ANALOG SYSTEMS : PROBLEM SET 9

Problem 1

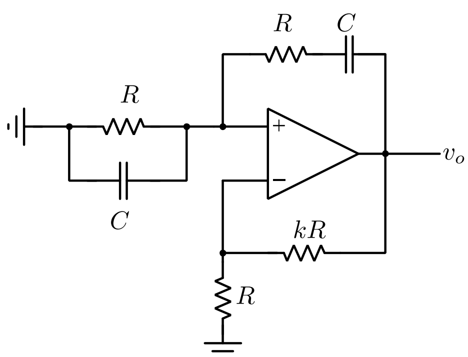


Figure 1: Circuit for Problem 1.

Fig. 1 shows a sinewave oscillator. Determine k so that it just begins to oscillate. All opamps are ideal.

Problem 2

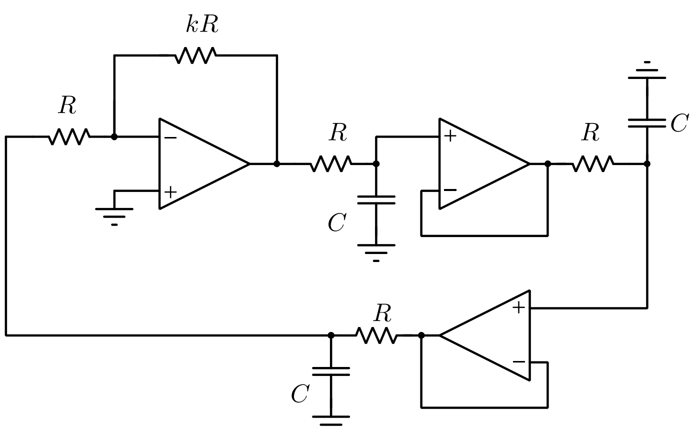


Figure 2: Circuit for Problem 2.

All opamps are ideal in Fig. 2. Determine k so that it just begins to oscillate.

Problem 3

Repeat for the circuit of Fig. 3.

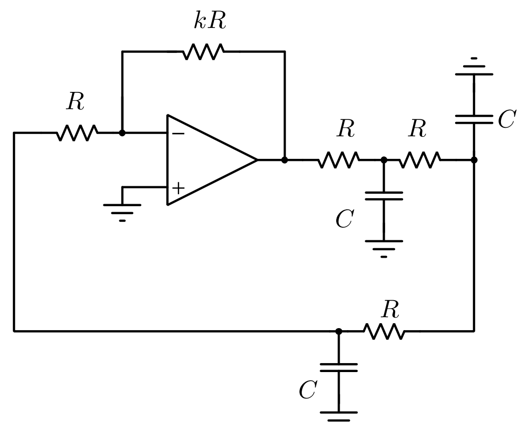


Figure 3: Circuit for Problem 3.

Problem 4

Fig. 4(a) shows an LCR network with a limiting VCCS. The Q of the RLC parallel network can be assumed to be $\gg 1$. The characteristic of the VCCS is shown in Fig. 4(b). The slope of the VCCS is denoted by G_1 and the maximum/minimum current it can source/sink is given by $\pm I_{max}$.

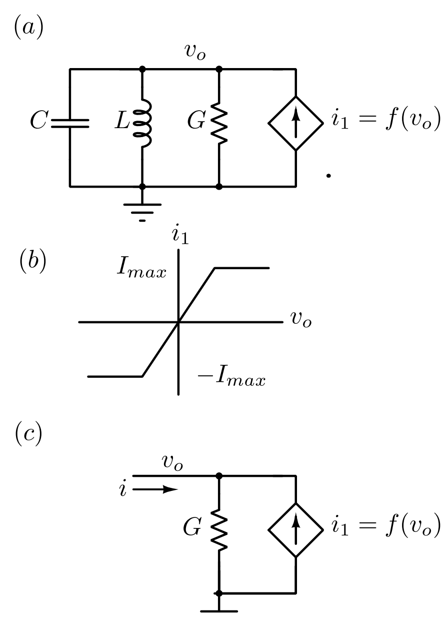


Figure 4: Circuit for Problem 4.

