scalar function and  $\mathbf{f}(x)$  denote a system of m equations of a vector  $x = [x_1, x_2, \dots x_n]^T$ . Then  $\nabla f$  and  $\nabla \mathbf{f}$  are defined as follows

$$\nabla f = \begin{bmatrix} \frac{\partial f}{\partial x_1} \\ \frac{\partial f}{\partial x_2} \\ \vdots \\ \frac{\partial f}{\partial x_n} \end{bmatrix}, \quad \nabla \mathbf{f} = \begin{bmatrix} \frac{\partial f_1}{\partial x_1} & \frac{\partial f_2}{\partial x_1} & \cdots & \frac{\partial f_m}{\partial x_1} \\ \frac{\partial f_1}{\partial x_2} & \frac{\partial f_2}{\partial x_2} & \cdots & \frac{\partial f_m}{\partial x_2} \\ \vdots & \vdots & \ddots & \vdots \\ \frac{\partial f_1}{\partial x_n} & \frac{\partial f_2}{\partial x_n} & \cdots & \frac{\partial f_m}{\partial x_n} \end{bmatrix}$$

Using these definitions, find the gradient of the following functions (a)  $\mathbf{f} = Ax$  (b)  $\mathbf{f} = x^T A$  (c)  $f = x^T x$  (d)  $f = x^T Ax$  and (e)  $f = \lambda^T Ax$ . In all cases, start with the definition and compute the gradient.

$$\nabla = \nabla \left\{ \begin{array}{c} \alpha_{11} \times_1 + \alpha_{12} \times_2 \\ - \end{array} \right\}$$

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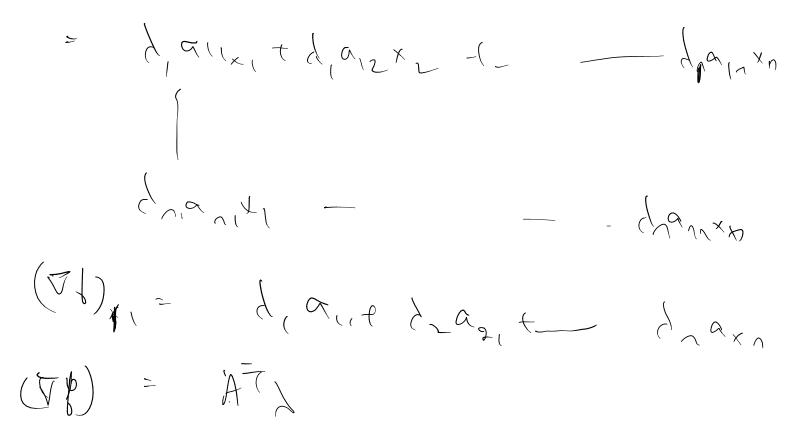
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 $d \int \int - x^{7} A = \left( x \right) x_{2} - y_{3} \tilde{\alpha}_{11} \tilde{\alpha}_{12} - e_{1} \tilde{m}$ 

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X, G,(x, + X, 0, 2x2 + x0 / 49, 2x  $\times$  ,  $\alpha$  ,  $\alpha$ × n a ni X 1 + - a, x, ta, x, ta, x, ta, x, ta, x, x, ta, x, x, y  $+\alpha_2, \chi_2 + \dots$   $A \times + \chi^{\tau} \times = (A + A^{\tau})_{\chi}$ (e)  $f = \lambda^T A x$ .



8. A is a  $m \times n$  matrix, with m > n and rank n. C is a  $p \times q$  matrix with p < q and rank p.  $b \notin \operatorname{col}\{A\}$  is a  $m \times 1$  vector. Use the method of Lagrange multipliers and find the equations that need to be solved in order to determine x in the following optimization problem.