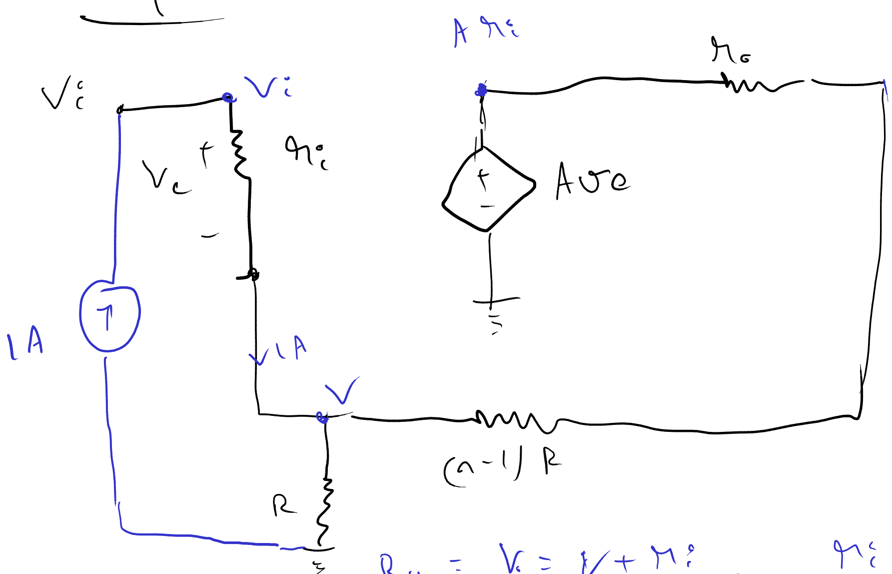
1.

Input Resistance



Assume 1A input source

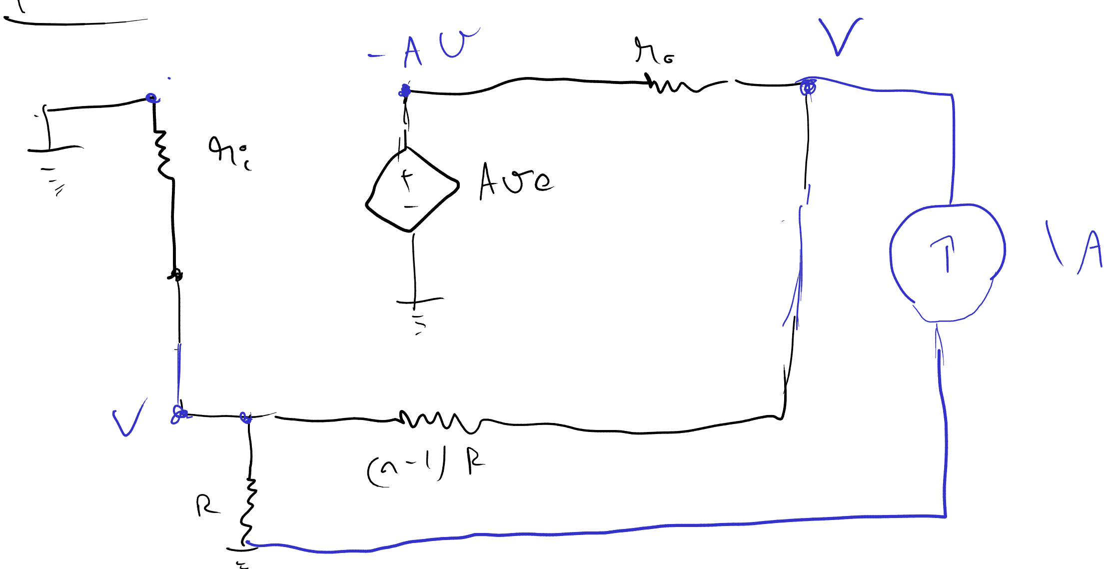
$$1A = \frac{V}{R} + \frac{V - A r_i}{r_o + (n-1)R}$$

$$1 + \frac{A r_i}{r_o + (n-1)R} = V\left[\frac{1}{R} + \frac{1}{r_o + (n-1)R}\right]$$

$$R_{th} = \frac{V_i}{1A} = \frac{V + r_i}{1A} = r_i + \frac{\frac{A r_i}{r_o + (n-1)R} + 1}{\frac{1}{R} + \frac{1}{r_o + (n-1)R}}$$

$$= r_i + R\left[\frac{r_o + (n-1)R + A r_i}{R + r_o + (n-1)R}\right] = r_i + R\left[\frac{A r_i + r_o + (n-1)R}{r_o + nR}\right]$$

Output Resistance



Assume 1A input,

$$1 = \frac{V}{(n-1)R + \frac{R r_i}{R + r_i}} + \frac{V + AV\left[\frac{\frac{r_i R}{r_i + R}}{\frac{r_i R}{r_i + R} + (n-1)R}\right]}{r_o}$$

$$V = \frac{1}{\frac{1}{(n-1)R + \frac{R r_i}{R+r_i}} + \frac{1 + \frac{A\frac{r_i R}{r_i+R}}{\frac{r_i R}{r_i+R} + (n-1)R}}{r_o}}$$

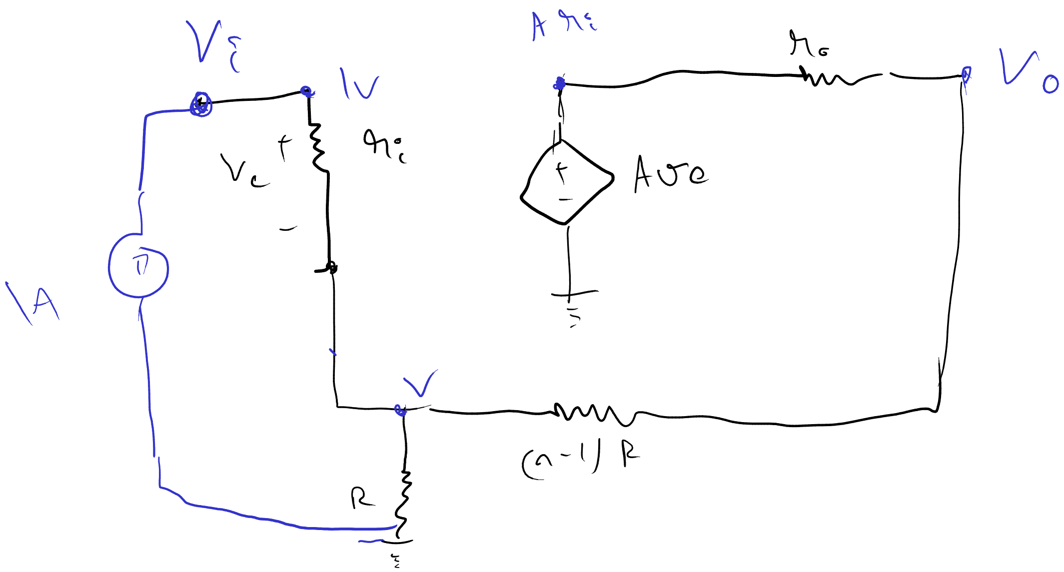
$$= \frac{1}{\frac{1}{(n-1)R + \frac{R r_i}{R+r_i}} + \frac{1 + \frac{A r_i R}{r_i R + (n-1)R(r_i+R)}}{r_o}}$$

$$= \frac{1}{\frac{R+r_i}{(R+r_i)(n-1)R + R r_i} + \frac{1}{r_o}\,\frac{r_i R + (n-1)R(r_i+R) + A r_i R}{r_i R + (n-1)R(r_i+R)}}$$

$$= \frac{\left[(R+r_i)(n-1)R + R r_i\right] r_o}{R r_o + r_i r_o + r_i R + (n-1)R(r_i+R) + A r_i R}$$

$$= \boxed{\frac{\left((R+r_i)(n-1)R + R r_i\right) r_o}{r_i r_o + R r_o + (n-1)R(r_i+R) + (A+1) r_i R}}$$

Gain



Consider 1A input. We have to find V_o & V_i

V_i

$$1A = \frac{V}{R} + \frac{V - Ar_i}{r_o + (n-1)R}$$

$$1 + \frac{Ar_i}{r_o + (n-1)R} = V\left[\frac{1}{R} + \frac{1}{r_o + (n-1)R}\right] \Rightarrow V = R\left[\frac{Ar_i + r_o + (n-1)R}{r_o + nR}\right]$$

$$V_i = V + r_i = r_i + \frac{\frac{Ar_i}{r_o + (n-1)R} + 1}{\frac{1}{R} + \frac{1}{r_o + (n-1)R}}$$

$$= r_i + R\left[\frac{r_o + (n-1)R + Ar_i}{R + r_o + (n-1)R}\right] = r_i + R\left[\frac{Ar_i + r_o + (n-1)R}{r_o + nR}\right] = V_i$$

$$V_o = V - \left(1 - \frac{V}{R}\right)(n-1)R$$

$$= V - (R - V)(n-1) = V - nR + R + nV - V$$

$$= nV - (n-1)R$$

$$V_o = n \, R \left[\frac{A r_i + r_o + (n-1)R}{r_o + nR} \right] - (n-1) R$$

$$= A \, \frac{nRr_i + nRr_o + \cancel{nR^2(n-1)} - (n-1)Rr_o + \cancel{n(n-1)R^2}}{r_o + nR}$$

$$= \frac{AnRr_i + Rr_o}{r_o + nR}$$

$$\frac{V_i}{V_o} = \frac{r_i + R\left[\frac{Ar_i + r_o + (n-1)R}{r_o + nR}\right]}{\frac{AnRr_i + Rr_o}{r_o + nR}} = \frac{r_i r_o + nRr_i + RAr_i + r_oR + (n-1)R^2}{AnRr_i + Rr_o}$$

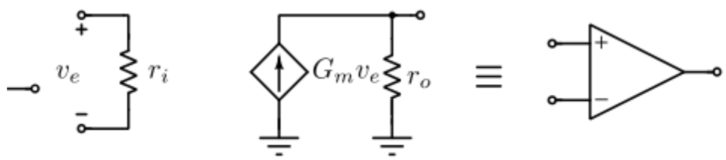
$$= \frac{r_i r_o + ARr_i + nRr_i + r_oR + (n-1)R^2}{AnRr_i + Rr_o}$$

$$\frac{V_o}{V_i} = \text{gain} = \boxed{\frac{AnRr_i + Rr_o}{r_i r_o + ARr_i + nRr_i + r_oR + (n-1)R^2}}$$

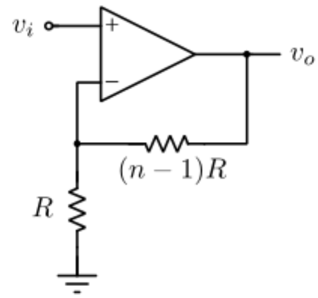
	$r_o \to 0$	$r_i \to \infty$	$A \to \infty$
Input Resistance $r_i + R\left[\frac{Ar_i + r_o + (n-1)R}{r_o + nR}\right]$	$r_i + \frac{[Ar_i + (n-1)R]}{n}$	∞	∞
Output resistance $\frac{(R+r_i)((n-1)R + Rr_i)\, r_o}{r_i r_o + R r_o + (n-1)R(r_i+R) + (A+1) r_i R}$	0	$\frac{r_o[nR]}{r_o + (n-1)R + (A+1)R} = \frac{r_o n R}{r_o + nR + AR}$	0
Gain $\frac{AnRr_i + Rr_o}{r_i r_o + ARr_i + nRr_i + r_o R + (n-1)R^2}$	$\frac{nAr_i}{r_i A + n r_i + (n-1)R}$	$\frac{AnR}{r_o + AR + nR}$	n

2.

