Group 61
Modelling and simulation

EES101

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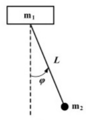
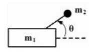
Contents

1 Introduction

In this assignment there is a helicopter which is hovering a mass as the figures below which are describing a Schematic and a top view of the system



(a) Schematic of a helicopter hovering a mass



(b) Top view of the system

Figure 1

The system will be described as two masses coupled by a rigid link. The following symbols will be used:

- p1: The position of the helicopter.
- ϕ, φ: The position of the handing mass
- u: The force acting on the helicopter.

The following matrix is the form of the generalized coordinates:

$$q = \begin{bmatrix} p1 \\ \phi \\ \varphi \end{bmatrix}$$

Where p1 will be describing the position of the helicopter

$$p1 = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

1.1 1a

p2 is a matrix which will be describing the position of the hovering mass position and to calculate the position:

$$p2 = \begin{pmatrix} x + l\ \cos(\text{theta})\ \sin(\varphi) \\ y + l\ \sin(\varphi)\ \sin(\text{theta}) \\ z - l\ \cos(\varphi) \end{pmatrix}$$

The following equation will be used to calculate the Euler-Lagrange:

$$\frac{d}{dt}\nabla_{\dot{q}}L - \nabla_q L = F \tag{1}$$

F is a vector which is acting on on the Helicopter and it will be described by the following:

$$\begin{bmatrix} u1 \\ u2 \\ u3 \\ 0 \\ 0 \end{bmatrix}$$

In order to get $\frac{d}{dt}\nabla_{\dot{q}}L$, we know that: $W(q)\dot{q} = \nabla_{\dot{q}}L$ and then we can use it as:

$$\frac{d}{dt}\nabla_{\dot{q}}L = \frac{d}{dt}(W(q)\dot{q}) = \frac{\partial}{\partial q}(W(q)\dot{q})\dot{q} + W(q)\ddot{q} \tag{2}$$

And the Euler-Lagrange equation becomes: $W(q)\ddot{q} + \frac{\partial}{\partial q}(W(q)\dot{q}) - \nabla_q L = F$

Which gives us the following:

$$W(q)\ddot{q} = -\frac{\partial}{\partial q}(W(q)\dot{q}) + \nabla_q L + F \tag{3}$$

We already know that $W(q) = M(q)$ and $\ddot{q} = \dot{v}$. The right side of equation (3) can be written as a function of q,$\dot{q}$, u
$M\dot{v} = b(q, \dot{q}, u)$
We were able calculate M with help of Matlab:

$$M = \left(\begin{array}{ccccc} m_1+m_2 & 0 & 0 & -l\,m_2\,\sin(\varphi)\,\sin \\ 0 & m_1+m_2 & 0 & l\,m_2\,\cos(\mathrm{theta}) \\ 0 & 0 & m_1+m_2 & 0 \\ -l\,m_2\,\sin(\varphi)\,\sin(\mathrm{theta}) & l\,m_2\,\cos(\mathrm{theta})\,\sin(\varphi) & 0 & m_2\,l^2\cos(\mathrm{theta})^2\sin(\varphi)^2 + m_2 \\ l\,m_2\,\cos(\varphi)\,\cos(\mathrm{theta}) & l\,m_2\,\cos(\varphi)\,\sin(\mathrm{theta}) & l\,m_2\,\sin(\varphi) & 0 \end{array}\right.$$

And, calculating M is the nothing but $b(q, \dot{q}, u)$

$$b(q, \dot{q}, u) = \left(\begin{array}{c} l\,m_2\,\cos(\mathrm{theta})\,\sin(\varphi)\,{\varphi_{\mathrm{dot}}}^2 + 2\,l\,m_2\,\cos(\varphi)\,\sin(\mathrm{theta})\,\varphi_{\mathrm{dot}}\,\theta_{\mathrm{dot}} + l\,m_2\,\cos(\mathrm{theta})\,\sin(\varphi \\ l\,m_2\,\sin(\varphi)\,\sin(\mathrm{theta})\,{\varphi_{\mathrm{dot}}}^2 - 2\,l\,m_2\,\cos(\varphi)\,\cos(\mathrm{theta})\,\varphi_{\mathrm{dot}}\,\theta_{\mathrm{dot}} + l\,m_2\,\sin(\varphi)\,\sin(\mathrm{theta} \\ -l\,m_2\,\cos(\varphi)\,{\varphi_{\mathrm{dot}}}^2 + \mathrm{uz} + g\,m_1 + g\,m_2 \\ -l^2\,m_2\,\varphi_{\mathrm{dot}}\,\theta_{\mathrm{dot}}\,\sin(2\,\varphi) \\ \frac{l\,m_2\left(l\,\sin(2\,\varphi)\,{\theta_{\mathrm{dot}}}^2 + 2\,g\,\sin(\varphi)\right)}{2} \end{array}\right.$$

(4)

The above matrix is the 5*1 and due to space constrains, it's not visible properly. please check the code in matlab file

1.2 1b

Because of using the constrained Lagrange equation, we need to write q differently:

q= $\begin{bmatrix} p1 \\ p2 \end{bmatrix} = \begin{bmatrix} x1 \\ y1 \\ z1 \\ x2 \\ y2 \\ z2 \end{bmatrix}$

Just like the past task, we calculated Euler-Lagrange after finding both the kinetic and potential energy, as well as deriving the Lagrange function. F(The external force) will be the same as the past task.

