

Homework 06

CS 624, 2024 Spring

1. Problem 26.1-7 (p714)

Suppose that, in addition to edge capacities, a flow network has vertex capacities. That is, each vertex v has a limit $l(v)$ on how much flow can pass through v . Show how to transform a flow network $G = (V, E)$ with vertex capacities into an equivalent flow network $G' = (V', E')$ without vertex capacities, such that a maximum flow in G' has the same value as a maximum flow in G . How many vertices and edges does G' have?

2. Exercise 1.4 ([aux16] p7)

Prove that all problems that are NP-complete are polynomially equivalent, in the sense that if A and B are NP-complete, then $A \leq_P B$ and $B \leq_P A$.

3. Exercise 2.1 ([aux16] p9)

Summary: Prove that the problem of satisfiability for expressions in *disjunctive normal form* is in P (that is, it is polynomial-time).

4. Exercise 3.6 ([aux16] p16)

Prove that the following are equivalent:

- (a) V_1 is a vertex cover of G .
- (b) $V - V_1$ is an independent set in G .

and, continuing, prove that the following are equivalent:

- (a) V_2 is an independent set in G .
- (b) V_2 is a clique in G^c .