Determine the condition on G_1 for oscillation to start up. Assuming that this condition is satisfied, draw the $i - v_o$ characteristic of the element shown in Fig. 4(c). Assume that $v_o = A\sin(\omega t)$. Determine and plot the amplitude of the fundamental component of the current i as A is varied from 0 to ∞. What will be amplitude of oscillation of the circuit in part (a) of the figure, in steady state? You may assume that v_o is a sinusoid at the fundamental frequency.

Problem 1

$(R + \frac{1}{sC})$

$\frac{1}{\frac{1}{R} + sC}$

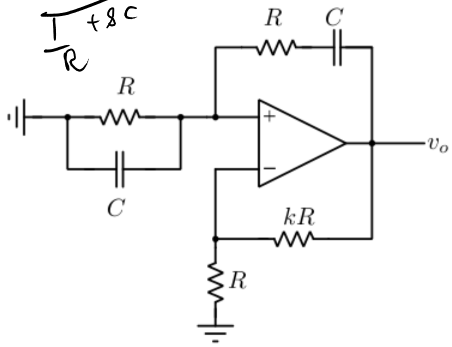


Figure 1: Circuit for Problem 1.

Fig. 1 shows a sinewave oscillator. Determine k so that it just begins to oscillate. All opamps are ideal.

$$\frac{v_o}{k} = v_o \frac{\left[\frac{1}{\left(\frac{1}{R} + sC\right)}\right]}{\frac{1}{\left(\frac{1}{R} + sC\right)} + R + \frac{1}{sC}}$$

$$\frac{1}{k} = \frac{1}{1 + \left(R + \frac{1}{sC}\right)\left(\frac{1}{R} + sC\right)}$$

$$\frac{1}{k} = \frac{1}{3 + \frac{1}{RsC} + sCR}$$

Use $s = j\omega$

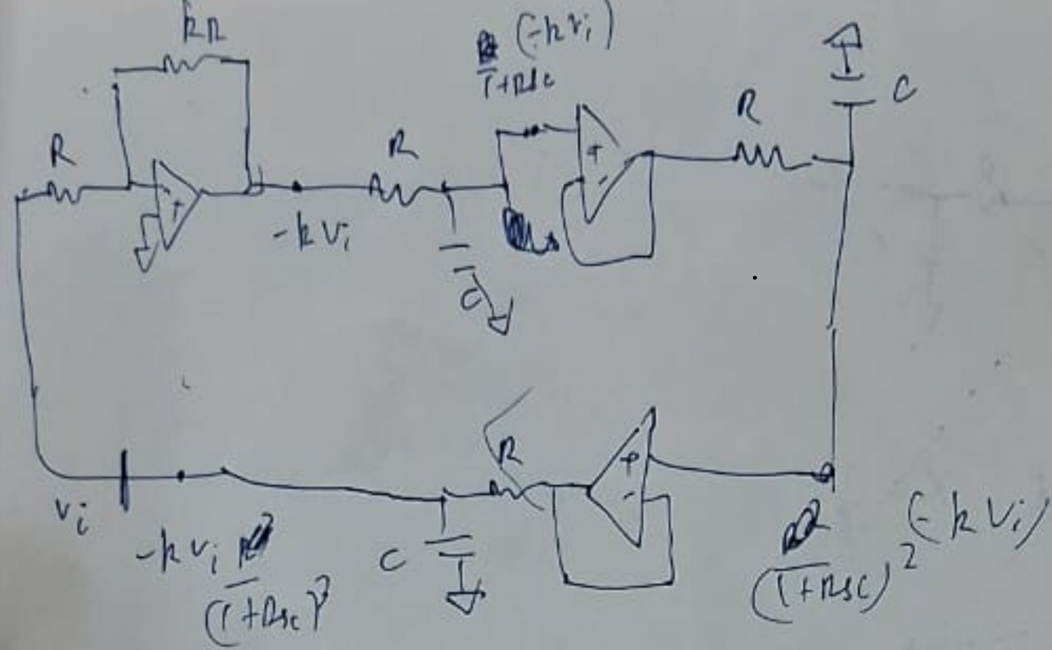
$$\frac{1}{k} = \frac{1}{3 - \frac{j}{RC\omega} + j\omega RC}$$