The Euler-Lagrange equation could be written as:

$$W(q)\ddot{q} = -\frac{\partial}{\partial q}(W(q)\dot{q}) + \nabla_q L + F \tag{5}$$

To calculate the constraint C , The new generalized coordinates has been used:

$$C(q) = \frac{(x_1 - x_2)^2}{2} + \frac{(y_1 - y_2)^2}{2} + \frac{(z_1 - z_2)^2}{2} - \frac{l^2}{2} \tag{6}$$

The result of using the constraint C and generalized coordinates will be as following:

$$M\dot{v} = \begin{pmatrix} \mathrm{ux} - Z\ (x_1 - x_2) \\ \mathrm{uy} - Z\ (y_1 - y_2) \\ \mathrm{uz} + g\,m_1 - Z\ (z_1 - z_2) \\ Z\ (x_1 - x_2) \\ Z\ (y_1 - y_2) \\ g\,m_2 + Z\ (z_1 - z_2) \end{pmatrix} \tag{7}$$

Comparison: In the first approach, the coordinates has some angles to define the position of the objects (hanging masses) with respect to the helicopter which made the model somehow complex but it's not as same as in the constrained model approach where it has the generalised coordinates as Cartesian coordinates. As the generalised coordinates act independently, so the model is simpler than the counterpart.

2 Explicit vs. Implicit model

2.1 (a)

From 1.b) problem model in the form mentioned below (equation-7), by considering variables W(q) and a(q), the relations which are all mentioned below which are obtained from 1.b) portion of the problem question where equation 9 represent the LHS and Equ 10 represent the RHS of the question equation.

$$\begin{bmatrix} M & a(q) \\ a(q)^T & 0 \end{bmatrix} \begin{bmatrix} \ddot{q} \\ z \end{bmatrix} = c(q, \dot{q}, u) \tag{8}$$

Here, the expression $c(q, \dot{q}, u)$ will be obtained by solving for those elements in the below expression which is a matrix. Its first (Input) element Q is obtained by estimating the equation (4) by solving for $W(q)\ddot{q}$ in the extended euler lagrange equation.

$$c(q, \dot{q}, u) = \begin{bmatrix} Q - \frac{\partial}{\partial q}(W(q)\dot{q})\dot{q} + \nabla qT - \nabla qV - \nabla qc.Z \\ -\frac{\partial}{\partial q}(\frac{\partial c}{\partial q}\dot{q})\dot{q} \end{bmatrix} \tag{9}$$

$$c(q,\dot{q},u) = \begin{pmatrix} \mathrm{ux} \\ \mathrm{uy} \\ \mathrm{uz} + g\,m_1 \\ 0 \\ 0 \\ g\,m_2 \\ \mathrm{dx}_2\,(\mathrm{dx}_1 - \mathrm{dx}_2) - \mathrm{dx}_1\,(\mathrm{dx}_1 - \mathrm{dx}_2) - \mathrm{dy}_1\,(\mathrm{dy}_1 - \mathrm{dy}_2) + \mathrm{dy}_2\,(\mathrm{dy}_1 - \mathrm{dy}_2) - \mathrm{dz}_1\,(\mathrm{dz}_1 - \mathrm{dz}_2 \end{pmatrix} \tag{10}$$

the above matrix expression is 6*1 matrix but due to the space constrain, the last row of the matix of not visible but it's clearly defined in the matlab code The left hand side of the equation(7) matrix can be as

$$\begin{pmatrix} m_1 & 0 & 0 & 0 & 0 & 0 & x_1 - x_2 \\ 0 & m_1 & 0 & 0 & 0 & 0 & y_1 - y_2 \\ 0 & 0 & m_1 & 0 & 0 & 0 & z_1 - z_2 \\ 0 & 0 & 0 & m_2 & 0 & 0 & x_2 - x_1 \\ 0 & 0 & 0 & 0 & m_2 & 0 & y_2 - y_1 \\ 0 & 0 & 0 & 0 & 0 & m_2 & z_2 - z_1 \\ x_1 - x_2 & y_1 - y_2 & z_1 - z_2 & x_2 - x_1 & y_2 - y_1 & z_2 - z_1 & 0 \end{pmatrix} \begin{bmatrix} \ddot{q} \\ z \end{bmatrix} \tag{11}$$