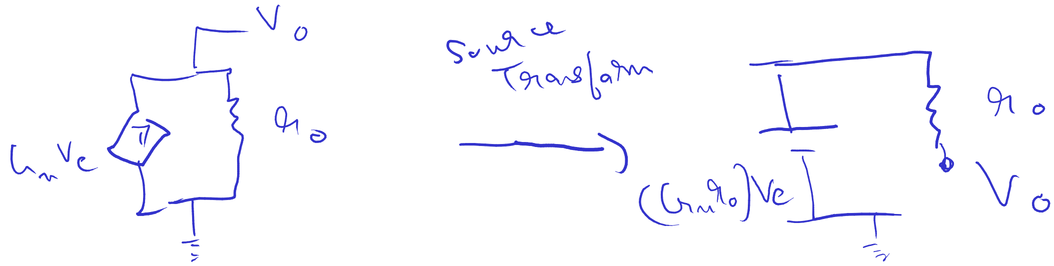
(a)



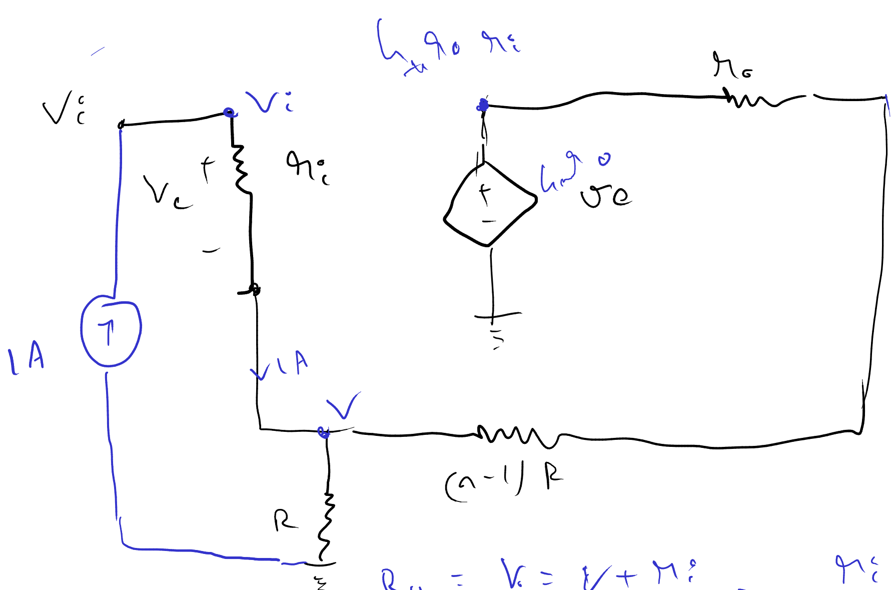
(b)



Note that



Input Resistance



Assume 1A input source

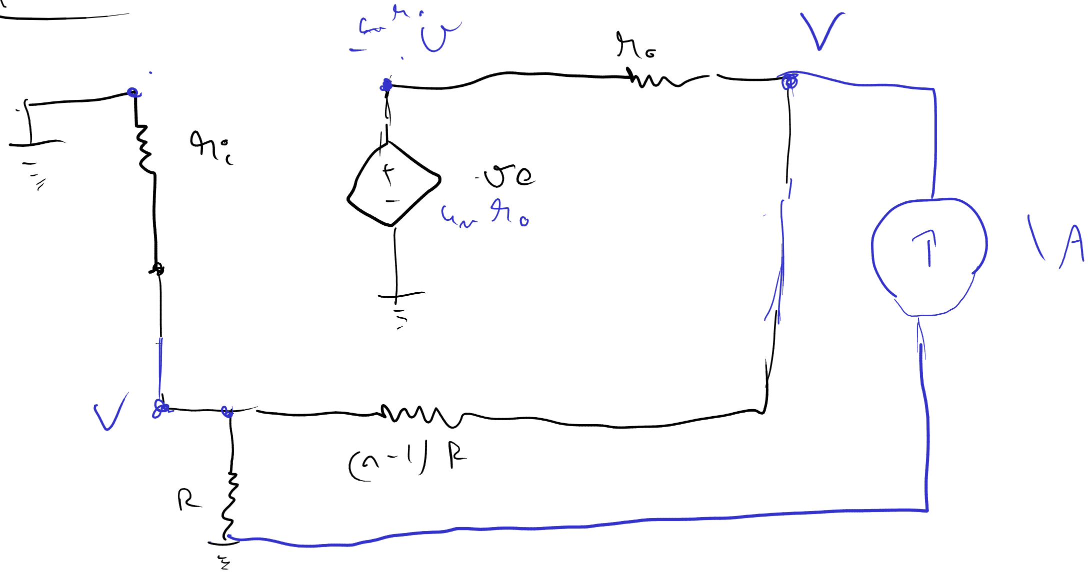
$$1 = \frac{V}{R} + \frac{V - r_i G_m r_o}{r_o + (n-1)R}$$

$$1 + \frac{r_i G_m r_o}{r_o + (n-1)R} = V\left[\frac{1}{R} + \frac{1}{r_o + (n-1)R}\right]$$

$$R_{th} = \frac{V_i}{1A} = \frac{V + r_i}{1A} = r_i + \frac{\frac{G_m r_o r_i}{r_o + (n-1)R} + 1}{\frac{1}{R} + \frac{1}{r_o + (n-1)R}}$$

$$= r_i + R\left[\frac{r_o + (n-1)R + r_i G_m r_o}{R + r_o + (n-1)R}\right] = \boxed{r_i + R\left[\frac{G_m r_i r_o + r_o + (n-1)R}{r_o + nR}\right]}$$

Output Resistance



Assume 1A input,

$$I = \frac{V}{(n-1)R + \frac{Rr_i}{R+r_i}} + \frac{V + g_m r_o V\left[\frac{\frac{r_iR}{r_i+R}}{\frac{r_iR}{r_i+R} + (n-1)R}\right]}{r_o}$$

$$V = \frac{1}{\frac{1}{(n-1)R + \frac{Rr_i}{R+r_i}} + \frac{1 + g_m r_o \frac{\frac{r_iR}{r_i+R}}{\frac{r_iR}{r_i+R} + (n-1)R}}{r_o}}$$

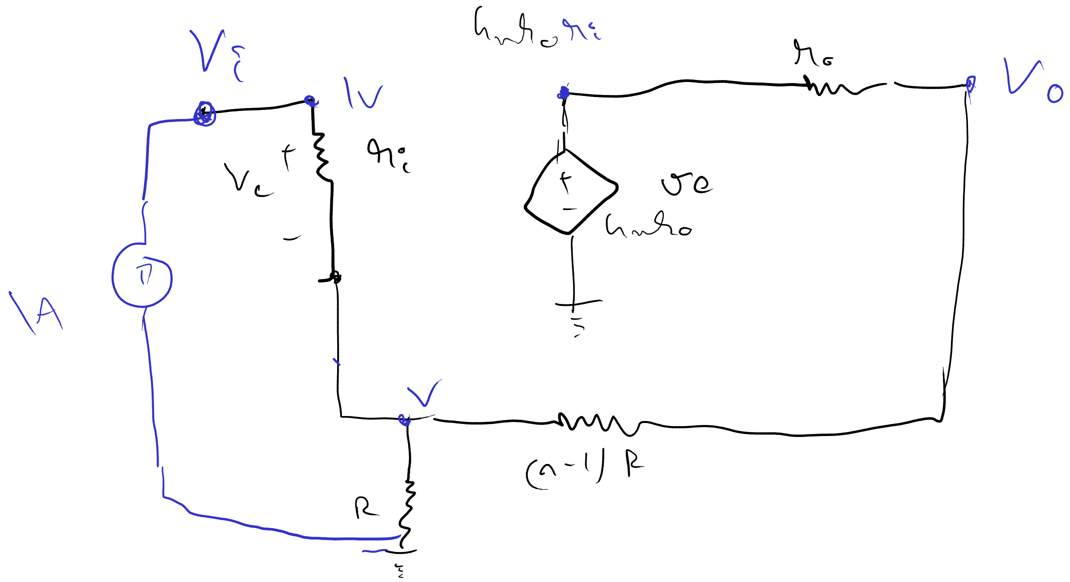
$$= \frac{1}{\frac{1}{(n-1)R + \frac{Rr_i}{R+r_i}} + \frac{1 + \frac{r_iR\,g_m r_o}{r_iR + (n-1)R(r_i+R)}}{r_o}}$$

$$= \frac{1}{\frac{R+r_i}{(R+r_i)(n-1)R + Rr_i} + \frac{1}{r_o}\,\frac{r_iR + (n-1)R(r_i+R) + r_iR\,g_m r_o}{r_iR + (n-1)R(r_i+R)}}$$

$$= \frac{\left[(R+r_i)(n-1)R + R r_i\right] r_o}{R r_o + r_i r_o + r_i R + (n-1)R(r_i+R) + r_i R\, g_m r_o}.$$

$$\boxed{= \frac{\left((R+r_i)(n-1)R + R r_i\right) r_o}{r_i r_o + R r_o + (n-1)R(r_i+R) + (g_m r_o + 1) r_i R}}$$

Gain



Consider 1A input. We have to find V_o & V_i

V_i

$$1A = \frac{V}{R} + \frac{V - r_i g_m r_o}{r_o + (n-1)R}$$

$$1 + \frac{r_i g_m r_o}{r_o + (n-1)R} = V\left[\frac{1}{R} + \frac{1}{r_o + (n-1)R}\right] \Rightarrow V = R\left[\frac{r_i g_m r_o + r_o + (n-1)R}{r_o + nR}\right]$$

$$V_i = V + r_i = r_i + \frac{\frac{g_m r_o r_i}{r_o + (n-1)R} + 1}{\frac{1}{R} + \frac{1}{r_o + (n-1)R}}$$

$$= r_i + R\left[\frac{r_o + (n-1)R + g_m r_i r_o}{R + r_o + (n-1)R}\right] = \boxed{r_i + R\left[\frac{g_m r_i r_o + r_o + (n-1)R}{r_o + nR}\right] = V_i}$$

$$V_o = V - \left(1 - \frac{V}{R}\right)(n-1)R$$

$$= V - (R - V)(n-1) = V - nR + R + nV - V$$

$$= nV - (n-1)R$$

$$V_o = n \quad R\left[\frac{A_n r_i r_o + r_o + (n-1)R}{r_o + nR}\right] - (n-1)R$$

$$= \frac{A_n r_o\, nR r_i + nR r_o + nR^2 \cancel{(n-1)} - (n-1)R r_o + n\cancel{(n-1)}R^2}{r_o + nR}$$

$$= \boxed{\frac{A_n r_o\, nR r_i + R r_o}{r_o + nR}}$$

$$\frac{V_i}{V_o} = \frac{r_i + R\left[\frac{A_n r_i r_o + r_o + (n-1)R}{r_o + nR}\right]}{\frac{A_n R r_i + R r_o}{r_o + nR}} = \frac{r_i r_o + nR r_i + R A_n r_o r_i + r_o R + (n-1)R^2}{A_n R r_i + R r_o}$$

$$= \frac{r_i r_o + A_n r_o R r_i + nR r_i + r_o R + (n-1)R^2}{A_n R r_i + R r_o}$$

$$\frac{V_o}{V_i} = \text{gain} = \boxed{\frac{A_n r_o\, nR r_i + R r_o}{r_i r_o + A_n r_o R r_i + nR r_i + r_o R + (n-1)R^2}}$$

	$r_o \to 0$	$r_i \to \infty$	$A \to \infty$
Input Resistance: $r_i + R\left[\dfrac{G_m r_i r_o + r_o + (n-1)R}{r_o + nR}\right]$	$r_i + \dfrac{[G_m r_o r_i + (n-1)R]}{n}$	∞	∞
Output resistance: $\dfrac{((R+r_i)(n-1)R + R r_i)\, r_o}{r_i r_o + R r_o + (n-1)R(r_i + R) + (G_m r_o + 1) r_i R}$	0	$\dfrac{r_o [nR]}{r_o + (n-1)R + (G_m r_o + 1)R} = \dfrac{r_o n R}{r_o + nR + R G_m r_o}$	0
Gain: $\dfrac{G_m r_o n R r_i + R r_o}{r_i r_o + G_m R r_i r_o + nR r_i + r_o R + (n-1)R^2}$	$\dfrac{R r_i}{nR r_i + (n-1)R^2} = \dfrac{r_i}{n r_i + (n-1)R}$	$\dfrac{G_m r_o n R}{r_o + r_o R G_m + nR}$	n