$$-\frac{1}{RC\omega} + \omega RC = 0$$

$$\boxed{\omega = \frac{1}{RC}}$$

$$\frac{1}{k} = \frac{1}{3 - 1} = \frac{1}{2} \qquad \boxed{k = 2}$$

2



$$LG(s) = \frac{V_{fb}}{V_i} = \frac{-k}{(1+RsC)^3}$$

$$= \frac{-k}{1+3sRC+3s^2R^2C^2+s^3R^3C^3}$$

$s = j\omega$

$$= \frac{-k}{1-3\omega^2R^2C^2+j\underbrace{[3\omega RC-\omega^3R^3C^3]}_{=0}}$$

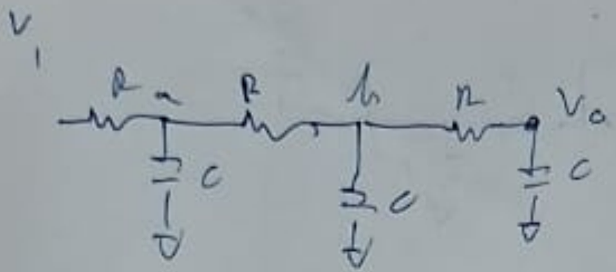
$$3\omega RC = \omega^3R^3C^3$$

$$\boxed{\omega = \sqrt{3}/RC}$$

Also, $|LG(s)| = 1$

$$\frac{-k}{1-3\,\frac{3R^2C^2}{R^2C^2}} = \frac{-k}{-8} = 1 \Rightarrow k = 8$$

(3)



$$\frac{a-V_1}{R} + \frac{a-b}{R} + asC = 0$$

$$\frac{b-a}{R} + sCb + \frac{b\,sC}{sRC+1} = 0$$

$$(b-a) + bsCR + \frac{bsCR}{sCR+1} = 0$$

$$b = \frac{a}{1+sCR+\frac{sCR}{sCR+1}}$$

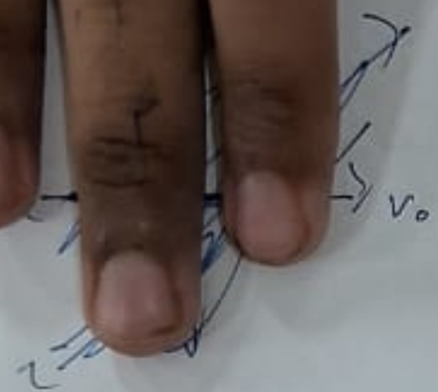
$$a\left[2 + sCR\right] = V_1$$

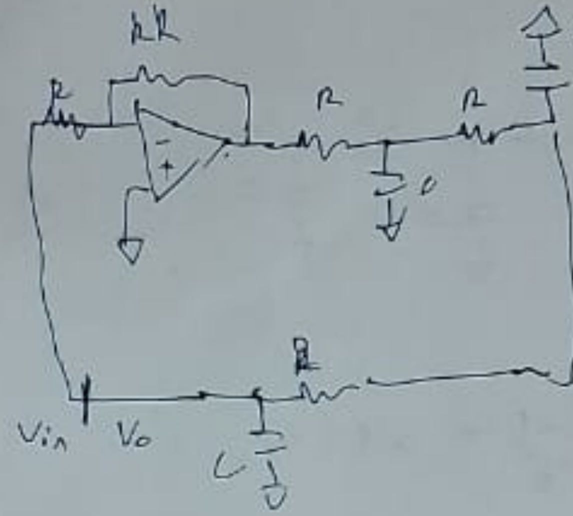
$$-b$$

$$b\left[1+sCR+\frac{sCR}{sCR+1}\right]\left[2+sCR\right] - b = V_1$$

$$b = \frac{V_1\,(sCR+1)}{\left((1+sCR)^2+sCR\right)(2+sCR)-(sCR+1)}$$

$$V_o = \frac{V_1}{\left((1+RsC)^2+RsC\right)(2+RsC)-(1+RsC)}$$





Now, Consider

$$H(s) = \frac{-k}{\left((1+sRC)^2 + sRC\right)(2+sRC) - (1+sRC)} = \frac{V_o}{V_{in}}$$