Further, the function of a(q) is nothing but the jacobian/differentiation of C with respect to q. It can expressed with matlab equations

$$a(q) = \begin{pmatrix} x_1 - x_2 \\ y_1 - y_2 \\ z_1 - z_2 \\ x_2 - x_1 \\ y_2 - y_1 \\ z_2 - z_1 \end{pmatrix} \tag{12}$$

2.2 (b)

In the following section, we going to calculate the explicit part of the expression that is stated in the question.

$$\begin{bmatrix} \ddot{q} \\ z \end{bmatrix} = \begin{bmatrix} M & a(q) \\ a(q)^T & 0 \end{bmatrix}^{-1} c(q,\dot{q},u) \tag{13}$$

the LHS of the above equation is the combination of qdotdot and the constrain Z, so its 7*1 matrix.

$$\begin{bmatrix} \ddot{q} \\ Z \end{bmatrix} = \begin{bmatrix} \ddot{x1} \\ \ddot{y1} \\ \ddot{z1} \\ \ddot{x2} \\ \ddot{y2} \\ \ddot{z2} \\ Z \end{bmatrix} \tag{14}$$

Likewise, the RHS side of the above equation can be calculated by multiplying the inverse of M and C from equation(12)

$$\begin{bmatrix} M & a(q) \\ a(q)^T & 0 \end{bmatrix}^{-1} c(q, \dot{q}, u) = \left(\begin{array}{c} \frac{\mathrm{ux}\left(m_1\,{x_1}^2+m_1\,{x_2}^2+m_1\,{y_1}^2+m_1\,{y_2}^2+m_2\,{y_1}^2+m_2\,{y_2}^2+m_1\,{z_1}^2+\cdots\right.}{{m_1}^2\,{x_1}^2-2\,{m_1}^2\,x_1\,x_2+{m_1}^2\,{x_2}^2+{m_1}^2\,{y_1}^2-2\,{m_1}^2\,y_1\,y_2+{m_1}^2\,{y_2}^2+{m_1}^2\,{z_1}^2-2\,{m_1}^2\,z_1\,z_2+{m_1}^2\,{z_2}^2+\cdots} \\ \frac{\mathrm{uy}\left(m_1\,{x_1}^2+m_1\,{x_2}^2+m_2\,{x_1}^2+m_2\,{x_2}^2+m_1\,{y_1}^2+m_1\,{y_2}^2+m_1\,{z_1}^2+\cdots\right.}{{m_1}^2\,{x_1}^2-2\,{m_1}^2\,x_1\,x_2+{m_1}^2\,{x_2}^2+{m_1}^2\,{y_1}^2-2\,{m_1}^2\,y_1\,y_2+{m_1}^2\,{y_2}^2+{m_1}^2\,{z_1}^2-2\,{m_1}^2\,z_1\,z_2+{m_1}^2\,{z_2}^2+\cdots} \\ \frac{(\mathrm{uz}+g\,m_1)\left(m_1\,{x_1}^2+m_1\,{x_2}^2+m_2\,{x_1}^2+m_2\,{x_2}^2+m_1\,{y_1}^2+m_1\,{y_2}^2+m_2\,y_1\cdots\right.}{{m_1}^2\,{x_1}^2-2\,{m_1}^2\,x_1\,x_2+{m_1}^2\,{x_2}^2+{m_1}^2\,{y_1}^2-2\,{m_1}^2\,y_1\,y_2+{m_1}^2\,{y_2}^2+{m_1}^2\,{z_1}^2-2\,{m_1}^2\,z_1\,z_2+{m_1}^2\,{z_2}^2+\cdots} \\ \frac{\cdots}{m_1\,{x_1}^2+m_1\,{x_2}^2+m_2\,{x_1}^2+m_2\,{x_2}^2+m_1\,{y_1}^2+m_1\,{y_2}^2+m_2\,y\cdots} \\ \frac{\cdots}{m_1\,{x_1}^2+m_1\,{x_2}^2+m_2\,{x_1}^2+m_2\,{x_2}^2+m_1\,{y_1}^2+m_1\,{y_2}^2+m_2\,y\cdots} \\ \frac{\cdots}{m_1\,{x_1}^2+m_1\,{x_2}^2+m_2\,{x_1}^2+m_2\,{x_2}^2+m_1\,{y_1}^2+m_1\,{y_2}^2+m_2\,y\cdots} \\ \frac{m\cdots}{m_1\,{x_1}^2+m_1\,{x_2}^2+m_2\,{x_1}^2+m_2\,{x_2}^2+\cdots} \end{array} \right. \tag{15}$$

Due to the space constrain, the above matrix expression(equ 14) is not visible. But it's well defined in the MATLAB code

COMPARISON: From the above equations(14), We can observe that the explicit model of the problem became too much complicated when comparing with the Implicit model of the equation. So, it's advisable to carry our modeling in implicit model rather than solving in explicit model