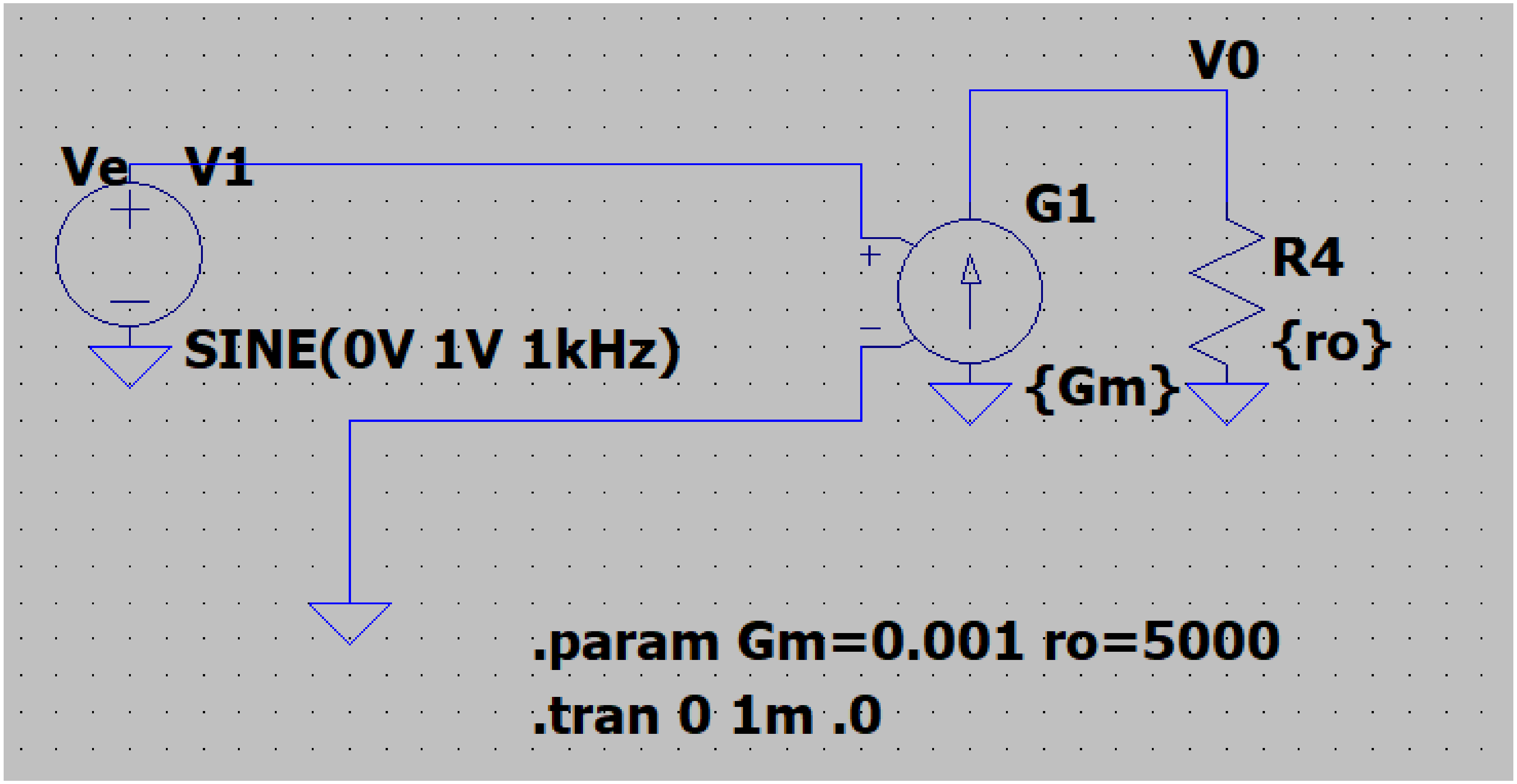
2. contd.

Rather than use a VCVS with a large (but uncertain) gain, this problem attempts to use a VCCS with a large (but uncertain) transconductance G_m. Further, the imprecise VCCS, shown in Fig. 2(a), is not all that ideal – its input and output resistances are finite. For simplicity, we use the same symbol for the imprecise VCCS as in Fig. 1(a). It is realized to make the VCVS shown in Fig. 2(b). What is v_o/v_i when $G_m = \infty$?