Let $sRC = j\omega_x$

$$= \frac{-k}{(1-\omega_x^2 + 3j\omega_x)(2+j\omega_x) - (1+j\omega_x)}$$

$$= \frac{-k}{(2-\omega_x^2 2 + 6j\omega_x + \cancel{j\omega_x} - j\omega_x^3 - 3\omega_x^2) - (1+\cancel{j\omega_x})}$$

$$= \frac{-k}{(1-\omega_x^2 5 + 6j\omega_x - j\omega_x^3)}$$

$6\omega_x = \omega_x^3 \quad \omega_x = \sqrt{6} = \omega_0 RC$

$\boxed{\omega_0 = \frac{\sqrt{6}}{RC}}$ $\quad \frac{-k}{1-5(6)} = 1, \quad \boxed{k=29} \quad \boxed{k=29}$

(4)

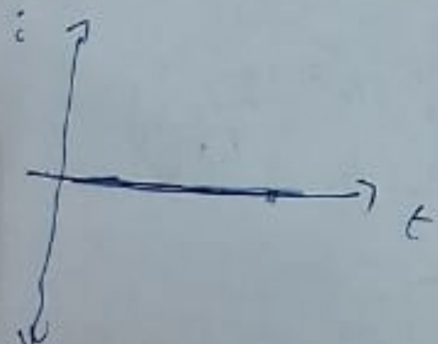
$$V_0 sC + \frac{V_0}{sL} + V_0 G_1 - V_0 G_{r1} = 0$$

We want $G_1 = -G_{r1}$, so that $\omega = \frac{1}{\sqrt{LC}}$

$G_1 = -G$

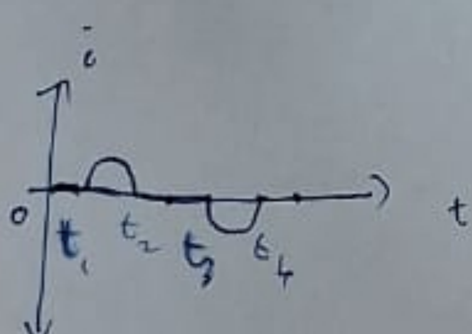
Now, considering I_{max},

if $A < \frac{I_{max}}{G_r}$, then $I = 0$ [As $h_{eff} = G_1 - G_{r1} = 0$]



If $A > \frac{I_{max}}{G_r}$ For $V < \frac{I_{max}}{G_r}$, $i = 0$.

$V > \frac{I_{max}}{G_r}$ $i = VG_r - i_{max}$



i starts increasing

at $t = \frac{1}{\omega}\sin^{-1}\left(\frac{i_{max}}{A}\right)$

$t_1 = \frac{1}{\omega}\sin^{-1}\frac{i_{max}}{A}$, $t_2 = \frac{T}{2} - t_1$, $t_3 = \frac{T}{2} + t_1$, $t_4 = T - t_1$

Amplitude of first component

$$t_1 = \frac{1}{\omega}\sin^{-1}\frac{I_{max}}{kA}$$

$$= \frac{2\int_{t_1}^{t_2}\left(\cancel{kA\sin\omega t} - I_{max}\right)e^{-j\omega t}\,dt + \int_{t_3}^{t_4}\left(\cancel{kA\sin\omega t} + I_{max}\right)e^{-j\omega t}\,dt}{T}$$

$$= \frac{\left(-I_{max}\right)\left(e^{-j\omega t_2} - e^{-j\omega t_1}\right)2}{2\pi/\omega \quad \cancel{\omega}}$$

$$= \frac{2 I_{max}}{2\pi/\omega}\left[2e^{-j\omega t_1}\right] = \frac{2I_{max}}{\pi}\cos\left(\sin^{-1}\left(\frac{I_{max}}{kA}\right)\right)$$

$$= \boxed{\frac{2I_{max}}{\pi}\sqrt{1-\frac{I_{max}^2}{k^2A^2}}}$$

As $A \to \infty$, Amplitude $\to \frac{2I_{max}}{\pi}$