

Recursive Problem Solving

CS 624 — Analysis of Algorithms

February 27, 2024

Problem: Heap Checking and Repair

Given an array A of integers:

- ▶ Does A represent a (max-)heap?
- ▶ If not, can it be turned into a heap with one call to Heapify?
- ▶ If so, at what index?

Problem: Making Change

Task: Pay for a cup of coffee that costs 63 cents.

- ▶ You must give exact change.
- ▶ You have an unlimited number of coins of the following denominations: 1 cent, 5 cents, 10 cents, and 25 cents.



1 cent



5 cents



10 cents



25 cents



Task Generalization

General task: Given nonnegative integers S (“sum”) and D_1, \dots, D_n (“denominations”), find nonnegative integers c_1, \dots, c_n (“counts”) such that $\sum_{i=1}^n c_i D_i = S$.

Instances:

- ▶ Paying for coffee with coins.
- ▶ “Is that American football score possible?”

Problem Variations

Here are some variations on the basic task:

- ▶ Determine if any solution exists.
- ▶ Find a solution, any solution.
- ▶ Find the solution that uses the fewest coins.
- ▶ Count the number of distinct solutions.
- ▶ Count the number of distinct minimal solutions (fewest coins).

Can you solve any of these? How?

Problem: Shopping

You are shopping at an antique store. The store has a set of n unique items. Each item k has a value to you of v_k and a cost c_k . You have a total budget of B , and you want to spend as much of it as possible, maximizing the total value of the items you purchase.

You must make n decisions $d_k \in \{0, 1\}$, such that $\sum_{k=1}^n d_k c_k \leq B$.
(That is, the total cost of your purchases must fit within your budget.)

Your goal is to maximize $\sum_{k=1}^n d_k v_k$.