→ When $G_m = \infty$,

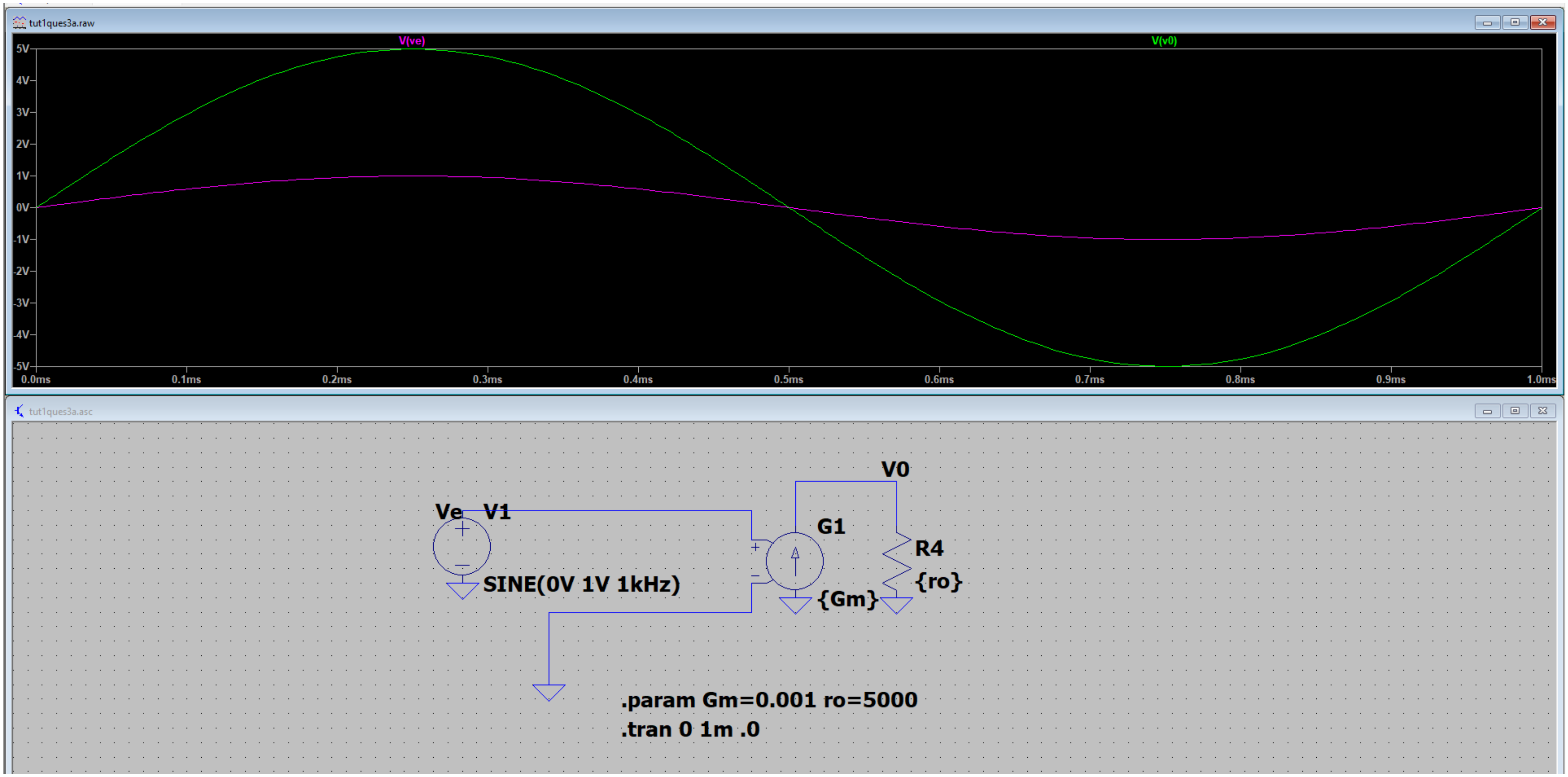
$$\boxed{\frac{v_o}{v_i} = n}$$

Question 3a

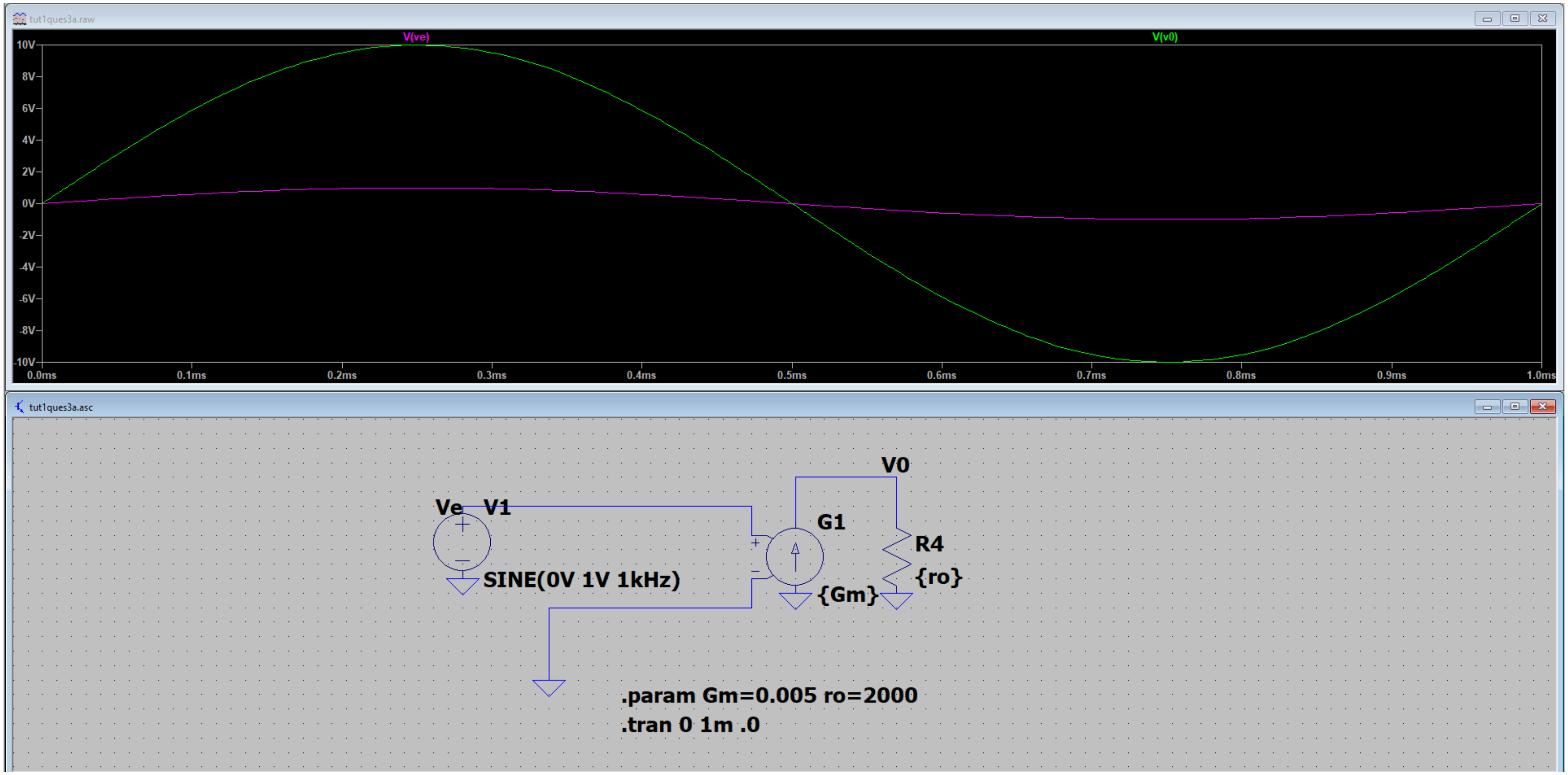


Theoretical Calculation of Gain = Gm * ro

- A0 = 1mS * 5k = 5 can be observed from plot

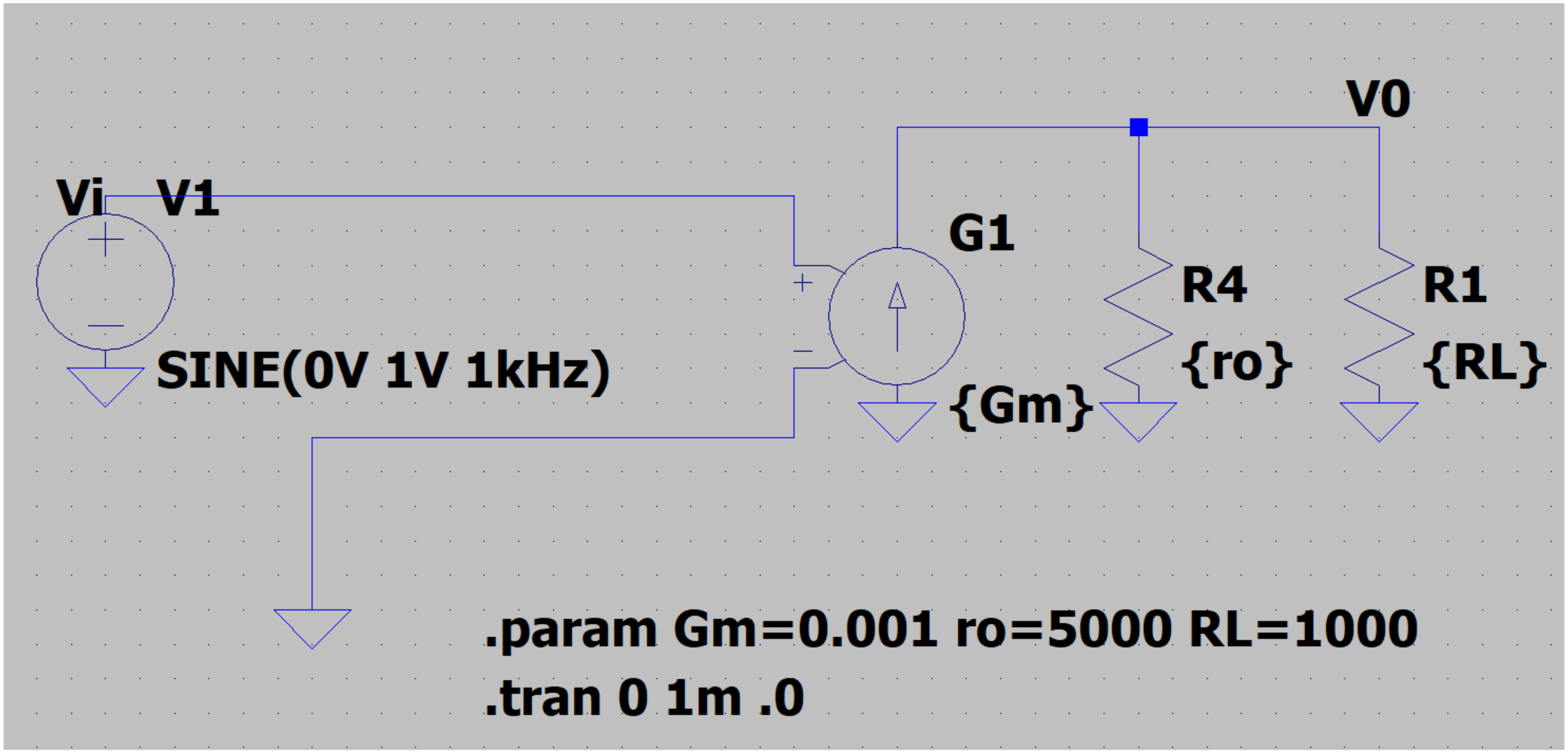


- A0 = 5S * 2k = 10 can be observed from plot



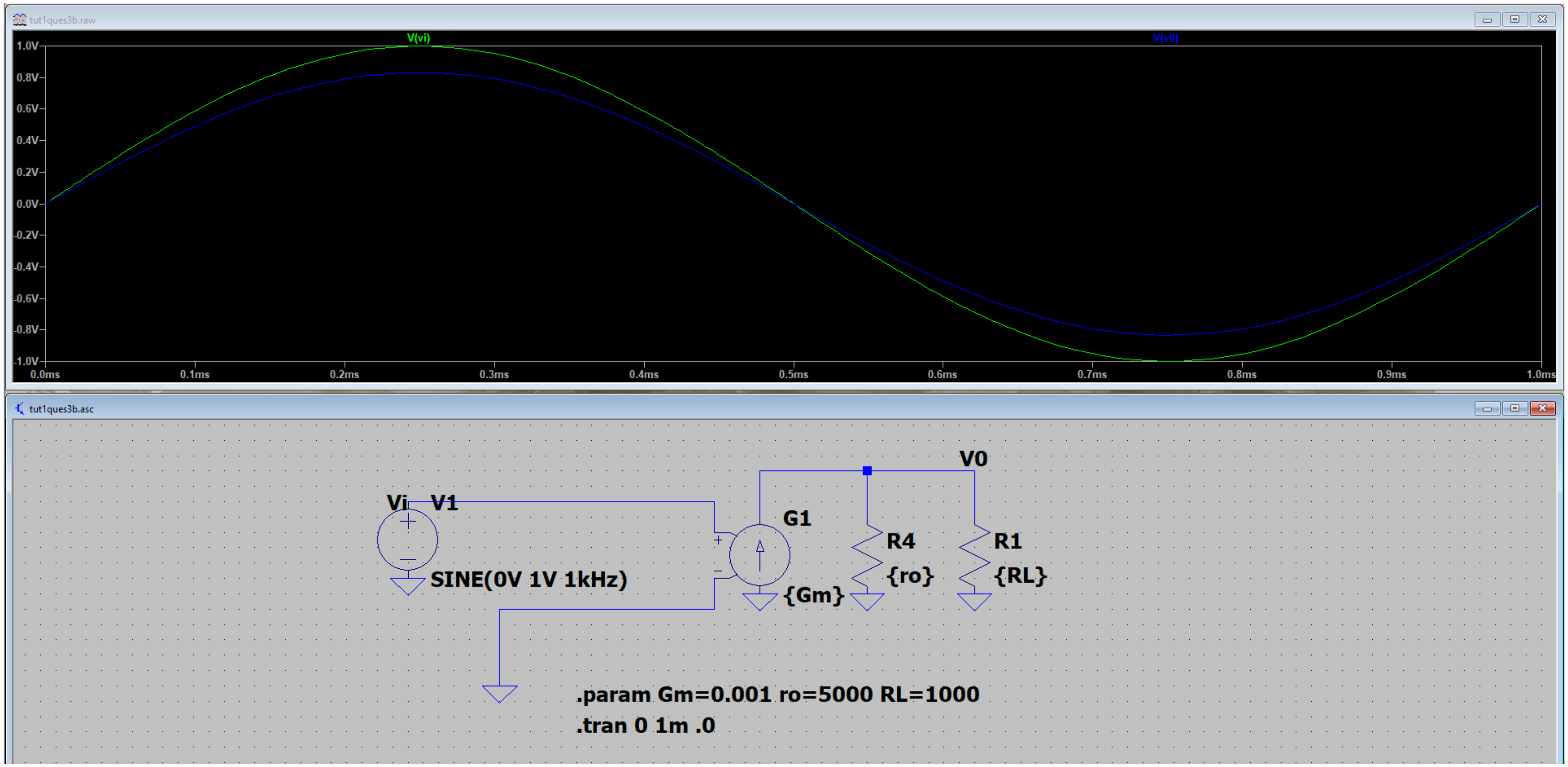
Explanation: Using LTSpice, it is verified that the gain graphically = Gm * ro, which is the theoretically expected value.

Question 3b

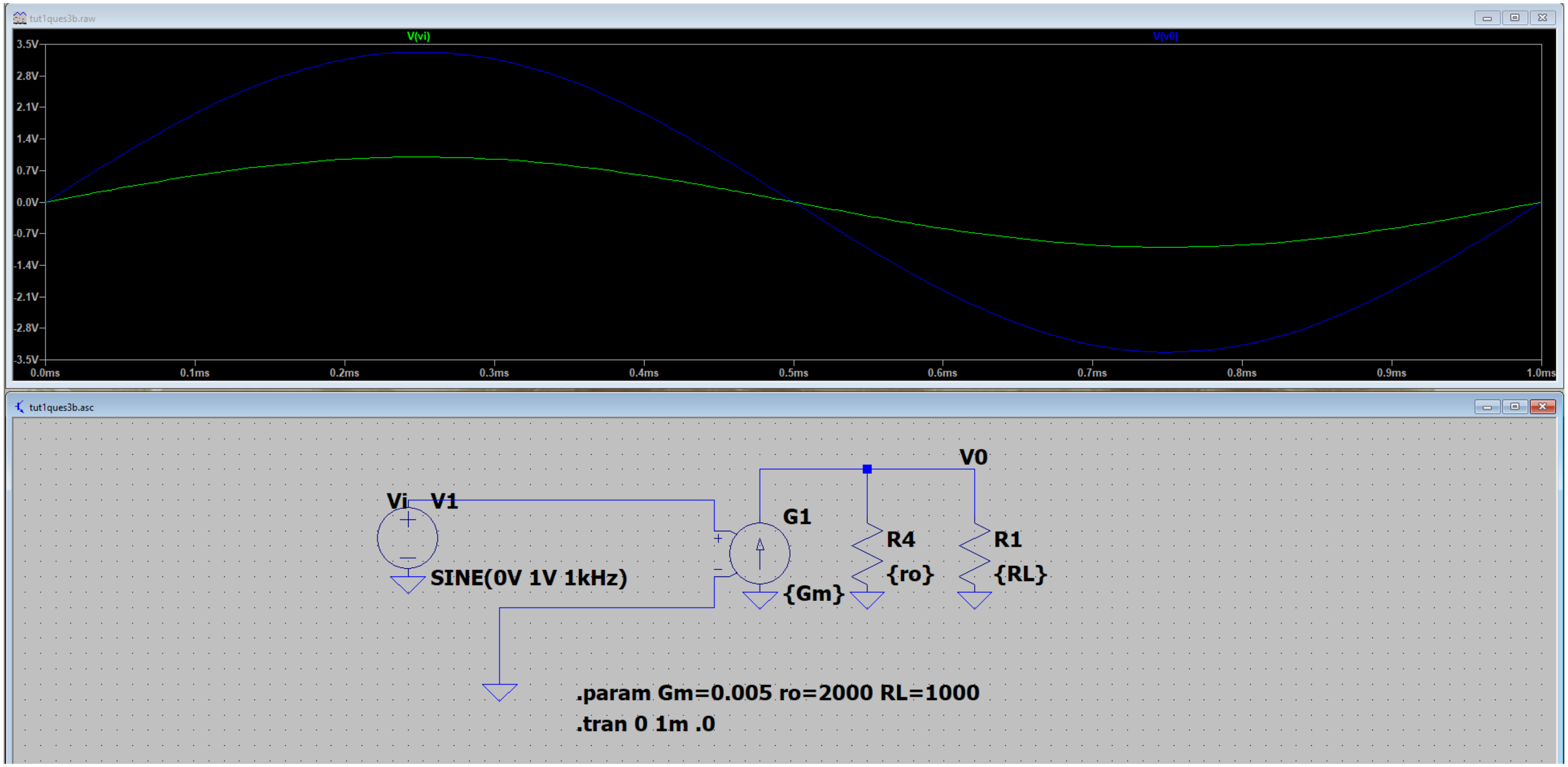


Theoretical Calculation of Gain = Gm * ((ro * RL)/(ro + RL))

- Theoretical gain A0 = 1mS * ((5000 * 1000)/(5000 + 1000)) = 0.83 can be observed from plot. Here, gain < 1, so amplification is not achieved.



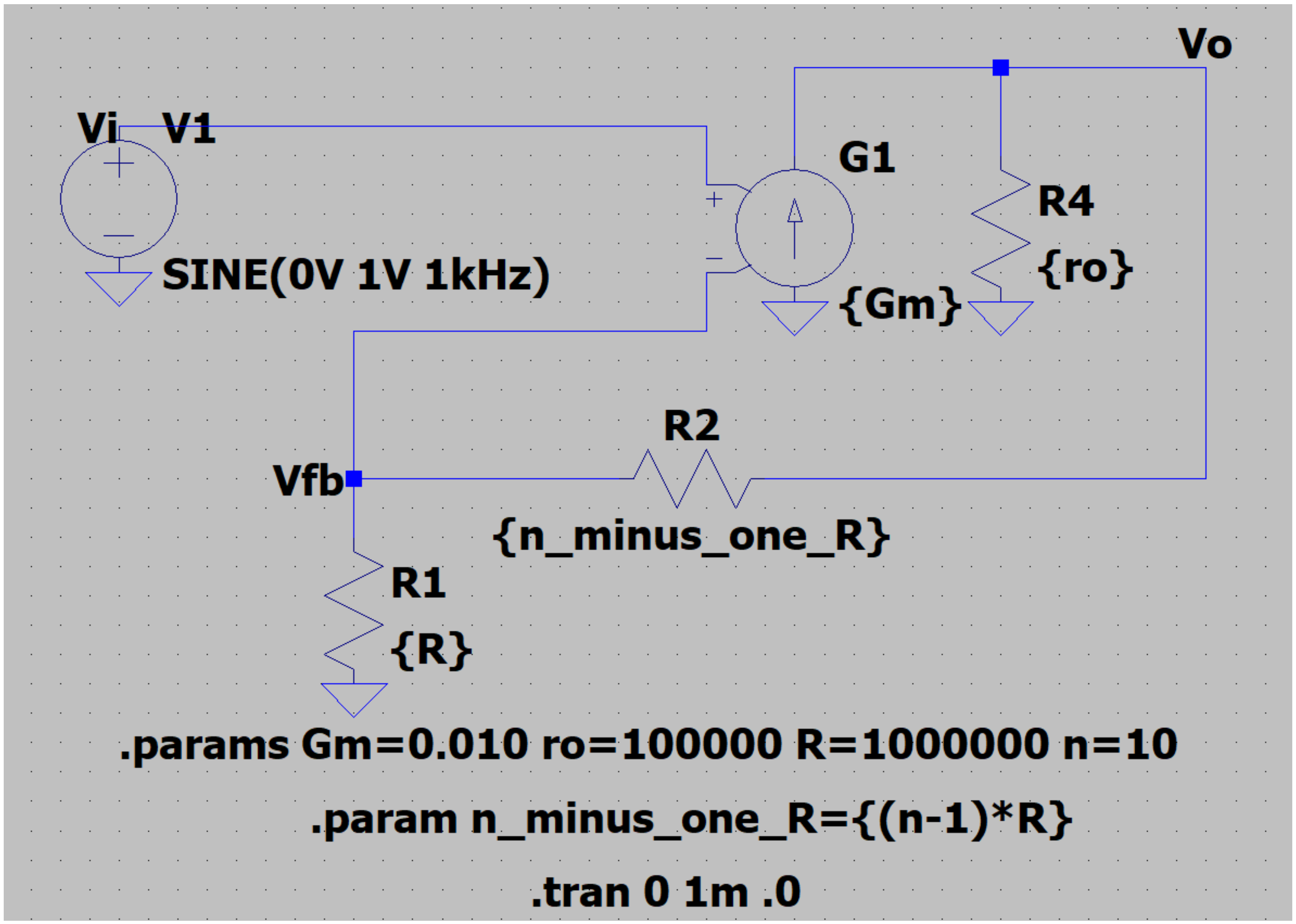
- Theoretical gain A0 = 5mS * ((2000 * 1000)/(2000 + 1000)) = 3.33 can be observed from plot



Explanation: Using LTSpice, it is verified that the gain graphically = Gm * (ro * RL)/(ro + RL)
Effect of RL = RL is a load resistance connected in parallel, so it effectively decreases the value of Output resistance ro. So, clearly, the **Gain decreases**.

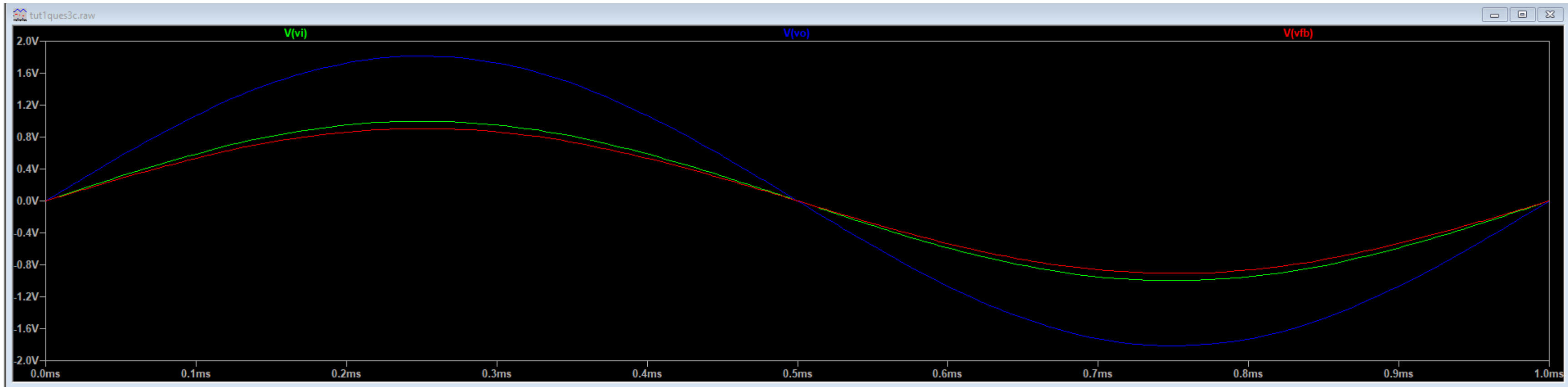
Ques 3c

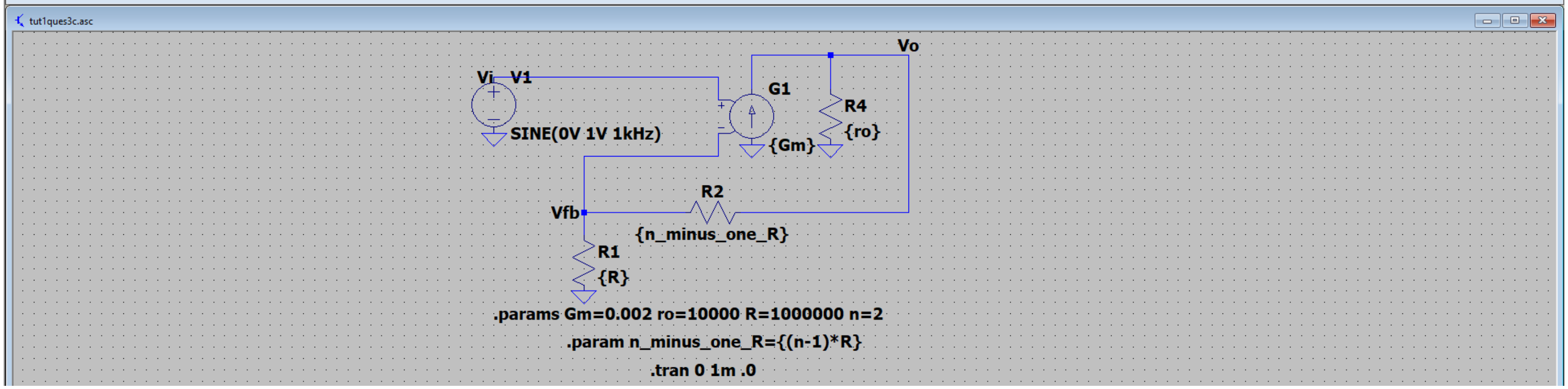
WITHOUT LOAD RESISTANCE



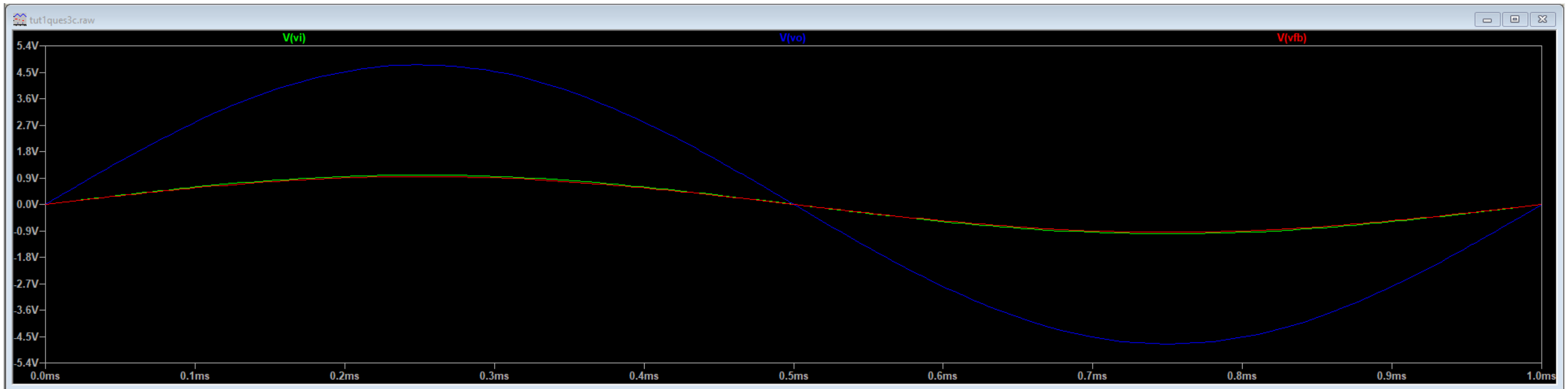
By theoretical calculation, closed loop gain of circuit would be 1/(1/n + 1/A0) = 1/(1/n + 1/(Gm * ro))

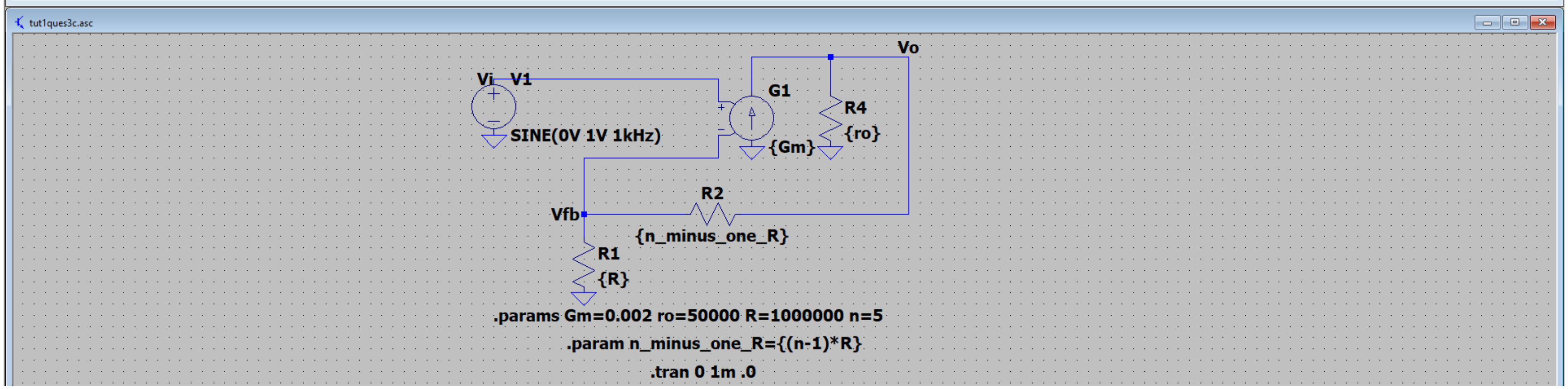
i) Closed Loop Gain = 1/(1/2 + 1/(2mS * 10k)) = 1/(½ + 1/20) = 1.81 by calculation, which can be verified graphically



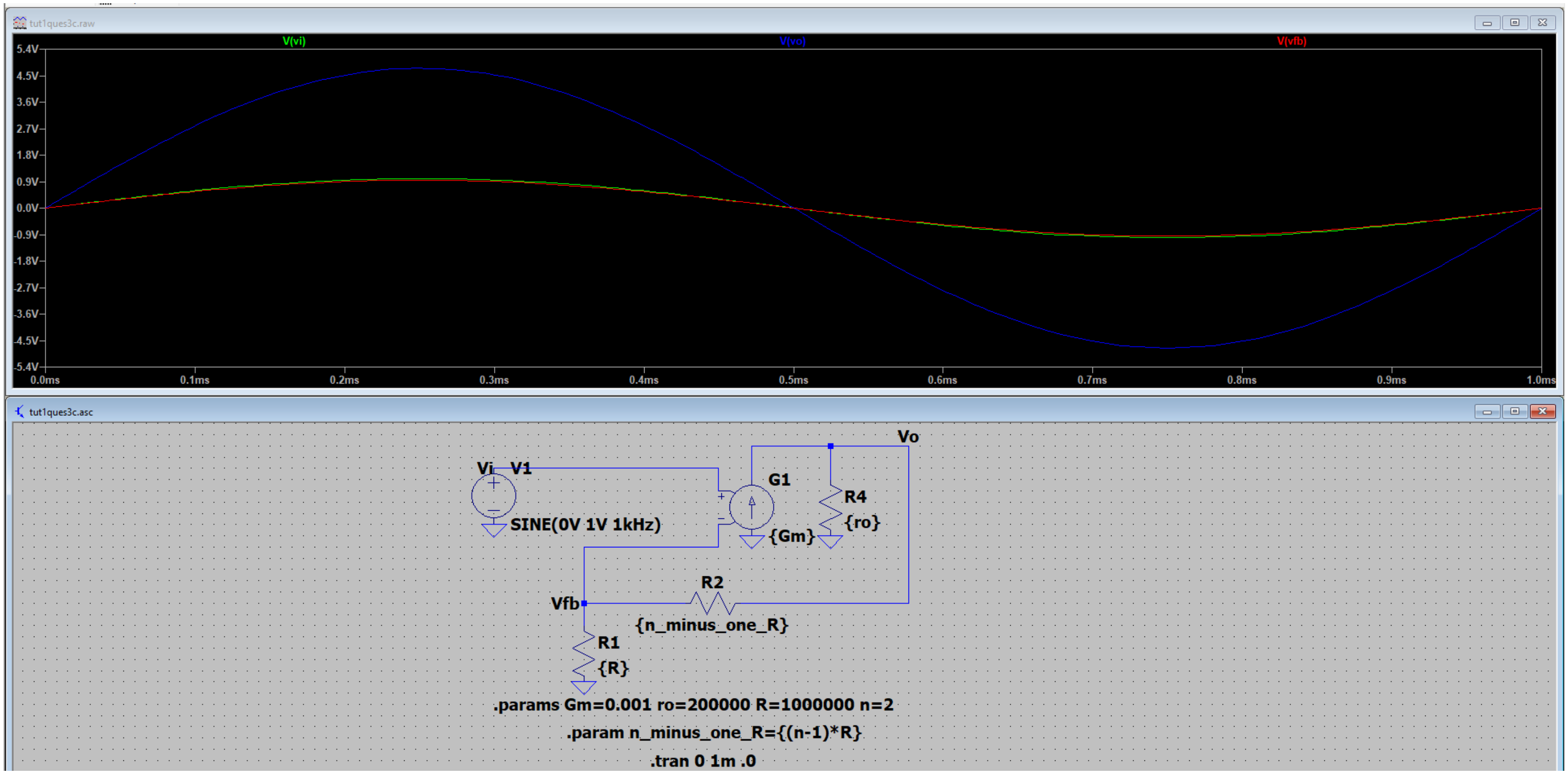


ii) Closed Loop Gain = 1/(1/5 + 1/(2mS * 50k)) = 1/(1/5 + 1/100) = 4.76 by calculation, which can be verified graphically

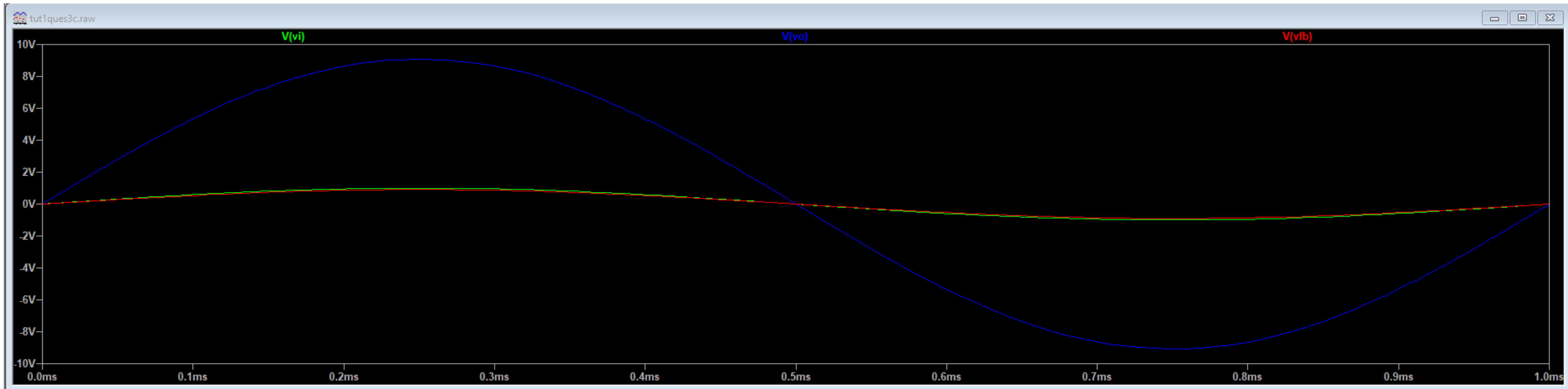


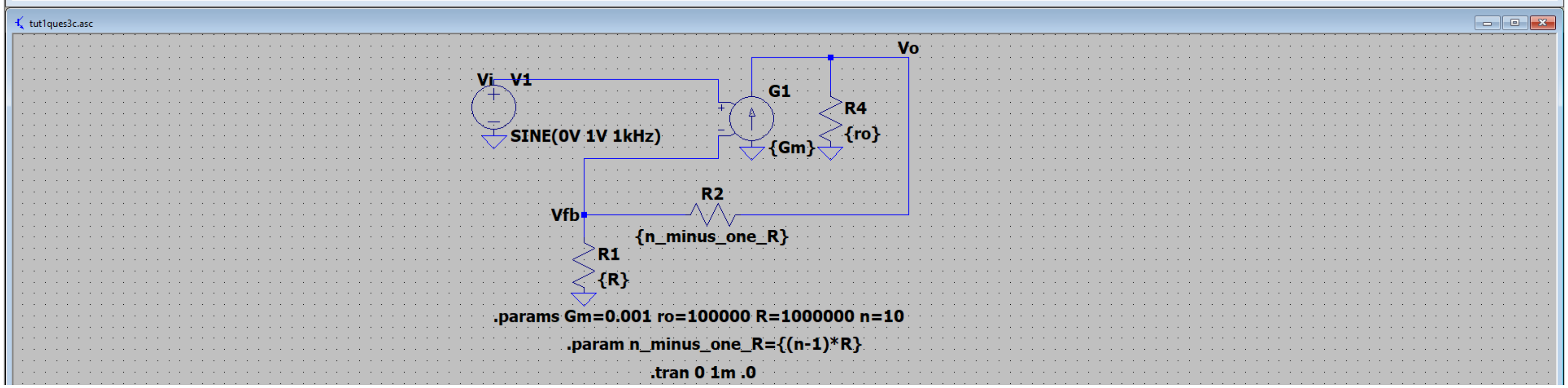


iii) Closed Loop Gain = 1/(1/2 + 1/(1mS * 200k)) = 1/(1/2 + 1/200) = 1.98 by calculation, which can be verified graphically

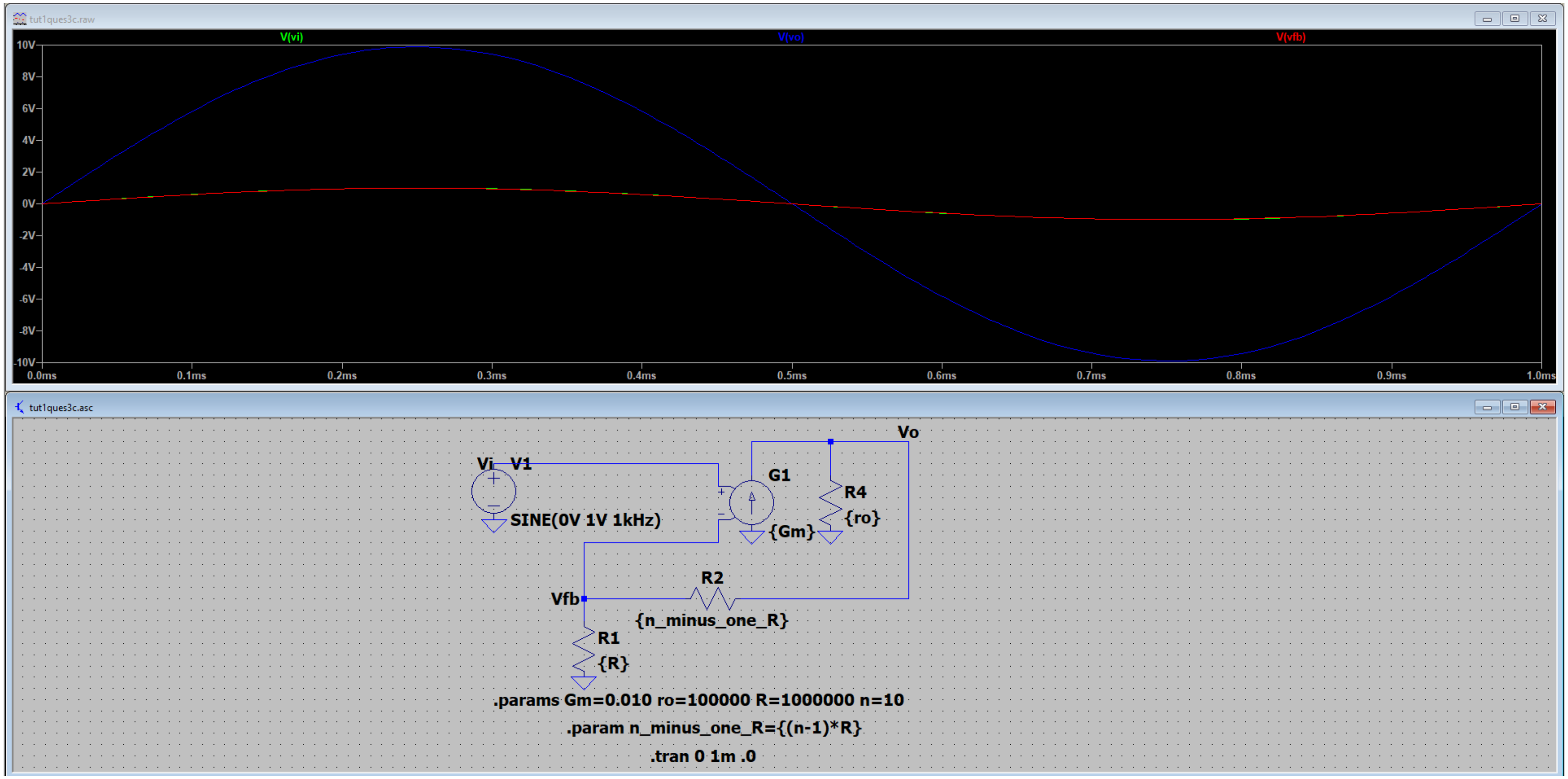


iv) Closed Loop Gain = 1/(1/10 + 1/(1mS * 100k)) = 1/(1/10 + 1/100) = 9.09 by calculation, which can be verified graphically

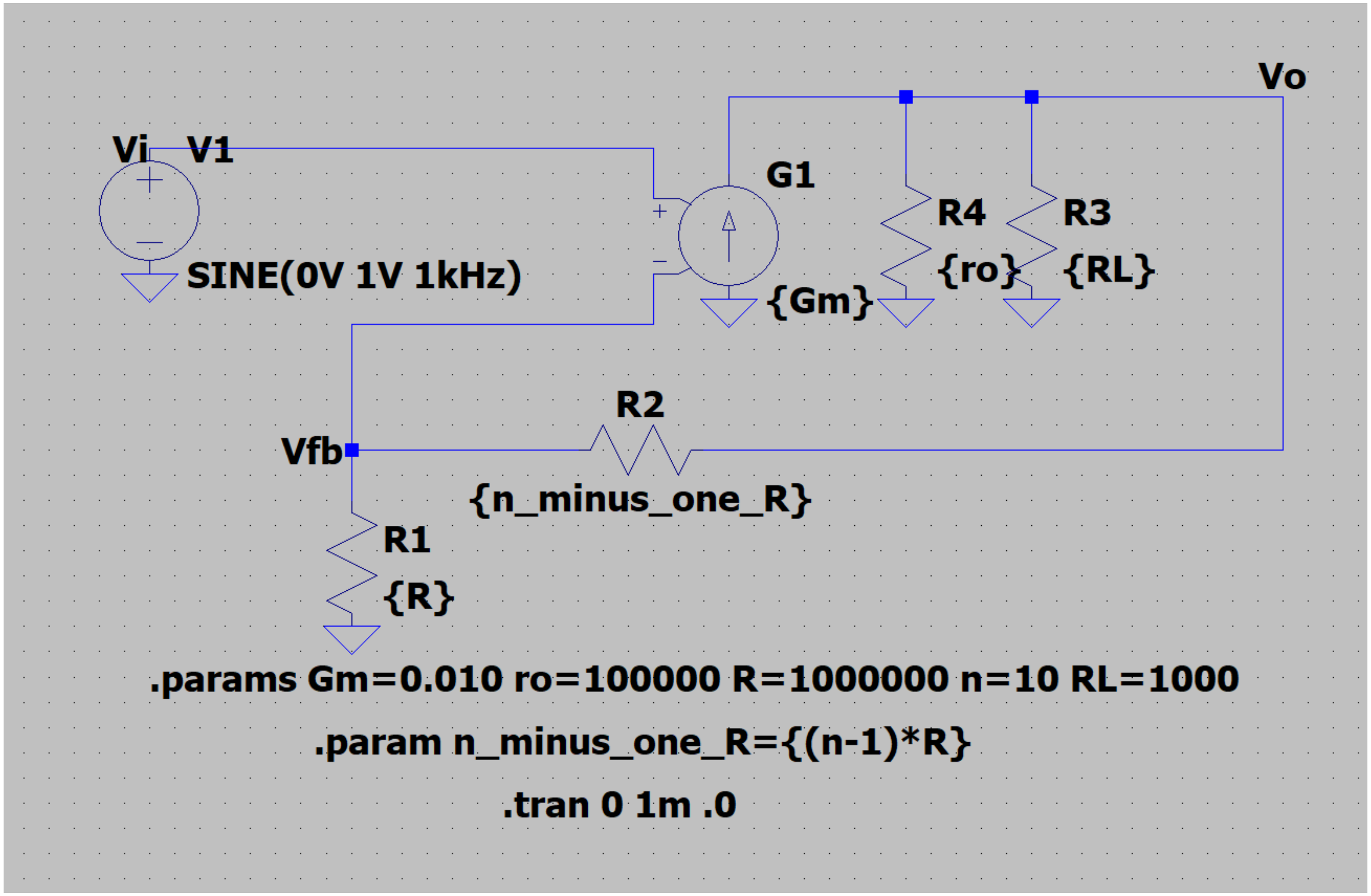




v) Closed Loop Gain = 1/(1/10 + 1/(10mS * 100k)) = 1/(1/10 + 1/100) = 9.90 by calculation, which can be verified graphically

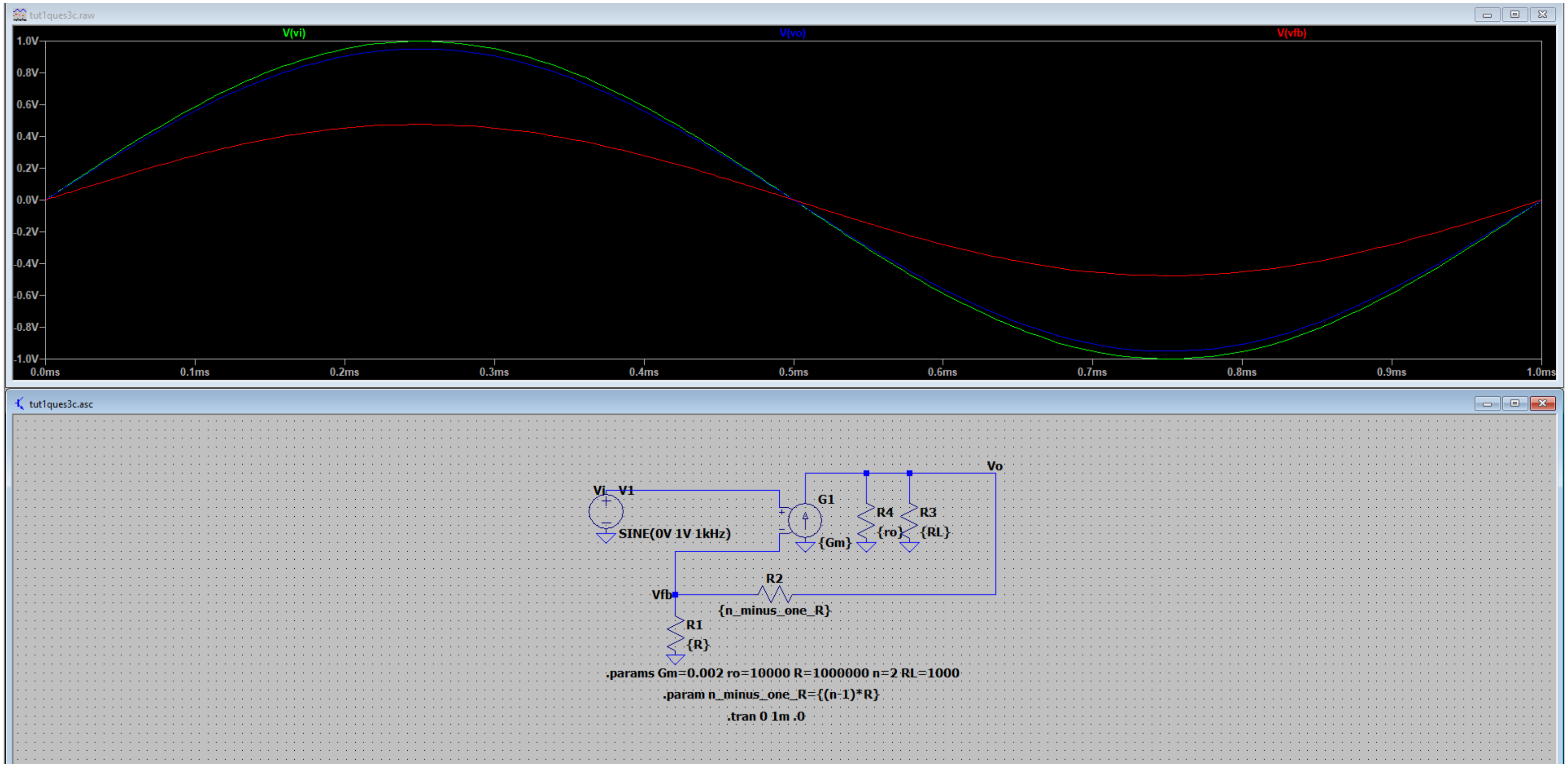


WITH LOAD RESISTANCE

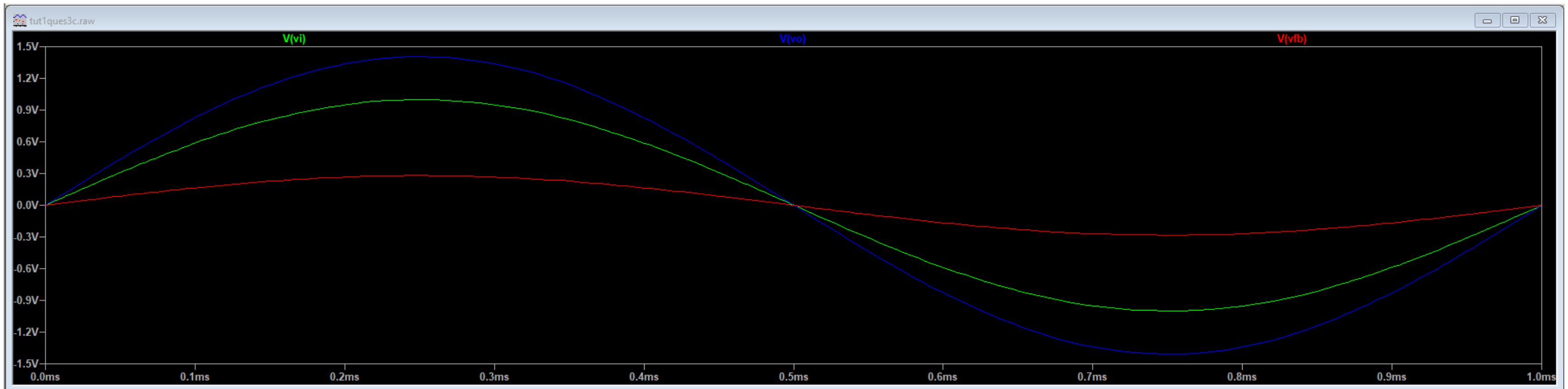


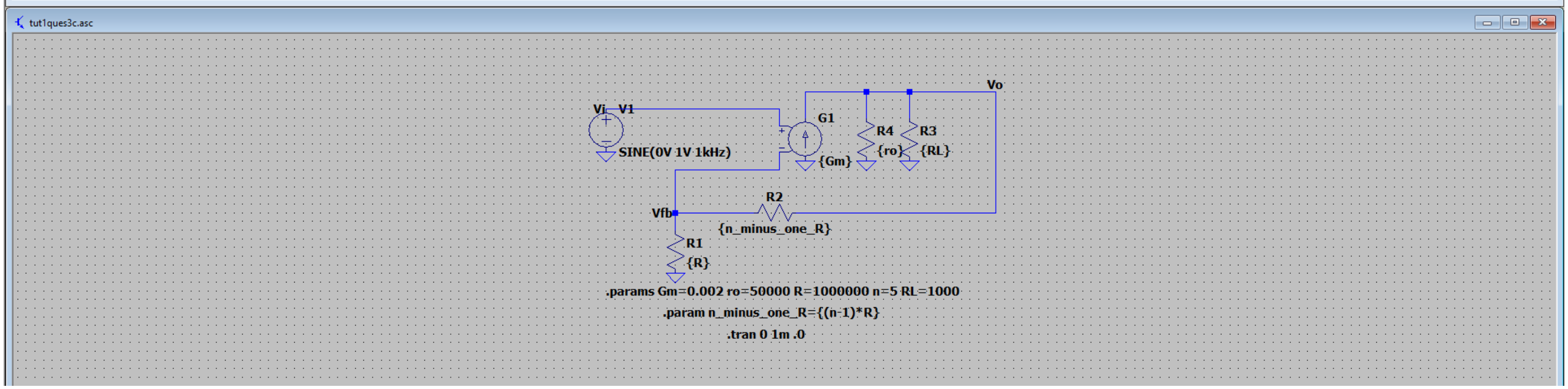
By theoretical calculation, closed loop gain of circuit would be 1/(1/n + 1/(Gm * (ro * RL/(ro + RL))))

i) Gain 0.95 by theoretical calculation, which matches with simulated values

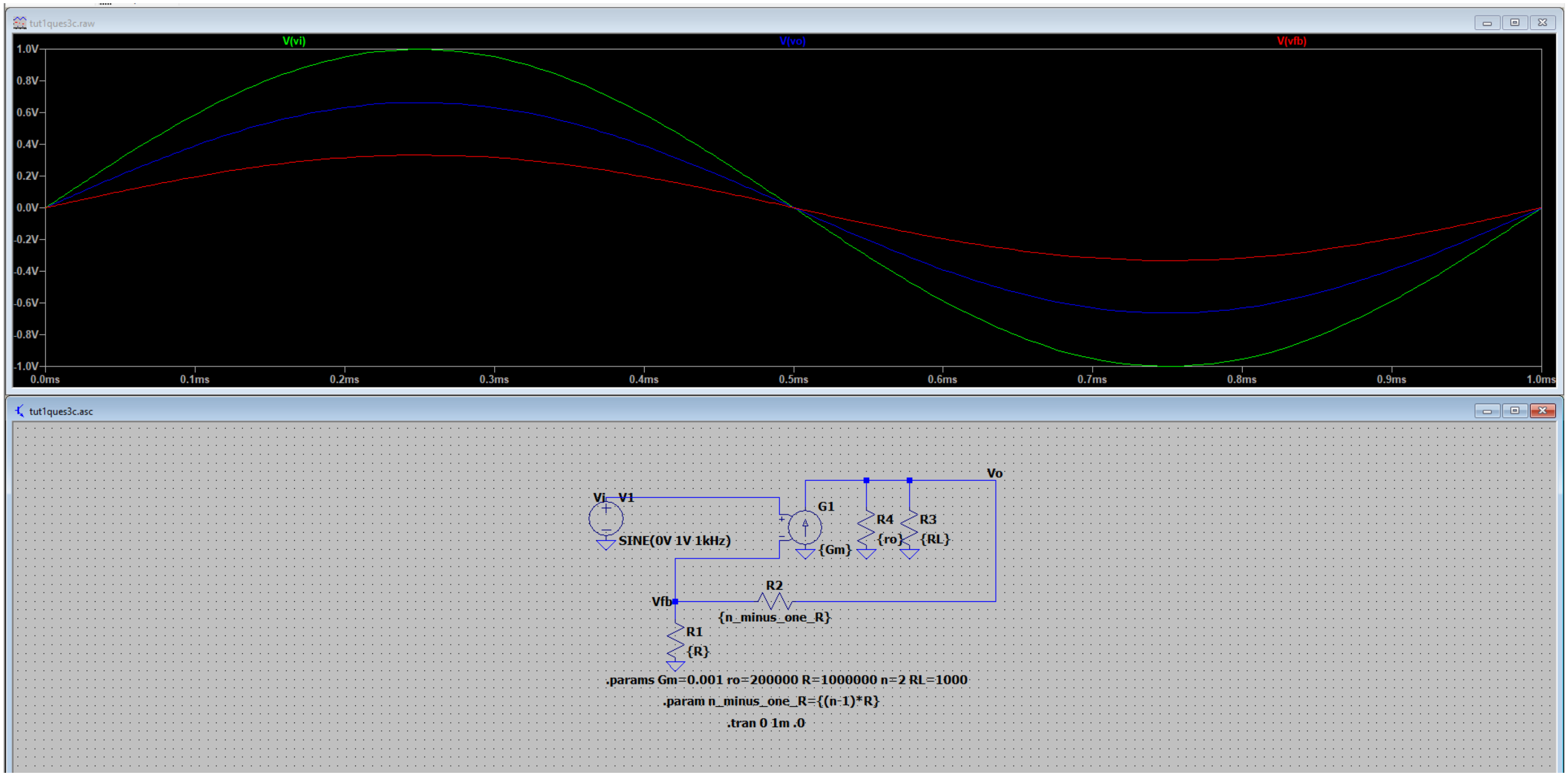


ii) Gain 1.408 by theoretical calculation, which matches with simulated values

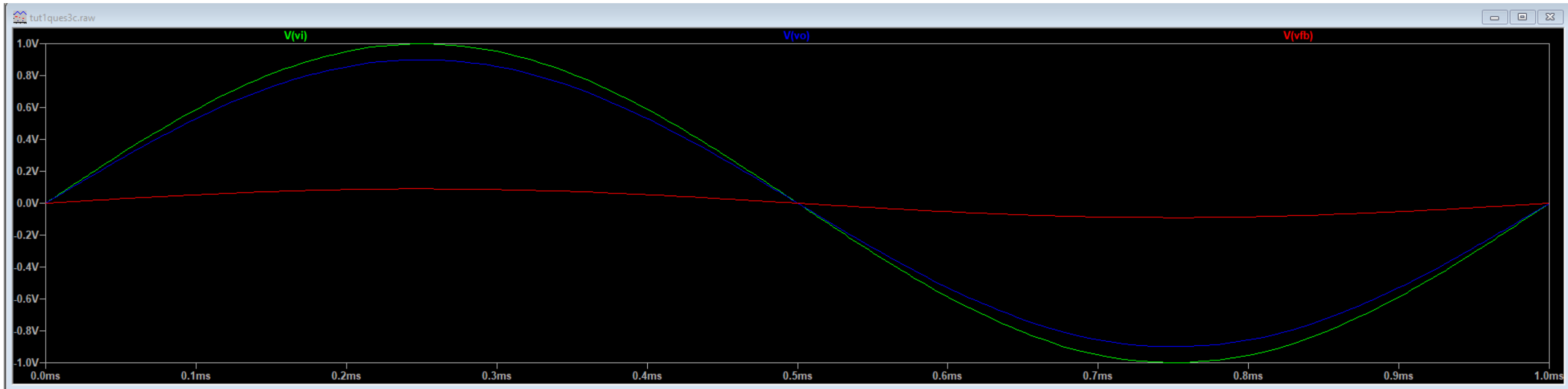


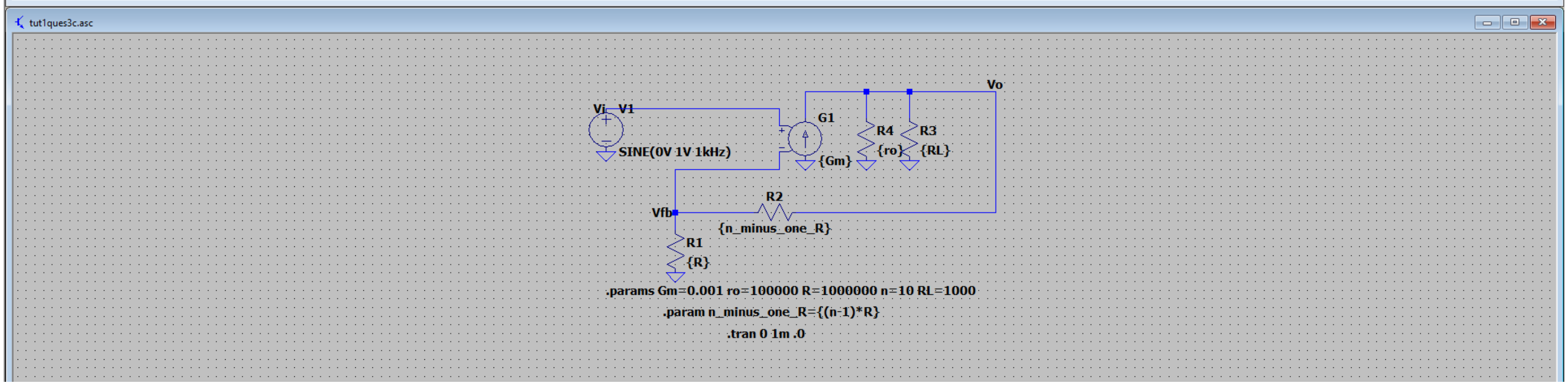


iii) Gain 0.66 by theoretical calculation, which matches with simulated values

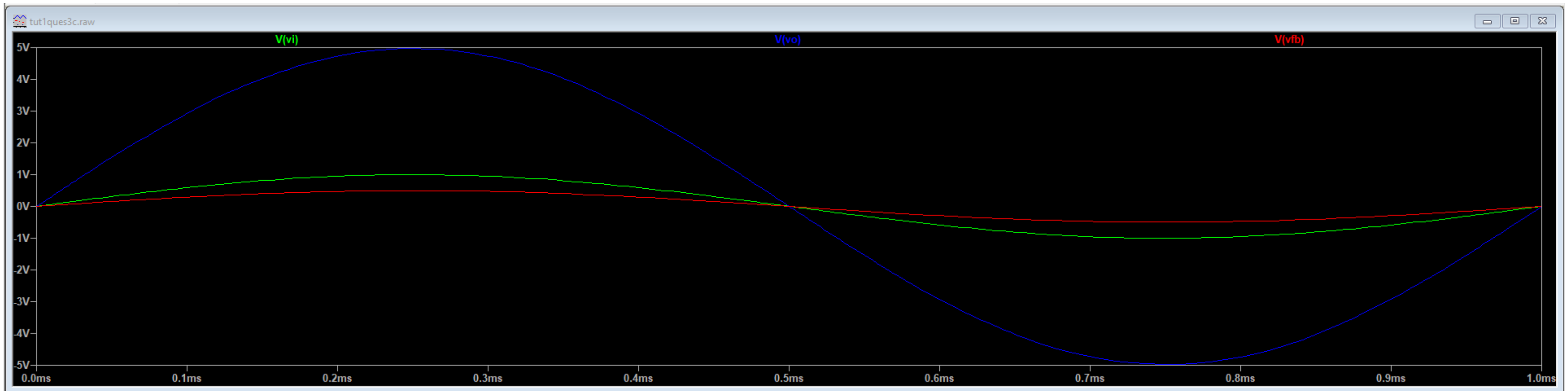


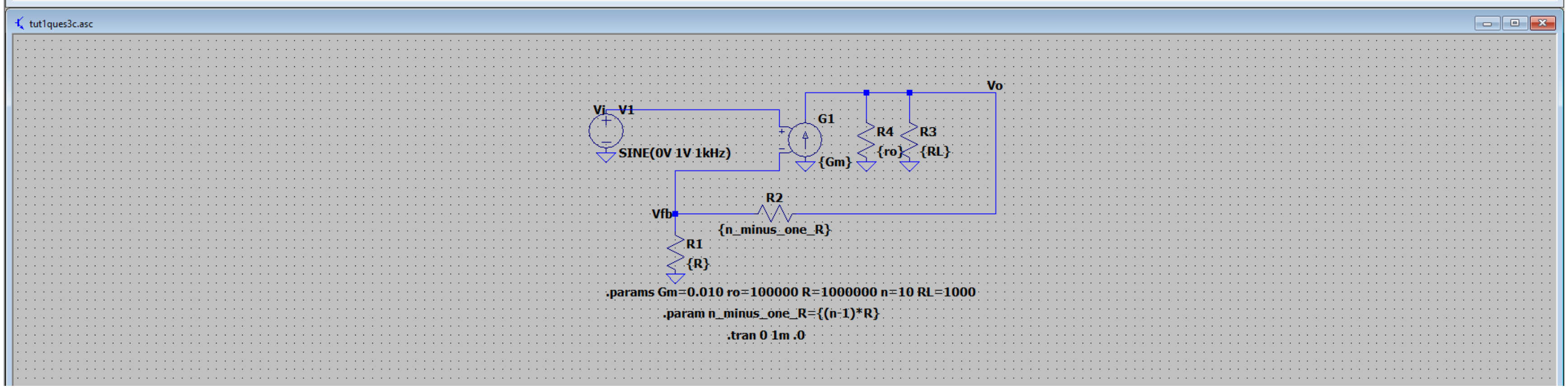
iv) Gain 0.90 by theoretical calculation, which matches with simulated values





v) Gain 4.97 by theoretical calculation, which matches with simulated values





Explanation: Compared to a and b, we see that the closed loop gain value is not the same as open loop gain, but depends on the value of n also.

It is verified using LTSpice that the gain is 1/(1/n + 1/(Gm * ro)) graphically. Now,

- As the value of VCVS gain (Gm * ro) increases, the closed loop gain tends to n. In the limit as VCVS gain becomes infinite, the closed loop gain = n
- As the value of n increases, the closed loop gain also increases